Expectations, Asset Prices, and Monetary Policy: The Role of Learning*

Simon Gilchrist† Masashi Saito‡

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Abstract

This paper studies the implications of financial market imperfections represented by a countercyclical external finance premium and gradual recognition of changes in the drift of technology growth for the design of an interest rate rule. Asset prices movements induced by changes in trend growth influence balance sheet conditions which determine the premium on external funds. The presence of financial market frictions provides a motivation for responding to the gap between the observed asset price and the potential asset price in addition to responding strongly to inflation. This is because the asset price gap represents distortions in the resource allocation induced by financial market frictions more distinctly than inflation. Policy maker's imperfect information about the drift of technology growth makes the calculation of potential imprecise and thus reduces the benefit of responding to the asset price gap. Asset price targeting which does not take into account changes in potential tends to be welfare reducing.

*The views expressed here are those of the authors and not necessarily those of the Bank of Japan or the Institute for Monetary and Economic Studies.

†Boston University and NBER, sgilchri@bu.edu

‡Bank of Japan, Institute for Monetary and Economic Studies, masashi.saitou@boj.or.jp

1 Introduction

Recent studies on asset prices and monetary policy considers the benefits of allowing the monetary authority to respond to asset prices in a monetary policy rule.[1] These studies frequently relies on two key assumptions: i) that asset price movements create distortions to economic activity through their effect on the ability of managers to finance investment and ii) there exists exogenous "bubbles" or non-fundamental asset price movements.[2] In such environments, non-fundamental increases in asset prices cause investment booms, an increase in output above potential and rising rates of inflation. In this framework, a strong inflation targeting policy is frequently found to be optimal to suppress the undesirable consequences of these asset price fluctuations. In other words, there is no need to respond to asset prices above and beyond what is implied by their ability to forecast inflation.

The notion that inflation targeting is a sufficient response to bubbles rests in part on the assumption that bubbles distort the economy by increasing managers' ability to invest without distorting their internal perceptions of the value of new investment. As Dupor (2005) emphasizes, these conclusions are tempered to the extent that bubbles directly influence managerial valuations of capital. More generally, non-fundamental movements in asset prices cause distortions to aggregate demand through their influence on markups and hence inflation and distortions to the consumption/investment decision through their influence on the cost of capital for investment activity. A monetary policy maker with one instrument – the nominal interest rate – faces a tradeoff between reducing distortions owing to variation in the markup and distortions owing to variations in the return on capital. In such environments, the policy maker may find monetary policy rules that respond to asset prices beneficial.

While much of the literature has focussed on non-fundamental movements in asset prices, it is often recognized that asset price booms occur in conjunction with changes in the underlying economic fundamentals (Beaudry and Portier (2004)). The fact that the late 1990s run up in US stock prices is closely to tied to perceived changes in

[1] Bernanke and Gertler (1999, 2001), Cechetti, Genberg, Lipsky and Wadhwani (2000), Gilchrist and Leahy (2002) and Tetlow (2005) provide recent examples.

[2] Mishkin and White (2003) provide a recent discussions of the evidence on stock market bubbles and their role in monetary policy for the U.S economy while Okina, Shirakawa and Shiratsuka (2001) provide a description of the Japanese stock market boom of the late 1980s along with an assessment of the conduct of monetary policy during this episode.

trend productivity growth is a case in point. Thus, a key question in this literature is whether or not the monetary authority can identify the source of movements in asset prices in an environment of technological change. As emphasized by Edge, Laubach and Williams (2004), it is plausible to believe that the underlying trend growth in productivity is unknown and that both private agents and policy makers learn over time about the true state of the economy. In this case, the benefits to responding to asset prices may depend on both the rate of learning, the information structure of the economy and the extent to which asset price movements cause distortions to economic activity through the financing mechanism described above.

To address these issues, we reconsider the design of monetary policy in an environment where asset prices reflect expectations about underlying changes in the trend growth rate of technology. Our economy is a standard New Keynesian framework augmented to include financial market imperfections through the financial accelerator mechanism described in Bernanke, Gertler and Gilchrist (1999) (henceforth BGG). In our framework, private agents and policy makers are uncertain about trend growth but gradually learn over time. This learning process is reflected in asset price movements. Revisions to expectations owing to learning influences asset prices and entrepreneurial net worth. Such revisions feedback into investment demand and are magnified through the financial accelerator mechanism.

Our findings reinforce previous results in the literature. In the absence of financial frictions, a policy of strong inflation targeting is optimal, even in situations where agents are uncertain about the true state of growth in the economy. In the absence of financial frictions, our economy has essentially one distortion, owing to variations in the markup, which influences input choices. Suppressing inflation stabilizes the markup. Adding asset prices to the monetary policy rule is unlikely to provide further benefits, even in situations where private agents are uninformed about the true state of growth of the economy.

In the presence of financial market imperfections, a strong inflation targeting policy eliminates much of the distortionary effect of asset price movements on economic activity. Nonetheless, with inflation stabilized, the economy still exhibits significant deviations of output from potential. By putting weight on asset prices in the monetary policy rule, the monetary authority can improve upon these outcomes. Stabilizing output relative to potential comes at the cost of increased volatility of inflation however. Thus, as in Dupor, the monetary authority faces a tradeoff owing to its desire to eliminate two distortions with one instrument.

Our policy analysis emphasizes the benefits to responding to an asset price gap – the deviation between the current asset price and the potential asset price that would occur in a flexible price economy without financial frictions. Computing such a gap requires the policy maker to make inferences regarding the true state of growth. We can thus distinguish between situations where the monetary authority has perfect information regarding underlying growth rates and situations where the policy maker is learning over time. We can similarly distinguish between environments where private agents are fully informed or are uninformed and learning over time.

Our results imply that the benefits to responding to asset prices depend on the information structure of the economy. The benefits of responding to the asset price gap are largest when the private sector is uninformed about the true state of the economy but policy makers are informed. At the other extreme, responding to the asset price gap may be detrimental when private agents are informed and the policy maker is uninformed about the true state of growth of the economy. In this case, the policy maker is responding to the "wrong" asset price gap.

We also consider alternative policies that do not require the policy maker to infer the current state of economic growth. These include responding to either asset price growth or output growth. Our findings suggest that both of these policies are likely to do well in our environment. On the other hand, we find that responding to the level of asset prices, as considered by much of the previous literature, is a particularly bad policy. Thus, the destabilizing effects of responding to asset price movements emphasized in previous studies may in part reflect the assumption that the monetary authority responds to the level of asset prices rather than their deviation from some underlying fundamental. If the latter is unobservable, responding to changes in asset prices is better than responding to the level itself.

1.1 Related Literature

Bernanke and Gertler (1999, 2001), Cecchetti et al. (2000), Gilchrist and Leahy (2002) and Tetlow (2005) introduce non-fundamental bubbles into an economy and study the benefits to allowing the monetary authority to respond to asset prices. According to Bernanke and Gertler, if the monetary authority is able to distinguish between fundamentals and bubbles then a policy of strong inflation targeting stabilizes the economy and asset prices are only useful to the extent that they provide information about inflation and the output gap. In this environment, bubbles are exogenous and affect the economy by increasing aggregate demand through a financial accelerator mechanism. A strong inflation targeting policy is sufficient to suppress this aggregate demand channel. Cecchetti et al. (2000) argue that there may be some benefit to responding to asset prices in such environments though it is likely small. This literature suggests that strong inflation targeting is likely to be a good policy even under two situations in which asset prices should contain a relatively large amount of information: an economy with financial frictions and shocks to net worth (Cecchetti et al. (2000)); an economy with news shocks that have a persistent impact on technology growth (Gilchrist and Leahy (2002)).

Our framework differs from this analysis in two fundamental ways. First, in our economy, deviations between asset prices and underlying cash flows occur because agents do not know the true state of growth of the economy but instead are learning about it over time. Recent studies by French (2001), Roberts (2001) and Kahn and Rich (2003) emphasize the distinction between transitory and persistent movements in the growth rate of the technology. Edge, Laubach and Williams (2004) study the effect of learning about transitory and persistent movement in technology growth in a model-based environment. As an example of such learning, these authors document that the productivity growth forecasts of professional forecasters and policy makers did not change until 1999, although the trend had shifted in the mid 1990s. They also demonstrate that a constant-gain Kalman filter tracks the actual forecasts of trend productivity in the 1970s and in the 1990s made by forecasters and policy makers well. Edge, Laubach and Williams (2004) apply constant gain learning in an RBC model to understand the effect of changes in the growth rate of technology on movements in real interest rates, output and hours. Our paper is also related to Tambalotti (2003) who considers the role of learning in a DSGE model with price rigidities but no capital accumulation, and Dupor (2005) who studies an environment where agents learn about fundamental and non-fundamental shocks to the return on capital.

Our framework is closely related to Edge, Laubach and Williams (2005) who allow for constant gain learning about trend productivity growth in a DSGE model with price rigidities and capital accumulation. We extend their framework by allowing both private agents and policy makers to learn about the true state of technology. We do so in an environment where learning influences asset values which feed back into the economy through the net worth channel emphasized by BGG. We show that this financial accelerator mechanism may be enhanced in the presence of learning. This stronger feedback mechanism raises the benefit to responding to asset prices, even in environments where the policy maker is itself uninformed about the true state of the economy.

Second, much of the previous literature focusses on the benefits to responding to the level of asset prices. In our framework, asset prices movements would occur in the absence of frictions in either information, price-setting or financial markets. Thus, we emphasize the importance of the monetary authority responding to asset prices relative to some underlying value such as the "asset price gap". Our finding that targeting the growth rate of asset prices is also beneficial is related to Tetlow (2005) who compares the benefit of responding to the growth rate of asset prices relative to the level of asset prices in a robust control framework. By emphasizing, the asset price gap as a variable in the monetary policy rule, we are able to study the benefits to responding to asset prices in situations where the monetary authority knows the true state of the economy compared to environments where the monetary authority is learning about the true state. We can also study the effect of asymmetries in information between the private sector and the policy maker on economic outcomes and the relative gains to various policy rules.

Our emphasis on asset price movements that are tied to fundamental changes in the underlying growth rate of the economy is related to the recent literature on the response of asset prices to news about future growth. Barsky and DeLong (1990, 1993) and Kiyotaki (1990) study the effects of learning about the persistent component of dividend growth on asset prices in a partial equilibrium model. When transitory and persistent shocks to dividend growth are not observed separately, investors extrapolate a transitory movement in dividend growth into the future, generating a large response in asset prices for a given change in dividend. The interest rate is fixed in these partial equilibrium models which helps to generate large movements in asset prices. Kiley (2000) provides a comparison of the asset pricing implications of partial versus general equilibrium models. Asset prices tend to fall in response to increases in the growth rate of dividends as interest rates rise in general equilibrium .

In a real business cycle framework that allows for capital accumulation, the increase in desired household consumption and leisure implies a rise in real interest rates and a reduction in investment and asset returns at the onset of a technology-driven increase in growth. Using a New Keynesian model, Gilchrist and Leahy (2002) show that asset prices may rise rather than fall in response to an increase in the future growth rate of technology, this positive response relies on an accommodative monetary policy characterized by weak inflation targeting. More recently, Christiano, Motto and Rostagno (2005) emphasize the role of monetary policy in generating an asset price boom in a model with habit formation and adjustment costs to investment. In their model, a favorable shock to technology tends to lower inflation and interest rates which raises asset values. Jaimovich and Rebelo (2006) consider real business cycle environments that may produce asset price booms following news about technology. In our framework, as in Gilchrist and Leahy (2002) asset prices are more likely to rise in response to news about future technology in the presence of financial frictions and accommodative monetary policy.

Finally, there is rich literature emphasizing the welfare benefits of monetary policy rules in environments that study learning and environments that allow for financial frictions. Dupor (2004) and Edge, Laubach and Williams (2005) solve a Ramsey problems to study optimal monetary policy while Tambalotti (2003) uses a second-order approximation to the utility function in a model without capital. More closely related to our work, Faia and Monacelli (2005) use a second-order approximation to the policy function in the BGG framework. Faia and Monacelli argue that including the level of asset prices in the interest rate rule with a modest coefficient increases welfare when the coefficient on inflation is relatively small. When the coefficient on inflation is large enough then including asset price in the policy rule does not change welfare. Although we focus on a quadratic loss function rather than formal welfare analysis, our results imply modest gains to responding to the asset price gap, even when the monetary authority is responding strongly to inflation. This difference in results may be partially attributable to our emphasis on asset price gaps rather than asset price levels as the variable in the policy rule.[3]

[3] Our findings that including the growth rate of asset prices or output growth also performs well is related to Orphanides and Williams (2002) who argue that in environments where the natural rate of output is unobservable, an interest rate rule which includes the growth rate of output (which does not require the knowledge of natural rates) is a robust policy.

2 Model

The core framework is a dynamic stochastic New Keynesian model. We include a financial accelerator mechanism, as developed in BGG. The financial accelerator mechanism links the condition of borrower balance sheets to the terms of credit, and hence to the demand for capital. Via the impact on borrower balance sheets, the financial accelerator magnifies the effects of shocks to the economy. Unanticipated movements in asset prices provide the main source of variation in borrower balance sheets.

2.1 Structure of the Economy

Within the model there exist both households and firms. There is also a government sector. Households work, save, and consume. There are three types of producers: (i) entrepreneurs; (ii) capital producers; and (iii) retailers. Entrepreneurs manage the production of wholesale goods. They borrow from households to finance the acquisition of capital used in the production process. Due to imperfections in the capital market, entrepreneurs' demand for capital depends on their respective financial positions - this is the key aspect of the financial accelerator. In turn, in response to entrepreneurial demand, capital producers build new capital. Finally, retailers package together wholesale goods to produce final output. They are monopolistically competitive and set nominal prices on a staggered basis. The role of the retail sector in our model is simply to provide the source of nominal price stickiness.

We now proceed to describe the behavior of the different sectors of the economy, along with the key resource constraints.

2.1.1 Households

Let C_t denote consumption, H_t denote household labor, and M_t/P_t denote real balances. Household preferences are given by:

$$E_t \left(\sum_{t=0}^{\infty} \beta^s U(C_t, H_t, M_t/P_t) \right) \tag{1}$$

with

$$U\left(C_t, H_t, \frac{M_t}{P_t}\right) = \ln C_t - \theta \frac{H_t^{1+\gamma}}{1+\gamma} + \xi \ln \frac{M_t}{P_t} \tag{2}$$

with $\theta, \xi \geq 0$.

Let W_t denote the nominal wage, Π_t real dividend payments from ownership of retail firms, T_t lump sum taxes, and B_{t+1} nominal bonds, then the household budget constraint is

$$C_t = \frac{W_t}{P_t} H_t + \Pi_t - T_t - \frac{M_t - M_{t-1}}{P_t} - \frac{B_{t+1} - R_{t+1}^n B_t}{P_t} \tag{3}$$

The household maximizes 1 subject to 3.

The optimality conditions for labor supply and the consumption/savings decision are

$$\frac{1}{C_t} = \beta E_t \left[\frac{1}{C_{t+1}} R_{t+1}^n \frac{P_t}{P_{t+1}} \right] \tag{1}$$

$$\frac{1}{C_t} \frac{W_t}{P_t} = \theta H_t^{\gamma} \tag{4}$$

where γ measures the inverse of the labor supply elasticity.

2.1.2 Entrepreneurs, Finance, and Wholesale Production

Entrepreneurs manage production and obtain financing for the capital employed in the production process. Entrepreneurs are risk neutral. To ensure that they never accumulate enough funds to fully self-finance their capital acquisitions, we assume they have a finite expected horizon. Each survives until the next period with probability η. Accordingly, the expected horizon is $1/(1-\eta)$. The entrepreneurs' population is stationary, with new entrepreneurs entering to replace those who exit. To ensure that new entrepreneurs have some funds available when starting out, we follow BGG by endowing each entrepreneur with H_t^e units of labor which is supplied inelastically as a managerial input to production. Entrepreneurs receive a small wage in compensation.

The entrepreneur starts any period t with capital, K_t, acquired in the previous period (shortly we describe the capital acquisition decision.) He then produces Y_t, using labor, L_t, and capital K_t. (For notational simplicity we omit entrepreneur-specific indices.) The labor input L_t is assumed to be a composite of household and managerial labor: $L_t = H_t^{e(\Omega)} H_t^{1-\Omega}$. We normalize H_t^e to unity. The entrepreneur's gross project output, GY_t consists of the sum of his production revenues and the market value of his remaining capital stock. In addition, we assume his project is subject to an idiosyncratic shock, ω_t, that affects both the production of new goods and the effective quantity of his capital. The shock ω_t may be considered a measure of the quality of his overall capital investment.

Let $P_{W,t}$ be the nominal price of wholesale output, Q_t the real market price of capital, δ the depreciation rate, and A_t a measure of multifactor productivity which is common to all entrepreneurs. Then, by definition, GY_t, equals the sum of output revenues, $(P_{W,t}/P_t)Y_t$, and the market value of the undepreciated capital stock, $Q_t\omega_t(1-\delta)K_t$:

$$GY_t \equiv \frac{P_{W,t}}{P_t}Y_t + Q_t\omega_t(1-\delta)K_t \tag{5}$$

where wholesale good production, Y_t, is given by [4]

$$Y_t = \omega_t(A_tL)^{\alpha}K_t^{1-\alpha} \tag{6}$$

We assume that ω_t is an i.i.d. (across firms and time) random variable, distributed continuously with mean equal to one, i.e. $E\{\omega_t\} = 1$.

At time t, the entrepreneur chooses labor and capital to maximize profits, conditional on K_t, A_t and ω_t. Accordingly, labor demand satisfies

$$\begin{aligned} \alpha(1-\Omega)\frac{Y_t}{H_t} &= \frac{W_t}{P_{W,t}} \\ \alpha\Omega\frac{Y_t}{H_t^e} &= \frac{W_t^e}{P_{W,t}} \end{aligned} \tag{7}$$

where W_t^e is the managerial wage.

We now consider the capital acquisition decision. At the end of period t, the entrepreneur purchases capital, K_{t+1}, which can be used in the subsequent period $t+1$ to produce output at that time. The entrepreneur finances the acquisition of capital partly with his own net worth available at the end of period t, N_{t+1}, and partly by issuing nominal bonds, B_{t+1}. Then capital financing is divided between net worth and debt, as follows:

[4] For technical convenience we assume that fixed costs are borne by the retail sector rather than the wholesale sector.

$$Q_t K_{t+1} = N_{t+1} + \frac{B_{t+1}}{P_t}. \tag{8}$$

The entrepreneur's net worth is essentially the equity of the firm, i.e., the gross value of capital net of debt, $Q_t K_{t+1} - (B_{t+1}/P_t)$. The entrepreneur accumulates net worth through past earnings, including capital gains. We assume that new equity issues are prohibitively expensive, so that all marginal finance is obtained through debt.

The entrepreneur's demand for capital depends on the expected marginal return and the expected marginal financing cost. The marginal return to capital (equal to the expected average return due to constant returns) is next period's ex-post gross output net of labor costs, normalized by the period t market value of capital:

$$R^k_{t+1} = \frac{\omega_{t+1}\left[\frac{P_{W,t+1}}{P_{t+1}}(1-\alpha)\frac{\overline{Y}_{t+1}}{K_{t+1}} + (1-\delta)Q_{t+1}\right]}{Q_t}$$

where $\overline{Y}_{t+1}$ is the average level of output per entrepreneur (i.e., $Y_{t+1} = \omega_{t+1}\overline{Y}_{t+1}$). Note that the marginal return varies proportionately with the idiosyncractic shock ω_{t+1}. Since $E_t\omega_{t+1} = 1$, we can express the expected marginal return simply as

$$E_t R^k_{t+1} = \frac{E_t\left[\frac{P_{W,t+1}}{P_{t+1}}(1-\alpha)\frac{\overline{Y}_{t+1}}{K_{t+1}} + (1-\delta)Q_{t+1}\right]}{Q_t} \tag{9}$$

The marginal cost of funds to the entrepreneur depends on financial conditions. We postulate an agency problem that makes uncollateralized external finance more expensive than internal finance. As in BGG, we assume a costly state verification problem. The idiosyncratic shock ω_t is private information for the entrepreneur, implying that the lender cannot freely observe the project's gross output. To observe this return, the lender must pay an auditing cost - interpretable as a bankruptcy cost - that is a fixed proportion μ_b of the project's ex-post gross payoff, $R^k_{t+1}Q_t K_{t+1}$. The entrepreneur and the lender negotiate a financial contract that: (i) induces the entrepreneur not to misrepresent his earnings; and (ii) minimizes the expected deadweight agency costs (in this case the expected auditing costs) associated with this financial transaction.

We restrict attention to financial contracts that are negotiated one period at a time and offer lenders a payoff that, in nominal terms, is independent of aggregate risk. Under these assumptions, it is straightforward to show that the optimal contract takes a very simple and realistic form: a standard debt with costly bankruptcy. If the entrepreneur does not default, the lender receives a fixed payment independent of ω_t. If the entrepreneur defaults, the lender audits and seizes whatever it finds.[5] It is true that we are arbitrarily ruling out the possibility of either entrepreneurs or households obtaining insurance against aggregate risks to their wealth.[6] We simply appeal to realism and features outside the model (e.g. difficulties in enforcing wealth transfers in bad times) to rule out aggregate state-contingent wealth insurance.

Overall, the agency problem implies that the opportunity cost of external finance is more expensive than that of internal finance. Because the lender must receive a competitive return, it charges the borrower a premium to cover the expected bankruptcy costs. Because the external finance premium affects the overall cost of finance, it therefore influences the entrepreneur's demand for capital.

In general, the external finance premium varies inversely with the entrepreneur's net worth: the greater the share of capital that the entrepreneur can either self-finance or finance with collateralized debt, the smaller the expected bankruptcy costs and, hence, the smaller the external finance premium. Rather than present the details of the agency problem here, we simply observe, following BGG, that the external finance premium, $s_t(\cdot)$, may be expressed as an increasing function of the leverage ratio, and hence the ratio of capital expenditures to net worth, $(Q_t K_{t+1})/N_{t+1}$:

$$\begin{aligned} s_t(\cdot) &= s\left(\frac{Q_t K_{t+1}}{N_{t+1}}\right) \\ s'(\cdot) &> 0,\ s(1)=1,\ s(\infty)=\infty. \end{aligned} \tag{10}$$

The specific form of $s_t(\cdot)$ depends on the primitive parameters of the costly state verification problem, including the proportional bankruptcy cost μ_b and the distribution of the idiosyncratic shock ω_t. In addition, note that $s_t(\cdot)$ depends only on the aggregate

[5]The loan contract guarantees the lender an average return (across contracts) which is nominally riskless. To be consistent with this assumption, the non-default payment to the lender depends on the expost realization of prices.

[6]Within our framework both entrepreneurs and households would like to hedge against aggregate shocks to their wealth: entreprenuers because the shadow value of wealth is countercyclical due to the credit market frictions and households because they are risk averse. Previous experiments suggest that under our baseline calibration insurance would likely flow on net from entrepreneurs to households. This would work to enhance the financial accelerator since it would imply a stronger procyclical movement in entrepreneurial balance sheets.

leverage ratio and not on any entrepreneur-specific variables. This simplification arises because, in equilibrium, all entrepreneurs choose the same leverage ratio, which owes to having constant returns in both production and bankruptcy costs due to risk neutrality (see, e.g., Carlstrom and Fuerst (1997) and BGG).

By definition, the entrepreneur's overall marginal cost of funds in this environment is the product of the gross premium for external funds and the gross real opportunity cost of funds that would arise in the absence of capital market frictions. Accordingly, the entrepreneur's demand for capital satisfies the optimality condition

$$E_t R_{t+1}^k = s_t\left(\cdot\right) E_t\left\{R_{t+1}^n \frac{P_t}{P_{t+1}}\right\} \tag{11}$$

where $E_t\{R_{t+1}^n(P_t/P_{t+1})\}$ is the gross cost of funds absent capital market frictions.

Equation (11) is interpretable as follows: at the margin, the entrepreneur considers acquiring a unit of capital financed by debt. The additional debt, however, raises the leverage ratio, increasing the external finance premium and the overall marginal cost of finance. Relative to perfect capital markets, accordingly, the demand for capital is lower, the exact amount depending on $s_t\left(\cdot\right)$.[7]

Equation (11) provides the basis for the financial accelerator. It links movements in the borrower financial position to the marginal cost of funds and, hence, to the demand for capital. Note, in particular, that fluctuations in the price of capital, Q_t, may have significant effects on the leverage ratio, $(B_{t+1}/P_t)/N_{t+1} = (B_{t+1}/P_t)/(Q_t K_{t+1} - (B_{t+1}/P_t))$. In this way, the model captures the link between asset price movements and financial conditions.

The other key component of the financial accelerator is the relation that describes the evolution of entrepreneurial net worth, N_{t+1}. Let V_t denote the value of entrepreneurial firm capital net of borrowing costs carried over from the previous period. This value is given by

$$V_t = R_t^k Q_{t-1} K_t - \left[s_{t-1}(\cdot) R_t^n \frac{P_{t-1}}{P_t}\right] \frac{B_t}{P_{t-1}}. \tag{12}$$

In this expression, R_t^k is the ex-post real return on capital, and $s_{t-1}(\cdot)R_t^n(P_{t-1}/P_t)$ is the ex-post cost of borrowing. Net worth may then be expressed as a function of V_t

[7] While we use the costly state verification problem to derive a parametric form for $s_t(\cdot)$, we note, however, that the general form relating external finance costs to financial positions arises across a broad class of agency problems.

and the managerial wage, (W_t^e/P_t),

$$N_{t+1} = \eta V_t + W_t^e/P_t \tag{13}$$

where the weight η reflects the number of entrepreneurs who survive each period.[8]

As equations (12) and (13) suggest, the main source of movements in net worth stems from unanticipated movements in returns and borrowing costs. In this regard, unforecastable variations in the asset price Q_t likely provide the principle source of fluctuations in R_t^k. It is for this reason that unpredictable asset price movements play a key role in the financial accelerator.

Entrepreneurs going out of business at time t consume their remaining resources. Let C_t^e denote the amount of the consumption composite consumed by the exiting entrepreneurs. Then

$$C_t^e = (1-\eta)V_t \tag{14}$$

is the total amount of equity that exiting entrepreneurs remove from the market.

2.1.3 Capital Producers

To construct new capital, producers use both investment goods and existing capital, which they lease from entrepreneurs. Each capital producer operates a constant returns to scale technology $\Phi(I_t/K_t)K_t$. Consistent with the notion of adjustment costs for investment, $\Phi(\cdot)$ is increasing and concave. Under constant returns to scale, the resulting economy-wide capital accumulation equation is

$$K_{t+1} = (1-\delta)K_t + \Phi(\frac{I_t}{K_t})K_t. \tag{15}$$

Individual capital producers choose inputs I_t and K_t to maximize expected profits from the construction of new investment goods. New capital goods are sold at a price Q_t. The optimality condition for net investment satisfies

$$Q_t\Phi'\left(\frac{I_t}{K_t}\right) = 1. \tag{16}$$

[8] In our quantitative exercises, W_t^e is of negligible size, and the dynamics of N_{t+1} is determined by V_t.

Equation (16) is a standard "Q-investment" relation[9]. The variable price of capital, though, plays an additional role in this framework: as we have discussed, variations in asset prices will affect entrepreneurial balance sheets, and hence, the cost of capital.

2.1.4 Retailers, Price-Setting, and Inflation

We assume there is a continuum of monopolistically competitive retailers of measure unity. Retailers buy wholesale goods from entrepreneurs/producers in a competitive manner and then differentiate the product slightly (e.g., by painting it or adding a brand name) at zero resource cost. We assume that the fixed (from the retailers' point of view) resource cost represents distribution and selling costs that are assumed to be proportional to the steady-state value of wholesale output.

Let $Y_t(z)$ be the good sold by retailer z. Final output is a CES composite of individual retail goods:

$$Y_t = \left[\int_0^1 Y_t(z)^{\frac{\vartheta-1}{\vartheta}} dz \right]^{\frac{\vartheta}{\vartheta-1}}. \tag{17}$$

The corresponding price of the composite final good, P_t, is given by $P_t = \left[\int_0^1 P_t(z)^{1-\vartheta} dz \right]^{\frac{1}{1-\vartheta}}$.

Households, capital producers, and the government buy final goods from retailers. Cost minimization implies that each retailer faces an isoelastic demand for his product given by $Y_t(z) = (P_t(z)/P_t)^{-\vartheta} Y_t$. Since retailers simply repackage wholesale goods, the real marginal cost to the retailers of producing a unit of output is simply the relative wholesale price, $(P_{W,t}/P_t)$.

As we have noted, the retail sector provides the source of nominal stickiness in the economy. We assume retailers set nominal prices on a staggered basis, following the approach in (Calvo 1983): each retailer resets his price with probability $(1 - \upsilon)$, independently of the time elapsed since the last adjustment. Thus, each period a measure $(1 - \upsilon)$ of producers reset their prices, while a fraction υ keeps their prices unchanged. Accordingly, the expected time a price remains fixed is $1/(1 - \upsilon)$. Thus, for example, if $\upsilon = .75$ per quarter, prices are fixed on average for a year.

[9] The second input into production, K_t, is required to preserve constant returns to scale. Let r_t^l denote the lease rate for existing capital; then profits equal $Q_t \Phi(I_t^n/K_t) K_t - I_t^n - r_t^l K_t$. The optimality condition for the choice of K_t determines the equilibrium lease rate r_t^l: $Q_t \left(\Phi\left(\frac{I_t^n}{K_t}\right) - \Phi'\left(\frac{I_t^n}{K_t}\right) \frac{I_t^n}{K_t} \right) = r_t^l$. At the steady-state, there are no adjustments costs so that $\Phi(0) = \Phi'(0) = 0$. As a result, lease payments $r_t^l K_t$ are second-order and are negligible in terms of both steady-state and model dynamics.

Since there are no firm-specific state variables, all retailers setting price at t will choose the same optimal value, P_t^*. The price index evolves according to

$$P_t = \left[v P_{t-1}^{1-\vartheta} + (1-v) \left(P_t^* \right)^{1-\vartheta} \right]^{\frac{1}{1-\vartheta}}. \tag{18}$$

Retailers free to reset choose prices to maximize expected discounted profits, subject to the constraint on the frequency of price adjustments. The optimal price is

$$P_t^* = \frac{\vartheta}{\vartheta - 1} \frac{E_t \sum_{i=0}^{\infty} v^i \Lambda_{t,i} P_{t+i}^W Y_{t+i} \left(\frac{1}{P_{t+i}} \right)^{1-\vartheta}}{E_t \sum_{i=0}^{\infty} v^i \Lambda_{t,i} Y_{t+i} \left(\frac{1}{P_{t+i}} \right)^{1-\vartheta}}, \tag{19}$$

where $\Lambda_{t,i} \equiv \beta^i \frac{U_{C_{t+i}}}{U_{C_t}} = \beta^i \frac{C_{t+i}}{C_t}$, and $\frac{\vartheta}{\vartheta-1}$ is the retailers' desired gross mark-up over wholesale prices. Note that if retail prices were perfectly flexible, equation (19) would simply imply $P_t^* = \frac{\vartheta}{\vartheta-1} P_{W,t}$, i.e., the retail price would simply be a proportional mark-up over the wholesale price. However, because their prices may be fixed for some time, retailers set prices based on the expected future path of marginal cost, and not simply on current marginal cost.

Combining equations (18) and (19) yields an expression that relates current inflation to real marginal cost and expected inflation, as described in the Appendix.

2.1.5 Resource Constraint

The economy-wide resource constraint is

$$Y_t = C_t + C_t^e + I_t + G_t \tag{20}$$

where G_t is government consumption and C_t^e is entrepreneurial consumption.

2.1.6 Government Budget Constraint

We assume that government expenditures are financed by lump-sum taxes and money creation as follows:

$$G_t = \frac{M_t - M_{t-1}}{P_t} + T_t. \tag{21}$$

Government expenditures are exogenous. Lump sum taxes adjust to satisfy the government budget constraint. Finally, the money stock depends on monetary policy, which we specify next.

2.1.7 Monetary Policy

We consider the following types of interest rate rule.

Interest Rate Rule with the Asset Price Gap

$$R_{t+1}^n = R^n \pi_t^{\phi_\pi} \left(\frac{Q_t}{Q_t^*} \right)^{\phi_Q}$$

where $\pi_t \equiv \frac{P_t}{P_{t-1}}$ is the gross inflation, R^n is the zero-inflation steady state level of nominal interest on one-period bond. We assume that the policy maker targets 0% inflation. Q_t^* is the flexible-price levels of Q *in the absence of financial accelerator.*

Q_t^* is computed under the information available to the policy maker. As we define below, we consider both the case of full information and imperfect information about the shocks to technology. When the policy maker has full information, we use $Q_{full,t}^*$ which is obtained by solving a flexible-price model under full information. When policy maker has imperfect information we use $Q_{imp,t}^*$ which is obtained by solving a flexible-price model under imperfect information.

There are two ways to construct potential Q. In the first, one could use the hypothetical levels of state variables that exist when the economy has been under flexible prices to compute Q. In the second, one may use the actual levels of state variables in the sticky-price economy and the decision rule under flexible prices. We currently adopt the first procedure. Neiss and Nelson (2003) uses the first and Woodford (2003) argue that the second approach is more realistic. The first is somewhat easier to work with however.[10]

Interest Rate Rule with Output Growth or Asset Price Growth The "gap" formulation of the policy rule requires policy makers to compute Q_t^*, the level of asset prices that would prevail in the flexible price economy absent the financial accelerator.

[10] We are currently investigating the robustness of our conclusions to the alternative approach.

An alternative would be to allow the policy maker to respond to the growth rate of asset prices: and actual output

$$R^n_{t+1} = R^n \pi_t^{\phi_\pi} \left(\frac{Q_t}{Q_{t-1}} \right)^{\phi_Q}$$

For comparison purposes, we also consider a monetary policy rule that responds to the growth rate of output:

$$R^n_{t+1} = R^n \pi_t^{\phi_\pi} \left(\frac{Y_t}{\mu Y_{t-1}} \right)^{\phi_Y}$$

where μ is the mean growth rate of technology.

Interest Rate Rule with Asset Price Level

$$R^n_{t+1} = R^n \pi_t^{\phi_\pi} \left(\frac{Q_t}{Q} \right)^{\phi_Q}$$

where $Q = 1$ is the nonstochastic steady state level of Tobin's Q.

2.1.8 Technology Shock Process

The growth rate of technology has both transitory and persistent components:

$$\ln A_t - \ln A_{t-1} = \mu_t + \varepsilon_t$$

The persistent component of technology (drift) in deviation from the mean μ follows an AR(1) process:

$$(\mu_t - \mu) = \rho_d(\mu_{t-1} - \mu) + v_t$$

Shocks to the transitory and persistent components are:

$$\begin{aligned} \varepsilon_t &\sim i.i.d.N(0, \sigma_\varepsilon^2) \\ v_t &\sim i.i.d.N(0, \sigma_v^2) \end{aligned}$$

2.2 Information Structure

Our technology process allows for two sources of variation: transitory shocks to the growth rate of technology, ε_t, and persistent shocks to the growth rate of technology, v_t.

We consider both full information structures where agents can fully observe both shocks separately and imperfect information structures where agents can observe the actual technology series, A_t, but cannot decompose movements in A_t into their respective sources.

Let $Z_t \equiv \frac{A_t}{A_{t-1}}$ denote technology growth, $\widetilde{z}_t \equiv \ln Z_t - \ln Z = \ln Z_t - \mu$ denote the % deviation of technology growth from the mean, and $\widetilde{d}_t \equiv \mu_t - \mu$ denote the % deviation of the persistent component of technology growth from the mean. Then we can write the technology process as:

$$\begin{aligned} \widetilde{z}_t &= \widetilde{d}_t + \varepsilon_t \\ \widetilde{d}_t &= \rho_d \widetilde{d}_{t-1} + \nu_t \end{aligned}$$

Under full information, agents observe both the shock to transitory component of technology growth, ε_t, and the shock to the persistent component of technology growth, v_t. Under imperfect information, agents observe $\widetilde{z}_t$ or the sum of two components, $\widetilde{d}_t + \varepsilon_t$, but never observe the two shocks (v_t, ε_t) separately.

2.3 Filtering under Imperfect Information

Let $E[\widetilde{d}_t|\widetilde{z}_t, \widetilde{z}_{t-1,...}] \equiv \widetilde{d}_{t|t}$ denote the inference of agents about current $\widetilde{d}_t$ based on the observations of current and past technology growth. We assume agents update inferences based on the steady-state Kalman filter:

$$\widetilde{d}_{t|t} = \lambda \widetilde{z}_t + (1-\lambda)\rho_d \widetilde{d}_{t-1|t-1}$$

where the gain parameter, λ, is given by

$$\lambda \equiv \frac{\phi - (1-\rho_d^2) + \phi\sqrt{(1-\rho_d^2)^2 \frac{1}{\phi^2} + 1 + \frac{2}{\phi} + 2\rho_d^2 \frac{1}{\phi}}}{2 + \phi - (1-\rho_d^2) + \phi\sqrt{(1-\rho_d^2)^2 \frac{1}{\phi^2} + 1 + \frac{2}{\phi} + 2\rho_d^2 \frac{1}{\phi}}}$$

and ϕ measures the signal to noise ratio:

$$\phi \equiv \frac{\sigma_v^2}{\sigma_\varepsilon^2}$$

Given $\widetilde{d}_{t|t}$, the inference about the shock to the transitory component of technology growth $E[\varepsilon_{A,t}|\widetilde{z}_t, \widetilde{z}_{t-1},...] \equiv \varepsilon_{A,t|t}$ is given by:

$$\varepsilon_{A,t|t} = \widetilde{z}_t - \widetilde{d}_{t|t}$$

and the inference about the shock to the persistent component of technology growth $E[v_t|\widetilde{z}_t, \widetilde{z}_{t-1},...] \equiv v_{t|t}$ is given by:

$$v_{t|t} = \widetilde{d}_{t|t} - \rho_d \widetilde{d}_{t-1|t-1}$$

It is straightforward to show that the gain, λ, is monotonically increasing in each of the signal to noise ratio, $\phi \equiv \frac{\sigma_v^2}{\sigma_\varepsilon^2}$, and the AR(1) coefficient on the persistent component, ρ_d.

The case of full information for one side and imperfect information for the other side When private agents observe the shocks (v_t, ε_t) but the policy maker does not, we have $\widetilde{d}_{t|t} = \widetilde{d}_t$ and $\varepsilon_{A,t|t} = \varepsilon_{A,t}, v_{t|t} = v_t$ for private agents, and $\widetilde{d}^*_{t|t} = \lambda \widetilde{z}_t + (1-\lambda)\rho_d \widetilde{d}^*_{t-1|t-1}$ and $\varepsilon^*_{A,t|t} = \widetilde{z}_t - \widetilde{d}^*_{t|t}, v^*_{t|t} = \widetilde{d}^*_{t|t} - \rho_d \widetilde{d}^*_{t-1|t-1}$ for the policy maker.

When private agents do not observe the shocks (v_t, ε_t) but the policy maker does, we have $\widetilde{d}_{t|t} = \lambda \widetilde{z}_t + (1-\lambda)\rho_d \widetilde{d}_{t-1|t-1}$ and $\varepsilon_{A,t|t} = \widetilde{z}_t - \widetilde{d}_{t|t}, v_{t|t} = \widetilde{d}_{t|t} - \rho_d \widetilde{d}_{t-1|t-1}$ for private agents and $\widetilde{d}^*_{t|t} = \widetilde{d}_t$ and $\varepsilon^*_{A,t|t} = \varepsilon_{A,t}, v^*_{t|t} = v_t$ for the policy maker.[11]

Properties of the inference about the persistent component of technology growth, $\widetilde{d}_{t|t}$ Agents tend to underestimate the persistence of a movement in technology growth when technology growth changes due to a persistent shock, while they tend to overestimate the persistence of an observed movement in technology growth when technology growth changes due to a transitory shock. Figure 1 illustrates the latter and Figure 2 the former.

Figure 1 shows the effect of a 1% shock to the transitory component of technology growth, ε_t. The dotted line is the actual level of the persistent component of technology growth, $\widetilde{d}_t \equiv \mu_t - \mu$, and the straight line is the inferred level of the persistent component of technology growth, $\widetilde{d}_{t|t}$. Both are in % deviation from the mean growth rate of technology, μ.

[11] In the case of imperfect information for both private agents and policy maker, we could also allow for the gain parameter, λ, to differ across private agents and the policy maker.

Figure 2 shows the effect of a 1% shock to the persistent component of technology growth, v_t, on both actual technology growth and the inference about the persistent component of technology growth.

3 Calibration

We adopt a fairly standard calibration of preferences, technology and the price-setting structure. The financial sector is calibrated to conform to a simplified version of BGG. These simplifications allow us to focus on the main distortion that is introduced by financial market imperfections – the introduction of a counter-cyclical premium on external funds which drives a wedge between the rate of return on capital and the real interest rate.

Preferences, Technology and Price-Setting A period is a quarter. The discount factor is $\beta = 0.984$. Labor share of income is $\alpha = \frac{2}{3}$. The inverse of labor supply elasticity is $\gamma = 0.8$. The depreciation rate is $\delta = 0.025$. Inverse of the elasticity of investment with respect to Q is $\eta_k \equiv -\frac{\phi''(\frac{i}{k}Z)\frac{i}{k}Z}{\phi'(\frac{i}{k}Z)} = 0.25$. For the pricing equation, the steady state markup is $\frac{\varepsilon}{\varepsilon-1} = 1.1$ while the probability that a producer does not adjust prices in a given quarter is $v = 0.75$.

Parameters Related to the Financial Accelerator When log-linearizing the model, we adopt a number of simplifications to the original financial sector specified in BGG. These simplifications allow us to focus on the primary distortion associated with the financial accelerator – namely that it introduces a time-varying counter-cyclical wedge between the household return, R_{t+1}, and the return on capital, R^k_{t+1}. We assume that variation in entrepreneurial consumption and real wage are negligible and can be ignored. We further assume that actual resource costs to bankruptcy are also negligible. Model simulations conducted under the original BGG framework imply that these simplifications are reasonable. The log-linearized model then implies that there are two key financial parameters to choose – the leverage ratio and the elasticity of the premium on external funds with respect to leverage. Increases in either the leverage ratio or the elasticity enhance the financial accelerator mechanism.

The steady state ratio of capital stock to net worth is chosen so that the steady state leverage ratio is 80%, $\frac{QK-N}{N} = 0.8$, which implies $\frac{QK}{N} = 1.8$. We also adopt a simplified function form for the premium on external funds:

$$s_t = \left(\frac{Q_t K_{t+1}}{N_{t+1}} \right)^{\chi}$$

In line with the calibration adopted by BGG, the elasticity of external finance premium with respect to leverage is set to 5%, $\chi = 0.05$. These parameters imply that the nonstochastic steady state level of external finance premium is $s \equiv \left(\frac{QK}{N}\right)^{\chi} = 1.0298$. In the case of no financial accelerator, $\chi = 0$.

Parameters Related to Technology Shocks and Filtering We set the mean technology growth rate at the mean TFP growth rate in the postwar U.S., $\mu = 0.00427$. Standard deviation of the shock to the transitory component of technology growth is $\sigma_{\varepsilon} = 0.01$ and the standard deviation of the shock to the persistent component of technology growth is $\sigma_v = 0.001$. The AR(1) coefficient on the persistent component of technology growth is $\rho_d = 0.95$. Given the variance of technology shocks chosen above, the signal to noise ratio is:

$$\phi \equiv \frac{\sigma_v^2}{\sigma_{\varepsilon}^2} = 0.01$$

Kalman gain parameter, λ, consistent with the choice of the shock parameters above is[12]:

$$\begin{aligned} \lambda &\equiv \frac{\phi - (1-\rho_d^2) + \phi\sqrt{(1-\rho_d^2)^2 \frac{1}{\phi^2} + 1 + \frac{2}{\phi} + 2\rho_d^2 \frac{1}{\phi}}}{2 + \phi - (1-\rho_d^2) + \phi\sqrt{(1-\rho_d^2)^2 \frac{1}{\phi^2} + 1 + \frac{2}{\phi} + 2\rho_d^2 \frac{1}{\phi}}} \\ &= 0.06138 \end{aligned}$$

4 Impulse Responses

In this section, we explore the role of imperfect information and its effect on output, inflation and investment in response to a particular shock to technology. We first

[12] This is within the range of values that are used in the literature. Edge, Laubach, and Williams (2005) use $\lambda = 0.025$ together with $\rho_d = 0.95$, Erceg, Gurrieri. and Gust (2005) use $\lambda = 0.1$ together with $\rho_d = 0.975$, and Tambalotti (2003) uses $\rho_d = 0.93$ together with $\frac{\sigma_v}{\sigma_{\varepsilon}} = 0.08$ which implies the signal to noise ratio of $\phi \equiv \frac{\sigma_v^2}{\sigma_{\varepsilon}^2} = 0.0064$.

consider the effects of a shock to the transitory component of technology growth. We begin with the case where agents have perfect information regarding the state of growth in technology. Within this perfect information environment, we explore the role of a weak versus strong inflation targeting policy, and the benefits to allowing the monetary authority to respond to the asset price gap. We then consider the same policies in the imperfect information environment.

4.1 Transitory Shocks to Technology Growth

4.1.1 Perfect Information for the Private Sector

Figure 3 presents the response of the economy to a 1% transitory shock to technology growth when the policy maker responds weakly to inflation, $\ln R^n_{t+1} = \ln R^n + 1.1 \ln \pi_t$. The figure contains the response of the economy with flexible prices and in the absence of financial frictions (the path denoted by RBC (full)), the economy with price rigidities but without financial accelerator (the path denoted by NK), and the economy with price rigidities and financial frictions (the path denoted by FA). We interpret the first as the potential.

Figure 3 shows that the economy under price rigidities has inflation that exceeds target (zero) and output above potential. The financial accelerator mechanism amplifies the response of output and inflation because a favorable shock to technology raises asset prices and reduces the external finance premium. This amplified response represents distortions in the economy activity. Asset prices and investment, variables that are closely linked to the financial accelerator mechanism, deviate from their efficient levels by a larger amount in the presence of the financial accelerator.

Figure 4 presents the response of the economy to the same shock when the policy maker aggressively responds to inflation, $\ln R^n_{t+1} = \ln R^n + 2.0 \ln \pi_t$. Output, investment, and asset prices in the sticky-price economy without the financial accelerator come very close to potential under this aggressive inflation targeting policy. In the economy with the financial accelerator, this rule brings the path of inflation close to the target. It also reduces the response of the premium on external funds and reduces the amount of over-investment that occurs. Nonetheless, this policy implies that there are still large deviations bewteen output, asset prices, and investment from their potential levels. These findings imply that a policy that responds aggressively to inflation is successful in decreasing the distortions arising from price rigidities, but is not enough to eliminate the distortions arising from financial market imperfections.

Figure 5 presents the economy's response when policy makers respond to both inflation and the asset price gap, $\ln R^n_{t+1} = \ln R^n + 2.0 \ln \pi_t + 1.5 \ln (Q_t/Q^*_t)$. Allowing the policy maker to respond to the asset price gap further reduces the investment distortion owing to the financial accelerator. As a result, output tracks potential more closely. This comes at the cost of deflation however.

4.1.2 Imperfect Information for the Private Sector

Figures 6-8 report the same exercises for the case of imperfect information on the part of private agents. For the policy that responds to the asset price gap, we assume that the monetary authority has the same information structure as the private agent so that the policy rule is now $\ln R^n_{t+1} = \ln R^n + 2.0 \ln \pi_t + 1.5 \ln \left(Q_t/Q^*_{t,imp}\right)$, where $Q^*_{t,imp}$ denotes the asset price that would occur under flexible prices and no financial accelerator with imperfect information.

With imperfect information, agents put some weight on the possibility that the shock to technology is due to a persistent increase in the growth rate. This growth effect raises desired consumption relative to the case of perfect information. In the real business cycle model under imperfect information, this increase in consumption is accompanied by a rise in real interest rates and reduction in investment spending relative to the case of perfect information. With weak inflation targeting (Figure 6), the rise in real interest rates is smaller than under flexible prices and consumption rises sharply without inducing an offsetting fall in investment. The accommodative stance of monetary policy also contributes to rising asset prices and a reduction in the premium on external funds. These combined effects imply a larger increase in output than what is observed in the case of perfect information. The inflation response is also much larger in the imperfect information case.

The less accommodative monetary policy implied by strong inflation targeting (Figure 7) is again very beneficial, leading to reductions in the response of both the markup and the external finance premium. In the sticky-price model without the financial accelerator, the model with imperfect information comes close to tracking the full-information RBC outcome. The model still implies distortions owing to the financial accelerator however, and as a result, there are benefits to responding to the asset price gap (Figure 8). By responding to the asset price gap, the premium on external funds is reduced and the over-investment that occurs because of the financial accelerator is largely eliminated. Output tracks potential more closely but this once again occurs at the cost of deflation.

Overall, the financial accelerator has similar effects on the premium on external funds under imperfect information as it does under perfect information. In response to a transitory shock, the primary effect of imperfect information is to cause a consumption boom which leads to increases in output and inflation. Although such a consumption boom can also influence asset prices and investment demand, imperfect information leads to an offsetting desire to wait to invest in response to a perceived increase in the growth rate of technology. As a result, the investment distortions owing to the financial accelerator are only slightly larger under imperfect information and weak inflation targeting, and are slightly lower under imperfect information and strong inflation targeting. In both cases, we find benefits to strong inflation targeting. In both cases, allowing the monetary authority to respond to asset prices reduces the over-investment that occurs because of the drop in the premium on external funds that occurs in response to asset price increases. Because responding to asset prices also produces deflation, the overall benefits will depend on the severity of financial distortions and the degree of price stickiness.

4.2 Persistent Shocks to Technology Growth

We now consider the effect of a persistent shock to the growth rate of output. We begin with the case of perfect information and then report the results obtained under imperfect information. We again consider policy rules that include weak inflation targeting $\ln R^n_{t+1} = \ln R^n + 1.1 \ln \pi_t$, strong inflation targeting $\left(\ln R^n_{t+1} = \ln R^n + 2.0 \ln \pi_t\right)$, and a rule that allows the monetary authority to respond to the asset price gap $\left(\ln R^n_{t+1} = \ln R^n + 2.0 \ln \pi_t + 1.5 \ln \left(Q_t/Q^*_t\right)\right)$.

4.2.1 Perfect Information for the Private Sector

Figure 9 plots the response to the persistent shock to the growth rate under weak inflation targeting. We again show impulse responses for the model without distortions (RBC), the model with sticky prices but no financial accelerator (NK) and the model with sticky prices and the financial accelerator (FA). With no distortions, the growth rate shock implies a boom in consumption but an initial fall in investment and asset prices. Over time, investment and asset prices rise. Adding sticky prices reduces the fall in investment and raises the output response. Inflation rises by 15 percentage points in this case. The financial accelerator causes a sharp drop in the premium on external funds, a further increase in investment and a substantial increase in asset prices. Thus, in the financial accelerator model with weak inflation targeting, asset prices rise rather than fall at the onset of the shock. The initial inflation response is also larger now – on the order of 20 percentage points.

Strong inflation targeting policy succeeds at dampening the inflation response (Figure 10). In the model without the financial accelerator, investment and asset prices fall upon impact, and the sticky price model replicates the RBC path. With the financial accelerator, we still observe a reduction in the premium on external funds and some distortions to asset prices and investment however. Allowing the monetary authority to respond to asset prices again provides some benefits in terms of further reducing the distortion in investment spending owing to the financial accelerator (Figure 11). This policy once again produces a disinflation however.

4.2.2 Imperfect Information for the Private Sector

With imperfect information, agents initially put a relatively large weight on the possibility that the growth rate shock is transitory. The initial response is thus closer to what we would observe in the transitory shock case. Over time, agents learn that the shock to the growth rate is persistent and the economic outcomes become more similar to those obtained in the case of a persistent shock under perfect information.

With weak inflation targeting (Figure 12), we again see a large albeit delayed increase in inflation. The markup moves countercyclically so that output is somewhat more procyclical with sticky prices (NK) than would be the case under flexible prices but imperfect information (RBC Imper). The financial accelerator also produces a countercyclical premium on external funds which implies a large distortion in investment spending relative to the RBC outcomes.

Strong inflation targeting (Figure 13) eliminates most of the movement in inflation and produces a response to the sticky price economy (NK) which mimics the RBC model under imperfect information (RBC Imper). Strong inflation targeting also reduces the size of asset price movements and reduces but does not eliminate the response of the premium on external funds.

Allowing the monetary authority to respond to the asset price gap is again beneficial (Figure 14). Such a policy further dampens asset price movements as well as the movements in the premium on external funds. Once again such a policy produces benefits in terms of stabilizing output relative to potential but comes at the cost of disinflation.

Imperfect information magnifies movements in the premium on external funds in response to persistent shocks to the growth rate of technology. These magnification effects are sizeable. For example, with strong inflation targeting, the decline in the premium is twice as large in the case of imperfect information relative to the case of perfect information. Because agents put a relatively low initial weight on the probability that the growth shock is persistent, imperfect information implies a series of positive shocks to expectations regarding the current state of technology. Such positive shocks raise the ex-post return on capital and enhance entrepreneurial net worth. These movements in net worth imply a strong hump-shaped countercylical response to the premium as well as a greater degree of procyclicality in asset prices than would be the case under perfect information. Because the financial accelerator mechanism is strengthened by imperfect information and learning on the part of private agents, we expect that the overall benefits to allowing the monetary authority to respond to asset prices to be greater in the case of imperfect information than the case of perfect information. We now turn to stochastic simulations to explore this issue further.

5 Stochastic Simulations

The previous section computed impulse response functions to technology shocks under alternative monetary policy rules. These results suggest potential benefits to strong inflation targeting as well as to allowing the monetary authority to respond to the asset price gap – the asset price relative to the price that would occur in the flexible price economy. The extent of these benefits depends on the nature of the shock, as well as the information structure of the economy. To further explore these issues, we now conduct stochastic simulations of the various models considered. The stochastic simulations depend on the combined effect of both transitory and persistent shocks to technology. When conducting such simulations, we parameterize the technology shock process in the manner described in our calibration.

5.1 Benefits to Inflation Targeting

We first consider the benefits to inflation targeting. As Bernanke and Gertler (1999) and Gilchrist and Leahy (2002) have emphasized, most of the destabilizing effects of asset prices on monetary policy can be eliminated using a monetary policy rule which responds strongly to inflation. The results emphasized in Bernanke and Gerlter (1999) are derived in an environment where exogenous movements in asset prices (bubbles) cause fluctuations in net worth. These "bubbles" do not alter entrepreneurs perceptions regarding the value of new investment however.

In our environment, misperceptions regarding the true state of technology cause fluctuations in asset values. These misperceptions also influence investment demand. We wish to consider whether the policy prescription of strong inflation targeting is robust to the information environment that we consider. To do so, we compare model performance under the two alternative monetary policy rules – weak inflation targeting

$$\ln R^n_{t+1} = \ln R^n + 1.1 \ln \pi_t$$

versus strong inflation targeting

$$\ln R^n_{t+1} = \ln R^n + 2.0 \ln \pi_t. \tag{22}$$

To compute the benefits to various policy rules, we use stochastic model simulations to compute the variance of output relative to Y^*_{full}, the level of output that would prevail in the flexible price equilibrium without financial frictions but with full information about the shocks to technology, and the variance of inflation. We also compute a loss function based on a weighted average of the variances of output and inflation:

$$Loss = 0.5 var(\ln Y - \ln Y^*_{full}) + 0.5 var(\pi)$$

We report the results of these simulations in table 1.

The first two rows of table one consider an environment where private agents have full information regarding the true growth rate of technology. For comparison purposes, we provide results for the sticky price model without the financial accelerator, as well as the model with the financial accelerator. The variance of inflation and output are reported in percentage points on a quarterly basis.

Strong inflation targeting provides substantial benefits in both the model with and without the financial accelerator. Without the financial accelerator, moving from weak to strong inflation targeting implies large reductions in both the variance of output and inflation. In fact, with strong inflation targeting the variance of output relative to the full employment level is very close to zero (0.0064). The variance of inflation is also very small (0.0438). These numbers are consistent with the model simulations showing that the sticky price model comes very close to reproducing the RBC outcome under a strong inflation targeting policy.

In the model with the financial accelerator, we also see substantial benefits to strong inflation targeting. Both output and inflation volatility are reduced with such a policy. Nonetheless, with the financial accelerator, output volatility is still significant – the variance of output relative to potential is larger with strong inflation targeting and the financial accelerator (0.47) than it is with weak inflation targeting and no financial accelerator (0.43). Thus, in response to technology shocks, the financial accelerator causes substantial increases in output volatility relative to the baseline sticky price model. This finding reinforces the intuition that the model with the financial accelerator has two distortions – one to the markup, and one to the return on capital. Inflation targeting does well at reducing the distortion owing to variation in the markup but does not eliminate the distortion to the return on capital. The presence of this distortion causes an increase in output volatility.

We now consider the role of imperfect information. These results are reported in the second two rows of table 1. Imperfect information implies an increase in the variance of output and a reduction in the variance of inflation. Under weak inflation targeting, the equal weighted loss function is actually lower with imperfect information than it is under perfect information. Because strong inflation targeting is clearly the dominant policy, it provides the more relevant comparison however. Here, the presence of imperfect information implies an increase in the loss function. In the sticky price model without the financial accelerator, the presence of imperfect information has only a small effect on the variances of output and inflation and hence the loss function. In the model with the financial accelerator, imperfect information leads to a large increase in output volatility with very little reduction in the variance of inflation. As a result, with the financial accelerator, the loss function is substantially higher under imperfect information (0.46) than under perfect information (0.26).

5.2 Benefits to Responding to the Asset Price Gap

We now perform stochastic simulations to consider whether a monetary policy that allows the nominal interest rate to respond to asset prices can improve upon a policy that simply targets inflation. Previous work analyzing the benefits of responding to asset prices have focussed on including the level of the asset price (Tobin's Q) in the monetary policy rule. As we document below, this Q-level based policy does not allow for the fact that asset prices move endogenously in the flexible price model. Following the output gap literature, we specify a policy rule which allows the monetary authority to respond to the gap between the current value of Tobin's Q and Q_t^*, the value of Tobin's Q implied by the flexible price equilibrium.

Because we have already shown strong inflation targeting to be beneficial, we restrict our attention to the case where the monetary authority responds strongly to inflation and then consider the additional gains from responding to the asset price gap, $\ln Q_t - \ln Q_t^*$:

$$\ln R_{t+1}^n = \ln R^n + 2.0 \ln \pi_t + \phi_Q (\ln Q_t - \ln Q_t^*) \tag{23}$$

We report results varying the coefficient ϕ_Q from 0.1 to 2.

An important question in this analysis is how to gauge the benefits of one policy relative to another. Because there is a consensus in the literature that there are substantial gains to conducting a strong inflation targeting policy, we use the gains from strong inflation targeting as the relevant benchmark. In particular, Table 2 reports the difference between outcomes obtained from pursuing policy 23 versus weak inflation targeting, divided by the difference between outcomes obtained from pursuing policy 22 versus weak inflation targeting. For example, when computing the loss function we compute

$$\text{Relative Gain(x)} = \frac{\text{Loss(Weak inflation targeting) - Loss(Policy x)}}{\text{Loss(Weak inflation targeting) - Loss(Strong inflation targeting)}}$$

We compute the relative gain for the reduction in output variance and inflation variance in an analogous manner. By computing relative gains, our results are easily summarized: if the relative gain is above one, the asset-price policy provides gains relative to strong inflation targeting. If the relative gain is negative, the policy provides outcomes that are strictly worse than weak inflation targeting.

In imperfect information environments, the policy maker may not have enough information to correctly compute Q^*_t. We thus distinguish between cases where the policy maker can correctly assess the state of growth of the economy, in which case $Q^*_t = Q^*_{full,t}$ which is the asset price that would prevail under full information, versus the case where the policy maker can observe the level of technology but cannot infer the true state of growth of the economy, in which case $Q^*_t = Q^*_{imp,t}$, the level of the asset price that occurs in the flexible price model with imperfect information.

When considering the benefits of such rules, we distinguish between environments where private agents have full versus imperfect information regarding the state of growth of the economy. Thus, our information structure allows for four distinct cases i) full information on the part of both private agents and policy makers, ii) full information for private agents but imperfect information for policy makers, iii) imperfect information for private agents and full information for policy makers and iv) imperfect information for both private agents and policy makers. Within these four cases we report results for the model with versus without the financial accelerator.

5.2.1 Full Information for the Private Sector

We first consider the case of full information on the part of private agents (Table 2). The top rows of Table 2 consider the case where the policy maker also has full information. In the sticky price model without the financial accelerator the relative gain is unity.[13] Thus, there are essentially no gains to allowing the monetary authority to respond to asset prices relative to a policy of strong inflation targeting. The intuition here is fairly simple: with strong inflation targeting, the monetary authority succeeds at stabilizing the markup which is the only distortion in the economy. With the markup stabilized, the actual path for asset prices is nearly identical to the flexible price path so putting weight on the asset price gap has no effect.

In contrast, in the model with the financial accelerator, responding to the asset price gap provides clear gains in terms of output gap stabilization – on the order of

[13] To deemphasize small differences in simulation results that may reflect sensitivity to numerical solution or simulation error, we report the relative gains rounded to the second decimal place. Our actual results suggest that the model does exhibit an extremely small but positive gain to allowing a positive coefficient on the asset price gap in the full information, no financial accelerator case. These gains imply a ratio that is always less than 1.005 however, implying that to a first approximation the gains are zero.

22% when $\phi_Q = 2$. Although strong inflation targeting stabilizes the markup, it does not eliminate the financial distortion which is reflected in the deviations of asset prices from the flexible-price equilibrium level. Thus, responding to asset prices helps reduce the financial distortion. As the coefficient on asset prices increases, the variance of output falls but the variance of inflation rises. With the financial accelerator, the monetary authority faces a tradeoff between output gap stabilization and inflation stabilization. Based on the loss function which puts equal weight on output gap and inflation volatility, our parameterization implies a modest gain to responding to asset prices, with a coefficient $0.1 < \phi_Q < 1$ minimizing this loss.

We now consider the case where agents have full information but policy makers have imperfect information. These results are reported in the bottom rows of Table 2. In the sticky price model without the financial accelerator, responding to the asset price gap is a strictly inferior policy which leads to large increases in the variances of output and inflation. In this environment, the asset price gap measured by the monetary authority is no longer correct and putting weight on it pushes the economy away from the RBC outcome which is essentially attainable under strong inflation targeting. With the financial accelerator, there is a small gain to allowing a very weak policy response to the asset price gap ($\phi_q = 0.1$) but a deterioration in terms of the variances of output and inflation for larger coefficients. When the monetary authority has imperfect information, it responds to the wrong measure of the asset price which offsets any potential gains to be achieved relative to the strong inflation targeting policy.

5.2.2 Imperfect Information for the Private Sector

We now consider the case where private agents have imperfect information (Table 3). We again begin with the case where the policy maker has full information. In the sticky price model without the financial accelerator, allowing the monetary authority to respond to asset prices produces a small gain in terms of reducing the variance of output. These gains are no longer present when the monetary authority also has imperfect information however. These results suggest that, in the absence of the financial accelerator, there are unlikely to be significant gains to allowing the monetary authority to respond to asset prices, even in the presence of imperfect information of private agents.

In the model with the financial accelerator, the gains to responding to asset prices are substantial. If the policy maker has full information, adopting a rule which responds to the asset price gap produces an incremental reduction in the variance of output of 50% when $\phi_q = 1.0$. Responding to the asset price gap reduces the variance of the output but increases the variance of inflation. Once again, the policy maker faces a clear tradeoff between inflation and output gap volatility.

If the policy maker has imperfect information, the gains obtained from responding to asset prices are somewhat lower than the case where the policy maker has perfect information but are still positive and economically interesting. When private agents have imperfect information, asset price volatility is increased relative to the case of perfect information. Thus the overall gains from responding to asset prices are now larger. These larger gains offset the loss associated with the fact that policy makers are responding to the "wrong gap". As a result, when private agents have imperfect information, responding to the asset price gap is beneficial even when policy makers also have imperfect information.

In summary, the results from tables 2 and 3 imply that there are gains associated with responding to the asset price gap in the presence of distortions in the return on capital caused by the financial accelerator. These gains are largest when the private sector has imperfect information and the policy maker is fully informed. Nonetheless, there are also gains from responding to the asset price gap when both the private sector and the policy maker have imperfect information. Finally, when choosing how to respond, the policy maker faces a tradeoff – by increasing the coefficient on the asset price gap in the Taylor rule it will reduce output gap volatility but increase inflation volatility.

5.3 Benefits to Policies that Do Not Require Inference Regarding the State of Technology

Monetary policy rules that allow the policy maker to respond to the asset price gap require inferences regarding the true state of technology. Because these policies are not necessarily robust to incorrect inference, it is also useful to consider policies that do not require the monetary authority to forecast the state of the economy. We consider three such rules:

i) Output growth:

$$\ln R^n_{t+1} = \ln R^n + 2.0 \ln \pi_t + \phi_Y(\ln Y_t - \ln Y_{t-1} - \mu)$$

ii) Q growth:

$$\ln R^n_{t+1} = \ln R^n + 2.0 \ln \pi_t + \phi_Q(\ln Q_t - \ln Q_{t-1})$$

iii) Q level:

$$\ln R^n_{t+1} = \ln R^n + 2.0 \ln \pi_t + \phi_Q \ln Q_t$$

Table 4 reports the relative gains from adopting these policy rules in the case where private agents have perfect information. Table 5 reports the gains for the case where private agents have imperfect information.

In the absence of the financial accelerator, none of these policies provide substantial gains relative to strong inflation targeting. Policies that respond to either output growth or Q growth lead to an increase in the variance of output but have little impact on the variance of inflation. This is true under either perfect or imperfect information. In the absence of the financial accelerator, strong inflation targeting does well at reducing variation in the markup which is the source of the distortion. As a consequence, there is little to be gained from adding additional variables to the Taylor rule.

With the financial accelerator, policies based on either output growth or the growth rate in asset prices (Q-growth) provide benefits relative to strong inflation targeting. In relative terms, these benefits are much larger when private agents have imperfect information regarding the true state of growth of the economy. Depending on the coefficient values, these policies can do as well as policies based on the asset price gap. Because these policies do not require the policy maker to make inference regarding the state of the economy, they are arguably more robust than policies based on the asset price gap itself.

Finally, we consider the policy which includes the level of the asset price (Q-level) in the Taylor rule. This policy has been considered in the past literature but researchers such as Bernanke and Gertler, and Gilchrist and Leahy have argued against it. Here we confirm their results, albeit for somewhat different reasons. When agents have imperfect information, targeting the level of Q provides clear benefits in terms of reducing output volatility in the model with the financial accelerator. It also leads to a large increase in inflation volatility. For coefficients above 0.5, the inflation outcome is actually worse than what is obtained under weak inflation targeting. Q-level targeting does not allow the monetary authority to adjust its policy owing to movements in asset prices that reflect changes in the desired level of investment spending in the fully flexible economy. Because asset prices are procyclical on average in the fully flexible economy, targeting the level of Q requires a strongly countercyclical policy which leads to significant deflations in expansionary environments. This deflationary response can be limited by adopting a policy that responds to either the asset price gap or the growth rate of asset prices.

6 Conclusion

This paper considers the design of monetary policy in an environment where agents learn about trend productivity growth. Owing to financial market imperfections, changes in trend productivity influence asset prices and hence borrower net worth. Productivity shocks which cause increases in asset prices reduce the premium on external funds and amplify the economy's response to productivity shocks. This amplification mechanism represents a distortion to underlying economic activity that can be only partially eliminated by a policy of strong inflation targeting. In this environment we show that allowing the monetary authority to respond to the asset price gap reduces output gap volatility but tends to increase the volatility of inflation. If private agents are unable to correctly infer the true state of technology growth, such a policy can improve outcomes as defined by a loss function which measures a weighted average of the variance of the output gap and inflation.

We also show that the gains from responding to the asset price gap are largest when the policy maker can correctly identify the true state of technology growth while private agents must infer it from past outcomes. These gains are reduced to the extent that the monetary authority must also infer potential based on past outcomes. We further show that policy rules that depend on either the growth rate of asset prices or the growth rate of output provide most of the benefit associated with including the asset price gap in the interest rate rule. Because asset prices respond to fluctuations in technology in the flexible price economy absent financial frictions, policies that respond to the level of asset prices, and hence fail to recognize fluctuations in potential, are particularly detrimental however.

This paper focusses on a quadratic loss function as the policy objective rather than a welfare-based measure of economic outcomes. Thus, future work should be oriented towards assessing the robustness of our conclusions for welfare calculations. In addition, although learning combined with the financial accelerator increases the procyclicality of asset prices, our underlying flexible price model still implies a fall in asset prices in response to an increase in trend growth. We are therefore also interested in exploring the robustness of our conclusions to alternative mechanisms that may provide a more realistic characterization of the link between asset prices and changes in expectations or "news" regarding economic fundamentals.

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Appendix

I. Equilibrium Conditions in Normalized Variables

This section lists the equilibrium conditions in terms of stationary variables.

Define the normalized variables:

$$c_t \equiv \frac{C_t}{A_t}, i_t \equiv \frac{I_t}{A_t}, y_t \equiv \frac{Y_t}{A_t}, k_t \equiv \frac{K_t}{A_{t-1}}, \text{ and } n_t \equiv \frac{N_t}{A_{t-1}}.$$

Note that K_t and N_t are determined in period $t-1$.

Define the technology growth as:

$$Z_t \equiv \frac{A_t}{A_{t-1}}.$$

The equilibrium conditions in terms of the normalized variables are as follows.

Consumption-savings:

$$\frac{1}{c_t} = \beta E_t \left[\frac{1}{c_{t+1}} \frac{1}{Z_{t+1}} R^n_{t+1} \frac{P_t}{P_{t+1}} \right]. \tag{1'}$$

Real interest rate:

$$R^n_{t+1} E_t \left[\frac{P_t}{P_{t+1}} \right] \equiv R_{t+1}. \tag{2}$$

Expected return on capital:

$$E_t R^k_{t+1} \equiv \frac{E_t \left[(1-\alpha) \frac{y_{t+1}}{k_{t+1}} Z_{t+1} mc_{t+1} + (1-\delta) Q_{t+1} \right]}{Q_t}, \tag{3'}$$

where $mc_{t+1} \equiv \frac{P^W_{t+1}}{P_{t+1}}$ is the real marginal cost.

Premium on external funds:

$$\frac{E_t R^k_{t+1}}{R_{t+1}} \equiv s_t, \tag{4}$$

which is determined by:

$$s_t = \left(\frac{Q_t k_{t+1}}{n_{t+1}} \right)^{\chi}. \tag{5'}$$

Evolution of net worth:

$$n_{t+1} = \eta \left[R^k_t Q_{t-1} k_t \frac{1}{Z_t} - E_{t-1} R^k_t \left(Q_{t-1} k_t \frac{1}{Z_t} - n_t \frac{1}{Z_t} \right) \right].$$

Or, using the definition of the external finance premium, $E_{t-1}R_t^k = s_{t-1}R_{t-1}$,

$$n_{t+1} = \eta \left[R_t^k Q_{t-1} k_t \frac{1}{Z_t} - s_{t-1}R_{t-1} \left(Q_{t-1} k_t \frac{1}{Z_t} - n_t \frac{1}{Z_t} \right) \right]. \tag{6'}$$

Investment-Q relationship:

$$Q_t = \frac{1}{\Phi' \left(\frac{i_t}{k_t} Z_t \right)}. \tag{7'}$$

Resource constraint:

$$y_t = c_t + i_t. \tag{8'}$$

Production function:

$$y_t = H_t{}^{\alpha} k_t^{1-\alpha} \frac{1}{Z_t^{1-\alpha}}. \tag{9'}$$

Labor market equilibrium condition:

$$\theta H_t^{\gamma} = \frac{1}{c_t} \alpha \frac{y_t}{H_t} mc_t. \tag{10'}$$

Price setting:

With Calvo-price setting, the optimal reset price is:

$$P_t^* = \frac{\vartheta}{\vartheta - 1} \frac{E_t \sum_{i=0}^{\infty} \upsilon^i \Lambda_{t,i} MC_{t+i} Y_{t+i} \left(\frac{1}{P_{t+i}} \right)^{1-\vartheta}}{E_t \sum_{i=0}^{\infty} \upsilon^i \Lambda_{t,i} Y_{t+i} \left(\frac{1}{P_{t+i}} \right)^{1-\vartheta}},$$

where $\Lambda_{t,i} \equiv \beta^i \frac{U_{C_{t+i}}}{U_{C_t}} = \beta^i \frac{C_{t+i}}{C_t}$ and $MC_t \equiv P_t mc_t = P_{t+1}^W$ denote the nominal marginal cost. Using the normalized variables,

$$P_t^* = \frac{\vartheta}{\vartheta - 1} \frac{E_t \sum_{i=0}^{\infty} \upsilon^i \left(\frac{c_{t+i}}{c_t} \frac{1}{A_t} \right)^{-1} MC_{t+i} y_{t+i} \left(\frac{1}{P_{t+i}} \right)^{1-\vartheta}}{E_t \sum_{i=0}^{\infty} \upsilon^i \left(\frac{c_{t+i}}{c_t} \frac{1}{A_t} \right)^{-1} y_{t+i} \left(\frac{1}{P_{t+i}} \right)^{1-\vartheta}}. \tag{11'}$$

Evolution of price index:

$$P_t = [\upsilon P_{t-1}^{1-\vartheta} + (1-\upsilon)(P_t^*)^{1-\vartheta}]^{\frac{1}{1-\vartheta}} \tag{12'}$$

Capital accumulation:

$$k_{t+1} = k_t \frac{1}{Z_t}(1-\delta) + \phi \left(\frac{i_t}{k_t} Z_t \right) k_t \frac{1}{Z_t}. \tag{13'}$$

Interest rate rule with Q gap:

$$R^n_{t+1} = R^n \pi_t^{\phi_\pi} \left(\frac{Q_t}{Q_t^*} \right)^{\phi_Q}.$$

where Q_t^* is the % deviations in the flexible-price levels of Q in the absence of financial accelerator.

Interest rate rule with output growth:

$$R^n_{t+1} = R^n \pi_t^{\phi_\pi} \left(\frac{\frac{y_t}{y_{t-1}} Z_t}{\exp(\mu)} \right)^{\phi_Y}.$$

Interest rate rule with Q growth:

$$R^n_{t+1} = R^n \pi_t^{\phi_\pi} \left(\frac{Q_t}{Q_{t-1}} \right)^{\phi_Q}.$$

Interest rate rule with Q level:

$$R^n_{t+1} = R^n \pi_t^{\phi_\pi} \left(\frac{Q_t}{Q} \right)^{\phi_Q}.$$

where Q is the nonstochastic steady state level of Q.

Process for technology growth:

$$\ln Z_t = \mu_t + \varepsilon_t,$$

and:

$$(\mu_t - \mu) = \rho_d(\mu_{t-1} - \mu) + v_t.$$

II. Nonstochastic Steady State

This section lists the conditions for the nonstochastic steady state in terms of normalized variables.

Normalize the steady state gross inflation rate at:

$$\pi = 1.$$

We specify the capital adjustment cost function such that:

$$Q = 1.$$

From (11'), $P = \frac{\vartheta}{\vartheta - 1} MC$, or:

$$mc = \frac{\vartheta - 1}{\vartheta},$$

where MC is the nominal marginal cost and mc is the real marginal cost ($mc \equiv \frac{MC}{P}$).

From (1') and (2'):

$$R = \frac{Z}{\beta}.$$

From (4) and (5') and using $Q = 1$, the non-stochastic steady state level of the external finance premium, s, is given by:

$$s = \frac{R^k}{R} = \left(\frac{k}{n}\right)^{\chi},$$

where the parameter χ and the steady state ratio of capital to net worth, $\frac{k}{n}$, are calibrated, as described in the paper.

From (6'):

$$\eta R^k \frac{1}{Z} = 1.$$

Note that R^k should also satisfy the condition above $\frac{R^k}{R} = \left(\frac{k}{n}\right)^{\chi}$. This implies that $R^k = \left(\frac{k}{n}\right)^{\chi} R = \frac{1}{\eta} Z$, or $\left(\frac{K}{N}\right)^{\chi} R = s\frac{Z}{\beta} = \frac{1}{\eta} Z$, or $s = \frac{\beta}{\eta}$. For $s > 1$, we need $\beta > \eta$.

From (2) and (3'):

$$\frac{y}{k} = \frac{1}{(1-\alpha) Z \cdot mc}[R^k - (1-\delta)].$$

From (13'):

$$\frac{i}{k} = 1 - \frac{1}{Z}(1-\delta).$$

From (8'):

$$\frac{c}{k} = \frac{y}{k} - \frac{i}{k}.$$

We also have:

$$\frac{y}{c} = \frac{\frac{y}{k}}{\frac{c}{k}}.$$

From (10'):

$$H = \left[\frac{\alpha}{\theta}\frac{y}{c}mc\right]^{\frac{1}{1+\gamma}}.$$

From (9'):

$$k = \frac{H}{\left(\frac{y}{k}\right)^{\frac{1}{\alpha}} Z^{\frac{1-\alpha}{\alpha}}}.$$

Then:

$$c = \frac{c}{k}k, i = \frac{i}{k}k, y = \frac{y}{k}k.$$

III. Log-Linearized Equilibrium Conditions

This section lists the equilibrium conditions in terms of log deviations in the normalized variables from nonstochastic steady state.

Let $\widetilde{z}_t$ denote the % deviation in technology growth from the mean:

$$\begin{aligned} \widetilde{z}_t &\equiv (\ln Z_t - \ln Z) \\ &= (\ln Z_t - \mu). \end{aligned}$$

Consumption-savings:

$$-\widetilde{c}_t = -E_t\widetilde{c}_{t+1} - E_t\widetilde{z}_{t+1} + \widetilde{r}^n_{t+1} - E_t\widetilde{\pi}_{t+1}.$$

Real interest rate:

$$\widetilde{r}^n_{t+1} - E_t\widetilde{\pi}_{t+1} \equiv \widetilde{r}_{t+1}.$$

Expected return on capital:

$$\begin{aligned} E_t\widetilde{r}^k_{t+1} &\equiv \frac{mc(1-\alpha)\frac{y}{k}Z}{mc(1-\alpha)\frac{y}{k}Z + (1-\delta)}(E_t\widetilde{y}_{t+1} - \widetilde{k}_{t+1} + E_t\widetilde{z}_{t+1} + E_t\widetilde{mc}_{t+1}) \\ &\quad + \frac{1-\delta}{mc(1-\alpha)\frac{y}{k}Z + (1-\delta)}E_t\widetilde{q}_{t+1} - \widetilde{q}_t. \end{aligned}$$

Premium on external funds:

$$E_t\widetilde{r}^k_{t+1} - \widetilde{r}_{t+1} \equiv \widetilde{s}_t,$$

which is determined by:

$$\widetilde{s}_t = \chi(\widetilde{q}_t + \widetilde{k}_{t+1} - \widetilde{n}_{t+1}).$$

In the case of no financial accelerator, we set $\chi = 0$.

Evolution of net worth:

Using the steady state condition $\eta R^k \frac{1}{Z} = 1$, the evolution of net worth is given by:

$$\widetilde{n}_{t+1} = \frac{k}{n}\widetilde{r}_t^k - \left(\frac{k}{n} - 1\right) E_{t-1}\widetilde{r}_t^k + \widetilde{n}_t - \widetilde{z}_t.$$

Or, using the definition of external finance premium $E_{t-1}\widetilde{r}_t^k \equiv \widetilde{s}_{t-1} + \widetilde{r}_t$:

$$\widetilde{n}_{t+1} = \frac{k}{n}\widetilde{r}_t^k - \left(\frac{k}{n} - 1\right)(\widetilde{r}_t + \widetilde{s}_{t-1}) + \widetilde{n}_t - \widetilde{z}_t.$$

Investment-Q relationship:

$$\widetilde{q}_t = \eta_k(\widetilde{i}_t - \widetilde{k}_t + \widetilde{z}_t),$$

where $\eta_k \equiv -\frac{\Phi''(\frac{i}{k}Z)\frac{i}{k}Z}{\Phi'(\frac{i}{k}Z)} = -\frac{\Phi''(Z-(1-\delta))\cdot(Z-(1-\delta))}{\Phi'(Z-(1-\delta))}$.

Resource constraint:

$$\widetilde{y}_t = \frac{c}{y}\widetilde{c}_t + \frac{i}{y}\widetilde{i}_t.$$

Production function:

$$\widetilde{y}_t = \alpha\widetilde{h}_t + (1-\alpha)\widetilde{k}_t - (1-\alpha)\widetilde{z}_t.$$

Labor market equilibrium condition:

$$\widetilde{y}_t + \widetilde{mc}_t - \widetilde{c}_t = (1+\gamma)\widetilde{h}_t.$$

Inflation:

$$\widetilde{\pi}_t = \kappa\widetilde{mc}_t + \beta E_t\widetilde{\pi}_{t+1},$$

where $\kappa \equiv \frac{(1-v)(1-\beta v)}{v}$.

Capital accumulation:

$$\widetilde{k}_{t+1} = \frac{1-\delta}{Z}(\widetilde{k}_t - \widetilde{z}_t) + \left(1 - \frac{1-\delta}{Z}\right)\widetilde{i}_t.$$

Interest rate rule with Q gap:

$$\widetilde{r}_{t+1}^n = \phi_\pi\widetilde{\pi}_t + \phi_Q(\widetilde{q}_t - \widetilde{q}_t^*).$$

When the policy maker observes shocks, $\widetilde{q}_t^* = \widetilde{q}_{full,t}^*$ where $\widetilde{q}_{full,t}^*$ is obtained by solving the flexible-price model without financial accelerator under full information. When the policy maker does not observe shocks, $\widetilde{q}_t^* = \widetilde{q}_{full,t}^*$ where $\widetilde{q}_{imp,t}^*$ are obtained by solving the flexible-price model without financial accelerator under imperfect information.

Interest rate rule with output growth:

$$\widetilde{r}_{t+1}^n = \phi_\pi \widetilde{\pi}_t + \phi_Y (\widetilde{y}_t - \widetilde{y}_{t-1} + \widetilde{z}_t).$$

Interest rate rule with Q growth:

$$\widetilde{r}_{t+1}^n = \phi_\pi \widetilde{\pi}_t + \phi_Q (\widetilde{q}_t - \widetilde{q}_{t-1}).$$

Interest rate rule with Q level:

$$\widetilde{r}_{t+1}^n = \phi_\pi \widetilde{\pi}_t + \phi_Q \widetilde{q}_t.$$

Process for technology growth:

$$\widetilde{z}_t = \widetilde{d}_t + \varepsilon_t,$$

and:

$$\widetilde{d}_t = \rho_d \widetilde{d}_{t-1} + \nu_t,$$

where $\widetilde{d}_t$ is defined as:

$$\widetilde{d}_t \equiv \mu_t - \mu.$$

IV. Solution to the Model

A. Solution to the model when the monetary authority does not respond to the asset price gap

When the interest rate rule does not include flexible-price level of asset price, we do not need to compute the flexible-price equilibrium in order to characterize the equilibrium in the sticky-price model.

A-1. When private agents observe shocks $(\nu_t, \varepsilon_{A,t})$

Solution to the model takes a stationary decision rule of the form:

$$X_t = B_1 X_{t-1} + B_2 u_t,$$

where:

$$X_t \equiv [\widetilde{c}_t; \widetilde{y}_t; \widetilde{h}_t; \widetilde{i}_t; \widetilde{k}_{t+1}; \widetilde{nw}_{t+1}; \widetilde{r}_t^k; \widetilde{s}_t; \widetilde{r}_{t+1}; \widetilde{r}_{t+1}^n; \widetilde{q}_t; \widetilde{mc}_t; \widetilde{\pi}_t; \widetilde{d}_t],$$

and:

$$u_t \equiv [\nu_t; \varepsilon_t].$$

A-2. When private agents do not observe shocks (ν_t, ε_t)

In characterizing a solution to the model under imperfect information, we assume certainty equivalence. The solution under imperfect information is characterized by the same decision rule coefficients, B_1 and B_2, as in the case of full information. We replace the unobservable variables, $\widetilde{d}_{t-1}, \nu_t$, and ε_t, on the RHS of the solution system with inferences, $\widetilde{d}_{t-1|t-1}, \nu_{t|t}$, and $\varepsilon_{t|t}$, which are determined by the following four equations. The first specifies the process of the persistent component of technology growth:

$$\widetilde{d}_t = \rho_d \widetilde{d}_{t-1} + \nu_t. \tag{A-1}$$

The second links observed technology growth, $\widetilde{z}_t = (\widetilde{d}_t + \varepsilon_t)$, to the inference about the persistent component of technology growth, $\widetilde{d}_{t|t}$:

$$\begin{aligned} \widetilde{d}_{t|t} &= \lambda_1 \widetilde{z}_t + (1 - \lambda_1)\rho_d \widetilde{d}_{t-1|t-1} \\ &= \lambda_1(\widetilde{d}_t + \varepsilon_t) + (1 - \lambda_1)\rho_d \widetilde{d}_{t-1|t-1}. \end{aligned} \tag{A-2}$$

where λ_1 is the Kalman gain that private agents use.

The third defines the inference about the shock to the persistent component of technology growth, $\nu_{t|t}$:

$$\nu_{t|t} = \widetilde{d}_{t|t} - \rho_d \widetilde{d}_{t-1|t-1}. \tag{A-3}$$

The fourth defines the inference about the shock to the transitory component of technology growth, $\varepsilon_{t|t}$:

$$\begin{aligned} \varepsilon_{t|t} &= \widetilde{z}_t - \widetilde{d}_{t|t} \\ &= (\widetilde{d}_t + \varepsilon_t) - \widetilde{d}_{t|t}. \end{aligned} \tag{A-4}$$

B. Solution to the model when the monetary authority responds to the asset price gap

The solution method below applies to a model in which the interest rate rule includes the asset price gap:

$$\widetilde{r}_t^n = \phi_\pi \widetilde{\pi}_t + \phi_Q(\widetilde{q}_t - \widetilde{q}_t^*).$$

B-1. When both private agents and policy maker observe shocks (ν_t, ε_t)

Solution to the sticky-price model takes a stationary decision rule of the form:

$$X_t = B_3 X_{t-1} + B_4 u_t,$$

where:

$$\begin{aligned} X_t \equiv & [\widetilde{c}_t; \widetilde{y}_t; \widetilde{h}_t; \widetilde{i}_t; \widetilde{k}_{t+1}; \widetilde{nw}_{t+1}; \widetilde{r}_t^k; \widetilde{s}_t; \widetilde{r}_{t+1}; \widetilde{r}_{t+1}^n; \widetilde{q}_t; \widetilde{mc}_t; \widetilde{\pi}_t; \widetilde{d}_t; \\ & \widetilde{c}_t^*; \widetilde{y}_t^*; \widetilde{h}_t^*; \widetilde{i}_t^*; \widetilde{k}_{t+1}^*; \widetilde{nw}_{t+1}^*; \widetilde{r}_t^{k*}; \widetilde{s}_t^*; \widetilde{r}_{t+1}^*; \widetilde{r}_{t+1}^{n*}; \widetilde{q}_t^*; \widetilde{mc}_t^*; \widetilde{\pi}_t^*; \widetilde{d}_t^*], \end{aligned}$$

and:

$$u_t \equiv [\nu_t; \varepsilon_t; \nu_t^*; \varepsilon_t^*].$$

The variables with * denote those in the flexible-price model without financial frictions (which policy maker uses when they compute the asset price gap), and the variables without * denote those in the sticky-price model.

Note that when we compute the impulse response or conduct stochastic simulations, the shocks are common across the sticky-price model and the flexible-price model: $\nu_t = \nu_t^*, \varepsilon_t = \varepsilon_t^*$.

B-2. When private agents observe shocks (ν_t, ε_t) but policy maker does not

The solution is characterized by the same decision rule coefficients, B_3 and B_4, as in the case of full information. We replace the unobservable variables, $\widetilde{d}_{t-1}^*, \nu_t^*$, and ε_t^*, on the RHS of the solution system with the inferences, $\widetilde{d}_{t-1|t-1}^*, \nu_{t|t}^*$, and $\varepsilon_{t|t}^*$, which are determined by the following equations:

$$\widetilde{d}_t^* = \rho_d \widetilde{d}_{t-1}^* + \nu_t^*. \tag{A-5}$$

$$\begin{aligned} \widetilde{d}_{t|t}^* &= \lambda_2 \widetilde{z}_t + (1-\lambda_2)\rho_d \widetilde{d}_{t-1|t-1}^* \\ &= \lambda_2(\widetilde{d}_t^* + \varepsilon_t^*) + (1-\lambda_2)\rho_d \widetilde{d}_{t-1|t-1}^*. \end{aligned} \tag{A-6}$$

where λ_2 is the gain parameter that the policy maker uses.

$$\nu_{t|t}^* = \widetilde{d}_{t|t}^* - \rho_d \widetilde{d}_{t-1|t-1}^*. \tag{A-7}$$

$$\begin{aligned}\varepsilon^*_{t|t} &= \widetilde{z}_t - \widetilde{d}^*_{t|t} \\ &= (\widetilde{d}^*_t + \varepsilon^*_t) - \widetilde{d}^*_{t|t}.\end{aligned} \tag{A-8}$$

B-3. When neither private agents nor policy maker observes shocks (ν_t, ε_t)

The solution is characterized by the same decision rule coefficients, B_3 and B_4, as in the case of full information.

We replace the unobservable variables, $\widetilde{d}_{t-1}, \nu_t, \varepsilon_t, \widetilde{d}^*_{t-1}, \nu^*_t$, and ε^*_t, on the RHS of the solution system with the inferences, $\widetilde{d}_{t-1|t-1}, \nu_{t|t}, \varepsilon_{t|t}, \widetilde{d}^*_{t-1|t-1}, \nu^*_{t|t}$, and $\varepsilon^*_{t|t}$, which are determined by 8 equations (A-1)-(A-8).

B-4. When private agents do not observe (ν_t, ε_t) but policy maker does

The solution is characterized by the same decision rule coefficients (B_3, B_4) as in the case of full information. We replace the unobservable variables, $\widetilde{d}_{t-1}, \nu_t$, and ε_t, on the RHS of the solution system with the inferences, $\widetilde{d}_{t-1|t-1}, \nu_{t|t}$, and $\varepsilon_{t|t}$, which are determined by 4 equations (A-1)-(A-4).

Table 1: Inflation Target

	No FA			FA		
	var(Y gap)	$var(\ln \pi)$	Loss	var(Y gap)	$var(\ln \pi)$	Loss
Private (full info)						
Weak inflation target	0.4309	2.8110	1.6210	1.9230	3.0220	2.4730
Strong inflation target	0.0064	0.0438	0.0251	0.4700	0.0558	0.2629
Private (imperfect info)						
Weak inflation target	0.5794	2.1030	1.3410	2.2470	2.2650	2.2560
Strong inflation target	0.0992	0.0275	0.0633	0.8700	0.0450	0.4575

Note: Y gap $= \ln Y - \ln Y^*_{full}$ where Y^*_{full} is the flexible-price equilibrium level of output in the absence of financial frictions and when policy maker has full information. The loss is calculated as $0.5var(\ln Y - \ln Y^*_{full}) + 0.5var(\ln \pi)$.

Table 2: Policy with Asset Price Gap: Full Information for the Private Sector

	No FA			FA		
	var(Y gap)	$var(\ln\pi)$	Loss	var(Y gap)	$var(\ln\pi)$	Loss
Policy (full info)						
Q gap = 0.1	1.00	1.00	1.00	1.03	1.01	0.95
Q gap = 0.5	1.00	1.00	1.00	1.10	1.01	1.04
Q gap = 1.0	1.01	1.00	1.00	1.13	0.98	1.03
Q gap = 1.5	1.01	1.00	1.00	1.17	0.95	1.02
Q gap = 2.0	1.02	1.00	1.00	1.22	0.92	1.02
Policy (imperfect info)						
Q gap = 0.1	0.98	1.00	1.00	1.03	1.00	1.01
Q gap = 0.5	0.85	1.00	0.98	0.97	1.00	0.99
Q gap = 1.0	0.59	0.99	0.94	0.94	0.98	0.97
Q gap = 1.5	0.31	1.00	0.91	0.94	0.94	0.94
Q gap = 2.0	0.21	1.00	0.90	0.79	0.88	0.85

Note: Y gap $= \ln Y - \ln Y^*_{full}$ where Y^*_{full} is the flexible-price equilibrium level of output in the absence of financial frictions and when policy maker has full information

Note: A value of larger than 1 implies that the policy is better than the strong inflation targeting. A negative value implies that the policy is worse than the weak inflation targeting.

Table 3: Policy with Asset Price Gap: Imperfect Information for the Private Sector

	No FA			FA		
	var(Y gap)	$var(\ln \pi)$	Loss	var(Y gap)	$var(\ln \pi)$	Loss
Policy (full info)						
Q gap = 0.1	1.02	1.00	1.00	1.09	1.00	1.04
Q gap = 0.5	1.12	0.99	1.01	1.36	1.00	1.14
Q gap = 1.0	1.12	0.99	1.01	1.50	0.95	1.18
Q gap = 1.5	1.06	0.99	1.00	1.51	0.94	1.16
Q gap = 2.0	0.97	0.99	0.99	1.53	0.86	1.12
Policy (imperfect info)						
Q gap = 0.1	0.92	1.00	0.99	1.20	1.01	1.08
Q gap = 0.5	0.94	1.00	0.99	1.22	1.01	1.09
Q gap = 1.0	0.96	1.00	0.99	1.38	0.97	1.13
Q gap = 1.5	0.98	1.00	1.00	1.44	0.93	1.12
Q gap = 2.0	0.96	1.00	1.00	1.49	0.87	1.09

Note: Y gap $= \ln Y - \ln Y^{*}_{full}$ where Y^{*}_{full} is the flexible-price equilibrium level of output in the absence of financial frictions and when policy maker has full information

Note: A value of larger than 1 implies that the policy is better than the strong inflation targeting. A negative value implies that the policy is worse than the weak inflation targeting.

Note: In the case of imperfect information for private agents, the base is the case of strong inflation target and private agents have imperfect information (not the case of full information for private agents). Thus we cannot compare the numbers in the case of imperfect information for private agents and the numbers in the case of full information for private agents.

Table 4: Alternative Policy Rules: Full Information for the Private Sector

	No FA			FA		
	var(Y gap)	$var(\ln \pi)$	Loss	var(Y gap)	$var(\ln \pi)$	Loss
Y growth = 0.1	0.99	1.00	1.00	1.04	1.01	1.01
Y growth = 0.5	0.85	1.01	0.99	1.04	1.01	1.02
Y growth = 1.0	0.57	1.00	0.95	1.04	1.01	1.02
Y growth = 1.5	0.23	0.98	0.88	0.97	0.99	0.98
Y growth = 2.0	-0.05	0.94	0.81	0.83	0.95	0.91
Q growth = 0.1	1.00	1.00	1.00	1.07	1.00	1.02
Q growth = 0.5	0.96	1.00	1.00	1.05	1.00	1.02
Q growth = 1.0	0.87	1.00	0.99	1.04	1.00	1.02
Q growth = 1.5	0.78	1.00	0.97	1.02	1.00	1.01
Q growth = 2.0	0.69	1.00	0.96	0.96	1.00	0.99
Q level = 0.1	0.99	1.01	1.00	1.01	1.01	1.01
Q level = 0.5	0.71	0.70	0.70	1.10	0.73	0.85
Q level = 1.0	0.13	-0.01	0.00	1.05	-0.31	0.13
Q level = 1.5	-0.78	-1.57	-1.46	0.91	-1.98	-1.03
Q level = 2.0	-2.16	-3.60	-3.41	0.71	-4.18	-2.57

Note: Y gap $= \ln Y - \ln Y^*_{full}$ where Y^*_{full} is the flexible-price equilibrium level of output in the absence of financial frictions and when policy maker has full information

Note: A value of larger than 1 implies an improvement over the strong inflation target policy. A negative value implies that the policy is worse than the weak inflation target policy.

Note: In the case of imperfect information for private agents, the base is the case of strong inflation target and private agents have imperfect information (not the case of full information for private agents). Thus we cannot compare the numbers in the case of imperfect information for private agents and the numbers in the case of full information for private agents.

Table 5: Alternative Policy Rules: Imperfect Information for the Private Sector

	No FA			FA		
	var(Y gap)	$var(\ln\pi)$	Loss	var(Y gap)	$var(\ln\pi)$	Loss
Y growth = 0.1	0.97	1.00	1.00	1.20	1.01	1.08
Y growth = 0.5	0.90	1.00	0.99	1.40	1.02	1.16
Y growth = 1.0	0.74	1.00	0.95	1.40	1.01	1.16
Y growth = 1.5	0.54	0.96	0.88	1.40	0.98	1.14
Y growth = 2.0	0.33	0.90	0.79	1.37	0.92	1.24
Q growth = 0.1	1.18	1.00	0.99	1.11	1.00	1.05
Q growth = 0.5	0.93	1.00	0.99	1.15	1.00	1.06
Q growth = 1.0	0.96	1.00	0.99	1.33	1.00	1.13
Q growth = 1.5	0.92	1.00	0.99	1.31	0.99	1.12
Q growth = 2.0	0.98	1.00	0.99	1.39	1.00	1.15
Q level = 0.1	0.96	1.01	1.01	1.21	1.02	1.09
Q level = 0.5	0.91	0.61	0.66	1.44	0.61	0.93
Q level = 1.0	0.49	-0.65	-0.44	1.52	-0.80	0.09
Q level = 1.5	0.13	-1.86	-1.49	1.48	-2.49	-0.97
Q level = 2.0	-0.78	-4.54	-3.83	1.30	-5.42	-2.85

Note: Y gap $= \ln Y - \ln Y^*_{full}$ where Y^*_{full} is the flexible-price equilibrium level of output in the absence of financial frictions and when policy maker has full information

Note: A value of larger than 1 implies an improvement over the strong inflation target policy. A negative value implies that the policy is worse than the weak inflation target policy.

Note: In the case of imperfect information for private agents, the base is the case of strong inflation target and private agents have imperfect information (not the case of full information for private agents). Thus we cannot compare the numbers in the case of imperfect information for private agents and the numbers in the case of full information for private agents.

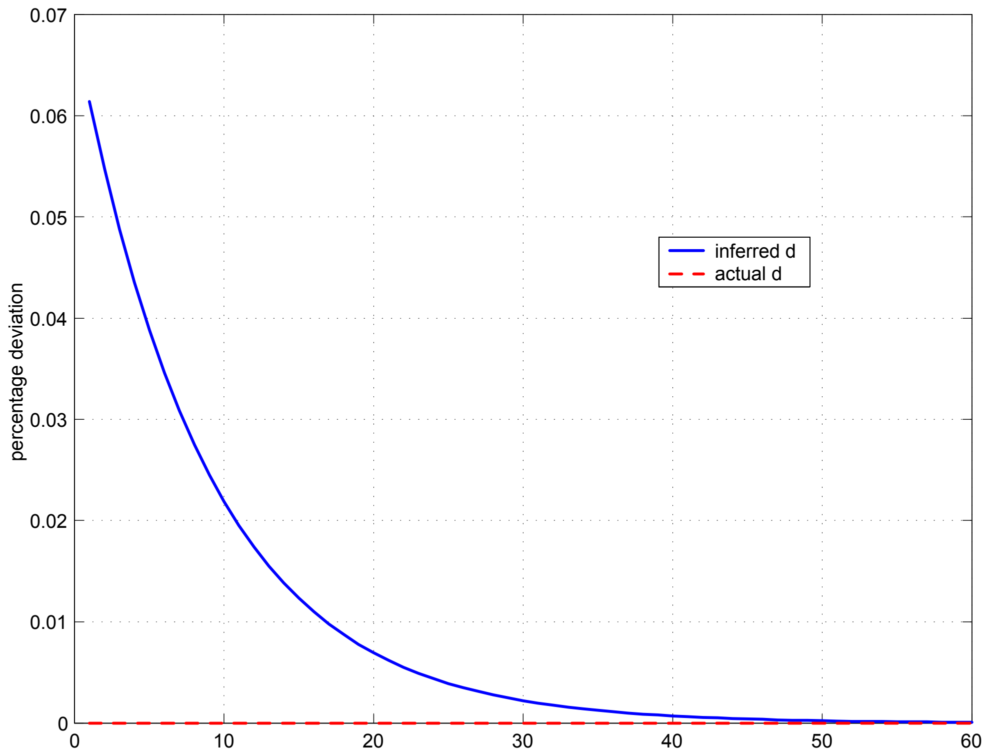

Figure 1: Actual vs Inferred Technology Growth: Shock to Transitory Component

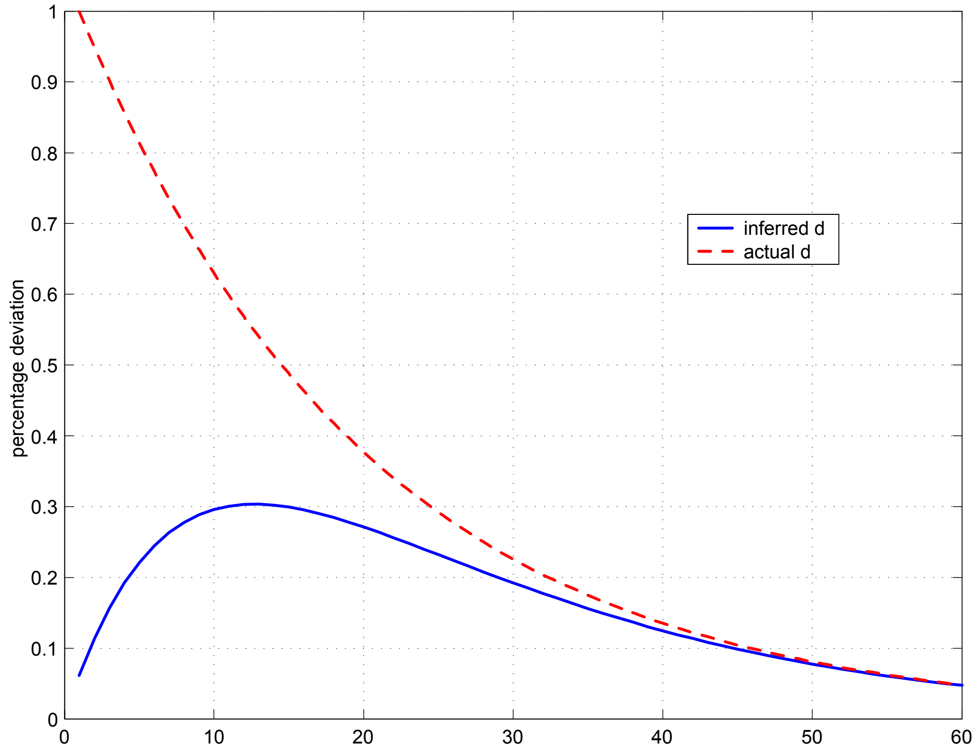

Figure 2: Actual vs Inferred Technology Growth: Shock to Persistent Component

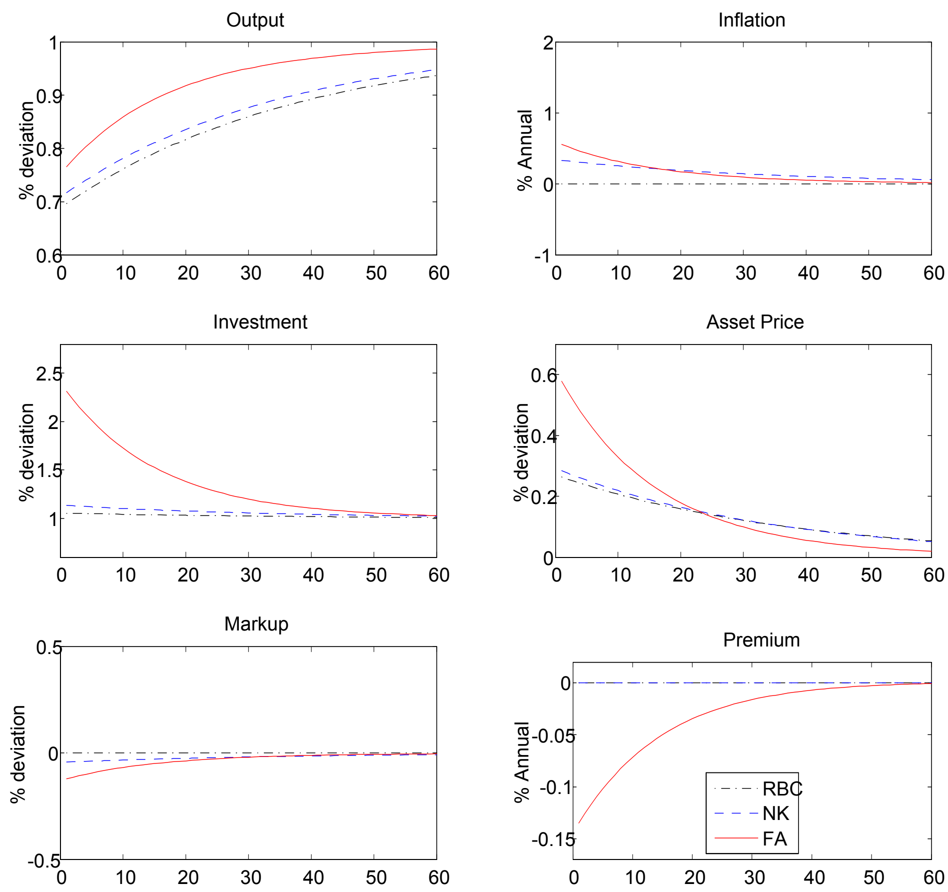

Figure 3: Weak Inflation Response: Perfect Information

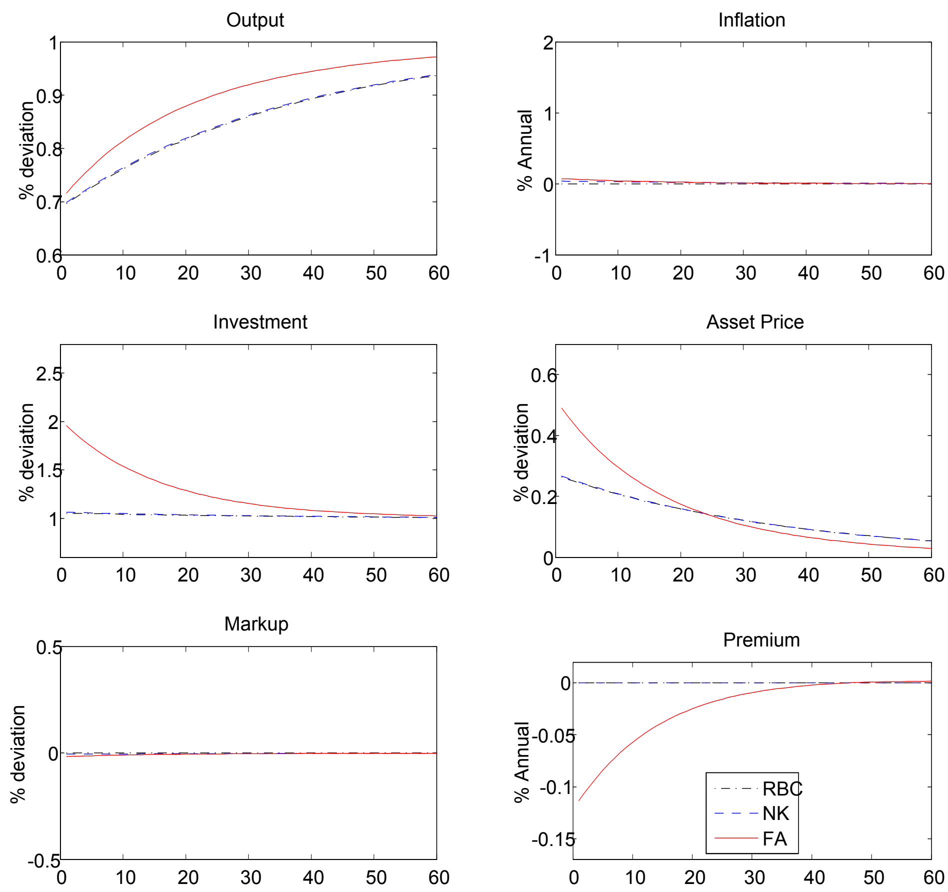

Figure 4: Strong Inflation Response: Perfect Information

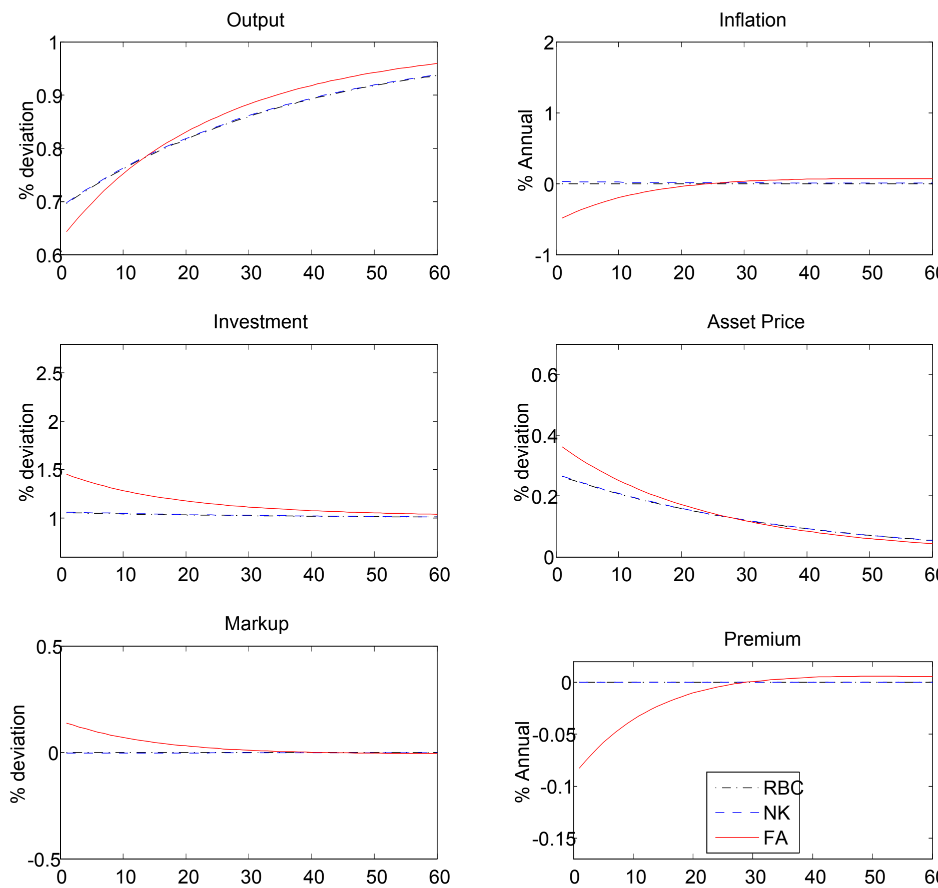

Figure 5: Inflation and Q gap Response: Perfect Information

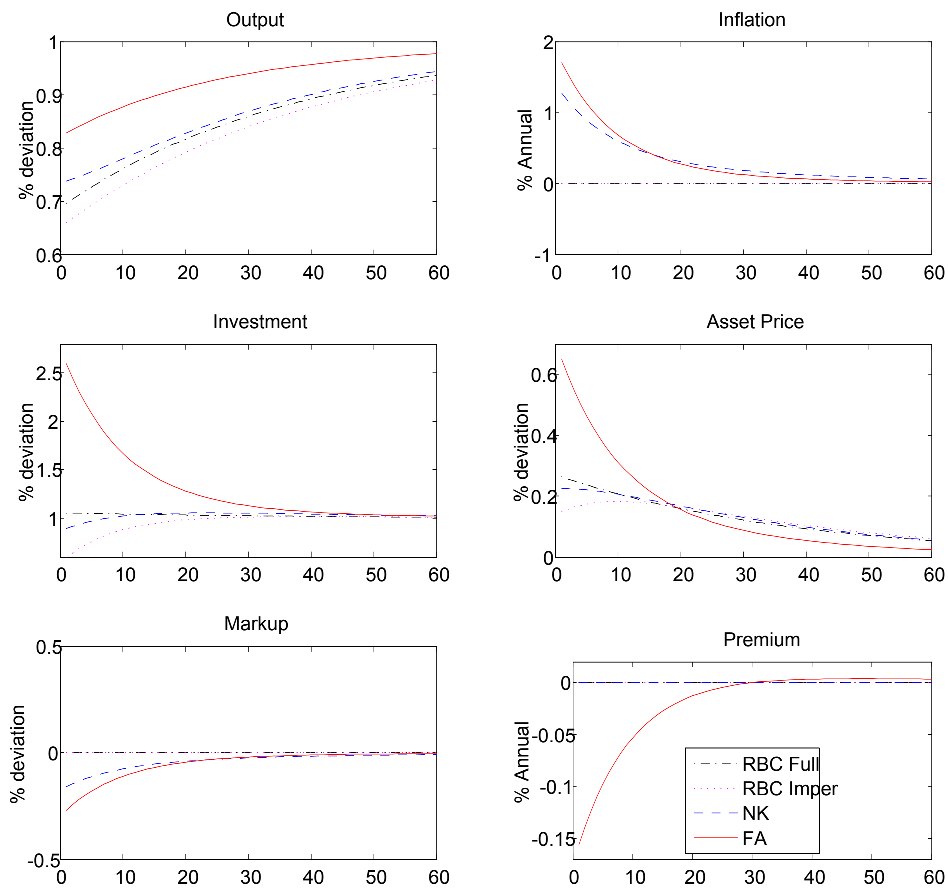

Figure 6: Weak Inflation Response: Imperfect Information

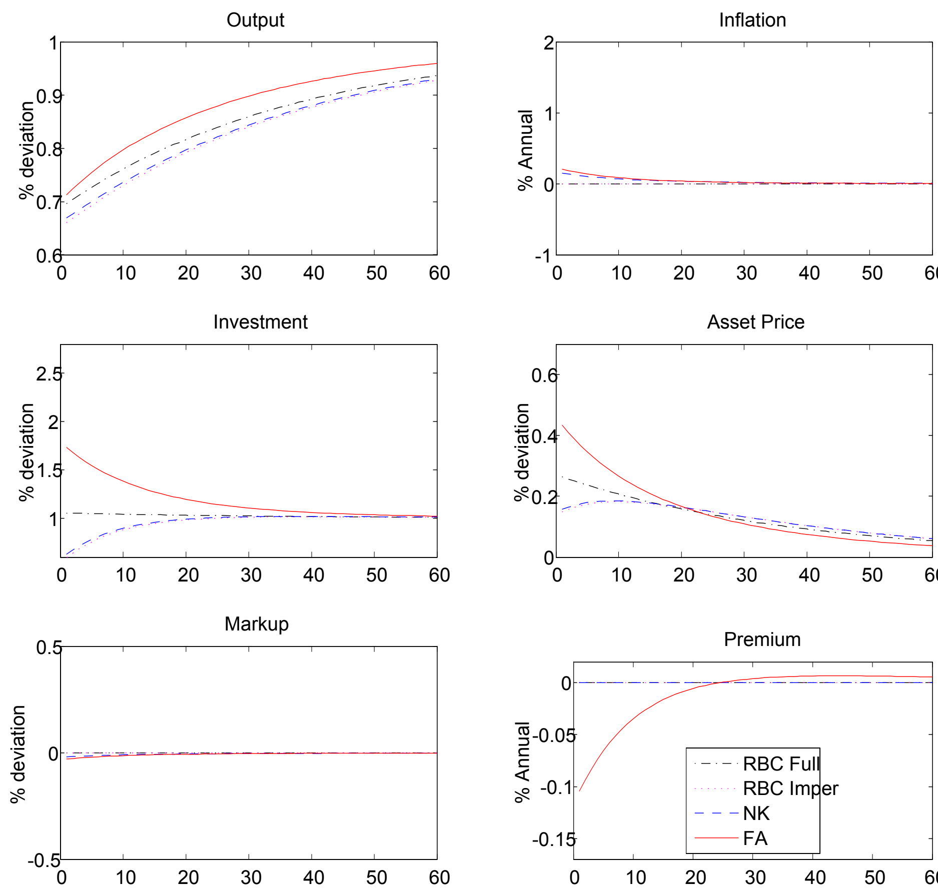

Figure 7: Strong Inflation Response: Imperfect Information

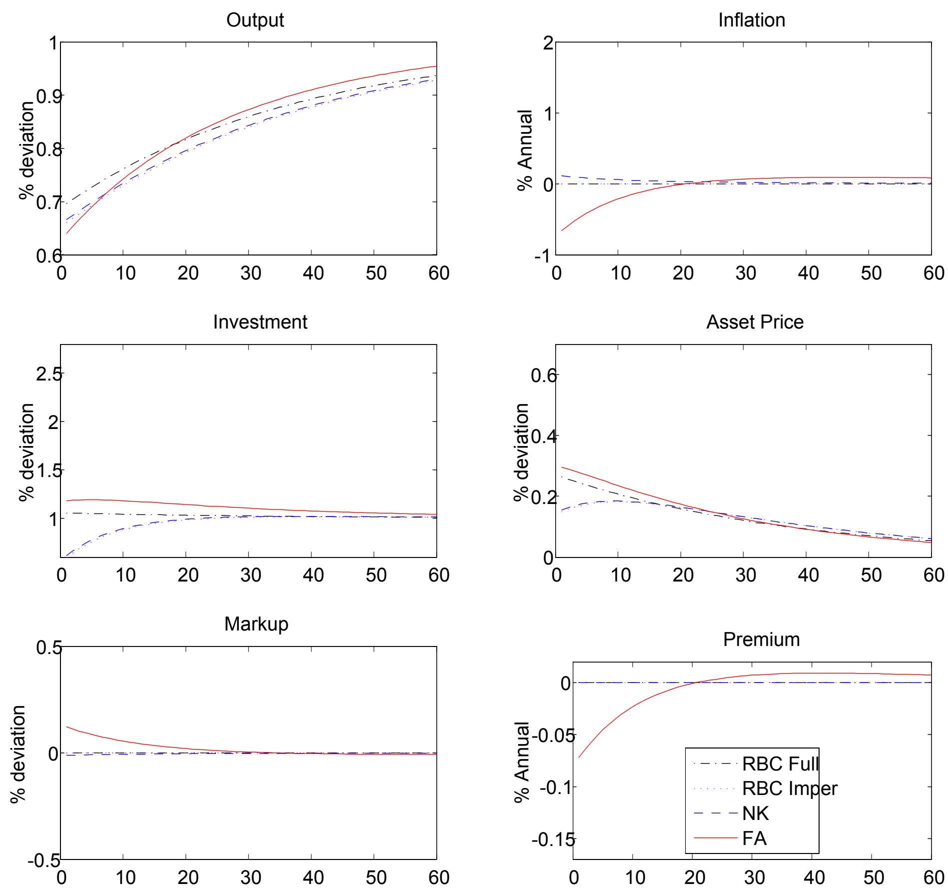

Figure 8: Strong Inflation and Q-gap Response: Imperfect Information.

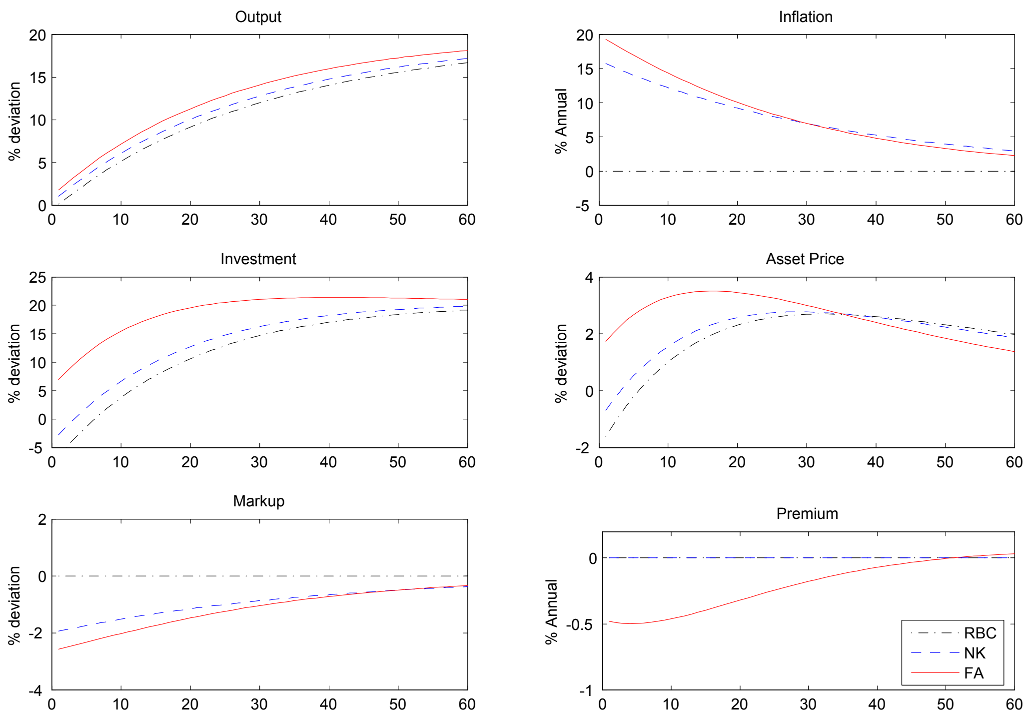

Figure 9: Weak Inflation Response: Full Information

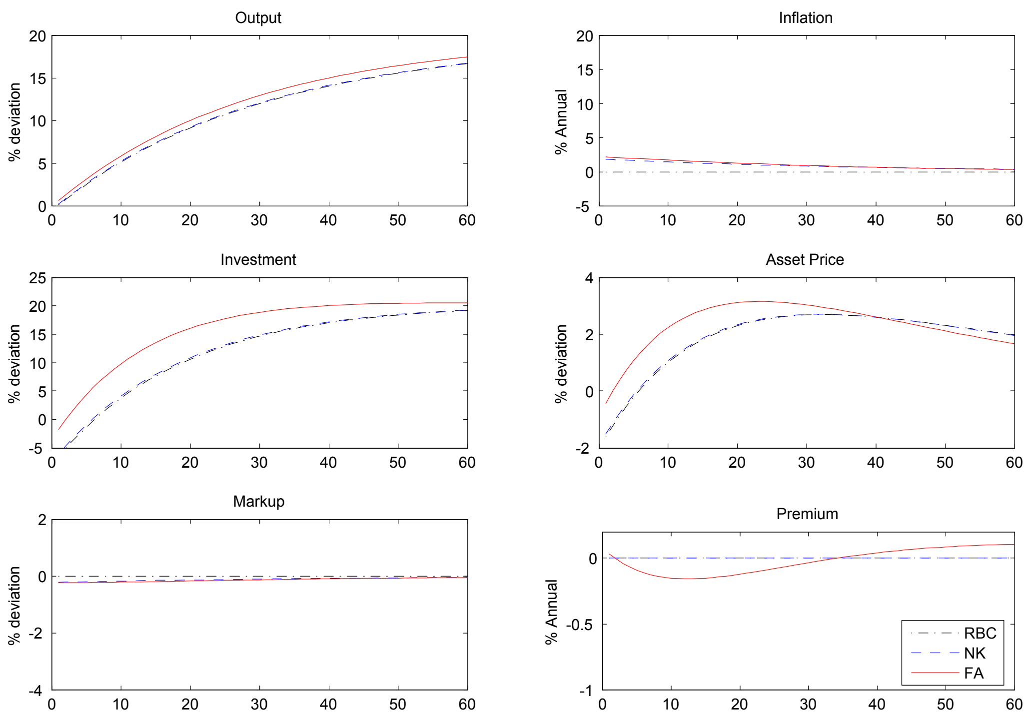

Figure 10: Strong Inflation Response: Full Information

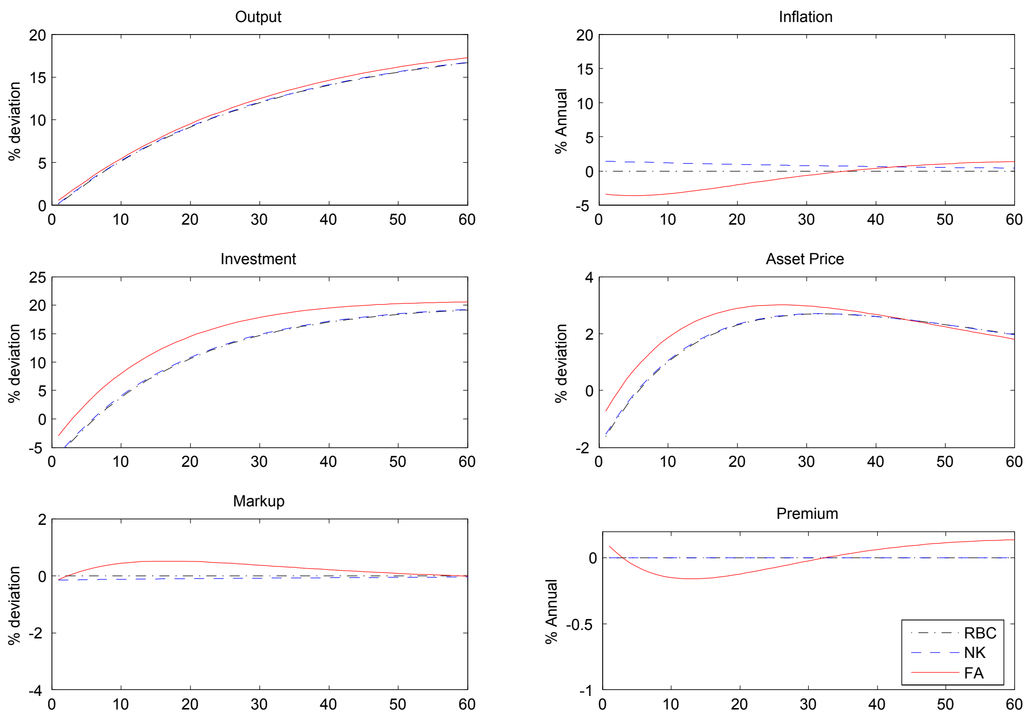

Figure 11: Inflation and Q-gap Response: Full Information

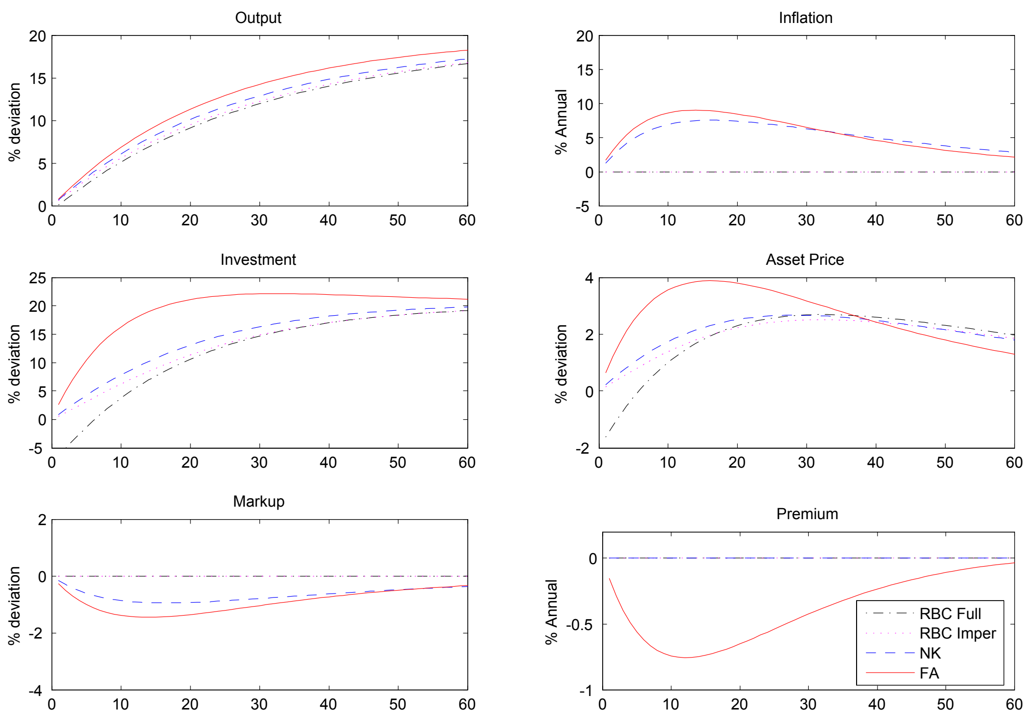

Figure 12: Weak Inflation Response: Imperfect Information

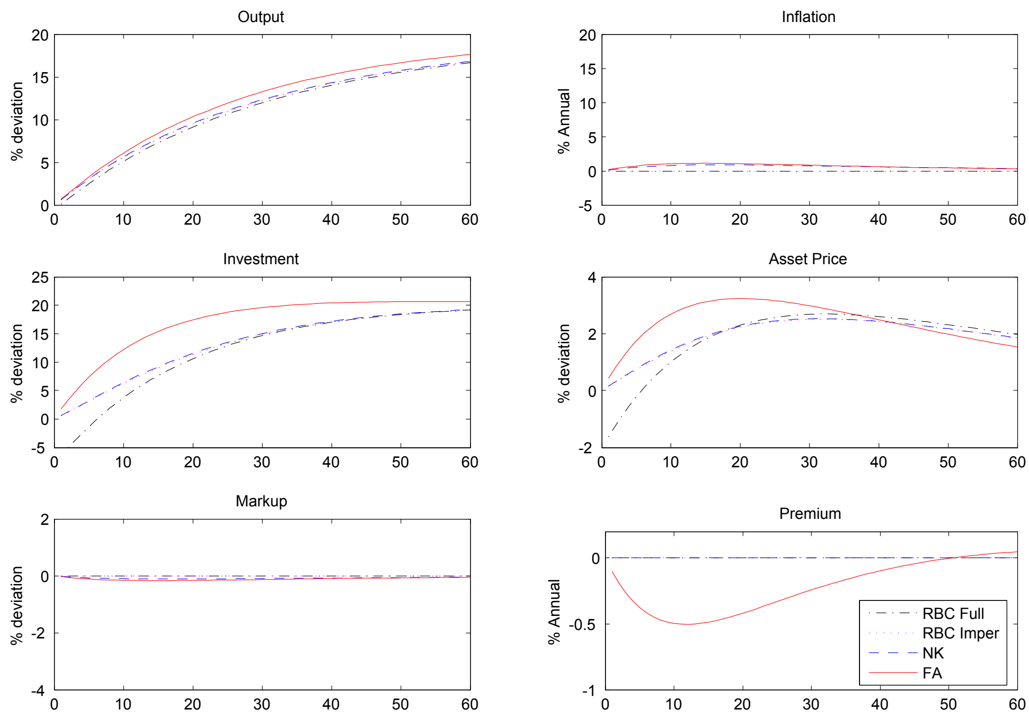

Figure 13: Strong Inflation Response: Imperfect Information.

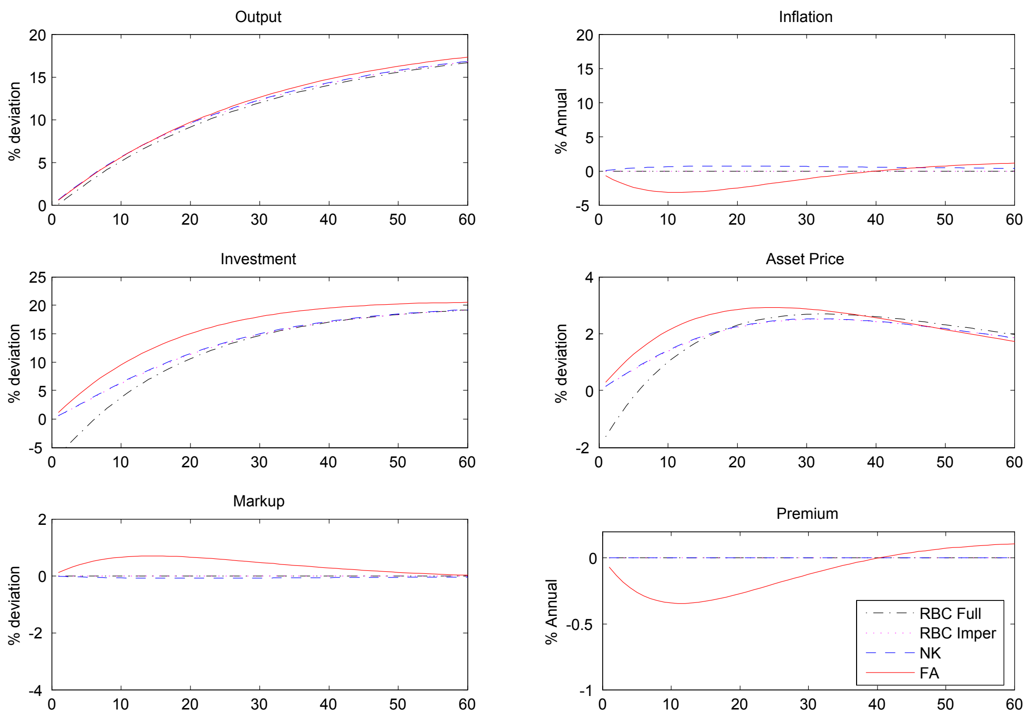

Figure 14: Inflation and Q-gap Response: Imperfect Information.