Technological Growth and Asset Pricing

(Preliminary and Incomplete)

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Abstract

In this paper we study asset pricing in the presence of technological growth. We present a model, where new technologies arrive periodically, giving firms the opportunity to plant new trees of a better vintage. The model gives rise to an endogenous technological cycle that may have a duration longer than the business cycle. In its initial phase a substantial part of the stock market valuation is driven by growth options. P/E ratios and expected returns are high, and attain a maximum once firms start to invest in the new technologies. As the economy experiences an investment driven boom, growth opportunities get exploited, leading to decreased P/E ratios and lower future expected returns. We show that the recurrence of such cycles can account *jointly* for a considerable number of well documented asset pricing phenomena in both the time series and the cross section of expected returns. We also show how the model can provide a micro-foundation for empirical evidence documenting that several asset pricing phenomena are driven by "long-run" risks. In two concrete applications we show how the model can account for the weakening of cross sectional predictability between 1986-2001 and the recently documented sucess of the consumption CAPM over longer horizons.

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1 Introduction

How does aggregate technological growth affect asset pricing? This question seems to be of paramount importance in light of the events of the late nineties. It is commonly accepted that the boom in the stock market during that period was related to rapid technological changes brought about by the IT Revolution and the Internet in particular. Rapidly rising price to earnings ratios were accompanied by strong and unusual returns, followed by a dramatic decline thereafter. These patterns are recurrent in times of technological progress. For example,very similar patterns were observed in the twenties, another period of rapid technological growth in distribution and production (radio, automobiles, department stores)

The process of invention, development and diffusion of new technologies has been widely studied in the economic literature. Hardly anyone would dispute that technological progress is the most important factor in determining living standards over the long run. It appears equally plausible that the anticipation of the benefits of technological advancement was a key determinant of asset price movements during many periods of economic history. Yet, little work has addressed the impact of technological growth on the pricing of risk.

The goal of this paper is two-fold.

The first goal is to build a theoretical model to study this interaction, assuming perfect rationality and capital markets. A key insight of the model is that the arrival of important technological innovations will lead to long lasting cycles in output growth, consumption growth and investment. Moreover, these long cycles will be shared by asset prices as well.

The second goal is to assess the empirical relevance of such long run variations, and their potential to explain jointly a wide number of asset pricing facts documented in the literature.

The key idea behind our theoretical framework is that technological growth is "general purpose" and permeates all industries simultaneously. The internet, for instance, fundamentally transformed the distribution channel of most industries. Similarly, the arrival of electricity transformed manufacturing and the automobile changed transportation in a number of sectors. Organizational, managerial, or financial innovations can equally well present sources of general purpose growth. For instance, the adoption of "just in time" principles in manufacturing, the widespread use of total quality management in production, or the managerial buyouts of the eighties may not present purely technological innovations, but have nevertheless altered the production and distribution process throughout the economy.

The proposed model captures this idea in a relatively simple way. We consider a standard Lucas tree economy and assume a continuum of firms that can plant trees. Technological innovation arrives at random times and presents all firms with (a firm specific) option to plant trees of a new "vintage" at a time of their choosing. Firms need to hire workers to plant these trees by paying them a wage that acts as a fixed cost. As a result, the arrival of a technological innovation generates growth options, which are progressively exploited over the medium run. This process gets repeated each time a new technological innovation arrives.

Almost all the results that are discussed in the paper revolve around one key intuition: Technological growth introduces a "life cycle" of growth options for all companies. On arrival of a new major innovation, growth options emerge in the price of all companies driving valuation ratios (such as the P/E ratio) up. Over time firms start to "exercise" their growth options and this leads to an acceleration of growth, followed by a decline the P/E ratios.

There are several distinct implications of this basic mechanism:

First, technological innovation introduces a slowly moving (stochastic) cycle into the behavior of both consumption growth and asset prices. We refer to such cycles as the "medium run" to keep with terminology in Comin and Gertler [2003]. In the first part of the cycle, firms wait and do not exercise their growth options. Since growth options are riskier than assets in place in our model, this drives the expected return of these companies up. As growth options get exercised however, the economy experiences strong growth, the P/E attains a maximum (and even starts to decline) and expected returns (going forward) become particularly low. This reversal in expected returns is particularly noticeable for the companies that can profit the most from the newly invented technology ("high tech firms"). We calibrate the model and argue that this change in the composition of value between growth options and assets in place over the technological cycle can account for the empirical finding that the aggregate P/E ratio and investment plans can predict (aggregate) excess returns - especially over longer horizons -.

Second, the model demonstrates how a substantial degree of covariation between financial markets and the real economy is attributable to these slowly moving, technological cycles. We use econometric methods that are tailored to decompose the comovement between different time series into medium-long and short run cycles, and find evidence that a substantial part of the comovement between the P/E ratio and economic growth is accounted for by lower frequencies (long-medium run cycles) in the data, just as the model predicts.

Third, we demonstrate how the model can account for certain interesting phenomena in the cross section of expected returns that have been established in the literature. In the model, firms obtain different growth opportunities in different cycles and this leads to heterogeneity in the cross section. Firms with numerous assets in place (compared to their growth options) will be less risky, larger and will have a high ratio of market to book value for their assets, so the model can produce both a size and value premium. A separate role for short run momentum and long run contrarian profits is also compatible with our framework.

In addition to these mostly well documented asset pricing phenomena, we also focus on some aspects of the cross section that we believe our model is particularly well suited to address.

First, we show how the model can account for the prolonged disappearance of the usual predictability relations between 1986-2001 (and their reappearance thereafter). This phenomenon is puzzling, because neither behavioral nor rational explanations can account for all its pieces. Existing models that produce countercyclical size premia over the business cycle will typically not be able to explain the prolonged duration of the phenomenon. At the same time, behavioral stories that might view the value and the size premium as anomalies that eventually had to disappear cannot account for their reappearance post 2001. Our model links the disappearance of cross sectional predictability to the technology driven long cycle of the US economy between 1986-2001. Specifically, we propose the following explanation: Sorting on size (or market to book) at the onset and the initial phases of a technological cycle will make it impossible to know if a given firm belongs to a given decile because it has very valuable assets in place or current growth options. Hence, sorting on size will produce inconclusive results about the expected return of the average firm in these deciles. When -however- most firms have used up their growth options and the technological cycle has progressed, size will be uniquely attributable to valuable assets in place, restoring the predictability relations. Calibrated versions of the model suggest that this process can easily last more than 10 years.

Second, we show how the model can account for the interesting findings in Parker and Julliard [2005] that the consumption CAPM seems to perform better at predicting cross sectional differences if one uses longer run consumption growth rates instead of short run consumption growth rates. Even though the conditional consumption CAPM holds in our framework, the unconditional CAPM does not. More importantly, a good fraction of the cross sectional differences will be driven by the prospects of the firms over the medium run cycle. Taking longer run consumption growth rates reveals more information about the true conditional beta of a portfolio, simply because over the longer run there is more significant correlation between the predictable component in consumption growth and cross sectional differences in expected returns.

In sum, the present paper presents an attempt to link many well documented asset pricing phenomena to macroeconomic movements that might outreach the duration of a typical business cycle. The model provides thus a micro-foundation for *why* medium-long cycles exist and *why* they matter for asset pricing.

The literature closest to this paper is the production based asset pricing literature (Berk, Green, and Naik [1999],Berk, Green, and Naik [2004], Kogan [2001], Kogan [2004], Gomes, Kogan, and Zhang [2003], Carlson, Fisher, and Giammarino [2004b], Zhang [2005], Cooper [2004]). To the best of our knowledge, the only other paper to address the cross section of expected returns in general equilibrium is Gomes, Kogan, and Zhang [2003]. The most significant difference between their model and ours is the presence of a true timing decision as to the exercise of the growth options. Gomes, Kogan, and Zhang [2003] assume that options arrive at exogenous random times, and the firms must decide "on the spot" if they want to proceed with the investment or not. If not, they have to wait again for a random amount of time. By contrast in our model, all firms are presented with a (firm specific) opportunity to plant a tree at the same time. However, they have full discretion as to the timing. This is not a mere technicality. In our model there is going to be some degree of simultaneity in the exercise of growth options and this will be the key mechanism behind our medium run cycles[1].

We also relate to the recent literature on long run risks (Bansal and Yaron [2004], Parker and Julliard [2005], Hansen, Heaton, and Li [2005] among others). These papers are mostly econometric

[1]We also improve on Gomes, Kogan, and Zhang [2003] in some other dimensions. In our model consumption is a random walk over the short run and presents some (but weak) predictability over longer horizons only. By contrast, in Gomes, Kogan, and Zhang [2003] the consumption process has stronger serial correlation, which appears less well supported by the data. Moreover, we are able to integrate habit formation in a tractable way into our model, which allows us to obtain a more realistic equity premium with a reasonable degree of risk aversion.

approaches to formalizing and measuring the notion of long run risk. Our purpose in this paper is different. We are interested in determining *why* long run risks exist in the consumption process and *why* they correlate with portfolios in the ways that can account for cross sectional asset pricing phenomena.

There is a vast literature in macroeconomics and growth that analyzes innovation, dissemination of new technologies or the impact of the arrival of new capital vintages. A highly partial listing would include Jovanovic and Rousseau [2004], Hobijn and Jovanovic [2001], Jovanovic and MacDonald [1994], Jovanovic and Rousseau [2003], Cooley and Yorukoglu [2003], Yorukoglu [1998], Benhabib and Rustichini [1991], Greenwood and Jovanovic [1999], Jovanovic and Stolyarov [2000], Rustichini and Siconolfi [2004], Atkeson and Kehoe [1999], Atkeson and Kehoe [1993], Helpman [1998]. Medium run cycles were originally introduced as a notion by Blanchard [1997] and more recently by Comin and Gertler [2003]. Our paper has a fundamentally different scope than this literature. In most of these models, uncertainty and the pricing of risk is not the focus of the analysis. By contrast these papers analyze innovation decisions in much greater depth than what we do. The trade-off is that they cannot allow for sufficiently rich uncertainty, and an endogenous determination of the stochastic discount factor as is possible in the slightly simpler setup of our paper. This is why most of this literature cannot be readily used for an in-depth asset pricing analysis, which is necessarily linked with the pricing of risk.

An important technical contribution of our work is that it provides a tractable solution to a general equilibrium model, where the micro-decsions are "lumpy" and exhibit optimal stopping features. The micro decision of the firm has a similar structure to the recent sequence of papers by Abel and Eberly [2003], Abel and Eberly [2002], Abel and Eberly [2004]. Just as firms in these papers adapt to the technological frontier at an optimally chosen time, firms in our framework decide on the optimal time to plant a new tree.

A somewhat related literature is concerned with the aggregation of S-s rules (Caballero and Engel [1999], Caballero and Engel [1991]) and optimal entry decisions of firms. A partial listing would include Dixit [1989], Dixit and Pindyck [1994], Dixit and Rob [1994], Leahy [1993], Caplin and Leahy [1997], Caballero and Pindyck [1996], Baldursson and Karatzas [1997], Novy-Marx [2003]. These are typically not general equilibrium models however, which makes it impossible to derive the stochastic discount factor and its correlation with asset prices endogenously.

A recent literature in Macroeconomics (Thomas [2002], Khan and Thomas [2003]) considers S-s rules in general equilibrium. This literature typically solves these models numerically and hence the level of theoretical analysis that can be performed is limited. Moreover, there is no role for capital vintages, medium run components or a discussion of asset pricing implications. By considering a different specification for adjustment costs we arrive at simple closed form solutions for all prices that can be analyzed quite closely. Additionally, we can address issues related to different capital vintages and technological cycles.

The present paper shares with many papers in the literature assumptions on external habit formation to produce large equity premia, while keeping the interest rate at relatively low levels (Campbell and Cochrane [1999], Chan and Kogan [2002]). By contrast to these papers however, we derive most of our results from endogenous investment decisions of firms - *not* habit formation and time varying risk aversion. In particular, by assuming external habit formation that is multiplicative in the utility function we effectively enforce constant relative risk aversion for the representative agent. We do this in order to isolate the *new* channels that are present in our paper, compared to previous literature.

Several recent papers have also attempted to address issues specific to the recent upswing in asset prices (Pastor and Veronesi [2004], Jermann and Quadrini [2002]). Our purpose in this paper is broader. We want to understand how technological growth interacts with asset prices, and how it leads to long cycles at a more general level than the specifics of any particular historical episode.

The structure of the paper is as follows: Section 2 presents the model and Section 3 its solution. Section 4 presents the qualitative implications of the model, while section 5 presents empirical evidence and quantitative implications. Section 6 concludes. All proofs are given in the appendix (not included in this version of the paper).

2 The model

2.1 Trees, Firms and Technological Epochs

2.1.1 Trees, Earnings, Epochs and the Firm's Optimization Problem

There exists a continuum of firms indexed by $j \in [0,1]$. Each firm owns a collection of trees that have been planted in different technological epochs[2], and its total earnings is just the sum of the earnings produced by the trees it owns. Each tree in turn produces earnings that are the product of three components: a) a vintage specific component that is common across all trees of the same technological epoch, b) a time invariant tree specific component and c) an aggregate productivity shock. To introduce notation, let $Y_{N,i,t}$ denote the earnings stream of tree i at time t, which was planted in the technological epoch $N \in (-\infty..-1,0,1,..+\infty)$. In particular, assume the following functional form for $Y_{N,i,t}$:

$$Y_{N,i,t} = \left(\overline{A}\right)^N \zeta(i)\theta_t \tag{1}$$

$\left(\overline{A}\right)^N$ captures the vintage effect. $\overline{A} > 1$ is a constant. $\zeta(\cdot)$ is a positive strictly decreasing function on $[0,1]$, so that $\zeta(i)$ captures a tree specific effect. θ_t is the common productivity shock and evolves as a geometric Brownian Motion:

$$\frac{d\theta_t}{\theta_t} = \mu dt + \sigma dB_t \tag{2}$$

where $\mu > 0, \sigma > 0$ are constants, and B_t is a standard Brownian Motion.

Technological epochs arrive at the Poisson rate $\lambda > 0$. Once a new epoch arrives, the index N becomes $N+1$, and every firm gains the option to plant a single tree of the new vintage at a time of its choosing. Since $\overline{A} > 1$, and N grows to $N+1$, equation (1) reveals that trees of a later epoch are on average "better" than previous trees.

Firm heterogeneity is introduced as follows: Once epoch N arrives, firm j draws a random number $i_{j,N}$ from a uniform distribution on $[0,1]$. This number informs the firm of the type of tree that it can plant in the new epoch. In particular a firm that drew the number $i_{j,N}$ can plant a tree with tree specific productivity $\zeta(i_{j,N})$. These numbers are drawn in an i.i.d fashion across epochs: It is possible that firm j draws a low $i_{j,N}$ in epoch n, a high $i_{j,N+1}$ in epoch $N+1$ etc.

[2] We shall also use word “round” to refer to an epoch.

To simplify the setup, we shall assume that once an epoch changes, the firm loses the option to plant a tree that corresponds to any previous epoch. It can only plant a tree corresponding to the technology of the current epoch.

Let:

$$X_{j,t} = \sum_{n=-\infty..N} \overline{A}^n \zeta(i_{j,n}) 1_{\{\widetilde{\chi}_{n,j}=1\}} \tag{3}$$

where N denotes the technological epoch at time t and $\widetilde{\chi}_{n,j}$ is an indicator function that is 1 if firm j decided to plant a tree in technological epoch N and 0 otherwise. A firm's total earnings are then given by:

$$Y_{j,t} = X_{j,t}\theta_t$$

Any given firm determines the time at which it plants a tree in an optimal manner. Planting a tree requires a fixed (labor) cost of w_t. This cost is the same for all trees of a given epoch. However, it may depend on the technological round and the entire path of θ_t and it is determined in general equilibrium.

Assuming complete markets, the firm's objective is to maximize its share price. Given that options to plant a tree arrive in an i.i.d fashion across epochs, there is no linkage between the decision to plant a tree in this epoch and any future epochs. Thus, the option to plant a tree can be studied in isolation in each round. Hence, the optimization problem of firm j in epoch N amounts to choosing the optimal stopping time τ:

$$P^o_{N,j,t} \equiv \sup_{\tau} E\left[1_{\{\tau<\tau_{N+1}\}}\left(\overline{A}^N \zeta(i_{j,N}) \int_{\tau}^{\infty} \frac{H_s}{H_t}\theta_s ds - \frac{H_\tau}{H_t} w_\tau\right)\right] \tag{4}$$

where H_s is the (endogenously determined) stochastic discount factor, τ_{N+1} is the random time at which the next epoch arrives, while $P^o_{N,j,t}$ denotes the (real) option of planting a new tree in epoch N.

2.1.2 Firm Prices

Given the setup, a firm's price will consist of three components: a) the value of assets in place, b) the value of the growth option in the current technological epoch and c) the value of the growth options in all subsequent epochs. To see this, let:

$$P^A_{j,t} \equiv X_{j,t}\left(E\int_t^{\infty} \frac{H_s}{H_t}\theta_s ds\right)$$

denote the value of assets in place (with $X_{j,t}$ as defined in [3]). Then the price of firm j, assuming it has not planted a tree (yet) in technological round N is

$$P_{N,j,t} = P^A_{j,t} + P^o_{N,j,t} + P^f_{N,t} \tag{5}$$

where:

$$P^f_{N,t} = E\left(\sum_{n=N+1..\infty} \frac{H_{\tau_n}}{H_t} P^o_{n,j,\tau_n}\right)$$

and τ_n denotes the time at which technological round n arrives. The first term on the right hand side of (5) is the value of assets in place, while the second term is the value of the growth option in the current epoch. The third term is the value of all future growth options. Naturally, for a firm that has planted a tree in the current technological epoch there is no longer a "live" option and hence its value is given by[3]:

$$P_{N,j,t} = P^A_{j,t} + E\left(\sum_{n=N+1...\infty} \frac{H_{\tau_n}}{H_t} P^o_{n,j,\tau_n}\right)$$

2.2 Aggregation

The total output in the economy at time t is given by

$$Y_t = \int_0^1 Y_t(j)dj = \left(\int_0^1 X_{j,t}dj\right)\theta_t \tag{6}$$

with $X_{j,t}$ defined in (3). It will be particularly useful to introduce one extra piece of notation. Let $K_{N,t} \in [0,1]$ denote the mass of firms that have updated their technology in technological epoch N up to time t. We show formally later that $K_{N,t}$ will coincide with the index of the most profitable tree that has not been planted yet (in the current epoch)[4].

[3] Clearly, the $P^A_{j,t}$ in this formula will now reflect the fact that the assets of the company have been increased by the addition of an extra tree.

[4] To see why, consider two firms j and j'. Assume that in the current epoch firm j has drawn a lower index i^j_N than firm j', so that $i_{j,N} < i_{j',N}$. By assumption $\zeta(\cdot)$ is a decreasing function and hence $\zeta(i_{j,N}) > \zeta(i_{j',N})$. This in turn implies that firm j has the ability to plant a better tree than firm j'. Since the costs of planting a tree in the current epoch are the same for the two companies, company j will always choose to plant a tree no later than company j'. Simply put, firms that can profit more from the new technology (since they have drawn a low i_N) have a strictly higher opportunity cost of waiting.

Since investment in new trees is irreversible, $K_{N,t}$ (when viewed as a function of time) will be an increasing process. Given the definition of $K_{N,t}$, the aggregate output is given as

$$Y_t = \left[\sum_{n=-\infty..N-1} \overline{A}^{(n-N)} \left(\int_0^{K_{n,\overline{\tau_n}}} \zeta(i)di \right) + \int_0^{K_{N,t}} \zeta(i)di \right] \overline{A}^N \theta_t$$

where $\overline{\tau_n} = \tau_{n+1}$ denotes the time at which epoch n ended (and epoch $n+1$ started). To analyze this decomposition it will be easiest to define

$$F(x) = \int_0^x \zeta(i)di$$

It can easily be verified that, $F_x \geq 0$ (since $\zeta(\cdot) > 0$) and $F_{xx} < 0$, (since $\zeta(\cdot)$ is declining). Hence $F(x)$ has the two key properties of a production function. Using the definition of $F(\cdot)$, Y_t can be rewritten as

$$Y_t = \left[\sum_{n=-\infty..N-1} \overline{A}^{(n-N)} F\left(K_{n,\overline{\tau_n}}\right) + F(K_{N,t}) \right] \overline{A}^N \theta_t \tag{7}$$

The aggregate output is thus the product of two components: A stationary component (inside the square brackets) and a trending component $\left(\overline{A}^N \theta_t\right)$ which captures the joint effects of aggregate technological progress and aggregate productivity growth. The term inside the square brackets is a weighted average of the contributions of the different vintages of trees towards the aggregate product. The weight on trees that were planted in previous epochs decays geometrically at the rate $\overline{A}$. In this sense, $\overline{A}$ is simultaneously the rate of technological progress (in terms of new trees) and technological obsolescence (in terms of existing ones).

Equation (7) implies our model (at the aggregate) resembles a simplified vintage model. The particular structure that we have introduced will allow us to both solve and characterize properties of the model in closed form. Moreover, it will allow us to obtain the entire cross sectional distribution of firm size, returns, book values and thus analyze both the macroeconomic and the asset pricing implications of the model jointly.

2.3 Markets

As is typically assumed in "Lucas Tree" models, each firm is fully equity financed and the representative agent holds all its shares. Moreover, claims to the output stream of these firms are the only assets in positive supply, and hence the total value of positive supply assets in the economy is:

$$P_{N,t} = \int_0^1 P_{N,j,t} dj$$

Next to the stock market for shares of each company there exists a (zero net supply) bond market, where agents can trade 0-coupon bonds of arbitrary maturity. We shall assume that markets are complete[5] Since markets are complete, the search for equilibrium prices can be reduced to the search for a stochastic discount factor H_t, which will coincide with the marginal utility of consumption for the representative agent. (See Karatzas and Shreve [1998], Chapter 4)

2.4 Consumers, Gardeners, and Preferences

To keep with Lucas's analogy of "trees", we shall assume that trees can only be planted by "gardeners". The economy is populated by a continuum of identical consumers/gardeners that can be aggregated into a single representative agent. The representative agent owns all the firms in the economy, and is also the only provider of labor services. Purely for simplicity, we will assume that labor is only used in order to plant the new trees and for no other purpose. If a company decides to plant a new tree, it needs to hire the representative agent, who is simultaneously a "gardener", and needs to be compensated.

An increase in the stock of trees between times t_1 and t_2 of the magnitude $\Delta K_{N,t}$ requires work by the gardener in the amount of Δl_t. In simple terms, there is a one-to-one relation between labor services provided and planting of trees. Providing these services will impose a direct disutility to the gardener of $\int_{t_1}^{t_2} e^{-\rho(t-t_1)} g(\cdot) dl_t$. The function $g(\cdot)$ can be left unspecified for now. The consumer/gardener receives a compensation of $\int_{t_1}^{t_2} \frac{H_s}{H_{t_1}} w_s dl_s$ (in net present value terms) in order to provide the labor services needed. We shall refer to w_t as adjustment or "gardening" costs.

The representative consumer's preference over consumption streams is characterized by a utility function of the form

$$U(C_t, M_t^C)$$

[5] In particular there exist markets where agents can trade securities (in zero net supply) that promise to pay 1 unit of the numeraire when technological round N arrives. These markets will be redundant in general equilibrium, since agents will be able to create dynamic portfolios of stocks and bonds that produce the same payoff as these claims. However, it will be easiest to assume their existence throughout to guarantee ex-ante that markets are complete.

where:

$$M_t^C = \max_{s \le t} \{C_s\} \tag{8}$$

denotes the running maximum of consumption up to time t, and $U\left(C_t, M_t^C\right)$ satisfies $U_C > 0$, $U_{CC} < 0$, $U_{M^C} < 0$, $U_{CM^C} > 0$. The main motivation for our use of habit formation, is that it will allow us to obtain reasonable levels of the risk premium. In the next section we elaborate more closely on some of the technically attractive properties of using the running maximum of consumption as the habit level.

The consumer maximizes expected discounted utility over consumption and labor supply plans in a complete market:

$$\max_{C_t, dl_t} E\left[\int_t^\infty e^{-\rho(s-t)}\left(U(C_t, M_t^C)ds - g(\omega_s)dl_s\right)\right] \tag{9}$$

$$s.t.$$

$$E\left(\int_t^\infty \frac{H_s}{H_t} C_s ds\right) \le \int_0^1 P_{N,j,t}dj + E\left(\int_t^\infty \frac{H_s}{H_t} w_s dl_s\right) \tag{10}$$

Note that this is no different than a standard consumption-portfolio-leisure choice problem with the sole exception that the disutility from labor is defined over the set of increasing processes.

2.5 Equilibrium

The equilibrium definition is standard. It requires that all markets clear and that all actions are optimal.

Definition 1 *A competitive equilibrium is a set of stochastic processes* $\langle C_t, l_t, K_{n,t}, H_t, w_t \rangle$ *s.t.*

a) C_t, l_t *solve the optimization problem (9) subject to (10)*

b) Firms solve the optimization problem (4) and $K_{n,t}$ *is defined as:*

$$K_{n,t} = \int_0^1 \widetilde{\chi}_{n,j,t} dj \tag{11}$$

where $\widetilde{\chi}_{n,j,t}$ *is an indicator that takes the value* 1 *if firm* j *has updated its technology in epoch* n *by time* t *and* 0 *otherwise.*

c) w_t *is determined so that the "labor" market clears, i.e :*

$$dl_t = dK_{n,t}$$

d) The goods market clears:

$$C_t = Y_t \text{ for all } t \geq 0 \tag{12}$$

e) The markets for all assets clear

If one could determine the optimal processes $K_{n,t}$, then the optimal consumption process could be readily determined by (12) and this would in turn imply that the equilibrium stochastic discount factor is given by:

$$H_t = e^{-\rho t} U_C \tag{13}$$

This observation suggests that the most natural way to proceed in order to determine an equilibrium is to make a conjecture about the stochastic discount factor H_t and w_t, solve for the optimal stopping times in equation (4), aggregate in order to obtain the processes $K_{n,t}$ for $n = N, ...\infty$, and verify that the resulting consumption process satisfies (13), while a consumer faced with the process of labor costs w_t will find it optimal to provide labor services equal to the increases in $K_{n,t}$. This is done in section 3.

2.6 Functional Forms and Discussion

Before proceeding with the solution of the model, we need to make certain assumptions on functional forms. The assumptions that we make are intended either a) to allow for tractability or b) to ensure that the solution of the model satisfies certain desirable properties.

The first assumption on functional form concerns the utility $U\left(C_t, M_t^C\right)$. We shall assume that

$$U\left(C_t, M_t^C\right) = \left(M_t^C\right)^{\gamma} \frac{C_t^{1-\gamma}}{1-\gamma} = \frac{\left(\frac{C_t}{M_t^C}\right)^{-\gamma} C_t}{1-\gamma}, \quad \gamma > 1 \tag{14}$$

It can be easily verified that $U_C > 0$, $U_{CC} < 0$, $U_{CM^C} > 0$, $U_{M^C} < 0$. This utility is a special case of the utilities studied in Abel [1990] and exhibits both "envy" ($U_{M^C} < 0$) and catching up with the Joneses ($U_{CM^C} > 0$) in the terminology of Dupor and Liu [2003]. The main difference is that the habit index is in terms of the past consumption maximum, not some exponential average of past consumption as in Campbell and Cochrane [1999] or Chan and Kogan [2002]. Using the running maximum of consumption M_t^C as the habit index is particularly attractive for our purposes, because of the analytic tractability that it will allow. As most habit level specifications already proposed in the literature, it has the very attractive property that it is "cointegrated" with aggregate consumption in the sense that the difference between $\log(C_t)$ and $\log(M_t^C)$ will be stationary. Moreover, the ratio between C_t and M_t^C will be bounded between 0 and 1 (as is the surplus in Campbell and Cochrane [1999]).

For our purposes, this utility specification will serve three purposes: First, it will allow us to match first and second moments of the equity premium and the short term interest rate. Second, it will imply that the growth cycles that will arise in the model will leave interest rates unaffected. To see this, note that

$$U_C = \left(\frac{C_t}{M_t^C}\right)^{-\gamma}$$

In equilibrium, it will turn out that

$$\frac{C_t}{M_t^C} = \frac{\theta_t}{\max_{s<t}\theta_s} \tag{15}$$

The right hand side of (15) is unaffected by the investment decisions of firms and this will in turn be true for the mean of the stochastic discount factor and therefore the real interest rate. This is a key advantage of this specification. Without habit formation most of the effects of technological innovations will work through the real interest rate, something that appears to be at odds with the experience of the nineties and the twenties, where most of the effects of technological growth were concentrated in the stock market[6]. Third, these preferences will imply a constant risk aversion and a constant Sharpe ratio. This feature will allow us to isolate the new effects introduced by our

[6] Keeping interest rates unaffected will overcome an additional severe problem of constant relative risk aversion (CRRA) utilities: The arrival of a technological innovation will boost growth expectations. Therefore, agents with CRRA utilities will try to smooth future consumption gains by dissaving. In general equilibrium, savings must remain at 0, and so real interest rates will have to rise in order to induce savings.With a coefficient of relative risk aversion above 1, the smoothing motive will be sufficiently strong as to push the interest rate so high, that the market price to earnings ratio will *decline.* This appears to be at odds with the stock market boom that was observed in periods of rapid technological innovation as the ninetiess and the twenties. If however, the representative agent has a utility that exhibits habit formation, then consumers understand that increases in consumption will be associated with increases in their habit level and this is enough to induce extra savings, so that the effect on interest rates will be moderate. In particular, our specification will guarantee that increases in consumption growth are exactly counterbalanced by increases in the habit level, so that interest rates are completely unaffected by the growth cycles that will arise endogenously in the model.

model, namely the effects associated with the time varying importance of growth options[7][8].

Our next choice of functional form concerns the specification of the disutility of labor for planting new trees. In the real world, developing and launching a new product requires the services of technical, legal, marketing, management specialists among others. Most of the work that they put into such development is irreversible. For instance, consider a corporation that hires a legal team to investigate whether environmental laws permit the launch of a new business. The company will have to pay their fees at the beginning of the project. If economic conditions make the project unprofitable, then the company cannot take the study that these lawyers produced and sell it to a secondary market: Hence these initial (labor) investment costs are irreversible. The same is true for the labor costs of scientists who produce studies on whether a given product presents hazards to the community, the marketing specialists who design the company's marketing strategy, the technical personnel who sets up the initial website of the company, and the architects and decorators who will design the company's buildings. All these labor costs are irreversible and thus are naturally modeled by the increasing process specification adopted in section 2.4. In general , we think of "gardening" services as compensation for the "know how" that is provided by experts who need to invent, create and install the new capital stock[9] [10].

[7] It turns out that our methods can be extended to the general functional form

$$U\left(C_t, N_t\right) = f\left(\frac{C_t}{M_t^C}\right) C_t$$

Since $0 < \frac{C_t}{M_t^C} \leq 1$, one can use similar methods to Campbell and Cochrane [1999] to produce time varying risk aversion and changing Sharpe ratios. We choose to use a simple polynomial for $f()$, so as to examine how many results we can obtain by the new channels introduced in this paper. However, we would like to point out that it is straightforward to combine the present framework with a framework involving time varying risk aversion.

[8] A minor technical detail about (14) is that the instantaneous interest rate might not be defined (on the measure 0 set where $C_t = M_t^C$). In technical terms, the Ito representation of the stochastic discount factor will contain a bounded variation component. (See Karatzas and Shreve [1998] for details). However, the price of any zero-coupon bond with fixed maturity is well defined and thus throughout when we refer to the "real" interest rate we will implicitly mean the yield on a one year zero-coupon bond.

[9] Even though we model this compensation as a fixed cost that is incurred at the start of a project, little would change if the completion of a project also involved time to build. The important assumption is that these costs are sunk and irreversible, so that companies are effectively presented with a timing decision that leads to the emergence of real options in their value

[10] Introducing fixed labor costs is common in many general equilibrium models. (See e.g. Khan and Thomas [2003], Thomas [2002]). If one were to modify the model to allow for some time to build, then costs could be taken out of

Our choice for the functional form of these costs is motivated by three main considerations: First, we want the magnitude of this compensation to share the same trend as aggregate output. Second, we want to keep the amount of labor provided stationary. Third, we want to keep the compensation constant within each epoch, in order to keep the analysis simple, tractable, and provide a link to the partial equilibrium literature on growth options.

To give a specification that satisfies all three objectives simultaneously, define

$$M_t = \max_{s \leq t} \theta_s \tag{16}$$

and let

$$g = U_C e \overline{A}^N M_{\tau_N} \tag{17}$$

where U_C denotes the marginal utility of consumption, $e > 0$ is a constant, $\overline{A}^N$ is the vintage specific productivity in the current epoch and M_{τ_N} is the value of the historical maximum of θ_t at the start of the technological epoch. Under this specification, the equilibrium cost to plant a tree will be

$$w_t = e\overline{A}^N M_{\tau_N}$$

Note that these costs will grow between epochs (since N will grow by 1, and $M_{\tau_{N+1}}$ will be higher than M_{τ_N}), however they will stay constant within an epoch. Moreover, they will share the same trend growth as consumption[11].

A final assumption that is made purely for technical convenience is that

$$\zeta(i) = \zeta_0(1-i)^s \tag{18}$$

where $\zeta_0, s > 0$ are constants.

3 Solution

3.1 Equilibrium Allocations

output directly as in Gomes, Kogan, and Zhang [2003].

[11] At a fundamental level, it appears sensible to make the disutility associated with adjustment grow with the rate of technological advancement, since more complex units of the capital stock probably require more elaborate education of the experts, who install these units. If education is painful, then the disutility of providing these services should grow with the general level of advancement.

We first start by making a guess about the stochastic discount factor and the adjustment costs in general equilibrium. In particular we assume that the equilibrium stochastic discount factor is:

$$H_t = e^{-\rho t}\left(\frac{\theta_t}{M_t}\right)^{-\gamma} \tag{19}$$

with M_t defined as in (16). Furthermore, assume that:

$$w_t = e\overline{A}^N M_{\tau_N} \tag{20}$$

Under these assumptions, we obtain the following result:

Proposition 1 *Define the constants Z^*, γ_1, γ_1^* and Ξ by*

$$\begin{aligned}
Z^* &= \frac{1}{\rho - \mu(1-\gamma) - \frac{\sigma^2}{2}\gamma(\gamma-1)} \\
\gamma_1 &= \frac{\left(\frac{\sigma^2}{2}-\mu\right) + \sqrt{\left(\frac{\sigma^2}{2}-\mu\right)^2 + 2\sigma^2(\rho+\lambda)}}{\sigma^2} \\
\gamma_1^* &= \frac{\left(\frac{\sigma^2}{2}-\mu\right) + \sqrt{\left(\frac{\sigma^2}{2}-\mu\right)^2 + 2\sigma^2\rho}}{\sigma^2} \\
\Xi &= \frac{e}{Z^*}\frac{\gamma_1}{\gamma_1 - 1}\frac{\gamma_1^* - 1}{\gamma_1^* + \gamma - 1}
\end{aligned}$$

and assume that:

$$\begin{aligned}
\gamma_1^* &> 1 \\
\frac{\Xi}{\zeta(0)} &> 1
\end{aligned}$$

Assume moreover that H_t is given by (19), and w_t is given by (20). Then, firm j faced with the optimal stopping problem (4) will plant a tree the first time that θ_t crosses the threshold $\overline{\theta}$

$$\overline{\theta} = M_{\tau_N}\frac{\Xi}{\zeta(i_{N,j})} \tag{21}$$

Formally, the optimal stopping time τ^* is given by

$$\tau^* = \inf\{t : \theta_t = \overline{\theta}\}$$

The solution to the optimal stopping problem of the firm has an intuitive "threshold" form: A firm should update when the aggregate productivity θ_t crosses the threshold $\overline{\theta}$ given by (21).

This threshold will depend on a) the productivity of the tree that firm j has the option to plant in the current epoch ($\zeta(i_{N,j})$), b) the running maximum of θ_t evaluated at the time at which the current epoch arrived (M_{τ_N}) and c) the constant Ξ that depends solely on parameters and hence is constant across firms and epochs.

This optimal policy has three intuitive and desirable properties.

First, the nature of the optimal policy is such that no firm will find it optimal to plant a tree immediately when the new epoch arrives, as long as[12]:

$$\frac{\Xi}{\zeta(0)} > 1 \tag{22}$$

Intuitively, condition (22) guarantees that even the firm that has the option to plant the tree with the highest productivity ($\zeta(0)$) will find it optimal to wait for a while.

Second, the firms that have the option to plant a more "productive" tree will always go first, since the threshold $\overline{\theta}$ will be lower for them. This is intuitive: A firm which can profit more from planting a tree has a higher opportunity cost of waiting and should always plant a tree first.

Third, and most importantly, there are going to be strong correlations between the optimal investment decisions of the firms. To see this, it is most useful to consider the implications of the optimal stopping rule (21) for the aggregate mass of companies ($K_{N,t}$) that have already planted a tree in the current epoch (N) by time t. To save some notation, we shall drop the subscript N in $K_{N,t}$ and just write K_t. To obtain the dynamics of K_t, let the function $\psi(K_t)$ be defined as:

$$\psi(K_t) = \frac{\Xi}{\zeta(K_t)} \tag{23}$$

Since $\zeta(K_t)$ is a decreasing function of K_t, the function $\psi(K_t)$ has an inverse, that we shall denote as $\phi(\cdot)$:

$$\phi(\cdot) = \psi^{-1}(\cdot)$$

[12] To see why this condition is sufficient to guarantee that no firm will immediately plant a new tree once a new epoch arrives, rewrite the optimal policy as:

$$\tau^* = \inf\left\{t : \frac{\theta_t}{\theta_{\tau_N}} = \frac{\overline{\theta}}{\theta_{\tau_N}}\right\}$$

where θ_{τ_N} is the value of θ_t evaluated at the beginning of the current epoch (τ_N). By (21)

$$\frac{\overline{\theta}}{\theta_{\tau_N}} = \frac{M_{\tau_N}}{\theta_{\tau_N}} \frac{\Xi}{\zeta(i_{N,j})} > 1$$

since $\frac{M_{\tau_N}}{\theta_{\tau_N}} \geq 1$ and $\min_{i_{N,j}} \frac{\Xi}{\zeta(i_{N,j})} = \frac{\Xi}{\zeta(0)} > 1$ by (22)

Then we have the following Lemma:

Lemma 1 *If firms follow the threshold policies of Proposition 1, then the evolution of K_t within technological epoch N is given by*

$$K_t = \begin{cases} 0 \text{ if } \frac{M_t}{M_{\tau_N}} < \frac{\Xi}{\zeta(0)} \\ \phi\left(\frac{M_t}{M_{\tau_N}}\right) \text{ otherwise} \end{cases} \tag{24}$$

In other words for the determination of the equilibrium K_t in round N, one needs information on $\frac{M_t}{M_{\tau_N}}$ alone[13]. Figure 1 gives a graphical illustration of the relation between K_t and $\frac{M_t}{M_{\tau_N}}$.

An important implication of the above discussion is that investment in new trees will go through "lumpy" cycles. As already discussed, there is going to be some time until the first company plants a tree. Conditional on reaching that threshold, however, a number of other firms will also find it optimal to invest, since their investment thresholds will be "close" to the investment threshold of the first firm. Hence, the model implies two distinct regimes in terms of "planting" new trees at the aggregate: In the first no firm finds it optimal to invest, while in the second regime a number of companies proceed with investment in new trees in close distance to each other.

To complete the characterization of equilibrium we need to show that our conjecture for H_t and w_t are correct. We first show the following:

Proposition 2 *If firms follow the threshold policies of Proposition 1, then*

$$\frac{C_t}{M_t^C} = \frac{\theta_t}{M_t}$$

with M_t, M_t^C defined in (16) and (8). Therefore

$$H_t = e^{-\rho t} U_C = e^{-\rho t}\left(\frac{C_t}{M_t^C}\right)^{-\gamma} = e^{-\rho t}\left(\frac{\theta_t}{M_t}\right)^{-\gamma}$$

as conjectured in (19).

[13] This is intuitive: A company which has the option to plant a tree with productivity $\zeta(i_{N,j})$ will do so the first time that $\frac{\theta_t}{M_{\tau_N}}$ reaches the level $\psi(i_{N,j})$ for the *first time*. An alternative way of saying this is that it will plant a tree the first time that $\frac{M_t}{M_{\tau_N}}$ reaches the level $\psi(i_{N,j})$. Hence the *maximum level of* θ_t (compared to its level at the beginning of the cycle) will be a sufficient statistic for all the trees that have been planted in this epoch. Obviosuly, the first tree will be planted the first time that $\frac{M_t}{M_{\tau_N}}$ reaches the level $\frac{\Xi}{\zeta(0)}$, and this explains why $K_t = 0$ as long as $\frac{M_t}{M_{\tau_N}} < \frac{\Xi}{\zeta(0)}$.

Finally, we have the following:

Proposition 3 *Let H_t be given by (19) and w_t by (20). Then, an optimal plan for the representative consumer within each epoch is to set:*

$$dl_t = dK_t$$

with dK_t defined as in (24)

These two propositions show that a) given the optimal policies of the firms, the resulting stochastic discount factor verifies the conjecture under which these policies were derived and b) the conjectured adjustment costs will induce the appropriate labor services by the representative consumer to attain the evolution of K_t given in (24).

The rest of the verification that H_t, w_t and the resulting processes for K_t, l_t, C_t constitute an equilibrium in the sense of definition 1 is standard. The reader is referred to Basak [1999] and the monograph of Karatzas and Shreve [1998] Chapter 4 for details.

3.2 Equilibrium Prices

The price of a firm in general equilibrium is given by (5). Equation (5) decomposes the price of a firm in three components: 1) the value of assets in place, 2) The value of growth options in the current technological epoch and 3) The value of growth options in all subsequent technological epochs. The next proposition gives a closed form solution for each of these components:

Proposition 4 *Let $\widehat{\gamma}_1, \widehat{Z}$ be given by*

$$\begin{aligned} \hat{\gamma}_1 &= \frac{\left(\frac{\sigma^2}{2} - \mu\right) + \sqrt{\left(\frac{\sigma^2}{2} - \mu\right)^2 + 2\sigma^2(\rho + \lambda\left(1 - \overline{A}\right))}}{\sigma^2} \\ \widehat{Z} &= -\frac{1}{\frac{\sigma^2}{2}\gamma_1(\gamma_1 - 1) + \mu\gamma_1 - \left[\rho + \lambda(1 - \bar{A})\right]} \end{aligned}$$

and assume that:

$$\hat{\gamma}_1 > 1, \;\; \widehat{Z} < 0$$

Then, the price of firm j in technological epoch N is given by (5) where

$$\begin{aligned} P_{j,t}^{A} &= Z^{*}X_{j,t}\theta_t\left[1+\frac{\gamma}{\gamma_1^{*}-1}\left(\frac{\theta_t}{M_t}\right)^{\gamma+\gamma_1^{*}-1}\right] \quad (25)\\ P_{N,j,t}^{o} &= Z^{*}\bar{A}^{N}\theta_t\left[\frac{1}{\gamma_1-1}\left(\frac{\theta_t}{M_t}\right)^{\gamma+\gamma_1-1}\left(\frac{M_t}{M_{\tau_N}}\right)^{\gamma_1-1}\left(\frac{e}{Z^{*}}\right)\psi(i_{j,N})^{-\gamma_1}\right]\left(1-1_{\{\widetilde{\chi}_{N,j}=1\}}\right)\\ P_{N,t}^{f} &= Z^{*}\bar{A}^{N}\theta_t\left\{\frac{\lambda\bar{A}\hat{Z}}{\gamma_1-1}\left[\left(\frac{\theta_t}{M_t}\right)^{\gamma+\gamma_1-1}-\frac{\gamma_1-1}{\hat{\gamma}_1-1}\left(\frac{\theta_t}{M_t}\right)^{\gamma+\widehat{\gamma}_1-1}\right]\left(\frac{e}{Z^{*}}\right)\left(\int_0^1\psi(i)^{-\gamma_1}di\right)\right\} \end{aligned}$$

The constants $Z^{*},\gamma_1^{*},\gamma_1$ are given in Proposition 1. $X_{j,t}$ is given by (3), $1_{\{\widetilde{\chi}_{N,j}=1\}}$ is the indicator function used in equation (3) that takes the value 1 *if firm j has planted a tree in the current epoch and* 0 *otherwise and $\psi(i_{j,N})$ is given by*

$$\psi(i_{j,N})=\frac{e}{Z^{*}}\frac{\gamma_1}{\gamma_1-1}\frac{\gamma_1^{*}-1}{\gamma_1^{*}+\gamma-1}\frac{1}{\zeta(i_{j,N})}$$

There are several observations about the equilibrium pricing function derived in Proposition 4. Since the price itself is nonstationary, it will be easiest to analyze the price to earnings ratio implied by this proposition, defined as:

$$PE_{N,j,t}\equiv\frac{P_{N,j,t}}{X_{j,t}\theta_t}=\frac{P_{j,t}^{A}}{X_{j,t}\theta_t}+\frac{P_{N,j,t}^{o}}{X_{j,t}\theta_t}+\frac{P_{N,t}^{f}}{X_{j,t}\theta_t} \quad (26)$$

The P/E ratio of firm j is stationary and is comprised of three terms corresponding to the P/E ratio of assets in place, growth options in the current epoch and growth options in all future epochs.

The P/E ratio of the assets in place is given by:

$$\frac{P_{j,t}^{A}}{X_{j,t}\theta_t}=Z^{*}\left[1+\frac{\gamma}{\gamma_1^{*}-1}\left(\frac{\theta_t}{M_t}\right)^{\gamma+\gamma_1^{*}-1}\right]$$

and is common across all firms in the economy since it depends only on $\left(\frac{\theta_t}{M_t}\right)$. This is the P/E ratio that would prevail in an economy, where no firm would have the option to invest ever again. Unsurprisingly, this expression is very similar to expressions obtained in Campbell and Cochrane [1999] or Chan and Kogan [2002]: The PE ratio is an increasing function of the "habit ratio" (or surplus) $\frac{\theta_t}{M_t}$. "Good times" are characterized by a value of $\left(\frac{\theta_t}{M_t}\right)$ close to 1, while "bad times" are characterized by low values of $\left(\frac{\theta_t}{M_t}\right)$.

The P/E ratio of the "current epoch" options is given by

$$\frac{P_{N,j,t}^{o}}{X_{j,t}\theta_t}=Z^{*}\left(\frac{\bar{A}^{N}}{X_{j,t}}\right)\left[\frac{1}{\gamma_1-1}\left(\frac{\theta_t}{M_t}\right)^{\gamma+\gamma_1-1}\left(\frac{M_t}{M_{\tau_N}}\right)^{\gamma_1-1}\left(\frac{e}{Z^{*}}\right)\psi(i_{j,N})^{-\gamma_1}\right]\left(1-1_{\{\widetilde{\chi}_{N,j}=1\}}\right) \quad (27)$$

The current epoch option clearly differs across firms, since $\psi(i_{j,N})$ differs across firms. It is also straightforward to see from the definition of $\psi(i_{j,N})$ that firms with a small index $i_{j,N}$ will have higher "current epoch" growth options (provided of course that they haven't already invested, in which case this option is 0). Moreover, every current period option is affected by variations in three separate components: $\left(\frac{\bar{A}^N}{X_{j,t}}\right)$, $\left(\frac{\theta_t}{M_t}\right)$,and $\left(\frac{M_t}{M_{\tau_N}}\right)$. It should be noted that each of these components is a stationary quantity, capturing different sources of variation. The first component $\left(\frac{\bar{A}^N}{X_{j,t}}\right)$ is a firm specific component that changes across epochs. To see why it is stationary it is easiest to consider its reciprocal $\left(\frac{X_{j,t}}{\bar{A}^N}\right)$. Using (3) we get:

$$\frac{X_{j,t}}{\bar{A}^N} = \sum_{n=-\infty..N} \overline{A}^{n-N} \zeta(i_{j,n}) 1_{\{\widetilde{\chi}_{n,j}=1\}}$$

In economic terms $\frac{X_{j,t}}{\bar{A}^N}$ is a geometrically weighted average of the productivity of the trees that firm j has planted sofar. Since past draws of $i_{j,n}$ are given a geometrically declining weight, $\frac{X_{j,t}}{\bar{A}^N}$ is stationary. A firm that hasn't planted a tree in several of the past periods will have a relatively large $\frac{\bar{A}^N}{X_{j,t}}$ and thus growth options will have a relatively larger weight in its price and its P/E ratio.

Just like assets in place, current growth options are influenced by variations in $\left(\frac{\theta_t}{M_t}\right)$ and are procyclical: their value increases in good times and declines in "bad times". This is intuitive: Growth options will deliver their payoffs, when θ_t attains a sufficiently large threshold. Therefore, increases in θ_t raise the likelihood that these options will be exercised. Similarly, declines in θ_t diminish this likelihood and hence their value.

Finally, $\frac{M_t}{M_{\tau_N}}$ is an increasing process that starts at 1 at the beginning of the epoch and grows (stochastically) until it is reset back to 1, once the new epoch arrives. Once $\frac{M_t}{M_{\tau_N}}$ reaches the critical level $\frac{\Xi}{\zeta(i_{N,j})}$, firm j exercises its option.

The P/E ratio of future growth options is given by

$$\frac{P^f_{N,t}}{X_{j,t}\theta_t} = Z^* \left(\frac{\bar{A}^N}{X_{j,t}}\right) \left\{ \frac{\lambda \bar{A} \hat{Z}}{\gamma_1 - 1} \left[\left(\frac{\theta_t}{M_t}\right)^{\gamma+\gamma_1-1} - \frac{\gamma_1 - 1}{\hat{\gamma}_1 - 1} \left(\frac{\theta_t}{M_t}\right)^{\gamma+\widehat{\gamma}_1-1} \right] \left(\frac{e}{Z^*}\right) \left(\int_0^1 \psi(i)^{-\gamma_1} di\right) \right\}$$

As might be expected, future growth options are influenced by two components: The first component is $\left(\frac{\bar{A}^N}{X_{j,t}}\right)$ which provides a measure of the relative importance of growth options compared to assets in place. The second component is the term included inside curly brackets, is common across firms and epochs, and is only (procyclicaly) influenced by $\left(\frac{\theta_t}{M_t}\right)$.

Of particular importance is the behavior of the P/E ratio of a firm around the time at which a new tree is planted. It turns out that the P/E ratio will decline discretely, even though the (net of adjustment cost) price will be continuous[14]. The intuition is clear. Current options decrease to 0, while assets in place increase. This makes the relative weight of growth options (current and future) decline and brings the P/E ratio down.

Figure 2 illustrates the typical pattern of the P/E ratios of two firms j and j' over an epoch. To make the comparison easier, assume that the two firms have the same relative weight of assets in place $\frac{X_{j,t}}{\bar{A}^N} = \frac{X_{j',t}}{\bar{A}^N}$ at the beginning of the cycle. However, assume that firm j was more fortunate than firm j' and has drawn $i_{j,N} < i_{j',N}$. Figure 2 shows that the P/E of firm j will be larger as a result at the beginning of the epoch. Over time $\left(\frac{M_t}{M_{\tau_N}}\right)$ increases and hence the current growth option of both companies increases (in light of equation [27]) . At some point $\frac{M_t}{M_{\tau_N}}$ crosses the critical threshold $\frac{\Xi}{\zeta(i_{N,j})}$ and firm j decides to plant a tree. This makes its P/E ratio jump downward. Firm j' will continue to have a high P/E ratio which will continue to grow up to the point where $\left(\frac{M_t}{M_{\tau_N}}\right)$ crosses the level $\frac{\Xi}{\zeta(i_{N,j'})}$. Thereafter the P/E ratio of this firm jumps downward too. Hence, the model produces a picture similar to "leapfrogging": The companies with the highest current growth options will exhibit the biggest runups and reversals in terms of their P/E ratios, followed by the firms with the next highest current growth options etc.

The next section analyzes the implications of the model further and relates it to qualitative features of the data. Thereafter, we address its quantitative implications.

4 Qualitative Implications of the Model

4.1 Medium Term Cycles in Consumption, Growth, Investment in New Trees and P/E Ratios

We start by showing how the model can account for medium run "cycles" in consumption growth rates, P/E ratios and investment[15].

[14] To see this, examine equation (26). The first term will remain identical around the time of the investment. The second term will become 0 and the third term will also decrease since $X_{j,t}$ will increase (and hence $\frac{\bar{A}^N}{X_{j,t}}$ will decrease).

[15] There is a small and growing literature that discusses the presence of such features of the data. Blanchard [1997] and Comin and Gertler [2003] discuss the presence of a medium run in aggregate macroeconomic series, while Bansal and Yaron [2004], Parker and Julliard [2005] discuss the asset pricing implications of such lower frequency

The easiest way to see why consumption growth will exhibit cycles is to use equation (7) together with $C_t = Y_t$ in order to obtain

$$\log(C_t) = \log(\theta_t) + \log\left[\sum_{n=-\infty..N-1} \overline{A}^n F\left(K_{n,\overline{\tau_n}}\right) + \overline{A}^N F(K_{N,t})\right]$$

We can then use Ito's Lemma to obtain

$$d\log(C_t) = \left(\mu - \frac{\sigma^2}{2}\right) dt + \sigma dB_t + \frac{F'(K_{N,t})}{\widetilde{F}} dK_{N,t} \tag{28}$$

where

$$\widetilde{F} = \sum_{n=-\infty..N-1} \overline{A}^{n-N} F\left(K_{n,\overline{\tau_n}}\right) + F(K_{N,t})$$

It is now easy to decompose the growth rate of consumption in three components: a constant drift $(\mu - \frac{\sigma^2}{2})$, a Brownian (unpredictable) increment σdB_t and then a term that captures the effect of investment in new trees $\frac{F'(K_{N,t})}{\widetilde{F}} dK_{N,t}$. It is clear that the first two components are driven by the (random walk) properties of $\log(\theta_t)$. The third term is the result of investment in new trees and it will be responsible for the persistence of growth rates.

Actually, consumption growth in our model has regime switching features: there are going to be stretches of low growth followed by stretches of high growth. Once a new epoch arrives, no firm will plant a tree for a while and thus the third term on the right hand side of (28) will be 0. However, after a certain point a number of firms will start planting trees in close proximity to each other. This will lead to increases in the mass of firms that have planted a tree in the current epoch $(dK_{N,t})$ and thus high growth.

Figure 3 illustrates this feature of the model. We plot the qualitative behavior of:

$$g_t = E\left[\log(C_{t+1}) - \log(C_t)\right] = E\left(\int_t^{t+1} d\log(C_t)\right) = \left(\mu - \frac{\sigma^2}{2}\right) + E\left(\int_t^{t+1} \frac{F'(K_{N,t})}{\widetilde{F}} dK_{N,t}\right)$$

for two epochs of different length. To pick a "typical" path we set $dB_t = 0$ throughout. In the left panel, we assume that an epoch is interrupted by the arrival of a new epoch when $K_t = 0.2$, while in the right panel the new epoch arrives when $K_t = 0.8$. These two panels show the rich growth dynamics that can result from this model. Even though the exact shapes will differ depending on assumptions about the shape of $\zeta(\cdot)$, the costs of updating etc., the general shape will exhibit

components. In the next section we also present some new empirical evidence in the same direction.

similar patters. Expected growth will peak as the first firms start to update their technologies. Interestingly, as the technological cycle progresses, expected growth diminishes, since the most productive trees have already been planted.

It is also interesting to note that expected consumption growth, the P/E ratio and investment in new trees will share a common cycle. Figure 4 illustrates this aspect of the model. It repeats the same exercise as Figure 3, but also plots the P/E ratio for the aggregate market, namely:

$$PE_t^M = \frac{\int_0^1 P_{N,j,t} dj}{Y_t}$$

As can be seen, the P/E ratio follows a similar pattern as expected consumption growth. It starts at a relatively low level at the beginning of an epoch as the current growth options (of all firms) are "alive" but discounted. However, as time passes, θ_t grows and starts approaching the thresholds at which firms will start "exercising" their growth options. At that point the P/E ratio peaks, since all options are still "alive" and about to be exercised. As θ_t grows further, several firms start investing and this transforms growth options into assets in place and leads to a decline in the P/E ratio. Over time K_t approaches 1 and the P/E ratio reaches the lowest level over the entire epoch, only to jump upward at the end of the period. The next proposition contains a precise mathematical formulation of these effects:

Proposition 5 *Let $K_{N,t}$ be given by (24) and fix an arbitrary $\left(\frac{\theta_t}{M_t}\right)$. Then:*

1. (Beginning of epoch) *If $K_{N,t} = 0$, PE_t^M is strictly increasing in $\left(\frac{M_t}{M_{\tau_N}}\right)$*

2. ("Investment Booms" and P/E ratio peaks) *There exists $K^* \geq 0$ such that, PE_t^M attains a maximum, i.e.*

$$\frac{\partial PE_t^M}{\partial \left(\frac{M_t}{M_{\tau_N}}\right)} = 0$$

3. (Mature Technologies and P/E ratio troughs) *For $K_t > K^*$, PE_t^M is a declining function of $\left(\frac{M_t}{M_{\tau_N}}\right)$.*

Note that $\left(\frac{M_t}{M_{\tau_N}}\right)$, the ratio of the current maximum of θ_t to its value at the beginning of the epoch is a strictly increasing process that starts at 1 at the beginning of the epoch. In a sense, $\left(\frac{M_t}{M_{\tau_N}}\right)$ is a measure of how much time has elapsed since the beginning of the epoch. Values close to

1 indicate that little time has passed, while values significantly larger than 1 indicate the opposite. Hence, one can literally re-interpret the above proposition in terms of the time that has elapsed since the beginning of the epoch.

4.2 The Time Series of Expected Returns over the Medium Term Cycle

In this section our aim is to show the implications of medium run cycles for expected returns. It will be most useful to establish up-front a Lemma that will prove particularly useful for the analysis that will follow.

Lemma 2 *The volatilities of the three components of a company's price are related by:*

$$\sigma_{j,t}^A \equiv \left(\frac{\sigma\theta_t}{P_{j,t}^A}\right)\frac{\partial P_{j,t}^A}{\partial\theta_t} < \sigma_t^f \equiv \left(\frac{\sigma\theta_t}{P_{N,j,t}^f}\right)\frac{\partial P_{N,j,t}^f}{\partial\theta_t} < \sigma_{j,t}^o \equiv \left(\frac{\sigma\theta_t}{P_{N,j,t}^o}\right)\frac{\partial P_{N,j,t}^o}{\partial\theta_t} \tag{29}$$

This Lemma ranks the three components of a company's valuation in terms of their volatilities. Assets in place are the least volatile followed by future growth options and then current growth options. As such, this Lemma provides a simple and intuitive way to determine the volatility of any firm. To this end, let the relative weights of the three components be defined as

$$w_{j,t}^A = \frac{P_{N,j,t}^A}{P_{N,j,t}}, w_{j,t}^o = \frac{P_{N,j,t}^o}{P_{N,j,t}}, w_{j,t}^f = \frac{P_{N,t}^f}{P_{N,j,t}} \tag{30}$$

It is now straightforward to determine the volatility of any given firm j as:

$$\sigma_{j,t} \equiv \frac{\sigma\theta_t}{P_{N,j,t}}\frac{\partial P_{N,j,t}}{\partial\theta_t} = w_{j,t}^A\sigma_{j,t}^A + w_{j,t}^o\sigma_{j,t}^o + w_{j,t}^f\sigma_t^f$$

Given that the (instantaneous) consumption CAPM holds, we also obtain the instantaneous expected excess return on any given stock as:

$$\mu_{j,t} - r = \gamma\sigma\sigma_{j,t} \tag{31}$$

These simple formulas suggest a particularly important set of implications for the "predictability" of returns over the course of a typical epoch. In the first stages of an epoch, most current period options have not been exercised. In accordance with (29) these growth options are the most volatile valuation component and hence command high expected returns. As time passes and θ_t grows, several firms approach the threshold at which they start to invest. Growth options increase in importance, and hence the (instantaneous) expected excess return of these firms becomes very high. Once however these firms start to invest in new trees, growth options get transformed into assets in place which (according to equation [29]) are the least risky valuation components. Therefore, the expected return of these companies drops discretely, and their expected returns going forward will be particularly low.

Even though we conducted this analysis at the level of an individual firm, it is clear that it carries over to the aggregate level. Aggregation will preserve this pattern of high and then low expected returns, because the investment decisions of firms will occur in close proximity as already discussed. The only difference is that there will be no abrupt reversals in expected returns, but instead aggregation will guarantee that expected (excess) returns decline gradually as more firms decide to invest.

These observations conform well with empirical evidence that investment plans have the ability to predict returns going forward (see Lamont [2000]). According to the model, periods with low investment activity will be associated with high expected (excess) returns going forward and vice versa. Moreover, a high P/E ratio at the aggregate is indication of both a high upcoming investment activity and low expected returns - a fact that is consistent with the data. Also consistent with empirical evidence is the prediction that firms with the most attractive growth options will be the most likely to invest, will experience the most dramatic runups in their price prior to investment and the most dramatic expected return reversals after investment (see Titman, Wei, and Xie [2004]). [16]

4.3 Cross Sectional Implications of the Medium Term Cycle

4.3.1 The Size and the Value Premium

[16] We would like to note that there is an additional source of variation in expected returns, resulting from changes in the habit level $\left(\frac{\theta_t}{M_t}\right)$. It is easy to show that increases in $\left(\frac{\theta_t}{M_t}\right)$ will make the volatility (and thus the expected return) of assets in place larger (as they do in the homogenous agents version of Chan and Kogan [2002]), whereas the volatility of future growth options will become smaller. We chose to focus on variations that result from changes in $K_{N,t}$ exclusively, as we feel that this predictability of returns over the "medium run" technological cycle is a result that is distinct from the pre-existing literature.

There is by now a well developed literature in finance that discusses patterns of expected returns in the cross section. By far the most well studied stylized facts are the size and the value premium. The size premium (Banz [1981], Fama and French [1992]) is an empirical regularity documenting that small cap stocks have higher average returns than large cap stocks. Similarly, the value premium is an empirical regularity documenting that stocks with high book to market value of equity tend to have higher returns than low book to market stocks.

The cross section of expected returns in this paper is compatible with both of these predictions. To see why there is a size premium, it will be easiest to start by first ignoring current period growth options, i.e. taking two firms j and j' that have already exercised their growth options in the current epoch. Since future growth options are identical across all firms, any differences in valuation between the two firms j and j' will be driven by differences in the value of their assets in place: If firm j has more assets in place this will also imply that the relative weight of assets in place in the company's total value is larger than for firm j'. In turn, this implies that the expected (excess) return of company j will be lower, since the expected excess return of assets in place is lower than for future growth options as we have already established.

If one introduces current period growth options , then the above reasoning ceases to be exact. A high market valuation could be the indication of either a lot of assets in place or the presence of very attractive current period growth options. To be concrete, assume that companies j and j' are in the highest size decile. However, company j could be in the high size decile because it has attractive current period growth options, while company j' could belong to the highest decile because it has a lot of assets in place. Hence, company j should be expected to have *a high* expected return, while company j' *a low* expected return.

Therefore, even though sorting by size will typically uncover firms with a lot of assets in place and low expected returns, the success of this sorting will depend on the relative importance of current growth options over the medium term cycle: The strength of cross sectional predictability will depend critically on whether a lot of firms have exercised their growth options or not. At the beginning of a cycle a lot of firms will not have exercised their growth options and hence sorting by size will produce groupings of companies with very dissimilar (expected) returns. Towards the end of a technological cycle however, most growth options will have been exercised and thus sorting by size will produce groups of companies with very similar characteristics.

This conforms very well with empirical evidence suggesting that cross sectional predictability relations almost disappeared from 1986-2001, only to resurface thereafter. If we accept that the 1990's was a period of rapid technological growth, then the present model may help explain this prolonged period of weak or no predictability at the cross section. In the next section we examine quantitatively, if the model can help account for both a size premium over longer stretches of time and a prolonged breakdown slightly before and during periods of rapid technological growth.

To understand why the model also accounts for a value premium, we start by defining the book value of a company as:

$$B_{N,j,t} = \sum_{n=-\infty..N} w_n 1_{\{\widetilde{\chi}_{n,j}=1\}}$$

where w_n are the costs of planting a tree in epoch n and $1_{\{\widetilde{\chi}_{n,j}=1\}}$ is the usual indicator that takes the value 1 if a company has planted a tree in that epoch and 0 otherwise.

The following Lemma will prove very useful:

Lemma 3 *Let $\overline{\theta}_{j,n}$ be the value of θ_t at which firm j planted a tree in period n (assuming that it did). Assume moreover that $\frac{\theta_t}{M_t} = 1$. Then:*

$$\frac{P^A_{j,t} - B_{N,j,t}}{\overline{A}^N \theta_t} = Z^* \left[1 + \frac{\gamma}{\gamma_1^* - 1}\right] \left(\sum_{n=-\infty..N} \overline{A}^{n-N} 1_{\{\widetilde{\chi}_{n,j}=1\}} \zeta(i_{j,n}) \left(1 - \frac{\gamma_1 - 1}{\gamma_1} \frac{\overline{\theta}_{j,n}}{\theta_t}\right)\right)$$

This Lemma expresses the difference between the market price of a company's assets in place and their book value. (The normalization by $\overline{A}^N \theta_t$ clearly does not affect the relative ordering, since it is common across firms). The Lemma shows that this quantity is a weighted average of the average productivity of the trees in a company. Ignoring current growth options, a firm with a high market to book value of equity (M/B) will have more productive trees in place than a company with a lower M/B. This will imply that companies with a higher M/B will tend to have planted more productive trees in the past , which in term implies that a) these firms should have a higher share of their valuation in the form of assets in place (since they own more valuable trees) and thus a lower expected return and b) these firms should have better measures of operating performance (Earnings to book). Both of these predictions are supported by the data. The former is supported by Fama and French [1992] while the latter is supported by Fama and French [1995].

We conclude by noting however, that both the size premium and the value premium induce similar orderings in our model. Alternatively put, they are not independent effects: In the simulations of the model that follow in the next section we were able to obtain both effects separately, but not in combination. Therefore, we focus exclusively on the size effect in what follows and note that similar results hold for the value premium.

4.3.2 Characteristics, Momentum, and Contrarian Profits

The previous section showed that sorting by size would produce an inverse ordering of expected returns, if one were to ignore current period growth options. If current period growth options are present, then any additional indication about a firm's growth options will help uncover a separate source of variation in expected returns in the cross section.

As an example, assume that there is a signal (like a firm's industry) that reveals information about a firm's growth options. Then, sorting by both size and industry will allow for two independent sources of variation in the cross section. Hence, finding that characteristics (such as industry) matter is by no means per se evidence of mispricing.

Interestingly, past expected returns will reveal information about the growth options of a firm. Consider for instance two firms j and j' having the same market size. As already discussed in the previous section, it could be that firm j has less assets in place than firm j' but higher current growth options (or vice versa). Size alone will not allow a clear prediction about which firm should have higher expected returns. If one had information however on the average returns of the two firms over the last few quarters, then one could infer which of the two firms has the highest growth options by just examining which of the two firms had higher average returns in the past. This suggests that momentum will be an independent effect in addition to size.

Similarly, one can predict that the firm which had high expected returns in the past is likely to invest in the near future, and thus experience a reversal in its expected return over the long run. Hence the model can explain both a momentum effect in the short run and a contrarian effect over the long run as found in Bondt and Thaler [1985]. Moreover, it can explain why sorting firms by capital expenditures will produce contrarian type effects as found in Titman, Wei, and Xie [2004].

5 Quantitative Analysis and Empirical Evidence

We calibrate the model so as to match three categories of data: a) the (unconditional) mean and standard deviation of the P/E ratio, the Book to Market ratio, the equity premium and the riskless rate, b) the mean, standard deviation of first differences in log consumption , and c) the stationary distribution of firm (log) size.

Then we present some evidence on the existence of medium run components in consumption growth and the P/E ratio and examine the ability of the model to a) match these cycles, b) produce *jointly* strong time series predictability in aggregate (excess) returns and moderate predictability for consumption growth (as in the data) and c) produce cross sectional predictability similar to the data.

We conclude by examining two applications that seem to provide particularly attractive manifestations of a medium term cycle. First, we show how a medium run cycle an help account for the disappearance of the usual predictability relationships between 1986 and 2001. And second, we show how the model can account for the patterns found in Parker and Julliard [2005] who demonstrate that computing correlations between returns and long run consumption growth can help improve the fit of the consumption CAPM.

5.1 Parameter Choice and Unconditional Moments

We start by matching unconditional means and standard deviations of the P/E ratio, the Book to Market ratio, the equity premium and the riskless rate.

Our choice of parameters is given in Table 1. The parameters μ, λ and $\overline{A}$ determine the mean growth rate of (log) consumption. The parameter μ is the "baseline" growth rate, while the parameter λ is the key parameter that controls the "frequency" of the medium run cycle. $\overline{A}$ controls the "amplitude" of the medium term cycle. These three parameters are chosen, so as to match as closely as possible the average rate of log consumption growth and some "medium run" properties of consumption (discussed in the next subsection). σ is the only parameter that controls the quadratic variation of (log) consumption in the continuous time limit, and is chosen to match the standard deviation of log consumption[17]. ρ and γ have a first order influence on the level of the real interest rates and the magnitude of the equity premium and are chosen so as to

[17] In the continuous time limit, the quadratic variation of log consumption is driven solely by σ. However, in discrete data there is going to be some extra volatility from the stochasticity of the "singular" increases in consumption resulting from the investment in new trees. For simplicity, we approximated the yearly volatility of consumption solely by the quadratic variation.

reproduce as closely as possible both the level and the standard deviation of these two quantities. The parameters ζ_0, s in equation (18) and the cost of adjustment e will influence the speed at which growth advances, once firms start to invest and -most importantly- the stationary distribution of firm size. We choose them so as to produce a distribution of log (size) as close as possible to the data.

Table 2 displays the unconditional moments implied by the model. To obtain these unconditional moments, we simulate 20 thousand years of data and drop the first 8 thousand to ensure that the data are from the stationary distribution. As can be seen from Table 2 the model performs well at reproducing the level of the equity premium, the level of the (aggregate) Book to Market ratio and their volatilities, as well as the mean growth rate of log consumption. The model is slightly less accurate at matching the mean of the P/E ratio, the level and the volatility of the real interest rate, which are all somewhat higher than in the data.

5.2 Medium Run Cycles: Empirical Evidence and Quantitative Implications of the Model

The key prediction of the model is the presence of a medium-long run cycle that is shared by consumption growth, P/E ratios and investment activity. We start by first presenting empirical evidence on the presence of such cycles and then compare the empirical evidence to the predictions of the model.

A particularly attractive way to present evidence on serial dependence is the so-called periodogram[18]: The periodogram is just a transformation of all the autocovariances of a stationary process. Most importantly, it allows an immediate way to examine how much of the variation in the data is explained by lower frequencies (medium-long term cycles) and how much is associated with high frequencies (short term cycles). A periodogram which is downward sloping is evidence of a persistent process. By contrast, a flat periodogram is evidence of white noise. The top panel in Figure 5 presents the (smoothed log) periodogram of yearly first differences in the logarithm of consumption between the end of world war I and today[19].

As can be easily seen, the log periodogram peaks at a frequency between 0.1-0.12 suggesting

[18] See Hamilton [1994] or Brockwell and Davis [1991] for an introduction to periodograms.

[19] The data for consumption are from the website of Robert Shiller. In order to avoid measurement error with pre-world war I consumption data, we kept only post-world war I data yielding 87 observations.

the presence of a (stochastic) cycle of about $1/0.12 \approx 8$ to $1/0.1 = 10$ years in consumption growth. To establish the statistical significance of this finding we used the bootstrap: Under the null hypothesis that consumption growth is white noise, one can bootstrap the first differences in log consumption to obtain confidence intervals. The two dashed lines present these confidence bounds which demonstrate that the peaks at the low frequencies are not the artefact of statistical error, but are statistically significant.

To compare, we also plotted the output of our model. This is done in the bottom part of the figure. We simulated paths of a length of 87 years and then computed their log periodogram along with 5-95% confidence bands. As can be seen, the model performs well at the low frequencies, since the periodogram of consumption growth exhibits a slight downward slope. The log periodogram of the actual consumption process is well within the confidence bands implied by the model, except for a few of the highest frequencies. This also indicates that the model does not produce too strong a serial dependence, that would be at odds with the data.

To summarize, the two subfigures in figure 5 jointly suggest that a) there is a small degree of persistence in consumption growth that makes its log periodogram have a slight downward slope, b) the periodogram of the data peaks at a frequency between 0.1-0.12 suggesting the presence of (stochastic) cycles that last between 8-l0 years and c) the calibrated model is successful at capturing these deviations from the pure random walk hypothesis, by producing a log periodogram very similar to the one found in the data.

To produce a formal statistical test, we also ran Bartlett's test for white noise. This test produces progressive sums of the (unsmoothed) periodogram[20]. Under the white noise hypothesis, the progressive sums of the periodogram should align on the 45 degree line. Figure 6 confirms formally that consumption growth is not pure white noise, since the cumulative periodogram runs well above the 45 degree line for frequencies between $0.08 - 0.2$. This further suggests that consumption growth has some serial dependence with (stochastic) cycles that last between $1/0.2 = 5$ and $1/0.08 \approx 12$ years. These cycles are somewhat longer than typical business cycles that last between 1.5 and 8 years. The p-value of the test is 0.0376, hence below the 5% critical level.

We turn next to the comovement between consumption growth and the price to earnings ratio. According to the model, there should be strong comovement between the two quantities over the

[20] The advantage of this test is that the periodogram need not be smoothed, and the confidence bands are uniform.

medium run cycle (see Figure 4 for an illustration). Figure 7 presents some “first pass” evidence to that effect. Using the band pass filter proposed by Baxter and King [1999] we first isolated business cycle frequencies (2-8 years) for both quantities, plotted them against each other and also computed their correlation. ln the bottom panel we repeated the same exercise over the medium run cycle (we kept frequencies between 8-30 years). The correlation between the two series is 0.32 over business cycle frequencies, and it jumps to 0.46 for medium run cycles.

An alternative way to determine precisely the cycles at which the comovement occurs is given in Figure 8. This figure presents the correlation between the (log) P/E ratio and consumption growth at different frequencies. This notion of correlation is sometimes referred to as “coherency” in the literature[21]. Schematically speaking, if two series are influenced by both short term and long term cycles, the coherency allows us to determine to what extent these series are *jointly* determined by long run cycles (low frequencies) or short cycles (high frequencies). As can be seen from the figure, there are two peaks of the coherency: One at a frequency of about $0.12 - 0.15$ and one at a frequency of about 0.35. The reciprocals of these numbers suggest that the log P/E ratio and consumption growth are most highly correlated over medium cycles that last about 8 years and also over shorter cycles that last about $3 - 4$ years.

To assess the statistical significance of these spikes in coherency we performed a Monte Carlo experiment. We first estimated an AR1 process for the log P/E ratio and then isolated the residuals of this process and consumption growth. Subsequently, we simulated 1000 paths of a length of 87 years, as follows: We drew an initial value for the log P/E ratio from its empirical distribution. Then we drew pairs of consumption growth and the innovation in the AR1 process for 87 years, creating artificial paths for consumption growth and artificial AR1 processes for the log P/E ratio. Repeating this exercise 1000 times we computed 5% and 95% confidence bands for the coherency between the two series. In short, these confidence bands were created under the assumption that the P/E ratio is at most contemporaneously correlated with consumption growth and has no predictive ability for the latter. As can be seen, the coherency peaks around 0.12 and 0.35 are well outside the confidence bands.

The finding that coherency is not constant (but instead has distinct peaks) has some independent interest, since it is a fact that appears hard to match with models that drive all the variation in

[21] See Hamilton [1994] or Brockwell and Davis [1991]

the P/E ratio from variations in risk aversion, while assuming unpredictable consumption growth. One can show formally[22] that in such models, the coherency should be constant.

A very practical implication of this argument is that consumption growth should be predictable by the log P/E ratio and especially so over the longer run. Figure 9 presents evidence to this effect. It presents results of regressions of T year ahead forecasts of consumption growth on the log P/E ratio.

The top subplot presents the coefficients of this regression. The confidence intervals were computed with a similar bootstrap procedure as for figure 8 (i.e. assuming no predictability). The figure demonstrates that the coefficients are flat for the first 2-3 years, increase sharply after that, remain relatively flat thereafter and then increase again between years 8-12. Once again, we find the same pattern as in Figure 8: consumption growth becomes progressively more predictable over horizons between 8 and 12 years.

To compare, the bottom panel plots the correlation coefficients between (log) P/E and consumption growth in the subsequent T periods for the data and the model. As can be seen, both the model and the data imply an inverse U pattern for these correlation coefficients. The pattern is the same for both the data and the model, with the only exception that the correlation coefficients for the model peak around 8 years and are stronger at shorter frequencies. However the order of magnitudes are fairly comparable.

The intuition for this inverse U-shape picture in our model is the following: Suppose that we are at the beginning of a cycle. Over a short interval of time it is unlikely that any firm will choose to invest. In the short run consumption will behave as a random walk, since it is going to be affected by movements in θ_t alone. Over a longer prediction horizon however, it becomes more likely that θ_t will increase enough to induce some firms to invest: The variance introduced with an increase in the prediction horizon will make it more likely that certain paths of θ_t will cross the investment thresholds with high probability. Hence, over short horizons consumption growth should be relatively unpredictable, becoming more predictable as the horizon lengthens.

Overall, we find evidence in the data that consumption growth is not a pure i.i.d process. More importantly, a good fraction of the variation is accounted for by cycles with an average duration between 8-12 years that appear to be shared by the P/E ratio. This is the notion of medium term

[22] Details of this calculation are available upon request.

cycles that were described in the theoretical section of the paper.

5.3 Variations in Expected Returns in the Time Series

We turn next to the implications of the model for return predictability over the medium run. We start by demonstrating how the growth options that arise over the medium run can generate substantial predictability in excess returns (especially for companies with substantial growth options) and then move on to investigate whether these strong patterns survive at the aggregate.

Consider first the expected return for a specific segment of the market, namely firms with small assets in place but large current growth options. Our interest in such firms is both theoretical and empirical. From a theoretical standpoint, we are interested in measuring the expected return (and its reversal) for a subset of companies for which the effects described in the model are strongest. From an empirical point of view, we believe that certain market indices (like the Nasdaq in the 1990's) consisted of stocks with these characteristics. Thus, it is interesting to investigate if the model can reproduce (in expectation) the astounding returns of such indices.

In particular, we perform the following thought experiment: We take a firm with size equal to the mean of the lowest size decile and assume that a new technological round arrives. We assume further that this firm "draws" the option to plant the highest productivity tree in the current epoch. We then compute its P/E ratio and its expected (instantaneous) return as a function of the expected remaining time to plant its tree. We also give its P/E ratio and its expected return after it has planted the tree[23]. The results are given in Table 3. The table shows the relative weights of the three valuation components and their expected returns.

The *expected* return of the entire firm remains at a level close to 17% for practically all 8 years, only to drop to about 10% after the firm invests. This very high expected return in the initial stage is driven by the almost 20% expected returns of current growth options, which make up the bulk of the valuation for this specific firm. We would also like to note that these numbers are unconditional expected returns, not the expected returns *conditional* on the firm investing at a given time T. The latter numbers are clearly going to be significantly larger as in Carlson, Fisher, and Giammarino [2004a]. Interestingly, the numbers that we find are similar in magnitude to the numbers given in

[23] To be consistent with the thought experiment, we condition on the event that the epoch does not change before the firm plants its tree.

Cochrane [2005] on the expected returns to venture capital investments. (Cochrane [2005] reports numbers around 25%)

It is noteworthy that expected returns rise slightly as the expected time to invest decreases, since the current period growth options become a progressively more important component of the price and thus both the P/E ratio and expected returns increase. Once the firm invests however, what used to be growth options becomes assets in place and thus we obtain the reversal in expected returns that is documented in the table.

The firm we considered sofar provides an isolated example of the model's ability to generate variations in expected returns over a medium term cycle. However, aggregation over all firms will preserve the same pattern. As already discussed in section 4.2, an increasing P/E ratio (at the aggregate) will be useful in predicting the relationship between assets in place and growth options over the medium run cycle and hence help predict returns, especially over longer horizons.

Table 4 provides evidence to this effect. It shows the results of standard predictive regression of cumulative (excess) returns for horizons of 1-7 years and compares the results to the data. Just as in the data, we obtain a negative coefficient which is increasing in absolute value over time. The intuition for this increase in predictability over longer horizons is identical to the one given in section 5.2: The increase in the variance of the distribution of $\log(\theta_{t+T}) - \log(\theta_t)$ makes it more likely that certain paths of θ_t will cross the investment thresholds of the various firms.

Quantitatively, the simulated point estimates are smaller than in the data. However, the coefficients found in the data typically lie within a two standard deviation band of the simulated point estimate. It is noteworthy, that the model can produce a non-negligible degree of predictability without having to resort to any variations in risk aversion (relative risk aversion is always fixed at 7 for all the exercises performed). The only channel at work is the variation in risk over the medium term cycle, associated with the emergence and eventual depletion of growth options.

5.4 Cross Sectional Variation and the Medium Run

In this section we explore the ability of the model to provide some new insights into cross sectional patterns of expected returns. Next to exploring the ability of the model to describe well documented patterns (like the size premium) we also focus on two specific applications, that the model is particularly well suited for: a) the prolonged breakdown of the usual predictability relationships during the so-called "long boom" of the US economy (1986-2001) and b) the ability of the standard consumption CAPM to perform better in the data, when one uses longer horizons for consumption growth.

5.4.1 The Size Premium and Extended Breakdowns

By now there exist numerous models that can account for both a size and a value premium. The present model can also account for such phenomena. An aspect of the model that is unique, however, is its potential to provide insights into why these patterns seem to have disappeared in the US economy from 1986-2001- a period that has been commonly referred to as the "long boom"-. We focus only on the size premium in this section since both the value and the size premium emanate from the same source in our paper. Alternatively put, if we sorted firms by book to market the results would be qualitatively similar.

Table 5 compares the output of long simulations of the model to the data. In their seminal paper Fama and French [1992] create portfolios by sorting stocks on size and then computing the average returns of these portfolios. We repeated the same exercise with our model by simulating a long path (20000 years) for 2000 companies to ensure stationarity and then sorting stocks into 12 size deciles for the last 500 years. To avoid simulation error we repeated this exercise 100 times and averaged over all simulations.

From the data of Fama and French [1992], it is easily seen that the difference in log size between the largest and smallest portfolio is about 3 times larger than that in our model and about 1.5 times larger if one ignores the extreme portfolios 1A and 10B. Similarly, the difference in returns between the largest and the smallest portfolio in the data is about 6 times larger than for our model and about 2.5 times larger if one ignores the extreme portfolios. The model can thus account for a good fraction of the size premium and performs better if one ignores the extreme portfolios in the data.

We next turn our attention to the predictability breakdowns implied by the model. To investigate these, we repeated the same exercise as above, however we only kept observations commencing with the beginning of an epoch and ending once 5% of the most productive firms have invested. This selection captures the initial phases of a cycle and the first stages of an investment driven boom. For most cycles, these observations span about 9-10 years on average, during which P/E is exceptionally high, much like the situation in the nineties. Table 5 shows that the size premium implied by the model is perfectly reversed for this subset of simulated observations: Market capitalization in an era of rapid technological growth will be associated with growth options, and not just assets in place. Of course, as options get exercised the size premium resurfaces, since size becomes uniquely associated with substantial assets in place.

Therefore, even if there is a size premium over the long run, it is to be anticipated that it will break down before and during the first stages of investment booms and stock market runups. Our simulations suggest that these periods could easily span 10 years and potentially more.

5.4.2 The Long Run Performance of the Consumption CAPM

As a last application, we analyze the ability of the model to reproduce the findings in Parker and Julliard [2005]. Parker and Julliard [2005] show that the consumption CAPM performs better if one uses longer horizons of consumption growth rates. Figure 10 replicates figure 1 in Parker and Julliard [2005] for our simulated dataset. In the top part of the figure, we first compute the covariances between the contemporaneous consumption growth rate and the excess returns of 25 size sorted portfolios for 500 years of simulated data. Subsequently we run a cross sectional regression of the average excess returns on the covariances of these portfolios with contemporaneous consumption growth. As Parker and Julliard [2005] we use quarterly returns. The top subfigure plots the resulting predicted returns against the actual returns. As can be seen, the fit is far from perfect[24], despite the fact that the *conditional* consumption CAPM holds in our framework. To obtain the bottom subfigure, we repeat the same procedure as for the top subfigure except that we use the long-horizon (5-year) consumption growth rate instead of the contemporaneous consumption growth rate. The fit of the CCAPM becomes significantly better, just as Parker and Julliard [2005] document for actual data.

The reason is that correlation with long run consumption growth "reveals" more information about the true conditional beta of each portfolio: Conditional betas in our framework vary because they capture differences in the relative importance of growth options. In turn, the importance of growth options is better revealed by examining the covariation of returns with the entire consumption growth path in the epoch, not just the one-period consumption evolution.

[24] This has a similar "flavor" with the findings in Gomes, Kogan, and Zhang [2003].

6 Conclusion

This paper is an attempt to understand the interactions between technological growth and asset pricing. We present a model that derives jointly long lasting technological cycles and their implications for asset prices. The key insight of the model is that technological progress will lead to a "life-cycle" for growth options. At the beginning of a cycle, growth options will emerge in the prices of most companies. This will increase their P/E ratios and their expected returns. The investment boom that will accompany the exploitation of growth opportunities will lead to a downturn in both P/E ratios and expected returns.

Technological cycles can thus introduce predictability of expected returns both in the cross section and at the aggregate. We demonstrate how most of the well established asset pricing phenomena (PE predictability, the size and the value premium, momentum in the short run and contrarian profits in the long run) can be addressed in a unified way in our model. Moreover, we present some new empirical evidence that shows that the link between the macroeconomy and financial valuation ratios appears to be quite strong over the medium run cycles that the model analyzes. Moreover, our model presents both theory and empirical evidence that the long held modeling assumption of consumption growth being i.i.d maybe a better description of its "short run" behavior, but maybe a bad assumption for its behavior over longer horizons. Indeed, it appears that the P/E ratio performs better as a predictor of growth over longer rather than shorter horizons.

In conclusion, our model provides a theoretical microfoundation for the presence of "long run" risks and their potential to provide a link between the macroeconomy and asset pricing over cycles that are longer than the average business cycle.

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μ	0.010	γ	7	$\zeta(0)$	1
σ	0.033	ρ	0.05	s	2
λ	0.050	$\bar{A}$	1.55	e	32

TABLE 1: Parameters used for the calibration

	Data	Model
Mean of consumption growth	0.022	0.030
Volatility of consumption growth	0.033	0.033
Mean of 1-year zero coupon yield	0.018	0.029
Volatility of 1-year zero coupon yield	0.030	0.066
Mean of Equity Premium	0.042	0.036
Volatility of Equity Premium	0.177	0.208
Mean of Price to Earnings Ratio	22.480	33.417
Volatility of (log) Price to Earnings Ratio	0.280	0.335
Mean of Book to Market	0.668	0.707
Volatility of Book to Market	0.230	0.316

TABLE 2: Unconditional Moments of the model and the data. Data for consumption growth are from the Website of Robert Shiller. Consumption growth refers to (yearly) differences in log consumption. Both means and standard deviations are computed for the entire sample and for post 1918 data and then averaged to avoid issues with mismeasurement of consumption in pre World War I data. The rest of the data are from Chan and Kogan [2002] except for the mean and the volatility of the book to market, which is taken from Pontiff and Schall [1998]. The unconditional moments for the model are computed from a Monte Carlo Simulation involving 20000 years of data, dropping the initial 8000 to ensure that initial quantitites are drawn from their stationary distribution.

Mean time to Plant a tree	8	6	4	2	0	After
Relative weight of assets in place	0.052	0.047	0.043	0.039	0.036	0.962
Relative weight of current growth option	0.641	0.668	0.694	0.719	0.743	0.000
Relative weight of future growth options	0.307	0.285	0.263	0.242	0.221	0.038
Expected return of assets in place	0.090	0.094	0.098	0.100	0.100	0.100
Expected return of current growth options	0.199	0.199	0.199	0.199	0.199	0.000
Expected return of future growth options	0.075	0.075	0.074	0.073	0.073	0.073
Expected Return	0.156	0.159	0.162	0.165	0.168	0.099

TABLE 3: Pattern of expected returns for a firm in the lowest size decile having drawn the most attractive growth option in the current epoch. The table gives the relative weight of the different valuation components and their evolution as a function of the expected time to invest. We condition on the epoch not changing until the firm invests. The table reports the total (not excess) return.

P/E Predictability of Excess Returns

	Data		Model	
Horizon(years)	Coefficient	R-square	Coefficient	R-square
1	-0.120	0.040	-0.041	0.010
			(0.068)	(0.012)
2	-0.300	0.100	-0.080	0.020
			(0.111)	(0.025)
3	-0.350	0.110	-0.108	0.029
			(0.145)	(0.034)
5	-0.640	0.230	-0.155	0.044
			(0.193)	(0.052)
7	-0.730	0.250	-0.186	0.054
			(0.220)	(0.057)

Table 4: Results of predictive Regressions. Excess returns in the aggregate stock market between t and $t+T$ for $T = 1, 2, 3, 5, 7$ are regressed on the P/E ratio at time t. A constant is included but not reported. The data column is from Chan and Kogan [2002]. The simulations were performed by drawing 100 time series of a length equal to the data and performing the same predictive regressions. We report the means of these simulations next to the data. The numbers in parentheses are the standard deviations of the estimates obtained in the simulations.

Portfolios formed on Size (Stationary Distribution)												
Deciles	1A	1B	2	3	4	5	6	7	8	9	10A	10B
Returns -Data	1.64	1.16	1.29	1.24	1.25	1.29	1.17	1.07	1.10	0.95	0.88	0.90
Returns -Simulated	0.70	0.69	0.68	0.67	0.66	0.64	0.63	0.62	0.61	0.60	0.60	0.59
Log Size - Data	1.98	3.18	3.63	4.10	4.50	4.89	5.30	5.73	6.24	6.82	7.39	8.44
Log Size - Simulated	3.34	3.80	4.07	4.31	4.56	4.83	5.10	5.39	5.69	6.01	6.29	6.60

Portfolios formed on Size (Breakdown Period)										
Deciles	1	2	3	4	5	6	7	8	9	10
Returns -Data	1.22	0.95	1.00	0.96	1.13	1.12	1.26	1.18	1.21	1.18
Returns -Simulated	0.77	0.84	0.86	0.87	0.88	0.88	0.88	0.87	0.87	0.86

Table 5: Portfolios sorted by size - model and data

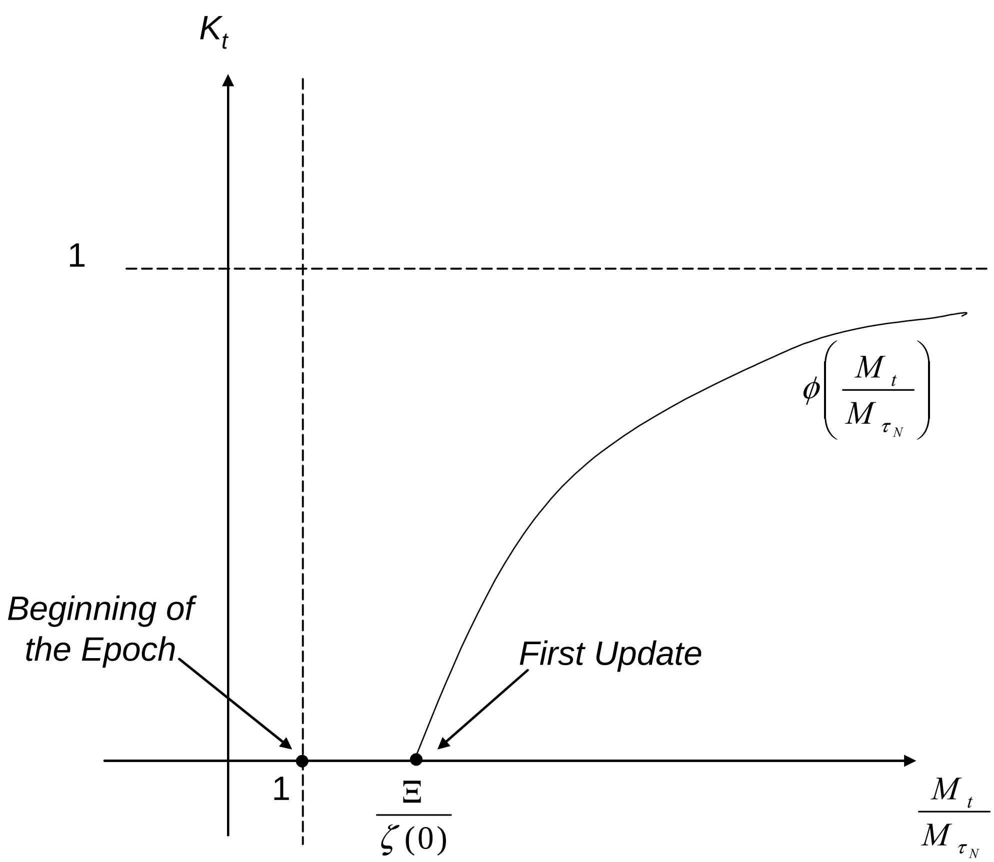


FIGURE 1: A plot of the $\phi(\cdot)$ function.

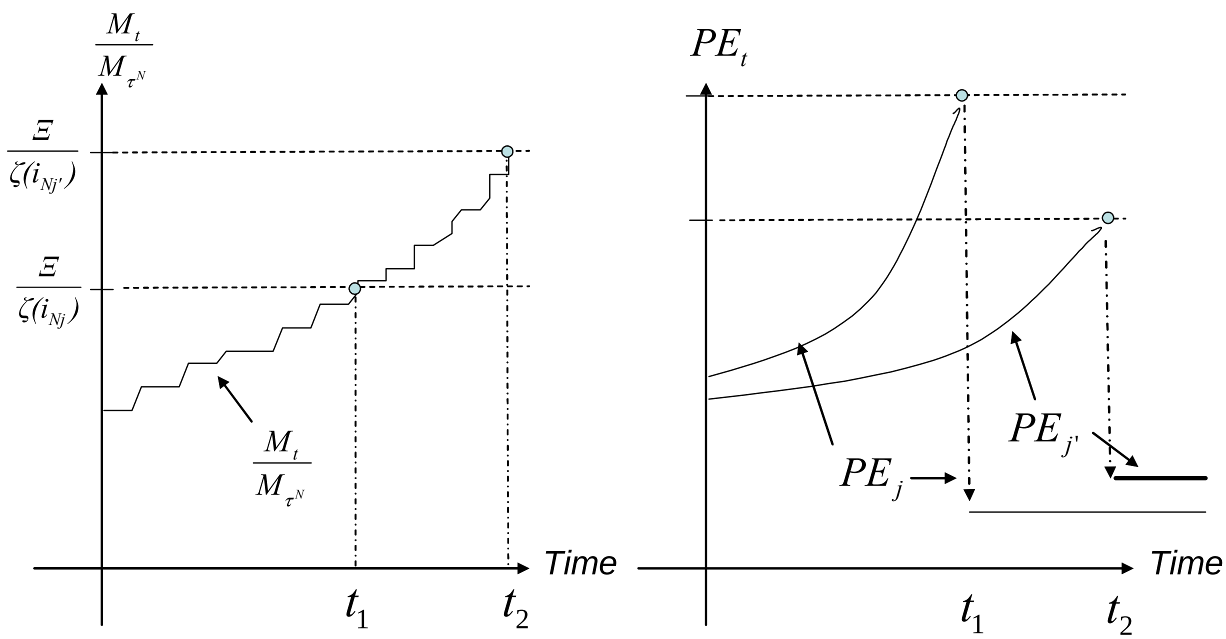


FIGURE 2: Paths of $\frac{M_t}{M_{\tau_N}}$ and P/E ratios for two firms j and j' with $\zeta(j) > \zeta(j')$ and equal assets in place. The left figure plots the path of $\frac{M_t}{M_{\tau_N}}$ as afunction of time, along with the times t_1 and t_2 at which firms decide to plant their trees. The right figure plots the path of the P/E ratio for these two firms.

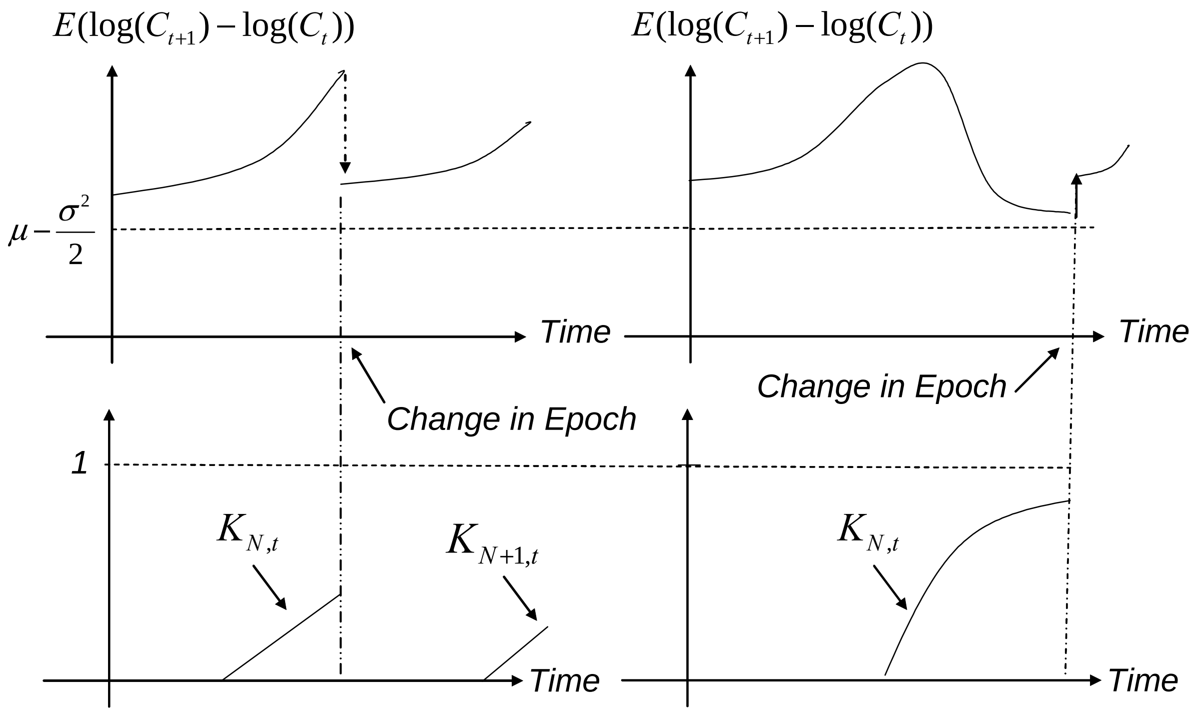


FIGURE 3: Expected Growth rate in consumption as a function of the duration of an epoch. The top left figure and the corresponding bottom left figure depict a scenario of a short epoch. The top right figure and the corresponding bottom right figure depict a long epoch. The top figures depict expected consumption growth as a function of time, while the bottom figures depict the respective paths for the mass of firms that have planted a tree in the current round $(K_{N,t})$.

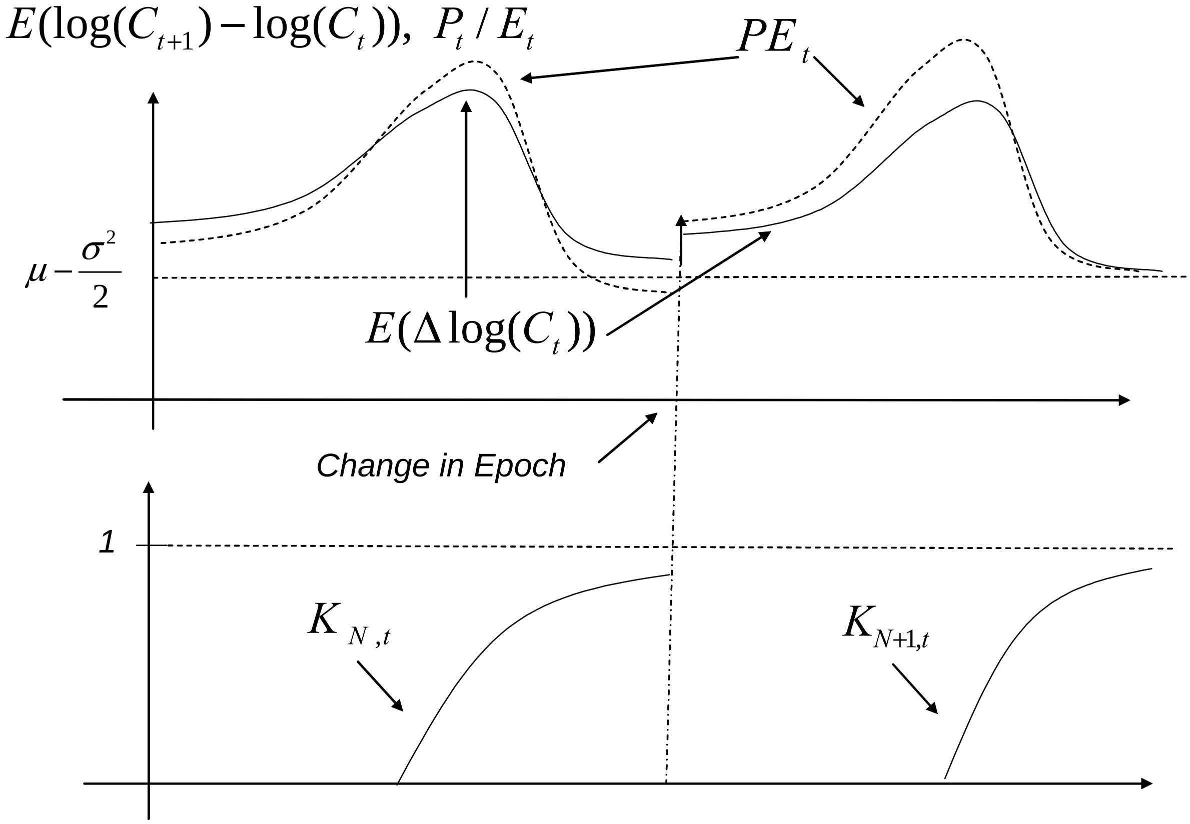
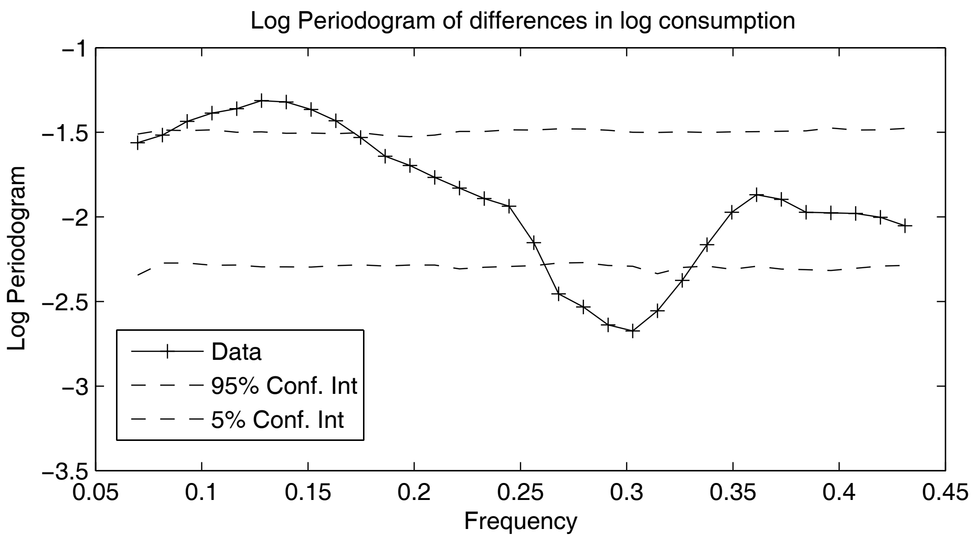


FIGURE 4: Path of P/E ratio and the expected growth rate of consumption over a technological epoch. The top panel depicts the P/E ratio and the expected growth rate of consumption, while the bottom panel plots the respective path of $K_{N,t}$.



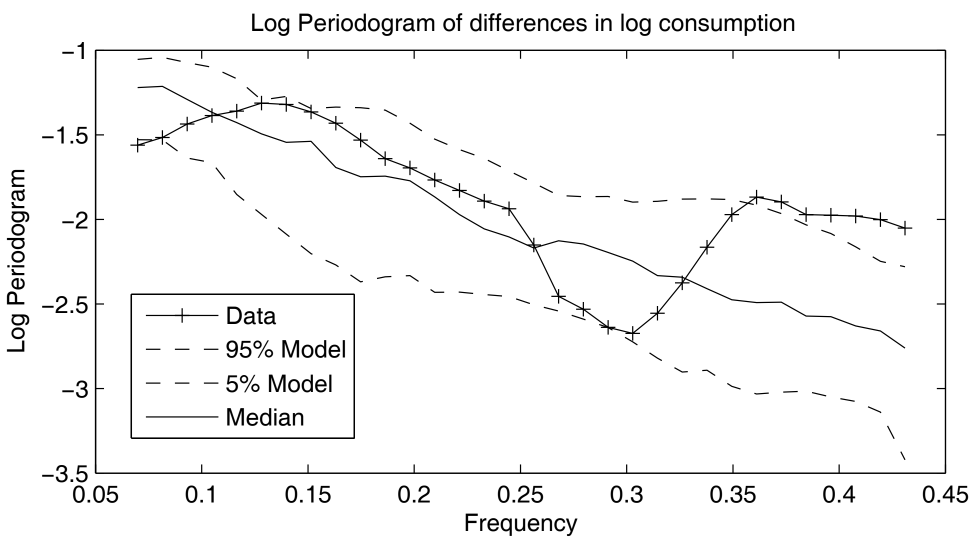


FIGURE 5: Log Periodogram of the consumption process for the data and the model. The top figure presents the log periodogram for post- world war I yearly differences in log consumption. Confidence bands are computed by bootstrapping the first differences in log consumption to produce 1000 artificial series with a length of 87 years. The bottom figure presents the same exercise for the model: 3000 years of simulated consumption growth were used to produce repeated series of a length of 87 years. The figure plots the median of these simulations, along with 5% and 95% range intervals. It also shows the log periodogram for the actual data. An equally weighted "nearest neighbor" kernel was used to perform the smoothing, equally weighting the 7 nearest frequencies.

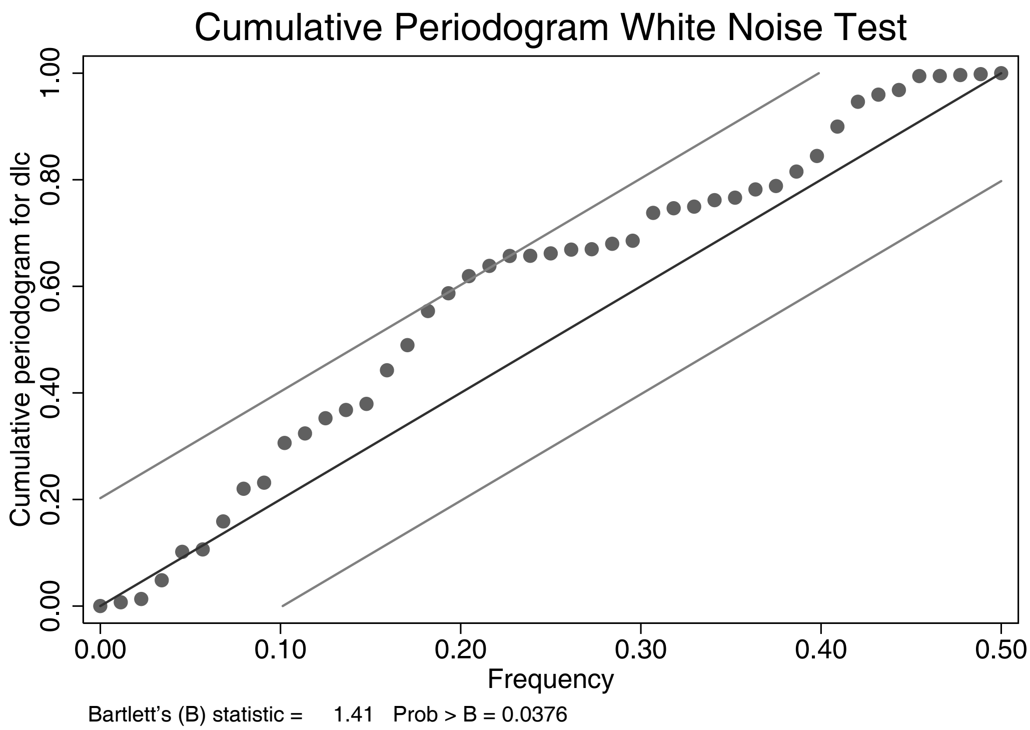


FIGURE 6: Bartlett's test for white noise based on the consumption cummulative periodogram. The test rejects the white noise hypothesis with a p-value of 0.0376

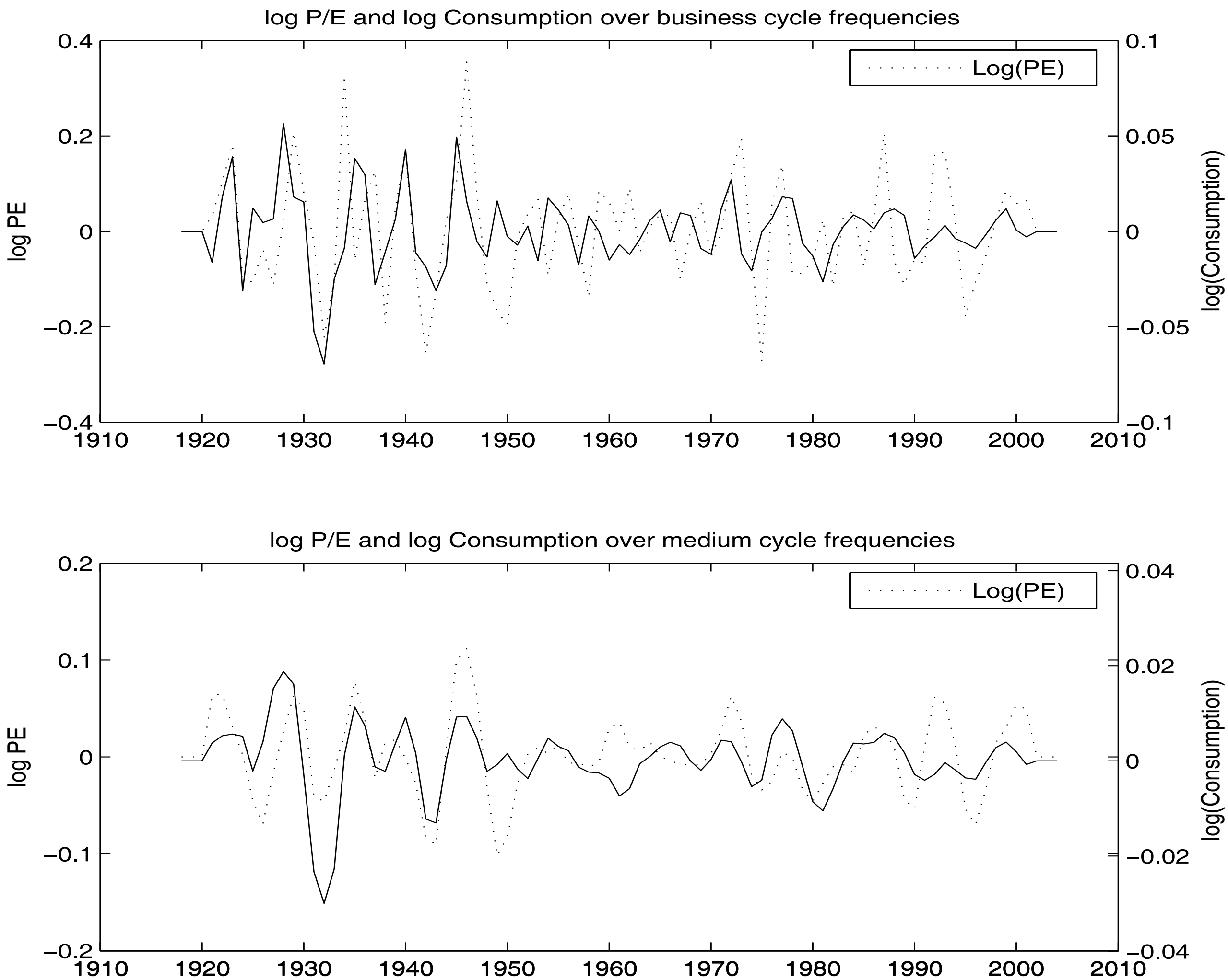


FIGURE 7: Consumption and the P/E ratio filtered at different frequencies. The top figure presents filtered (log) consumption and the (log) P/E ratio keeping frequencies between 2-8 years (business cycle frequencies), while the bottom figure retains frequencies between 8-30 years. All data are yearly. The P/E ratio is evaluated at the beginning of the period. The correlation between the two series is 0.32 for the top panel and 0.46 for the bottom panel. A Baxter King filter is used for both series.

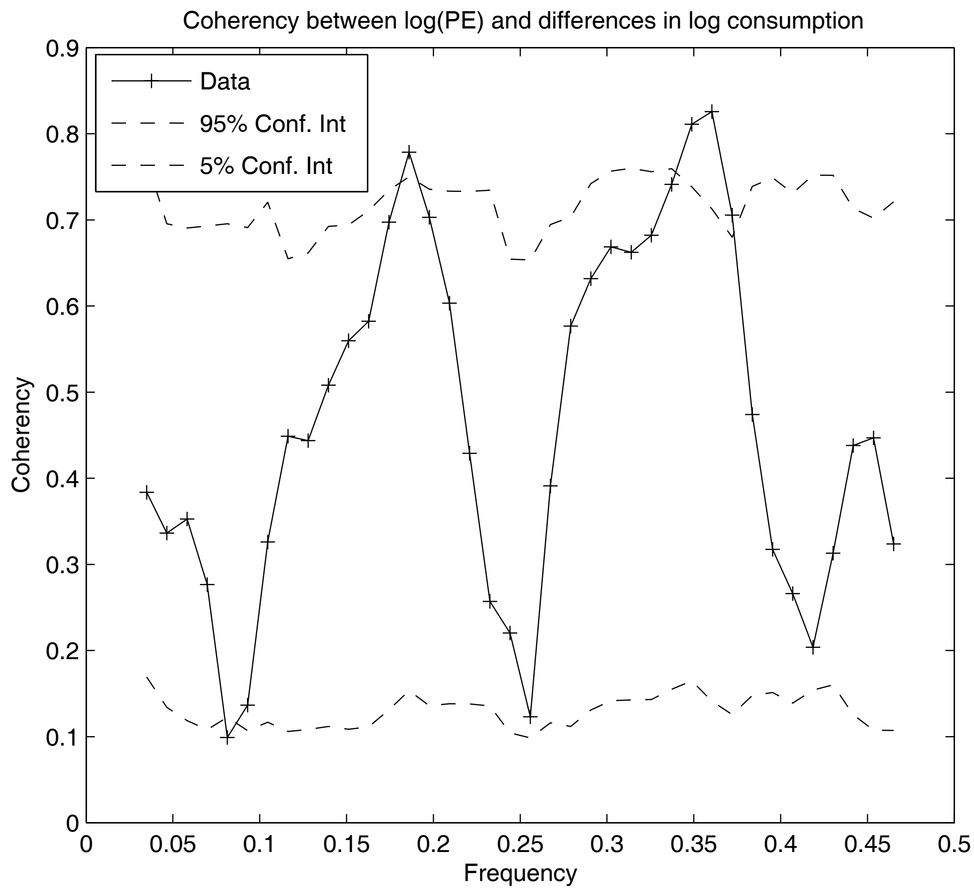
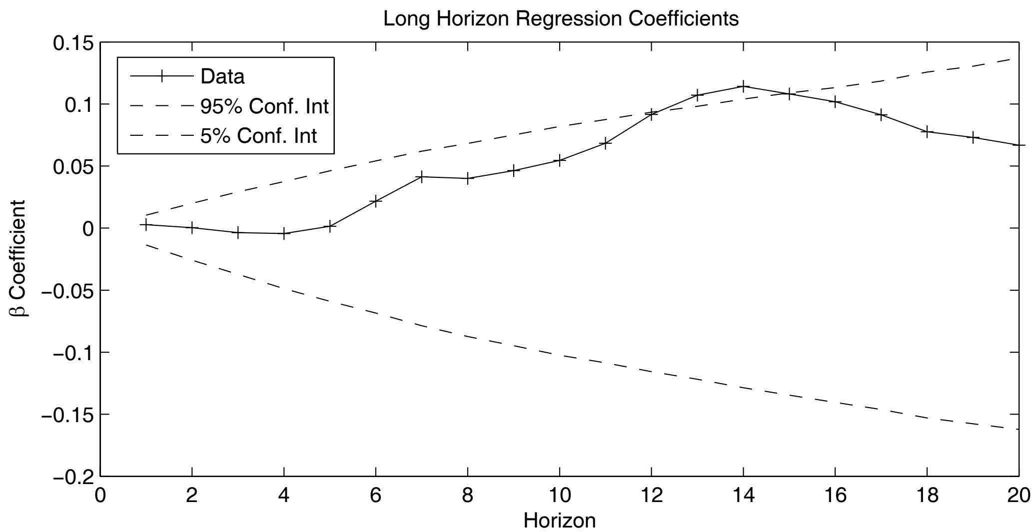


FIGURE 8: Coherency (Correlation at different frequencies) between (log) P/E and the differences in (log) consumption. Confidence bands are computed using a Monte Carlo simulation of 5000 artificially created series under the assumption that (log) consumption is a random walk, the (log)P/E ratio is an AR1 process, and thus consumption growth is unpredictable.



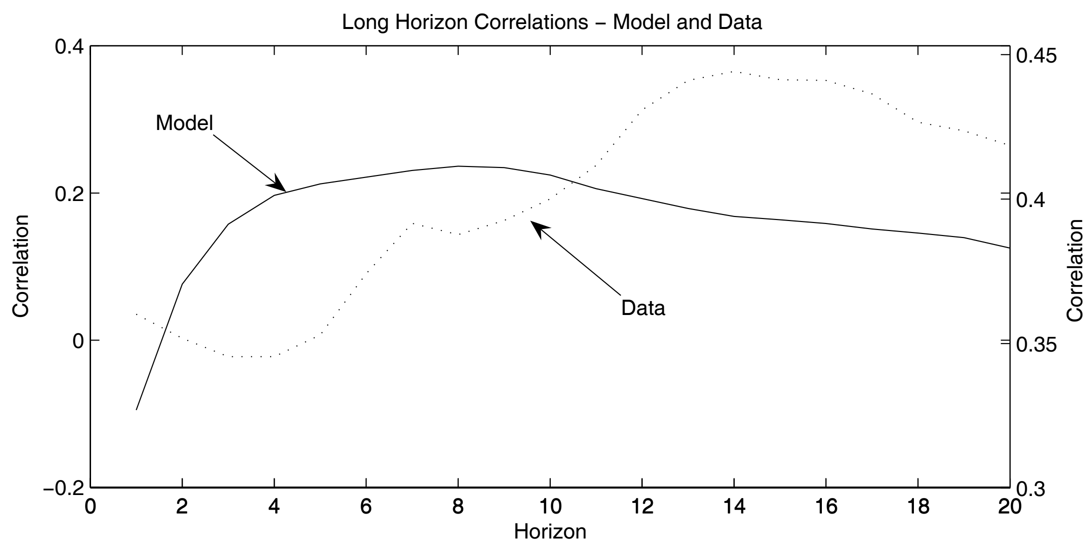
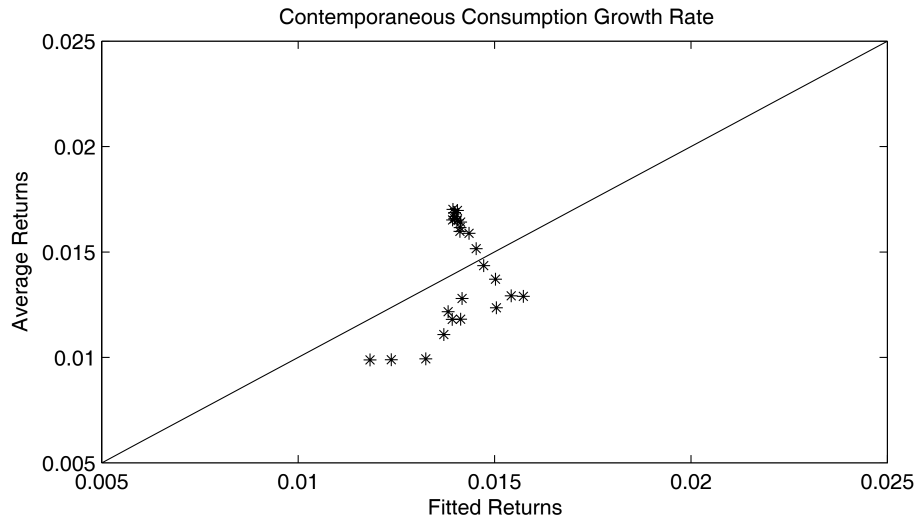


FIGURE 9: Long Horizon Predictability Regressions of cummulative consumption growth between t and $t+T$ on the (log) P/E ratio at time $t-1$. The top figure presents results for the data. Confidence intervals are computed by Monte Carlo simulations. We create artificial time series by bootstraping yearly differences in log consumption and the residuals of an AR1 process fitted to the (log) P/E ratio. (For details see text). The bottom figure compares the correlation coefficients for the same predictive regressions in the data and the model.



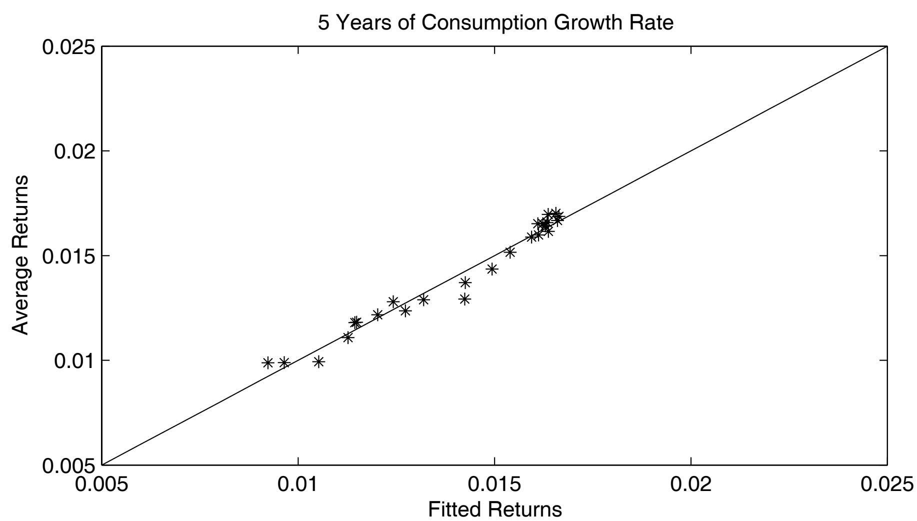


FIGURE 10: The consumption CAPM using 1-quarter consumption growth and 5-year consumption growth rates to evaluate the covariation between consumption growth and excess returns.