Monetary Policy Under Uncertainty in Micro-Founded Macroeconometric Models

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Abstract

Over the past decade there has been remarkable progress in developing empirical micro-founded macroeconomic models for monetary policy analysis that feature coherence both to economic theory and to the data. In this paper, we estimate a second-generation micro-founded model of the US economy using Bayesian methods. We examine the characteristics of optimal monetary policy in the model, where the policy objective is the maximization of welfare of the representative household. Our results point to the central role of labor markets and wage-setting in affecting welfare and the design of monetary policy. We show that a parsimonious implementable rule targeting wage inflation closely mimics the outcomes of the fully optimal Ramsey policy. We then examine the impact of parameter uncertainty as measured by the sampling variation we estimate, and we find that our simple rule is robust to this uncertainty.

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However these results depend on a number of key assumptions made in specifying the model, and this specification uncertainty may have large effects on policy and welfare.

Uncertainty is not just an important feature of the monetary policy landscape; it is the defining characteristic of that landscape.

Alan Greenspan (2003)

1 Introduction

1997 Macroeconomics Annual helped initiate a research program which has become extraordinarily fruitful. The contributions by Goodfriend and King (1997) and Rotemberg and Woodford (1997) summarized and extended the literature to develop what has become a standard framework for policy analysis, built around dynamic general equilibrium models with nominal rigidities. A mature literature has developed analyzing the performance of monetary policy under uncertainty using small-scale macroeconomic models with explicit micro-foundations.¹ Recently, there has also been progress on improving the empirical performance of dynamic general equilibrium models. Recent research has incorporated such frictions into these models as nominal wage and price rigidities, habit formation, variable capacity utilization, and investment adjustment costs. These frictions allow the model to account for the observed behavior of macroeconomic data while maintaining a coherent micro-founded framework.² In this paper we contribute to the emerging literature analyzing optimal monetary policy in larger scale empirical general equilibrium models.³

¹This literature is enormous, including the key contributions summarized in Woodford (2003) and the papers in Taylor (1999).

²The relevant literature is large, but in addition the works already cited key papers which we draw on include Christiano, Eichenbaum, and Evans (2005), Altig, Christiano, Eichenbaum, and Linde (2004), and Smets and Wouters (2003a). Some important advances in the estimation of DSGE models are due to Schorfheide (2000) and Del Negro and Schorfheide (2004).

³Recent examples include Onatski and N. Williams (2004), Levin and Lopez-Salido (2004), Laforte (2003), Christiano, Motto, and Rostagno (2004), and Schmitt-Grohé and Uribe (2004).

We estimate a micro-founded dynamic general equilibrium macroeconometric model using Bayesian methods on postwar US data, and use this model to analyze the performance of monetary policy under uncertainty. The model contains many of the frictions we noted above, which have proven useful in matching the properties of the model to those of the data and, in particular, to the persistence evident in many economic times series. In addition, the model is supplemented with a number of structural shocks, including shocks to productivity, labor supply, preferences, price and wage markups, government spending, and monetary policy. We obtain plausible estimates of the key structural parameters, and in particular we find a fair amount of price and wage stickiness. Our model estimation which yields not only our baseline estimates but also the full distribution of the model parameters and, thus, provides measures of the uncertainty in the estimates. We find that most of the parameters are tightly estimated, and we use micro evidence to inform our priors about those parameters which are estimated less sharply.

To explore the problem of optimal monetary policy choice, we assume that the policymaker aims to maximize the utility of the representative household. We use a second-order approximation of household welfare as the policy objective, and solve for the optimal Ramsey policy. We find that there are significant welfare costs of fluctuations in the model, which are largely driven by our estimated wage stickiness. Under flexible wages (but sticky prices), the welfare costs are near zero. However wage stickiness leads to undesirable fluctuations in labor as well as wage dispersion, both of which swamp the welfare effects of price inflation alone. These results point to the importance of the labor market in our analysis, with the degree and nature of wage stickiness a key determinant of welfare and hence optimal policy.

Given the prime importance of wage stickiness, we find that the fully optimal Ramsey policy can be well approximated by an interest rate rule responding only to wage inflation. We optimize the coefficients of a rule of this form, finding that a rule which depends solely on the lagged interest rate and wage inflation achieves nearly the same welfare level as the Ramsey policy. Moreover since this policy rule responds only to observable variables it is relatively robust and immune to a number of problems which may plague other policy rule specifications. In particular, our optimal simple rule does not require the construction of a measure of the output gap or the natural rate of interest, and it can be implemented without knowledge of the deep structural parameters of the model.⁴

The robustness of our optimal rule also comes out when we study the effects of parameter uncertainty alters some of our conclusions. In our analysis of policy under uncertainty, we recognize the links in micro-founded models between the parameters of the policy objective function and the estimated parameters of the model emphasized by Levin and J. Williams (2004). However we find that parameter uncertainty as measured by our estimates is relatively unimportant for optimal policy. Not only are many of our parameters estimates precise, but welfare is insensitive to changes in most parameters over a fairly large range. We do find that a number of key parameters, such as those governing habit formation, wage and price setting, and investment adjustment costs may lead to large losses far from our benchmark estimates. While the critical values are unlikely under our particular specification, these findings point to the importance of our particular modeling choices.

More broadly, our analysis of the benchmark model highlights some of the strengths and weaknesses of standard monetary policy analysis. One strength is that, by incorporating significant frictions into the model, we are able to fit the data reasonably well and still retain explicit micro-foundations. A weakness is that, in order to do so we need to make sharp choices in the specification of the model, which are not always clearly guided by theory or microeconomic evidence. Moreover, a key advantage of basing optimal policy on a microfounded model is that the policy objective is naturally determined by the model specification. But, this implies that uncertainty regarding model specification translates into uncertainty

⁴While we focus on simple rules here, an alternative is to consider the formulation of a simplified objective function of the "flexible inflation targeting" form which policymakers could seek to optimize. Our findings suggest that stabilizing wage inflation may yield good results in terms of welfare.

regarding the representation of the policy objective in terms of observable variables. While the standard approach is able to account for uncertainty in estimated parameters, it does not account for uncertainty regarding the specification of the model. This point was recently made by Thomas Sargent who pointed out in Evans and Honkapohja (2005), that within likelihood-based approaches for macro models, "it seems difficult to complete a self-contained analysis of sensitivity to key features of a specification."⁵

To account for "specification uncertainty," we examine the policy implications of a variety of theoretically justified variations of our benchmark model. As we've noted, wage stickiness is key in our results, and particular assumptions on the form of wage rigidity are key. A seemingly minor issue with sizeable welfare effects relates to how wages are indexed. More broadly, our benchmark model assumes the commonly-used Calvo (1983) specification for wage stickiness. However the Taylor (1980) contracting approach has similar microfoundations and leads to many similar expressions. However the empirical performance and welfare implications of the model may be sensitive to which type of nominal rigidity is assumed.⁶ Moreover, our benchmark model abstracts from monetary frictions. An alternative specification following Christiano, Eichenbaum, and Evans (2005) considers the effect of money on households via money demand an firms via a working capital channel. Additionally, our benchmark model assumes internal habit formation, but external habit is a plausible alternative. Finally, as we noted above the model is supplemented with a number of shocks. In our welfare analysis we make a sharp distinction between efficient and inefficient shocks, with a goal of policy being to offset the inefficient shocks. However some of the shocks we label efficient may actually reflect distortions, and vice versa.

⁵An classic reference on related issues is Learner (1978).

⁶See Chari, Kehoe, and McGrattan (2000), Kiley (2002), and Guerrieri (2001) for example.

2 The Model

The benchmark model is a modified version of the Christiano, Eichenbaum, and Evans (2005) model and is close to that studied by Smets and Wouters (2003a). This model includes a number of frictions and additions which can induce intrinsic persistence in the propagation of shocks. The frictions include sticky prices and sticky wages, both with partial indexation, and adjustment costs in investment. The model also allows for habit persistence and variable capacity utilization with utilization costs. Further, in order to empirically confront seven data series in estimation, as in Smets and Wouters (2003a) the model is supplemented with ten structural shocks, six of which are allowed to be serially correlated. In this section, we describe the full nonlinear model; the log-linearized model that we estimate is described in the appendix.

2.1 Preferences and Technology

The model consists of a continuum of households, indexed by j, each of whom value consumption, $C_t(j)$, and suffer disutility from labor, $L_t(j)$. Preferences incorporate a form of "internal habit persistence" in consumption, where utility in period t depends negatively on the habit stock, which is assumed to equal consumption in the preceding period, $C_{t-1}(j)$.⁷ Each household supplies a differentiated labor input that is used in the production of intermediate goods. There are two preference shocks: one to the subjective rate of discount, ϵ_t^B , and one to the disutility of labor, ϵ_t^L , each of which is assumed to follow an AR(1) process. Preferences for the representative household, indexed by j, at period t = 0 are given by:

$$W_0(j) = E \sum_{t=0}^{\infty} \epsilon_t^B \beta^t V_t(j),$$

⁷We also considered a model with external habit persistence. External habit led to nearly identical estimates, but implies a new role for policy in undoing the externality distortions. As we discuss below, we chose to focus on settings with undistorted steady states. See Benigno and Woodford (2004) for discussion of policy with a distorted steady state.

where period utility is given by:

$$V_t(j) = \frac{1}{1 - \sigma} (C(j)_t - \theta C(j)_{t-1})^{1 - \sigma} - \frac{\epsilon_t^L}{1 + \chi} L_t(j)^{1 + \chi}, \tag{1}$$

where $\beta < 1$ is the subjective discount factor, $0 \leq \theta < 1$ measures the degree of habit persistence, $\sigma > 0$ is the curvature parameter for consumption utility, and $\chi > 0$ is the parameter determining the disutility of labor.

There exists a continuum of plants of unit measure that possess the production technology for intermediate goods that takes the Cobb-Douglas form with fixed costs Φ . There is a common technology shock, ϵ_t^A , which is assumed to follow an AR(1) process and have unit mean. For each plant, existing capital can be utilized at varying rate $U_t(i)$ at a cost in terms of foregone final output equal to:

$$\Psi(U_t(i)) = \mu \frac{U_t(i)^{1+\psi^{-1}}}{1+\psi^{-1}},$$
(2)

where ψ is the inverse of the elasticity of utilization cost with respect to utilization, and μ is chosen so that steady state utilization costs are zero.⁸

Denote by $N_t(i)$ plant i's labor input given by

$$N_t(i) = \left[\int_0^1 L_t(i,j)^{\frac{1}{1+\lambda_{w,t}}} dj\right]^{1+\lambda_{w,t}}$$

where $L_t(i, j)$ is the input of labor of type j at plant i, and $\lambda_{w,t}$ is assumed to be an i.i.d. random variable that affects all plants with mean λ_w and variance σ_w^2 . The plant's output is given by:

$$Y_t(i) \le \epsilon_t^A (U_t(i) K_{t-1}(i))^{\alpha} N_t(i)^{1-\alpha} - \Phi,$$
(3)

⁸Note that we explicitly specify a utilization cost function, as opposed to Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2003a), who simply state properties of the function necessary for estimation. This specificity is necessary for our optimal policy analysis. The same applies to the adjustment cost function. In both cases, the functional forms yield identically the same properties as assumed in these other papers.

where $K_{t-1}(i)$ is the plant's end-of-period capital stock available for use in period t, and Φ is plant-level fixed costs.⁹

There are adjustment costs in investment at each plant, where the costs are assumed to be a function $S(\cdot)$ of the growth rate of investment and subject to an aggregate shock ϵ_t^I which is assumed to have unit mean and follow an AR(1) process. The capital evolution equation for plant *i* is thus:

$$K_t(i) = (1 - \delta)K_{t-1}(i) + [1 - S(\epsilon_t^I I_t(i) / I_{t-1}(i))]I_t(i),$$
(4)

where δ is the depreciation rate. In the following, we assume the quadratic adjustment cost specification given by:

$$S(\epsilon_t^I I_t(i) / I_{t-1}(i)) = \zeta^{-1} \frac{1}{2} \left(\epsilon_t^I \frac{I_t(i)}{I_{t-1}(i)} - 1 \right)^2.$$
(5)

Note that adjustment costs are assumed to be zero at the steady state, S(1) = 0, and are only of second-order at the steady state.

Aggregate final good output, Y_t is created by (costlessly) combining a continuum of intermediate goods, $Y_t(i)$, indexed by *i*:

$$Y_{t} = \left[\int_{0}^{1} Y_{t}(i)^{\frac{1}{1+\lambda_{p,t}}} di\right]^{1+\lambda_{p,t}},$$
(6)

where $\lambda_{p,t}$ is assumed to be an i.i.d. random variable with mean λ_p and variance σ_p^2 . Final goods output is equal to consumption, aggregate investment, I_t , government spending, G_t , and capital utilization costs:

$$Y_t = C_t + G_t + I_t + \Psi(U_t)K_{t-1},$$
(7)

where fiscal policy is assumed to be Ricardian, and variations in government spending around steady-state, ϵ_t^G , are modeled as an AR(1) process.

⁹We assume that there is a common rental market for capital, unlike some recent papers, such as Altig, Christiano, Eichenbaum, and Linde (2004) and Woodford (2005), which assume firm specific capital. Given the adjustment costs, our assumption does not imply implausible movements in investment. However firmspecific capital may alter the identification of some key parameters, as Altig, Christiano, Eichenbaum, and Linde (2004) show.

2.2 Equilibrium Allocation and Prices

In the following, we omit household and plant indices where no confusion will result. The household's budget constraint is standard and households trade in a complete market to allocate their consumption over time. Let MUC_t denote the marginal utility associated with an incremental increase in consumption in period t, accounting for its effect on period utility in the period t + 1:

$$MUC_{t} = \epsilon_{t}^{B} \left(C_{t} - \theta C_{t-1} \right)^{-\sigma} - \beta \theta E_{t} \epsilon_{t+1}^{B} \left(C_{t+1} - \theta C_{t} \right)^{-\sigma}$$

$$\tag{8}$$

The consumption Euler equation summarizes the representative household's optimal saving behavior:

$$MUC_t = E_t \left[MUC_{t+1}R_t P_t / P_{t+1} \right] \tag{9}$$

where R_t is the gross nominal interest rate and P_t is the price level.

Because of their market power, households are wage setters in the labor market. However they cannot reset their wages every period, but instead face nominal wage rigidity of the Calvo (1983) type in which they can only reset wages with probability $1 - \xi_w$. We allow for partial indexation, so households that cannot re-optimize have their wages grow at a rate equal to the rate of inflation raised to the power $\gamma_w \in [0, 1]$.¹⁰ Households set wages subject to their individual labor demand curves, which arise from the firms' input demands. The evolution of the aggregate nominal wage index is:

$$W_t^{-1/\lambda_{w,t}} = \xi_w W_{t-1}^{-1/\lambda_{w,t}} \left(\frac{P_{t-1}}{P_{t-2}}\right)^{-\gamma_w/\lambda_{w,t}} + (1-\xi_w) \tilde{w}_t^{-1/\lambda_{w,t}},\tag{10}$$

where we have allowed $\lambda_{w,t}$ to vary over time to reflect changes in market power, but is assumed to be i.i.d. around a constant mean: $\lambda_{w,t} = \lambda_w + \eta_t^{w,11}$ Here \tilde{w}_t is the optimal

¹⁰As long as γ_w (and γ_p below) is less than unity this indexation assumption leads to a long-run Phillips curve. It is straightforward to add indexation to the unconditional mean of wage inflation raised to the power $1 - \gamma_w$ which implies a vertical long-run Phillips curve. The only effect will be in the determination of the optimal steady state inflation rate, which is zero under our indexation assumption.

¹¹As will be clear below, the i.i.d. restriction is necessary in order to identify the shock.

nominal wage chosen by those households who can optimize in the period, which satisfies:

$$\frac{\tilde{w}_t}{P_t} E_t \sum_{i=0}^{\infty} \beta^i \xi_w^i L_{t+i}^j \epsilon_{t+i}^b \left[\left(\frac{P_t P_{t+i-1}}{P_{t-1} P_{t+i}} \right)^{\gamma_w} \frac{MUC_{t+i}}{1 + \lambda_{w,t+i}} - \epsilon_t^L (L_{t+i}^j)^{\chi} \right] = 0.$$
(11)

As in usual Calvo pricing models, (11) incorporates forward-looking expectations of future nominal wages, but now includes lagged inflation via the partial indexation.

Households own the capital stock K_t , which they accumulate using the capital accumulation technology, and rent to firms at rental rate R_t^k . This leads to three key relationships. First, we let Q_t be the real share value per unit of capital which is determined by an asset pricing Euler equation from (9):

$$Q_{t} = \eta_{t}^{Q} E_{t} \left[\beta \frac{MUC_{t+1}}{MUC_{t}} \left(Q_{t+1}(1-\delta) + Z_{t+1}R_{t+1}^{k} - \Psi(U_{t+1}) \right) \right],$$
(12)

where η_t^Q is as an i.i.d. equity premium shock. As Smets and Wouters (2003a) acknowledge, this shock is not micro-founded but is taken to reflect variations in the external finance premium. The optimal investment decision leads to an investment Euler equation:

$$1 - Q_t S'\left(\frac{\epsilon_t^I I_t}{I_{t-1}}\right) \frac{\epsilon_t^I I_t}{I_{t-1}} = E_t \left[\frac{MUC_{t+1}}{MUC_t} Q_{t+1} S'\left(\frac{\epsilon_{t+1}^I I_{t+1}}{I_t}\right) \frac{\epsilon_{t+1}^I I_{t+1}}{I_t} \frac{I_{t+1}}{I_t}\right]$$
(13)

This equation balances the costs and benefits of investment, with lagged investment and the shocks showing up through the effects of the costs of adjustment. Finally, the first order condition for utilization gives:

$$R_t^k = \Psi'(U_t). \tag{14}$$

On the production side, there are a continuum of unit measure of intermediate goods producers who are monopolistic competitors. Their products are aggregated into a single final good which is used for consumption and investment via a Dixit-Stiglitz aggregator. As in the labor market, the power in the aggregator is assumed to be stochastic and is i.i.d. around a constant mean: $\lambda_{p,t} = \lambda_p + \eta_t^p$. The intermediate goods producers possess the production technology given in equation (3). The firm's cost minimization condition is:

$$\frac{W_t L_t}{R_t^k U_t K_{t-1}} = \frac{1-\alpha}{\alpha}.$$
(15)

Thus firms equate the marginal rate of transformation between labor and effective capital $(U_t K_{t-1})$ to the relative factor prices, and the capital-labor ratio is identical across firms. Marginal costs are then given by:

$$MC_{t} = \frac{W_{t}^{1-\alpha}(R_{t}^{k})^{\alpha}}{\epsilon_{t}^{A}} \alpha^{-\alpha} (1-\alpha)^{\alpha-1}.$$
 (16)

As households do in the labor market, firms also face Calvo-type nominal rigidities in price setting, and the model allows for partial indexation. Each firm may change its price in a given period with probability $1 - \xi_p$, but those firms that do not re-optimize see their prices increase by a rate equal to the rate of inflation raised to the power γ_p . The evolution of the aggregate nominal price index is:

$$P_t^{-1/\lambda_{p,t}} = \xi_p P_{t-1}^{-1/\lambda_{p,t}} \left(\frac{P_{t-1}}{P_{t-2}}\right)^{-\gamma_p/\lambda_{p,t}} + (1-\xi_p) \tilde{p}_t^{-1/\lambda_{p,t}}$$
(17)

Here \tilde{p}_t is the optimal price chosen by those firms who can optimize in the period, which satisfies:

$$E_{t} \sum_{i=0}^{\infty} \beta^{i} \xi_{p}^{i} \lambda_{p,t+i} Y_{t+i}^{j} \epsilon_{t+i}^{b} MUC_{t+i} \left[\frac{\tilde{p}_{t}}{P_{t}} \left(\frac{P_{t} P_{t+i-1}}{P_{t-1} P_{t+i}} \right)^{\gamma_{p}} - (1 + \lambda_{p,t+i}) \frac{MC_{t+i}}{P_{t+i}} \right] = 0.$$
(18)

Equilibrium in the goods market obtains when production equals the demand for goods by households and the government. Finally, we close the model by specifying an empirical monetary policy reaction function. We specify policy in terms of a generalized Taylortype rule, where the policy authority sets nominal rates in response to inflation and the output gap. To do this, we define a model-consistent output gap as the difference between actual and potential output, where potential output is defined as what would prevail under flexible prices and wages and in the absence of the three "cost-push" shocks $(\eta_t^W, \eta_t^Q, \eta_t^P)$ that cause variations in wage and price markups and the equity premium. Thus the model is supplemented with flexible-price versions of (9)-(17) which determine the potential output Y_t^* . Then the policy rule is assumed to take the following form:

$$r_{t} = r_{i}r_{t-1} + (1 - r_{i})\left(\bar{\pi}_{t} + r_{\pi}(\pi_{t-1} - \bar{\pi}_{t}) + r_{y}\log(Y_{t-1}/Y_{t-1}^{*})\right)$$

$$+ r_{\Delta\pi}(\pi_{t} - \pi_{t-1}) + r_{\Delta y}\left\{\log(Y_{t}/Y_{t}^{*}) - \log(Y_{t-1}/Y_{t-1}^{*})\right\} + \eta_{t}^{R}.$$
(19)

Here $r_t = \log R_t$ is the short-term interest rate, $\pi_t = \Delta \ln(P_t/P_{t-1})$ is the inflation rate, $\bar{\pi}_t$ is an AR(1) shock to the inflation objective with correlation ρ_{π} , while η_t^R is an i.i.d. policy shock. Note that the rule allows for the Taylor (1993) effects, along with interest rate smoothing, and responses to changes in inflation and the output gap.

3 Estimation

In this section, we describe the Bayesian estimation methodology that we use, and we report the results and compare them to estimates from the literature. We first briefly describe the data, then the formulation of the prior, then turn to estimation of the model.

3.1 The Data

We estimate the log-linearized version of the model (a detailed description of the log-linearization is provided in the appendix). Seven series of US data from 1955Q1-2001Q4 are used in the estimation: consumption, investment, output, nonfarm business hours of work, the real wage (nonfarm wage divided by GDP price deflator), the inflation rate (the log-change in the GDP price deflator), and the federal funds rate.¹² The first five series are measured in log deviations from linear trends, multiplying the resulting series by 100 to express them in percentage points. Government spending is taken to be the residual between GDP and the sum of consumption and investment. The inflation rate and the fed funds rate are demeaned and measured at quarterly rates.

 $^{^{12}\}mathrm{We}$ use the same data set as Altig, Christiano, Eichenbaum, and Linde (2004), obtained from Martin Eichenbaum's web page.

Note that there are 10 shocks in the model, but we have only seven data series. Because available data such as capacity utilization are imperfect proxies for the corresponding model concepts, we use the Kalman filter to estimate latent variables.

3.2 Prior

Taken together, there are 31 parameters to estimate: the standard deviations of the 10 shocks in the model, six of which are assumed to follow AR(1) processes and therefore have estimated autocorrelation parameters, 10 structural parameters related to preferences and technology, and 5 parameters describing monetary policy.

We formulate our prior in a manner analogous to Smets and Wouters (2003a). First, we fix a number of model parameters based on values typically used in the literature: $\lambda_w = 0.05$, $\alpha = 0.36$, $\beta = 0.99$, and $\delta = 0.025$. We calibrate the output shares of consumption, investment, and government spending from the long-run averages of the U.S. data: $c_y = \bar{C}/\bar{Y} = 0.56$, $i_y = 0.24$, and $g_y = 0.20$.¹³ As is standard in the literature, we set independent priors for each of the parameters which are combined to form the prior for the model. In setting the prior we drew on the brief survey of the literature summarized in Onatski and N. Williams (2004).¹⁴ The prior is given in Table 1.

Overall, our prior is consistent with the previous literature and is relatively uninformative for most of the parameters, with some exceptions noted below. For the parameters of the shock processes, where we had little guidance from the literature, so set relatively loose priors. For the standard deviations, we used gamma distributions with standard deviations

¹³The mean ratio of net exports to GDP was zero to two decimal points over our sample.

¹⁴This survey includes studies focusing on real models (such as King and Rebelo (1999) and Boldrin, Christiano, and Fisher (2001)) as well as papers focusing on monetary policy in smaller models (such as Rotemberg and Woodford (1997), Judd and Rudebusch (1998), Sack (1998)). We drew more heavily on the papers which have estimated this or similar models, either on US or European data. The key papers we looked at here were Smets and Wouters (2003a), (Smets and Wouters 2003b), Christiano, Eichenbaum, and Evans (2005), Altig, Christiano, Eichenbaum, and Linde (2004), and the results in Onatski and N. Williams (2004).

Param	eter	Distribution	Mean	Std Dev
ζ	Investment adjustment costs	Normal	0.5	0.2
σ	Consumption utility	Normal	2	0.5
θ	Consumption habit	Beta	0.7	0.15
χ	Labor utility	Normal	1.2	0.5
$\phi - 1$	Fixed cost-1	Gamma	0.075	0.0125
ψ^{-1}	Capital utilization costs	Log Normal	6.4	5
ξ_w	Calvo wages	Beta	0.5	0.25
ξ_p	Calvo prices	Beta	0.5	0.25
γ_w	Wage indexation	Beta	0.5	0.25
γ_p	Price indexation	Beta	0.375	0.1
r_{π}	Policy, lagged inflation	Normal	2	0.5
$r_{\Delta\pi}$	Policy, change in π	Normal	0.2	0.1
r_i	Policy, lagged interest rate	Normal	1	0.15
r_y	Policy, lagged output gap	Gamma	0.25	0.25
$r_{\Delta y}$	Policy, change in gap	Gamma	0.25	0.25
σ_a	Productivity	Gamma	0.6	0.6
σ_{π}	Inflation objective	Gamma	0.1	0.1
σ_b	Preference	Gamma	0.3	0.3
σ_g	Govt. spending	Gamma	0.3	0.3
σ_l	Labor supply	Gamma	3	3
σ_i	Investment	Gamma	0.1	0.1
σ_r	Interest rate	Gamma	0.1	0.1
σ_q	Equity premium	Gamma	5	5
σ_p	Price markup	Gamma	0.2	0.2
σ_w	Wage markup	Gamma	0.2	0.2
ρ_a	Productivity	Beta	0.5	0.25
$ ho_{\pi}$	Inflation objective	Beta	0.85	0.1
$ ho_b$	Preference	Beta	0.5	0.25
$ ho_g$	Government spending	Beta	0.5	0.25
$ ho_l$	Labor supply	Beta	0.5	0.25
$ ho_i$	Investment	Beta	0.5	0.25

Table 1: Specification of Prior

equal to the means. We gauged the relative magnitudes of the shocks from Onatski and N. Williams (2004) and Smets and Wouters (2003a). For all but one persistence parameter, we used a wide beta distribution. The exception was the inflation objective shock. Since it and the interest rate shock enter additively in the policy rule (37) a tighter prior is necessary to distinguish between them. For the structural parameters, we chose the parameters of the distributions to cover with reasonably high probability the range of estimates we found in the literature. A couple of parameters deserve note. For the capacity utilization cost, there has been a wide dispersion of estimates, ranging from 0.5 by Altig, Christiano, Eichenbaum, and Linde (2004) to a value of 100 on a boundary by Christiano, Eichenbaum, and Evans (2005). We chose a lognormal distribution to reflect a potentially large tail. For the policy responses to the output gap or changes in the gap, we expected small positive numbers and hence chose a loose gamma distribution. For the fixed cost, we chose a relatively tight prior centered on 7.5% of output. This reflects the estimates of Basu (1996) and Basu and Fernald (1997) who find fixed costs of 3-10%. As we discuss below, this prior restriction was important to obtain plausible estimates. Finally, for the Calvo pricing parameter we based our prior mean on the studies of Golosov and Lucas (2003) and Klenow and Kryvtsov (2004) who find that firms change prices roughly once every 1.6 quarters.

3.3 Estimation Results

With the prior specified, we now turn to the estimation of the model. As in Smets and Wouters (2003a), we first look for a parameter vector which maximizes the posterior mode, given our prior and the likelihood based on the data.¹⁵ We then sample from the posterior distribution using a Metropolis-Hastings Markov chain Monte Carlo (MCMC) algorithm in

¹⁵We took great efforts to explore the parameter space sufficiently to locate a global maximum. In particular, we sampled 200 values from the prior distribution, and used these as starting values for Chris Sims's optimization algorithms designed to avoid common problems with likelihood functions (available on his web page). We re-ran it in combination with a standard hill-climber algorithm until it settled on the maximal value. We used the resulting mode as the starting point for our MCMC sampling.



Figure 1: Estimated posterior distributions (blue solid lines) and prior distributions (green dashed) for the shock parameters.

order to make small sample inferences about the parameters. We sampled 10 separate chains for 45,000 periods each, discarding the first 15,000 periods. Thus we were left with 300,000 points from the posterior distribution. In order to assess convergence of the Markov chains, we use the potential scale reduction statistic described by Gelman, Carlin, Stern, and Rubin (2004) which gave clear indications of convergence for all the parameters.

Table 2 reports the estimates and 5% and 95% bounds from the posterior distributions of the model parameters; the final column reports the mode estimates from Smets and Wouters (2003a) for comparison. Figure 1 plots the prior and posterior distributions for the parameters describing the shock processes; Figure 2 plots the same for the for the structural

	Tal	<u>ble 2: Es</u>	timation	Results		
Parameter		Mode	Mean	5% Bound	95% Bound	SW Mode
ζ	Investment adjustment	0.5368	0.5413	0.2647	0.8042	0.156
σ	Consumption Utility	2.0434	2.1673	1.6769	2.7404	1.375
θ	Consumption Habit	0.2978	0.2930	0.2048	0.3784	0.543
χ	Labor Utility	1.2961	1.3592	0.8308	2.0107	2.977
ϕ	Fixed cost	1.0818	1.0843	1.0628	1.1077	2.268
ψ	Capital utilization	0.1986	0.2127	0.1238	0.3234	0.327
ξ_w	Calvo wages	0.7834	0.7648	0.6797	0.8291	0.718
ξ_p	Calvo prices	0.8298	0.8344	0.8109	0.8593	0.923
γ_w	Wage indexation	0.7856	0.8200	0.5026	0.9982	0.712
γ_p	Price indexation	0.1142	0.0787	0.0013	0.2112	0.440
r_{π}	Policy, lagged inflation	2.7078	2.7497	2.2288	3.3214	1.687
$r_{\Delta\pi}$	Policy, change in π	0.3053	0.2951	0.1987	0.3888	0.113
r_i	Policy, lagged interest rate	0.8248	0.8305	0.7770	0.8806	0.957
r_y	Policy, lagged output gap	0.0001	0.0559	0.0039	0.1566	0.097
$r_{\Delta y}$	Policy, change in gap	0.4846	0.5059	0.4074	0.6333	0.163
σ_a	Productivity	0.5941	0.5950	0.5454	0.6492	0.315
σ_{π}	Inflation objective	0.1085	0.1172	0.0932	0.1449	0.017
σ_b	Preference	0.1186	0.1251	0.0683	0.2052	0.836
σ_{g}	Govt. spending	0.2846	0.2872	0.2631	0.3138	0.361
σ_l	Labor supply	2.2233	2.3529	1.7341	3.1204	2.862
σ_i	Investment	1.0261	1.0451	0.8660	1.2290	0.063
σ_r	Interest rate	0.0000	0.0002	0.0000	0.0000	0.101
σ_q	Equity premium	3.7041	4.0473	2.6499	6.4839	7.755
σ_p	Price markup	0.2025	0.2050	0.1796	0.2327	0.160
σ_w	Wage markup	0.2897	0.2945	0.2676	0.3225	0.273
ρ_a	Productivity	0.9608	0.9629	0.9464	0.9781	0.926
ρ_{π}	Inflation objective	0.9959	0.9959	0.9890	0.9996	0.916
$ ho_b$	Preference	0.9461	0.9462	0.8936	0.9893	0.836
$ ho_g$	Govt. spending	0.9422	0.9453	0.9153	0.9727	0.959
$ ho_l$	Labor supply	0.9818	0.9836	0.9633	0.9973	0.934
$ ho_i$	Investment	0.7516	0.7347	0.5928	0.8381	0.919



Figure 2: Estimated posterior distributions (blue solid lines) and prior distributions (green dashed) for the structural parameters.

parameters.¹⁶ Most of the parameters are tightly estimated; exceptions include the utility parameters, σ and χ , investment adjustment costs, ζ , and the standard deviations of the shocks to the disutility of labor and Tobin's Q, σ^l and σ_q , respectively. Most posterior distributions have a single interior mode; the exceptions are the price and wage indexation parameters, γ_p and γ_w , respectively, and σ_r , the standard deviation of the interest rate shock. Our prior insures that σ_r never hits zero, but its modal value is only 4×10^{-6} . Except for the investment shock, the AR(1) shocks are highly persistent, with the inflation objective shock in particular being nearly a unit root process.

The estimates of structural parameters are reasonable and are consistent with other estimates. Our estimate of investment adjustment costs are noticeably lower than SW, and more in line with those estimated by Altig, Christiano, Eichenbaum, and Linde (2004), henceforth ACEL. This largely reflects our much lower estimates of the fixed costs, which we find account for 8% of output versus 120%. Based on the micro evidence we discussed above, we found estimates above 10% to be unreasonable, and hence had tight priors for this parameter.¹⁷ In Figure 2 we see that under our priors, the data were not terribly informative about these parameters. By contrast, the data were indeed informative about the Calvo and indexation parameters. The Calvo wage parameter is nearly the same as in SW and is similar to Christiano, Eichenbaum, and Evans (2005) and ACEL. The Calvo wage parameter is larger than that in the micro studies noted above, but accords with the estimates in the homogenous capital model of ACEL.¹⁸ We find considerably less price indexation than SW, but closer to the findings of Edge, Laubach, and J. Williams (2003). Finally, the policy rule coefficients are stabilizing and similar to other estimates that cover a long time frame. The estimated rule features a fair amount of inertia and a strong inflation response.

¹⁶Note that the plots show the marginal posterior distributions, while the mode estimate is for the full joint posterior. So the peak of the marginal posterior in each plot need not agree with the mode estimate.

 $^{^{17}}$ When we loosened our prior on the fixed costs ϕ we found much larger adjustment costs.

¹⁸A main reason they consider a model with firm-specific capital is to bring the estimated price stickiness closer to the micro data.

4 Optimal Monetary Policy

In this section, we compute the optimal monetary policies based on the point estimates reported above, and abstract for the time being the issues of parameter and specification uncertainty. We start by assuming that the central bank's follows the optimal policy under commitment that maximizes the utility of the representative household. We then consider alternative specifications of policy in terms of instrument rules, where the overnight interest rate is determined by a small number of variables as in the Taylor Rule.

4.1 Welfare

We measure the performance of policy in terms of conditional household welfare, measured using a second-order approximation to the model equations, following the approach described in Woodford (2003).¹⁹ Welfare depends negatively on unwanted fluctuations in aggregate consumption and aggregate labor input and dispersion in labor input resulting from sticky wages. We follow Levin and Lopez-Salido (2004) and approximate welfare using a second-order approximation to the model using the DYNARE software. We generally report welfare in terms of the percentage increase in permanent consumption needed to restore welfare to what would obtain in the deterministic steady state.

In order to separately analyze the problems of distortions to the steady-state and the effects of nominal rigidities on fluctuations around the steady-state, we assume that the allocation that would obtain under flexible wages and prices in the benchmark model is Pareto optimal. Given the presence of monopoly power of firms and workers, we assume that labor and production subsidies perfectly offset the steady state distortionary effects of markups. There are no other distortions in the benchmark model, so given these two subsidies the flexible price equilibrium is Pareto optimal and monetary policy is solely concerned with

¹⁹Kim, Kim, Schaumburg, and Sims (2003) and others have noted some of the potential difficulties with unconditional welfare analysis.

fluctuations in allocations and prices around the steady state.

Under these assumptions, we can obtain valid welfare approximations by using a quadratic expansion of welfare and the linearized model. Onatski and N. Williams (2004) derive an explicit welfare expansion for the model, showing that it can be represented in the form:

$$W_t \approx \frac{1}{2} \begin{bmatrix} S_t \\ S_{t-1} \end{bmatrix}' \Lambda_{SS} \begin{bmatrix} S_t \\ S_{t-1} \end{bmatrix} + \Lambda_{\pi} (\pi_t - \gamma_p \pi_{t-1})^2 + \Lambda_w (w_t - w_{t-1} + \pi_t - \gamma_w \pi_{t-1})^2$$

where we collect the logarithmic deviations from steady state of a number of variables and shocks in the vector S_t :

$$S_t = (y_t, k_t, k_{t-1}, u_t, \epsilon_t^g, \epsilon_t^i, i_{t-1}, \epsilon_t^a, \epsilon_t^b, \epsilon_t^L).$$

The first component of the loss function captures fluctuations in consumption, while the other terms weight price and wage inflation. As in Woodford (2003) price inflation affects welfare via the dispersion of output across firms, while as in Erceg, Henderson, and Levin (2000) and Woodford (2003) wage inflation enters via the dispersion of labor supply across households. The consumption utility terms are more complex than in smaller models, due to the endogenous capital accumulation and the investment adjustment costs. This implies that fluctuations in the different components of output are weighed differently. The lags of the real variables enter due to the habit persistence in consumption. The loss function is large and includes nonzero weights on many terms. Explicit expressions for Λ_{SS} , Λ_{π} and Λ_w are given in Onatski and N. Williams (2004), and they display rather complex dependence on the structural parameters. As we discuss below, this is important in our analysis of parameter uncertainty as it implies an uncertain objective function for policymakers.

4.2 Optimal Inflation Rate

In our benchmark model, the optimal inflation rate is zero. This result is driven by the effects of inflation on wage and price dispersion and the absence of any effects of nominal interest rates, beyond the effects of real rates, on the allocation of resources (see Levin and Lopez-Salido (2004) for a detailed analysis of the optimal inflation rate in a similar DSGE model). Correspondingly, in the following, we assume that policy sets the long-run inflation goal at zero. In the later sections, we consider an alternative specification of the model that imply a non-zero optimal inflation rate.

4.3 The Optimal Ramsey Policy

Given the second-order approximation to household welfare, we derive the first-order conditions that describe the setting of monetary policy aimed at maximizing household welfare. In this section, we assume that the policymaker knows the structure of the model, all parameter values, and observes current period data in making decisions. Throughout we assume that the central bank possesses a commitment technology and focus exclusively on the characterization of policy under commitment. To compute the equilibrium under optimal policy, we replace the instrument rule given by equation (19) with the first-order optimization conditions relating Lagrange multipliers to endogenous variables.²⁰

In response to shocks to preferences and technology, the Ramsey policy yields paths for consumption and aggregate labor that mimic that in the flexible price economy, but deviate from it in the direction of smoothing the pace of wage inflation in order to reduce distortions owing to relative wage dispersion. Figures 3 and 4 show the impulse responses to the nine shocks under the Ramsey policy, the estimated policy, and in the flexible-price equilibrium. Looking at the responses to a positive shock to productivity, the Ramsey policy is constrains labor demand, resulting in a more muted response of real wages than in the flexible-price equilibrium. A similar is pattern is seen in response to other shocks to fundamentals. The estimated policy rule, however, does a relatively poor job in mimicking the responses under

 $^{^{20}}$ We do not address the implementation of optimal policy, which may be nontrivial. See Svensson and Woodford (2004) and Giannoni and Woodford (2004).



23 Figure 3: Impulse responses for one standard deviation shocks; Ramsey policy (red solid lines), estimated policy (blue dashed), and flexible-wage and -price equilibrium (dash-dot green lines).

the optimal policy, both in terms of the initial responses to the shocks and the dynamic paths following the shocks.

In response to positive shocks to wages and Tobin's Q, the Ramsey policy calls for a sharp increase in interest rates that causes a large decline in consumption. The contractionary policy response reduces the responses of wages. In contrast, the estimated policy rule accommodates the shocks to a greater degree and allows larger rises in wage inflation. The optimal policy response to a transitory shock to prices, however, is do virtually nothing. The estimated policy rule reacts to the rise in inflation, sending real rates higher and reducing consumption and aggregate labor.

The "cost of fluctuations" under the Ramsey policy is equivalent in terms of utility to a 1.4 percent permanent decrease in consumption. Table 3 reports the welfare and macroeconomic outcomes under different assumptions regarding monetary policy. The second column reports welfare in terms of the percent decrease in steady-state consumption that would equate welfare in the stochastic economy to welfare in the deterministic steady state (a larger absolute number is a worse outcome). The welfare loss under the Ramsey policy is considerably higher than that found by Lucas. This difference reflects the inclusion of the effects of undesired fluctuations in labor on welfare and the effects of wage and price dispersion. By comparison, the estimated policy rule yields a far greater loss equivalent to a 2.2 percent permanent reduction in consumption relative to the Ramsey policy. Based on 2004 annual nominal consumption of about \$28,000 per person, the loss under the Ramsey policy is about \$390 per person per year, and that under the estimated policy is equivalent to a loss of \$610 per year, implying that a switch from the estimated policy to the Ramsey policy would improve welfare on the order of \$220 annually per person, or about \$64 billion per year for the United States as a whole.

Interestingly, this sizable increase in the loss under the estimated policy rule is not apparent in terms of the unconditional standard deviations of the output gap, inflation and



Figure 4: Impulse responses for one standard deviation shocks; Ramsey policy (red solid lines), estimated policy (blue dashed), and flexible-wage and -price equilibrium (dash-dot green lines). prior distributions (green dashed) for the structural parameters.

	Coefficients		Welfare	Standard Deviation				
Policy	r_i	r_{π}	r_w	r_y		$y - y^*$	π	r
Ramsey policy					-1.40	2.56	0.30	5.29
Estimated policy rule					-2.18	2.73	0.33	0.42
Taylor rule		1.5		0.5	-2.06	1.24	0.59	0.74
Benchmark wage inflation rule	1.0		3.8		-1.49	3.16	0.29	2.09

the interest rate, standard measures of loss in the monetary policy literature. Indeed, the standard deviations of the output gap and the inflation rate are comparable under the two policies while that of inflation and the interest rate is much lower under the estimated policy. Evidently, these variables alone do not provide sufficient summary statistics for household welfare in this model.

As we noted above, wage stickiness is the key to the large welfare costs we find. The labor input and wage inflation both receive significant weight in our welfare measure, and these drive the results. As we show next this makes simple rules targeting wage inflation perform nearly as well as the fully optimal rule. However as we discuss later, it also implies that the wage setting specification is crucial to the model.

4.4 Simple Rules

We also consider a variety of policy rules that take the general form:

$$r_t = r_i r_{t-1} + r_\pi \pi_t + r_w \left(\Delta w_t + \pi_t \right) + r_y (y_t - y_t^*), \tag{20}$$

where the nominal interest rate depends on its own lag, the rates of price and wage inflation (the sum of the growth of real wages w_t and prices), the output gap, and/or the level of detrended output. The lower part of Table 3 reports the coefficients and outcomes under a number of such rules. The first is the standard Taylor Rule (1993), according to which the interest rate is determined by contemporaneous inflation and the output gap. As reported in the table, the Taylor rule yields a modest improvement in household welfare over the estimated rule. It is possible to improve the welfare performance of the standard parameterization of the Taylor rule by increasing the coefficients to extremely large values, but even then such a policy yields welfare equivalent to a 2 percent permanent reduction in consumption.

Given the importance of wage dispersion on utility, a better alternative to the Taylor rule is a rule that responds directly to *wage* inflation, as proposed by Erceg, Henderson, and Levin (2000). We consider a first-difference form of such a rule, where the change in the nominal interest rate is determined by the nominal rate of wage inflation. This rule is operational in the sense of McCallum (1999) in that the policy instrument is determined only by observable variables, as opposed to model-specific constructed data such as the natural rates of interest and output.²¹ The parameter of the rule we report in the table was chosen to maximize welfare in the benchmark model. Moreover as we show below since this rule does not directly depend on the structural parameters or the shocks, it is robust to a wide range of parameter uncertainty.

The benchmark wage inflation monetary policy rule yields welfare nearly the same as the fully-optimal Ramsey policy. The benchmark rule yields welfare equivalent to less than a 0.1 percent permanent reduction in consumption, about \$30 per person per year (or \$7 billion annually for the entire country) compared to the Ramsey policy. In the following analyses of parameter and specification uncertainty, we use this benchmark wage inflation rule as a proxy for the optimal rule based on the mean model parameter estimates.

The impulse responses under the benchmarks wage inflation rule mimic closely those of the Ramsey policy, as seen in Figures 5 and 6. In response to the technology shock, the benchmark rule initially tightens monetary policy instead of loosens it as is optimal. As a result, labor hours and consumption fall by too much at the onset of the shock. Nonetheless,

 $^{^{21}}$ However the rule does respond to current data, rather than the lagged data which was an additional condition discussed by McCallum (1999).



 $\frac{28}{\text{Figure 5: Impulse responses for one standard deviation shocks; Ramsey policy (red solid lines), optimized}$ wage inflation policy rule (blue dashed), and flexible-wage and -price equilibrium (dash-dot green lines).



29 Figure 6: Impulse responses for one standard deviation shocks; Ramsey policy (red solid lines), optimized wage inflation policy rule (blue dashed), and flexible-wage and -price equilibrium (dash-dot green lines).

after a few quarters, the benchmark policy rule gets back on track and the paths of labor hours, consumption, and price and wage inflation follow the optimal paths closely. The outcomes under the preference, government spending, and labor supply shocks follow the same pattern, but in these cases, the benchmark is initially too loose. The benchmark policy does less well at getting the responses to the Tobin's Q and price shocks right, but the magnitude of the errors in allocations and prices is still relatively modest.

5 Parameter Uncertainty

In the preceding section, we assumed that the model parameters were known with certainty. Of course, even if the model is correctly specified, in practice parameters are known imperfectly owing to sampling variation. In this section, we analyze the implications of parameter uncertainty for policy and develop a measure of the social value of reducing uncertainty in terms of household welfare. The latter analysis provides information on the value, from the perspective of monetary policy, in improving estimates of various parameters and suggests where are the highest payoffs, and for which parameters further gains in accuracy are relatively unimportant.

As emphasized by Levin and J. Williams (2004), in models with explicit microfoundations such as the one studied here, uncertainty about structural parameters implies uncertainty regarding the weights in the second-order approximation to welfare. Our analysis takes into account the cross-equation restrictions between the parameters that determine the model dynamics and the weights in the objective function.²²

In considering parameter uncertainty, one confronts a number of conceptual issues. First, we abstract from learning and assume that the degree of uncertainty remains constant for the foreseeable future. This assumption is unrealistic, and assuming that the true structure of the economy is unchanging biases upward our measure of the costs of uncertainty. Second,

 $^{^{22}}$ See also Walsh (2005).

once one allows for uncertainty regarding model parameters, one must take a stand on exactly what information related to the model parameters that the policymaker does and does not possess. In the following, we focus on policy rules that are based on observable data only and do not respond to constructed variables such as the output gap that implicitly convey information regarding model parameters. Finally, steady-state welfare depends on some of the parameters that we consider. To avoid comparisons between apples and oranges, we focus on the consumption-equivalent welfare loss instead of the levels of welfare in the following.²³

5.1 Monetary Policy under Individual Parameter Uncertainty

In this section, we explore the sensitivity of household welfare to variations in the structural parameters for the wage inflation policy rule optimized to the baseline parameter estimates discussed in Section 3. We compare the performance of this rule to the first-best benchmark of the Ramsey policy optimized to the assumed values of the model parameters. As we vary a specific parameter value, we hold all other parameter values at their respective mean estimates. We report the difference in welfare between that found under the benchmark rule and the first-best policy for the specified parameter. Thus, we measure the potential benefit from switching from the benchmark rule to the first-best policy assuming the true parameter value were known. If the resulting plotted line is horizontal, estimation error of the parameter value yields no costs in terms of welfare, while a steeply curved line indicates that parameter estimation error carries high costs and that better estimates potentially would have a large social benefit. In some cases we also consider the levels of the welfare costs, which has slightly different implications. For some parameters the welfare costs increase under both the benchmark rule and the optimized rule, but there is little change between

²³Although most model parameters-including those describing the shock processes, wage and price adjustment, investment adjustment costs, and utilization costs-have no implications for the deterministic steady state and therefore welfare comparisons are straightforward, this is not the case for all parameters. Preference parameters, however, do affect the steady state levels of labor, consumption, the capital stock and, by implication, welfare. And, the share of fixed costs affects the steady-state allocation and welfare as well.



Figure 7: Parameter uncertainty: price and wage setting. The difference between welfare under the benchmark policy rule and the Ramsey policy is plotted as the red curve. The two vertical lines indicate the 5 and 95 percent bounds from the posterior distributions.

them. This suggests that knowledge of these parameters may be important for determining the size of the welfare costs of fluctuations, but better knowledge of the parameters will not change policy.

The solid lines in Figure 7 plot the results for the four parameters related to price and wage setting. The results for the Calvo parameter are shown in the upper panels; the results for the indexation parameters are shown in the lower panels. The left-hand panels refer to price-setting parameters; those on the right to wage-setting. The vertical lines indicate the 5% and 95% posterior intervals for the parameters calculated from the Markov Chain Monte Carlo simulations. In Figure 8 we plot the values welfare losses in each case as well.

Based on the posterior distributions, sampling uncertainty alone implies modest costs of uncertainty regarding parameters describing price and wage setting. The relative welfare loss from following the benchmark rule is nearly constant over the 90% posterior intervals for the two price-setting parameters, which are estimated with precision based on the posterior distribution. Similarly, the welfare loss is relatively constant over the posterior intervals for



Figure 8: Parameter uncertainty: price and wage setting. The welfare under the Ramsey policy (red line) and the benchmark policy rule (black dashed). The two vertical lines indicate the 5 and 95 percent bounds from the posterior distributions.

the wage-setting parameters. The one exception is the Calvo wage parameter where values at the upper end of the posterior distribution entail significant costs. Although the degree of wage indexation is imprecisely estimated, the relative welfare loss is nearly invariant to the value of this parameter. Taken at face value, these results indicate that in the context of the benchmark model and our estimation methodology, uncertainty of the magnitude that we estimate regarding price- and wage-setting parameters is relatively unimportant for welfare-maximizing policy. However the levels of the losses deteriorate with some parameters, particularly for the wage stickiness parameter, as shown in Figure 8. Even though the relative difference between our benchmark policy rule and the Ramsey policy is small, the results imply significant variations in the welfare costs due to changes in this parameter. Thus, as discussed above, wage stickiness is important for determining the size of the welfare costs of fluctuations. However for our estimated uncertainty about this parameter, there is little effect on policy.

Looked at from a broader, or an explicitly min-max, perspective, however, reducing un-



Figure 9: The difference between welfare under the benchmark policy rule and the Ramsey policy is plotted as the red curve. The two vertical lines indicate the 5 and 95 percent bounds from the posterior distributions.

certainty about price and wage-wetting parameters potentially could yield moderate benefits in terms of monetary policy design and welfare. For example, consider the case that the true Calvo price parameter, ξ_p , is as low as some of the micro evidence suggests. According to the posterior distribution, such a low value is extremely unlikely. But, if true, knowledge of that parameter could be used to design a monetary policy that yields an improvement in welfare equivalent to about a 0.2 percentage point increase in consumption. Similarly, if the true degree of price inflation indexation were significantly higher than we estimate, then knowledge of that parameter would have a large welfare payoff, compared to the outcome under the benchmark rule. Finally, knowledge that the Calvo wage parameter is higher than the point estimate would be very valuable so that the high losses under the benchmark rule could be avoided.

Uncertainty about parameters related to preferences and technology is generally unimportant for the design of welfare-maximizing policy. Figure 9 plots the results for the parameters related preferences and technology. Given the estimated precision of these parameter estimates, parameter uncertainty has trivial implications for welfare and therefore for policy. Although χ , the parameter measuring the disutility of labor, is imprecisely estimated, it has a modest effect on relative welfare.

However, considered from a min-max perspective, the parameter ϕ , which measures the degree of increasing returns, has a significant effect on relative welfare under the benchmark policy. As noted before, with a loose prior, we would estimate a value for this parameter near 2. Assuming that the results of the literature indicating at most modest increasing returns are true, the resulting reduction in uncertainty has a large effect on welfare in this model assuming policy is designed to be optimal at the baseline estimates. Moreover our estimate of the habit persistence parameter is on the low side of recent estimates, which tend to find values in the 0.5-0.7 range. Once again, we find such values to be unlikely but we find significant drops in welfare as the habit parameter increases past 0.5.

Our results in this section can also be informative about the relative importance of some of the different frictions. For example, investment adjustment costs decrease with our parameter ζ . We see that welfare decreases substantially as this parameter increases. Thus investment adjustment costs are not only important to allow the model to fit the data, but they have a significant impact on welfare.

5.2 Monetary Policy under Joint Parameter Uncertainty

In this section we extend the previous results to consider the full effect of uncertainty as measured by our estimated posterior distribution. We compute the welfare losses associated with joint parameter uncertainty of all the structural parameters together. For this purpose, we randomly select 5000 draws of the parameter vector from the posterior distribution described above. For each draw, we compute welfare under the Ramsey policy for those parameters and welfare under the benchmark policy. We then average the results over the



Figure 10: The distribution of welfare losses for the estimated posterior distribution. Left panel: Welfare losses relative to the steady state for the Ramsey policy tuned to each parameter draw (red line) and the benchmark wage targeting rule (dashed black line) Right panel: Relative welfare loss between the Ramsey and benchmark policy.

set of draws and refer to this figure as the expected relative welfare loss. This approach incorporates the covariance between the model parameters, allowing for the possibility that particular combinations of parameter realizations may have more sizeable effects on policy than in the analysis above.

Based on a preliminary set of 2000 draws, we find that the expected welfare loss under the benchmark policy is only modestly worse than that under the Ramsey policy. This is shown in Figure 10 which plots the distributions of the welfare losses under the Ramsey policy tuned to each parameter realization compared to those of the benchmark wage inflation targeting rule. In the left panel we see that the distributions of losses are very similar, centered on means around 1.4 % of consumption, but with a relatively long left tail. Thus sampling uncertainty can easily make the welfare costs of fluctuations more than double what we estimate. However parameter uncertainty in itself has little effect on policy, as the right panel of the figure shows that the benchmark policy rule does nearly as well as the Ramsey policy as the relative welfare losses are mostly in the 0-0.2% range. This finding

supports the parameter-by-parameter analysis above and confirms the conclusion that the welfare benefits for policy of reducing sampling variation in our baseline model under the benchmark policy are quite modest.

6 Specification Uncertainty

This section expands upon the analysis of the preceding section by considering specification uncertainty in the sense of Leamer (1978). In specifying the benchmark model, we made numerous choices that affect the parameter estimates and the structure of the model and the determinants of welfare. In this section, we analyze the sensitivity of optimal policies to alternate assumptions regarding the model specification and evaluate the marginal benefit of reducing uncertainty of each of the key specification issues in terms of social welfare. As in the preceding section, this analysis provides information on the value, from the perspective of monetary policy, in improving our knowledge of specification issues and suggests where the highest payoffs are for further research in this area. While the list of specifications we consider is far from exhaustive, it provides some examples of the type of specification uncertainty that may be important for policy analysis.

6.1 Innovation Uncertainty

We start by considering alternative assumptions regarding the set of shocks included in the model and the characterization of shocks in terms of their relationship to welfare. In computing welfare, we have had to take a stand on each shock as to whether it reflects shifts in fundamentals, the effects of distortions, or measurement error. In particular, we have assumed that the wage and price shocks and the shocks to the equity premium are distortionary, while the remaining shocks reflect shifts in fundamentals. We now revisit these assumptions and evaluate the performance of the benchmark policy under alternative assumptions regarding the nature of innovations.

	Ramsey	Benchmark	
Experiment	Policy	Policy Rule	Difference
Benchmark specification	-1.40	-1.49	-0.09
No shock to Tobin's Q	-1.46	-1.57	-0.11
Distortionary time preference shocks	-2.10	-2.19	-0.09
Distortionary labor disutility shocks	-1.86	-1.96	-0.09
Distortionary investment shocks	-0.95	-1.07	-0.09
Combined distortionary shocks	-2.11	-2.23	-0.12
No price shocks	-1.33	-1.40	-0.07
No wage shocks	-0.24	-0.30	-0.06
No price or wage shocks	-0.17	-0.21	-0.04

 Table 4: Welfare under Innovation Uncertainty

The benchmark model is admittedly profligate in specifying shocks. In particular, the equity premium shock η^Q has a large estimated variance and is important for monetary policy and welfare, but arguably lacks any microfoundations. Importantly, we have assumed that this shock does not affect fundamentals, but instead represents a type of "animal spirits" or inefficient fluctuations in an external finance premium that monetary should counteract. In contrast, we have assumed that investment adjustment costs shocks reflect fundamentals and their effects should be accommodated. We therefore consider an alternative model specification in which these η^Q shocks do not exist, that is, Tobin's Q strictly follows fundamentals. We estimated this alternative model. The resulting posterior mode estimate of ζ falls from the benchmark value of 0.54 to 0.16, implying significantly *higher* costs of adjusting investment. In addition, the investment adjustment cost shock becomes more variable and less persistent, as it absorbs more of the variation that had been captured by the i.i.d. equity premium shock. Estimates of the remaining parameters are nearly identical to the benchmark estimates. Detailed estimation results are reported in Table 5.

The benchmark rule is also nearly optimal in this alternative model. The second line of Table 4 reports the results from this experiment. Welfare under the Ramsey policy and the benchmark policy fall by about the same amount relative to the benchmark specification, despite the change in the adjustment cost parameter and the shift of one source of output variation from non-fundamentals to fundamentals.

We then examined how the benchmark policy under alternative assumptions regarding the nature of other shocks. In the benchmark model, shocks to the time preference, the disutility of labor, and the investment adjustment costs reflect fundamental movements in the economy that should not be counteracted by monetary policy, leaving little scope for output stabilization relative to the stabilization of wage and price inflation. We now modify this assumption and assume that the shocks reflect non-fundamentals. We considered each shock in isolation, then combined the three. The results are shown in the middle section of Table 4. Again, the benchmark policy is nearly efficient.

Finally, we consider an alternative assumption regarding the shocks to prices and wages. In the benchmark model, they are viewed as being distortionary movements in markups. Here, we instead assume that these shocks represent measurement error evident in estimating the model, but have no effects on the actual allocation of resources. In examining this alternative assumption, we do not re-estimate the model, but instead simply zero out these residuals. As before, we consider each shock in isolation and the combined effect. The results are shown in the lower part of Table 4.

Eliminating both the price and wage shocks reduces by half the welfare gap between the Ramsey policy and the benchmark rule. The remaining gap is only 0.04 percent in terms of foregone consumption, or about \$11 per person per year. The price shocks have relatively little effect on welfare. The wage shocks, however, are an important source of welfare loss under both the Ramsey and the benchmark policies, but have little effect on the relative performance of the benchmark policy rule.

In summary, the benchmark rule is remarkably robust to changes in assumptions regarding the nature of shocks hitting the economy. We now turn to alternative forms of specification uncertainty.

6.2 Monetary Frictions and Working Capital

The model we have described completely abstracts from monetary frictions, again following much of the recent literature. However if monetary frictions are important, either on the consumer side or via a cash flow channel in firm investment, we are potentially missing some important effects. We now introduce those frictions in the model, following Christiano, Eichenbaum, and Evans (2005).

First, to introduce a role for cash balances in transactions we alter the household preferences to:

$$E\sum_{t=0}^{\infty}\epsilon_t^B\beta^t\left[V_t(j)+\epsilon_t^m\frac{(M_t/P_t)^{1-\kappa}}{1-\kappa}\right],$$

where M_t represents nominal cash balances, and ϵ_t^m is an autocorrelated shock to money demand. Households invest the remainder of their assets $A_t - M_t$ (which we relate to broad money) with a financial intermediary earning the nominal interest rate R_t . The first order condition for the household portfolio allocation is then:

$$\epsilon_t^B \epsilon_t^m (M_t/P_t)^{-\kappa} = (R_t - 1) M U C_t.$$
(21)

For the firm side, we suppose that firms must borrow from financial intermediaries to pay their wage bill. The loan is paid back at the end of the period at gross interest rate R_t , making firms' costs $R_tW_tL_t + R_t^kU_tK_{t-1}$. Thus the cost minimization condition (15) now becomes:

$$\frac{R_t W_t L_t}{R_t^k U_t K_{t-1}} = \frac{1-\alpha}{\alpha} \tag{22}$$

In turn, the marginal cost changes from (16) to:

$$MC_t = \frac{(R_t W_t)^{1-\alpha} (R_t^k)^{\alpha}}{\epsilon_t^A} \alpha^{-\alpha} (1-\alpha)^{\alpha-1}.$$
 (23)

Market clearing in the loan market then implies:

$$W_t L_t = A_t - M_t,$$

where A_t represents level of broad money after the infusion of money via the central bank.

Since we specify that policy is conducted via an interest rate rule, we do not need to concern ourselves with the market clearing in the loan market. This would only serve to pin down the value of broad money A_t . Instead, we simply append the portfolio allocation equation (21) to the model to determine the household's cash balances (which now affect welfare), and we make the changes (39) and (23) reflecting the role of working capital. Our estimation procedure now incorporates data on cash balances, and we now estimate the additional parameters (κ , ρ_m , σ_m).²⁴ In the third column of Table 5 we report the estimates of the model with monetary frictions.

The modified model has two implications for policy. First, owing to the effects of nominal interest rates on costs and money balances, the optimal inflation rate is below zero. In the estimated model, this influence is however dominated by the desire to avoid deflation-induced dispersion in prices and wages, and the optimal inflation rate is almost exactly zero.

Second, there is a cost to highly variable nominal interest rates that is absent in the benchmark model and a resultant benefit to smoothing interest rates. As a result, the optimal policy responds less aggressively to shocks. This is reflected in a deterioration in the welfare performance of the benchmark rule, which falls to about -0.3 percent of consumption relative to the Ramsey policy. A policy that responds less aggressively to wage inflation performs better in the model with monetary frictions and working capital.

 $^{^{24}}$ The monetary data is only available from 1959 onward, so we shorten the sample by four years. Linearized expressions are again given in the appendix.

		Benchmark	No Tobin's	Money
Para	meter	Specification	Q Shock	Frictions
ζ	Investment adjustment	0.5368	0.1594	0.6142
σ	Consumption Utility	2.0434	2.2123	1.9962
θ	Consumption Habit	0.2978	0.2862	0.3272
χ	Labor Utility	1.2961	1.7631	1.5225
κ	Money Utility	_	—	11.3829
ϕ	Fixed cost	1.0818	1.0860	1.0794
ψ	Capital utilization	0.1986	0.1793	0.1470
ξ_w	Calvo wages	0.7834	0.7718	0.7629
ξ_p	Calvo prices	0.8298	0.8378	0.8258
γ_w	Wage indexation	0.7856	0.7898	0.8557
γ_p	Price indexation	0.1142	0.1687	0.1904
σ_a	Productivity	0.5941	0.5941	0.5804
σ_{π}	Inflation objective	0.1085	0.1115	0.1103
σ_b	Preference	0.1186	0.1448	0.1184
σ_{g}	Govt. spending	0.2846	0.2835	0.2855
σ_l	Labor supply	2.2233	2.8811	2.3013
σ_i	Investment	1.0261	1.2214	1.0122
σ_r	Interest rate	0.0000	0.0000	0.0000
σ_q	Equity premium	3.7041	_	3.5533
σ_p	Price markup	0.2025	0.1979	0.1652
σ_w	Wage markup	0.2897	0.2866	0.2871
σ_m	Money demand	_	_	19.9002
ρ_a	Productivity	0.9608	0.9683	0.9568
$ ho_{\pi}$	Inflation objective	0.9959	0.9959	0.9948
$ ho_b$	Preference	0.9461	0.9489	0.9454
$ ho_g$	Govt. spending	0.9422	0.9465	0.9499
$ ho_l$	Labor supply	0.9818	0.9619	0.9831
$ ho_i$	Investment	0.7516	0.4637	0.7667
$ ho_m$	Money demand	—	_	0.9877
r_{π}	Policy, lagged inflation	2.7078	2.3811	2.6745
$r_{\Delta\pi}$	Policy, change in π	0.3053	0.2885	0.2890
r_i	Policy, lagged interest rate	0.8248	0.7901	0.8305
r_y	Policy, lagged output gap	0.0001	0.0001	0.0001
$r_{\Delta y}$	Policy, change in gap	0.4846	0.6111	0.4724

Table 5: Posterior mode estimates under different specifications of the model.

6.3 Alternative Models of Wage Setting

A key result in our analysis is the importance of stabilizing wages owing to the distortions associated with wage dispersion under Calvo-style contracts. Given the importance of this channel we consider alternative specifications of wage setting that have significant effects on the welfare implications of sticky wages and for optimal policy. In particular, we consider three alternative specifications in which the effects of wage dispersion on welfare are muted relative to the Calvo-style model.

We first consider a modest modification to the indexation of wages in the model and assume that non-optimized wages are indexed to last period's *wage* inflation rate, as opposed to price inflation. This modification to the model reduces the effects of fluctuations in wage inflation on wage dispersion and thereby improves welfare. We estimated a version of the benchmark model where wage indexation depends a combination of past price and wage inflation. We found that the weight is primarily on past price inflation, providing support for the benchmark model specification. Nonetheless, one may not be convinced by this finding and remained concerned about uncertainty regarding the form of wage indexation.

Under the Ramsey policy, the consumption-equivalent loss in conditional welfare is somewhat smaller than in the benchmark model; the performance of the benchmark rule, however, is considerably worse under this form of wage indexation. The second line of Table 4 reports the results from this specification for a number of policy rules. Under the benchmark policy, the consumption-equivalent loss in conditional welfare is 0.4 percentage points larger than under the Ramsey policy. Indeed, The benchmark policy does only slightly better than the estimated policy rule, which performs poorly in the benchmark model. Thus, the benchmark policy is not robust to this seemingly innocuous change in the model specification. Evidently, under this form of wage indexation, a better policy is to respond both to wage and price inflation.

	Ramsey	Benchmark	Estimated
Experiment	Policy	Policy Rule	Rule
Benchmark specification	-1.40	-1.49	-2.18
Wage-wage indexation	-1.24	-1.68	-1.74
Taylor wage contracts	-0.28	-0.61	-0.43
Taylor wage and price contracts	-0.18	-0.59	-0.35
No wage dispersion effects	-0.12	-0.41	-0.42

Table 6: Welfare under Uncertainty Regarding Wage Setting

As a larger change to the benchmark model, we next consider a Taylor (1980) contracting approach. This wage setting specification has microfoundations similar to the Calvo model and leads to the many similar expressions. However the empirical performance of the model may be sensitive to which type of nominal rigidity is assumed.²⁵ We first note how some of the key equations of the model change under staggered wage contracting, then we turn to the results.

Following Chari, Kehoe, and McGrattan (2000) we suppose that household wage setting is structured via staggered contracts which are reset every M periods. Thus at t the distribution of wages is given by $\{W_{t,j}\}$ where $j = 1, \ldots, M$ denotes the number of periods since the last re-set. We again allow for partial indexation so the evolution of an individual household's wage is given by:

$$W_{t,j} = \left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma_p} W_{t-1,j-1} \text{ if } j \neq M$$
$$= \tilde{w}_t \text{ if } j = M.$$

The aggregate wage index changes from (10) to:

$$W_t^{-1/\lambda_{w,t}} = \frac{1}{M} \sum_{j=1}^M W_{t,j}^{-1/\lambda_{w,t}}.$$
(24)

²⁵See Chari, Kehoe, and McGrattan (2000), Kiley (2002), and Guerrieri (2001) for example.

Finally, the optimal wage choice satisfies:

$$\frac{\tilde{w}_t}{P_t} E_t \sum_{i=0}^{M-1} \beta^i L_{t+i}^j \epsilon_{t+i}^b \left[\left(\frac{P_t P_{t+i-1}}{P_{t-1} P_{t+i}} \right)^{\gamma_w} \frac{MUC_{t+i}}{1 + \lambda_{w,t+i}} - \epsilon_t^L (L_{t+i}^j)^{\chi} \right] = 0.$$
(25)

We implement the Taylor contract approach by assuming that price and wage contracts last for four quarters, roughly in line with our estimates of the Calvo update parameters, ξ_p and ξ_w of about three-quarters. We do not change any other parameter estimates and revert to the benchmark assumption that wages are indexed to past price inflation. We first consider the case where only wages are set according to staggered contracts; we then consider the case where wages and prices are both set this way. The results are reported in Table 6. As expected, replacing Calvo-style wages with Taylor-style significantly reduces wage dispersion and hence the welfare costs associated with fluctuations.

With Taylor-style staggered wages, the relative performance of the benchmark rule falls dramatically. Indeed, the estimated rule outperforms the benchmark rule with Taylor-style contracts. This result demonstrates the critical role of wage dispersion on the design of welfare-maximizing policies. Additionally switching to Taylor-style price setting reinforces this result. With both wages and prices set according to Taylor-style contracts, the estimated rule yields welfare equivalent to only a 0.17 percent reduction in consumption relative to the Ramsey policy. The benchmark policy, in contrast yields welfare equivalent to a 0.41 percent reduction in consumption relative to the Ramsey policy. Because wage inflation is no longer the dominant determinant of welfare, the benchmark rule can be improved by adding a response to price inflation as well as wage inflation.

Finally, we consider a variant of the quadratic adjustment cost model of Rotemberg (1982). This model yields no wage dispersion and thus can be viewed as the opposite extreme example of the Calvo model, with the Taylor staggered contract model lying between. We find that in the absence of a wage dispersion effect, the welfare costs of fluctuations fall dramatically, to about a 0.1 percent reduction in consumption. Although the benchmark

policy still performs relatively well, the welfare gap between it and the optimal Ramsey policy rises to about 0.3 percent of consumption, about the same as the estimated rule.

These results again bring out the central importance of determinants of wages and labor market imperfections in general for analyzing the welfare costs of macroeconomic fluctuations and the optimal design of monetary policy. While the wage setting models we've considered have been stylized aggregate specifications, our results suggest that there would be significant payoff to incorporating more sophisticated micro-founded labor market dynamics into models designed for policy analysis.

7 Conclusion

Over the past decade there has been remarkable progress in developing empirical microfounded macroeconomic models for monetary policy analysis. In this paper we have drawn on and extended this literature to consider the design of policy under uncertainty. By confronting a fully specified dynamic stochastic general equilibrium with the data we not only insure the plausibility of the model, but we also directly gauge the uncertainty associated with the model. Moreover we've traced through how different assumptions about the model specification get translated into different estimates of the key structural parameters, which in turn may imply different optimal policy responses.

Overall, our results have emphasized the central importance of the labor market for welfare and for policy analysis. We find relatively large welfare costs of fluctuations in our model, and this is largely driven by our estimates of significant wage stickiness. Our main policy implication is that a simple benchmark policy rule which targets wage inflation is both efficient and robust. In nearly all of the cases we considered, the rule performs almost as well as a fully optimal policy, and this performance is relatively unaffected by uncertainty about the structural parameters or the shocks hitting the economy. However the performance of the rule deteriorates more dramatically for different assumptions about wage setting.

Thus we find that a better understanding of the labor market dynamics and wage setting behavior may have a large value for policy makers. Of course, the importance of wage setting and labor dynamics for economic fluctuations has long been recognized, and wage determination is again front and center as an active research area (see Hall (2005) for a discussion). However there is as yet no consensus model of the labor market which can be tractably incorporated in a model for monetary policy analysis. Our results suggest that this will be a fruitful area for future research.

Our approach in this paper has been to examine monetary policy under uncertainty within a single basic model. We started by assuming that uncertainty was reflected by the variation in the values of parameters in the model. We then extended the analysis to uncertainty regarding the shock processes and aspects of the specification of the model. Although we stopped there, specification uncertainty clearly encompasses a far broader range of issues than we were able to consider in one paper. In addition to the labor market issues discussed above, other examples include issues related to an open economy (see Lubik and Schorfheide (2005) for example), financial market frictions that affect household and firms, and imperfect knowledge of the economy associated with learning. Research has found that the policy implications of uncertainty of this broader type far outweigh those associated with parameter and moderate specification uncertainty within a given model (see Levin and J. Williams (2003) and Onatski and N. Williams (2003)). This analysis, however, has not been conducted in a welfare-theoretic framework and represents a natural direction for future research.

Appendix

A The Linearized Model

A.1 Benchmark Model

We estimate and analyze the log-linearized version of the model described above. We use lower-case letters indicates the log-deviation from steady state. In the case of shocks, ϵ_t^x and η_t^x refer to the shocks normalized to the log-linear equations.

To save on notation, we define \bar{R}^k as the mean real rate of return on capital which is assumed to satisfy $\beta = 1/(1 - \delta + \bar{R}^k)$, and ϕ equals 1 plus the share of fixed costs in production. Furthermore, we denote c_y , g_y , and k_y as the steady state ratios of consumption, government spending, and capital to output, respectively.

The following ten equations in ten endogenous variables $\{c_t, i_t, q_t, k_t, r_t^k, u_t, l_t, y_t, w_t, \pi_t\}$

are the linearized counterparts to the equations described in the previous subsection:

$$c_{t} = E_{t} \frac{1}{1+\theta+\beta\theta^{2}} \Big\{ \theta c_{t-1} + (1+\beta\theta^{2}+\beta\theta)c_{t+1} - \beta\theta c_{t+2}$$

$$- \frac{1-\theta}{\sigma} \Big((1-\beta\theta)(r_{t}-\pi_{t+1}) - \epsilon_{t}^{b} + (1+\beta\theta)\epsilon_{t+1}^{b} - \beta\theta\epsilon_{t+2}^{b} \Big) \Big\},$$

$$(26)$$

$$q_t = -(r_t - E_t \pi_{t+1}) + \frac{1}{1 - \delta + \bar{R}^k} \Big\{ (1 - \delta) E_t q_{t+1} + \bar{R}^k E_t r_{t+1}^k \Big\} + \eta_t^q,$$
(27)

$$i_{t} = \frac{1}{1+\beta} E_{t} \Big\{ i_{t-1} + \beta i_{t+1} + \zeta q_{t} + \beta (\epsilon_{t+1}^{i} - \epsilon_{t}^{i}) \Big\},$$
(28)

$$k_t = (1-\delta)k_{t-1} + \delta i_t, \tag{29}$$

$$u_t = \psi r_t^k, \tag{30}$$

$$l_t = -w_t + r_t^k + u_t + k_{t-1}, (31)$$

$$y_t = c_y c_t + g_y \epsilon_t^g + \delta k_y i_t + \bar{R}^k k_y u_t, \qquad (32)$$

$$y_t = \phi \Big[\epsilon_t^a + \alpha (u_t + k_{t-1}) + (1 - \alpha) l_t \Big],$$
(33)

$$w_t = \frac{1}{1+\beta} E_t \Big\{ \beta w_{t+1} + w_{t-1} + \beta \pi_{t+1} - (1+\beta \gamma_w) \pi_t + \gamma_w \pi_{t-1}$$
(34)

$$-\frac{\lambda_w(1-\beta\xi_w)(1-\xi_w)}{(\lambda_w+(1+\lambda_w)\chi)\xi_w}\Big(w_t-\chi l_t-\epsilon_t^L+\frac{\beta\theta}{1-\beta\theta}(\epsilon_t^b-\epsilon_{t+1}^b)-\eta_t^w\tag{35}$$

$$- \frac{\sigma}{(1-\theta)(1-\beta\theta)} ((1+\beta\theta^2)c_t - \theta c_{t-1} - \beta\theta c_{t+1}) \Big) \Big\},$$

$$\pi_t = \frac{1}{1+\beta\gamma_p} \Big\{ \beta E_t \pi_{t+1} + \gamma_p \pi_{t-1} + \frac{(1-\beta\xi_p)(1-\xi_p)}{\xi_p} (\alpha r_t^k + (1-\alpha)w_t - \epsilon_t^a + \eta_t^p) \Big\} (36)$$

As shown in Onatski and N. Williams (2004), equation (32) corrects a slight error in Smets and Wouters (2003a) due to the capital utilization costs which enters as the final term.

In addition, there are ten equations for the shock processes, six of which, $\{\epsilon_t^a, \epsilon_t^\pi, \epsilon_t^b, \epsilon_t^g, \epsilon_t^i, \epsilon_t^i\}$, follow AR(1) processes of the form:

$$\epsilon_t^x = \rho_x \epsilon_{t-1}^x + \nu_t^x,$$

where ν_t^x is a mean zero innovation with variance σ_x^2 . The remaining four shocks, $\{\eta_t^p, \eta_t^q, \eta_t^r, \eta_t^w\}$, are assumed to be i.i.d. with mean zero and variance σ_x^2 . The innovations are assumed to have zero contemporaneous correlation.

The full model also includes counterparts to equations (26)-(33) that describe the loglinearized equations for the flexible-price allocation. In these equations, the shocks η_p , η_q , and η_w are set to zero, as is the inflation rate. The nominal interest rate is replaced by the flexible-price real interest rate, r_t^* . This yields nine equations and nine additional variables, $\{c_t^*, i_t^*, y_t^*, w_t^*, l_t^*, q_t^*, r_t^{**}, k_t^*\}$, where the asterisk superscript denotes the flexible-price value of the variable.

We close the model by including the linearized counterpart to the policy rule:

$$r_{t} = r_{i}r_{t-1} + (1 - r_{i})\left(\bar{\pi}_{t} + r_{\pi}(\pi_{t-1} - \bar{\pi}_{t}) + r_{y}(y_{t-1} - y_{t-1}^{*})\right)$$

$$+ r_{\Delta\pi}(\pi_{t} - \pi_{t-1}) + r_{\Delta y}(y_{t} - y_{t}^{*} - (y_{t-1} - y_{t-1}^{*})) + \eta_{t}^{i}.$$
(37)

A.2 Model with Monetary Frictions

Here we note the modifications to the expression above when we consider the role of monetary frictions. Linearizing (21) gives:

$$-\kappa m_t + \epsilon_t^m = \frac{\bar{R}}{\bar{R} - 1} r_t + \frac{\sigma}{(1 - \theta)(1 - \beta\theta)} ((1 + \beta\theta^2)c_t - \theta c_{t-1} - \beta\theta E_t c_{t+1}) - \frac{\beta\theta}{1 - \beta\theta} (\epsilon_t^b - E_t \epsilon_{t+1}^b)$$
(38)

where m_t is the log-deviation of real cash balances. Here \bar{R} is the steady state gross nominal rate which satisfies $\bar{R} = 1 + \bar{R}^k - \delta$. Linearizing (22) we see that we replace (31) with:

$$l_t = -w_t - r_t + r_t^k + z_t + k_{t-1}$$
(39)

Linearizing (23) we see that we replace (36) with:

$$\pi_t = \frac{1}{1+\beta\gamma_p} \Big\{ \beta E_t \pi_{t+1} + \gamma_p \pi_{t-1} + \frac{(1-\beta\xi_p)(1-\xi_p)}{\xi_p} (\alpha r_t^k + (1-\alpha)(w_t + r_t) - \epsilon_t^a + \eta_t^p) \Big\}.$$

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