

Endogenizing Firm Scope: Trade Liberalization and the Size Distribution of Multiproduct Firms*

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Abstract

We develop a theory of multiproduct firms and endogenous firm scope that can explain a well-known empirical puzzle: larger firms appear to be less efficient in that they have lower values of Tobin's Q . We extend this theory to study the effects of trade liberalization and market integration on the size distribution of firms. We show that a symmetric bilateral trade liberalization leads to a less skewed size distribution. The opposite result obtains in the case of a unilateral trade liberalization in the liberalizing country. In our model, trade liberalization not only affects the distribution of observed productivities but also productivity at the firm level. In the empirical section, we show that the key predictions are consistent with the data.

1 Introduction

Standard economic models of firm heterogeneity where firms differ in their (constant) marginal costs predict that more efficient firms are larger and exhibit a higher value of Tobin's Q , the ratio between a firm's market value and its book value. Hence, there should be a positive relationship between firm size and Tobin's Q in the data. Yet there is strong empirical evidence showing that the opposite is true (Lang and Stulz, 1994; Eeckhout and Jovanovic, 2002). In Figure 1 we plot the logarithm of Tobin's Q on the logarithm of firm sales, using Compustat data for the year 2004. The figure shows a clear negative relationship between firm size and Tobin's Q . This "size discount" is robust to controlling for industry fixed effects; see the Appendix for details.

The relationship between intrinsic firm efficiency, observed productivity, and firm size is fundamental in understanding the productivity effects of economic policies such as trade liberalization and market integration. There is a large and growing literature that is concerned

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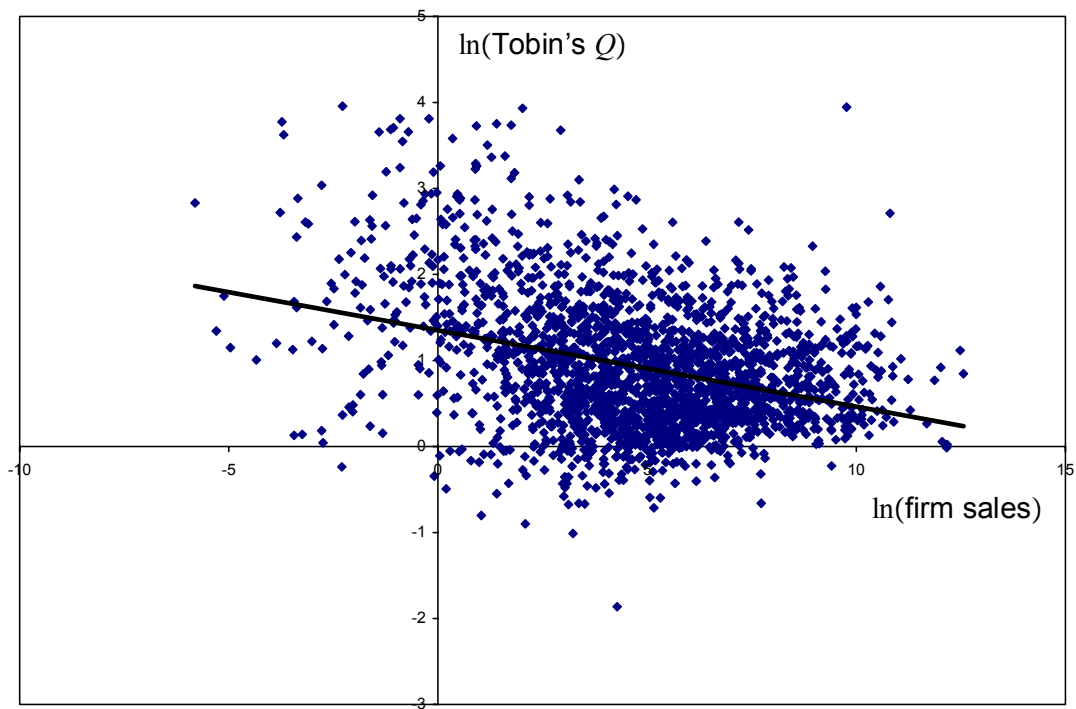


Figure 1: The relationship between the logarithm of Tobin's Q and the logarithm of firm sales.

with the productivity implications of international trade (e.g., Melitz, 2003; Melitz and Ottaviano, 2005). But this literature predicts (as do other standard models of firm heterogeneity) a positive relationship between firm size and Tobin's Q . Moreover, because a firm's productivity is assumed to be exogenous, the only productivity effects arising in these models are due to selection effects resulting from firm entry and exit. This literature is thus unable to explain the finding that trade liberalization and market integration have productivity effects at the plant level (e.g., Pavcnik, 2002; Treffer, 2004).

In this paper, we develop a model that allows us to explain the size-discount puzzle and analyze the productivity effects of trade liberalization and market integration. There are three key ingredients. First, each firm chooses how many product lines to manage. Second, there are decreasing returns to the span of control at the firm level: the more product lines a firm chooses to manage, the less good it is at managing each one of it, and so the higher are its marginal costs. This ingredient is consistent with the finding by Schoar (2002) that the total factor productivity of a firm's existing product lines decreases when new product lines are added. Third, firms differ in their organizational capabilities: the greater is a firm's organizational capability, (i) the lower are its marginal costs, holding fixed the number of product lines, and (ii) the less responsive are marginal costs to increases in the number of product lines.

In equilibrium, each firm chooses the number of product lines so that the profit of the marginal product line is equal to the negative externality that the marginal product line exerts on the sum of the profits of the inframarginal product lines. Suppose firm 1 chooses the number of product lines optimally. Suppose firm 2 has better organizational capability than firm 1 and chooses the number of product lines in such a way that its marginal costs are the same as those of firm 1. In this case, the profit of the marginal product line is the same for both firms but – since firm 2 has greater organizational capability – the marginal product line of firm 2 imposes a smaller negative externality on the sum of the profits of its inframarginal product lines. This implies that firm 2 should optimally add product lines so that its marginal costs are higher than those of firm 1. Hence, firms with greater organizational capability have higher marginal costs – and thus lower values of Tobin's Q – than firms with inferior organizational capability. This solves the size-discount puzzle.

We embed our theory of multiproduct firms in a two-country model of international trade in order to analyze the effects of trade liberalization and market integration. We show that a symmetric trade liberalization leads to a less skewed size distribution: large firms downsize by selling product lines while small firms expand the number of product lines. Our model thus generates a surge of (partial) firm acquisitions and divestitures following a trade liberalization, which is consistent with the data (e.g., Breinlich, 2005). A trade liberalization affects productivity both at the level of the firm and the industry. Average industry productivity can be shown to increase as high-cost firms contract while low-cost firms expand. Following a unilateral trade liberalization, the opposite result obtains in the liberalizing country: the size distribution becomes more skewed following a trade liberalization.

In the empirical part of the paper, we use Compustat data on publicly traded U.S. manufacturing companies. Our empirical results confirm the predictions of our model: a unilateral decrease in U.S. tariffs is associated with greater skewness in the size distribution of U.S. companies, while a multilateral reduction in shipping costs is associated with less skewness.

Plan of the Paper. In the next section, we present our theory of multiproduct firms in a simple environment where each firm is a monopolist for each of its products. We show that firms with greater organizational capability choose to have higher marginal costs and thus a lower value of Tobin's Q. In section 3, we extend the model by allowing firms to export their products to a foreign market. We show that a reduction in trade costs leads to a merger wave and a decrease in the skewness of the firm size distribution. In section 4, we introduce monopolistic competition (and free entry) into the two-country version of our model. We show that a symmetric trade liberalization leads to a less skewed distribution, while the opposite result obtains in the liberalization country following a unilateral trade liberalization. In section 5, we test and confirm the predictions of our model on the size distribution. We conclude in section 6.

2 A Theory of Endogenous Firm Scope

This section is organized as follows. We first introduce our theory of multiproduct firms that differ in their organizational capabilities and that choose how many product lines to manage. We then analyze how firms with different organizational capabilities solve the fundamental trade off between firm scope and productivity.

2.1 The Model

There is a mass M of firms that differ in their organizational capabilities. A firm's organizational capability is denoted by $\theta \in [\underline{\theta}, \bar{\theta}]$, where $\underline{\theta} > 0$, and the distribution of organizational capabilities in the population of firms is given by the distribution function G . Each firm can manage any number $n \geq 1$ of product lines. (For simplicity, we will treat n as a continuous variable.) We assume that firms have constant marginal costs at the product level but decreasing returns to the span of control at the firm level: the more products a firm manages, the higher are its marginal costs for each product line.

The firm faces two types of costs. First, there is a fixed cost r per product line. This can be thought of as either a cost of inventing a product or as a cost of purchasing an existing product line. Second, there is a constant marginal cost $c(n; \theta)$ associated with the production of each unit of output. This marginal cost function has the following properties. First, an increase in the number of product lines increases a firm's marginal cost, $\partial c(n; \theta) / \partial n > 0$. This property is suggested by Schoar's (2002) empirical finding that adding new product lines decreases the total factor productivity of all inframarginal product lines. Second, we want to abstract from exogenous cost differences amongst single-product firms and focus instead on the idea that organizational capability is about how good firms are at coordinating the production of multiple products. We thus assume that $c(1; \theta)$ is independent of θ and that $\partial^2 c(n; \theta) / \partial n \partial \theta < 0$. This implies that, holding fixed the number $n > 1$ of product lines, firms with greater organizational capability have lower marginal costs: $\partial c(n; \theta) / \partial \theta < 0$ for $n > 1$. To capture these properties and for simplicity, we assume that organizational capability θ is the inverse of the elasticity of marginal cost with respect to the number of product lines:

$$c(n; \theta) = c_0 n^{1/\theta}. \tag{1}$$

On the demand side, product lines are symmetric, and there are no demand linkages. For each product line, a firm faces inverse market demand $P(q)$, where q is the quantity sold of that product. We assume that demand is downward-sloping, $P'(q) < 0$ for all q such that $P(q) > 0$. Further, we impose a mild regularity condition on the inverse demand function which is familiar from Cournot oligopoly and requires that demand is not too convex:

$$P'(q) + qP''(q) < 0 \text{ for all } q > 0 \text{ such that } P(q) > 0. \quad (2)$$

Each firm's optimization problem consists in choosing the number of product lines, n , and the quantity for each product line k , q_k , so as to maximize its profit. (Since each firm is a monopolist for each of its product lines, it could equivalently choose price p_k rather than quantity.)

2.2 The Optimal Choice of Firm Scope

Consider a firm with organizational capability θ . We first analyze the firm's quantity-setting problem, holding fixed the number n of product lines. Since the firm has the same (constant) marginal cost for each product line and the demand function is the same for each product line, the firm will optimally sell the same quantity of each product line. Let $q(c(n; \theta))$ denote the profit-maximizing quantity per product line of a firm with organizational capability θ that manages n product lines. Since there are no demand linkages between product lines, the firm's quantity-setting problem can be analyzed separately for each product line. Hence,

$$q(c(n; \theta)) \equiv \arg \max_q [P(q) - c(n; \theta)]q.$$

The first-order condition is given by

$$P(q(c(n; \theta))) - c(n; \theta) + q(c(n; \theta))P'(q(c(n; \theta))) = 0. \quad (3)$$

We consider now the firm's optimal choice of the number of product lines. Given the optimal output policy, the profit of a firm with organizational capability θ that manages n product lines is given by

$$n [\pi(c(n; \theta)) - r],$$

where

$$\pi(c(n; \theta)) \equiv [P(q(c(n; \theta))) - c(n; \theta)] q(c(n; \theta)) \quad (4)$$

is the firm's gross profit per product line. From the envelope theorem, $\pi'(c(n(\theta); \theta)) = -q(c(n(\theta); \theta))$, and so the first-order condition for the optimal choice of the number of product lines, $n(\theta)$, can be written as

$$[\pi(c(n(\theta); \theta)) - r] - n(\theta)q(c(n(\theta); \theta)) \frac{\partial c(n(\theta); \theta)}{\partial n} = 0. \quad (5)$$

The impact of an additional product line on the firm's profit can be decomposed into two effects. The first term on the l.h.s. of equation (5) is the net profit of the marginal product line. The second term summarizes the negative externality that the marginal product line imposes on

the $n(\theta)$ inframarginal product lines: the production cost of each product line increases by $q(c(n(\theta); \theta))\partial c(n(\theta); \theta)/\partial n$ since the firm is now less good at managing each one of them.

From the cost function (1), $n(\theta)\partial c(n(\theta); \theta)/\partial n = (1/\theta)c(n(\theta); \theta)$. Hence, the optimal choice of the number of product lines, $n(\theta)$, enters the first-order condition (5) only through the induced marginal cost $c(n(\theta); \theta)$. This means that the firm's problem can equivalently be viewed as one of choosing c rather than n . Indeed, using the gross profit function (4), the first-order condition can be rewritten as

$$\Psi(c(\theta); \theta) \equiv [P(q(c(\theta))) - c(\theta)] q(c(\theta)) - r - \frac{c(\theta)}{\theta} q(c(\theta)) = 0, \quad (6)$$

where $c(\theta) \equiv c(n(\theta); \theta)$.

Henceforth, we will assume that the fixed cost r is not too large so that the firm can make a strictly positive profit by managing a single product line, i.e.,

$$\pi(c_0) = [P(q(c_0)) - c_0] q(c_0) > r.$$

We are now in the position to state our central result on the relationship between a firm's organizational capability and its observed productivity.

Proposition 1 *The optimal choice of product lines is such that the induced marginal cost $c(\theta)$ is weakly increasing in the firm's organizational capability θ . Specifically, there exists a unique cutoff $\tilde{\theta}$ given by*

$$\tilde{\theta} \equiv \frac{c_0 q(c_0)}{[P(q(c_0)) - c_0] q(c_0) - r}$$

such that $c(\theta) = c_0$ for all $\theta \leq \tilde{\theta}$, and $c(\theta)$ is strictly increasing in θ for all $\theta \geq \tilde{\theta}$.

Proof. See appendix. ■

For a given number n of product lines, the negative externality that the marginal product line exerts on the inframarginal product lines is the smaller, the greater is the firm's organizational capability. Not surprisingly then, firms with greater organizational capability will optimally choose a weakly larger number of product lines than firms with inferior organizational capability: $n(\theta) = 1$ for $\theta \leq \tilde{\theta}$, and $n(\theta)$ is strictly increasing in θ for $\theta \geq \tilde{\theta}$. Perhaps paradoxically, however, for $\theta \geq \tilde{\theta}$, $n(\theta)$ is increasing so fast with θ that firms with greater organizational capability will, in fact, exhibit higher unit costs. To see this, consider two firms, firm 1 and firm 2, with organizational capability $\theta_1 \geq \tilde{\theta}$ and $\theta_2 > \theta_1$, respectively. From the first-order condition (6), firm 1 will optimally choose $n(\theta_1)$ such that its marginal cost $c(\theta_1)$ satisfies $\Psi(c(\theta_1); \theta_1) = 0$. Suppose now firm 2 were to choose the number of product lines such that its induced marginal cost is also equal to $c(\theta_1)$. If so, the two firms would sell the same quantity $q(c(\theta_1))$ per product line, and thus fetch the same price $P(q(c(\theta_1)))$. Hence, the net profit of the marginal product line, $[P(q(c(\theta))) - c(\theta)] q(c(\theta)) - r$, would be the same for the two firms. However, as can be seen from equation (6), the absolute value of the negative externality that the marginal product line imposes on the inframarginal product lines, $\chi(c(\theta); \theta) \equiv (1/\theta)c(\theta)q(c(\theta))$, is smaller for the firm with the greater organizational capability,

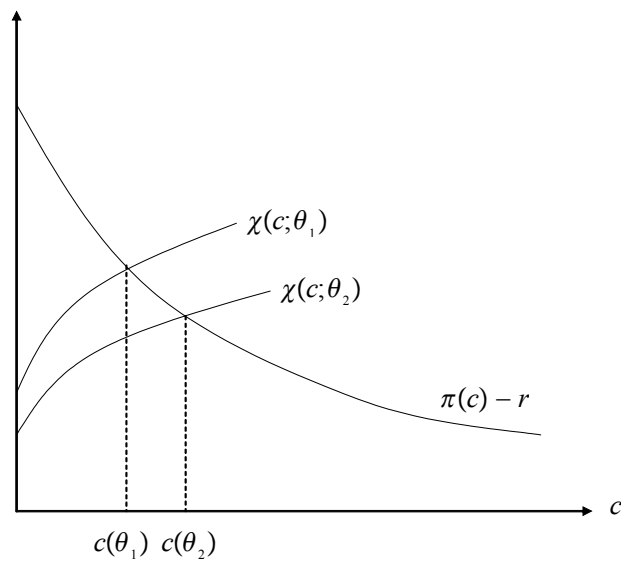


Figure 2: The induced choice of marginal cost balances the net profit per product line, $\pi(c) - r$, and the negative externality on production costs, $\chi(c; \theta)$. A firm with greater organizational capability, $\theta_2 > \theta_1$, chooses to have higher marginal costs, $c(\theta_2) > c(\theta_1)$.

and so $\Psi(c(\theta_1); \theta_2) > 0$. Hence, firm 2 can increase its profit by further adding product lines, even though this implies higher unit costs, $c(\theta_2) > c(\theta_1)$. This is illustrated graphically in figure 2.

Proposition 1 shows that observed unit cost is *inversely* related to the firm's intrinsic efficiency (its organizational capability θ). This raises a potentially important conceptual issue for empirical work that attempts to identify a firm's intrinsic efficiency from its costs. Our model shows that even if unit costs are observable such an exercise is valid only if one corrects for the number of product lines:

$$\theta = \frac{\ln(n)}{\ln\left(\frac{c}{c_0}\right)}.$$

In practice, it is often hard to measure costs correctly. A popular alternative measure of firm efficiency is Tobin's Q , the market-to-book ratio

$$T(\theta) \equiv \frac{m(\theta)}{b(\theta)},$$

where $m(\theta)$ is the market value of the firm (including its assets) and $b(\theta)$ the book value of the assets used by the firm (independently of whether the assets are rented or owned). The firm's assets are its product lines as well as any capital it uses for production. Suppose each firm has a Cobb-Douglas production function and α is the capital share in production costs. Then, the firm's book value is given by

$$b(\theta) = n(\theta)r + n(\theta)\alpha c(\theta)q(c(\theta)),$$

where the first term is the book value of the product lines and the second term the book value of the capital used for production. The market value of the firm (and its assets) is given by

$$m(\theta) = n(\theta)P(q(c(\theta)))q(c(\theta)) - n(\theta)(1 - \alpha)c(\theta)q(c(\theta)),$$

where the first term is revenue and the second term labor costs. The next lemma shows that the market-to-book ratio is negatively related to a firm's intrinsic efficiency.

Lemma 1 *A firm's market-to-book ratio (Tobin's Q), $T(\theta)$, is decreasing in the firm's organizational capability θ .*

Proof. See appendix. ■

Our model predicts a relationship between organizational capability θ and various measures of firm size. Let

$$S(\theta) \equiv n(\theta)q(c(\theta))P(q(c(\theta)))$$

denote the sales of a firm with organizational capability θ .

Lemma 2 *A firm's sales $S(\theta)$, book value $b(\theta)$, and market value $m(\theta)$ are increasing in the firm's organizational capability θ .*

Proof. See appendix. ■

Lemma 1 establishes a relationship between Tobin’s Q and organizational capability, while lemma 2 establishes a relationship between firm size and organizational capability. As shown in the following proposition, our model can explain the size-discount puzzle found in the data.

Proposition 2 *A firm’s market-to-book ratio (Tobin’s Q), $T(\theta)$, is inversely related to various measures of firm size: sales $S(\theta)$, book value $b(\theta)$, and market value $m(\theta)$.*

Proof. This follows immediately from lemmas 1 and 2. ■

The empirical evidence on the relationship between market-to-book ratio and firm size is consistent with our model, but contradicts standard models of firm heterogeneity where firms differ in their constant marginal costs.¹ While there is strong empirical evidence showing a negative relationship between Tobin’s Q and firm size, there are a number of empirical papers (e.g., Schoar, 2002) that find a positive relationship between firm size and total factor productivity. There is, however, good reason to be skeptical about any cross-firm comparison in measured total factor productivity: the data does not contain information on input quality. In particular, it is well known that large firms pay higher wages, and many authors have argued that this is, at least in part, because they hire better workers. This implies that any empirical study of total factor productivity that does not account for input quality overestimates the total factor productivity of large firms compared to small firms. Our model naturally gives rise to the positive relationship between average wages and firm size found in the data if managing many product lines requires the firm to hire more highly talented workers to oversee and coordinate production.

Our model also predicts a negative relationship across firms between the number of product lines, $n(\theta)$, and sales per product, $P(q(c(\theta)))q(c(\theta))$. Indeed, taking the derivative of sales per product with respect to θ and using the first-order condition for optimal output, (3), yields $dP(q(c(\theta)))q(c(\theta))/d\theta = c(\theta)q'(c(\theta))c'(\theta)$, which is strictly negative for $\theta > \tilde{\theta}$ since $q'(c(\theta)) < 0$ and $c'(\theta) > 0$. Noting that $n(\theta)$ is increasing in θ , the asserted negative relationship between $n(\theta)$ and $P(q(c(\theta)))q(c(\theta))$ then follows. This prediction is consistent with the empirical evidence presented in Berger and Ofek (1995), who document that the mean sales per product line of single-product firms are about 20 percent higher than those of multi-product firms.

In this section, we have assumed that each firm acts as a monopolist for each one of its product lines. Alternatively, we could have assumed monopolistic competition between firms. If the residual demand curve that firms face for each product line satisfies the mild regularity condition we imposed on $P(\cdot)$, proposition 1 carries over to this setting: firms with greater organizational capability have higher unit costs than firms with inferior organizational capability.

¹Consider, for example, Melitz (2003). Since consumers have CES preferences, a firm with efficiency φ charges a constant markup over marginal cost, $p(\varphi)/c(\varphi) = \rho > 1$, and output is of the form $q(\varphi) = \gamma c(\varphi)^{-\varepsilon}$, where $\gamma > 0$ and $\varepsilon > 1$. Tobin’s Q can then be written as

$$T(\varphi) = \frac{[\rho - (1 - \alpha)]}{\frac{r}{\gamma c(\varphi)^{1-\varepsilon}} + \alpha},$$

which is decreasing in firm efficiency φ , while firm sales are increasing in φ .

3 Trade Costs and the Size Distribution of Firms

In this section, we extend our model by introducing a second country to which firms can export. We then study the effects of changes in trade costs on firm scope, aggregate productivity, and the size distribution of firms.

For notational simplicity, we assume that market demand is the same in both countries. (None of our results depend on this assumption.) A firm that exports to the foreign country incurs two types of trade costs: a specific tariff and iceberg-type transport costs. Specifically, if $c(n; \theta)$ denotes the marginal cost of production of a type- θ firm managing n product lines, then

$$\tau c(n; \theta) + t$$

is this firm's marginal cost of serving the foreign market, where $\tau \geq 1$ and $t \geq 0$. We assume that $\tau - 1$ and t are sufficiently small so that each firm finds it optimal to sell in both countries.

In this section, we are concerned with the short-run effects of a change in trade costs. By short run, we mean that the mass M of firms and the mass $N > M$ of product lines is fixed. We may think of M and N being in pre-shock long-run equilibrium. While the mass N of product lines is fixed in the short-run, firms can buy and sell product lines at an *endogenous* market price r . Trade in product lines correspond to partial acquisitions and divestitures, which are about half of all M&A activity in the US (Maksimovic and Phillips, 2001).

A firm makes output decisions separately for each country. If \tilde{c} is the firm's marginal cost of serving a particular market, then

$$q(\tilde{c}) \equiv \arg \max_q [P(q) - \tilde{c}] q$$

denotes the firm's profit-maximizing output for that market. The first-order condition for optimal output choice is given by

$$P(q(\tilde{c})) - \tilde{c} + q(\tilde{c})P'(q(\tilde{c})) = 0. \quad (7)$$

Let $\pi(c(n; \theta))$ denote the gross profit per product line of a type- θ firm managing n product lines:

$$\pi(c(n; \theta)) = [P(q(c(n; \theta))) - c(n; \theta)] q(c(n; \theta)) + [P(q(\tau c(n; \theta) + t)) - \tau c(n; \theta) - t] q(\tau c(n; \theta) + t),$$

where the first term is the gross profit in the domestic market and the second term is the gross profit in the foreign market. The firm's problem of choosing the optimal number $n(\theta; t)$ of production lines can then be written as

$$\max_n n [\pi(c(n; \theta)) - r].$$

Let $n(\theta)$ denote the solution to this problem. The first-order condition is given by

$$\begin{aligned} \Phi(c(\theta); \theta; \tau; t) &\equiv \left[P(q(c(\theta))) - \left(1 + \frac{1}{\theta}\right) c(\theta) \right] q(c(\theta)) \\ &\quad + \left[P(q(\tau c(\theta) + t)) - \left(1 + \frac{1}{\theta}\right) \tau c(\theta) - t \right] q(\tau c(\theta) + t) - r \\ &= 0 \end{aligned} \quad (8)$$

where $c(\theta) \equiv c(n(\theta); \theta)$. It is straightforward to show that propositions 1 and 2 carry over this setting: firms with greater organizational capability have higher marginal costs and lower values of Tobin's Q . For convenience, we will assume that $\underline{\theta}$ is sufficiently large so that for all firms with organizational capability $\theta \in [\underline{\theta}, \bar{\theta}]$, the implicit choice of $c(\theta)$ is given by the solution to the first-order condition $\Phi(c(\theta); \theta; \tau; t) = 0$, and so $n(\theta) \equiv [c(\theta)/c_0]^\theta > 1$.

Since the mass M of firms and the mass $N > M$ of product lines are fixed in the short run, the endogenous market price of a product line, r , must adjust to ensure market clearing. The clearing condition for the market for product lines is given by

$$N = M \int_{\underline{\theta}}^{\bar{\theta}} n(\theta) dG(\theta). \quad (9)$$

Definition 1 *A short-run equilibrium is the collection $\{q(\cdot), n(\cdot), c(\cdot), r\}$ satisfying the cost function (1), the first-order condition for optimal output, (7), the first-order condition for the choice of the number of product lines, (8), and the merger market clearing condition (9).*

We now consider a small increase in the specific tariff t . We will show that, under some reasonable condition on demand, the rise in trade costs will lead to a more skewed size distribution of firms: (large) high- θ firms will expand by purchasing product lines from (small) low- θ firms. Hence, $c(\theta)$ will increase for high- θ firms and decrease for low- θ firms.

Applying the implicit function theorem to the first-order condition for the optimal choice of the number of product lines, (8), we obtain $dc(\theta)/dt = -\Phi_t(c(\theta); \theta; \tau; t)/\Phi_c(c(\theta); \theta; \tau; t)$, where Φ_s denotes the partial derivative of Φ with respect to $s \in \{c, t\}$. Since the first-order condition defines a profit maximum, $\Phi_c(c(\theta); \theta; \tau; t) < 0$, and so the sign of $dc(\theta)/dt$ is equal to the sign of $\Phi_t(c(\theta); \theta; \tau; t)$. We have

$$\Phi_t(c(\theta); \theta; \tau; t) = -q(\tau c(\theta) + t) - \frac{dr}{dt} + \left(\frac{-\tau q'(\tau c(\theta) + t)}{q(c(\theta)) + \tau q(\tau c(\theta) + t)} \right) \left(\frac{c(\theta)}{\theta} [q(c(\theta)) + \tau q(\tau c(\theta) + t)] \right).$$

The first term is the change in the gross profit per product line in the foreign market. The second term is the change in the endogenous market price of a product line. The third term is the change in the externality associated with an additional product line, holding fixed the number of product lines. This last term is the product of two factors: (i) the absolute value of the fractional change in the firm's shipped world output² due to the increase in t , and (ii) the increase in total production costs induced by adding another product line (i.e., the size of the externality). From the first-order condition (8), factor (ii) is equal to the net profit per product line. Hence, we can rewrite the expression as

$$\Phi_t(c(\theta); \theta; \tau; t) = -q(\tau c(\theta) + t) - \frac{dr}{dt} + \left(\frac{-\tau q'(\tau c(\theta) + t)}{q(c(\theta)) + \tau q(\tau c(\theta) + t)} \right) [\pi(c(\theta)) - r], \quad (10)$$

where θ enters only through $c(\theta)$.

²Because of iceberg-type transport costs, the firm ships $\tau q(\tau c(\theta) + t)$ units of output to the foreign country, but only $q(\tau c(\theta) + t)$ units arrive there.

Taking the derivative with respect to θ , yields

$$\frac{d\Phi_t(c(\theta); \theta; \tau; t)}{d\theta} = [\pi(c(\theta)) - r] c'(\theta) \frac{d}{dc} \left(-\frac{\tau q'(\tau c(\theta) + t)}{q(c(\theta)) + \tau q(\tau c(\theta) + t)} \right).$$

This expression obtains because of an envelope-type result:

$$-\tau q'(\tau c(\theta) + t) = \left(-\frac{\tau q'(\tau c(\theta) + t)}{q(c(\theta)) + \tau q(\tau c(\theta) + t)} \right) \frac{d}{dc} [\pi(c(\theta)) - r],$$

where the l.h.s. is the effect of an increase in marginal cost on the induced change in the (gross) profit per product line in the foreign country, i.e., on the first term in (10), whereas the r.h.s. is the absolute value of the fractional change in shipped world output due to the increase in t (i.e., factor (i) in the third term in (10)) multiplied by the effect of an increase in marginal cost on the size of the externality (i.e., on factor (ii) in the third term in (10)). Since $c'(\theta) > 0$ from proposition 1 and since the net profit per product line is positive, $d\Phi_t(c(\theta); \theta; \tau; t)/d\theta$ is positive if the condition $d\{-\tau q'(\tau c(\theta) + t)/[q(c(\theta)) + \tau q(\tau c(\theta) + t)]\} > 0$ holds. As transport costs become small, $\tau \rightarrow 1$ and $t \rightarrow 0$, this condition becomes $d\{-q'(c)/q(c)\} > 0$. We will assume that $P''(q)$ and $P'''(q)$ are not too large so that $-\tau q'(\tau c + t)/[q(c) + \tau q(\tau c + t)]$ is strictly increasing in c . In particular, this assumption holds if demand is linear.

We have thus shown that, under our assumption on demand, $dc(\theta)/dt$ is strictly increasing in θ . Since the mass of product lines is fixed in the short run, $dc(\theta)/dt$ cannot be positive for all θ since this would mean that all firms are adding product lines. Similarly, $dc(\theta)/dt$ cannot be negative for all θ since this would mean that all firms are selling product lines. Hence, the endogenous market price of a product line, r , will adjust so that there exists a threshold type $\hat{\theta} \in (\underline{\theta}, \bar{\theta})$ such that all firms with organizational capability $\theta \in [\underline{\theta}, \hat{\theta})$ respond to an increase in t by selling product lines (and so $dc(\theta)/dt < 0$), whereas all firms with organizational capability $\theta \in (\hat{\theta}, \bar{\theta}]$ respond to an increase in t by buying product lines (and so $dc(\theta)/dt > 0$).

We summarize the effect of an increase in the specific tariff t in the following proposition.

Proposition 3 *Assume $d\{-\tau q'(\tau c + t)/[q(c) + \tau q(\tau c + t)]\}/dc > 0$ for all $c \geq c_0$, and consider a small increase in the specific tariff t . In short-run equilibrium, there exists a threshold type $\hat{\theta} \in (\underline{\theta}, \bar{\theta})$ such that $dn(\theta)/dt < 0$ for all firms with organizational capability $\theta \in [\underline{\theta}, \hat{\theta})$, whereas $dn(\theta)/dt > 0$ for all firms with organizational capability $\theta \in (\hat{\theta}, \bar{\theta}]$.*

The proposition implies that any change in trade costs induces a “merger wave” in the short run. Following an increase in trade costs, small firms sell product lines to large firms, and so the size distribution of firms becomes more skewed, while the opposite result obtains following a reduction in trade costs. Proposition 3 is concerned with the effect of changes in the specific tariff. As the following proposition shows, the same qualitative result obtains following an increase in the iceberg-type transport costs.

Proposition 4 *Suppose that the demand condition of proposition 3 holds, and assume that $d[-q'(c)/q(c)]/dc > 0$ for all $c \geq c_0$. Consider a small increase in the iceberg transport*

cost parameter τ . In short-run equilibrium, there exists a threshold type $\hat{\theta} \in (\underline{\theta}, \bar{\theta})$ such that $dn(\theta)/d\tau < 0$ for all firms with organizational capability $\theta \in [\underline{\theta}, \hat{\theta})$, whereas $dn(\theta)/d\tau > 0$ for all firms with organizational capability $\theta \in (\hat{\theta}, \bar{\theta}]$.

Proof. See appendix. ■

While the phrasing of the proposition suggests that proposition 4 requires a stronger condition on demand than proposition 3, this is not the case. In fact, for small trade costs, $\tau \approx 1$ and $t \approx 0$, the prediction of proposition 4 obtains under a fairly weak condition, namely if the absolute value of the elasticity of output with respect to marginal cost is increasing in marginal cost, $d[-cq'(c)/q(c)]/dc > 0$. In the remainder of this section, we assume that the demand conditions of propositions 3 and 4 are satisfied. The following corollary is an immediate implication of propositions 3 and 4 and lemma 1.

Corollary 1 *Consider a reduction in trade costs, i.e., either a decrease in t or in τ . Then, firms with large market-to-book ratios $T(\theta)$ purchase product lines from firms with small market-to-book ratios.*

To the extent that much of the merger and acquisition activity is due to “globalization” (or, alternatively, positive productivity shocks), our model predicts that firms with high values of Tobin’s Q buy corporate assets from firms with low Tobin’s Q . This is indeed consistent with the empirical evidence summarized by Andrade, Mitchell, and Stafford (2001).

Propositions 3 and 4 suggest that an increase in trade costs induces a more skewed size distribution of firms. This intuition is indeed correct, as the following proposition shows, if one measures the size of a firm with organizational capability θ by its domestic sales (or revenue) $S(\theta) \equiv n(\theta)P(q(c(\theta)))q(c(\theta))$.

Proposition 5 *An increase in trade costs – either in the specific tariff t or in the iceberg-type transport cost τ – increases (decreases) the domestic sales of a type- θ firm, $S(\theta)$, if and only if it induces an increase (decrease) in the optimal choice of the number of product lines $n(\theta)$. Hence, following an increase in trade costs, there exists a threshold type $\hat{\theta} \in (\underline{\theta}, \bar{\theta})$ such that the domestic sales of all (small) firms of type $\theta \in [\underline{\theta}, \hat{\theta})$ fall, while those of all (large) firms of type $\theta \in (\hat{\theta}, \bar{\theta}]$ rise.*

Proof. See appendix. ■

4 Monopolistic Competition: Trade Liberalization and the Size Distribution of Firms

In this section, we turn to the effects of trade liberalization and market integration on firm scope and the size distribution of firms in a two-country model with monopolistic competition. We are concerned with the effects of trade liberalization both in the short run, where the

number of firms and the aggregate number of product lines is fixed, and the long run, where the number of firms and the aggregate number of product lines are endogenous.

There are two countries, country 1 and country 2, and a population of firms in each. Firms can sell in both countries but can produce only in their home country. In this section, we will refer to $c(n; \theta)$, which is again given by (1), as the firm's marginal cost, and to the additive cost parameter t as the transport cost or tariff. The transport cost or tariff is indexed by a country pair (i, j) : t_{ij} is the transport cost or tariff per unit of output from country i to country j . Transport costs and tariffs have to be incurred only for exports from one country to the other, and so $t_{11} = t_{22} = 0$, $t_{12} > 0$, and $t_{21} > 0$. Countries differ only in their tariffs.

In each country, there is a mass L of identical consumers with the following linear-quadratic utility function:

$$U = \int x(k)dk - \int [x(k)]^2 dk - 2\sigma \left[\int x(k)dk \right]^2 + H,$$

where $x(k)$ is consumption of product line k , H is consumption of the Hicksian composite commodity, and $\sigma > 0$ is a parameter that measures the degree of product differentiation. Assuming that consumer income is sufficiently large, each consumer's inverse demand for product line k is then given by

$$p(k) = 1 - 2x(k) - 4\sigma \int x(l)dl.$$

We assume that each firm can set a different output (or price) in the two countries. Since each product line is of measure zero, a firm's choice of output for one product line does not affect its choice of output for another product line. Consider now a firm with marginal cost c from country i selling in country j (which may or may not be the same country). It can be shown that its profit-maximizing output $q_{ij}(c)$ and gross profit per product line $\pi_{ij}(c)$ from sales in country j are given by

$$q_{ij}(c) = \frac{L}{4}(a_j - t_{ij} - c), \quad i, j = 1, 2,$$

and

$$\pi_{ij}(c) = \frac{L}{8}(a_j - t_{ij} - c)^2, \quad i, j = 1, 2,$$

respectively, where a_j is the endogenous residual demand intercept in country j . This endogenous demand intercept is given by

$$a_j = \frac{1 + \sigma \int (c + t_{1j})\mu_{1j}(dc) + \sigma \int (c + t_{2j})\mu_{2j}(dc)}{1 + \sigma \int \mu_{1j}(dc) + \sigma \int \mu_{2j}(dc)}, \quad (11)$$

where μ_{ij} is the Borel measure over marginal costs of those product lines that are produced in country i and sold in country j . To simplify notation, we will henceforth normalize market size $L \equiv 8$.

As the following lemma shows, each product line will be sold in each country, provided transport costs and tariffs are sufficiently small.

Lemma 3 *Suppose the two countries impose identical tariffs, $t_{12} = t_{21} = t$, so that the demand intercept is the same in both countries, $a_1 = a_2 = a$. Then, if the common tariff t is sufficiently small, all firms will choose to sell in both countries.*

Proof. See appendix. ■

Henceforth, we will assume that all firms sell in both countries. The first-order condition for the optimal choice of the number of product lines then becomes

$$\begin{aligned}\Omega^i(c_i(\theta); \theta; t_{12}, t_{21}) &\equiv \left\{ [a_i - c_i(\theta)]^2 + [a_j - t_{ij} - c_i(\theta)]^2 - r_i \right\} \\ &\quad - \frac{2c_i(\theta)}{\theta} \{ [a_i - c_i(\theta)] + [a_j - t_{ij} - c_i(\theta)] \} \\ &= 0,\end{aligned}\tag{12}$$

where $c_i(\theta) = c_0 [n_i(\theta)]^{1/\theta}$ is the implicit choice of marginal cost by a firm with organizational capability θ based in country i , and r_i the fixed cost per product line in country i . As in section 3, we assume that the domain of organizational capabilities, $[\underline{\theta}, \bar{\theta}]$, is such that this first-order condition determines the optimal choice of $c_i(\theta)$ for all $\theta \in [\underline{\theta}, \bar{\theta}]$. Applying the implicit function theorem to (12), we obtain

$$c'_i(\theta) = \frac{c_i(\theta) \{ [a_i - c_i(\theta)] + [a_j - t_{ij} - c_i(\theta)] \}}{\theta^2 \{ [a_i - c_i(\theta)] + [a_j - t_{ij} - c_i(\theta)] \} + \theta \{ [a_i - 2c_i(\theta)] + [a_j - t_{ij} - 2c_i(\theta)] \}}.\tag{13}$$

Since each firm makes positive sales from selling in each country, $a_i > c_i(\theta)$ and $a_j > t_{ij} + c_i(\theta)$, the first-order condition (12) implies that $\theta \{ [a_i - c_i(\theta)] + [a_j - t_{ij} - c_i(\theta)] \} > 2c_i(\theta)$. It then follows that proposition 1 carries over the two-country setting with monopolistic competition: $c'_i(\theta) > 0$.

Let M_i denote the mass of firms producing in country i , and N_i the mass of product lines managed by firms from country i . The endogenous demand intercept in country i can then be written as

$$a_i = \frac{1 + \sigma \int [M_i n_i(\theta) c_i(\theta) + M_j n_j(\theta) c_j(\theta)] dG(\theta) + \sigma N_j t_{ji}}{1 + \sigma(N_1 + N_2)}, \quad i \neq j, \quad i = 1, 2.\tag{14}$$

Aggregating the endogenous numbers of product lines over all M_i firms from country i yields the mass N_i of product lines managed by these firms:

$$N_i = M_i \int_{\underline{\theta}}^{\bar{\theta}} n_i(\theta) dG(\theta), \quad i = 1, 2.\tag{15}$$

Below, we will turn to the short-run and long-run effects of trade liberalization. A change in trade costs will lead to different responses across firms in their choice of the number of product lines, and these different responses will alter the endogenous demand intercept a . The following lemma shows how a changes when high- θ firms add product lines while low- θ firms reduce the number of product lines.

Lemma 4 *Suppose there exist marginal types $\widehat{\theta}_1$ and $\widehat{\theta}_2$ such that all firms in country $i \in \{1, 2\}$ with organizational capability $\theta > \widehat{\theta}_i$ divest product lines, $\Delta n_i(\theta) < 0$ for $\theta > \widehat{\theta}_i$, while all other firms in country i add product lines, $\Delta n_i(\theta) > 0$ for $\theta < \widehat{\theta}_i$, holding the total mass of product lines in each country i fixed, $\int \Delta n_i(\theta) dG(\theta) = 0$. Then, the weighted average (by the number of product lines) marginal costs of firms producing in country i decreases:*

$$\int \frac{d}{dn} [nc_i(n; \theta)] \Big|_{n=n_i(\theta)} \Delta n_i(\theta) dG(\theta) < 0.$$

Hence, the endogenous demand intercept a_i decreases, $\Delta a_i < 0$.

Proof. See appendix. ■

We now turn to the short-run and long-run effects of trade liberalization and market integration.

4.1 The Short-Run Effects of Trade Liberalization

In short-run equilibrium, the mass of firms producing in country i , M_i , is fixed, as is the mass of product lines managed by these firms, N_i . Since the location of production of a product line is assumed to be fixed in the short run, the endogenous (short-run) market price of a product line, r_i , may differ across countries. We can then define a *short-run equilibrium* as a collection $\{c_i(\cdot), n_i(\cdot), a_i, r_i\}_{i=1}^2$ satisfying the cost equation (1), the first-order condition for the optimal choice of the number of product lines, (12), the equation for the endogenous demand intercept, (14), and the merger market condition (15).

We now analyze the short-run effects of symmetric and unilateral tariff changes on firm scope and the size distribution of firms. For this purpose, we assume that, prior to the change in tariffs, the two countries are identical: $N_1 = N_2 = N$, $M_1 = M_2 = M$, and $t_{12} = t_{21} = t$. We first consider a small symmetric reduction in the common tariff t .

Proposition 6 *Suppose that the countries impose identical tariffs, $t_{12} = t_{21} = t$, and consider the short-run effects of a small symmetric trade liberalization, $dt < 0$. There exists a marginal type $\widehat{\theta} \in (\underline{\theta}, \bar{\theta})$ such that all firms with organizational capability $\theta > \widehat{\theta}$ respond by divesting product lines, while all firms with organizational capability $\theta < \widehat{\theta}$ respond by purchasing additional product lines.*

Proof. See appendix. ■

In response to a symmetric trade liberalization, large firms decide to downsize by divesting product lines. If the market price of a product line were unchanged, all firms would actually want to purchase product lines. However, the number of product lines is fixed, and so the price per product line r increases in response to a symmetric trade liberalization. But given this endogenous price increase, only the firms with the lowest marginal costs (i.e., the firms with inferior organizational capability) find it optimal to add product lines. Proposition 6 thus mirrors our earlier result, proposition 3, on the effects of a change in trade costs when each firm is a monopolist for each of its product lines. Proposition 6 in conjunction with lemma 4 implies

that a symmetric trade liberalization reduces the weighted (by number of product lines) average production costs in the industry. To the extent that the Canadian-U.S. free-trade agreement can be viewed as a symmetric trade liberalization, this last prediction is consistent with Treffer (2004), who attributes a 15 percent increase in average labor productivity in Canada to the free-trade agreement.

Next, we consider a small unilateral reduction in the tariff imposed by country 1 on imports from country 2, t_{21} .

Proposition 7 *Suppose that the countries initially impose identical tariffs, $t_{12} = t_{21} = t$, and consider the short-run effects of a small unilateral trade liberalization by country 1, $dt_{21} < 0$. In the liberalizing country 1, there exists a marginal type $\hat{\theta}_1 \in (\underline{\theta}, \bar{\theta})$ such that all firms with organizational capability $\theta > \hat{\theta}_1$ respond by purchasing additional product lines, while all firms with organizational capability $\theta < \hat{\theta}_1$ respond by divesting product lines. In contrast, in country 2, there exists a marginal type $\hat{\theta}_2 \in (\underline{\theta}, \bar{\theta})$ such that all firms with organizational capability $\theta > \hat{\theta}_2$ respond by divesting product lines, while all firms with organizational capability $\theta < \hat{\theta}_2$ respond by purchasing additional product lines.*

Proof. See appendix. ■

The short-run effects of a unilateral trade liberalization are very different from those of a symmetric trade liberalization. In the liberalizing country 1, increased competition with foreign firms induces the largest firms to add product lines while the smallest firms become even smaller as they divest product lines. Hence, a country that unilaterally reduces its trade barriers with the rest of the world will experience a steepening of the size distribution of its firms. The improved access of country-2 firms to country 1's market has the opposite impact on firms in that country: the size distribution of firms producing in country 2 becomes flatter as large firms contract and small firms expand.

4.2 The Long-Run Effects of Trade Liberalization

In our analysis of the effects of trade liberalization on firm scope and the size distribution of firms, we have assumed so far that the mass of firms and the aggregate mass of product lines produced in each country is fixed. Here, we consider a different set of assumptions: we assume that, both the mass of firms and the aggregate mass of product lines will adjust in response to changes in tariffs. We are thus concerned with the long-run effects of trade liberalization.

Specifically, there is a sufficiently large mass of ex ante identical potential entrants. If a firm decides to enter, it has to pay a fixed entry cost ϕ ; if it decides not to enter, it obtains a payoff normalized to zero. After paying the entry cost, a firm receives a random draw of its organizational capability θ from the c.d.f. $G(\cdot)$. A firm then decides on the number of its product lines. In both countries, the fixed development cost per product line is r . We assume that the life span of each product line is limited, which implies that, in long-run equilibrium, the market price of each product line is equal to the exogenous development cost r , and the merger market does not play any allocative role. Since potential entrants are ex ante identical, the

expected net profit of each entrant in country i must be equal to zero in long-run equilibrium:

$$\int_{\underline{\theta}}^{\bar{\theta}} n_i(\theta) \left\{ [a_i - c_i(\theta)]^2 + [a_j - t_{ij} - c_i(\theta)]^2 - r_i \right\} dG(\theta) - \phi = 0, \quad i = 1, 2. \quad (16)$$

We define a *long-run equilibrium* as a collection $\{c_i(\cdot), n_i(\cdot), a_i, N_i, M_i\}_{i=1}^2$ satisfying the cost equation (1), the first-order condition (12), the equation for the endogenous demand intercept, (14), the adding-up condition (15), and the free-entry condition (16).

We now analyze the long-run effects of (unanticipated) symmetric and unilateral tariff changes on firm scope and the size distribution of firms. For this purpose, we assume that the industry is in a long-run equilibrium, both before and after the change in tariffs. As before, we assume that, prior to the change in tariffs, the two countries are identical, and so $N_1 = N_2 = N$, $M_1 = M_2 = M$, and $t_{12} = t_{21} = t$. We first consider a small symmetric reduction in the common tariff t .

Proposition 8 *Suppose that the countries impose identical tariffs, $t_{12} = t_{21} = t$, and consider the long-run effects of a small symmetric trade liberalization, $dt < 0$. There exists a marginal type $\tilde{\theta} \in [\underline{\theta}, \bar{\theta}]$ such that all firms with organizational capability $\theta > \tilde{\theta}$ have a reduced number of product lines, $dn(\theta) < 0$, while all firms with organizational capability $\theta < \tilde{\theta}$ have an increased number of product lines, $dn(\theta) > 0$.*

Proof. See appendix. ■

Qualitatively, the long-run effects of a trade liberalization are similar to the short-run effects: there is a tendency for small firms with inferior organizational capability (but low marginal cost) to increase the number of product lines, while the reverse holds for large firms with superior organizational capability (but high marginal cost). In contrast to the short run, however, it is conceivable that $n(\theta)$ moves in the same direction for all firms, namely when $\tilde{\theta} = \underline{\theta}$ or $\tilde{\theta} = \bar{\theta}$.

Next, we consider the long-run effects of a small unilateral reduction in the tariff imposed by country 1 on imports from country 2, t_{21} .

Proposition 9 *Suppose that the countries initially impose identical tariffs, $t_{12} = t_{21} = t$, and consider the long-run effects of a small unilateral trade liberalization by country 1, $dt_{21} < 0$. In the liberalizing country 1, there exists a marginal type $\hat{\theta}_1 \in [\underline{\theta}, \bar{\theta}]$ such that all firms with organizational capability $\theta > \hat{\theta}_1$ have an increased number of product lines, $dn_1(\theta) > 0$, while all firms with organizational capability $\theta < \hat{\theta}_1$ have a reduced number of product lines, $dn_1(\theta) < 0$. In contrast, in country 2, there exists a marginal type $\hat{\theta}_2 \in [\underline{\theta}, \bar{\theta}]$ such that all firms with organizational capability $\theta > \hat{\theta}_2$ have a reduced number of product lines, $dn_2(\theta) < 0$, while all firms with organizational capability $\theta < \hat{\theta}_2$ have an increased number of product lines, $dn_2(\theta) > 0$.*

Proof. See appendix. ■

The long-term implications of a unilateral trade liberalization for the size distribution of firms are similar to those of the short-run. In the liberalizing country, production becomes

more concentrated in the largest firms while production becomes less concentrated in the other country. As was the case for symmetric liberalization, it is conceivable that all firms within a country contract or expand.

5 Empirics

In this section, we use firm-level panel data to test our model's predictions on the effects of changes in the international trading environment on the size distribution of firms. According to Proposition 6, a reduction in trade costs that is symmetric across countries will induce large, high- θ firms to shed product lines and small, low- θ firms to add product lines. As a result, a symmetric fall in trade costs is associated with less skewness of the size distribution of firms within an industry. According to Proposition 7, a unilateral reduction in a country's trade barriers induces large, high- θ firms in the liberalizing country to add product lines and the small, low- θ firms to drop product lines, thereby causing the size distribution of firms to become more skewed.

Our empirical analysis investigates the link between the level of trade barriers and the skewness of the size distribution of U.S. manufacturing companies. On one hand, we consider changes in *shipping costs* between the U.S. and its trading partners. We assume that a reduction in shipping costs are driven by technology and are therefore symmetric between countries. On the other hand, we consider changes in U.S. *tariffs* imposed on imports from the rest of the world to be primarily unilateral reductions in trade costs.

To measure the degree of dispersion we consider the shape of the size distribution of U.S. firms – i.e., the relationship between the logarithm of an individual firm's domestic sales and the logarithm of its rank within the industry in terms of its sales. To assess the predictions of our model, we consider versions of the following specification:

$$\begin{aligned} \ln SALES_{jit} = & \alpha_{it} + \beta_0 \ln RANK_{jit} + \beta_1 (\ln RANK_{jit})^2 + \\ & \beta_2 \ln FREIGHT_{it} \ln RANK_{jit} + \beta_3 \ln TARIFF_{it} \ln RANK_{jit} + \varepsilon_{jit}, \end{aligned} \quad (17)$$

where $SALES_{jit}$ is the sales of firm j in industry i at time t , $RANK_{jit}$ is the rank of this firm in the size distribution (the largest firm has $RANK_{jit} = 1$), $FREIGHT_{it}$ is an ad-valorem measure of freight and insurance costs into and out of the United States in industry i at time t , $TARIFF_{it}$ is an ad-valorem measure of tariffs imposed on imports into the United States in industry i at time t , α_{it} is an industry-time fixed effect, and ε_{jit} are unobserved determinants of a firm's sales. We allow for non-linearities in the relation between size and rank by including $(\ln RANK_{jit})^2$, and we allow for the intercept (α_{it}) to vary within a year across industries and to vary within an industry across years.

The gradient of $\ln SALES_{jit}$ with respect to $\ln RANK_{jit}$ (which is negative by construction) summarizes the size distribution of firms:

$$\frac{\partial \ln SALES_{jit}}{\partial \ln RANK_{jit}} = \beta_0 + \beta_1 \ln RANK_{jit} + \beta_2 \log FREIGHT_{it} + \beta_3 TARIFF_{it}.$$

As the gradient becomes steeper (negative, but with greater absolute value), a larger share of production is concentrated in the relatively larger firms. Our model predicts that a symmetric

rise in shipping costs (an increase in $FREIGHT_{it}$) should be associated with a steeper gradient. Hence, the model predicts $\beta_2 < 0$. Our model predicts that a unilateral rise in U.S. tariffs (an increase in $TARIFF_{it}$) should be associated with a flatter size distribution of firms. Hence, the model predicts $\beta_3 > 0$.

To estimate (17) we require only firm-level sales data ($SALES_{jit}$), shipping costs ($FREIGHT_{it}$), and tariffs ($TARIFF_{it}$). Our firm-level data was collected from the *Compustat* database from which we obtained an unbalanced panel of 4,445 firms in 111 three-digit manufacturing SIC industries over the years 1989-2001. We observe each firm's sales in the U.S. market (exports and any other sales in foreign markets are removed). A firm's rank in the size distribution (at the three-digit industry level) was then computed. Our measure of freight and shipping costs and our measure of tariffs is calculated from the Feenstra et al. (2002) dataset.³ The variable $FREIGHT_{it}$ is calculated as freight and insurance charges (C.I.F. imports less F.O.B. imports) divided by F.O.B. imports by industry and year, while our variable $TARIFF_{it}$ is the value of duties paid divided by F.O.B. imports. Descriptive statistics for this data (including controls) are shown in Table 1.

Table 1	<i>mean</i>	<i>stdev</i>	min	max
$\ln RANK$	3.078	1.446	0	5.790
$\ln SALES$	4.149	2.500	-6.908	11.823
$\ln FREIGHT$	0.956	0.708	-2.351	3.055
$\ln GAP$	-0.268	0.182	-0.510	0.023
$\ln GDP$	8.994	0.112	8.851	9.199
$RINT$	4.801	0.675	3.220	5.670

The results of estimating equation (17) are shown in Table 2. Note that all-specifications include industry-year fixed effects and that the standard errors (shown in parentheses) allow for both heteroskedasticity and clustering by industry-year. For purposes of comparison, we first show in column 1 the result of regressing the logarithm of a firm's size as a function of the logarithm of a firm's size ranking and its square within a three digit industry. The fact that the coefficient on the quadratic term ($(\ln RANK_{jit})^2$) is negative and significant indicates that the size-distribution is not well described by a Pareto distribution.

The baseline results are shown in column 2. The coefficient on $\ln FREIGHT$ is negative and statistically significant at a very high level of confidence. As predicted by our model, a symmetric increase (decrease) in trade costs is associated with greater (less) dispersion within an industry as the largest firms expand (contract) and the smallest firms contract (expand). By contrast, the coefficient on $\ln TARIFF$ is positive and statistically significant, indicating that a unilateral increase (decrease) in U.S. tariffs is associated with a slightly flatter (steeper) size distribution.⁴

³The time span of our data is driven by the time span of this dataset. For years outside of this range industry codes are different and therefore difficult to concord into industry classifications that are consistent with those of *Compustat*.

⁴We conjecture that the much smaller size of this coefficient might reflect is collinearity with falling tariffs abroad, a hypothesis that we are unable to test due to a lack of foreign tariff data.

While we do allow for a full set of industry-year fixed effects, the potential for spurious correlation needs to be addressed. A possible alternate hypothesis is that changes in the availability of credit over the time period might be correlated with trends in trade costs. As a check, we include three additional variables. The first is the interaction between the logarithm between GDP and a firm's rank. The second is the interaction between the real interest rate $RINT_t$ and $\ln RANK_{jit}$. Our measure of the real interest rate is the difference between the nominal interest rate charged to low-risk corporate borrowers and the contemporaneous rate of inflation. Finally, our third control is included to allow for the possibility that changes in credit market conditions might make credit constraints facing small firms relatively more severe, we include the interaction between logarithm of the difference between the nominal interest rates charged to high and low risk borrowers, GAP_t , on the one hand, and $\ln RANK_{jit}$ on the other. All three macroeconomic measures were collected from the *Economic Report of the President*.

Table 2	(1)	(2)	(3)
$\ln RANK$	-1.193 (0.047)	-0.677 (0.091)	4.550 (2.450)
$(\ln RANK)^2$	-0.180 (0.009)	-0.230 (0.011)	-0.236 (0.011)
$\ln FREIGHT * \ln RANK$		-0.301 (0.043)	-0.301 (0.042)
$\ln TARIFF * \ln RANK$		0.064 (0.018)	0.048 (0.018)
$\ln GDP * \ln RANK$			-0.568 (0.270)
$int * \log rank$			-0.003 (0.041)
$\log gap * \log rank$			-0.540 (0.168)
R^2	0.789	0.794	0.795
n	34,917	34,917	34,917

The results of estimating the extended specification are shown in column 3. The coefficient on the interaction between $\ln RANK_{jit}$ and $\ln FREIGHT_{it}$ is virtually unchanged, while the coefficient on the interaction between $\ln RANK_{jit}$ and $\ln TARIFF_{it}$ is slightly smaller in terms of its magnitude but continues to be positive and statistically different from zero. With respect to the controls, movements in real interest rates do not appear to have any impact on the size distribution of firms, but variation across time in real GDP and the gap between high and low risk bonds are both associated with changes in the size distribution of firms. In particular, in periods of high aggregate demand sales become increasingly concentrated in large firms. Further, when credit conditions are tight as indicated by a rising spread between high and low risk bonds, the size distribution is becoming more skewed toward the largest firms in an industry.

A number of other robustness checks were considered. In the interest of space, we simply describe these alternative specifications. First, since the appropriate level of industrial aggre-

gation in a multiproduct setting is not obvious, we also constructed the size distribution of firms at the two-digit SIC level. The coefficient estimates for equation (17) obtained using this alternate sample were nearly identical to those reported in Table 2. Second, in another specification we considered only a balanced sample of firms that continuously reported positive sales to *Compustat* throughout the period. The results were qualitatively unchanged for this narrower sample. Third, we considered a specification in which time trends were interacted with the logarithm of a firm's rank in the distribution. Again, this specification yielded nearly identical coefficient estimates as the baseline specification.

6 Conclusion

To be written....

7 Appendix

The Relationship between Tobin's Q and Firm Size. To examine the relationship between Tobin's Q (i.e., the ratio between market value and book value) and firm size, we use the *Compustat* database. We use data for the most recent year available, namely 2004. We follow Jovanovic and Rousseau (2002) in calculating market value as the market value of common equity (product of items 24 and 25), plus the book value of preferred shares (item 130) and short- and long-term debt (items 34 and 9). Book value is computed similarly but uses book (rather than market) value of common equity (item 60). Our measure of firm size is firm sales (item 9). Our variable $\ln T_i$ is then the logarithm of the ratio of firm i 's market value and book value, while $\ln SALES_i$ is the logarithm of firm i 's sales. Deleting outliers where $\ln T_i \geq 4$, we are left with 5,965 observations.

We regress $\ln T_i$ on $\ln SALES_i$ and a set of industry fixed effects (according to the firm i 's main line of business). Using 2-digit SIC fixed effects, the coefficient on $\ln SALES_i$ is -0.5648 with a standard error of 0.0036. Using 4-digit SIC fixed effects, the coefficient on $\ln SALES_i$ becomes -0.4489 with a standard error of 0.0037.

Proof of proposition 1. Recall that

$$\Psi(c; \theta) \equiv q(c) \{P(q(c)) - (1 + 1/\theta)c\} - r.$$

The first-order condition (6) then states that $\Psi(c(\theta); \theta) = 0$. We proceed in several steps.

Step 1. We show that $\Psi(c; \theta)$ is strictly decreasing in c whenever $\Psi(c; \theta) \geq 0$. Taking the derivative with respect to c , we obtain

$$\begin{aligned} \Psi_c(c; \theta) &= -(1 + 1/\theta)q(c) + [P(q(c)) - (1 + 1/\theta)c + q(c)P'(q(c))]q'(c) \\ &= -(1 + 1/\theta)q(c) - (1/\theta)cq'(c) \\ &= -q(c) - \frac{1}{\theta} \{q(c) + cq'(c)\}, \end{aligned}$$

where the second equality follows from using the first-order condition for output, equation (3). Suppose the expression in curly brackets is nonnegative. Then, $\Psi_c(c; \theta) < 0$. Suppose now that

the expression in curly brackets is negative. Since $\Psi(c; \theta) \geq 0$ implies that

$$\frac{1}{\theta} < \frac{P(q(c)) - c}{c},$$

we then obtain

$$\Psi_c(c; \theta) < -q(c) - \left(\frac{P(q(c)) - c}{c} \right) \{q(c) + cq'(c)\} \equiv \delta$$

We will now show that $\delta \leq 0$, and so $\Psi_c(c; \theta) < 0$. Applying the implicit-function theorem to the first-order condition for optimal output choice, (3),

$$q'(c) = \frac{1}{2P'(q(c)) + q(c)P''(q(c))}. \quad (18)$$

Inserting this expression into the equation for δ , we obtain

$$\delta = -\frac{1}{2P'(q(c)) + q(c)P''(q(c))} \left\{ q(c) \frac{P(q(c))}{c} [2P'(q(c)) + q(c)P''(q(c))] + P(q(c)) - c \right\}.$$

Since our assumption on demand implies that $2P'(q(c)) + q(c)P''(q(c)) < 0$, we have $\delta \leq 0$ if and only if the expression in curly brackets is nonnegative. It follows that $\delta \leq 0$ if

$$\frac{P(q(c))}{c} [2P'(q(c)) + q(c)P''(q(c))] + \frac{P(q(c)) - c}{q(c)} \leq 0.$$

From (3), this inequality holds if

$$\frac{P(q(c))}{c} [2P'(q(c)) + q(c)P''(q(c))] - P'(q(c)) \leq 0.$$

Since $P(q(c)) \geq c$ and $2P'(q(c)) + q(c)P''(q(c)) \leq 0$, the last inequality is implied by $P'(q(c)) + q(c)P''(q(c)) \leq 0$, which holds by assumption. Hence, $\delta \leq 0$, and so $\Psi_c(c; \theta) < 0$ whenever $\Psi(c; \theta) \geq 0$. In particular, $\Psi_c(c(\theta); \theta) < 0$ for any $\theta > 0$. It follows that for each θ , there exists at most one value of c such that $\Psi(c; \theta) = 0$. (In fact, there exists exactly one such value of c for all those θ such that $\Psi(c_0; \theta) \leq 0$, while there exists no such value of c for all θ such that $\Psi(c_0; \theta) > 0$.)

Step 2. It can easily be verified that

$$\Psi_\theta(c; \theta) = \frac{cq(c)}{\theta^2} > 0.$$

Step 3. We now show that $c(\theta) = c_0$ if and only if $\theta \leq \tilde{\theta}$. It is straightforward to check that $\tilde{\theta}$ is the unique solution to $\Psi(c_0; \theta) = 0$. Since $\Psi_\theta(c; \theta) > 0$, it follows that $\Psi(c_0; \theta) \leq 0$ for all $\theta \leq \tilde{\theta}$, and $\Psi(c_0; \theta) > 0$ for all $\theta \geq \tilde{\theta}$. Moreover, since $\Psi_c(c; \theta) < 0$ whenever $\Psi(c; \theta) \geq 0$, it follows that $\Psi(c; \theta) < 0$ for all $\theta \leq \tilde{\theta}$ and all $c > c_0$. Hence, the corner solution $c(\theta) = c_0$ obtains for all $\theta \leq \tilde{\theta}$. In contrast, for all $\theta > \tilde{\theta}$, $c(\theta)$ is given by the first-order condition $\Psi(c(\theta); \theta) = 0$.

Step 4. We finally show that $c(\theta)$ is strictly increasing in θ for all $\theta \geq \tilde{\theta}$. Using the implicit function theorem, we have

$$\frac{dc(\theta)}{d\theta} = -\frac{\Psi_{\theta}(c(\theta); \theta)}{\Psi_c(c(\theta); \theta)} > 0,$$

where the inequality follows from $\Psi_{\theta}(c(\theta); \theta) > 0$ and $\Psi_c(c(\theta); \theta) < 0$. Since $c(\theta)$ is uniquely defined by the first-order condition (for $\theta \geq \tilde{\theta}$), this comparative static result holds globally. ■

Proof of lemma 1. Tobin's Q is given by

$$T(\theta) \equiv \frac{[P(q(c(\theta))) - (1 - \alpha)c(\theta)]q(c(\theta))}{r + \alpha c(\theta)q(c(\theta))}, \quad (19)$$

which is independent of θ for $\theta \leq \tilde{\theta}$ since then $c(\theta) = c_0$. Assume now that $\theta > \tilde{\theta}$ so that $c'(\theta) > 0$.

Step 1. Consider the numerator in (19),

$$[P(q(c(\theta))) - (1 - \alpha)c(\theta)]q(c(\theta)). \quad (20)$$

We claim that this term is strictly decreasing in θ . Since the net profit per product line is strictly decreasing in θ , this claim is correct if $c(\theta)q(c(\theta))$ is nondecreasing in θ . Suppose now instead that $c(\theta)q(c(\theta))$ is strictly decreasing in θ . Then, the term in (20) is strictly decreasing in α . Since $\alpha \in [0, 1]$, this implies that the term is strictly decreasing in θ if revenue per product line,

$$P(q(c(\theta)))q(c(\theta)),$$

is decreasing in θ . But we have

$$\begin{aligned} \frac{dP(q(c(\theta)))q(c(\theta))}{d\theta} &= \{P'(q(c(\theta)))q(c(\theta)) + P(q(c(\theta)))\} q'(c(\theta))c'(\theta) \\ &= c(\theta)q'(c(\theta))c'(\theta) \\ &< 0, \end{aligned}$$

where the second equality follows from the first-order condition for the optimal output choice, equation (3), and the inequality from $c'(\theta) > 0$ and $q'(c(\theta)) < 0$.

Step 2. Since the numerator in (19) is decreasing in θ , $dT(\theta)/d\theta < 0$ if

$$\frac{d}{d\theta} \left\{ \frac{[P(q(c(\theta))) - (1 - \alpha)c(\theta)]q(c(\theta))}{\alpha c(\theta)q(c(\theta))} \right\} < 0, \quad (21)$$

or

$$\frac{d}{d\theta} \left\{ \frac{P(q(c(\theta)))}{\alpha c(\theta)} - \frac{1 - \alpha}{\alpha} \right\} < 0.$$

Taking the derivative of $P(q(c(\theta)))/c(\theta)$ with respect to θ , we obtain

$$\begin{aligned} \frac{d}{d\theta} \left[\frac{P(q(c(\theta)))}{c(\theta)} \right] &= \left(\frac{c'(\theta)}{[c(\theta)]^2} \right) \{P'(q(c(\theta)))q'(c(\theta))c(\theta) - P(q(c(\theta)))\} \\ &= \left(\frac{c'(\theta)}{[c(\theta)]^2} \right) \left\{ \frac{P'(q(c(\theta)))c(\theta)}{2P'(q(c(\theta))) + q(c(\theta))P''(q(c(\theta)))} - P(q(c(\theta))) \right\}, \end{aligned}$$

where the second equality follows from the first-order condition for optimal output choice, equation (3). Our assumption on demand, equation (2),

$$\frac{P'(q(c(\theta)))c(\theta)}{2P'(q(c(\theta))) + q(c(\theta))P''(q(c(\theta)))} < c(\theta).$$

Since $c'(\theta) \geq 0$ and $c(\theta) < P(q(c(\theta)))$, it follows that

$$\frac{d}{d\theta} \left[\frac{P(q(c(\theta)))}{c(\theta)} \right] < 0, \quad (22)$$

and so equation (21) does indeed hold. ■

Proof of lemma 2. *Step 1.* We first show that a firm's sales,

$$S(\theta) = n(\theta)q(c(\theta))P(q(c(\theta))) = \left(\frac{c(\theta)}{c_0} \right)^\theta q(c(\theta))P(q(c(\theta))),$$

are increasing in θ . Taking the derivative with respect to θ , we obtain

$$\begin{aligned} & \left(\frac{c(\theta)}{c_0} \right)^\theta \ln \left(\frac{c(\theta)}{c_0} \right) q(c(\theta))P(q(c(\theta))) + \frac{\theta}{c_0} \left(\frac{c(\theta)}{c_0} \right)^{\theta-1} c'(\theta)q(c(\theta))P(q(c(\theta))) \\ & + \left(\frac{c(\theta)}{c_0} \right)^\theta q'(c(\theta))c'(\theta) [P(q(c(\theta))) + q(c(\theta))P'(q(c(\theta)))] \end{aligned}$$

Clearly, the first term is strictly positive for $\theta > \tilde{\theta}$ (and equal to zero for $\theta \leq \tilde{\theta}$). We now show that the sum of the second and third terms is also strictly positive for $\theta > \tilde{\theta}$. Collecting terms and noting that $P(q(c(\theta))) + q(c(\theta))P'(q(c(\theta))) = c(\theta)$, this sum can be written as

$$\left[\frac{c(\theta)}{c_0} \right]^{\theta-1} \frac{c'(\theta)}{c_0} \left\{ \theta q(c(\theta))P(q(c(\theta))) + [c(\theta)]^2 q'(c(\theta)) \right\}.$$

Since $c'(\theta) > 0$ for $\theta > \tilde{\theta}$ (proposition 1), this expression is positive if the expression in curly brackets is positive. From the first-order condition (6), $\theta > c(\theta)/[P(q(c(\theta))) - c(\theta)]$, and so the expression in curly brackets is strictly positive if

$$c \left[\frac{q(c)P(q(c))}{P(q(c)) - c} + cq'(c) \right] \geq 0,$$

where $c \equiv c(\theta)$. Using the first-order condition for optimal output, (3), and (18), this inequality can be rewritten as

$$\begin{aligned} & \frac{c}{P'(q(c)) [2P'(q(c)) + q(c)P''(q(c))]} \\ & \times \left\{ -c [P'(q(c)) + q(c)P''(q(c))] + q(c)P'(q(c)) [2P'(q(c)) + q(c)P''(q(c))] \right\} \\ & \geq 0. \end{aligned}$$

It can easily be verified that this inequality is implied by our assumption on demand, $P'(q) + qP''(q) \leq 0$. Hence, $S(\theta)$ is increasing in θ .

Step 2. We now show that a firm's book value,

$$b(\theta) = n(\theta)r + n(\theta)\alpha c(\theta)q(c(\theta)),$$

is increasing in θ . Since $n'(\theta) \geq 0$ (with a strict inequality if and only if $\theta \geq \tilde{\theta}$), it suffices to show that

$$\frac{d}{d\theta} \{n(\theta)c(\theta)q(c(\theta))\} > 0$$

for $\theta \geq \tilde{\theta}$. This inequality can be rewritten as

$$\frac{d}{d\theta} \left\{ S(\theta) \left(\frac{c(\theta)}{P(q(c(\theta)))} \right) \right\} > 0.$$

But $S(\theta)$ is increasing in θ , as we have shown in step 1. Moreover, from equation (22) in the proof of lemma 1, $c(\theta)/P(q(c(\theta)))$ is increasing in θ . Hence, the inequality does indeed hold.

Step 3. Finally, we show that a firm's market value,

$$m(\theta) = n(\theta)P(q(c(\theta)))q(c(\theta)) - n(\theta)(1 - \alpha)c(\theta)q(c(\theta)),$$

is increasing in θ . It is immediate to see that $m(\theta)$ is constant for $\theta \leq \tilde{\theta}$. We need to show that $m(\theta)$ is strictly increasing in θ for $\theta \geq \tilde{\theta}$. We can rewrite the market value as the sum of the firm's net profit and its book value:

$$m(\theta) = n(\theta) \{ [P(q(c(\theta))) - c(\theta)] q(c(\theta)) - r \} + b(\theta).$$

Clearly, a high- θ can always replicate the choice of product lines by a small- θ firm, but at lower unit costs, and so a firm's net profit is increasing in θ . Moreover, $b'(\theta) > 0$ for $\theta \geq \tilde{\theta}$, as we have shown in step 2. Hence, the firm's market value is strictly increasing in θ for $\theta \geq \tilde{\theta}$. ■

Proof of proposition 4. Applying the implicit function theorem to the first-order condition for the optimal choice of the number of product lines, (8), we obtain $dc(\theta)/d\tau = -\Phi_\tau(c(\theta); \theta; \tau; t)/\Phi_c(c(\theta); \theta; \tau; t)$, where Φ_s denotes the partial derivative of Φ with respect to $s \in \{c, \tau\}$. Since the first-order condition defines a profit maximum, $\Phi_c(c(\theta); \theta; \tau; t) < 0$, and so the sign of $dc(\theta)/d\tau$ is equal to the sign of $\Phi_\tau(c(\theta); \theta; \tau; t)$. We have

$$\begin{aligned} \Phi_\tau(c(\theta); \theta; \tau; t) &= -c(\theta)q(\tau c(\theta) + t) - \frac{dr}{d\tau} - \frac{c(\theta)}{\theta}q(\tau c(\theta) + t) - \frac{\tau [c(\theta)]^2}{\theta}q(\tau c(\theta) + t) \\ &= -c(\theta)q(\tau c(\theta) + t) - \frac{dr}{d\tau} + [\pi(c(\theta)) - r] \left(\frac{q(\tau c(\theta) + t)}{q(c(\theta)) + \tau q(\tau c(\theta) + t)} \right) \\ &\quad \times \left\{ \frac{-\tau q'(\tau c(\theta) + t)}{q(\tau c(\theta) + t)} - 1 \right\}, \end{aligned}$$

where the second equality follows from (8). Taking the derivative with respect to θ , and collecting terms, we obtain

$$\begin{aligned} \frac{d\Phi_\tau(c(\theta); \theta; \tau; t)}{d\theta} &= c'(\theta) [\pi(c(\theta)) - r] \left\{ \frac{d}{dc} \left(\frac{-\tau c(\theta)q'(\tau c(\theta) + t)}{q(c(\theta)) + \tau q(\tau c(\theta) + t)} \right) \right. \\ &\quad \left. + \frac{d}{dc} \left(\frac{-q(\tau c(\theta) + t)}{q(c(\theta)) + \tau q(\tau c(\theta) + t)} \right) \right\}. \end{aligned} \quad (23)$$

The first term in curly brackets is strictly positive:

$$\frac{d}{dc} \left(\frac{-\tau c(\theta) q'(\tau c(\theta) + t)}{q(c(\theta)) + \tau q(\tau c(\theta) + t)} \right) > \frac{d}{dc} \left(\frac{-\tau q'(\tau c(\theta) + t)}{q(c(\theta)) + \tau q(\tau c(\theta) + t)} \right) > 0,$$

where the second inequality is the condition on demand from proposition 3. As regards the second term in curly brackets in equation (23),

$$\frac{d}{dc} \left(\frac{-q(\tau c(\theta) + t)}{q(c(\theta)) + \tau q(\tau c(\theta) + t)} \right) \geq 0$$

if

$$-\frac{\tau q'(\tau c(\theta) + t)}{q(\tau c(\theta) + t)} \geq -\frac{q'(c(\theta))}{q(c(\theta))}.$$

But this last inequality is implied by our condition on demand, $d[-q'(c)/q(c)]/dc > 0$, and the fact that $\tau \geq 1$. Hence, the curly bracket in equation (23) is strictly positive. Since the net profit per product line is strictly positive and $c'(\theta) > 0$, it follows that $d\Phi_\tau(c(\theta); \theta; \tau; t)/d\theta > 0$, and so $dc(\theta)/d\tau$ is strictly increasing in θ . Since the mass of product lines is fixed, the endogenous market price of a product line, r , will adjust so that there exists a threshold type $\hat{\theta} \in (\underline{\theta}, \bar{\theta})$ such that all firms with organizational capability $\theta \in [\underline{\theta}, \hat{\theta})$ respond to an increase in τ by selling product lines (and so $dc(\theta)/d\tau < 0$), whereas all firms with organizational capability $\theta \in (\hat{\theta}, \bar{\theta}]$ respond to an increase in τ by buying product lines (and so $dc(\theta)/d\tau > 0$). ■

Proof of proposition 5. Taking the derivative of domestic sales with respect to $\kappa \in \{t, \tau\}$ yields

$$\frac{dS(\theta)}{d\kappa} = \frac{dn(\theta)}{d\kappa} \left\{ P(q(c(\theta)))q(c(\theta)) + [P(q(c(\theta))) + q(c(\theta))P'(q(c(\theta)))] \frac{c(\theta)}{\theta} q'(c(\theta)) \right\}, \quad (24)$$

where $c(\theta; t) \equiv c_0 n(\theta)^{1/\theta}$. We need to show that the term in curly brackets is strictly positive.

Applying the implicit function theorem to the first-order condition for output choice, equation (7), we obtain

$$q'(c(\theta; t)) = \frac{1}{2P'(q(c(\theta; t))) + q(c(\theta; t))P''(q(c(\theta; t)))} > \frac{1}{P'(q(c(\theta; t)))}, \quad (25)$$

where the inequality follows from our assumption on demand, equation (2). This implies that the markup $P(q(c)) - c$ is decreasing in marginal cost c since

$$\frac{d[P(q(c)) - c]}{dc} = P'(q(c))q'(c) - 1 < 0,$$

where the first inequality follows from (25) and $P'(q(c)) < 0$. Hence, $P(q(c)) - c \geq P(q(c+t)) - (c+t)$, and so from the first-order condition for the optimal number of product lines, (8),

$$P(q(c(\theta))) \geq \left(1 + \frac{1}{\theta}\right) c(\theta). \quad (26)$$

The term in curly brackets in equation (24) can thus be re-written as

$$\begin{aligned}
& \left\{ P(q(c(\theta)))q(c(\theta)) + \frac{c(\theta)^2}{\theta}q'(c(\theta)) \right\} \\
\geq & \left\{ \frac{P(q(c(\theta)))q(c(\theta))P'(q(c(\theta))) + c(\theta)^2/\theta}{P'(q(c(\theta)))} \right\} \\
= & \left\{ \frac{-P(q(c(\theta))) [P(q(c(\theta))) - c(\theta)] + c(\theta)^2/\theta}{P'(q(c(\theta)))} \right\} \\
\geq & \left\{ \frac{-(1 + 1/\theta)c(\theta)^2/\theta + c(\theta)^2/\theta}{P'(q(c(\theta)))} \right\} \\
> & 0,
\end{aligned}$$

where the first inequality follows from equation (25), the equality follows from the first-order condition for the optimal choice of quantity, equation (7), and the second inequality follows from equation (26). Hence, $dS(\theta)/d\kappa$, $\kappa \in \{t, \tau\}$ has the same sign as $dn(\theta)/d\kappa$. The sign of $dn(\theta)/d\kappa$ then follows from propositions 3 and 4. ■

Proof of lemma 3. Suppose a firm with organizational capability θ chose not to sell in the foreign market. Then, the first-order condition for the optimal number of product lines is given by

$$\left[(a - c(\theta))^2 - r \right] - \frac{2c(\theta)}{\theta} (a - c(\theta)) = 0,$$

which is the same as (6), but applied to linear demand. From this first-order condition, the firm's induced marginal cost $c(\theta)$ would satisfy $c(\theta) \leq a\theta/(2 + \theta) \leq a\bar{\theta}/(2 + \bar{\theta})$, where the first inequality is strict if $r > 0$. Hence, if $t < 2a/(2 + \bar{\theta})$, the firm could increase its profit by selling in the foreign market, even without changing its number of product lines. ■

Proof of lemma 4. The first step consists in showing that $\frac{d}{dn} [nc_i(n; \theta)]|_{n=n_i(\theta)}$ is positive and strictly increasing in θ . To see this, note that

$$\begin{aligned}
\frac{d}{dn} n_i(\theta)c_i(n_i(\theta); \theta)|_{n=n(\theta)} &= \frac{d}{dn} c_0 [n]^{(1+\theta)/\theta} \Big|_{n=n_i(\theta)} \\
&= \left(\frac{1 + \theta}{\theta} \right) c_0 [n_i(\theta)]^{1/\theta} \\
&= \left(\frac{1 + \theta}{\theta} \right) c_i(\theta) \\
&> 0.
\end{aligned}$$

The second step consists in showing that $(1 + \theta)c(\theta)/\theta$ is strictly increasing in θ . We have

$$\frac{d}{d\theta} \left(\frac{1 + \theta}{\theta} \right) c_i(\theta) = \left(\frac{1 + \theta}{\theta} \right) c'(\theta) - \frac{c(\theta)}{\theta^2}.$$

Using equation (13), it can easily be seen that $c'(\theta) > \theta^{-1}(1 + \theta)^{-1}c(\theta)$. The claim then follows. We have thus shown that $\frac{d}{dn} [nc_i(n; \theta)]|_{n=n_i(\theta)}$ is positive and strictly increasing in θ .

The next step consists in showing that $\int \frac{d}{dn} [nc_i(n; \theta)]|_{n=n_i(\theta)} \Delta n_i(\theta) dG(\theta) < 0$. But this follows immediately from the following observations: (i) $\frac{d}{dn} [nc_i(n; \theta)]|_{n=n_i(\theta)}$ is positive and strictly increasing in θ , (ii) $\Delta n_i(\theta) > 0$ for $\theta < \hat{\theta}$ and $\Delta n_i(\theta) < 0$ for $\theta > \hat{\theta}$, and (iii) $\int \Delta n_i(\theta) dG(\theta) = 0$.

The final step consists in showing that $\Delta a_i < 0$ for each country i . But this follows immediately from the previous results and the equilibrium condition for a_i , equation (14). ■ **Proof of proposition 6.** We need to show that $dc(\theta)/dt$ is positive for high- θ (i.e., high- c) firms and negative for low- θ (i.e., low- c) firms. Under symmetric tariffs, the first-order condition (12) can be rewritten as

$$\begin{aligned} \Omega(c(\theta); \theta; t) &\equiv \left\{ (a - c(\theta))^2 + (a - t - c(\theta))^2 - r \right\} \\ &\quad - \frac{2c(\theta)}{\theta} \{(a - c(\theta)) + (a - t - c(\theta))\} \\ &= 0, \end{aligned} \tag{27}$$

Applying the implicit function theorem to this equation, we obtain

$$\frac{dc(\theta)}{dt} = -\frac{\Omega_t(c(\theta); \theta; t)}{\Omega_c(c(\theta); \theta; t)},$$

where the subscript $s \in \{t, c\}$ indicates the partial derivative with respect to variable s . Note that $\Omega_c(c(\theta); \theta; t) < 0$ since $\Omega(c(\theta); \theta; t) = 0$ is a profit maximum. Consequently, the sign of $dc(\theta)/dt$ is equal to the sign of $\Omega_t(c(\theta); \theta; t)$. Market clearing for product lines requires that some firms sell product lines while others purchase product lines, and so the sign of $\Omega_t(c(\theta); \theta; t)$ will vary with θ . In the following, we will show that $d\Omega_t(c(\theta); \theta; t)/d\theta > 0$.

Taking the partial derivative of $\Omega(c(\theta); \theta; t)$, as defined by equation (27), with respect to the cost parameter t , yields

$$\begin{aligned} \Omega_t(c(\theta); \theta; t) &= 2 \left\{ (a - c(\theta)) + (a - t - c(\theta)) - \frac{2c(\theta)}{\theta} \right\} \frac{da}{dt} \\ &\quad - 2(a - t - c(\theta)) + \frac{2c(\theta)}{\theta} - \frac{dr}{dt}. \end{aligned} \tag{28}$$

From the first-order condition (27),

$$\frac{2c(\theta)}{\theta} = \frac{(a - c(\theta))^2 + (a - t - c(\theta))^2 - r}{(a - c(\theta)) + (a - t - c(\theta))}.$$

Inserting this expression into equation (28) and simplifying, we obtain

$$\Omega_t(c(\theta); \theta; t) = \left\{ \frac{2(a - c(\theta))(a - t - c(\theta)) + r}{(a - c(\theta)) + (a - t - c(\theta))} \right\} \left[2\frac{da}{dt} - 1 \right] + t - \frac{dr}{dt}.$$

Observe that θ enters this equation only through the endogenous marginal cost $c(\theta)$. Hence,

$$\begin{aligned} \frac{d\Omega_t(c(\theta); \theta; t)}{d\theta} &= \frac{d}{dc} \left\{ \frac{2(a - c(\theta))(a - t - c(\theta)) + r}{(a - c(\theta)) + (a - t - c(\theta))} \right\} \left[2 \frac{da}{dt} - 1 \right] \frac{dc(\theta)}{d\theta} \\ &= -2 \left\{ \frac{[(a - c(\theta))^2 + (a - t - c(\theta))^2] - r}{[(a - c(\theta)) + (a - t - c(\theta))]^2} \right\} \left[2 \frac{da}{dt} - 1 \right] \frac{dc(\theta)}{d\theta}. \end{aligned}$$

From the first-order condition (27), the expression in curly brackets is strictly positive. Since $dc(\theta)/d\theta > 0$, the sign of $d\Omega_t(c(\theta); \theta; t)/d\theta$ is thus equal to the sign of $[1 - 2da/dt]$.

We claim that $da/dt < 1/2$. To see this, suppose first that $da/dt = 1/2$. Then, $d\Omega_t(c(\theta); \theta; t)/d\theta = 0$, and so three cases may arise: (i) $dc(\theta)/dt > 0$ for all θ , (ii) $dc(\theta)/dt < 0$ for all θ , or else (iii) $dc(\theta)/dt = 0$ for all θ . But cases (i) and (ii) cannot occur since there is a fixed number of product lines. Hence, we case (iii) must apply: $dc(\theta)/dt = 0$ for all θ ; that is, there is no trade in product lines. But then, from equation (14), $da/dt = \sigma N/[1 + 2\sigma N] < 1/2$. A contradiction. Next, suppose that $da/dt > 1/2$. Then, $d\Omega_t(c(\theta); \theta; t)/d\theta < 0$. Hence, there exists a threshold type $\hat{\theta} \in (\underline{\theta}, \bar{\theta})$ such that – following a small *increase* in t – all firms with $\theta < \hat{\theta}$ purchase product lines (and so $dc(\theta)/d\theta < 0$) while all firms with $\theta > \hat{\theta}$ sell product lines (and so $dc(\theta)/d\theta > 0$). From lemma 4, it follows that this “reshuffling” of product lines reduces the endogenous demand intercept a . From (14), the direct effect of an increase in t on a , holding $n(\theta)$ fixed, satisfies $\partial a/\partial t < 1/2$. Hence, the total effect of a small increase in t on a satisfies $da/dt < 1/2$. A contradiction. We have thus shown that $da/dt < 1/2$, and so there exists a threshold type $\hat{\theta}$, such that – in response to a small increase in t – all firms with $\theta < \hat{\theta}$ sell product lines while all firms with $\theta > \hat{\theta}$ acquire product lines. The reverse conclusion holds if $dt < 0$. ■

Proof of proposition 8. We need to show that there exists a $\tilde{\theta} \in [\underline{\theta}, \bar{\theta}]$ such that $dc(\theta)/dt$ is positive for $\theta > \tilde{\theta}$ and negative for $\theta < \tilde{\theta}$. As shown in the proof of proposition 6, the sign of $dc(\theta)/dt$ is equal to the sign of $\Omega_t(c(\theta); \theta; t)$, where

$$\begin{aligned} \Omega_t(c(\theta); \theta; t) &= 2 \left\{ (a - c(\theta)) + (a - t - c(\theta)) - \frac{2c(\theta)}{\theta} \right\} \frac{da}{dt} \\ &\quad - 2(a - t - c(\theta)) + \frac{2c(\theta)}{\theta} \end{aligned}$$

since $dr/dt = 0$ in the long run. Using the same steps as in the proof of proposition 6,

$$\Omega_t(c(\theta); \theta; t) = \left\{ \frac{2(a - c(\theta))(a - t - c(\theta)) + r}{(a - c(\theta)) + (a - t - c(\theta))} \right\} \left[2 \frac{da}{dt} - 1 \right] + t,$$

and

$$\frac{d\Omega_t(c(\theta); \theta; t)}{d\theta} = -2 \left\{ \frac{[(a - c(\theta))^2 + (a - t - c(\theta))^2] - r}{[(a - c(\theta)) + (a - t - c(\theta))]^2} \right\} \left[2 \frac{da}{dt} - 1 \right] \frac{dc(\theta)}{d\theta}. \quad (29)$$

We now claim that $da/dt < 1/2$ in the long run. To see this, suppose otherwise that $da/dt \geq 1/2$. Then, the profit of each firm of type θ would strictly increase following a small

increase in t , even holding fixed the choice of the number of product lines, $n(\theta)$:

$$\frac{d}{dt} \{(a - c(\theta))^2 + (a - t - c(\theta))^2\} > 0 \text{ for all } \theta.$$

But this is inconsistent with free entry.

Since $da/dt < 1/2$, equation (29) implies that $d\Omega_t(c(\theta); \theta; t)/d\theta > 0$. Hence, the assertion of the proposition follows. ■

Proof of proposition 7. We need to show that $dc_1(\theta)/dt_{21}$ is negative for high- θ (i.e., high- c) firms and positive for low- θ (i.e., low- c) firms, while the opposite holds for $dc_2(\theta)/dt_{21}$. From the first-order condition (12), $\Omega^i(c_i(\theta); \theta; t_{12}, t_{21}) = 0$, and so

$$\frac{2c_i(\theta)}{\theta} = \frac{(a_i - c_i(\theta))^2 + (a_j - t_{ij} - c_i(\theta))^2 - r_i}{(a_i - c_i(\theta)) + (a_j - t_{ij} - c_i(\theta))}. \quad (30)$$

Applying the implicit function theorem to the first-order condition, we obtain

$$\frac{dc_i(\theta)}{dt_{21}} = -\frac{\Omega_{t_{21}}^i(c_i(\theta); \theta; t_{12}, t_{21})}{\Omega_c^i(c_i(\theta); \theta; t_{12}, t_{21})},$$

where the subscript $s \in \{t, c\}$ indicates the partial derivative with respect to variable s . Note that $\Omega_c^i(c_i(\theta); \theta; t_{12}, t_{21}) < 0$ since $\Omega^i(c_i(\theta); \theta; t_{12}, t_{21}) = 0$ is a profit maximum. Consequently, the sign of $dc_i(\theta)/dt_{21}$ is equal to the sign of $\Omega_{t_{21}}^i(c_i(\theta); \theta; t_{12}, t_{21})$. Market clearing for product lines requires that some firms sell product lines while others purchase product lines, and so the sign of $\Omega_{t_{21}}^i(c_i(\theta); \theta; t_{12}, t_{21})$ will vary with θ . In the following, we will show that $d\Omega_{t_{21}}^1(c_i(\theta); \theta; t_{12}, t_{21})/d\theta < 0$ and $d\Omega_{t_{21}}^2(c_i(\theta); \theta; t_{12}, t_{21})/d\theta > 0$.

Consider first country 1. Using the first-order condition (12) and initial symmetry between countries, we obtain

$$\begin{aligned} \Omega_{t_{21}}^1(c(\theta); \theta; t_{12}, t_{21}) &= \left[2(a - c(\theta)) - \frac{2c(\theta)}{\theta} \right] \left[\frac{da_1}{dt_{21}} + \frac{da_2}{dt_{21}} \right] - 2t \frac{da_2}{dt_{21}} - \frac{dr_1}{dt_{21}} \\ &= \left[2(a - c(\theta)) - \frac{(a - c(\theta))^2 + (a - t - c(\theta))^2}{(a - c(\theta)) + (a - t - c(\theta))} \right] \left[\frac{da_1}{dt_{21}} + \frac{da_2}{dt_{21}} \right] \\ &\quad - 2t \frac{da_2}{dt_{21}} - \frac{dr_1}{dt_{21}}, \end{aligned}$$

where the second equality follows from equation (30). Taking the partial derivative of this expression with respect to c , yields

$$\frac{d\Omega_{t_{21}}^1(c(\theta); \theta; t_{12}, t_{21})}{d\theta} = -2 \left\{ \frac{(a - c(\theta))^2 + (a - t - c(\theta))^2 - r}{[(a - c(\theta)) + (a - t - c(\theta))]^2} \right\} \left[\frac{da_1}{dt_{21}} + \frac{da_2}{dt_{21}} \right] \frac{dc(\theta)}{d\theta}.$$

From the first-order condition, the expression in curly brackets is strictly positive. Since $dc(\theta)/d\theta > 0$, the sign of $d\Omega_{t_{21}}^1(c(\theta); \theta; t_{12}, t_{21})/d\theta$ is thus equal to the sign of $-[da_1/dt_{21} + da_2/dt_{21}]$.

Consider now country 2. We have

$$\begin{aligned}\Omega_{t_{21}}^2(c(\theta); \theta; t_{12}, t_{21}) &= \left[2(a - c(\theta)) - \frac{2c(\theta)}{\theta}\right] \left[\frac{da_1}{dt_{21}} + \frac{da_2}{dt_{21}} - 1\right] + 2t \left[1 - \frac{da_1}{dt_{21}}\right] - \frac{dr_2}{dt_{21}} \\ &= \left[2(a - c(\theta)) - \frac{(a - c(\theta))^2 + (a - t - c(\theta))^2}{(a - c(\theta)) + (a - t - c(\theta))}\right] \left[\frac{da_1}{dt_{21}} + \frac{da_2}{dt_{21}} - 1\right] \\ &\quad + 2t \left[1 - \frac{da_1}{dt_{21}}\right] - \frac{dr_2}{dt_{21}},\end{aligned}$$

where the second equality follows again from equation (30). Taking the partial derivative of this expression with respect to c , yields

$$\frac{d\Omega_{t_{21}}^2(c(\theta); \theta; t_{12}, t_{21})}{d\theta} = -2 \left\{ \frac{(a - c(\theta))^2 + (a - t - c(\theta))^2 - r}{[(a - c(\theta)) + (a - t - c(\theta))]^2} \right\} \left[\frac{da_1}{dt_{21}} + \frac{da_2}{dt_{21}} - 1 \right] \frac{dc(\theta)}{d\theta}.$$

From the first-order condition, the expression in curly brackets is strictly positive. Since $dc(\theta)/d\theta > 0$, the sign of $d\Omega_{t_{21}}^2(c(\theta); \theta; t_{12}, t_{21})/d\theta$ is thus equal to the sign of $[1 - da_1/dt_{21} - da_2/dt_{21}]$.

We claim that $0 < da_1/dt_{21} + da_2/dt_{21} < 1$. To see this, suppose first that $da_1/dt_{21} + da_2/dt_{21} \geq 1$. Then, $d\Omega_{t_{21}}^1(c(\theta); \theta; t_{12}, t_{21})/d\theta < 0$ and $d\Omega_{t_{21}}^1(c(\theta); \theta; t_{12}, t_{21})/d\theta \leq 0$. Hence, there exists a threshold type $\hat{\theta}_1 \in (\underline{\theta}, \bar{\theta})$ in country 1 such that firms of type $\theta > \hat{\theta}_1$ in country 1 will sell product lines to firms of type $\theta < \hat{\theta}_1$. In country 2, either $n_2(\theta)$ remains unchanged, namely if $da_1/dt_{21} + da_2/dt_{21} = 1$, or else there also exists a threshold type $\hat{\theta}_2 \in (\underline{\theta}, \bar{\theta})$ such that firms of type $\theta > \hat{\theta}_2$ in country 2 will sell product lines to firms of type $\theta < \hat{\theta}_2$. From lemma 4, it follows that this “reshuffling” of product lines reduces the endogenous demand intercepts a_1 and a_2 . Moreover, from (14), the “direct” effect of an increase in t_{21} on the demand intercepts satisfies $\partial a_1/\partial t_{21} < 1/2$ and $\partial a_2/\partial t_{21} = 0$. It follows that the total effect of a small increase in t_{21} on the demand intercepts satisfies $da_1/dt_{21} + da_2/dt_{21} < 1$. A contradiction. A similar argument can be used to show that $da_1/dt_{21} + da_2/dt_{21} \leq 0$ leads to a contradiction. ■

Proof of proposition 9. We need to show that there exist thresholds $\hat{\theta}_1 \in [\underline{\theta}, \bar{\theta}]$ and $\hat{\theta}_2 \in [\underline{\theta}, \bar{\theta}]$ such that $dc_1(\theta)/dt_{21}$ is negative for $\theta > \hat{\theta}_1$ and positive for $\theta < \hat{\theta}_1$, while the opposite holds for $dc_2(\theta)/dt_{21}$. As shown in the proof of proposition 7, the sign of $dc_i(\theta)/dt_{21}$ is equal to the sign of $\Omega_{t_{21}}^i(c_i(\theta); \theta; t_{12}, t_{21})$, where

$$\begin{aligned}\Omega_{t_{21}}^1(c(\theta); \theta; t_{12}, t_{21}) &= \left[2(a - c(\theta)) - \frac{(a - c(\theta))^2 + (a - t - c(\theta))^2}{(a - c(\theta)) + (a - t - c(\theta))}\right] \left[\frac{da_1}{dt_{21}} + \frac{da_2}{dt_{21}}\right] \\ &\quad - 2t \frac{da_2}{dt_{21}},\end{aligned}$$

and

$$\begin{aligned}\Omega_{t_{21}}^2(c(\theta); \theta; t_{12}, t_{21}) &= \left[2(a - c(\theta)) - \frac{(a - c(\theta))^2 + (a - t - c(\theta))^2}{(a - c(\theta)) + (a - t - c(\theta))}\right] \left[\frac{da_1}{dt_{21}} + \frac{da_2}{dt_{21}} - 1\right] \\ &\quad + 2t \left[1 - \frac{da_1}{dt_{21}}\right],\end{aligned}$$

since r is fixed in the long run. As we have shown in the proof of proposition 7,

$$\frac{d\Omega_{t_{21}}^1(c(\theta); \theta; t_{12}, t_{21})}{d\theta} = -2 \left\{ \frac{(a - c(\theta))^2 + (a - t - c(\theta))^2 - r}{[(a - c(\theta)) + (a - t - c(\theta))]^2} \right\} \left[\frac{da_1}{dt_{21}} + \frac{da_2}{dt_{21}} \right] \frac{dc(\theta)}{d\theta}.$$

and

$$\frac{d\Omega_{t_{21}}^2(c(\theta); \theta; t_{12}, t_{21})}{d\theta} = -2 \left\{ \frac{(a - c(\theta))^2 + (a - t - c(\theta))^2 - r}{[(a - c(\theta)) + (a - t - c(\theta))]^2} \right\} \left[\frac{da_1}{dt_{21}} + \frac{da_2}{dt_{21}} - 1 \right] \frac{dc(\theta)}{d\theta}.$$

We now claim that $da_1/dt_{21} + da_2/dt_{21} < 1$ in the long run. To see this, suppose otherwise that $da_1/dt_{21} + da_2/dt_{21} \geq 1$. Consider the change in the profit per product line of a country-1 firm with marginal cost $c(\theta)$:

$$\frac{d[\pi_{11}(c(\theta)) + \pi_{12}(c(\theta))]}{dt_{21}} = 2(a - c(\theta)) \left[\frac{da_1}{dt_{21}} + \frac{da_2}{dt_{21}} \right] - 2t \frac{da_2}{dt_{21}}.$$

Free entry implies that this expression cannot be strictly positive for all values of $c(\theta)$. Hence, $da_2/dt_{21} > 0$. Consider now change in the profit per product line of a country-2 firm with marginal cost $c(\theta)$:

$$\frac{d[\pi_{22}(c(\theta)) + \pi_{21}(c(\theta))]}{dt_{21}} = 2(a - t - c(\theta)) \left[\frac{da_1}{dt_{21}} + \frac{da_2}{dt_{21}} - 1 \right] + 2t \frac{da_2}{dt_{21}}.$$

Free entry implies that this expression cannot be strictly positive for all values of $c(\theta) \leq a - t$ (which holds by assumption). Hence, $da_2/dt_{21} \leq 0$. A contradiction.

We now claim that $da_1/dt_{21} + da_2/dt_{21} > 0$ in the long run. To see this, suppose otherwise that $da_1/dt_{21} + da_2/dt_{21} \leq 0$. Free entry implies that $d[\pi_{11}(c(\theta)) + \pi_{12}(c(\theta))]/dt_{21}$ cannot be strictly negative for all values of $c(\theta)$. Hence, $da_2/dt_{21} \leq 0$. Free entry also implies that $d[\pi_{22}(c(\theta)) + \pi_{21}(c(\theta))]/dt_{21}$ cannot be strictly negative for all values of $c(\theta)$. Hence, $da_2/dt_{21} > 0$. A contradiction.

Since $0 < da_1/dt_{21} + da_2/dt_{21} < 1$, it then follows that $d\Omega_{t_{21}}^1(c(\theta); \theta; t_{12}, t_{21})/d\theta < 0 < d\Omega_{t_{21}}^2(c(\theta); \theta; t_{12}, t_{21})/d\theta$. ■