Machines as Engines of Growth

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Abstract

This paper builds a model of industrialization and growth. In this model machines replace workers in a growing number of tasks. This enables the economy to experience long-run growth, as machines become servants of humans, and as their number can grow unboundedly. Growth is created by the feedback between industrialization and wages. High wages create incentives to use machines and industrialization raises wages. The model shows that industrialization takes off only if the economy is productive enough, so that wages are sufficiently high. The model also shows that monopoly power can stifle growth, as it lowers wages. A reduction of monopoly power can therefore lead from stagnation to industrialization.

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1. Introduction

During the last two hundred years global output per capita has grown by more than 8. In the more developed countries output per capita has grown by twice as much. Such rapid growth has never been experienced before and is therefore a new historical phenomenon of less than two centuries, which began with the industrial revolution, or at 1820, according to Maddison (1995). This paper is part of the effort to explain this new historical phenomenon, by focusing on industrialization. The paper claims that growth has been made possible by creating machines that can perform various jobs that humans had performed before, and replace them in their work. Prominent examples are the steam engine, the car and the computer. Hence, machines have become our servants and have enabled us to increase production significantly. Unlike scarce humans, machines are available in increasing numbers, since they are easily created. Hence, productivity is increased by using this ever growing army of servants, machines.

This paper builds a growth model that formalizes this idea. It describes a world where the final good is produced by many intermediate goods. Initially, each intermediate good is produced by workers and by some small amount of capital, mainly tools and structures. A machine that replaces these workers can be invented, but that machine is costly, as it consists of some amount of capital that must be purchased. Machines are used and there is demand for them only if their cost is lower than the alternative cost of production by labor. This leads to an important implication of this approach, namely that machines are invented and used only when wages are sufficiently high. Otherwise it does not pay to buy the machine and producers keep using labor instead. Hence, according to this approach, growth is enhanced by high wages. This is because new technologies are not only embodied in capital, but they require an increase of this factor of production. Hence, invention of such technologies depends on factor prices.

Growth depends positively on wages, but affects wages as well. Increasing the number of intermediate goods that are produced by machines leads to an increase of output of these intermediate goods. As a result, wages of workers in other sectors rise. This creates a feedback between growth and wages. This feedback can explain how growth continues over time. Note, that in this model replacing workers by machines does not only substitute factors of production along the same technology, but requires a change of technology as well. This explains why more and more capital can replace workers in this model without ever reaching the point of low marginal productivity.

The model shows that long-run growth prevails if overall productivity is high enough, namely if wages are sufficiently high. If not, the process of growth and industrialization might come to a stop at some point and if overall productivity is very low, industrialization might not even start. This is an interesting result, as it shows that a one-time increase in productivity might change the long-run rate of growth. The model has another interesting result on the effect of monopoly. If producers of intermediate goods have monopoly power, growth is reduced and might even stop completely. The reason is that monopoly power enables producers to maintain high income on behalf of workers' wages, and lower wages deter growth.

These results can shed light on the possible origins of the industrial revolution. One possibility is that the increase in productivity after the discovery of America pushed the global economy from a stagnant pre-industrial equilibrium to a new equilibrium, of on-going industrialization. Another possibility is that the collapse of Feudalism, with its established monopoly rights, and the opening of free labor markets, led to the industrial revolution by raising the cost of labor. These hypotheses, which are suggested by the model, are of course very preliminary and deserve more research.

The paper also considers introduction, in addition to the physical good, of services, which are produced by labor mainly. This leads to two interesting results. The first is that the share of labor in the service sector increases over time. This is because the price of services rises by less than income. The second result is that despite the decline of labor income in manufacturing, the share of labor income in the overall economy does not fall, as is indeed observed in reality.

The model presented in this paper can be viewed as combining together the theory of capital deepening of Solow (1956) together with the theory of technical innovations, namely of R&D based growth, which has been developed by Romer (1990), Segestrom, Anant, and Dinopolous (1990), Grossman and Helpman (1991), Aghion and Howitt (1992) and Jones (1995, a, b). In this paper growth is driven by accumulation of capital, namely of machines, but capital accumulation also changes the production function continuously, as it requires inventing new machines all the time. Hence, the role of technology in this model is very different from R&D based growth models. First, growth does not depend on the ability to innovate or on the size of the R&D sector. Second, this model tries to answer a question unanswered by the R&D models: how do innovations increase productivity? How can obscure scribbles of inventors increase productivity of millions of workers? This paper's answer to this question is that innovations create machines that perform jobs previously done by workers. Innovators invent servants that help us in production. This is not only an explanation to the content of innovations, but it has significant implications for the dynamics of economic growth, as shown below.

The idea of machines that replace workers in performing various jobs has appeared before in Champernowne (1963) and in Habbakuk (1962). This idea is also modeled in Zeira (1998), but that paper studies a very different issue, of technology adoption and output differences across countries. The current paper uses this idea in a very different framework, of global growth, with endogenous invention of technologies. Beaudry and Collard (2002) use a similar idea as well, in analyzing growth dynamics.

The paper is constructed as follows. Section 2 presents the benchmark model. Section 3 describes industrialization and determination of factor prices and Section 4 examines the dynamics of long-run growth. Section 5 discusses the effect of monopoly power on growth. Section 6 presents the various explanations the model offers to the industrial revolution. Section 7 adds a service sector to the economy. Section 8 discusses various other issues, like optimality, divergence and energy prices. Section 9 summarizes.

2. The Model

In this section we describe the benchmark model of growth and industrialization. Consider a closed economy, which produces one final good, which is used both for consumption and for investment. The final good is produced out of a continuum of intermediate goods, ordered on [0, 1]. Production of the final good in period t, Y_t, is described by the following Cobb-Douglas production function:[1]

[1] Alternative production functions, like CES, yield the same results.

(1) $$\log Y_t = \log a + \int_0^1 \log x_{i,t} di,$$

where $x_{i,t}$ is the amount of the intermediate good i used and a is a productivity parameter, which holds for the aggregate economy. It is later shown that this productivity parameter plays an important role in the dynamics of the model.

Each intermediate good can be produced by one of two potential technologies, pre-industrial (manual) or industrial, namely by a machine. Both technologies operate in fixed proportions. In the pre-industrial technology one unit of the intermediate good i is produced by l_i units of labor and k_i units of capital. Capital fully depreciates after one period of time, as time units are assumed to be long. Capital in this pre-industrial technology consists of structures and tools, but not machines. The industrial technology introduces a machine that can produce the same intermediate good. Thus the machine that consists of m_i units of capital can replace the old technology and produce one unit of the intermediate good i. Under this technology capital fully depreciates after one period of production as well. It is assumed that the invention of such a machine is costless, so that a machine is invented once there is demand for it. This assumption is made mainly to fully differentiate between this model and the R&D based growth literature.

We next make a few assumptions on m_i, which lead to the result of long-run growth. First assume that:

(2) $$m_i \xrightarrow[i \to 1]{} \infty.$$

Namely, machines required to produce intermediate goods, which are close to 1, become increasingly complicated and costly. In other words, some jobs, like a CEO, or an engineer, are very hard to replace by a machine. But also assume that this complexity does not make overall machinery too expensive. Hence, the second assumption is that the sum of machine costs over all potential machines is bounded:

$$\int_0^1 \log m_i di < \infty. \tag{3}$$

Denote this upper bound by log *b*. This second assumption is sufficient to have long-run growth, as shown below. If it does not hold, growth stops at some level of output. To these two assumptions on $\{m_i\}$ we add a third one, which is not necessary for any of the results that follow, but it simplifies the presentation of the analysis. Assume that the capital cost per worker of industrializing production is increasing in *I*, namely:

$$\frac{m_i - k_i}{l_i}$$

is increasing in *i*.

We next describe individuals in the economy. Assume that it consists of a mass *L* of identical individuals with infinite horizons. Each person supplies 1 unit of labor in each period and has the following utility from consumption:

$$U = \sum_{t=0}^{\infty} \frac{\log(c_t)}{(1+\rho)^t}. \tag{4}$$

The use of logarithmic utility is for simplification only and the results of the paper hold for any utility function.

3. Industrialization and Factor Prices

The main decision facing producers is choice of technology, namely whether to stick to the old pre-industrial technology or to industrialize. The decision depends on factor prices, since industrialization involves reduction of labor, but at the expense of purchasing more capital. Producers of i adopt the new technology and industrialize in period t if:

$$R_t m_i \leq R_t k_i + w_t l_i,$$

where w_t is the wage rate and R_t is the gross rental rate of capital or the gross interest rate, paid in period t on capital invested in period t – 1. Written differently, production of i is industrialized if:

$$\frac{w_t}{R_t} \geq \frac{m_i - k_i}{l_i}.$$

Thus, the set of intermediate goods produced by machines in period t, namely the industrial set I_t, is equal to:

$$I_t = \left\{ i : \frac{m_i - k_i}{l_i} \leq \frac{w_t}{R_t} \right\}. \quad (5)$$

Hence, the degree of industrialization depends crucially on the wage rate relative to the rate of return. Higher wages create incentive to invent and use more technologies, as these enable reduction of costly labor input. Lower wages on the other hand deter industrialization, as workers are inexpensive relative to costly machines.

Note, that under the above assumption on the relative cost of machines that $(m_i - k_i)/l_i$ is increasing in i, the industrial set I_t can be described in a simple way. It is equal to $I_t = [0, f_t]$, where the industrialization frontier f_t is defined by:

$$\frac{m_{f_t} - k_{f_t}}{l_{f_t}} = \frac{w_t}{R_t}. \quad (6)$$

The degree of industrialization in the economy therefore depends on the factor prices of labor and capital. We next turn to describe how these are determined.

Perfect competition in the markets for intermediate goods leads to the following profit maximization condition:

(7) $$p_{i,t} = \frac{\partial Y_t}{\partial x_{i,t}} = \frac{Y_t}{x_{i,t}}.$$

This describes the demand for the intermediate good. Its supply is perfectly elastic due to fixed marginal productivity. Hence the price of intermediate good is:

(8) $$p_{i,t} = \min\{R_t m_i, R_t k_i + w_t l_i\} = \begin{cases} R_t m_i \text{ if } i \le f_t \\ R_t k_i + w_t l_i \text{ if } i > f_t. \end{cases}$$

Substituting (8) in (7) and then in (1) we get the following relationship between the wage rate, the interest rate and the degree of industrialization:

(9) $$\begin{aligned} &\int_0^1 \min\{\log(R_t m_i), \log(R_t k_i + w_t l_i)\} di = \\ &= \int_0^{f_t} \log(R_t m_i) di + \int_{f_t}^1 \log(w_t l_i + R_t k_i) di = \log a. \end{aligned}$$

This equation describes the factor price frontier.

An alternative way to present the factor price frontier is by the following function:

$$H(w,R) = \int_0^1 \min\{\log(Rm_i), \log(Rk_i + wl_i)\} di.$$

It can be shown that H is concave and increasing in both w and R. From equation (9) it follows that:

(10) $$H(w_t, R_t) = \log a.$$

The factor price frontier can also be written as an explicit function: $w_t = h(R_t)$, where h is defined by $H[h(R), R] = \log a$.

In order to get a better understanding of the factor price frontier note that equation (9) can be rewritten as:

(11) $$\log R_t = \log a - \int_0^1 \min\{\log m_i, \log(k_i + \frac{w_t}{R_t} l_i)\} di.$$

The integral on the RHS of (11) is the area below the two curves in Figure 1. It is clearly an increasing function of the ratio of factor prices w_t/R_t. Denote it by $\varphi(w_t / R_t)$. Due to our assumptions on m_i this function is bounded, since:

$$\varphi\left(\frac{w_t}{R_t}\right) = \int_0^1 \min\{\log(m_i), \log(k_i + \frac{w_t}{R_t} l_i)\} di \le \int_0^1 \log m_i di = \log b < \infty.$$

Furthermore, as is clear from Figure 1 this function is converging to the upper bound, namely to log b. Note that if the factors' price ratio w_t/R_t is very low no intermediate good is industrialized and the industrialization frontier is zero, $f_t = 0$. This is the case of a pre-industrialized economy, like the world prior to the industrial revolution.

[Insert Figure 1 here]

The factor price frontier can therefore be described by the function φ as well:

(12) $$\log R_t = \log a - \varphi\left(\frac{w_t}{R_t}\right).$$

This relationship is described in Figure 2. Clearly, the interest rate is a diminishing function of w_t/R_t, it is bounded from below and as the factor price ratio rises to infinity, log R_t converges to $\log a - \log b$. Note that if the assumption of bounded sum of costs of machines in equation (2) does not hold, then φ is unbounded and R_t converges to 0 as the wage rate rises to infinity.

[Insert Figure 2 here]

Figure 2 describes the relationship between the logarithm of the gross rate of interest and the factor price ratio. Note that to the left of point A the economy is in a pre-industrial state, while to the right of A it is industrializing in greater and greater parts of the economy. From this relationship we can also derive the factor price frontier $w_t = h(R_t)$ itself. Similarly it is a decreasing function and R is bounded below by a/b, so that when R approaches a/b the wage rate goes to infinity. These conclusions are summarized in Figure 3.

[Insert Figure 3 here]

The function H also helps in describing the labor market equilibrium condition:

(13) $$L = \int_{f_t}^{1} l_i x_{i,t} di = \int_{f_t}^{1} \frac{l_i Y_t}{w_t l_i + R_t k_i} di = Y_t H_w .$$

This condition determines output Y_t. The capital market equilibrium condition is also described by use of the function H:

(14) $$K_t = \int_{0}^{f_t} m_i x_{i,t} di + \int_{f_t}^{1} k_i x_{i,t} di = \int_{0}^{f_t} \frac{Y_t m_i}{R_t m_i} di + \int_{f_t}^{1} \frac{Y_t k_i}{w_t l_i + R_t k_i} di = Y_t H_R .$$

Hence, the capital labor ratio is described by: $k_t = K_t / L = H_R / H_w = -h'(R_t)$. Namely, the capital labor ratio is the slope of the factor price frontier. Figure 3 describes in addition to the capital labor ratio also output per worker. Note that the total gross income is equal to gross output:

$$w_t L + R_t K_t = Y_t (w_t H_w + R_t H_R) = Y_t .$$

Hence output per worker can also be described by Figure 3:

$$y_t = \frac{Y_t}{L} = w_t + R_t \frac{K_t}{L} = w_t + R_t k_t .$$

The capital output ratio can also be described by Figure 3 and is denoted x_t:

$$x_t = \frac{y_t}{k_t}.$$

Clearly x_t is increasing with R_t and converges to a/b as R_t goes down to a/b.

4. The Dynamics of Industrialization

The rest of the solution of the model is similar to the standard analysis of the representative agent economy. There are two dynamic conditions. One is the first order condition of utility maximization and the second is the goods market equilibrium. The utility maximization FOC is:

$$\frac{c_{t+1}}{c_t} = \frac{R_{t+1}}{1+\rho}. \tag{15}$$

The goods market equilibrium condition is:

$$c_t = y_t - k_{t+1} = w_t + R_t k_t - k_{t+1} = h(R_t) - R_t h'(R_t) + h'(R_{t+1}). \tag{16}$$

The dynamic Rational Expectations solution to these two dynamic equations, which satisfies the No-Ponzi-Game condition, is a saddle path that converges to a steady state. We next show that there are two main cases of these dynamics.

In order to analyze the dynamics of this economy, where consumption can grow permanently, we define a new variable, the ratio between consumption and capital:

$$v_t = \frac{c_t}{k_t}.$$

Substituting in equations (15) and (16) we derive two dynamic equations of the system with the variables v_t, R_t. The equation that describes the dynamics of R is:

$$\frac{k_{t+1}}{k_t} = \frac{-h'(R_{t+1})}{-h'(R_t)} = \frac{y_t}{k_t} - v_t = x_t - v_t. \tag{17}$$

The dynamics of v are described by:

(18) $$\frac{v_{t+1}}{v_t} = \frac{R_{t+1}}{1+\rho}\frac{k_t}{k_{t+1}} = \frac{R_{t+1}}{1+\rho}\frac{1}{x_t - v_t}.$$

In order to analyze the dynamics of the economy, we draw the phase diagram of the system in Figures 4a and 4b. The curve $R_{t+1} = R_t$ is derived from (17) and is described by $v_t = x_t - 1$ for $R_t > a/b$, which is an increasing curve, and also by the vertical line $R_t = a/b$. The curve $v_{t+1} = v_t$ is derived from (18) and is described by:

(19) $$v_t = x_t - \frac{R_{t+1}}{1+\rho} = x_t - \frac{R_{t+1}(R_t, v_t)}{1+\rho}.$$

It can be shown that this curve, which is the solution to equation (19), has a smaller slope than the $R_{t+1} = R_t$ curve.

Next we differentiate between two cases. In the first case productivity is low, so that $a/b \leq 1+\rho$. This case is described in Figure 4a, where the two curves of the phase diagram intersect at $R_t = 1+\rho$. The dynamic path of the economy is described by the saddle path in Figure 4a. Note that in this case the growth rate falls to zero at the steady state, since the steady state rate of interest is equal to ρ. Hence consumption does not grow at the steady state, and since v is constant at the steady state, capital per worker k and output do not grow as well. Hence, this case describes an economy where industrialization and growth of output per capita stop as the economy reaches the steady state. An even more extreme sub-case is when industrialization does not begin at all. This occurs when productivity a is very small, so that steady state wages are so low, that there is no industrialization at all. This happens if the steady state satisfies $m_0 > k_0 + l_0(w/R)$, as shown in Figure 1 when the two curves do not intersect. In this case the economy remains in a pre-industrialized equilibrium.

[Insert Figures 4a and 4b here]

The second case occurs when productivity a is sufficiently high and $a/b > 1+\rho$. This case is described in Figure 4b. Here the economy converges along the saddle path in Figure 4b to a steady state described by $R^* = a/b$ and to $v^* = \frac{a}{b}\frac{\rho}{1+\rho}$. Therefore, the economy experiences in this case long-run growth. Formally, the rate of growth of consumption converges to g, where:

$$(20) \qquad g = \frac{a}{b(1+\rho)} - 1 > 0.$$

Since v_t converges to a finite number and so does x_t, it follows that both output and capital grow permanently and that their long-run rates of growth are equal to g as well. Hence, this model of machines that replace workers can generate long-run growth.

Furthermore, this model shows that the long-run rate of growth depends crucially on the overall productivity of the economy a. A one-time shock to productivity can lead to increased growth over a long period of time and even to permanent growth. The reason for this result is the following mechanism. The rise in productivity raises labor costs and increases the incentives to use machines. Once these are used wages rise by even more, since intermediate goods cooperate in the production of the final good, and increasing production of some by machines increases the marginal productivity of labor in the other intermediate goods. That creates incentive to invest in more costly machines and put them into use. And so the process of industrialization is rolling on, creating incentives for further industrialization at each step on the way.

Note that growth in this model is enabled by innovations of new machines that replace human labor in various tasks. But unlike the R&D based literature, growth is not determined by the supply of new innovations, but rather by the demand for them. This demand depends mostly on the price of labor, which becomes a crucial variable in this analysis. If productivity is sufficiently high, so that wages are high as well, long-run growth is possible. As equation (20) shows, the rate of growth itself depends not only on productivity, but on the saving behavior of individual, namely on ρ. Note that this result has resemblance to the AK literature. But in other aspects this model differs significantly from the AK literature. The difference can be best seen in the effect of monopoly power on growth, which is discussed in the next section.

5. Monopolies, Wages and Growth

In this section we consider a situation where producers of intermediate goods enjoy a monopoly power. This monopoly power can reflect social norms, like Feudalism, or other conditions of competition. We assume that the monopoly power is exogenously determined and examine how it affects the growth process. Intuitively, the main intuition of the model implies that the effect of monopoly power on growth is negative. Monopoly power enables producers to reduce wages and that tends to impede growth in our framework. The rest of the section formalizes this insight.

Assume that producers of intermediate goods have a monopoly power so that they earn a profit, which is a share z of revenues. One possible situation that can lead to this outcome is the case of N ($N > 1$) producers of each intermediate good, who participate in a Cournot competition. It can be shown that in a symmetric equilibrium each producer earns a profit, which is equal to a share z of revenues, where:

$$z = \frac{2}{N} - \frac{1}{N^2}.$$

If producers earn a profit of rate z, the price of each intermediate good is equal to:

$$p_{i,t} = \begin{cases} \dfrac{w_t l_i + R_t k_i}{1-z} & \text{if } \; i \geq f_t \\ \dfrac{R_t m_i}{1-z} & \text{if } \; i < f_t. \end{cases} \qquad (21)$$

Combining (21) with equation (7) and substituting in equation (1) we get the following factor price frontier:

$$\int_0^1 \min\{\log(R_t m_i), \log(R_t k_i + w_t l_i)\} di - \log(1-z) = \log a. \qquad (22)$$

Hence, in a monopolistic economy the factor price frontier is affected not only by productivity a, but by the degree of monopoly power z as well. As monopoly power increases the factor price frontier is shifted to the left. It is clear from (22) that as wages rise to infinity, the gross rate of interest R converges to:

$$\frac{a}{b}(1-z).$$

The solution of the rest of the dynamics of the model is similar to what is described in Section 4, except that a/b is replaced by $a(1\text{-}z)/b$.[2] Thus if the economy experiences long-run growth its steady state rate of growth is:

$$g = \frac{a(1-z)}{b(1+\rho)} - 1.$$

[2] Note that although agents are not identical under monopoly, as workers and producers earn different incomes, the dynamic equations of the model are the same. Since consumption dynamics are linear for each individual: $c_{t+1} = c_t R_{t+1} (1+\rho)^{-1}$, they can be aggregated across individuals. The goods market equilibrium condition is also the same: $k_{t+1} = y_t - c_t$.

Furthermore, the condition for long-run growth is more restrictive under monopoly. The economy can have long-run growth only if:

$$\frac{a}{b}(1-z) > 1+\rho.$$

It therefore follows that monopoly power impedes growth. If the economy experiences long-run growth the rate of growth is lower due to monopoly. Monopoly power can also shift the economy from long-run growth to stagnation if it is high enough, namely if:

$$z \geq 1-\frac{b}{a}(1+\rho).$$

Hence, in this model monopoly reduces growth and might even cause stagnation. The mechanism through which this effect operates is by lowering wages, which is detrimental to growth.[3]

6. The Industrial Revolution

This section examines how this model can contribute to the understanding of the industrial revolution. We know from various sources, like Maddison (1995), that economic growth has been a fairly recent phenomenon. It started somewhere in the beginning of the 19th century and has been going steadily since then. It is also clear that growth is inherently related to the process of industrialization. Hence, this model of growth through industrialization seems suitable to study the industrial revolution. We should therefore ask what, according to this model, can push the economy from a pre-industrial equilibrium into industrialization.

[3] The effect of wages on growth is also studied recently by Saint-Paul (2005), but through its effect on consumption and demand.

Theoretically, the model offers two potential explanations, namely two exogenous events that could have triggered the industrial revolution. One is a rise in productivity a. Such a rise in productivity that takes the economy over the threshold of $b(1 + \rho)$ can start a process of long-run growth, as shown in Section 4. Thus, a rise in productivity could have triggered the industrial revolution. It increases the cost of labor, creates incentives to use machines, which as a result are invented, produced and become used all over the world. The second potential explanation to the industrial revolution could be a reduction in monopoly power. A stagnant economy can start industrialization and economic growth by reducing its monopoly power, as shown in Section 5. The reduction of monopoly power raises wages, which creates incentives to industrialization and growth.

What are the historical equivalents of an increase in productivity or of a reduction in monopoly power prior to the industrial revolution? Two possible answers come to mind. One is that the rise in productivity in Western Europe could have been the result of the discovery of America. This contributed to sea faring, to agriculture, through discovery of new plants and animals, and also by adding new territories, as described in Maddison (2001, p. 18). This discovery raised incomes and as a result the cost of labor increased as well. The rise in income after the discovery of America is documented in Maddison (2001). Between 1500 and 1820 income per capita in Western Europe, North America and Japan increased by more than 60%. This gives some indication to an increase in productivity. Hence, the discovery of America, and the rise in productivity it created, could be one potential trigger to the beginning of the industrial revolution.

The other historical development that could have triggered the industrial revolution is the decline of Feudalism. This happened first in England following the

Cromwell Revolution, then in France during the Revolution, and in Germany under Bismarck. During the 19th century all over Europe the old system of control by few over land and over production was crumbling down. Our model claims that these historical developments could also trigger and enable the industrial revolution.

This paper is of course not capable of assessing these two explanations to the beginning of the industrial revolution, namely the discovery of America and the collapse of Feudalism. It is possible that the two historical developments together contributed to it. It is also possible that the two events were not completely independent of one another, and the discovery of America contributed to the decline of Feudalism. This should be kept in mind when we try to test some of the ideas of this paper by looking at the historical data. Also, the relationships between the discovery of America, the collapse of Feudalism and the industrial revolution have been noted before.[4] This paper has two specific contributions in this respect. The first is exposing the direction of the effect from these two events to the industrial revolution, and the second is pointing at the cost of labor as the main mechanism of effect.

7. Goods and Services

In this section we present another extension of the model, which adds a second final good to the analysis. In addition to the physical good produced by labor and capital as in the benchmark model, this section introduces a service good, which is produced by labor only. This addition eliminates one result of the benchmark model which is at odds with the stylized facts of economic growth. The benchmark model predicts that the share of

[4] One example that comes to mind is of course the Communist Manifesto by Marx and Engels (1998).

capital in output, which is equal to R_t/x_t, is rising to 1 during the growth process, so that the share of labor declines to zero. These results do not fit the patterns of growth in the last two centuries, where the shares of labor and capital in output have been rather stable around 2/3 and 1/3 respectively. Once we add the service good to the model the share of labor does not diminish in the economy as a whole, but only in the manufacturing sector. Furthermore, the share of labor does not diminish because the number of workers in the service sector increases continually, which fits the empirical observations as well.[5]

Consider a model with two final goods. One is a physical good produced by many intermediate goods, themselves produced by labor and capital, as described in Section 2. This good is used for consumption and for investment. The second good is a service good, produced by labor only, where each unit of the good is produced precisely by one unit of labor. The service good is used for consumption only, but not for investment. Utility is derived from consumption of the physical good c and consumption of the service good s:

$$(23) \qquad \sum_{t=0}^{\infty} \frac{\log c_t + \alpha \log s_t}{(1+\rho)^t}.$$

It is further assumed that the size of the population is fixed and equal to 1.

Due to perfect competition in the labor market and to the linear technology of production of services, the price of the service good is the wage rate w_t. It follows that the demand for the service good satisfies:

$$(24) \qquad s_t = \frac{\alpha c_t}{w_t}.$$

[5] For another discussion of the dynamics of the share of labor see Zuleta (2003).

In other words, $\alpha/(1+\alpha)$ is the share of services in total consumption expenditures. Maximizing utility we get the first order condition:

(25) $$\frac{c_{t+1}}{c_t} = \frac{R_{t+1}}{1+\rho},$$

which is the same as in the benchmark model. Capital accumulation is similar as well:

(26) $$K_{t+1} = Y_t - c_t.$$

The main departure from the equilibrium of the benchmark model is that the supply of labor for production of the physical good is no longer equal to the overall supply of labor in the economy. It is determined by:

(27) $$L_t = 1 - s_t = 1 - \frac{\alpha c_t}{w_t}.$$

Equations (25), (26) and (27), completely determine the rational expectations dynamic path of the economy.

Since the full dynamic analysis is a bit cumbersome, we only present some aspects of the dynamic path. Assume that we are in the case of sustainable growth, namely that $a/b > 1+\rho$. From (25) it follows that the rate of growth of consumption of the physical good converges to:

$$g = \frac{a/b}{1+\rho} - 1 > 0.$$

The rate of growth of capital is given by:

$$\frac{K_{t+1}}{K_t} = \frac{Y_t}{K_t} - \frac{c_t}{K_t} = x_t - \frac{c_t}{K_t}.$$

Since x_t converges to a finite number *a/b*, it follows that the rate of growth of capital must converge to that of consumption of the physical good. Hence the ratio between consumption of goods and capital converge to:

$$\frac{\rho a}{b(1+\rho)}.$$

Note that from equation (24) and from Figure 3 we get:

$$L_t = 1 - s_t = 1 - \frac{\alpha c_t}{k_t(x_t - R_t)} = 1 - \alpha \frac{c_t}{K_t} \frac{L_t}{x_t - R_t}.$$

Hence, labor input in the industrial sector satisfies:

(28) $$1 = L_t\left(1 + \alpha \frac{c_t}{K_t} \frac{1}{x_t - R_t}\right).$$

Since $x_t - R_t$ converges to zero, it follows from (28) that L_t converges to zero as well. Namely, the share of labor in manufacture declines to zero, while the share of labor in services increases continuously to 1.

We next examine the share of capital in income. The ratio of income to capital income satisfies:

$$\frac{R_t K_t + w_t}{R_t K_t} = 1 + \frac{\alpha}{R_t} \frac{c_t}{K_t} \frac{w_t}{\alpha c_t} = 1 + \frac{\alpha}{R_t} \frac{c_t}{K_t} \frac{1}{s_t} \xrightarrow[t\to\infty]{} 1 + \frac{\alpha\rho}{1+\rho} = \frac{1+\rho+\alpha\rho}{1+\rho}.$$

Hence the share of capital in income converges to:

$$\frac{1+\rho}{1+\rho+\alpha\rho}.$$

Note that if $\alpha = 2$, so that the share of services in consumption is 2/3, and if $\rho = 3$, which is reasonable for a period of 30 years, we get that the share of capital in income converges to .4. Hence, the model, despite its great simplifications, leads to results which are close to the empirically observed stylized facts.

This extension of the model, therefore, avoids the result that the share of labor in income is diminishing to zero. It also has an additional interesting result. The share of labor in services increases continually. This has two intuitive explanations. One is that less and less workers are required in manufacturing, since they are replaced by machines. Second, the price of the service good rises by less than income, as shown in Figure 3 and as is clear intuitively. Hence, the demand for the service good increases and its production increases with it.

8. Discussion

This section discusses briefly some implications of the model. It first examines the optimality of equilibrium and shows that the market equilibrium is indeed optimal. It then examines whether the model can account not only for global growth, but also to the large and increasing differences between regions since the industrial revolution. It then discusses the effect of energy prices on growth. They are related in this model since replacing workers by machines also involves replacing human energy by thermal energy. It is shown that energy prices have a negative effect on growth. Finally this section discusses the issues of factor shares along the growth path, and the interpretation of the growth of TFP.

7.1. Optimality of Equilibrium

The market equilibrium described which is described in Sections 3 and 4 is also optimal. To see this divide the optimization of utility into two parts: temporal and intertemporal maximization. The temporal maximization is:

(29) $$\max_{I_t, x_{i,t}} \left\{ \int_0^1 \log x_{i,t} di : \int_{I_t} m_i x_{i,t} di + \int_{I_t^c} k_i x_{i,t} di = K_t, \int_{I_t^c} l_i x_{i,t} di = L \right\}.$$

The maximization yields two shadow prices, $\lambda_{1,t}$ and $\lambda_{2,t}$, which are equal to the real wage and gross interest rate, w_t and R_t, respectively. The first order conditions of (29) correspond to the equilibrium conditions derived in Section 3. In the second stage the central planner maximizes the intertemporal utility:

(30) $$\sum_{t=0}^{\infty} \frac{\log c_t}{(1+\rho)^t} = \sum_{t=0}^{\infty} \frac{\log(y_t - k_{t+1})}{(1+\rho)^t}.$$

It can be shown that the first order conditions of this maximization are the same as the dynamic conditions of the market equilibrium. Hence, the market equilibrium is optimal.

7.2. Divergence between Regions

So far this model has been used to describe global economic growth, namely it implicitly assumed that the closed economy is the world. Next we show that the model can be applied to explain large and growing differences across countries. As shown in many empirical studies, like Maddison (1995), Pritchett (1998), and Bourguignon and Morrison (2002), gaps between regions in the world have been increasing significantly since the beginning of the industrial revolution. This section shows how this model can account for such findings.[6]

Consider a world with two countries, or regions, *A* and *B*. The two countries are similar except in their basic productivity *a*, and it is assumed that $a_A > a_B$. Furthermore, assume that:

(31) $$\frac{a_A}{b} > 1 + \rho > \frac{a_B}{b}.$$

[6] Zeira (1998) already uses a similar model to explain differences across countries. This section further strengthens this result.

Assume also that there is full capital mobility in the world. For simplicity assume that the intermediate goods are not tradable. The equilibrium in this economy is easy to solve. Clearly the gross interest rate must satisfy: $R_t > a_A / b$. Hence, economy A grows a positive rate, which is higher than $a_A b^{-1}(1+\rho)^{-1} - 1$. Economy B is stagnant and gets stuck at a fixed level of wages and output per capita.[7]

This model can therefore account for very different growth performances of regions in the world, due to differences in basic productivity. It is interesting to examine the data presented by Maddison (2001) with respect to two main regions. Region A is Western Europe, Western Offshoots and Japan. Region B is the rest of the world. In 1500 GDP per capita in A was 704, while GDP per capita in B was 535. Until 1820 GDP per capita in A rose to 1,130, more than 60%, while GDP per capita in B rose only to 573, a rise of 7%. This shows that at the outset of the industrial revolution productivity difference between regions were significant.

7.3. Energy and Growth

Actually the machines that replace humans in various tasks and jobs require energy to work. In a way machines that replace workers also replace the source of energy, from human energy to fossil energy, either coal or oil. Thus, if we want to model the process of replacing workers by machines more realistically, we should add the energy requirements of machines as well. Next we extend the model in this direction in a very simplified way. Assume that when an intermediate good i is produced by machines it requires a machine

[7] This equilibrium has one aspect which is not realistic, namely that consumption in both A and B grows at the same rate. This means that consumption in the stagnant economy has very low levels in period 0. This result is due to the Ramsey framework and to having the same interest rate in both economies. One way to avoid this type of result is to assume a different structure of population. An economy of overlapping generations with utility from bequests, and with minimum subsistence consumption can yield the same results with respect to growth, but with consumption in the stagnant economy being stagnant as well.

of size m_i and an input of energy of size e_i. Assume that the price of energy is q, and that it is fixed over time. Clearly, the condition for industrialization is similar to the benchmark model:

$$R_t m_i + qe_i \leq R_t k_i + w_t l_i. \tag{32}$$

The condition that determines the factor price frontier is:

$$\int_0^1 \min\{\log(R_t m_i + qe_i), \log(R_t k_i + w_t l_i)\} di = \log a. \tag{33}$$

It can be shown that the equilibrium is similar to the industrial growth equilibrium described above in Section 5. But the long run growth in this case depends crucially on the price of energy. If the price rises during the period of industrialization it can hold it down and even stop it. Thus according to this model the growth process is inherently bounded by the supply of energy on our planet. Of course, we can assume that the stock of energy on our planet is large enough, and that even when it is depleted we will be able to find other ways of harnessing solar energy to our use. But this brief analysis demonstrates that the price of energy is crucial for the process of industrialization and economic growth.

9. Summary

This paper presents a model of industrialization, by assuming that it consists of inventing new machines that replace workers in performing a growing set of tasks. In this process of economic growth the wage rate plays a critical role. Wages serve as an incentive for adopting new technologies. But wages are also affected by adoption of technologies, since performance of some tasks by machines enables the workers, who perform the remaining tasks, to have higher wages, due to higher marginal productivity. This feedback between wages and technology is the main mechanism that drives the results of this paper. It explains how the growth process can continue for long periods, where wages induce innovations, and these in turn raise wages. This role of wages also explains why monopoly deters growth. Monopoly power enables producers to earn monopoly rent, which reduces wages.

Finally we discuss briefly the type of innovations analyzed in this paper, namely machines that replace human labor. Although this is a specific type of innovation, it can be shown to be quite general. An innovation that replaces a machine by a better one, also enables the workers operating it to produce more, namely to use less labor in production. Note that innovations of new consumption goods also tend to replace labor this way or the other. A dishwasher, TV dinner, radio, cinema, all replace labor, either at home, or in other locations. We do not have to go back to the time of the Ludites, to realize that new machines that replace human labor have been a central element in economic growth since the industrial revolution. This paper shows that embodying this insight into growth theory can help us significantly in understanding the growth process.

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Figures

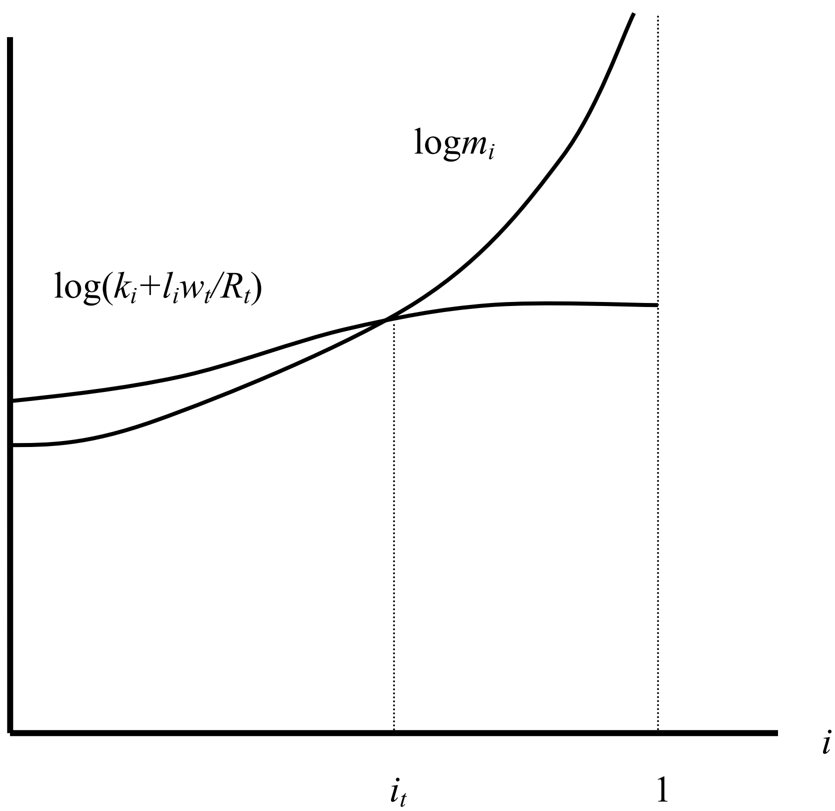

Figure 1

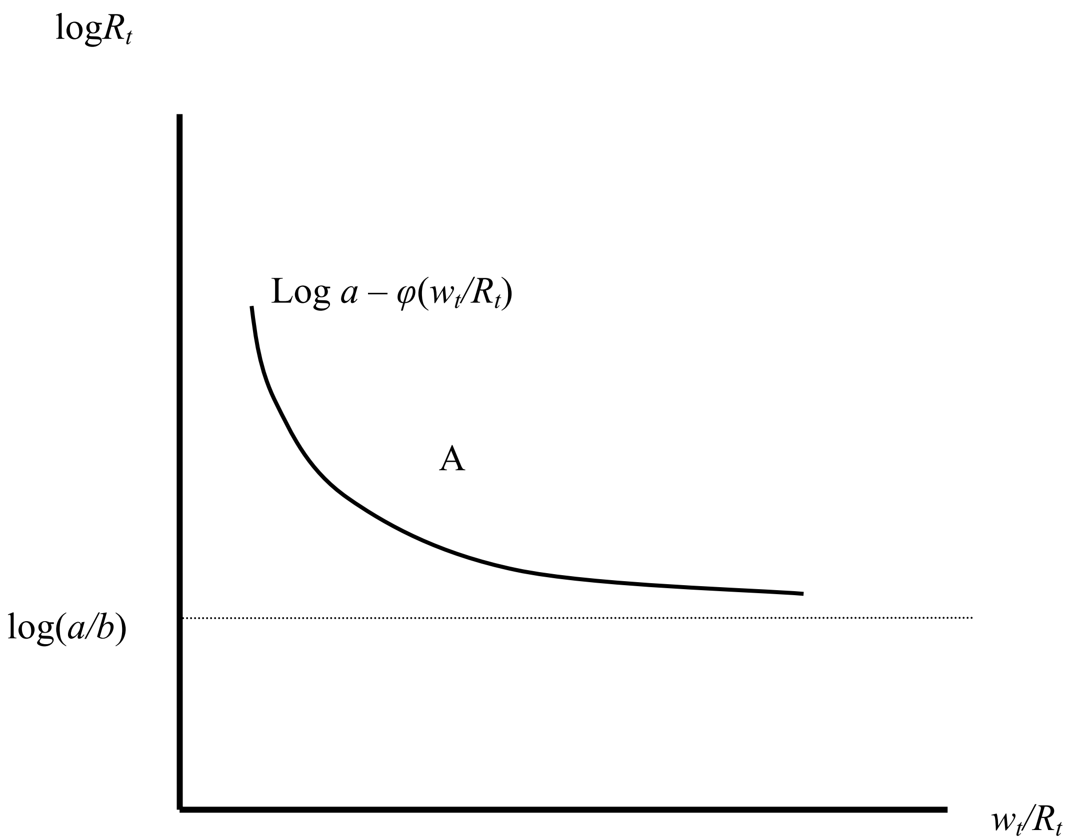

Figure 2

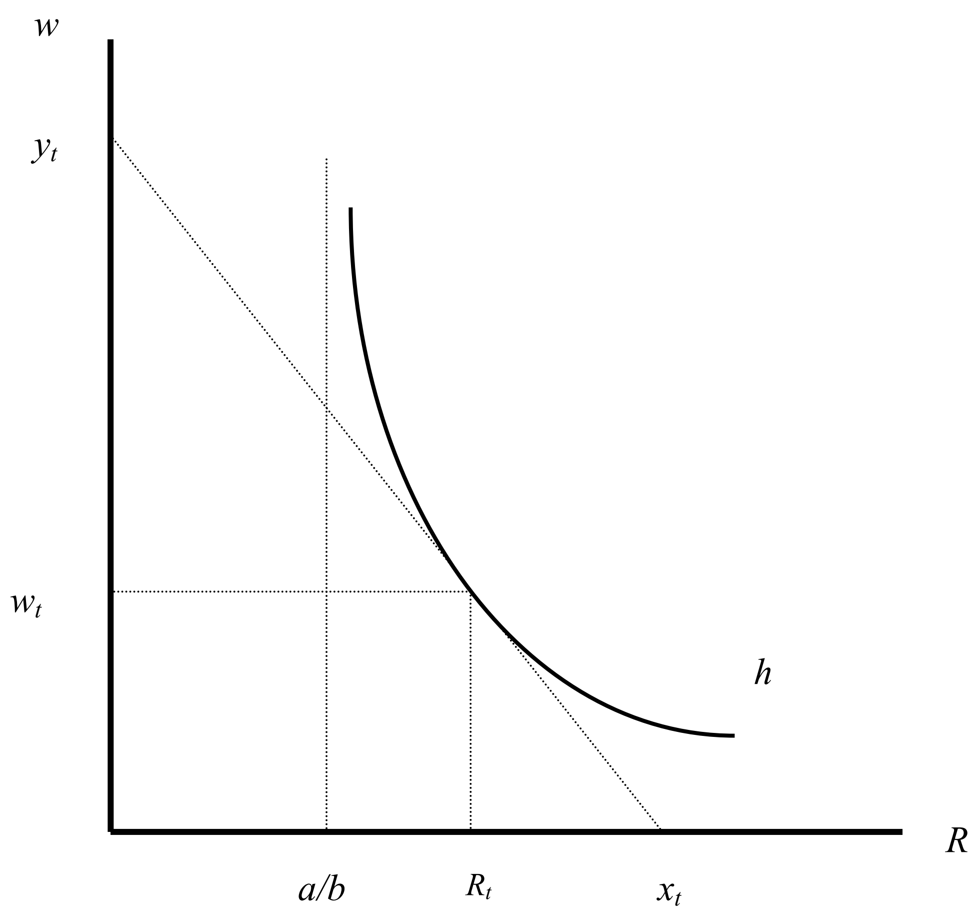

Figure 3

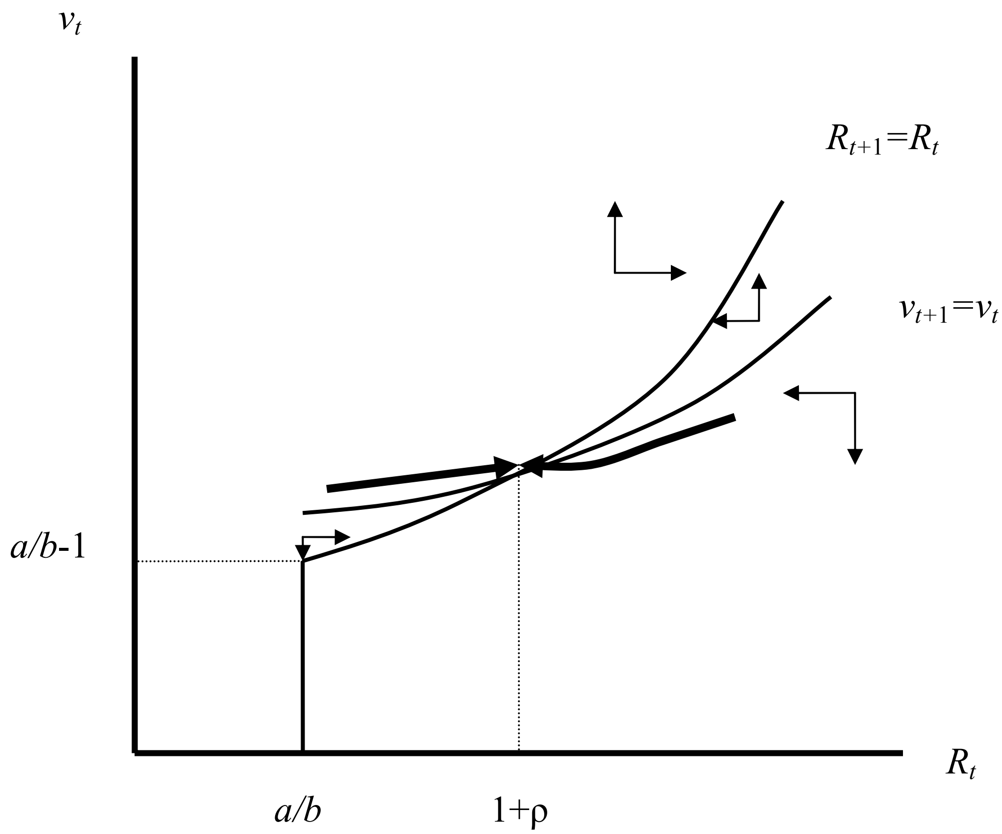

Figure 4a

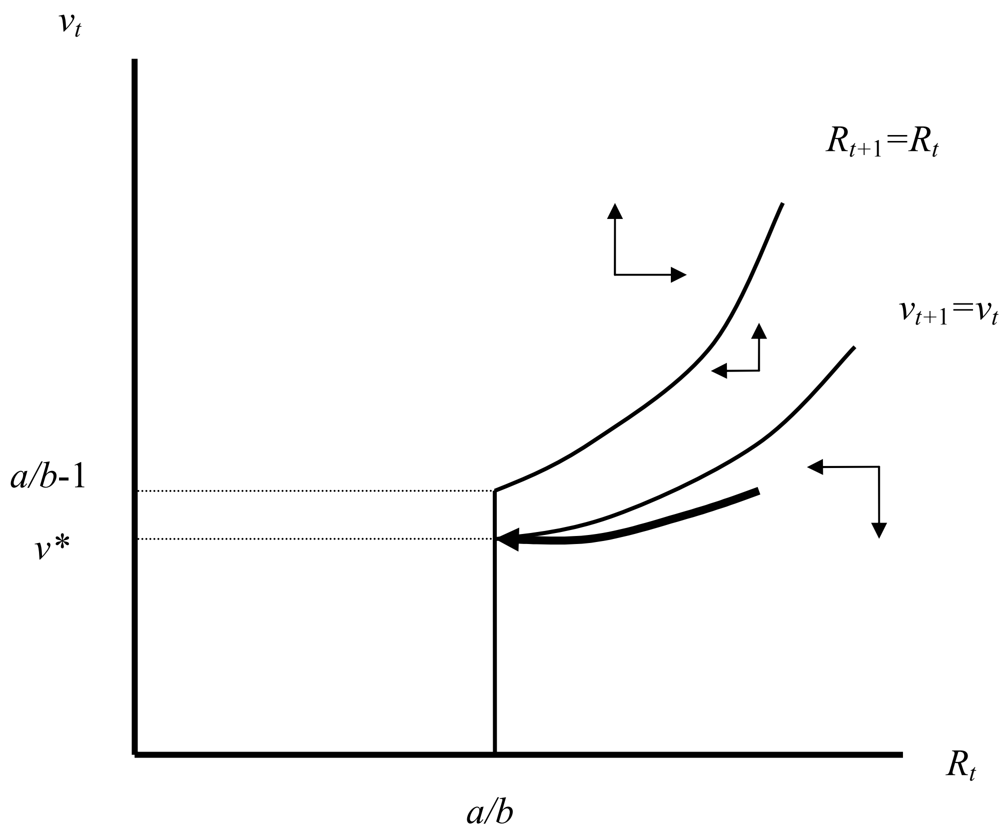

Figure 4b