

INTERNATIONAL SEIGNIORAGE PAYMENTS

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What are the "liquidity services" provided by "over-priced" assets? What determines the choice of the international currency? How do international seigniorage payments affect the choice of monetary policies? What are the optimal inflation rates in the global economy? Does a country gain when other use its currency? These questions are analyzed here in a model in which demand uncertainty (taste shocks) and sequential trade are key. I apply the analysis to the recent policy discussion concerning the accumulation of foreign debt by the US. It is argued that the recent experience of stable demand in the US may explain why: (a) there is a sizeable excess returns of gross US assets over gross US liabilities, (b) the US is cheap relative to the prediction of income-price regressions, (c) most of US liabilities are in dollar terms and (d) a common currency increases trade. In the steady state the stable demand country (the US) gets seigniorage payments from foreigners with less stable demand. But this does not mean that the US gains from having an international currency.

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1. INTRODUCTION

"In the next decade the U.S. economy will undergo a fundamental restructuring: from spending more than the national income, the economy must turn to earning its way, servicing debts by trade surpluses". This was not written today. It is the opening sentence to Dornbusch, Krugman and Park (1989).² One of the main prediction in their paper is that "Inevitably, the U.S. will begin to balance its trade in manufactured goods - indeed, the legacy of debt from the 1980s will insure that the U.S. runs unprecedented surpluses in its manufactures trade by the end of this century". This has not happened. During the period 1987 - 2006 the manufacturing sector real output increased at a lower rate than real GDP (67% versus 79%) and the US deficit has increased.

Maybe the US debt problem is not as severe as standard models suggests. Here I use a non-standard model to explore this possibility focusing on the return on US liabilities and seigniorage payments.

Recently there has been a large increase in US liabilities. The total liabilities of the US government held by foreigners (including cash) were 4.5% of GDP in 1985 and 15.6% of GDP in 2004.³ Foreign holdings of US equities went up from 2.6% of US GDP in 1984 to 16.2% in 2004. Foreign holdings of long-term corporate debt went up from 0.8% of US GDP in 1984 to 12.2% in 2004. The net position in long-term

² I am indebted to Bob Driskill for this reference.

³ The data are from the Bureau of Economic Analysis international investment position. http://www.bea.doc.gov/bea/di/intinv03_t2.xls

securities of US residents went down from -374 billion dollars in 1994 (5.2% of GDP) to -2334 billion dollars in 2004 (20% of GDP).⁴

The average long run rate of return on US assets is critical in assessing the consequences of the accumulation of US assets by foreigners. One possible scenario may assume that these assets earn the "world interest rate". An alternative scenario may assume that the rate of return on US assets held by foreigners is lower than the "world rate" and in spite of that foreigners are willing to hold US obligations in a long run steady state equilibrium. This paper illustrates the possibility of the alternative scenario.

In the alternative scenario foreigners pay seigniorage to the US. The standard definition of seigniorage is revenues generated by the creation of high-powered money. A broader definition may include other assets that are overpriced relative to standard asset pricing models (Lucas [1978, 1982]). For example, McGrattan and Prescott (2003) argue that short term US government securities provide liquidity and are therefore overpriced. They use the rate of return on longer terms bonds as their proxy for the risk free rate and estimate it at about 4%. Mehra and Prescott (1985) estimated a less than 1% real rate of return on shorter term US treasury bills. This suggests that agents who hold US treasury bills pay seigniorage to the US government of about 3% per year.

Seigniorage may be paid on other assets. In a recent article Gourinchas and Rey (GR, 2005) found strong evidence of sizeable excess returns of gross US assets over gross US liabilities. They found that

⁴ See Tables 1 and 3 in Report (2005).

during the period 1952 - 2004 the average annualized real rate of return on gross liabilities was 3.61% while the average annualized real rate of return on gross assets was 5.72%. The difference of 2.11% is considerable. This difference is especially large when looking at the post Bretton-Woods period: 1973 - 2004. The post Bretton-Woods average asset return is 6.82% while the corresponding total liability return is only 3.50%. The excess return in the post Bretton Woods era is thus 3.32%.

In Appendix A I provide a conceptual framework and some preliminary calculations of the seigniorage that the US may expect to receive from foreigners. The calculations assume risk neutrality, expected excess return equal to the post Bretton-Woods average (3.32%) and quantities at their 2004 levels. If we adopt the narrow definition of seigniorage (payments on cash) we get roughly 0.2% of US GDP. If we adopt a broad definition we get 2% of US GDP. The broad definition includes payments both to the US government and to US private agents. The US government may expect to collect about 0.7% of US GDP from securities and cash held by foreigners.

The question of the appropriate definition of seigniorage revenues is closely related to the question of the appropriate definition of money: M1, M2 or a broader definition. We may say that an asset provides some "liquidity services" if its rate of return is less than the prediction of standard asset pricing models. Indeed Barnett et al. (1992) construct a weighted monetary aggregate along these lines. According to this view, the GR findings suggest that on average US assets provide more "liquidity services" than other assets.

Of-course saying that an asset provides "liquidity services" is not an explanation of why agents are willing to hold it. It is just a name for the unexplained residual. Here I attempt an explanation that uses the following main assumptions: heterogeneous risk neutral agents, demand uncertainty (taste shocks) and sequential trade.

In the sequential trade model I use, uncertainty about demand causes price dispersion. This allows for the distinction between the rate of return on the asset and its "liquidity". The rate of return depends on the asset and not on the individual who holds it. Liquidity is an individual specific attribute. It depends on the probability that you will be able to use the asset to buy at the low price when you want to consume.

In the model sellers choose both the price and the currency that they are willing to accept. They may choose a low price or a high price and may state that they are willing to accept the equivalent amount in terms of any currency or just in terms of a particular currency. Since in the equilibrium we study high price sellers accept any currency, the liquidity of an asset depends on the probability that you find the good at the low price as well as the probability that the low price seller will accept the asset. An individual who typically buys in the high demand state, has a low chance of finding the good at the low price and is therefore willing to pay only a relatively small premium on the generally accepted currency.

For the sake of concreteness, I assume two types of agents: Japanese and Americans. Both are risk neutral. The demand of the Japanese is erratic and plays the role of "aggregate demand shifter". The demand of the Americans is stable. The Americans are more likely to

buy at the low aggregate demand state. This means that they have a higher chance of finding the good at the low price and therefore they are willing to pay a relatively high premium for the generally accepted asset. I focus on an equilibrium in which the yen has a lower inflation rate but is less "liquid" than the dollar. For the Japanese the liquidity of the dollar exactly compensates for its lower rate of return and in equilibrium they hold both currencies. Since the Americans have a higher probability of finding the good at a low price they are willing to pay a higher "liquidity premium" for the dollar and therefore they accept dollars only.

As was mentioned above, price dispersion is required for our definition of "liquidity". In Prescott's (1975) "hotels" model there is price dispersion. Versions of the Prescott model have been studied by, among others, Bryant (1980), Rotemberg and Summers (1990), Dana (1998) and Deneckere and Peck (2005). Here I use a flexible price version of the model: The uncertain and sequential trade (UST) model in Eden (1990, 1994) and Lucas and Woodford (1993).

The UST model uses the survival probability function of demand (one minus the cumulative distribution) to define a sequence of Walrasian markets. The number of markets that open for trade is an increasing function of the realization of demand. Only one market opens if the lowest possible realization of demand occurs. Additional markets open if demand is higher. At each stage of the trading process sellers know that they can sell at the "current" market but are not sure whether additional markets will open or not. In equilibrium they are indifferent between selling at the "current market" price to betting on the event

that additional markets will open and they will be able to sell at a higher price.

Using the UST model we get the following main results.

- (a) Demand Uncertainty reduces welfare.
- (b) The US may suffer from trade with the unstable demand country.
- (c) The foreign country may benefit from full or partial dollarization.
- (d) There is an "inflation bias" in the US arising from an incentive to raise seigniorage revenues from foreigners and minimize trade.
- (e) Efficiency requires national monies with differential and low inflation rates.

On the positive side we get:

- (f) The US is relatively cheap.
- (g) US liabilities are in dollar terms (Tille [2003] and Lane and Milesi-Ferretti [2005]).
- (h) A common currency increases trade (Rose and van Wincoop [2001], Frankel and Rose [2002]).
- (i) The dollar rate of inflation is higher than the yen rate of inflation.
- (j) Japanese pay seigniorage to the US.

2. THE MODEL

I consider a single good overlapping generations model. There are two countries. The demand in the home country (US) is stable and the demand in the foreign country (Japan) is unstable. Otherwise the countries are symmetric. I start with the case of autarky assuming a single asset called money.

Autarky in the foreign country:

A new generation is born each period. Individuals live for two-periods. They work in the first period of their life and if they want they consume in the second period.

The representative agent's utility function is random and depends on the realization of a "taste shock" he experiences in the second period of his life. The random utility function for an agent born at t is: $\theta_{t+1}\beta c_{t+1} - v(L_t)$, where L_t is the amount of first period labor, c_{t+1} is the amount of second period consumption, β is a discount factor and θ_{t+1} is a random variable that can take the realizations 1 with probability π and 0 otherwise. It is assumed that one unit of labor produces one unit of output. For simplicity I assume a quadratic cost function:

$$v(L) = (\frac{1}{2})L^2.$$

The realization of the taste shock θ is known only after production has been made. Output produced will be sold only when $\theta = 1$. It is assumed that when the old generation experiences $\theta = 0$ they transfer their balances to the young generation as an accidental bequest and do not derive utility from that bequest. This is the case analyzed in Abel (1985). An alternative formulation may assume that agents derive utility from bequest as in Barro (1974), but the weight they assign to the utility of future generation is random. The main results will not change if this more general specification is employed.

Buyer h (an old agent) starts period t with M_t^h yen and gets in addition, a perfectly anticipated lump sum transfer of G_t yen. The average per-buyer beginning of period balances is: $M_t = (\frac{1}{N})\sum_{h=1}^N M_t^h$ and the average post transfer balances is: $M_{t+1} = G_t + M_t$ yen. The

deterministic rate of change in the money supply is: $M_{t+1}/M_t = 1 + \mu$. In what follows I assume a representative agent ($N = 1$).

The representative young agent born at time t takes the yen prices of the consumption good (P_t, P_{t+1}) and the yen amount of the transfer payment (G_{t+1}) as given. When at time t , the old generation experiences $\theta_t = 1$, he sells his output and gets $P_t L_t$ yen for it. When $\theta_t = 0$, he does not sell but get a bequest of M_{t+1} yen. If he supplies L_t units of labor (= output), his expected next period post transfer balances are:

$$(1) \quad B_{t+1} = \pi(P_t L_t + G_{t+1}) + (1 - \pi)(M_{t+1} + G_{t+1})$$

The worker will use these balances in the next period if $\theta_{t+1} = 1$. He therefore chooses L by solving:

$$(2) \quad \max_L -v(L_t) + \pi\beta B_{t+1}/P_{t+1} \\ = -v(L_t) + \pi^2\beta(P_t L_t + G_{t+1})/P_{t+1} + (1 - \pi)\pi\beta(M_{t+1} + G_{t+1})/P_{t+1} ,$$

where the equality uses (1). The first order condition for this problem is:

$$(3) \quad v'(L_t) = \pi^2\beta P_t/P_{t+1}$$

We may think of $\pi^2\beta P_t/P_{t+1}$ as the (expected discounted) real price or the real wage. The first order condition (3) says that the marginal cost must equal the real price. The term $\beta P_t/P_{t+1}$ on the right hand side of (3) is standard. The term π^2 plays a role because a unit produced yields utility to the producer only if it is sold (with probability π) and only

if he will want to consume (also with probability π). The probability of this joint event is π^2 . The real wage is therefore the standard $\beta P_t/P_{t+1}$ with probability π^2 and zero otherwise.

We require market clearing when $\theta_t = 1$. That is,

$$(4) \quad P_t L_t = M_t(1 + \mu).$$

I focus on an equilibrium in which inflation is constant and the price level is proportional to the post transfer money supply. I thus assume a normalized price p such that:

$$(5) \quad P_t = p M_t(1 + \mu)$$

Substituting (5) in the first order condition (3) leads to:

$$(6) \quad v'(L_t) = L = \pi^2 \beta P_t / P_{t+1} = \pi^2 \beta p M_t(1 + \mu) / p M_t(1 + \mu)^2 = \pi^2 \beta / (1 + \mu).$$

Thus by varying μ the monetary authorities can vary L .

With the risk of repetition I now set the problem in normalized magnitudes. This will become useful later when full integration is considered.

Normalized magnitudes are nominal magnitudes divided by the post transfer money supply, $M_t(1 + \mu)$. A normalized yen (NY) is $M_t(1 + \mu)$ regular yen and in a steady state its purchasing power does not change over time. The price of consumption is P_t yen per unit or $p = P_t / M_t(1 + \mu)$ normalized yen per unit. To think directly in terms of normalized yen note that when the price of a unit is say half of a

normalized yen it means that you have to pay half of the post-transfer money supply to get the unit. Since the money supply changes over time we must renormalize every period. A normalized yen (NY) in the current period that is carried to the next period is worth

$$\omega = M_t(1+\mu) / M_{t+1}(1+\mu) = (1 + \mu)^{-1} \text{ in terms of next period's NYs.}$$

It follows that a worker (young agent) who sells a unit for p NYs will have in the next period $p\omega$ NYs. The expected real wage conditional on selling is $p\omega Z$, where Z is the expected purchasing power of a normalized yen defined by:

$$(7) \quad Z = \pi/p.$$

When $\theta_t = 1$ the worker sells his output (L_t) and gets on average $(\omega p L_t)Z$ units of consumption in period $t+1$. In addition he gets a transfer payment of $\omega\mu$ (in terms of next period's normalized yen) that will buy on average $(\omega\mu)Z$ units. His expected consumption when $\theta_t = 1$ is therefore: $(\omega p L_t)Z + (\omega\mu)Z$ units. When $\theta_t = 0$ the worker does not sell his output but receives a bequest of ω in terms of next period's NY. In addition to the bequest he receives a transfer payment of $\omega\mu$ NY and his expected consumption when $\theta_t = 0$ is therefore: $\omega(1 + \mu)Z = Z$. The worker's maximization problem is therefore:

$$(8) \quad \max_L \pi\omega(pL + \mu)\beta Z + (1 - \pi)\beta Z - v(L).$$

The first order condition for (8) requires that the marginal cost equals the expected discounted real wage:

$$(9) \quad v'(L) = L = \beta\pi\omega pZ = \beta\pi^2\omega,$$

where the last equality uses (7). We require market clearing when demand is strictly positive ($\theta = 1$):

$$(10) \quad pL = 1.$$

Note that the equilibrium conditions (9) and (10) are the same as (4) and (6) but their derivation does not require algebra.

Steady state welfare is measured by:

$$(11) \quad \text{Welfare} = \beta\pi(1/p) - (\frac{1}{2})L^2.$$

Autarky in the home country:

There are no taste shocks in the home country and $\pi = 1$. Otherwise the two countries are symmetric. The home country uses dollars and ND denotes normalized dollars. Table 1 calculates the equilibrium magnitudes for different values of ω assuming $\pi = 1$ and $\beta = 1$.

Table 1: Autarky in the home country ($\pi = 1$; $\beta = 1$)

$\omega = 1/(1 + \mu)$	p	L	Welfare
1	1	1	0.5
0.90	1.111	0.90	0.495

Table 2 repeats the calculations for $\pi = 0.9$.

Table 2: Autarky in the foreign country ($\pi = 0.9$; $\beta = 1$)

$\omega = 1/(1 + \mu)$	p	L	Welfare
$1/\pi$	$1/\pi$	π	0.405
1	1.235	0.81	0.401
0.90	1.372	0.729	0.390

Note that labor supply is lower in the foreign country. This is because the uncertainty about demand leads to less than full capacity utilization and to a lower expected real wage.

I now turn to discuss efficiency. To maximize steady-state welfare a planner will solve:

$$(12) \quad \max \pi\beta L - v(L)$$

The first order condition for this problem is:

$$(13) \quad v'(L) = \pi\beta.$$

Since in equilibrium $v'(L) = \beta\pi^2\omega$ efficiency requires:

$$(14) \quad \beta\pi^2\omega = \pi\beta \text{ or } \omega = 1/\pi.$$

Thus, when $\pi < 1$, efficiency requires deflation. This result is similar to the well-known result by Friedman (1969) but here, as in other OG models, the optimal deflation rate does not depend on the discount factor. The argument for deflation is however, analogous to

Friedman's argument. When there is zero inflation there is a difference between the social and the private value of a unit produced. The social value of a unit produced is $\pi\beta$ because it will be consumed, by an old agent, with probability π . From the individual's point of view a unit produced yields utility only if he sells and only if he wants to consume. This joint event occurs with probability π^2 . Therefore when inflation is zero a unit produced is worth to the individual only $\beta\pi^2$ units of consumption. Deflation is required to correct for the difference between the social and the individual's point of view.

Since discounting does not play an important role in the analysis I assume, in what follows, $\beta = 1$.

3. A FULLY INTEGRATED WORLD ECONOMY

I now allow for trade between the two countries under the assumption of costless transportation (and travel) and a single world currency: The dollar.⁵ At the beginning of each period the buyers in the

⁵ A real version of this model that allows for transportation costs is in Eden (2005). Small transportation costs can be used to select among equilibria but otherwise do not affect the main results. In the real version of the model I also distinguish between the case in which goods must be displayed on location before the beginning of trade to the case in which orders are placed first and delivery occurs later. Here I focus on the second delivery to order case. We may think, for example, of the market for resorts. Buyers from all over the world may make reservations on the internet. Those who make early reservations may get relatively cheap vacations. Other examples may be trade in intermediate goods. Ethier (1979) and Sanyal and Jones (1982) emphasize the fact that much of international trade is in intermediate

home country get a transfer payment and as a result the world money supply grows at the rate of μ . Foreigners do not get a transfer payment and accumulate dollars only by selling output. This is important for our results.

After receiving the transfer payment the representative buyer in the home country holds m normalized dollars and the representative buyer in the foreign country holds $1 - m$ normalized dollar, where as before, a normalized dollar (ND) is the post transfer supply of dollars. I start with a steady state analysis in which m does not change over time.

Trade occurs sequentially. At the beginning of the period buyers who want to buy form a line. When $\theta = 0$, only US buyers want to consume and therefore only US buyers get in line. When $\theta = 1$ buyers from both countries get in line. The place in the line is determined by a lottery that treats all buyers symmetrically. Since the number of buyers is large I assume that any segment of the line represents the population of active buyers.

Active buyers arrive at the market place one by one according to their place in the line. They see all prices and choose to buy at the cheapest available price.

The amount of money that will be spent is m ND if only the home country buyers want to consume and 1 ND if all buyers want to consume. We say that the first m NDs buy in the first market at the price of p_1 ND per unit. If $\theta = 1$ an additional amount of $1 - m$ NDs will arrive, open the second market and buy at the price p_2 .

inputs and not in final goods. But for simplicity, I keep the assumption that there is one final good produced by labor only.

When aggregate demand is low and only one market opens the probability of buying at the first market price is unity. When demand is high and two markets open the probability of buying at the first market price is m (= the fraction of dollars that will buy in the first market). The expected purchasing power of a normalized dollar if exactly s markets open (z_s) is therefore:

$$(15) \quad z_1 = 1/p_1 \text{ and } z_2 = m/p_1 + (1-m)/p_2$$

The unconditional expected purchasing power of a normalized dollar is:

$$(16) \quad Z = (1 - \pi)z_1 + \pi z_2, \text{ for a home country buyer and}$$

$$Z^* = \pi z_2, \text{ for a foreign country buyer.}$$

Note that a buyer in the home country will buy regardless of the realization of θ and therefore Z is a weighted average of z_1 and z_2 . A foreign buyer will buy only if $\theta = 1$. In this case two markets will open and therefore Z^* is a weighted average between zero and z_2 .

Sellers (workers) take prices as given. They know that they can sell (in the first market) at the price p_1 with probability 1 and (in the second market) at the price p_2 with probability π . I use k_s to denote the supply of the home country seller to market s . The home country seller solves:

$$(17) \quad \max_{k_s} - v(k_1+k_2) \\ + (1 - \pi)\omega(p_1k_1 + \mu)Z + \pi\omega(p_1k_1 + p_2k_2 + \mu)Z.$$

The first term in (17) is the cost of producing $k_1 + k_2$ units. The last two terms are the expected consumption. When only one market opens the seller sells only k_1 units and his revenues is $p_1 k_1$ ND. In addition he gets a transfer payment of μ ND so his next period money balances are $\omega(p_1 k_1 + \mu)$ NDs. When both markets open the seller's revenues are $p_1 k_1 + p_2 k_2$ and his next period balances are $\omega(p_1 k_1 + p_2 k_2 + \mu)$. To convert next period's balances to expected consumption we multiply by Z .

The representative young agent in the foreign country solves:

$$(18) \quad \max_{k_1^*, k_2^*} -v(k_1^* + k_2^*) \\ + (1 - \pi)\omega[p_1 k_1^* + (1 - m)]Z^* + \pi\omega(p_1 k_1^* + p_2 k_2^*)Z^*$$

Note that the expected purchasing power function is different (Z^* instead of Z) and the foreign agent does not get a transfer payment from the government but may get a bequest.

It is convenient to use: $L = k_1 + k_2$ and $L^* = k_1^* + k_2^*$ for the supply of labor. In the real version of this model (in Eden [2005]) I show that when transportation is costly the home country sellers supply only to the first market. Motivated by this analysis I focus here on a steady state in which the home country seller supplies to the first market only ($L = k_1$) and the post transfer balances held by the buyer in the home country do not change over time and are given by:

$$(19) \quad m = \omega(p_1 L + \mu)$$

I now turn to describe the first order condition for the problems (17) and (18). The expected real revenue per unit is $\omega p_1 Z$ ($\omega p_1 Z^*$) if the unit is supplied to the first market and $\pi \omega p_2 Z$ ($\pi \omega p_2 Z^*$) if it is supplied to the second market. At the optimum the marginal cost ($v'[L] = L$) must equal the expected real wage:

$$(20) \quad L = \omega p_1 Z = \pi \omega p_2 Z; \quad L^* = \omega p_1 Z^* = \pi \omega p_2 Z^*$$

In addition to the first order conditions (20), steady state equilibrium requires the clearing of markets that open (that experience a strictly positive demand). Thus,

$$(21) \quad p_1(L + k_1^*) = m; \quad p_2(k_2^* = L^* - k_1^*) = 1 - m.$$

A steady state equilibrium is a solution $(L, L^*, k_1^*, p_1, p_2, m)$ to (19) - (21).

Claim 1: There exists a unique steady state equilibrium for the single currency world.

The proof of this and all other claims is in Appendix B.

Table 3 illustrates the steady state solutions for two values of μ . The last two columns are the steady state welfare in each country computed by: $W = c - (\frac{1}{2})L^2$, $W^* = \pi(L + L^* - c_2) - (\frac{1}{2})(L^*)^2$, where $c = (1 - \pi)c_1 + \pi c_2$, $c_1 = m/p_1$ and $c_2 = m[(m/p_1 + (1-m)/p_2)]$.

Table 3: The fully integrated single currency world ($\pi = 0.9$)

μ	m	L	L*	W	W*
0	0.501	0.955	0.855	0.456	0.447
0.05	0.501	0.912	0.817	0.498	0.404

Comparing Tables 3 and 1 reveals that when $\mu = 0$ both employment and welfare in the home country are higher under autarky. Buyers in the home country suffer from the price dispersion introduced by the foreigners because sometimes they cannot buy at the cheaper price. Note that imposing a moderate inflation tax works in the direction of compensating the home country.

Two currencies:

I now introduce an additional currency: the yen. I start by assuming that US sellers accept dollars only. This assumption will be justified in equilibrium. As before, in the steady state Americans hold a fraction m of the dollar money supply and Japanese hold a fraction $1 - m$ of the dollar money supply where these fractions are derived endogenously. In addition Japanese hold yen. Dollars promise a higher chance of buying in the first market and in this sense they are more liquid. Therefore Japanese sellers will accept both currencies only if the rate of inflation of the yen is lower than the rate of inflation of the dollar.

The pre-transfer supply of dollars at time t is M_t and the pre-transfer supply of yen is M_t^* . At the beginning of period t the home country buyer gets a lump sum transfer of μM_t dollars and the foreign

country buyer gets a lump sum transfer of $\mu^* M_t^*$ yen. The growth rates (μ, μ^*) are deterministic.

In the steady state, the yen and the dollar inflation rates are constant and there exist normalized dollar prices, p_s , and normalized yen prices p_s^* such that:

$$(22) \quad P_{st} = p_s M_t (1 + \mu); \quad P_{st}^* = p_s^* M_t^* (1 + \mu^*).$$

The dollar price of yen (e_t) is determined in a foreign exchange market that opens before the realization of the taste shock θ_t . I assume that the Japanese sellers accept both currencies in the second market but accept only dollars in the first market. The assumption that Japanese accept only dollars in the first market is not required for the main results.⁶ Since nothing happens between the selling of the goods and the opening of the foreign exchange market in the next period, we require:

⁶ This assumption can be motivated as follows. Since Americans do not hold yen, a seller who gets a yen offer will conclude that this must be a state of high demand. He may therefore not deliver at the low price claiming that he is stocked out and offer to deliver at the high price. It follows that a seller cannot commit in a credible time consistent manner to a low yen price. A rigid price alternative that allows the seller to commit to prices and quantities is more complicated but will not change the main results. The main results requires that the dollar has an advantage in buying at the low price. This advantage follows from the result that Americans sell in the low price market and accept dollars only.

$$(23) \quad P_{2t}^* = P_{2t}/e_{t+1} = p_2 M_t (1 + \mu) / e_{t+1}$$

This leads to:

$$(24) \quad P_{2t}^* / P_{2t-1}^* = (1 + \mu)(e_t / e_{t+1}) = 1 + \mu^* \text{ or } e_{t+1} / e_t = (1 + \mu) / (1 + \mu^*).$$

Thus as in standard models, the rate of growth of the exchange rate depends on the ratio of the money supplies growth rates. Note that

(24) implies:

$$(25) \quad \alpha = e_t M_t^* (1 + \mu^*) / M_t (1 + \mu) = e_{t-1} M_t^* / M_t.$$

Thus the dollar value of the yen supply is a constant fraction α of the dollar supply.

Taking the inflation of the dollar as given, the Japanese central bank determines α by an appropriate choice of the yen inflation rate (a higher α requires a lower yen inflation rate). It is convenient however to treat α as the policy choice variable and the yen inflation rate as an endogenous variable. An alternative that treats the yen inflation rate as the policy choice variable will make no difference for the analysis.

The expected purchasing power of a normalized dollar is given by (16). Since yen can buy goods in the second market only, the expected purchasing power of a normalized yen is:

$$(26) \quad X = 1/p_2^* \text{ for a home country buyer and} \\ X^* = \pi(1/p_2^*) \text{ for a foreign country buyer.}$$

The first order conditions (20) describe the labor supply choices under the assumption that sellers accept dollars only. Since yen are not strictly preferred to dollars these first order conditions still hold. Here we add conditions that justify the assumed choice of currencies. We require that the US sellers cannot benefit by selling in yen:

$$(27) \quad L = \omega p_1 Z = \pi \omega p_2 Z \geq \omega^* p_1^* X = \pi \omega^* p_2^* X$$

And we require that Japanese sellers are indifferent between dollars and yen:

$$(28) \quad L^* = \omega p_1 Z^* = \pi \omega p_2 Z^* = \pi \omega^* p_2^* X^*$$

Steady state equilibrium requires (19), the first order conditions (27) - (28) and the market clearing conditions

$$(29) \quad p_1(L + k_1^*) = m ; p_2(L^* - k_1^*) = 1 - m + \alpha,$$

where $\alpha = e_t M_t^* (1 + \mu^*) / M_t (1 + \mu)$ is the supply of yen in terms of normalized dollars. I require in addition that $0 \leq m, \alpha \leq 1$.

We now show (the proof is in Appendix B) the following Proposition.

Proposition 1: There exists a unique steady state equilibrium for the two currencies world with the following properties:

- (a) $L \geq L^*$ and $\mu^* \leq \mu$ with the inequalities being strict when $\pi < 1$;
- (b) An increase in α leads to an increase in m , and an increase in labor supplies in both countries;
- (c) US sellers strictly prefer dollars to the equivalent yen amount;
- (d) When $\mu = 0$ and $\alpha = 1$ the steady state equilibrium allocation solves the following planner's problem:

$$(30) \quad \max c - v(L) \quad \text{s.t.} \quad c + c^* = L + L^* ; \quad \pi c^* - v(L^*) \geq x.$$

The intuition is as follows. Foreign workers may not want to consume and therefore have less incentive to work. The yen inflation must be lower because lower price sellers do not accept it. When α increases foreign agents substitute yen for dollars and $1 - m$ goes down. As a result the dollar promises a higher chance of buying in the first market and a lower yen inflation is required to compensate for the difference in liquidity. The lower yen inflation leads to a higher expected real wage in Japan. The expected real wage in the US also goes up as a result of the increase in m and the increase in the probability that US buyers will buy at the cheaper price. The increase in the expected real wage leads to an increase in labor supply in both countries. The "liquidity premium" on the dollar is sufficient to make Japanese sellers accept both currencies. US sellers are willing to pay a higher "liquidity premium" on holding dollars because they buy in both states and the advantage of the dollar is larger in the low demand state

(where the probability of buying at the cheaper price is unity for the dollar and zero for the yen).

Note that since the sellers are happy with their choice of currencies there will be no transactions in the foreign exchange market.

The planner's problem (30) is that of maximizing the welfare in the home country subject to a worldwide resource constraint and a requirement that the level of welfare in the foreign country is given. Similar to (13), the first order conditions for this problem are:

$$(31) \quad v'(L) = 1 ; v'(L^*) = \pi.$$

Since $\alpha = 1$ means autarky, (d) says that an efficient outcome can be obtained under autarky with an appropriate choice of monetary policies. The choice $\mu = 0$ insures efficiency in the home country. It turns out that to support $\alpha = 1$ the foreign country must choose the efficient rate of inflation $\omega^* = 1/\pi$ or $\mu^* = \pi - 1$.

Table 4 computes the equilibrium magnitudes for various μ and α . The first four rows assume $\mu = 0$ and allow for four different values of α ($\alpha = 0, 0.1, 0.8, 1$). Note that $\alpha > 0$ requires deflation of the yen ($\mu^* < 0$) and an increase in α requires more deflation (lower μ^*). The intuition is as follows. An increase in α reduces the probability that a dollar will buy in the second market and therefore increases the liquidity premium on the dollar. An increase in α reduces welfare in the foreign country and increases welfare in the home country because it reduces the probability that Japanese buyers will buy at the low price.

When $\mu > 0$, increasing α (and holding μ constant) has an ambiguous effect on welfare. It reduces both the inflation tax paid by

foreigners and the probability that a foreign buyer will buy at the low price. The first inflation tax effect works to improve welfare in the foreign country and reduce welfare in the home country. The second, term of trade effect, works in the opposite direction. The inflation tax effect dominates when μ is large. This can be seen in the last four rows of Table 4 when $\mu = 0.1$.

Increasing μ (and holding α constant) has also two effects on welfare. It increases the inflation tax collected from foreigners (when $\alpha < 1$) and it creates a distortion in the labor supply choice. When α is low the inflation tax effect dominates and therefore an increase in μ increases welfare in the home country and reduces welfare in the foreign country. When α is large the distortion effect dominates and an increase in μ reduces welfare in both countries.

Figures 1 and 2 describe welfare in both countries as a function of π . The measure plotted is welfare relative to the no-taste shock case (Since in the no shock case, $W = \frac{1}{2}$, I plot $2W, 2W^*$). This is done for $\mu = 0$ and two values for α : $\alpha = 0.1$ and $\alpha = 0.8$. Note that a decrease in π has an adverse effect on welfare. The effect on welfare is more pronounced in Japan but has also a considerable effect on the US. Welfare in the US is lower and welfare in Japan is higher for small α . This is special to the case $\mu = 0$ when no inflation tax is imposed.

Table 4: The fully integrated world economy with two currencies

 $(\pi = 0.9)$

μ	α	m	μ^*	L	L*	W	W*
0	0	0.501		0.955	0.855	0.456	0.447
0	0.1	0.551	-0.06	0.960	0.860	0.460	0.443
0	0.8	0.900	-0.09	0.991	0.891	0.491	0.414
0	1	1	-0.1	1	0.9	0.5	0.405
0.05	0	0.526		0.912	0.817	0.498	0.404
0.05	0.1	0.574	-0.01	0.916	0.821	0.495	0.407
0.05	0.8	0.905	-0.05	0.944	0.849	0.495	0.407
0.05	1	1	-0.05	0.952	0.857	0.499	0.404
0.1	0	0.549		0.872	0.781	0.531	0.366
0.1	0.1	0.594	0.03	0.876	0.785	0.522	0.375
0.1	0.8	0.910	-0.00	0.902	0.811	0.497	0.400
0.1	1	1	-0.01	0.909	0.818	0.496	0.402

* The first two columns are the choice of the two policy-makers: μ , α . We then have the following endogenous variables: the fraction of the post transfer dollar supply held by the buyers in the home country (m), the equilibrium rate of change in the yen supply (μ^*), labor supply in the home country (L), labor supply in the foreign country L* and welfare in the two countries (W, W*).

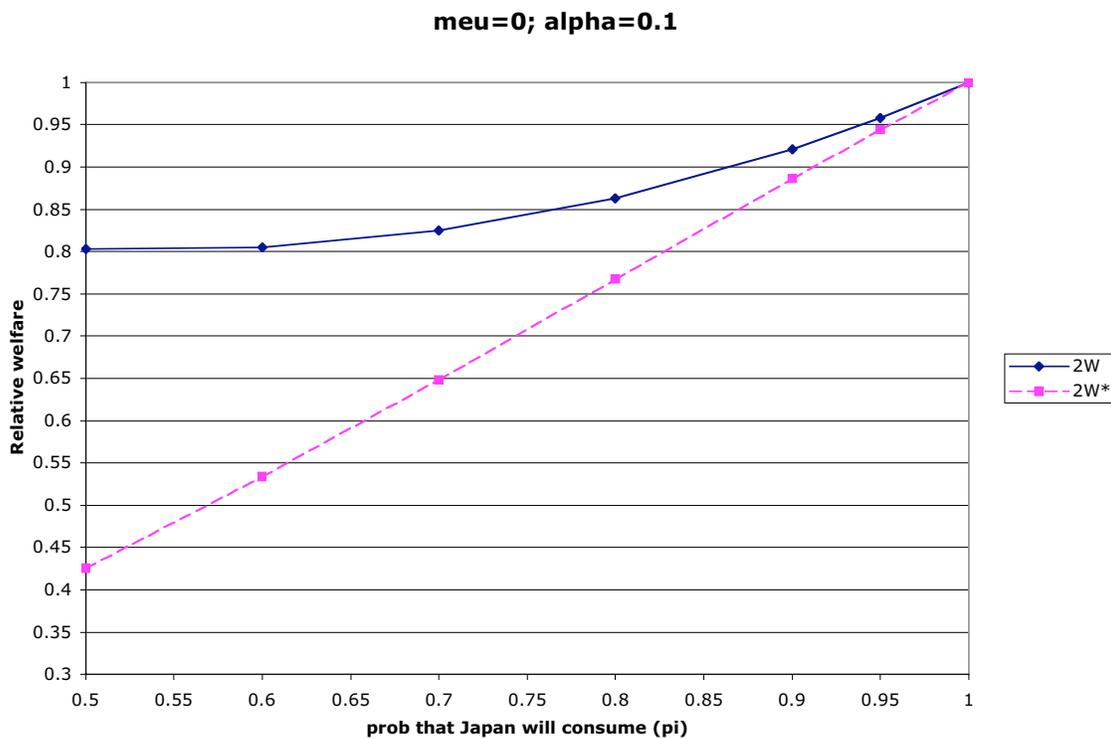


Figure 1: Welfare relative to the no-shock case ($W/0.5, W^*/0.5$) when $\mu=0$ and $\alpha = 0.1$

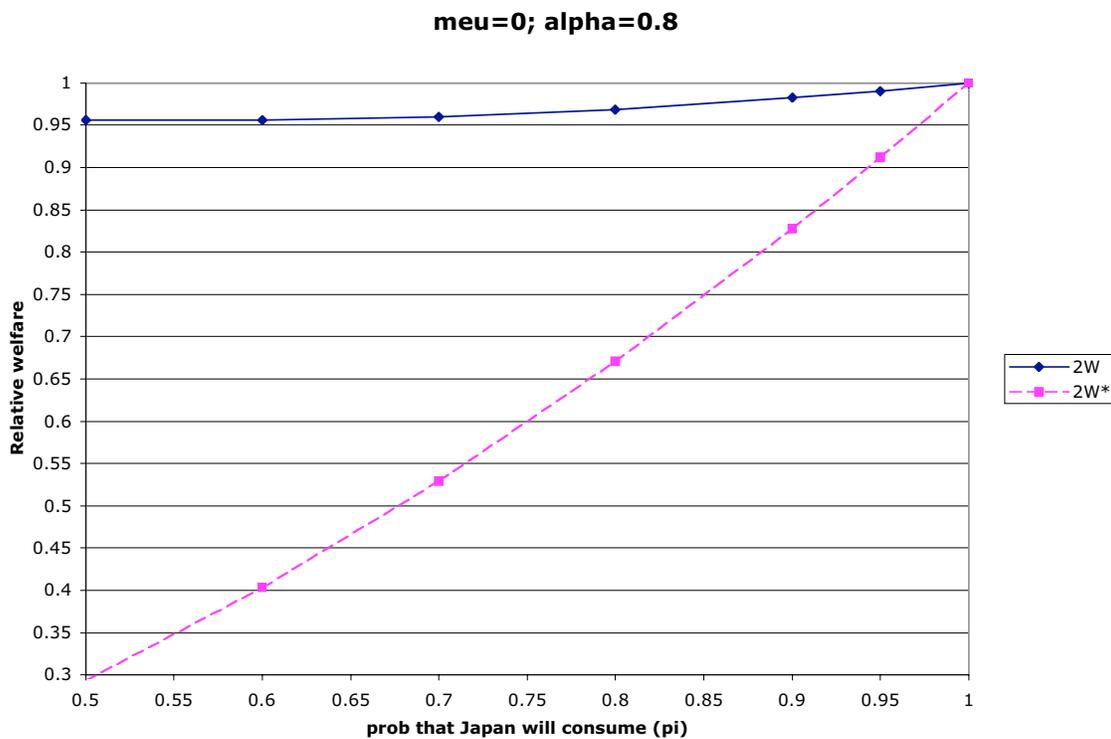


Figure 2: Welfare relative to the no-shock case ($W/0.5, W^*/0.5$) when $\mu=0$ and $\alpha = 0.8$

A Sequential policy game

I now turn to a brief description of a sequential game between the policy makers. Since there are 193 countries in the world I assume that the US moves first and chooses μ , knowing the reaction function of the rest of the world. The rest of the world then chooses $\alpha(\mu)$.

Figure 3 illustrates the reaction function $\alpha(\mu; \pi)$ of the representative foreign government for the US choice of μ . This is done for two cases: $\pi = 0.9$ and $\pi = 0.95$. The foreign country trade-off is between the terms of trade (the probability of buying at the low price) and the inflation tax. When $\mu = 0$ there is no inflation tax and therefore the foreign country focus on the terms of trade which are best when $\alpha = 0$. When μ is positive a higher α means less inflation tax but also less favorable terms of trade. When μ is sufficiently high the inflation tax dominates and the foreign country chooses $\alpha = 1$. Note that when π increases from 0.9 to 0.95 the term of trade effect becomes less important and the foreign government chooses higher α for any given μ to avoid the inflation tax. In the limit case when $\pi = 1$, the foreign government will choose $\alpha = 1$ regardless of μ .

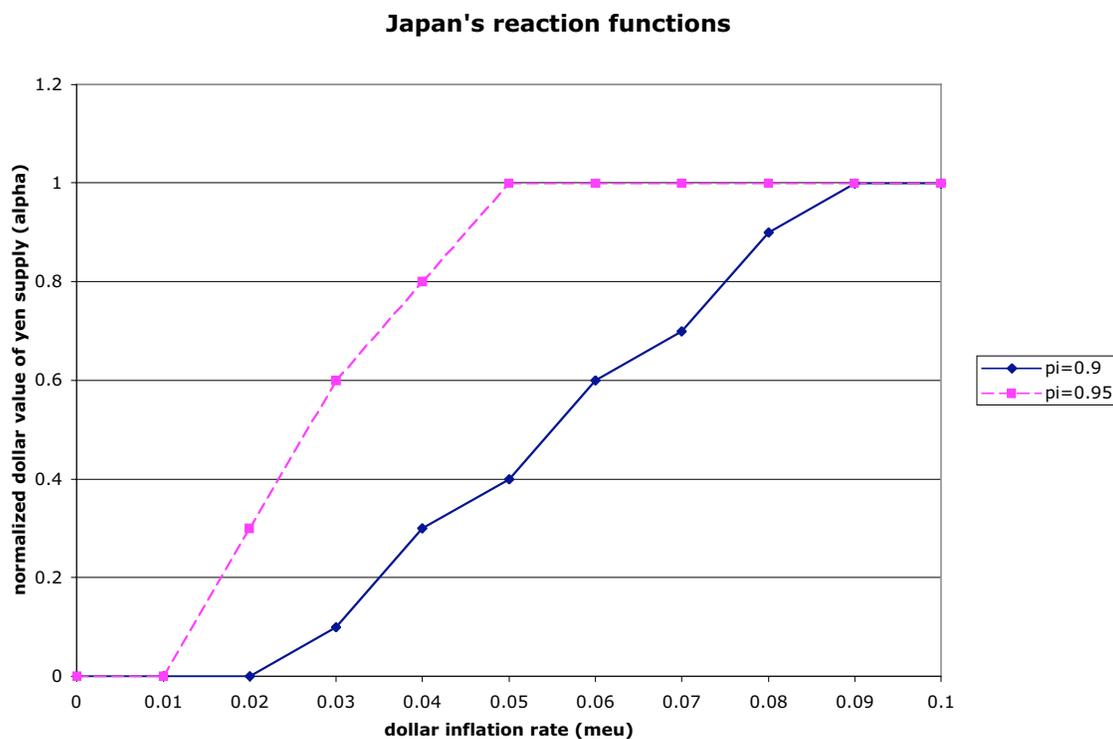


Figure 3: $\alpha(\mu)$ for $\pi = 0.9$ (the solid line) and $\pi = 0.95$

Table 4 shows that choosing $\mu = 0$ is not optimal from the US point of view. When $\mu = 0$, the rest of the world will choose $\alpha(0) = 0$ and welfare in the US will be $W = 0.456$. The US can do better by choosing $\mu = 0.1$ for example. In this case, the rest of the world will choose $\alpha(0.1) = 1$ and the US welfare will be $W = 0.496$.

A more detailed calculations reveals that the optimal choice of the US is $\mu = 0.08$ when $\pi = 0.9$ and $\mu = 0.05$ when $\pi = 0.95$. In the first case the optimal reaction is $\alpha(0.08; 0.9) = 0.9$. In the second the optimal reaction is $\alpha(0.05; 0.95) = 1$. Relative to the first best (autarkic case) the optimal choice of μ leads to a loss of welfare of 0.7% when $\pi = 0.9$ and 0.2% when $\pi = 0.95$. For the sake of comparison, a choice of $\mu = 0.01$ implies a loss of welfare of 7% when $\pi = 0.9$ and 3% when $\pi = 0.95$.

It seems that the optimal μ is "too high" relative to recent observations. We may increase π and get a lower μ but this will lead to $\alpha = 1$. It seems that a successful calibration of the model will require some modification. For example, we may add a "transaction motive" for holding money. This will increase the welfare cost of inflation and reduce the optimal μ .

Net export and the real exchange rate

Table 4 shows that when the dollar inflation is low and there is partial or full dollarization in Japan, the US suffers from trade. This is because of an adverse effect on the terms of trade: as a result of trade US buyers are sometimes forced to import at the high price.

Table 5 uses Table 4 to illustrate the adverse effect on the terms of trade by calculating measures of net exports for the home country. Net export is measured here by the difference between output and consumption. The physical unit measure of net exports ($x_s = L - c_s$) varies with the states of nature while the nominal measure ($p_1L - m$) does not.⁷

When $\mu = 0$, the nominal measure is always zero. But the physical unit measure in the high demand (x_2) is strictly positive and decreasing

⁷ Some "real measures" that ignore the variations in the terms of trade may also remain constant over time and states. For example if we measure "real net export" by the dollar value of net export divided by the price charged by US sellers we will get $L - m/p_1$ which does not vary over time and states. If we use a price index that is a weighted average of the prices quoted by foreign sellers and domestic sellers (say $p = \delta p_1 + (1-\delta)p_2$ where δ remains constant over time) we will also get a measure that does not vary over time and states.

in α . This occurs because in the high demand state, there is cross-hauling. The home country exports the good at the low price and pay the high price for some of its imports.

When $\mu = 0.05$ the nominal measure of net export is negative and decreasing in absolute value with α . This is the inflation tax imposed on foreigners. But the physical unit measure of export in the high demand state is positive reflecting the terms of trade effect.

When $\mu = 0.1$ the inflation tax effect dominates and all measures of net exports are negative. Note that net export are decreasing with μ but are not monotonic in α . Again this is because of the two effects of increasing α : The inflation tax effect and the terms of trade effect.

Note that in a steady state with low inflation rate (say $0 < \mu \leq 0.05$ and $\alpha < 1$ in Table 5) net export in the US are positive in the high demand state and buyers in the US will pay higher prices on average. Thus, a high demand state may be characterized by an increase in US CPI and an increase in net exports but not by a devaluation of the dollar: The rate of change of the exchange rate (24) is independent of the state of nature.

The volume of trade in our model may be measured by the absolute value of exports from the home country in the two states: $|x_1| + |x_2|$. Holding μ constant the Table reveals a negative correlation between α and $|x_1| + |x_2|$. This is consistent with the observation that the adoption of a common currency increases trade (Rose and Wincoop [2001]).

The last two columns in Table 5 calculate measures of the real exchange rates. The average price of consumption in state 2 (in terms of normalized dollars) is $CPI_2 = mp_1 + (1 - m)p_2$ for an American and

$CPI_2^* = [(1-m)CPI_2 + \alpha p_2]/(1 - m + \alpha)$ for a Japanese. The ratio CPI_2^*/CPI_2 is in the seventh column. It shows that increasing α increases this measure of the real exchange rate. The last column computes CPI_2^*/CPI where $CPI = (p_1 + CPI_2)/2$ is the average across states price paid by the Americans.

Table 5: Net export for the home country ($\pi = 0.9$)

μ	α	x_1	x_2	Ex	$p_1L - m$	CPI_2^*/CPI_2	CPI_2^*/CPI
0	0	0	0.047	0.043	0	1	1.027
0	0.1	0	0.043	0.039	0	1.011	1.035
0	0.8	0	0.010	0.009	0	1.088	1.094
0	1	0	0	0	0	1.111	1.111
0.05	0	-0.043	0.002	-0.002	-0.024	1	1.026
0.05	0.1	-0.035	0.005	0.001	-0.021	1.012	1.035
0.05	0.8	-0.005	0.004	0.003	-0.005	1.089	1.095
0.05	1	0	0	0	0	1.111	1.111
0.1	0	-0.078	-0.035	-0.040	-0.045	1	1.024
0.1	0.1	-0.064	-0.026	-0.030	-0.041	1.012	1.035
0.1	0.8	-0.009	-0.001	-0.002	-0.009	1.090	1.095
0.1	1	0	0	0	0	1.111	1.111

* The first two columns are the policy choices (μ, α). We then have real net export in the low demand state ($x_1 = L - c_1$) and real net export in the high demand state ($x_2 = L - c_2$). The column that follows calculates the expected real net export: $Ex = (1 - \pi)x_1 + \pi x_2$. The sixth column is the normalized dollar measure of net export: $p_1L - m$. The last two columns are measures of the real exchange rate. CPI_2^* is the average price paid by Japanese in state 2, CPI_2 is the average price paid by Americans in state 2 and CPI is the unconditional average price paid by Americans, all in terms of normalized dollars.

To complete the picture, Table 6 calculates inflation tax revenues. The third column is the inflation tax collected by the foreign government from printing its own money. Then we have the inflation tax collected by the home government from its own residents and (in the fourth column) from foreign residents. An increase in α reduces the inflation tax collected by the foreign government and reduces the inflation tax paid by foreigners to the home country. An increase in μ increases the inflation tax collected by the home country's government.

Table 6*: Steady state inflation tax revenues

μ	α	$\omega^* \mu^* X^*$	$\omega \mu Z$	$(1 - m) \omega \mu Z$
0	0	0	0	0
0	0.1	-0.086	0	0
0	0.8	-0.089	0	0
0	1	-0.09	0	0
0.05	0	-0.012	0.044	0.039
0.05	0.1	-0.018	0.044	0.032
0.05	0.8	-0.041	0.045	0.005
0.05	1	-0.045	0.045	0
0.1	0	0.050	0.083	0.068
0.1	0.1	0.040	0.082	0.056
0.1	0.8	-0.001	0.082	0.008
0.1	1	-0.007	0.083	0

* The first two columns are the policy choice variables (μ, α). The third column is the inflation tax collected by the foreign government from printing its own money ($\omega^* \mu^* X^*$). The fourth column is the inflation tax collected by the government in the home country from its own residents ($m \omega \mu Z$) and the fourth column is the inflation tax collected by the government in the home country from foreign residents ($[1-m] \omega \mu Z$).

We have focused on steady state analysis. I now turn to a relatively simple example of a transition from one steady state to another.

Transition to autarky:

What will happen if foreigners choose to reduce their holdings of US assets? This may occur if there is a change in fundamentals. For the sake of concreteness I assume a change in π from $\pi = 0.9$ to $\pi = 1$. I use sub "+" to denote the new steady state (with $\pi = 1$) and sub "-" to denote the old steady state (with $\pi = 0.9$). Variables with no subs denote the transition period.

I assume that after the change in π both countries adopt the efficient autarkic solution for the new steady state:

$m_+ = \alpha_+ = 1$ and $\mu_+ = \mu_+^* = 0$. For the sake of concreteness I assume that the levels of the variables in the old steady state were: $\mu_- = 0.5$ and $\alpha_- = 0.8$. Table 4 implies in this case: $m_- = 0.905$ and $\mu_-^* = -0.05$. It is assumed that the change become public knowledge before production choices are made but after the current period transfer is made.

The transition period will last exactly one period. In the transition period 1 ND will be spent with certainty: m_- ND by US buyers and $1 - m_-$ ND by Japanese buyers. Labor supply and normalized price in the US are determined by (9): $L(\omega) = \omega$ and $p(\omega) = 1/L = 1/\omega$.

A US policy maker that wants to maximize the sum of the transition period's utilities (of US buyers and sellers) will solve:

$$(32) \quad \max_{\omega} [m_{-}/p(\omega)] - (\frac{1}{2})[L(\omega)]^2 = \omega m_{-} - (\frac{1}{2})\omega^2$$

The first order condition for this problem is:

$$(33) \quad \omega = m_{-}$$

In our numerical example (33) implies: $\mu \approx 0.1$. The intuition is as follows. Since some of the output produced will be consumed by foreign buyers, it is optimal to produce less than the autarkic level. This can be achieved by increasing the expected rate of inflation. Table 7 illustrates. An increase in μ lowers output and increases the seller's utility. It also reduces the buyer's utility because of the decrease in the purchasing power of the dollar. The sum of utilities is maximized at $\mu = 0.1$.

Table 7*: Transition period in the US from the old steady state with ($\mu_{-} = 0.05$, $\alpha_{-} = 0.8$, $\pi = 0.9$) to the new autarkic steady state with ($\mu_{+} = 1$, $\alpha_{+} = 1$, $\pi = 1$).

μ	buyer's utility	seller's utility	total
0	0.905	-0.5	0.405
0.05	0.862	-0.454	0.408
0.1	0.823	-0.413	0.410
0.15	0.787	-0.378	0.409

* The first column is the choice of μ during the transition period. The second column is the buyer's utility during the transition period (m_{-}/p). The third column is the seller's utility ($-(\frac{1}{2})L^2$). The fourth column is sum of the second and the third columns.

Discussion:

There is a long and somewhat sparse literature on the subject of international seigniorage payments. McKinnon (1969, pp. 17-23) and Grubel (1969, pp. 269-72) argued that competition will drive international seigniorage payments to zero. Cohen (1971) examined the career of the sterling as an international currency and argued that what was once a virtual monopoly has been completely eradicated by competition from the dollar in his sample period (1965-69). Why did the sterling lose to the dollar and not to the Swiss franc or the yen? How does this competition on seigniorage payments suppose to work?

The complete elimination of seigniorage payments requires the adoption of Friedman's rule of zero nominal interest rate in the country that issues the international currency. Clearly we have never been in equilibrium according to this definition. It seems that the sterling was replaced by the dollar even when the nominal interest rate on the dollar was positive and the rate of inflation of other currencies like the yen and the Swiss frank was lower.

The analysis here suggests that the dollar took over because in the twentieth century, the US economy emerged as the most stable economy (relative to Europe, Japan and Switzerland). This is especially true after 1980 where the US experienced a period of sustained growth. The analysis here suggests that the Euro may take over if the European economy becomes more stable than the US economy. This scenario seems unlikely in the foreseeable future.

Although the paper focus on a broad definition of money it has bearing on the issues of dollarization and currency unions that

typically focus on narrow definitions of money. Fischer (1982) argues that countries choose to have national monies to avoid paying seigniorage to a foreign government. He notes that the problem of choosing between national and foreign money is related to the choice between fixed and flexible exchange rates, the optimum currency area and the optimal inflation tax. Fischer also notes that transaction costs and the lack of the ability to commit to a monetary policy could make the use of foreign money optimal.

Transaction costs and the ability to commit play a major role in Alesina and Barro (2001, 2002) analysis of dollarization and currency unions. They argue that seigniorage should be part of the overall negotiations. This may be feasible in the case of currency unions they consider. But here we discuss the holding of dollar denominated assets by agents from all (193) countries. Cooperation in this case is more difficult. To illustrate, suppose that many small countries adopt partial or full dollarization and "export" demand uncertainty to the US. The US may demand direct compensation for lowering the dollar inflation rate and the inefficient seigniorage payments. But since there are many countries that will benefit from the low inflation of the dollar it may be difficult to enforce a direct compensation scheme.

It is shown that under portfolio autarky with $\mu = 0$ and $\alpha = 1$, the allocation is Pareto efficient and each country's rate of inflation is optimal. But when the US chooses the efficient zero inflation rate, it is optimal for the foreign country to choose $\alpha = 0$. We get an "inflation bias" in a sequential game in which countries want to maximize the steady state welfare of their own citizens. The reason for the "inflation bias" in the US is the desire to collect inflation tax

from foreigners and to minimize trade. Unlike Kydland and Prescott (1977) and Barro and Gordon (1983) it occurs here under perfect commitment.

I now turn to the positive implications of the model. Assuming a broad definition of money, our model is consistent with the observation that in general the rate of return on US foreign assets has exceeded that on US foreign liabilities (Gourinchas and Rey [2005], Lane and Milesi-Ferretti [2005]) and that US assets are only partially linked to the dollar but US liabilities are almost entirely dollar-denominated (Tille [2003]). In our model, US liabilities are in dollar terms and they earn a rate of return that is less than the rate of return on the foreign asset.

Our model has price dispersion in equilibrium and can therefore explain deviations from PPP. In particular, it is consistent with the observation that the US is cheap relative to the prediction of income-price regressions. See Balassa (1964), Samuelson (1964) and Rogoff (1996).

As in Rose and Wincoop (2001), the adoption of a common currency increases trade in our model. This does not hold in all models. Recently, Bacchetta and van Wincoop (2000) used a cash-in-advance model to analyze the implications of a monetary union and demand uncertainty that arises as a result of money supply shocks. They find that exchange-rate stability is not necessarily associated with more trade. Devereux and Engel (2003) find that the implications of risk for foreign trade are highly sensitive to the choice of currency at which prices are set. In these models prices are rigid and firms satisfy demand. In the UST model used here prices can be changed during trade and sellers are not

committed to satisfy demand (indeed, low price sellers are stocked out in the high demand state).

Americans work hard in our model because they are relatively certain about the prospect of enjoying the fruits of their labor. This is not unlike the tax explanation in Prescott (2004). Nothing will change in our model if instead of a taste shock we assume that the Japanese government imposes a random tax on accumulated wealth.

The recent increase in the importance of US assets in the global portfolio may be viewed as a transition to a steady-state equilibrium rather than a temporary phenomenon. This of course assumes that the US will continue to be the most stable economy in the world. In a recent article Caballero et al. (2006) attributes the increase in the importance of US assets to an unexpected reduction in the growth rate of European and Japanese output and (or) a collapse of the asset markets in the rest of the world. They use a "real" overlapping generations model that does not explain the excess return (GR) puzzle that is the focus of this paper.

In a recent newsletter Gourinchas (2006) suggests that a theoretical resolution of the puzzle requires two ingredients: consumption and portfolio home biases. Here preferences do not exhibit home bias. In equilibrium a large percentage of consumption is produced at home and only Japanese hold yen. This may look like a home bias but it is not: At the same rate of return all agents will specialize in dollars.

Blanchard Giavazzi and Sa (2005) follow the partial-equilibrium portfolio balance literature of Kouri (1982). In their model, interest rates are exogenous and in the steady state the rates of return on all

assets are equal to the world interest rate (r). The steady state level of the trade deficit (D) depends on the steady state level of the net debt (F) and the standard formula, $rF + D = 0$, holds. This says that the larger is the net debt, the larger is the trade surplus required in steady state to finance interest payments on the debt. Here rates of return are endogenous and do not converge to a single rate in the steady state. We show that a steady state equilibrium with both $F > 0$ and $D > 0$ is possible. In this steady state the deficit is financed by seigniorage payments.

The models in Blanchard et.al and Caballero et al. are "real" models. We have a monetary model. It is my conjecture that the main results will not change if instead of money we have physical capital with explicit taxation replacing the implicit inflation tax we use here. In such a model sellers will accept claims on the income generated by capital but claims on Japanese capital will be less liquid than claims on American capital. This has to be worked out. The advantage of using a monetary model is that it allows for a discussion of measurement issues (like the difference between the real and the monetary measures of the net export in Table 5).

I now turn to discuss modeling aspects of the paper. As in other UST models, we may think of the sellers' choice among alternative hypothetical markets as a choice of price tags: The seller chooses a price tag on each unit taking the probability of making a sale into account. Here we added the choice of currency. A seller may choose to accept the equivalent value of any currency or just a particular currency. The choice of currency may be different for different individuals because the liquidity of the currency depends on the

probability of finding the good at the low price that is different across agents. We thus use price dispersion to model "liquidity premium" as an individual specific attribute of an asset.

Our approach is related to the random matching models pioneered by Kiyotaki and Wright (1993). In both models uncertainty about trading opportunities plays a key role. In the random matching models agents are uncertain about whether they will meet someone that they can actually trade with. But whenever a meeting takes place it is bilateral. In the UST model sellers are also uncertain about the arrival of trading partners but whenever a meeting occurs there are a large number of agents on both sides of the market. As a result there is a difference between the assumed price determination mechanisms. In the random matching models prices are either fixed or are determined by bargaining (as in Trejos and Wright [1995] and Shi [1995]). In the UST model prices clear markets that open.

At the end of their paper Kiyotaki and Wright (1993) consider an economy with two currencies: red and blue. The red currency circulates with a higher probability and in equilibrium yields a lower rate of return. The high return asset is less acceptable or less liquid. Similarly, here the currency that promises the higher chance of buying at the low price yields a lower rate of return. The difference is that here the international currency is more liquid than the domestic currency. This is not natural if we think of bilateral meetings. It makes sense in our setup where trade is done on the internet and a broad definition of money is used.

Matsuyama, Kiyotaki and Matsui (1993), Zhou (1997), Wright and Trejos (2001) and Liu and Shi (2005) use the random matching approach to

study international currency. Wright and Trejos (2001) show that there can be three distinct type of equilibria, where in every case monies circulate locally, and either one, both, or neither circulate internationally. The assumed matching process plays a key role in determining the type of equilibria possible. For example, in the absence of inflation tax equilibrium with two national monies and no international money exists if the two countries are similar and the probability of meeting a foreigner is low. In our model the key difference between the two countries is in the probability of the taste shock. The example in Table 4 suggests that in the absence of inflation tax it is not possible to get equilibrium with national monies only (unless $\pi = 1$ and the two countries are completely symmetric).

The difference in the taste shock probability limits the applicability of Gresham's law. In our model we get a steady state equilibrium with two monies even when $\mu \neq \mu^*$. This is different from Karekan and Wallace (1981). In their model, there is no difference between the currencies. As a result there is a continuum of equilibria that differ in the nominal exchange rates. At any given equilibrium, the nominal exchange rate is constant over time and therefore the currency whose supply grows at a faster rate will represent an increasing fraction of the currency portfolio held by agents.

Our overlapping generations model does not distinguish between money and bonds. The framework in Lagos and Wright (2005) may be a good way of doing it. In their framework, random matching occurs during the "day" and Walrasian auction occurs during the "night". We may replace the Walrasian auction that occurs during the night with sequential trade. That is, after interacting in a decentralized market with

anonymous bilateral matching during the day, agents go on the internet and place orders as in our model. During the night it is easy to transfer funds from one account type to another and therefore we may assume that in fact everyone accepts bonds. My conjecture is that in such a model the rate of inflation of the other currency (the yen) may but need not be less than the rate of inflation of the dollar but the real rate of return on foreign bonds should still be higher than the real rate on dollar denominated bonds. But this of-course should be worked out. Other useful extensions may include longer horizon agents with a smoothing of consumption motive and the introduction of physical capital.

APPENDIX A: REVENUES FROM ASSET CREATION

As was said in the introduction, Gourinchas and Rey (GR, 2005) found strong evidence of sizable excess returns of gross US assets over gross US liabilities. Here I provide a preliminary calculations about the implied seigniorage paid to US agents.

According to the broad definition, foreign agents pay seigniorage on US assets that promise rates of returns that are less than the rates that they can get on investment in their own country. To illustrate, I assume a single good, two periods exchange economy with four assets: an international currency (the dollar), US (home country) government bonds, US private bonds and foreign bonds. I assume that the rates of return on these assets are exogenous and work out the implications of the assumption that budget constraints are satisfied and markets are cleared.

At time t the representative agent in the home (foreign) country gets an endowment of Y_t (Y_t^*) units of the consumption good. The representative agent in the home country gets also a transfer payment from his government. The real value of the transfer payment is: G_t . There are no explicit taxes and no government in the foreign country. The transfers are financed by seigniorage revenues.

I use m , b^s , b^p and f to denote the real value of money, US government bonds, US private bonds and foreign bonds. The real rates of return these assets are: r^m , r^s , r^p , r^f . It is assumed that: $E(r^m) \leq E(r^p) \leq E(r^s) \leq E(r^f)$, where E denotes expectations taken in the first period. The representative US consumer first period budget constraint is given by:

$$(A1) \quad f = Y_1 + G_1 - b^s - b^p - m - c_1$$

His second period consumption is:

$$(A2) \quad c_2 = Y_2 + G_2 + b^s(1 + r^s) + b^p(1 + r^p) + m(1 + r^m) + f(1 + r^f)$$

Substituting (A1) in (A2) yields:

$$(A3) \quad c_1(1 + r^f) + c_2 \\ = (Y_1 + G_1)(1 + r^f) + Y_2 + G_2 - b^s(r^f - r^s) - b^p(r^f - r^p) - m(r^f - r^m)$$

Similarly, for the representative foreign agent we have:

$$(A4) \quad c_1^*(1 + r^f) + c_2^* = Y_1^*(1 + r^f) + Y_2^*$$

$$- b^{g*} (r^f - r^g) - b^{p*} (r^f - r^p) - m^* (r^f - r^m)$$

Market clearing requires:

$$(A5) \quad c_1 + c_1^* = Y_1 + Y_1^*; \quad c_2 + c_2^* = Y_2 + Y_2^*; \\ b^p + b^{p*} = 0.$$

We can now add (A3) and (A4) to get:

$$(A6) \quad [b^g (r^f - r^g) + b^p (r^f - r^p) + m (r^f - r^m)] \\ + [b^{g*} (r^f - r^g) + b^{p*} (r^f - r^p) + m^* (r^f - r^m)] \\ = (Y_1 + G_1 + Y_1^*) (1 + r^f) + Y_2 + G_2 + Y_2^* - (c_1 + c_1^*) (1 + r^f) - c_2^*$$

Substituting the market-clearing conditions yields:

$$(A7) \quad [b^g (r^f - r^g) + m (r^f - r^m)] + [b^{g*} (r^f - r^g) + m^* (r^f - r^m)] \\ = G_1 (1 + r^f) + G_2$$

The right hand side of (A7) is the future value of the government transfers. The left hand side is the seigniorage revenue: The terms in the first bracket are the seigniorage paid by US consumers and the terms in the second bracket are the seigniorage paid by foreigners. Note that when $r^f > r^g$, seigniorage is paid on government bonds as well as on real balances.

The market clearing condition $b^p + b^{p*} = 0$, implies:

$$(A8) \quad b^p (r^f - r^p) = - b^{p*} (r^f - r^p)$$

Thus if $b^{p*} > 0$ and $r^f > r^p$, foreigners pay "seigniorage" to private US agents. The term "seigniorage" may be appropriate because it reflects the ability of US private agents to create an asset that is "overpriced".

I focus on the expected value of the total seigniorage paid by foreigners:

$$(A9) \quad b^{g*} E(r^f - r^g) + m^* E(r^f - r^m) + b^{p*} E(r^f - r^p)$$

We may assume that all the transfers are made to the old generation. I now turn to a calibration exercise.

Calibration: As was said before, GR estimated that the average real rate of return on foreign assets held by US residents between 1973 - 2004 was 6.82%. The estimated real rate of return on US foreign liabilities in this period is: 3.50%. The estimated premium is thus 3.32%. The estimated premium for the entire sample (1952-2004) is 2.11%.

I start by using the post Bretton Woods data for forming expectations. I also assume that there is no difference between US government securities and US private securities and that the expected rate of inflation is 2%. I thus assume:

$$(A10) \quad E(r^f) = 6.82\%, \quad E(r^g) = E(r^p) = 3.50\%; \quad E(r^m) = -2\%;$$

The seigniorage paid by foreigners to US agents (government and private) requires an estimate of $b^* = b^{g*} + b^{p*}$ and m^* . We can find recent levels

of gross liabilities in a report prepared by the Department of the Treasury, Federal Reserve Bank of New York and Board of Governors of the Federal Reserve System (Report [2005]). In this report there are data on foreign holdings of U.S. securities for the years 2002 - 2004 (Table 1 on page 3). The total was 4338 billion dollars for 2002, 4979 billion dollars for 2003 and 6006 billion dollars for 2004. These estimates include Equities, government and corporate debt. Our analysis does not distinguish between debt and equity. I will therefore lump them together and use $b^* = 6006$ billion for the current estimate on total US securities held by foreigners. Multiplying this amount by the expected premium of 3.32% we arrive at a seigniorage figure of 199 billion which is 1.7% of 2004 US GDP. If we use the more moderate "liquidity premium" of 2.11% we get about 1% of 2004 GDP.

We add to that the amount that foreigners expect to pay on their holding of currency: $m^*E(r^f - r^m)$. The amount of cash held by foreigners in 2004 is estimated to be close to 333 billion dollars. Multiplying this by the premium $r^f - r^m = 8.82\%$ we arrive at a seigniorage figure of about 29 billions dollars or 0.25% of US 2004 GDP. Adding this to the seigniorage on other assets we get a total close to 2% of US GDP. If we use GR estimates for the entire sample (1952 - 2004) we get a total seigniorage of about 1.3%. These are big numbers. They are close to Switzerland's entire GDP (which is roughly 2.2% of US GDP).

An alternative view may focus on seigniorage earned on Government securities and cash. GR finds that during the post Bretton Woods era (1973 - 2004) the real rate of return on foreign bonds held by US residents was 4.05% while the real rate of return on US bonds held by foreigners was only 0.32%. The excess return on bonds is thus 3.73%. The

BEA estimate that about 1.5 billions dollar worth of US government securities were held by foreigners in 2004 (close to 13% of GDP). The expected seigniorage to the US government is close to 56 billions which is close to 0.5% of GDP. Adding to this the seigniorage on cash held by foreigners we arrive at a total of 0.7% of US GDP.

APPENDIX B

Proof of Claim 1:

The first order conditions (20) imply:

$$(B1) \quad p_1 = \pi p_2.$$

We substitute (B1) in (15)-(16) to get:

$$z_1 = 1/p_1 ; z_2 = m/p_1 + (1-m)\pi/p_1 \text{ and}$$

$$(B2) \quad z = (1/p_1)[1 - \pi + \pi^2 + \pi(1 - \pi)m]$$

$$z^* = (1/p_1)[\pi^2 + \pi(1 - \pi)m]$$

Substituting (B2) in (20) yields:

$$(B3) \quad L = \omega[1 - \pi + \pi^2 + \pi(1 - \pi)m] ; \quad L^* = \omega[\pi^2 + \pi(1 - \pi)m]$$

From (19) we get: $p_1 L = m/\omega - \mu = m(1 + \mu) - \mu$. Using (B3) leads to:

$$(B4) \quad p_1 = [m(1 + \mu) - \mu] / \omega[1 - \pi + \pi^2 + \pi(1 - \pi)m].$$

Substituting $p_1 L = m(1 + \mu) - \mu$ in the market clearing condition

$p_1(L + k_1^*) = m$ leads to: $p_1 k_1^* = (1 - m)\mu$. We now substitute this in the

market clearing condition $p_1(L^* - k_1^*) = \pi(1 - m)$ to get:

$p_1 L^* = (\pi + \mu)(1 - m)$. Using (B3) leads to:

$$(B5) \quad p_1 = (\pi + \mu)(1 - m) / \omega[\pi^2 + \pi(1 - \pi)m].$$

Equating (B4) to (B5) leads to:

$$(B6) \quad \begin{aligned} & [\pi^2 + \pi(1 - \pi)m] / [1 - \pi + \pi^2 + \pi(1 - \pi)m] \\ & = (\pi + \mu)(1 - m) / (m + \mu m - \mu) \end{aligned}$$

Lemma: There exists a unique solution to (B6).

Proof: When $L > 0$, (19) implies $m > \omega\mu$ and the right hand side of (B6) is positive. When $m - \omega\mu$ is small the right hand side (RHS) is large and when $m = 1$ the RHS is equal to zero. The left hand side (LHS) is π when $m = 1$ and $\pi^2/(1 - \pi + \pi^2)$ when $m = 0$. Furthermore, the RHS is decreasing in m and the LHS is increasing in m . Therefore there exists a unique solution \bar{m} in Figure B1. \square

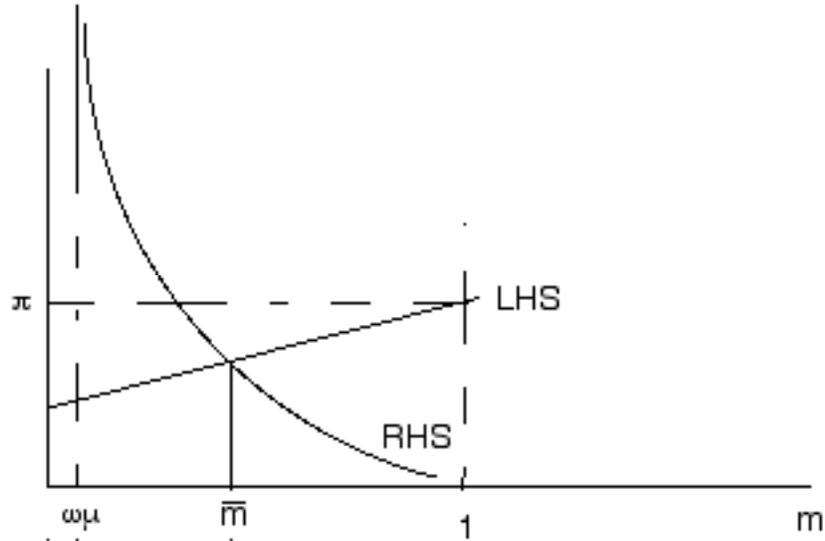


Figure B1

We now substitute the solution \bar{m} in (B3) to solve for the steady state magnitudes L and L^* . We proceed by solving for p_1 from (B5) and $p_2 = \pi p_1$. To solve for k_1^* we use the market clearing condition:

$$p_2(L^* - k_1^*) = 1 - m. \quad \square$$

Proof of Proposition 1:

I start by solving for the steady state level of m . As before we get: $p_1 L = m(1 + \mu) - \mu$ from (19). Substituting this in the condition for clearing the first market, $p_1(L + k_1^*) = m$, leads to:

$p_1 k_1^* = (1 - m)\mu$. We now substitute this in the second market clearing condition, $p_1(L^* - k_1^*) = \pi(1 - m) + \pi\alpha$, to get:

$p_1 L^* = (\pi + \mu)(1 - m) + \pi\alpha$. We also verify that equation (B3) and (B4) still hold. Using (B3) leads to:

$$(B7) \quad p_1 = [(\pi + \mu)(1 - m) + \pi\alpha] / \omega[\pi^2 + \pi(1 - \pi)m].$$

Equating (B4) to (B7) leads to:

$$(B8) \quad \frac{[\pi^2 + \pi(1-\pi)m]}{[1-\pi + \pi^2 + \pi(1-\pi)m]} \\ = \frac{[(\pi + \mu)(1 - m) + \pi\alpha]}{(m + \mu m - \mu)}$$

I now turn to show that there exists a unique solution to (B8). The left hand side of (B8) is the same as the LHS of (B6) and therefore the LHS curve is unchanged. As before, when $m - \omega\mu$ is small the right hand side (RHS) is large. But now when $m = 1$ the RHS is equal to $\pi\alpha$ rather than zero. Since $0 \leq \alpha \leq 1$, $\pi\alpha \leq \pi$ and there exists a unique solution, \bar{m} in Figure B2.

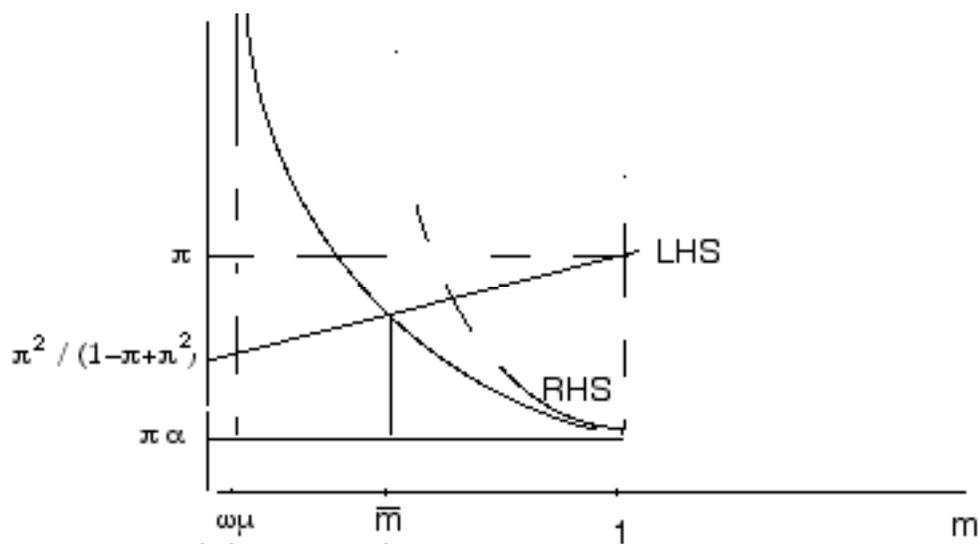


Figure B2

We now use the solution \bar{m} to solve for the steady state magnitudes. We have thus shown existence and uniqueness.

I now choose ω^* so that the Japanese seller is indifferent between yen and dollars. From (26) we get:

$$(B9) \quad \omega p_2 Z^* = \omega^* p_2^* X^* = \pi \omega^*$$

We use (B1) and (B2) to get: $\omega p_2 Z^* = \omega[\pi + (1-\pi)m]$. Substituting this in (B9) leads to:

$$(B10) \quad \omega^*(m) = \omega \left(1 + \frac{(1-\pi)m}{\pi} \right)$$

Condition (B10) implies that $\omega^*(m)$ is an increasing function and $\omega^* > \omega$. Note that (B3) implies $L \geq L^*$ and (B10) implies $\mu^* \leq \mu$. We have thus shown part (a).

To show (b) note that an increase in α increases the RHS of (B8) for all m and therefore shifts the RHS curve in Figure B2 to the right. This leads to an increase in the steady state level of m . Note also that (B3) implies that L and L^* are monotonic in m . Therefore as α grows and m grows and labor supplies in both countries grow.

I now turn to show that the US seller strictly prefers dollars. A US seller that sells for dollars will have the expected real wage:

$$(B11) \quad \omega p_1 Z = \omega[1 - \pi + \pi^2 + \pi(1 - \pi)m]$$

The expected real wage when selling in yen is:

$$(B12) \quad \omega^* p_1^* X = \omega^* \pi p_2^* (1/p_2^*) = \omega^* \pi = \omega \pi \left(1 + \frac{(1-\pi)m}{\pi} \right)$$

where the last equality uses (B10). Subtracting (B12) from (B11) leads to:

$$(B13) \quad \omega p_1 Z - \omega^* p_1^* X = \omega(1 - m)(1 + \pi^2 - 2\pi) \geq 0.$$

When $\pi < 0$, this difference is strictly positive and decreasing in π . We have thus shown (c).

To show (d) note that $m = 1$, $\alpha = 1$ solve (B8). Substituting $m = 1$ in (B3) leads to: $L = 1$ and $L^* = \pi$ that satisfy the first order conditions (31). \square

REFERENCES

- Abel Andrew B. "Precautionary Saving and Accidental Bequest" The American Economic Review, Vol. 75, No. 4 (Sep., 1985), pp. 777-791.
- Alesina Alberto and Robert J. Barro., "Dollarization" American Economic Review, Papers and Proceedings, May 2001, Vol. 91, No. 2, 381-385.
- _____ "Currency Unions" The Quarterly Journal of Economics, Vol. 117, Issue 2, May 2002, pp. 409-436.
- Bacchetta Philippe and Eric van Wincoop., "Does Exchange-Rate Stability Increase Trade and Welfare?" American Economic Review, 90 (Dec. 2000), 1093-1109.
- Balassa, B. "The Purchasing Power Parity Doctrine: A Reappraisal" Journal of Political Economy, 72 (1964), 584-96.
- Barnet William A., Douglas Fisher and Apostolos Serletis "Consumer Theory and the Demand for Money" Journal of Economic Literature, Dec. 1992, pp. 2086-2119.
- Barro Robert J. "Are Government Bonds Net Wealth?" Journal of Political Economy, November/December 1974, 82, 1095-1117.
- _____ and David B. Gordon., "A positive Theory of Monetary Policy in a Natural Rate Model" The Journal of Political Economy, Vol. 91, Number 4, August 1983, 589-610.
- Blanchard Olivier, Francesco Giavazzi and Filip SA "International Investor, the U.S. Current Account, and the Dollar" Brookings Papers on Economic Activity, 1:2005.

- Bryant, J. "Competitive Equilibrium with Price-Setting Firms and Stochastic Demand" *International Economic Review*, 21 (1980), pp. 519-531.
- Caballero Ricardo J., Emmanuel Farhi and Pierre-Olivier Gourinchas., "An Equilibrium Model of 'Global Imbalances' and Low Interest Rates" mimeo, April 19, 2006.
- Dana James D. Jr. „Advance-Purchase Discounts and Price Discrimination in Competitive Markets“ *Journal of Political Economy*, Vol.106, Number 2, April 1998, 395-422.
- Deneckere Raymond and James Peck "Dynamic Competition with Random Demand and Costless Search: A Theory of Price Posting" mimeo, 2005.
- Devereux Michael B. and Charles Engel., "Monetary Policy in the Open Economy Revisited: Price Setting and Exchange-Rate Flexibility" *Review of Economic Studies*, 70 (2003), 765-783.
- Dornbusch Rodiger, Paul Krugman and Yung Chul Park. "Meeting World Challenges: U.S. Manufacturing in the 1990s" mimeo 1989, with a forward by Colby H. Chandler.
- Eden, Benjamin. "Marginal Cost Pricing When Spot Markets are Complete" *Journal of Political Economy*, Dec.1990. Vol. 98, No.6,1293-1306.
- _____ "The Adjustment of Prices to Monetary Shocks When Trade is Uncertain and Sequential" *Journal of Political Economy*, Vol. 102, No.3, 493-509, June 1994.
- _____ "Inefficient Trade Patterns: Excessive Trade, Cross-hauling and Dumping" mimeo, February 2005.
- Ethier, Wilfred J. "Internationally Decreasing Costs and World Trade." *Journal of International Economics*, 1979, 9(1), pp. 1 -24.
- Frankel Jeffrey A. and Andrew Rose "An Estimate of the Effect of Common Currencies on Trade and Income" *The Quarterly Journal of Economics*, Vol. 117, Issue 2, May 2002, pp. 437-466.
- Fischer, Stanley "Seigniorage and the Case for a National Money" *Journal of Political Economy*, 1982, Vol. 90, no.2.
- Friedman, Milton "The Optimum Quantity of Money", in *The Optimum Quantity of Money and Other Essays*, Adline, 1969, pp. 1- 50.
- Gourinchas Pierre-Olivier "The Research Agenda: On Global Imbalances and Financial Factors" *EconomicDynamics News letter*, Volume 7, Issue 2, April 2006.

- _____ and Helene Rey "From World Banker to World Venture Capitalist: US External Adjustment and The Exorbitant Privilege" NBER WP 11563, August 2005.
- Grubel Herbert G. "The Distribution of Seigniorage from International Liquidity Creation" in Robert A. Mundell and Alexander K. Swoboda, eds., Monetary Problems of the International Economy (Chicago: University of Chicago Press, 1969)
- Karekan, J. and N. Wallace, "On the Indeterminacy of Equilibrium Exchange Rates" Quarterly Journal of Economics, 96 (1981), pp. 207-222.
- Kiyotaki, N. and R. Wright, "A Search-theoretic Approach to Monetary Economics" American Economic Review, 83 (1993), 63 - 77.
- Kouri, Pentti. "Balance of Payments and the Foreign Exchange Market: A Dynamic Partial Equilibrium Model" In Economic Interdependence and Flexible Exchange Rates, Edited by Jagdeep S. Bhandari and Bulford H. Putnam. MIT Press, 1982, pp. 116-156.
- Kydland, F.E. and Prescott E.C. "Rules Rather than Discretion: The Inconsistency of Optimal Plans", The Journal of Political Economy, Vol. 85, No. 3. (Jun., 1977), pp. 473-492.
- Lane Philip R. and Gian Maria Milesi-Ferretti "Financial Globalization and Exchange Rates" IMF working paper, January 2005.
- Liu Qing and Shouyong Shi "Currency Areas and Monetary Coordination" mimeo, 2005.
- Lucas, Robert E., Jr. "Asset Prices in an Exchange Economy", Econometrica, 1978, Vol. 46(6), pp. 1426-1445.
- _____ "Interest Rates and Currency Prices in a Two-Country World" Journal of Monetary Economics 10 (1982) 335-359.
- _____ and M. Woodford., "Real Effect of Monetary Shocks in an Economy with Sequential Purchases" NBER Working Paper No. 4250, January 1993.
- Matsuyama, M., Kiyotaki, N. and A. Matsui, "Towards a Theory of International Currency" Review of Economic Studies, 60 (1993), 283-307.
- McGrattan Ellen, R. and Prescott Edward C. "'Average Debt and Equity Returns: Puzzling?" American Economic Review, Papers and Proceedings, May 2003, Vol. 93 No.2, pp. 392-397.

- McKinnon, Ronald I. Private and Official International Money: The Case of the Dollar, Princeton Essays in International Finance No. 74 (Princeton: International Finance Section, 1969)
- Obstfeld Maurice and Kenneth Rogoff "The Unsustainable US Current Account Position Revisited", mimeo November, 2005. (This is a revised version of NBER WP# 10869, November, 2004).
- _____ "The Six Major Puzzles in International Macroeconomics: Is There a Common Cause?" NBER Macroeconomic Annual, 15, 2000.
- Prescott, Edward. C., "Efficiency of the Natural Rate" Journal of Political Economy, 83 (Dec. 1975): 1229-1236.
- _____ "Why Do Americans Work So Much More Than Europeans?" Federal Reserve Bank of Minneapolis Quarterly Review, 28 (2004): 2-12.
- Report on Foreign Portfolio Holdings of U.S. Securities as of June 2004. Prepared by the Department of the Treasury, Federal Reserve Bank of New York and Board of Governors of the Federal Reserve System. June 2005 [Report (2005)]
<http://www.treas.gov/tic/shc2003r.pdf>.
- Rogoff, K. "The Purchasing Power Parity Puzzle" Journal of Economic Literature, 34 (1996), 647-68.
- Rose, Andrew K. and Eric van Wincoop., "National Money as a Barrier to International Trade: The Real Case for Currency Union" American Economic Review, Papers and Proceedings, May 2001, 386-390.
- Rotemberg, Julio, J. and Summers Larry.H., "Inflexible Prices and Procyclical Productivity," November 1990., The Quarterly Journal of Economics, 105, 851-74.
- Samuelson, P. "Theoretical Notes on Trade Problems" Review of Economic and Statistics, 46 (1964), 145-54.
- Sanyal, Kalyan K. and Jones, Ronald W. "The Theory of Trade in Middle Products." American Economic Review, 1982, 72(1), pp. 16-31.
- Shi, S. "Money and Prices: A Model of Search and Bargaining" Journal of Economic Theory, 27 (1995), 467-496.
- Tille, C. "The Impact of Exchange Rate Movements on U.S. Foreign Debt" Current Issues in Economics and Finance, 2003, Vol. 9, No. 1 pp.1-8 (New York: Federal Reserve Bank).
- Trejos, A. and R. Wright "Search, Bargaining, Money and Prices" Journal of Political Economy, 103 (1995), 118-41.

Wright, R. and A. Trejos, "International Currency" Advances in Macroeconomics 1 (no. 1, 2001), article 3.

Zhou, R., "Currency Exchange in a Random Search Model" Review of Economic Studies, 64 (1997), 289-310.