Market Liquidity and Funding Liquidity*

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June 2005

Abstract

We provide a model that links a security's market liquidity — i.e., the ease of trading it — and traders' funding liquidity — i.e., their availability of funds. Traders provide market liquidity and their ability to do so depends on their funding, that is, their capital and the margins charged by their financiers. In times of crisis, reductions in market liquidity and funding liquidity are mutually reinforcing, leading to a liquidity spiral. The model provides a natural explanation for the empirically documented features that market liquidity (i) can suddenly dry up (i.e. is fragile), (ii) has commonality across securities, (iii) is related to volatility, (iv) experiences "flight to liquidity" events, and (v) comoves with the market. Finally, the model shows how the Fed can improve current market liquidity by committing to improve funding in a potential future crisis.

Keywords: Liquidity Risk Management, Liquidity, Liquidation, Systemic Risk, Leverage, Margins, Haircuts

*We are grateful for helpful comments from Yakov Amihud, David Blair, Bernard Dumas, Christian Julliard, John Kambhu, Markus Konz, Guillaume Plantin, Dimitri Vayanos, and Jiang Wang. We would also like to thank seminar participants at the New York Federal Reserve Bank and the New York Stock Exchange, Citigroup, Bank of International Settlement, University of Zürich, INSEAD, Northwestern University, Stockholm Institute for Financial Research and conference participants at the American Economic Association Meeting 2005, FMRC conference in honor of Hans Stoll at Vanderbilt, and NBER Market Microstructure Meetings.

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1 Introduction

Trading requires capital. When a trader — e.g. a dealer, hedge fund, or investment bank — buys a security, he can use the security as collateral and borrow against it, but he cannot borrow the entire price. The difference between the security's price and collateral value, denoted the margin, must be financed with the trader's own capital. Similarly, shortselling requires capital in the form of a margin; it does not free up capital. Hence, at any time the total margins on all positions cannot exceed the trader's capital.

Our model shows that the funding of trades affects, and is affected by, market liquidity in a profound way. When funding liquidity is tight, traders become reluctant to take on positions, especially "capital-intensive" positions in high-margin securities. This lowers market liquidity. Further, low future market liquidity increases the risk of financing a trade, thus increasing the margins.

Based on the links between funding and market liquidity, we provide a unified explanation for the main empirical features of market liquidity. In particular, the model implies that market liquidity (i) can suddenly dry up, (ii) has commonality across securities, (iii) is related to volatility, (iv) experiences "flight to liquidity" events, and (v) comoves with the market.

Our model is similar in spirit to Grossman and Miller (1988) with the new feature that dealers face the real-world funding constraint discussed above. In our model, a group of "initial customers" face a supply shock at time one, which affects their willingness to hold shares. A group of "complementary customers" face the opposing shock, but these agents arrive in the market only at a later time. A group of dealers bridge the gap between the initial and complementary customers by smoothing the price and thus providing market liquidity.

We derive the competitive equilibrium of the model and explore its liquidity implications. We define market liquidity as the difference between the transaction price and the fundamental value, and funding liquidity as a dealer's scarcity (or shadow cost) of capital. Naturally, as long as dealer capital is abundant, market liquidity is at its highest level and insensitive to marginal changes in capital and margins. However, when dealers hit their capital constraints — or risk hitting their capital constraints over the life of a trade — then they are forced to reduce their positions and market liquidity is reduced.

We show that, under a certain condition, there are multiple equilibria. In one equilibrium markets are liquid leading to favorable margin requirements for dealers, which in turn helps dealers make markets liquid. In another equilibrium markets are illiquid, resulting in larger margin requirements (or dealer losses), thus restricting dealers from providing market liquidity. The necessary and sufficient condition for such a multiplicity is, loosely said, that increased market illiquidity leads to either higher margin requirements or losses on dealers' existing positions.

Importantly, in case of multiple equilibria, any equilibrium selection has the property that there must be a risk of sudden market liquidity dry-ups. In particular, for any equilibrium selection there exists a level of dealer funding such that market liquidity drops off discontinuously for any infinitesimal drop in funding. This sudden dry-up (or fragility) of market liquidity is due to the fact that with high dealer capital, markets must be in the liquid equilibrium, and, if dealer capital in reduced enough, the market must eventually switch to the low-liquidity/high-margin equilibrium.

Further, when markets are illiquid, market liquidity is highly sensitive to further changes in funding conditions. This is due to two liquidity spirals: first, a "margin spiral" emerges if margins are increasing in market illiquidity because a reduction in dealer wealth lowers market liquidity, leading to higher margins, tightening dealers' funding constraint further, and so on. Second, a "loss spiral" arises if dealers hold a large initial position that is negatively correlated with customers demand shock. In this case, a funding shock increases market illiquidity, leading to dealer losses on their initial position, forcing dealers to sell more, causing a further price drop, and so on. The liquidity spirals imply, paradoxically, that a shock to the customers' demand for immediacy leads to a reduction in the provision of immediacy in such stress times.

Our model also provides a natural explanation for the commonality of liquidity across assets since shocks to the funding constraint of the dealer sector affect all securities. This may help explain why market liquidity is correlated across stocks (Chordia, Roll, and Subrahmanyam (2000), Hasbrouck and Seppi (2001) and Huberman and Halka (2001)), and across stocks and bonds (Chordia, Sarkar, and Subrahmanyam (2005)).

Next, our model predicts that market liquidity declines as fundamental volatility increases, which is consistent with the empirical findings of Benston and Hagerman (1974) and Amihud and Mendelson (1989). To see the intuition for this result, note first that fundamental volatility trivially leads to price volatility, which leads to higher margins. Consequently, it is more capital intensive for dealers to trade in volatile securities, therefore dealers provide less market liquidity in such securities. The reduced market liquidity further increases the risk of financing such trades, thus further increasing margins, and so on. This reasoning applies both when comparing the market liquidity across securities in the cross section, and when explaining changes in market liquidity in the time series.

The model implies that the liquidity differential between high-volatility and low-volatility securities increases as dealer capital deteriorates — a phenomenon often referred to as "flight to quality" or "flight to liquidity." According to our model, this happens because a reduction in dealer capital induces traders to provide liquidity mostly in securities that do not use much capital (low volatility stocks since they have lower margins). Acharya and Pedersen (2005) document empirical evidence consistent with flight to liquidity.

Since market making firms are often long the market, capital constraints are more likely to be hit during market downturns. Under this premise, our model explains why sudden liquidity dry-ups occur more often when markets decline.

Next, we analyze how margins are set and describe circumstances under which margins are destabilizing. The objective of margins is to almost perfectly shield financiers from default risk. Margin are typically set equal to asset's value at risk (VaR) which corresponds to the largest possible price drop within a certain confidence interval. We show that margins stabilize the price and decrease with market illiquidity, if financiers know that prices diverge due to temporary market illiquidity and know that liquidity will be improved shortly as complementary customers arrive. This is because a larger price discount due to current illiquidity reduces the size of future price declines. In other words, current price discounts provide a "cushion" against further price drops, which reduces the margin. This cushioning effect disappears, however, if financiers do not know when the trade will converge (i.e. when complementary customers arrive), leading to a constant margin. If the financier cannot distinguish price movements due to fundamental and liquidity reasons and if fundamentals have time-varying volatility, then margins can increase in volatility. This can lead to the destabilizing effects discussed above. Hence, we predict that dealers face more destabilizing margins in specialized markets in which financiers cannot easily distinguish fundamental shock from liquidity shocks or cannot predict when a trade converges.

Our analysis also has implications for central bank policy. Central banks can mitigate market liquidity problems in several ways. If a central bank is better than typical financiers of dealers at distinguishing liquidity shocks from fundamental shocks, then the central bank can convey this information and urge financiers to relax their funding requirements — as the Federal Reserve Bank of New York did during the 1987 stock market crash. Central banks can also directly improve market liquidity by boosting dealer's funding conditions during a liquidity crisis, or by simply stating the intention to provide extra funding during times of crisis which will loosen margin requirements immediately.

In summary, our model provides insights on the interaction of funding liquidity and market liquidity, and can help explain the major empirical features of liquidity. Furthermore, the model suggests a novel line of empirical work, namely to link empirically measures of dealer's funding to measures of market liquidity.

The remainder of the paper proceeds as follows. Section 2 describes the institutional features associated with the financing of trading activity for market makers, banks, and hedge funds, and discusses funding liquidity risk. Section 3 lays out our basic model and shows how the link between funding and market liquidity leads to fragility and liquidity spirals. Section 4 derives the model's cross-sectional implications, in particular, commonality of liquidity and flight to quality. Section 5 shows under which circumstances margin requirements are stabilizing or destabilizing. Section 6 shows how our framework relates to the liquidity concepts discussed within the literatures on market microstructure, corporate finance, banking, limits of arbitrage, macroeconomics, and general equilibrium. Section 7 concludes and discusses the liquidity implications of central bank policy in light of the endogenous margins

The appendix contains proofs.

2 The Funding Liquidity of Traders

There are several types of providers of market liquidity, that is, traders that act as intermediaries by buying or selling. The main types of such traders are market makers, proprietary traders, and hedge funds. These traders are subject to funding constraints on their trading activity, and we refer to the risk of a binding funding constraint as funding liquidity risk. To set the stage for our model, we review the main real-world funding constraints for securities firms.

2.1 Margins and Capital Constraints: The Case of Hedge Funds

We first consider the funding issues faced by hedge funds since they have relatively simple balance sheets and face little regulation. Below, we discuss the funding issues (including regulation) for banks, and market makers.

A hedge fund must finance its activities using its capital. A hedge fund's capital W_t at time t consists of its equity capital supplied by the partners and of possible long-term debt financing that can be relied upon during a potential funding crisis. Since a hedge fund is a partnership, the equity is not locked into the firm indefinitely as in a corporation. The investors (that is, the partners) can withdraw their capital at certain times, but — to ensure funding — the withdrawal is subject to so-called lock-up periods (typically at least a month, often several months or even years). A hedge fund usually cannot issue long-term unsecured bonds, but some (large) hedge funds manage to obtain debt financing in the form of medium term bank loans or in the form of a guaranteed line of credit.[1] Hedge funds lever their capital using collateralized borrowing financed by the hedge fund's prime broker(s). The prime brokerage business is opaque since the terms of the financing are subject to negotiation and are secret to outsiders. We describe stylized financing terms and, later, we discuss caveats.

If a hedge fund buys at time t a long position of $x_t^j > 0$ shares of a security j with price p_t^j, then this requires the hedge fund to come up with $x_t^j p_t^j$ dollars. The security can, however, be used as collateral for a new loan of, say, l_t^j dollars. The difference between the price of the security and the collateral value is denoted the margin requirement $m_t^{j+} = p_t^j - l_t^j$. Hence, this position uses $x_t^j m_t^{j+}$ dollars of the

[1] A line of credit may have a "material adverse change" clause or other covenants subject to discretionary interpretation of the lender. Such covenants imply that the line of credit may not be a reliable source of funding during a crisis.

fund's capital. We note that the collateralized funding implies that the capital use depends on margins, not notional amounts.

The margins on fixed income securities and over-the-counter (OTC) derivatives are set through a negotiation between the hedge fund and the broker that finances the trade, often the hedge funds' prime broker. The margins are typically set such as to make the market almost risk free for the broker. Hence, the collateral value of a long position l_t^j for borrowing between time t and $t+1$ is the smallest possible value that the security might have to be sold for at time $t+1$ with a certain confidence in case the borrower defaulted on the loan. Hence, the margin is essentially the positions' value-at-risk (VaR).[2]

In the U.S., margins on equities are subject to Regulation T, which stipulates that all non-broker/dealers must have an initial margin of 50% of the market value of the underlying stock, both for long and short positions. Hedge funds can circumvent Regulation T for instance by organizing the transaction as a total return swap, which is a derivative that is functionally equivalent to buying the stock.

The margin on exchange traded futures (or options) is set by the exchange. The principle for setting the margin for futures or options is the same as that described above. The margin is set such as to make the exchange almost immune to default risk of the counterparty, and hence riskier contracts have larger margins.

At the end of the financing period, time $t+1$, the position is "marked-to-market," which means that the hedge fund receives any gains (or pays any losses) that have occurred between t and $t+1$, that is, the fund receives $x_t^j(p_{t+1}^j - p_t^j)$, and the fund pays interest on the loan, $r_t x_t^j l_t^j$, where r_t is the funding rate. If the trade is kept on, the broker keeps the margin to protect against losses going forward from time $t+1$. The margin can be adjusted if the risk of the collateral has changed unless the counterparties have contractually fixed the margin for a certain period.

If the hedge fund wants to sell short a security, then the fund asks one of its brokers to locate a security that can be borrowed, and then the trader sells the borrowed security. Duffie, Gârleanu, and Pedersen (2002) describes in detail the institutional arrangements of shorting. The broker requires a collateral that we denote by c_t^j. The collateral value of a short position c_t^j is the highest possible value that the security might have to be bought back for at time $t+1$ with a certain confidence. As with a long position, this makes the transaction almost risk free for the broker. This collateral requirement implies that the short sale uses $x_t^j(c_t^j - p_t^j)$ of the hedge fund's capital. The margin on a short position — i.e. the per share capital use — is denoted by $m_t^{j-} = c_t^j - p_t^j$. We note that a short sale does not raise capital for a hedge fund; it

[2] The value at risk is the largest loss with a certain statistical confidence, e.g. 1 percent. This is also sometimes referred to as the broker's "potential future exposure" (PFE). Often brokers also take into account the time it takes between a fail by the hedge fund is noticed and the security is actually sold. Hence, the margin of a one-day collateralized loan depends on the estimated risk of holding the asset over a time period that is often set to be five to ten days.

uses capital.

A hedge fund must be able to finance all its security positions at any point of time. This means that the total capital use must be smaller than the available net capital plus available long-term debt funding. That is, at any time t,

$$\sum_j \left(x_t^{j+} m_t^{j+} + x_t^{j-} m_t^{j-}\right) \leq W_t \tag{1}$$

where $x_t^{j+} \geq 0$ and $x_t^{j-} \geq 0$ are the positive and negative parts of $x^j = x_t^{j+} - x_t^{j-}$, respectively.

So far, we focussed on situations in which margins are covered using risk-free assets (cash). A dealer can also post risky assets to cover margins, provided that they have not been used as collateral otherwise. However, the dealer has to post a higher security value if he uses risky assets, since a so-called haircut is subtracted. For example, a dealer who bought x^j shares of stock j and has to come up of with margins of $x_t^j m_t^j$, can cover it with $x_t^{j'}$ of his uncollateralized bonds j'. Since the bond is risky a haircut $h_t^{j'}$ is subtracted and his funding constraint becomes $x_t^j m_t^j \leq W_t - x_t^{j'} h_t^{j'}$. Moving the haircut term to the left hand side reveals that the haircut behaves like a margin. Hence, the dealer essentially still faces funding constraint (1). Indeed, the dealer could have alternatively used the bonds j' to raise cash and then use this cash to cover the margins for asset j. We therefore use the terms margins and haircuts interchangeably.

Our description of the determination of margins was described as if margins are set separately for each security position. It is, however, sometimes possible to "cross-margin", i.e. to jointly finance several trades that are part of the same strategy. This leads to a lower total margin if the risks of the various positions are partially offsetting. For instance, much of the interest rate risk is eliminated in a "spread trade" with a long position in one bond and a short position in a similar bond. Hence, the margin/haircut of a jointly financed spread trade is smaller than the sum of the margins of the long and short bonds. For a strategy that is financed jointly, we can reinterpret security j as such a strategy. Prime brokers compete by, among other things, offering low margins and haircuts — a key consideration for hedge funds — which means that it is becoming increasingly easy to finance more and more strategies jointly. In the extreme, one can imagine a joint financing of a hedge fund's total position such that the funding constraint becomes

$$M_t\left(x_t^1, \ldots, x_t^J\right) \leq W_t \tag{2}$$

where M is the margin requirement of the portfolio x, that is, the most the portfolio can lose over the next funding period with a certain confidence (the portfolio's value-at-risk). Currently, it is often not practical to jointly finance a large portfolio. This is because a large hedge fund finances its trades using several brokers, both a hedge fund and a broker can consist of several legal entities (possibly located in different jurisdictions), certain trades need separate margins paid to exchanges (e.g. futures and options) or to other counterparties of the prime broker (e.g. securities lenders), prime brokers may not have sufficiently sophisticated models to evaluate the diversification benefits (e.g. because they don't have enough data on the historical performance of newer products such as CDOs), and because of other practical difficulties in providing joint financing. Further, if the margin requirement m relies on assumed stress scenarios in which the securities are perfectly correlated (e.g. due to predatory trading), then (2) coincides with (1).

2.2 Funding Requirements for Banks

A bank's capital W consists of equity capital plus its long-term borrowings (including credit lines secured from individual or syndicates of commercial banks), reduced by assets that cannot be readily employed (e.g. goodwill, intangible assets, property, equipment, and capital needed for daily operations), and further reduced by uncollateralized loans extended by the bank to others (see e.g. Goldman Sachs 2003 Annual Report). Banks also raise money using short-term uncollateralized loans such as commercial papers and promissory notes, and, in the case of commercial banks, demand deposits. These sources of financing cannot, however, be relied on in times of funding crises since lenders may be unwilling to continue lending, and therefore this short term funding is not included in W.

The financing of a bank's trading activity is largely based on *collateralized* borrowing. Banks can borrow securities to short from mutual funds and pension funds, for instance, and can finance long positions using collateralized borrowing from corporations with excess cash, other banks, insurance companies, and the Federal Reserve Bank. These transactions typically require margins which must be financed by the bank's capital W as captured by the funding constraint (1).

The financing of a bank's proprietary trading is more complicated than that of a hedge fund, however. For instance, banks may negotiate zero margins with certain counterparties, banks can often sell short shares held in house, that is, held in a customers margin account (i.e. held in "street name") such that the bank does not need to borrow the share externally. Further, a bank receives margins when financing hedge funds (i.e. the margin is negative from the point of view of the bank). However, often the bank wants to pass on the trade to an exchange or another counterparty and hence has to pay a margin to the exchange. In spite of these caveats, we believe that in times of stress, banks face margins and are ultimately subject to a funding constraint in the spirit of (1) (see, e.g., Goldman Sachs, 2003 Annual Report, page 62).

In addition, banks have to satisfy certain regulatory requirements. Commercial banks are subject to the Basel accord, supervised by the Federal Reserve system for US banks. In short, the Basel accord of 1988 requires that a bank's "eligible capital" exceeds 8% of the "risk-weighted asset holdings," which is the sum of each asset holding multiplied by its risk weight. The risk weight is 0% for cash and government securities, 50% for mortgage-backed loans, and 100% for all other assets. The requirement posed by the 1988 Basel accord corresponds to Equation (1) with margins of 0%, 4%, 8%, respectively. In 1996, the accord was amended, allowing banks to measure market risk using an internal model similar to (2) rather than using standardized risk weights.

U.S. broker-dealers, including banks acting as such, are subject to the Securities and Exchange Commission's (SEC's) "net capital rule" (SEC Rule 15c3-1). This rule stipulates, among other things, that a broker must have a minimum "net capital," which is defined as equity capital plus approved subordinate liabilities minus "securities haircuts" and operational charges. The haircuts are set as security-dependent percentages of the market value. The standard rule requires that the net capital exceeds at least $6\frac{2}{3}\%$ (15:1 leverage) of aggregate indebtedness (broker's total money liabilities) or alternatively 2% of aggregate debit items arising from customer transactions. This constraint is similar in spirit to (1).[3] As of August 20, 2004, SEC amended (SEC Release No. 34-49830) the net capital rule for Consolidated Supervised Entities (CSE) such that CSE's may, under certain circumstances, use their internal risk models similar to (2) to determine whether they fulfill their capital requirement.

2.3 Funding Requirements for Market Makers

There are various types of market-making firms. Some are small partnerships, whereas others are parts of large investment banks. The small firms are financed in a similar way to hedge funds in that they rely primarily on collateralized financing; the funding of banks was described in Section 2.2.

Certain market makers, such as NYSE specialists, have an obligation to make a market and a binding funding constraint means that they cannot fulfill this requirement. Hence, avoiding the funding constraint is especially crucial for such market makers.

Market makers are in principle subject to the SEC's net capital rule (described in Section 2.2), but the rule has special exceptions for market makers. Hence, market makers' main regulatory requirements are those imposed by the exchange on which they operate. These constraints are often similar in spirit to (1).

2.4 Funding Liquidity Risk

Funding liquidity risk is the risk that a trader's funding constraint is binding. As discussed in Sections 2.1–2.3, the real-world funding constraint is captured by Equation (1), Equation (2), or something in between. In the remainder of this paper, we focus on (1) since it is more realistic in several interesting cases as discussed above.

[3] Let L be the lower of $6\frac{2}{3}\%$ of total indebtedness or 2% of debit items and h^j the haircut for security j; then the rule requires that $L \leq W - \sum_j h^j x^j$, that is, $\sum_j h^j x^j \leq W - L$.

Funding risk stems from the risks that net capital decreases, short term borrowing availability is reduced, and margins increase. Net capital decreases if the institution experiences trading losses or otherwise incurs losses. Short term borrowing can dry up for a bank if it cannot sell commercial paper or, in the case of a commercial bank, because of deposit withdrawals — a bank run. Margins and haircuts increase if the collateral becomes more risky, which happens if fundamental uncertainty increases (e.g. after the crash in 1987 and after September 11, 2001) or if market liquidity is reduced such that liquidation of the collateral is more difficult. For instance, Long Term Capital Management (LTCM) estimated that in times of severe stress, haircuts on AAA-rated commercial mortgages would increase from 2% to 10%, and similar haircut increases for other securities (HBS Case N9-200-007(A)).

Naturally, financial institutions try to manage their funding liquidity risk. For instance, Goldman Sachs (2003 Annual Report, page 62) states that it seeks to maintain net capital in excess of total margins and haircuts that it would face in periods of market stress plus the total draws on unfunded commitments at such times. Hence, Goldman Sachs recognizes that it may not have access to short-term borrowing during a crisis, that margins and haircuts may increase during such a crisis, and that counterparties may withdraw funds at such times.

The risk of a funding crisis is not purely academic. For instance, in the 1987 stock market crash numerous market makers hit (or violated) their funding constraint (1):

> "By the end of trading on October 19, [1987] thirteen [NYSE specialist] units had no buying power"
> — SEC (1988), page 4-58

Several of these firms managed to reduce their positions and continue their operations. Others did not. For instance, Tompane was so illiquid that it was taken over by Merrill Lynch Specialists and Beauchamp was taken over by Spear, Leeds & Kellogg (Beauchamp's clearing broker).

Also, market makers outside the NYSE experienced funding troubles: the Amex market makers Damm Frank and Santangelo were taken over; at least 12 over-the-counter (OTC) market makers ceased operations; and several trading firms went bankrupt.

These funding problems were due to (i) reductions in capital arising from trading losses and default on unsecured customer debt, and (ii) an increased funding need stemming from increased inventory and increased margins. Margins were increased since agents financing the trading perceived that the collateral had an increased risk and a reduced market liquidity. One New York City bank, for instance, increased margins/haircuts from 20% to 25% for certain borrowers, and another bank increased margins/haircuts from 25% to 30% for all specialists (SEC (1988) page 5-27 and 5-28). Other banks reduced the funding period by making intra-day margin calls, and at least two banks made intra-day margin calls based on assumed 15% and 25% losses, thus effectively increasing the haircut by 15% and 25%. Also, some broker-dealers experienced a reduction in their line of credit.

Another stark example of funding liquidity risk is LTCM, which — in spite of their numerous measures to control funding risk — ultimately could not fund its positions and was taken over by 14 banks in September 1998.

The goal of this paper is to study market liquidity provision by dealer who face funding liquidity risk and consider the equilibrium implications for market liquidity. We turn next to our model.

3 Fragility of Liquidity and Liquidity Spirals

We start by considering a simple model with trade in one security. There are three groups of agents: initial customers, complementary customers, and dealers. At time 1, the initial customers arrive in the market with a need to trade, and, at time 2, the complementary customers arrive with the opposite trading need. The dealers provide immediacy by always being available to trade in the market. At time 3, the security pays off v, a random variable defined on a probability space $(\Omega, \mathcal{F}, \mathcal{P})$. Later, we generalize the model in various directions. [4]

At any time $t = 1, 2$ the initial customers want to sell $S(z, E_t[v] - p_t)$ shares of the security, where p_t is the price and time t, and the $\mathcal{C}^1$ function $S(z, L) : [0, \infty)^2 \to \mathcal{R}$ is increasing in the size of the "demand shock" z, and strictly decreasing in the second argument $E_t\left[v\right] - p_t$ with $S(z, 0) > 0$ and $S(z, \bar{\Lambda}(z)) = 0$ for some $\bar{\Lambda}(z) \in \mathcal{R}$. Said differently, customers naturally sell less if the price is lower and refrain from selling altogether if the price is too low. The initial customers supply function with these properties can be easily derived from, for instance, a risk averse utility function combined with a hedging need or a specification of holding costs.

At time 2, the complementary customers arrive with a need to buy $S(z, E_2[v] - p_2)$ shares. Hence, at time 2, the equilibrium price is naturally $p_2 = E_2[v]$. Hence, we focus on the price at time 1, and we use the simplified notation $p = p_1$.

The market liquidity of the security at time 1 is denoted Λ, defined as

$$\Lambda := |E_1\left[v\right] - p|. \tag{3}$$

Since the initial customers are sellers, the time-1 equilibrium price must satisfy $p \leq E_1[v]$, that is, $\Lambda = E_1[v] - p$. Of course, the model would be identical if buying customers arrived before selling ones. In that case, everything would be the same except that $\Lambda = p - E_1(v)$.

There is a unit mass of identical dealers. A dealer is risk neutral, has an initial endowment of x_0 shares and B dollars, and chooses his security positions at times 1

[4] In Section 4 we introduce multiple assets and in Section 5.2 we consider an infinite horizon setting in which complementary customers arrive at a random point in time.

and 2. Since the price at time 2 is equal to the fundamental in our baseline setting, the dealer only trades between time 1 and 2. The dealer solves:

$$\max_x x(E_1[p_2] - p) = x(E_1[v] - p) = x\Lambda \tag{4}$$

subject to his capital constraint

$$xm(\sigma, \Lambda) \leq W(\Lambda) := \max\{0, B + x_0(E_1[v] - \Lambda)\} \tag{5}$$

where the margin $m : [0, \infty)^2 \to (0, \infty)$ is a $\mathcal{C}^2$ function, which depends on fundamental volatility σ and the market liquidity Λ, W is the dealer's wealth, and x_0 is his initial holding. The margin is increasing in fundamental volatility $\partial m/\partial\sigma > 0$.

We consider competitive equilibria of the economy:

Definition 1
(i) *An* equilibrium *is a market illiquidity* Λ *such that a market-clearing position of* $x = S(z, \Lambda)$ *is a solution to the dealer's problem.*
(ii) *An equilibrium with* $\Lambda > 0$ *is* unstable *if a small increase in the dealer's position improves market liquidity enough to relax the dealer's capital constraint, that is,* $\exists\delta > 0$ *such that* $\forall\varepsilon \in (0, \delta) : S(z, \Lambda - \varepsilon)m(\sigma, \Lambda - \varepsilon) < W(\Lambda - \varepsilon)$*; otherwise an equilibrium is* stable*.*

Perfect market liquidity, i.e. $\Lambda = 0$, can occur in equilibrium if dealers have enough capital, that is, if

$$S(z, 0)m(\sigma, 0) \leq B + x_0 E_1[v]\,. \tag{6}$$

If the dealer's capital constraint is binding in equilibrium then the market is illiquid, $\Lambda > 0$. The following proposition provides a more extensive characterization of the set of equilibria and gives a necessary and sufficient condition for fragility, that is, the property that a small change in fundamentals can lead to a large jump in liquidity:

Proposition 1
(i) *If* $\Lambda \to S(z, \Lambda)m(\sigma, \Lambda) + x_0\Lambda$ *is decreasing, there exists a unique stable equilibrium for each level of dealer wealth* B*. The equilibrium market illiquidity* $\Lambda^*(B)$ *is continuously decreasing in dealer wealth* B*.*
(ii) (Fragility) *Otherwise, there exists equilibrium selections* $\Lambda^*(B)$ *such that market illiquidity* $\Lambda^*(B)$ *is decreasing in dealer wealth* B*, but all equilibrium selections are discontinuous. In particular, there must be a wealth level* B' *such that illiquidity jumps discontinuously if wealth drops by any amount, that is,* $\exists L > 0$ *such that* $\forall\varepsilon > 0 : \Lambda^*(B) > \Lambda^*(B - \varepsilon) + L$*. Further, there are multiple stable equilibria for an open set of wealth levels unless* $\exists\Lambda' \in [0, \bar{\Lambda}(z))$ *such that* $\Lambda \to S(z, \Lambda)m(\sigma, \Lambda) + x_0\Lambda$ *is decreasing on* $[0, \Lambda']$ *and increasing on* $[\Lambda', \bar{\Lambda}(z)]$*.*

The condition for fragility — that $\Lambda \to S(z,\Lambda)m(\sigma,\Lambda) + x_0\Lambda$ is not decreasing — is intuitive once we decipher it. It basically means that either the margin m is sufficiently increasing in market illiquidity or the dealer's initial position x_0 is sufficiently positive. An increasing margin leads to fragility because dealer losses can lead to self-perpetuating reductions in market liquidity and associated increases in margins. Similarly, a large x_0 leads to fragility because dealer losses leads to self-perpetuating reductions in market liquidity and associated further losses on the dealer's initial position.

Once the economy enters into the illiquid equilibrium, market liquidity becomes highly sensitive to shocks because of natural amplification mechanisms. We distinguish two different amplification mechanisms, the "margin spiral" and the "loss spiral."

Proposition 2
(i) *If $\Lambda = 0$ then, generically, Λ is insensitive to local changes in dealer wealth, fundamental value, customer demand, and fundamental volatility.*
(ii) (Liquidity Spirals) *If $\Lambda > 0$ in a stable equilibrium then the local sensitivity of Λ with respect to dealer wealth, fundamental value, customer demand, and fundamental volatility are:*

$$\begin{aligned} \frac{d\Lambda}{dB} &= \frac{-1}{-\frac{\partial S}{\partial \Lambda}m - \frac{\partial m}{\partial \Lambda}S - x_0} \\ \frac{d\Lambda}{dE_1[v]} &= \frac{-x_0}{-\frac{\partial S}{\partial \Lambda}m - \frac{\partial m}{\partial \Lambda}S - x_0} \\ \frac{d\Lambda}{dz} &= \frac{\frac{\partial S}{\partial z}m}{-\frac{\partial S}{\partial \Lambda}m - \frac{\partial m}{\partial \Lambda}S - x_0} \\ \frac{d\Lambda}{d\sigma} &= \frac{\frac{\partial m}{\partial \sigma}S}{-\frac{\partial S}{\partial \Lambda}m - \frac{\partial m}{\partial \Lambda}S - x_0} \end{aligned}$$

where $-\frac{\partial S}{\partial \Lambda}m - \frac{\partial m}{\partial \Lambda}S - x_0 > 0$. Multiplier effects arise if margin increase in illiquidity $\frac{\partial m}{\partial \Lambda} > 0$ ("margin spiral") or if dealers have existing positions $x_0 > 0$ ("loss spiral").

Figure 1 illustrates both "liquidity spirals." A decline in funding B forces dealers to provide less market liquidity. If margins increase in illiquidity Λ, then this initial decline tightens dealers' funding constraint further. This in turn forces them to cut down on their trading and so on, leading to a "margin spiral." In addition, there is a "loss spiral" if dealers hold a positive initial position $x_0 > 0$ because funding problems lead to attempts to sell, which lowers market liquidity, leading to dealer losses, and so on. Note that if market making firms are net long the market — which is often the case — then the model implies that market liquidity is low when the market is down, consistent with empirical evidence.

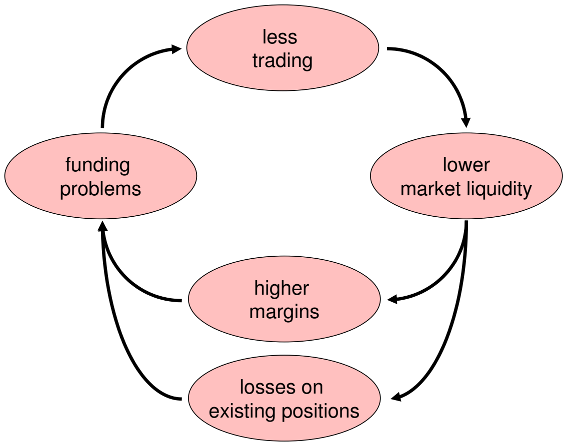

Figure 1: Liquidity Spirals

To illustrate mathematically how these derivatives can be interpreted as multipliers, we note that $-\frac{\partial S}{\partial \Lambda}m > 0$ (since the initial customers supply is decreasing in illiquidity), and that, for any $a > 0$ and z with $|z| < a$, it holds that

$$\frac{1}{a-z} = \frac{1}{a} + \frac{z}{a^2} + \frac{z^2}{a^3} + ...$$

Each term in this infinite series corresponds to one loop around the circle in Figure 1. The total effect of the margin and the dealer's position is a multiplier if and only if $\frac{\partial m}{\partial \Lambda}S + x_0 > 0$. Of course, a multiplier can arise even if one spiral is at work and the other is working in opposite direction to mitigate shocks (i.e. if $x_0 < 0$ or if $\frac{\partial m}{\partial \Lambda} < 0$) as long as the former effect is stronger.

Interestingly, Proposition 2 also reveals that the margin spiral and loss spiral amplify one another if they are both at work. Hence, the total effect of a margin spiral and a loss spiral is greater than the sum of their separate effects. Mathematically this can be seen by using simple convexity arguments, and it can be seen intuitively from the flow diagram of Figure 1.

The result that market liquidity can suddenly dry up discontinuously — i.e. liquidity is fragile — is consistent with anecdotal evidence. Similarly, the finding that the volatility of market liquidity is particularly high during illiquid times is in line with casual observation. It is illustrative to see how fragility and the liquidity spirals arise in the subsequent simple numerical example.

Numerical Example

The initial customers' supply is linear, $S(z, \Lambda) = z - 2\Lambda$ with an endowment shock of $z = 20$. This supply is, for instance, the optimal supply of initial customers with quadratic holding costs or with constant absolute risk aversion combined with normal returns. A linear supply function guarantees that fragility is not simply the consequence of a non-linear supply schedule; fragility and spirals must be due to the provision of market liquidity.

The demand curve of the dealers depends on their funding. As soon as the price is lower than the fundamental value, i.e. $\Lambda > 0$, the risk-neutral dealers take on their maximum position in a one-period model.[5] They exploit maximum leverage by making use of secured collateralized borrowing, thus demanding $\{B + x_0 (E_1 [v] - \Lambda)\} / m(\sigma, \Lambda)$ shares.

Funding effects depend on the specification of the margin function $m(\sigma, \Lambda)$ and profits and losses depends on the initial position x_0. To separate these effects, we first consider two specifications of the margin function in the context of zero initial positions, and then we consider the effect of the initial position in the context of a constant margin.

Fragility. We first consider the case in which dealers have no initial position $x_0 = 0$ and the margin is increasing in market illiquidity $m(\sigma, \Lambda) = 4 + \Lambda$. Figure 2 depict the initial customers' supply curve $S = 20 - 2\Lambda$ and the dealers' demand curve for different levels of funding B. Dealers' demand is given by $x = \frac{B}{m(\sigma,\Lambda)} = \frac{B}{4+\Lambda}$ for any $\Lambda > 0$.

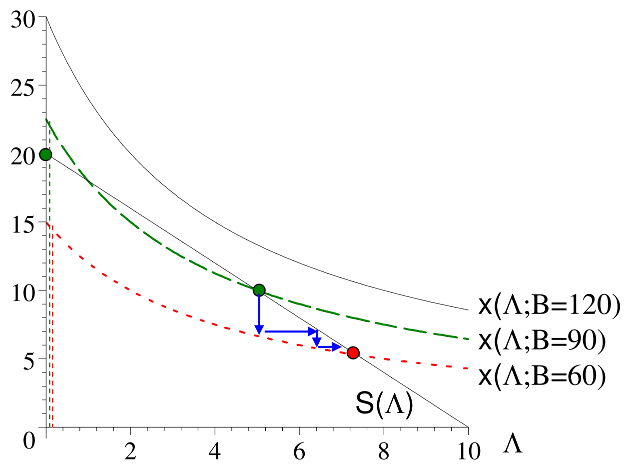

Figure 2: Fragility and Margin Spiral. The figure plots the initial customers' supply S and the dealers' demand x as functions of market illiquidity Λ when the margin is $m(\sigma, \Lambda) = 4 + \Lambda$ and the dealers' initial position is $x_0 = 0$.

[5]This outcome is not necessarily true in a dynamic model since dealers are worried that funding might be even more scarce in some future state of the world.

When the dealer is well funded $B = 120$, the dealers' demand curve is given by top solid curve in Figure 2. At this level of funding, the dealer would have a larger demand than the initial customers' supply for any $\Lambda > 0$. Hence, perfect liquidity provision $\Lambda = 0$ is the unique equilibrium. The dealers fully absorb initial customers' total selling pressure of $z = 20$ and market illiquidity is zero.

As funding declines to $B = 90$, the perfect liquidity equilibrium remains because, at $\Lambda = 0$ the dealer can still fund a position of $x = 20$ which fully absorbs the customers' demand shock. However, a second stable equilibrium with less liquidity provision $\Lambda = 5$ also emerges. In this funding-constrained illiquid equilibrium, the dealer's funding constraint is binding because margins are higher. The higher margin prevents the dealers from providing full market liquidity, and the low market liquidity in turn justifies the higher margins. Hence, funding and market liquidity reinforce each another. Low market liquidity (high Λ), leads to higher margins m which tightens dealers' funding and reduces dealers' market liquidity provision.

We note that with $B = 90$ there is also a third equilibrium in which $\Lambda = 1$. This equilibrium is not stable, however. Indeed, if Λ dropped slightly below 1 then dealers could trade more, pushing Λ further down, a process that would stop only when $\Lambda = 0$. Alternatively, if Λ increased above 1 then dealers would violate their funding constraint, thus need to reduce their position, pushing Λ further up until $\Lambda = 5$. Naturally, we are only interested in the properties of stable equilibria.

If funding is as low as $B = 60$ then perfect liquidity $\Lambda = 0$ is no longer an equilibrium and only a funding-constrained illiquid equilibrium remains. In summary, for sufficiently high B, the perfect-liquidity equilibrium is the unique outcome. For B in an intermediate range there are two stable equilibria; the perfect-liquidity equilibrium and a funding-constrained illiquid equilibrium. For low B the unique equilibrium is funding constrained.

Figure 2 highlights the "disconnect" between the perfect-liquidity equilibrium and the funding-constrained illiquid equilibria. Hence, a marginal reduction in B cannot always lead to a smooth reduction in market liquidity. There must be a level of funding such that an infinitesimal drop in funding leads to a large drop in market liquidity. This discontinuity can help explain the sudden market liquidity dry ups, that is, the fragility of liquidity.

Margin Spiral. Continuing with the example of Figure 2, we consider the effect on market liquidity of a marginal drop in wealth. Of course, a reduction of B does not affect the market illiquidity Λ as long as the market remains in the perfect-liquidity equilibrium with $\Lambda = 0$.

More interestingly, after a wealth shock that leads to a sudden discontinuous drop in market liquidity as the equilibrium switches to the funding-constrained illiquid equilibrium, market liquidity becomes very sensitive to further changes in wealth B. This is because a further reduction in B is amplified by a "liquidity spiral." This is seen in Figure 2 where wealth drops from 90 to 60, which leads to a violation of dealers' funding constraint. Therefore, dealers have to cut back on their positions even if prices (i.e. Λ) were to stay the same. This leads to an excess supply so the price declines (Λ increases), which in turn leads to higher margins, further tightening the dealers' funding constraint, and so on. The new equilibrium is only reached at $\Lambda = 7.3$ with a position of $x = S = 5.2$.

Similarly, a change in supply shock z (not illustrated) would also set off a liquidity spiral: as the supply curve shifts outwards, the excess supply leads to larger Λ, which tightens the funding constraint and lowers dealers' demand, which in turn leads to an increase in Λ, and so on.

Unique Equilibrium and No Spiral. Figure 3 depicts the case in which margins decline as the market becomes less liquid. More specifically, the margin is $m(\sigma, \Lambda) = \max\{4 - \Lambda, 0\}$ and we keep the assumption for now that $x_0 = 0$.

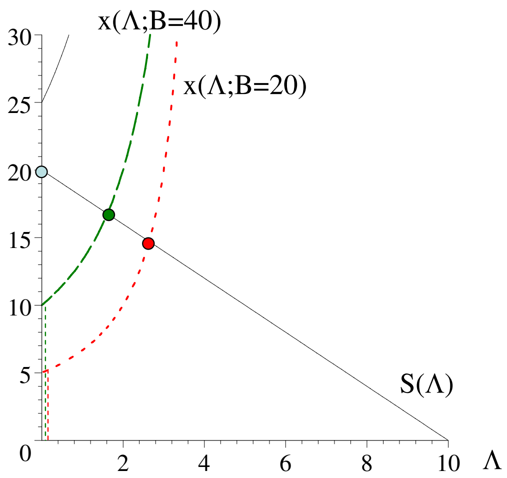

Figure 3: No Fragility. The figure plots the initial customers' supply S and the dealers' demand x as functions of market illiquidity Λ when the margin is $m(\sigma, \Lambda) = \max\{4 - \Lambda, 0\}$ and the dealers' initial position is $x_0 = 0$.

The upward sloping curves are the dealers' demand curves for different levels of funding B. Clearly, there is a unique equilibrium — shown as a circle — for each level of funding. Further, the equilibrium market liquidity is a continuous function of wealth B.

Finally, there is no liquidity spiral; on the contrary the margin function mitigates a wealth shock. To see this, consider a reduction in funding B. This downward shift in the dealer's demand curve would, for fixed Λ, lead to excess supply. Hence, Λ increases, which reduces the margin, thus relaxing the dealers' funding constraint.

Hence, in this case, sudden liquidity dry-ups (fragility), and liquidity spirals do not emerge.

Note that one should not conclude from our numerical example that fragility occurs if and only if m is increasing in Λ; Proposition 1 provides precise conditions for fragility.

Loss Spiral. In the previous examples, the dealers' initial position, x_0, is zero, so that their wealth is independent of current prices. This allowed us to abstract from the effects of endogenous dealer profits and losses. In this example, we let $x_0 = 5$. To focus exclusively on the effect of this initial position, we let the margin be constant, $m = 4$, i.e. independent of Λ. Hence, dealers' demand, $\frac{B+x_0(E_1[v]-\Lambda)}{m}$, is linearly decreasing in Λ with a slope $-\frac{x_0}{m}$.

Figure 4 depicts the initial customers' supply schedule $S(20, \Lambda) = 20 - 2\Lambda$ and the dealers' demand schedule for different wealth levels. The solid line plots the case of dealer wealth $B + x_0 E_1[v] = 70$. In equilibrium the dealers absorb $\frac{40}{3}$ shares of the initial customers' selling pressure at an illiquidity level of $\Lambda = \frac{10}{3}$.

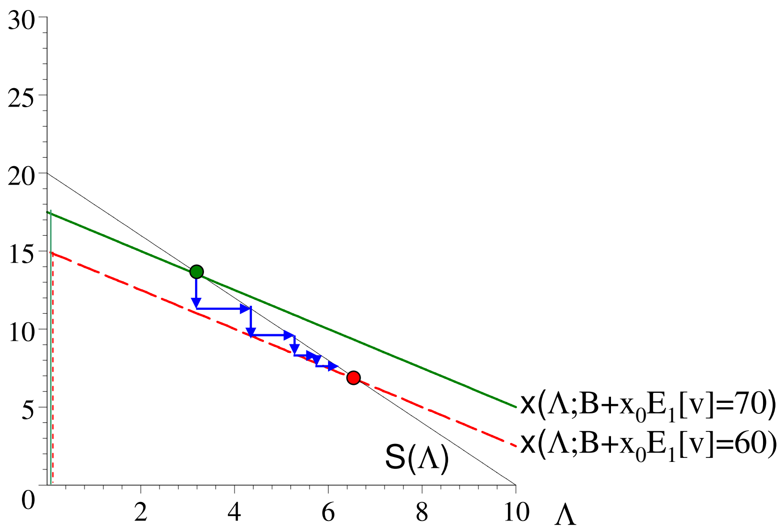

Figure 4: Loss Spiral. The figure plots the initial customers' supply S and the dealers' demand x as functions of market illiquidity Λ when the margin is $m(\sigma, \Lambda) = 4$ and the dealers' initial position is $x_0 = 5$.

The dashed line in Figure 4 reflects the dealers' demand function after reduction in their cash holding B by 10 units. This tightens their constraints and they are no longer able to purchase $\frac{40}{3}$ shares. Even if prices were to stay the same, i.e. illiquidity were fixed at $\Lambda = \frac{10}{3}$, dealers' absorption capacity is reduced by $\frac{\Delta B}{m} = \frac{10}{4}$ shares. This initial reduction is already significant due to the leverage effect. Prices adjust in addition, triggering the loss-spiral. To see this, note that the excess supply by the initial customers leads to more illiquidity (lower prices) which erodes dealers' current wealth $B + x_0 (E_1 [v] - \Lambda)$ further. This forces dealers to reduce their position even further, which in turn increases illiquidity and so on. Figure 4 illustrates this loss spiral which arises because the slopes of demand curve and of supply curve have the same sign. In our numerical example, a reduction of B by 10 units doubles illiquidity from $\frac{10}{3}$ to $\frac{20}{3}$.

A similar loss spiral arises in case of a drop in the fundamental value of the stock $E_1 [v]$. This is because the associated wealth loss is x_0 times the drop in fundamental value. Hence, $\frac{d\Lambda}{dE_1[v]} = x_0 \frac{d\Lambda}{dB}$. The larger is x_0, the larger is the initial wealth loss, and, further, the more pronounced is the multiplier $\frac{d\Lambda}{dB}$ because the initial position amplifies any price drops.

An increase in the supply shock z also leads to a multiplier effect spiral. A higher z (parallel shift of the supply curve) results in an increase in Λ (reduction in price), which leads to dealer losses on the existing positions x_0, leading to a further increases in Λ, forcing the constrained dealer to reduce his position x_0 even further. Interestingly, an increased supply z leads to a reduction in the provision of immediacy.

Also, an increase in fundamental volatility σ translates into a higher margin m, which flattens the demand curve $x (\Lambda, W)$ by rotating it around the intercept with the horizontal axes. Holding illiquidity fixed, dealers' demand is depressed after increasing σ. This increases Λ, which depresses dealers' demand and so on.

4 Commonality and Flight to Quality

In order to consider cross-sectional variation in liquidity, we now generalize the model to have multiple securities, indexed $j = 1, \ldots, J$. Security j has a final payoff of v^j and a fundamental volatility σ^j. The initial customers have a demand shock z^j for asset j and thus want to sell $S(z^j, \Lambda^j)$ securities, where S is defined above. The complementary customers have the reverse shock, and, therefore, the price of security j is equal to $E_2 [v^j]$ at time 2.

The dealer maximizes expected profit

$$\max_{(x^j)} E_1 \left[\sum_j x^j (v^j - p^j) \right] = \sum_j x^j \Lambda^j$$

subject to the capital constraint

$$\begin{aligned} \sum_j m(\sigma^j, \Lambda^j) x^j &\leq \max\{0, B + \sum_j x_0^j (E_1 \left[v^j\right] - \Lambda^j)\} && (7) \\ x^j &\geq 0 && (8) \end{aligned}$$

where m is the margin as a function of fundamental volatility σ and market liquidity Λ. We assume as before that m is increasing in fundamental volatility σ and, further, that the margin elasticity with respect to Λ is less than one, $\frac{\partial m}{\partial \Lambda}\frac{\Lambda}{m} < 1$, because the margin is in part due to fundamental volatility.

Since both the dealer's objective function and constraint are linear, he optimally invests all his capital in securities that have the greatest expected profit Λ^j per capital use m^j. We define the shadow cost of capital ϕ as the maximum attainable profit per used capital

$$\phi = \max_j \frac{\Lambda^j}{m^j} \tag{9}$$

Hence, the dealer invests positive amounts, $x^j > 0$, only in securities with

$$\frac{\Lambda^j}{m^j} = \phi \tag{10}$$

and he does not invest in securities with $\frac{\Lambda^j}{m^j} < \phi$.

The dealer's shadow cost of capital ϕ captures well the notion of funding liquidity. Indeed, a high ϕ means that the available funding — from capital W and from collateralized financing with margins m^j — is low relative to the needed funding, which depends on the investment opportunities deriving from demand shocks z^j.

Consider the equilibrium market liquidity for a given cost of capital ϕ for the dealer. If the dealer does not invest in security j, then market illiquidity is the smallest value such that in equilibrium $S(z^j, \Lambda) = 0$. We denote this value by $\bar{\Lambda}(z^j)$. The dealer will refrain from investing in this asset if his expected profit per capital use in this security is less than his shadow cost of capital ϕ, that is, if

$$\frac{\bar{\Lambda}(z^j)}{m(\sigma^j, \bar{\Lambda}(z^j))} < \phi. \tag{11}$$

Otherwise, the dealer invests in this security, thus improving market liquidity until the point at which Λ^j solves[6]

$$\frac{\Lambda^j}{m(\sigma^j, \Lambda^j)} = \phi \tag{12}$$

The market liquidity corresponding to ϕ is denoted by $\Lambda^j(\phi)$. That is, $\Lambda^j(\phi)$ is the minimum of $\bar{\Lambda}(z^j)$ and the solution to (12).

We can also express the price p^j as a function of funding liquidity ϕ, expected value of fundamentals $E_1[v^j]$ and the margin:

$$p^j = E_1\left[v^j\right] - \phi m^j(\sigma^j, \Lambda^j(\phi)). \tag{13}$$

[6]It can be shown that there is a unique solution to (12).

Intuitively, the prices are closer to fundamentals if funding is less tight (ϕ small), or if the margin m^j is small.

We can characterize the equilibrium funding liquidity as follows. We insert the $\Lambda^j(\phi)$ and the equilibrium condition $x^j = S(z^j, \Lambda^j)$ into the dealer's funding constraint to obtain the following inequality in ϕ:

$$\sum_j m(\sigma^j, \Lambda^j(\phi)) S(z^j, \Lambda^j(\phi)) \leq \max\{0, B + \sum_j x_0^j (E_1 \left[v^j\right] - \Lambda^j(\phi))\} \tag{14}$$

Hence, the equilibrium shadow cost of capital ϕ is characterized as follows: either (i) $\phi = 0$, $\Lambda^j = 0$, and

$$\sum_j m(\sigma^j, 0) S(z^j, 0) \leq B + \sum_j x_0^j E_1 \left[v^j\right], \tag{15}$$

(ii) $\phi \in (0, \bar{\phi})$, where $\bar{\phi} := \max_j \frac{\bar{\Lambda}(z^j)}{m(\sigma^j, \bar{\Lambda}(z^j))}$ and

$$\sum_j m(\sigma^j, \Lambda^j(\phi)) S(z^j, \Lambda^j(\phi)) = B + \sum_j x_0^j (E_1 \left[v^j\right] - \Lambda^j(\phi)), \text{ or} \tag{16}$$

(iii) $\phi = \bar{\phi}$ and

$$0 \geq B + \sum_j x_0^j (E_1 \left[v^j\right] - \Lambda^j(\phi)). \tag{17}$$

As in the case of one security, there can be one or more equilibria, that is, one or more solutions ϕ exits to these equations. Also, fragility generalizes to the case of multiple securities. Rather than repeating these results, we focus on the model's implications for cross-sectional variation in market liquidity. Interestingly, the model implies that market liquidity is common across securities:

Proposition 3 (Commonality of market liquidity)
If $B, E_0\left[v^1\right], \ldots, E_0\left[v^J\right]$ are random, the market liquidity of any two securities j and k comove,

$$Cov\left(\Lambda^j, \Lambda^k\right) \geq 0,$$

and market liquidity comoves with funding liquidity,

$$Cov\left(\Lambda^j, \phi\right) \geq 0.$$

This commonality arises in a straightforward way from the fact that any securities market liquidity depends on the funding condition of the dealer sector.

The model further links market illiquidity to fundamental volatility. To see this, note that if an asset has higher fundamental volatility σ^j it also has higher margins, everything else equal. This, in turn, implies a higher equilibrium market illiquidity. Further, the market illiquidity of securities with higher fundamental volatility has a disproportionally stronger response to a capital shock. The next proposition demonstrates these volatility implications for market liquidity.

Proposition 4 *Suppose that asset k has lower fundamental volatility than asset j, $\sigma^k < \sigma^j$. Then*
(i) (Quality=Liquidity) *Assets with lower fundamental volatility have better market liquidity. Specifically,*

$$\Lambda^j \geq \Lambda^k$$

if $x^j, x^k > 0$, or if these securities have the same demand shock $z^j = z^k$.
(ii) (Flight to Quality) *The market liquidity differential between high and low fundamental volatility securities is bigger when dealer funding is tight if m is not too nonlinear. Specifically,*

$$\left|\frac{\partial \Lambda^j}{\partial B}\right| > \left|\frac{\partial \Lambda^k}{\partial B}\right|$$

if $x^j, x^k > 0$ and if $\partial^2 m/\partial \Lambda^2$ and $\partial^2 m/\partial \sigma \partial \Lambda$ are small.

Hence, not only are more volatile securities on average more illiquid (part (i)), they are also more sensitive to changes in dealers' funding condition (part (ii)). The excess sensitivity of "risky" securities to the funding of dealers captures the notion of flight to quality/flight to liquidity. In our model, this phenomenon arises because dealers need a higher compensation for providing liquidity in high-margins securities when capital is tight. Empirically, a relation between market liquidity and volatility has been documented by Benston and Hagerman (1974) and Amihud and Mendelson (1989), and flight to liquidity has been documented by Acharya and Pedersen (2005).

Numerical Example

To see the intuition for commonality in liquidity and flight to quality, we extend our numerical example of Section 3 to a case with two assets that differ in their fundamental volatility. The fundamental volatility of asset 1 is $\sigma^1 = 1$, while asset 2 has fundamental volatility $\sigma^2 = 2$.

To focus on commonality and flight to quality while abstracting from fragility and liquidity spirals, we assume that the the dealers' initial position is $x_0^1 = x_0^2 = 0$ and that the margin is solely governed by fundamental volatility $m(\sigma, \Lambda) = \sigma$. This ensures that the equilibrium is unique in our example. The initial customers' have a supply curve for each asset j of $S\left(z^j, \Lambda\right) = z^j - 2\Lambda$, where the endowment shocks are $z^1 = z^2 = 20$.

Figure 5 depicts the assets' market illiquidity for different funding levels B. For any given funding level, Λ^2 is always above Λ^1. That is, the high-fundamental-volatility asset 2 is always less liquid than the low-fundamental-volatility asset 1. This observation corresponds to our result that relates fundamental volatility to market liquidity ("Quality=Liquidity.").

The graph also illustrates our result on "Flight to Quality." To see this, let us look at the relative sensitivity of Λ with respect to changes in B: For funding levels above 60, market liquidity is perfect for both assets, i.e. $\Lambda^1 = \Lambda^2 = 0$. In this high range

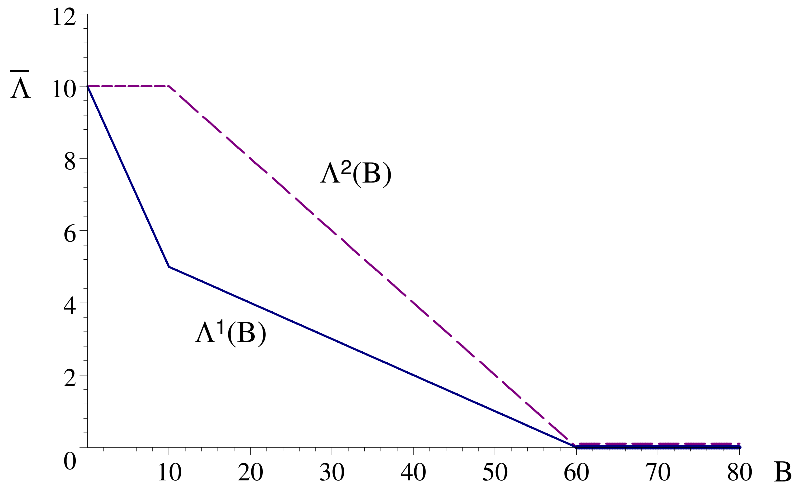

Figure 5: Commonality in Liquidity and Flight to Liquidity. The figure plots that market illiquidity Λ^j of assets 1 and 2 as functions of dealer wealth B. Asset 1 has lower fundamental risk than asset 2, $\sigma^1 = 1 < 2 = \sigma^2$.

of B, market liquidity is insensitive to marginal changes in funding. As funding falls below 60, market illiquidity of both asset increases since dealers must take smaller stakes in both asset. Importantly, as B decreases, $\Lambda^2(B)$ increases more steeply than $\Lambda^1(B)$, that is, asset 2 is more sensitive to funding declines than asset 1. This is because dealers cut back more on the "funding intensive" asset 2 with high margin requirement. The dealers want to maximize their profit per dollar margin, Λ^j/m^j and therefore Λ^2 must be twice as high as Λ^1 to compensate dealers for using twice as much capital for margin.

As funding B declines below 10, dealers put all their funds only into asset 1 and initial customers cannot sell any of their supply shock of $z^2 = 20$. Hence, the market illiquidity of asset 2 is at its worst $\Lambda^2 = \bar{\Lambda} = 10$. As funding drops further below 10, the market illiquidity of asset 1 naturally increases further. Hence, in this range the assets' liquidity differential narrows, but this effect is, of course, solely driven by the fact that asset 1 is maximally illiquid.

Figure 5 also depicts our funding liquidity measure ϕ, the marginal value of an extra dollar of funding. Recall that, as long as dealers are the marginal investors in asset j, the shadow cost of funding is equal to the expected profit per capital usage, $\phi = \frac{\Lambda^j}{m^j}$. Since dealers are marginal in asset 1 for any B and $m^1 = 1$, funding illiquidity in this case coincide with the market illiquidity of asset 1, $\phi(B) = \Lambda^1(B)$ for all $B \geq 0$.

This graph makes clear that funding B (or, equivalently, the shadow cost of funding ϕ) is a common factor that drives the market illiquidity of both assets. This naturally implies a commonality of market liquidity across asset 1 and asset 2 as well as a positive covariance between funding illiquidity and market illiquidity for random B.

5 On Margin Setting

Our analysis so far shows how phenomena such as fragility of market liquidity and liquidity spirals depend on the nature of the margin requirements $m(\sigma, \Lambda)$. Hence, it is important to determine the circumstances under which margin rules fuel liquidity crisis by requiring more capital as prices diverge, and the circumstances under which margin requirements are stabilizing.

As discussed in Section 2, margins are set using a value-at-risk approach.[7] Hence, at any time t, a financier will loan an amount l_t to a dealer financing a security with price p_t, where l_t is determined using a value-at-risk with a confidence of $\pi \in [0, 1]$:

$$Pr(p_{t+1} < l_t) = \pi \tag{18}$$

Since the margin for a long position is $m_t = p_t - l_t$, the margin requirement is determined equivalently by

$$Pr\left(-(p_{t+1} - p_t) > m_t\right) = \pi \tag{19}$$

Using that the price can be decomposed as $p_t = v_t - \Lambda_t$ (where $v_t = E_t[v]$), we can also write

$$Pr\left(-(v_{t+1} - v_t) + (\Lambda_{t+1} - \Lambda_t) > m_t\right) = \pi \tag{20}$$

An extreme case is risk-free financing, that is, a loan value equal to the lowest possible value of $E_2[v]$. This is captured by (19) with $\pi = 0$. In this case, the margin is determined by

$$\begin{aligned} m_t &= p_t - \min p_{t+1} \\ &= (v_t - \min v_{t+1}) + (\max \Lambda_{t+1} - \Lambda_t) \end{aligned} \tag{21}$$

Intuitively, (20) and (21) show that the margin is the sum of the fundamental risk (i.e. changes in v_t) and the market liquidity risk (i.e. changes in Λ_t).

While this general result on margin setting provides insight, it does not alone determine whether margins are stabilizing or destabilizing. Interestingly, the margin can have either property depending on the risks in the economy and on the financier's information set as we illustrate in the following sections.

[7] The value at risk approach can be seen as the outcome of credit rationing due to adverse selection and moral hazard in the lending market as described by Stiglitz and Weiss (1981). Also, Geanakoplos (2003) considers endogenous contracts in a general-equilibrium framework.

5.1 Stabilizing Margins: the Cushioning Effect

We first show that the margin m can be decreasing in market illiquidity if the financiers can perfectly distinguish between a permanent fundamental shock and temporary selling pressure and can perfectly anticipate future market liquidity.

To make this point we consider the 3-period model of Section 3 and assume that the financier knows the expected fundamental value $E_1[v]$ at time 1 when the dealer puts on his trade. Hence, the financier knows that the price is depressed because of a temporary demand shock and that the market will become perfectly liquid at time 2 such that price at time 2 will be the fundamental value $p_2 = E_2[v]$.

Under these assumptions, at time 1, the financier will loan a dealer an amount l that depends on the time-2 conditional distribution of the fundamental. In particular, since the dealer uses a value-at-risk with a confidence level of π, the loan value l is given by

$$Pr(E_2[v] < l) = \pi. \tag{22}$$

Importantly, this loan value does not depend on current market liquidity Λ. The margin of the dealer is the difference between his purchase price $p = E_1(v) - \Lambda$ and the loan value l, that is,

$$m = E_1[v] - l - \Lambda. \tag{23}$$

Hence, we have

Proposition 5 *If the financier knows the fundamental value and knows that next period's market liquidity is perfect, $\Lambda_{t+1} = 0$, then the current margin m_t is linearly decreasing in current market illiquidity Λ_t.*

The proof follows directly from equation (23). The intuition for this simple result is clear: current market illiquidity pushes the price away from its fundamental which makes the trade more profitable for the dealer and, importantly, less risky since the price is known to "bounce back" to its fundamental value. Said differently, the current illiquidity discount in the price provides a "cushion" against future fundamental risk.

Obviously, this result relies on the strong assumptions that the financier can perfectly distinguish between permanent and transitory price movements and that the financier knows that the market becomes liquid next period. Next, we relax these assumptions.

5.2 Constant Margins

In the real world, dealers and financiers do not know exactly when a trade will converge, that is, when the complementary customers will arrive. This uncertainty implies that a depressed price does not necessarily imply that the financier's losses are cushioned since the price could be even more depressed in the future. This risk can offset the cushioning effect and imply a constant margin requirement.

To capture this effect, we extend our model to an infinite horizon economy. The initial customers arrive at time 1 as before, but the complementary customers now arrive randomly. Specifically, if the complementary customers has not arrived prior to time t, then they arrive at time t with constant probability α, where $\alpha \in (0, 1)$.[8] After complementary customers arrive, the market becomes liquid, that is, $\Lambda_t = 0$.

We assume that dealers can hedge the fundamental risk away. More specifically, there are two "mirror" assets with perfectly negatively correlated fundamentals, $\Delta v_t^1 = -\Delta v_t^2$. The fundamental of security $i = 1, 2$ evolves as a random walk with i.i.d. increments, that is, the distribution of Δv_t^i is constant over time.[9]

The initial customers have a supply of security $i = 1, 2$ of $\bar{S}(z^i, \Lambda_t^i, \mathcal{L}(\Lambda_{t+1}^i))$ depending on their identical supply shock $z^1 = z^2 = z$, the current market illiquidity Λ_t^i and the distribution of the future market illiquidity $\mathcal{L}(\Lambda_{t+1}^i)$. Since in a stationary equilibrium $\Lambda_t^i = \Lambda^i$ for all t before the complementary customers arrive, we can write the supply simply as a function of the constant market illiquidity Λ^i, $\bar{S} = S(z^i, \Lambda^i)$, where S has the natural properties given in Section 3. When complementary customers arrive, the supply and demand of both groups of customers cancel such that the price is $p_t = v_t$.

In the stationary equilibrium of this economy, the dealers hold the same long position in both securities, and the illiquidity either vanishes (i.e. $\Lambda_{t+1} = 0$) in the next period or the dealers face the same situation as in the current period (i.e. $\Lambda_{t+1} = \Lambda_t$). The margin m_t at any time t prior to the arrival of the complementary customers is set based on the following condition:

$$\begin{aligned}
\pi &= Pr(-\Delta p_{t+1} > m_t) && (24)\\
&= Pr(-\Delta p_{t+1} > m_t \mid \Lambda_{t+1} = 0)\alpha + Pr(-\Delta p_{t+1} > m_t \mid \Lambda_{t+1} > 0)(1 - \alpha) && (25)\\
&= Pr(-\Delta v_{t+1} > m_t + \Lambda_t)\alpha + Pr(-\Delta v_{t+1} > m_t)(1 - \alpha) && (26)
\end{aligned}$$

In other words, the financier's value at risk is the mixture of the risk if the complementary customers arrive next period or not. Clearly, the riskier scenario in the one in which the complementary customers do not arrive. Indeed, $Pr(-\Delta v_{t+1} > m_t + \Lambda_t)$ is small relative to $Pr(-\Delta v_{t+1} > m_t)$ for large Λ_t. Therefore the margin largely depends on the fundamental risk of the security. This is because the risk that the complementary customers do not arrive next period "switches off" the cushioning effect described in Section 5.1. We can make this statement precise, for instance, in the case of the risk-free financing $\pi = 0$ (which makes most sense in the case of a bounded support of the fundamental risk) as described in the following proposition:

[8]The model with known arrival of the complementary customers is a special case of this model with $\alpha = 1$.

[9]Gromb and Vayanos (2002) consider a similar model with two perfectly negatively correlated securities. Their model is a finite horizon one and the cushioning effect partially remains.

Proposition 6 *If the fundamental values of securities 1 and 2 have bounded support $\Delta v_t^1 = -\Delta v_t^2 \in [-\sigma, \sigma]$, $z^1 = z^2$, and the financier uses a VaR with $\pi = 0$, then there is a stationary equilibrium with a margin $m = \sigma$ that does not depend on market liquidity. In this equilibrium, market illiquidity is decreasing in dealer wealth, and the dealer's marginal value of a dollar is $1 + \frac{\Lambda}{\sigma}$.*

These results are intuitive: The margin is constant because complementary customers might not arrive next period, implying an equally distorted price, and because fundamental risk $var_t(\Delta v_{t+1})$ is constant over time.

Further, the dealer's marginal value of a dollar before the complementary customers arrive is the value of the dollar itself, namely 1, plus the expected profit of Λ times the maximum leverage ratio of $1/\sigma$. Since Λ is higher if wealth is lower, so is the marginal value of a dollar. This marginal value can be seen as the funding liquidity.

This dynamic model can be solved in closed form if we parameterize the supply function as $\bar{S} = z - \frac{1}{\gamma} E_t [\Lambda_t - \Lambda_{t+1}]$, where the constant $\gamma > 0$ can be thought off as the holding cost of customers. In a stationary equilibrium we have $S = z - \frac{\alpha}{\gamma}\Lambda$, which implies

$$\Lambda = \frac{\gamma}{\alpha}\left(z - \frac{W_0}{2\sigma}\right). \tag{27}$$

Hence, in equilibrium the dealer's marginal value of a dollar is

$$1 + \frac{\gamma}{\alpha\sigma}\left(z - \frac{W_0}{2\sigma}\right). \tag{28}$$

In the setting of Proposition 6 liquidity is constant. One way to make margins dependent on the risk that future market liquidity might worsen is to relax the assumption that both assets are perfectly negative correlated. We can do so by assuming that the perfect hedge of both assets breaks at random points in time.[10] Such "hedge-breaks" can lead to an erosion of dealers' wealth which affects their future liquidity provision. This additional risk induces financiers to set initially higher margins compared to a setting without hedge breaks. The model with hedge-breaks is outlined in the appendix.

Aside: Constant Proportional Margins. In the real world, the margin is sometimes a constant proportion of market value (rather than a constant dollar amount per share), that is, $m_t = fp_t$ for some constant $f \in (0, 1)$. For instance, $f = 0.5$ for stocks because of Regulation T as discussed in Section 2. This margin requirement can arise because of regulation or because the fundamental risk is proportional to value as in a geometric Brownian motion. Interestingly, a constant proportional margin has a

[10] Alternatively, one could also introduce additional supply shocks by customers.

stabilizing effect for long positions and a destabilizing effect for short positions. To see that, note that for a long position, a constant proportional margin implies that

$$m_t = fp_t = fE_t[v] - f\Lambda_t \tag{29}$$

which decreases in Λ_t, that is, a depression of the price lowers the margin, which enables dealers to buy more shares and stabilize the price.

When initial customers want to buy — not sell as we usually consider for ease of exposition — then the price is temporarily increased, $p_t = E_t(v) + \Lambda_t$, and the dealers will take short positions. With constant proportional margins on a short position, we have

$$m_t = fp_t = fE_[t](v) + f\Lambda_t \tag{30}$$

which *increases* in the market illiquidity Λ_t. Hence, an increase of the price raises the margin, thus making it harder for dealers shortsell and stabilize the price.

Further, with constant proportional margin, the dollar margin on a long position goes down when the position loses, while the dollar margin on a short position goes up when the position loses. This asymmetry between the implications of constant proportional margin for, respectively, long and short position implies that dealers (and other traders) will be more reluctant to take short positions than long ones.

5.3 Destabilizing Margins

In the real world, the financier of a trader often has less information about the trade than the trader does. Hence, the financier may worry, for example, that a price drop is due to a fundamental shock rather than a temporary demand effect. This information disadvantage of the financier is worst in markets in which the trading activity is very specialized. Indeed, if the financier fully understood the dealer's trade, then the financier could do the trade himself rather than just finance it.

To capture this real-world problem, we consider the benchmark 3-period model with the twist that initial customers may or may not arrive at time 1, and only dealers know whether these customers arrive. The ex-ante price at time zero is equal to the fundamental $p_0 = v_0$. At time 1, if initial customers arrive, the price is $p = v_1 - \Lambda$; otherwise, the price is simply the fundamental $p = v_1$. While the dealers know whether the initial customers arrived, their financiers do not.

We further assume that fundamental volatility has an autoregressive conditional heteroskedasticity (ARCH) structure. Specifically, the conditional expected value of the final payoff $v_t = E_t[v]$ evolves according to

$$v_{t+1} = v_t + u_{t+1} = v_t + \sigma_{t+1}\varepsilon_{t+1} \tag{31}$$

where ε_t is i.i.d. standard normal and the volatility σ_t has dynamics

$$\sigma_{t+1}^2 = (1-\theta)\bar{\sigma}^2 + \theta u_t^2 \tag{32}$$

where $\bar{\sigma}^2 = E[\sigma_t^2]$ is the mean variance and $\theta \in (0,1)$. A positive θ means that shocks to fundamentals also increase future volatility.

The financier sets the margin based on a value-at-risk calculation conditional on his information, which is the observed prices at times 0 and 1, p_0 and $p_1 = p$. The financier does not know whether to attribute a price change $p - p_0$ to a fundamental shock u_1 alone, or to a combination of u_1 and a liquidity shock Λ. A fundamental shock is a permanent reduction in the value and leads to an increase in fundamental risk; a liquidity shock, on the other hand, is temporary in that the price will bounce back and has no effect on fundamental risk:

$$\begin{aligned}
\pi &= Pr(p_2 - p < -m \,|\, p) \\
&= Pr(p_2 - p < -m \,|\, p, z = 0) Pr(z = 0 \,|\, p) \\
&\quad + Pr(p_2 - p < -m \,|\, p, z \neq 0) Pr(z \neq 0 \,|\, p) \\
&= Pr(v_2 - v_1 < -m \,|\, v_1 = p) Pr(z = 0 \,|\, p) \\
&\quad + Pr(v_2 - v_1 < -m - \Lambda \,|\, v_1 = p + \Lambda) Pr(z \neq 0 \,|\, p)
\end{aligned}$$

The financier's value at risk largely derives from the risk that a price change was due to a u shock if $Pr(z = 0 \,|\, p)$ is not too small:

$$\pi \cong Pr(v_2 - v_1 < -m \,|\, v_1 = p) Pr(z = 0 \,|\, p) \tag{33}$$

$$= \Phi\left(-\frac{m}{E(\sigma_2 | u_1 = p - p_0)}\right) Pr(z = 0 \,|\, p) \tag{34}$$

where Φ is the normal cumulative distribution function. Hence,

$$m \cong E(\sigma_2 | u_1 = p - p_0)\, \Phi^{-1}\left(1 - \frac{\pi}{Pr(z = 0 \,|\, p)}\right)$$

If liquidity shock occurs such that $p = v_1 - \Lambda$ then

$$E(\sigma_2 | u_1 = p - p_0) = \sqrt{(1-\theta)\bar{\sigma}^2 + \theta(p - p_0)^2} \tag{35}$$

$$= \sqrt{(1-\theta)\bar{\sigma}^2 + \theta(v_1 - \Lambda - v_0)^2} \tag{36}$$

$$= \sqrt{(1-\theta)\bar{\sigma}^2 + \theta(\Lambda - u_1)^2} \tag{37}$$

Hence, the financiers' expected fundamental volatility is increasing in market illiquidity Λ if $\Lambda - u_1 > 0$. Therefore, the margin

$$m \cong \sqrt{(1-\theta)\bar{\sigma}^2 + \theta(\Lambda - u_1)^2}\; \Phi^{-1}\left(1 - \frac{\pi}{Pr(z = 0 \,|\, p)}\right)$$

is increasing in Λ if $Pr(z = 0 \,|\, p)$ is relatively insensitive to $p - p_0$.

Proposition 7 *If the financier does not know whether a demand shock has occurred, fundamental volatility is time-varying $\theta > 0$, and $Pr(z = 0)$ is close to 1, then the margin is increasing in market illiquidity Λ for a nontrivial set of fundamental shocks u_1.*

Intuitively, market illiquidity Λ combined with a negative fundamental shock $u_1 < 0$ leads to a price drop which worsens the financier's VaR scenario and hence increases the margin. This is because the financier fear that the price drop could be due to a fundamental shock, implying that the price will not bounce back and volatility will increase.

We note that increasing margins can arise simply because financiers compute the value at risk using a volatility estimate based on recent returns. In this case, a liquidity shock associated with a large price movement will lead to an increase of margin requirements. Indeed, some brokers and futures exchanges (e.g. EUREX) set margins based on value at risk calculated using past data and typically use forward looking information (e.g. like option-implied volatility) only to further increase margin requirements.[11] In light of our asymmetric information framework, one can view this behavior as a rule-of-thumb that protect a financier who cannot determine whether shocks were due to fundamental shocks or liquidity shock and cannot perform the full Bayesian updating in real time.

6 Related Literature

In this section, we review how our paper links the various literatures that touches on market liquidity and funding liquidity.

The *market liquidity* literature shows that a security can be costly to trade — that is, has less than perfect market liquidity — because of exogenous order processing costs, private information (Kyle (1985) and Glosten and Milgrom (1985)), inventory risk of market makers (e.g. Stoll (1978), Ho and Stoll (1981,1983) Ho and Stoll (1981) and Grossman and Miller (1988)), search frictions (Duffie, Gârleanu, and Pedersen (2003, 2003a)), or predatory trading (Brunnermeier and Pedersen (2005)). This literature assumes that intermediaries have no funding constraints, with a few exceptions. Attari, Mello, and Ruckes (2005) and Brunnermeier and Pedersen (2005) consider the strategic interactions of large traders who may face funding problems and market-liquidity reducing predation. Weill (2004) considers a capital limit for market makers. Grossman and Vila (1992) study the optimal trading with leverage constraints. Our fragility results are related to Gennotte and Leland (1990) which focusses on portfolio-insurance traders and asymmetric information. Our result on multiplicity due to dealer losses is

[11]We thank Markus Konz from the futures exchange EUREX for describing how margin requirements are set.

similar to Chowdhry and Nanda (1998). None of these papers consider margin-induced multiplicity and multiplier effects, nor do they explain the differential market liquidity changes during "flight to liquidity" events.[12] Our model of market liquidity is similar to Grossman and Miller (1988) with the additional funding constraint. We complement this literature by endogenizing the funding of intermediaries, including the determination of margins, and linking this funding problem to a unified explanation for the time variation and cross-sectional variation of market liquidity.

Empirically, Chordia, Roll, and Subrahmanyam (2000), Hasbrouck and Seppi (2001) and Huberman and Halka (2001) document that there is commonality of stocks' market liquidity, that is, market liquidity is correlated across stocks.[13] Our model shows that this commonality in market liquidity can be driven by the underlying funding liquidity of the market making sector. Acharya and Pedersen (2005) find that when aggregate market liquidity falls, it falls primarily for illiquid assets — a notion often termed "flight to liquidity." In our model, flight to liquidity arises when dealers have funding problems. This is because dealers prefer to invest their limited capital in liquid markets with small margins/haircuts. Chordia, Sarkar, and Subrahmanyam (2005) show that increases in volatility lead to reductions in market liquidity of bonds and stocks.

The second liquidity concept, *funding liquidity*, is studied in the corporate finance literature. This literature explains how funding liquidity problems arise because of agency problems combined with contract and market incompleteness (see e.g. Hart (1995) and references therein). Similarly, in our model, agents can only collateralize part of the value of an asset, that is, there are positive haircuts.

The banking literature emphasizes funding liquidity shocks of banks due to early withdrawals by their customers. The role of banks is to insure across customers' time preference shocks by investing in projects and assets with the right maturity structure. However, as Bryant (1980) and Diamond and Dybvig (1983) show, banks are subject to bank-runs if they offer demand deposit contracts (and markets are incomplete). In the case of a bank-run, they are forced to liquidate long-run investment projects at a low liquidation value. Most of the banking literature follows Diamond and Dybvig (1983) in assuming an exogenous liquidation technology — that is, market liquidity is not endogenized. In recent work, however, Allen and Gale consider a banking model in which the sell-off of the bank's long-run asset may depress its liquidation value, and analyze the implications for bank runs (Allen and Gale (1998)), fragility of equilibria (Allen and Gale (2005)), and constrained Pareto efficiency (Allen and Gale (2004)). Similarly,

[12]In Vayanos (2004) liquidity *premia* are time varying because costly withdrawal of funds are more likely during volatile times. Liquidity is defined as a constant trading cost so liquidity does not change during volatile times in that model.

[13]Market liquidity also affects the overall price level. Amihud and Mendelson (1986) show that assets with a low level of liquidity have on average a higher return. Pastor and Stambaugh (2003) and Acharya and Pedersen (2005) document that aggregate time variation in market liquidity appears to be priced.

Shleifer and Vishny (1992) show, within a corporate finance context, that a funding crisis can lead to "fire sales" of machines since potential buyers of these industry-specific machines also face similar funding problems at the same time. Our model applies the corporate finance insights of Shleifer and Vishny (1992) and Allen and Gale to study endogenous market liquidity when the market making sector faces margins constraints of different forms. In addition to highlighting the effect of destabilizing margin requirements, we consider a multi-asset economy and derive cross-sectional asset-pricing implications. Holmström and Tirole's (1998, 2001) research focuses primarily on funding liquidity. They show that corporations with agency problems have a preference for government bonds because they provide a cushion for future funding liquidity problems. Hence, government bonds trade at a premium.

Our paper is also related to parts of the literature on the "*limits to arbitrage*."[14] Shleifer and Vishny (1997) show, among other things, that a demand shock can be amplified if losses lead to withdrawal of capital from fund managers. We show that a similar effect can arise due to leverage and document how the multiplier is exacerbated by the degree of leverage (Proposition 2) and that this funding effect can lead to fragility (Proposition 1). Liu and Longstaff (2004) derive the optimal dynamic arbitrage strategy under funding constraints in a setting with exogenous price process. Gromb and Vayanos (2002) derive welfare results in a model in which arbitrageurs face margin constraints similar to those in our model. Unlike in our paper, they focus on welfare, while we focus on fragility, liquidity spirals and flight to quality. Our commonality results can also be related to certain work on contagion (see e.g. Allen and Gale (2000), Kyle and Xiong (2001), and Brunnermeier and Pedersen (2005)).

Our paper also touches on the literature that studies the *amplification mechanism* over the business cycle due to agency costs. This literature was initiated by Bernanke and Gertler (1989) who show that higher net wealth during booms reduces agency costs, while the lower net wealth during recessions leads to higher agency costs and tighter collateral constraints (see also Eisfeldt (2004)). Kiyotaki and Moore (1997) show in a deterministic model how a temporary productivity shock can lead to a dynamic multiplier. This happens because the shock lowers the net worth of "farmers," thus constraining their production, which reduces their future net worth, and so on. This reduction in the farmers' current and future demand for the asset reduces its price. Krishnamurthy (2003) points out that incomplete hedging possibilities are crucial for this amplification result. We consider a model with uncertainty and multiple assets, which enables us to study the interaction between risk and credit constraints, and cross-sectional implications such as flight to quality.

Finally, Geanakoplos (1997) derives endogenous contracts due to collateral requirements in a model of general-equilibrium with incomplete markets (*GEI*). Lustig (2004) considers the collateral effect on the equity premium, and Lustig and Nieuwerburgh

[14]See also DeLong, Shleifer, Summers, and Waldmann (1990) and Abreu and Brunnermeier (2002).

(2005) perform an empirical study using housing as the collateral asset. Geanakoplos (2003) relates liquidity to the GEI literature using an illustrative example. In his paper agents trade because of differences of opinions and margins are set using an endogenous contract. This contract turns out to be similar to our risk free VaR constraint. Since he has a finite horizon model in which all agents are perfectly informed, margins are decreasing in illiquidity due to the cushioning effect described in our Section 5.1.[15] Our paper extends these ideas by showing how fragility and liquidity spirals can arise due to destabilizing margin requirements, by considering how such margin requirements can arise due to asymmetric information between the financiers and the dealers, by modeling market liquidity explicitly, and by showing how flight to quality can arise endogenously.

7 Conclusion and Central Bank Policy

One central message of this paper is that dealers' funding drives many empirically observed market liquidity phenomena. Dealers' funding, in turn, depend on their capital and the terms of their financing, that is, the margin that they face. Fragility and liquidity spirals arise if margin requirements are destabilizing or if dealers' existing positions are correlated with customers' demand shock as described in Section 3.

Margin requirements can be destabilizing if financiers do not have the information to judge whether a price move is due to fundamental news or temporary price pressure (Section 5.3). Hence, a central bank concerned about market liquidity during a temporary market displacement can try to signal to financiers that a price drop is a liquidity event, not a permanent reduction of fundamentals. If this signal is credible, margins will be reduced and market liquidity will recover.

More generally, our analysis shows how a central bank can indirectly affect market liquidity by boosting the funding of dealers. In the US, the Fed only lends to commercial banks, but sometimes the Fed "asks" commercial banks to extend credit to securities firms. Hence, the Fed can improve the dealers' funding by ensuring that they get loans (i.e. increase W) or by ensuring that they get reduced margins and haircuts.

The Federal Reserve Bank of New York (FRBNY) did just that during the 1987 crash:

> "calls were placed by high ranking officials of the FRBNY to senior management of the major NYC banks, indicating that ... they should encourage their Wall Street lending groups to use additional liquidity being

[15]The "crash" derived in his three period example relies on the assumption that default is impossible during the first period, but possible in the second period. Hence, the margin increases from first to second period since fundamental volatility increases substantially. This is different from a setting in which margin increase with market illiquidity.

supplied by the FRBNY to support the securities community"
— SEC (1988), page 5-25

Moreover, the central bank can improve current liquidity by stating an intention to fund dealers if needed in the future, without actually taking immediate action. The Federal Reserve also used this implicit channel to further improve liquidity in 1987.

> "the statement issued by the Chairman of FRB indicating that the FRB would be ready 'to serve as a source of liquidity to support the economy and financial system' was considered significant."
> — SEC (1988), page 5-27

Note that margins and haircuts depend on the feared future liquidity meltdowns. Hence, if the central bank can credibly signal that it will improve funding in a crisis, then the market expects reduced liquidity shortage. This, in turn, reduces current margins and haircuts, thereby boosting current funding and market liquidity.

A Proofs

Proof of Proposition 1. We consider the dealer's funding constraint with the equilibrium demand $x = S(z,\Lambda)$

$$S(z,\Lambda)m(\sigma,\Lambda) + x_0\Lambda \leq B + x_0E_1[v]. \tag{A1}$$

Inequality (A1) can be written as

$$f(\Lambda) \leq b, \tag{A2}$$

where $b := B + x_0E_1[v]$ and

$$f(\Lambda) := S(z,\Lambda)m(\sigma,\Lambda) + x_0\Lambda. \tag{A3}$$

A stable equilibrium is either (1) $\Lambda = 0$ if $f(0) \leq b$; (2) a $\Lambda > 0$ with $f(\Lambda) = b$ and $f'(\Lambda) < 0$; or (3) $\Lambda = \bar{\Lambda}(z)$ if $f(\bar{\Lambda}(z)) \geq b$.

(i) For dealer wealth levels b with $b > f(0) = S(z,0)m(\sigma,0) - x_0E_1(v)$ it is a stable equilibrium that $\Lambda = 0$. For $b \in [f(\bar{\Lambda}(z)), f(0))$, the dealer's funding constraint is binding and Λ is the unique solution to

$$f(\Lambda) = b \tag{A4}$$

in the interval $[0, \bar{\Lambda}(z)]$. For b smaller than $f(\bar{\Lambda}(z))$, the dealer is in default and the unique equilibrium is $\Lambda = \bar{\Lambda}(z)$.

Clearly, $\Lambda^*(b)$ is continuously decreasing (e.g. using the implicit function theorem).

(ii) Suppose first that $\exists\Lambda'$ such that f is decreasing on $[0,\Lambda']$ and increasing on $[\Lambda', \bar{\Lambda}(z)]$. Then, as above, for $b > f(0)$, $\Lambda = 0$ is a stable equilibrium and for $b \in [f(\Lambda'), f(0)]$, the dealer's funding constraint is binding and Λ is the unique solution to equation (A4) in the interval $[0,\Lambda']$. Solutions in $[\Lambda', \bar{\Lambda}(z)]$ are instable since f' is positive. For $b < f(\Lambda')$, the dealer is in default and $\Lambda = \bar{\Lambda}(z)$. Hence, there is a unique stable equilibrium for all values of b and the equilibrium is fragile around $b = f(\Lambda')$ because market liquidity jumps from Λ' to $\bar{\Lambda}(z)$.

Suppose next that f is increasing for $\Lambda \in [0, \Lambda_1]$ and decreasing for $\Lambda \in [\Lambda_1, \Lambda_2]$, where $0 < \Lambda_1 < \Lambda_2 \leq \bar{\Lambda}(z)$. Then, for $b \in (\max\{f(0), f(\Lambda_2)\}, f(\Lambda_1))$ there are at least two stable equilibria: First, $\Lambda = 0$ is an equilibrium because $b > f(0)$. Second, since $b \in (f(\Lambda_2), f(\Lambda_1))$ there exists Λ such that $f(\Lambda) = b$ and $f'(\Lambda) < 0$.

Finally, the remaining case is that f is decreasing for $\Lambda \in [0, \Lambda_1]$, increasing for $\Lambda \in [\Lambda_1, \Lambda_2]$, and decreasing for $\Lambda \in [\Lambda_2, \Lambda_3]$, where $0 < \Lambda_1 < \Lambda_2 < \Lambda_3 \leq \bar{\Lambda}(z)$. There are two sub-cases:

(a) $f(\Lambda_3) > f(0)$: then there are at least two stable equilibria for $b \in (f(\Lambda_3), f(\Lambda_2))$: $\Lambda = 0$ (because $b > f(0)$) and $\Lambda \in [\Lambda_2, \Lambda_3]$ with $f(\Lambda) = b$.

(b) $f(\Lambda_3) < f(0)$: then there are at least two stable equilibria for

$$b \in (\,\max\{f(\Lambda_1), f(\Lambda_3)\}\,,\, \min\{f(0), f(\Lambda_2)\}\,)$$

namely the solutions to $f(\Lambda) = b$ in each of the intervals $[0, \Lambda_1]$ and $[\Lambda_2, \Lambda_3]$.

We need to show that $\Lambda^*(b)$ can be chosen decreasing and cannot be chosen continuous. For the former, let $\Lambda^*(b) = 0$ for $b > f(0)$ and, $\Lambda^*(b) = \min\{\Lambda : f(\Lambda) = b, f'(\Lambda) \leq 0\}$ for $b \leq f(0)$. If the minimum is over the empty set then let $\Lambda^* = \bar{\Lambda}$. To see that $\Lambda^*(b)$ is decreasing, consider $b_1 < b_2 < f(0)$. Then, since f is continuous there must be $\Lambda \in (0, \Lambda^*(b_1)\,)$ such that $f(\Lambda) = b_2$ and $f'(\Lambda) \leq 0$ and, since Λ^* is the smallest such value, $\Lambda^*(b_2) < \Lambda^*(b_1)$.

To see that Λ^* cannot be chosen continuous, note that there must be discontinuities when f turns from decreasing to increasing. ■

Proof of Proposition 2. When the funding constraint binds, we use the implicit function theorem to compute the derivatives. For this, we totally differentiate the funding constraint

$$\frac{\partial m}{\partial \sigma} S d\sigma + \left\{ \frac{\partial S}{\partial \Lambda} m + \frac{\partial m}{\partial \Lambda} S + x_0 \right\} d\Lambda + \frac{\partial S}{\partial z} m dz = x_0 dE_1[v] + dB. \tag{A5}$$

We have that $-\frac{\partial S}{\partial \Lambda} m - \frac{\partial m}{\partial \Lambda} S - x_0 > 0$ because $f' < 0$ in a stable equilibrium, where f is defined in (A3). ■

Proof of Proposition 3. Since $B, z^1, \ldots, z^J, E_0[v^1], \ldots, E_0[v^J]$ are random, the equilibrium shadow cost of capital ϕ is random. Further, for each $i = j, k$, $\Lambda^i(\phi)$ is increasing in ϕ since, when the dealer is marginal, $\Lambda^i / m(\sigma^i, \Lambda^i) = \phi$ implies that

$$\frac{d\Lambda^i}{d\phi} = \frac{m^i}{1 - \phi \partial m^i / \partial \Lambda^i} > 0 \tag{A6}$$

and, when the dealer is not investing in asset i, $\frac{\partial \Lambda^i}{\partial \phi} = 0$.

Therefore, $Cov\left[\Lambda^j(\phi), \Lambda^k(\phi)\right] \geq 0$ since any two functions which are both increasing in the same random variable are positively correlated. ■

Proof of Proposition 4. (i) If $z^j = z^k$, then it can be seen that if $x^k = 0$ then $x^j = 0$ and hence each security is equally illiquid. If $x^j = 0$ and $x^k > 0$ then clearly $\Lambda^j \geq \Lambda^k$. Finally, if $x^j > 0$, then $x^k > 0$. When $x^j, x^k > 0$ then — whether or not $z^j = z^k$ — we can view Λ^m, $m = j, k$, as an implicit function of σ^m using $\Lambda^m / m(\sigma^m, \Lambda^m) = \phi$ for fixed ϕ. This yields that

$$\left.\frac{\partial \Lambda^m}{\partial \sigma^m}\right|_{\text{fixed } \phi} = \frac{\phi \partial m / \partial \sigma}{1 - \phi \partial m / \partial \Lambda} > 0. \tag{A7}$$

Hence, $\Lambda^j \geq \Lambda^k$.

(ii) For $m = j, k$ we have

$$\left|\frac{\partial \Lambda^m}{\partial B}\right| = \frac{\partial \Lambda^m}{\partial \phi}\left|\frac{\partial \phi}{\partial B}\right|, \tag{A8}$$

$$= \frac{m^m}{1 - \phi \partial m^m / \partial \Lambda^m}\left|\frac{\partial \phi}{\partial B}\right|. \tag{A9}$$

The result $\left|\frac{\partial \Lambda^j}{\partial B}\right| > \left|\frac{\partial \Lambda^k}{\partial B}\right|$ now follows from the fact that the derivative of $\frac{m^m}{1 - \phi \partial m^m / \partial \Lambda}$ with respect to σ, for fixed ϕ, is

$$\frac{\frac{dm}{d\sigma}\left(1 - \phi \frac{\partial m}{\partial \Lambda}\right) + m\phi\left(\frac{\partial^2 m}{\partial \Lambda^2}\frac{\partial \Lambda}{\partial \sigma} + \frac{\partial^2 m}{\partial \sigma \partial \Lambda}\right)}{(1 - \phi \frac{\partial m}{\partial \Lambda})^2}, \tag{A10}$$

which is positive for small $\partial^2 m / \partial \Lambda^2$ and $\partial^2 m / \partial \sigma \partial \Lambda$ because $\frac{dm}{d\sigma}\left(1 - \phi \frac{\partial m}{\partial \Lambda}\right) > 0$. ■

Proof of Proposition 6. In a stationary equilibrium, market illiquidity $\Lambda_t = \Lambda$ is constant until the complementary customers arrive. Hence, the margin $m_t = \sigma + \max \Lambda_{t+1} - \Lambda_t$ is simply equal to σ since market liquidity either collapses to zero or stays constant.

To solve the dealer's problem, we use dynamic programming and introduce his value function U. The dealers solves

$$U(W_t, A_t) = \max_{x_t} E_t\left[U(W_{t+1}, A_{t+1})\right] \tag{A11}$$

$$\text{s.t. } x_t' m_t \leq W_t, \tag{A12}$$

where A_{t+1} is the indicator that the complementary customers have arrived before time $t+1$, $W_{t+1} = W_t + x_t'(p_{t+1} - p_t)$, and we use vector notation for $x_t = (x_t^1, x_t^2)'$, Λ, p, and m. When the complementary customers arrive, the dealer's value function is simply his wealth, that is, $U(w, 1) = w$ is a boundary condition. Further, since the problem is linear in wealth, the value function before complementary customers arrive is $U(w, 1) = uw$, where $u > 1$ is a constant.

Using this, the dealer's problem before the complementary customers arrive, $A_t = 0$, can be written

$$\begin{aligned} uW_t &= U(W_t, 0) \\ &= \max_{x_t} \alpha E_t[W_t + x_t'(v_{t+1} - v_t + \Lambda)] + (1-\alpha)E_t u[W_t + x_t'(v_{t+1} - v_t)] \\ &= \max_{x_t} \ (\alpha + (1-\alpha)u)W_t + \alpha x_t'\Lambda, \end{aligned}$$

subject to

$$(x_t^1 + x_t^2)\sigma \leq W_t. \tag{A13}$$

We see that dealers can optimally choose

$$x_t^1 = x_t^2 = W_t/(2\sigma). \tag{A14}$$

(Other positions with the same value of $x_t^1 + x_t^2$ give the same expected utility, but are not consistent with this equilibrium.)

Further, the dealer's value function coefficient u can be solved as

$$u = 1 + \frac{\Lambda}{\sigma}. \tag{A15}$$

Equilibrium is characterized by market clearing $x_t = S_t$. That is, equilibrium market illiquidity Λ solves $W_0/(2\sigma) = \bar{S}(z, \Lambda_t, \mathcal{L}(\Lambda_{t+1})) = S(z, \Lambda)$. Further, using the implicit function theorem, we get

$$\frac{\partial \Lambda}{\partial W_0} = \frac{1}{2\sigma \frac{\partial S}{\partial \Lambda}} < 0, \tag{A16}$$

since $\frac{\partial S}{\partial \Lambda} < 0$ by assumption.

If $S = z - \frac{1}{\gamma}E_t\left[\Lambda_t - \Lambda_{t+1}\right]$ then in a stationary equilibrium we have $S = z - \frac{\alpha}{\gamma}\Lambda$, which implies

$$\Lambda = \frac{\gamma}{\alpha}\left(z - \frac{W_0}{2\sigma}\right). \tag{A17}$$

Hence, the equilibrium dealer value function coefficient is

$$u = 1 + \frac{\gamma}{\alpha\sigma}\left(z - \frac{W_0}{2\sigma}\right). \tag{A18}$$

■

Model with "hedge-break". In Proposition 6 we assumed that the two securities are perfectly negatively correlated, i.e. $\Delta v_t^1 = -\Delta v_t^2$. We now assume that this is only the case with probability $1-\varepsilon$, while with probability ε, they are distributed such that $\Delta v^j + \Delta v^{-j} \in (-2f\sigma, 2f\sigma)$, where f is some constant in $[0,1]$. After the securities move apart no further hedge breaks can occur, and, therefore, we are in the model with no liquidity risk characterized by Proposition 6. We denote all variables after the worst possible hedge break with a hat. We focus on the case in which ε goes to zero and supply and customers have a linear supply/demand $S = z - \frac{1}{\gamma}E_t\left[\Lambda_t - \Lambda_{t+1}\right]$.

Proposition 8 *If $\varepsilon = 0$ and $\pi = 0$, then the margin is*
(i) $m = \frac{1}{2}\left(\sigma + \sqrt{\sigma^2 + 2\gamma f W/\alpha}\right)$, $\Lambda > 0$, and, $\hat{\Lambda} > 0$ for $W/2 < z\sigma + z^2\gamma f/\alpha$;
(ii) $m = \frac{\alpha\sigma^2 + \gamma z f \sigma - \gamma W/2}{\alpha\sigma - \gamma z}$, $\Lambda = 0$ and $\hat{\Lambda} > 0$ for $z\sigma + z^2\gamma f/\alpha \leq W/2 \leq z\sigma(1+f)$;
(iii) $m = \hat{m} = \sigma$, $\Lambda = 0$ and $\hat{\Lambda} = 0$ for $W/2 \geq z\sigma(1+f)$.
After the hedge-break the equilibrium is described by Proposition 6. In particular, the margin $\hat{m} = \sigma$.

Proof. The maximum loss is given by two components: First there is the fundamental loss of $xf\sigma$, but there is an additional loss $x\left(\hat{\Lambda} - \Lambda\right)$, since the decline in illiquidity erodes the *marked to market value* of the position. Formally, the minimum wealth after the worst hedge break $\hat{W}$ is given by

$$\frac{\hat{W}}{2} = \frac{W}{2} - x\left(f\sigma + \hat{\Lambda} - \Lambda\right) \tag{A19}$$
$$= \frac{W}{2} - xm + (1-f)\,\sigma x. \tag{A20}$$

Linear supply $S = z - \frac{1}{\gamma}E_t\left[\Lambda_t - \Lambda_{t+1}\right]$ by initial customers and stationarity imply that $\Lambda = \frac{\gamma}{\alpha}(z - x)$ and $\hat{\Lambda} = \frac{\gamma}{\alpha}(z - \hat{x})$.

Range (i): $W/2 < z\sigma + z^2\gamma f/\alpha$
In this range funding is constrained before and after hedge-break. Hence, $\Lambda > 0$**,** $\hat{\Lambda} > 0$ and $W/2 = xm$ and $\hat{W}/2 = x\hat{m}$, where $\hat{m} = \sigma$ from Proposition 6.
Equation (A20) implies in this case that $\hat{x} = (1-f)\,x$.

$$\frac{W}{2} = x\left(\sigma + \hat{\Lambda} - \Lambda\right). \tag{A21}$$

Substituting in for Λ and $\hat{\Lambda}$ and solving for x yields

$$x = \frac{-\sigma + \sqrt{\sigma^2 + 2\gamma f W/\alpha}}{2\gamma f/\alpha} \text{ and} \tag{A22}$$
$$m = \frac{1}{2}\left(\sigma + \sqrt{\sigma^2 + 2\gamma f W/\alpha}\right). \tag{A23}$$

It follows directly that

$$\frac{\partial x}{\partial W} > 0,\ \frac{\partial x}{\partial \sigma} \leq 0,\ \frac{\partial x}{\partial \alpha} > 0, \text{ and} \frac{\partial x}{\partial f} < 0. \tag{A24}$$

Range (ii): $z\sigma + z^2\gamma f/\alpha \leq W/2 \leq z\sigma(1+f)$
In this range, funding is only constrained after hedge-break. Prior to hedge break there is full liquidity provision, i.e. $x = z$ and $\Lambda = 0$. Replacing $\hat{W}/2$ using equation (A20) in $\hat{W}/2 = \sigma\hat{x}$,

we obtain an equation for $\hat{x}$ as a function of m. Replacing m with $\sigma + \hat{\Lambda}$ and substituting the obtained equation of $\hat{x}$ into, $\hat{\Lambda} = \frac{\gamma}{\alpha}(z - \hat{x})$ leads to

$$\hat{\Lambda} = \frac{\gamma}{\alpha\sigma}\left[-\frac{W}{2} + z\,(m + f\sigma)\right], \tag{A25}$$

and

$$\begin{aligned} m &= \sigma + \frac{\gamma}{\alpha\sigma}\left[-\frac{W}{2} + z\,(m + f\sigma)\right] && \text{(A26)} \\ &= \frac{\alpha\sigma^2 + \gamma z f\sigma - \gamma W/2}{\alpha\sigma - \gamma z}. && \text{(A27)} \end{aligned}$$

Range (iii): $W/2 \geq z\sigma\,(1+f)$
Funding is never constrained in this range of W. Hence $\Lambda = 0$, $\hat{\Lambda} = 0$ and $m = \hat{m} = \sigma$. In addition, $\frac{\hat{W}}{2} = \frac{W}{2} - \sigma f z$ and $\frac{\hat{W}}{2} \geq z\sigma$. Consequently, $W/2 \geq z\sigma\,(1+f)$.

After putting all three ranges together one sees that m is a continuous function in wealth W. ■

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