Endogenizing Firm Scope: Multiproduct Firms in International Trade

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VERY PRELIMINARY AND INCOMPLETE

Abstract

We develop a theory of multiproduct firms that differ in their organizational capabilities. In our model, a firm's unit cost is the endogenous outcome of its choice of the number of its product lines. The more product lines a firm manages, the higher are its units costs, but this trade-off is less severe for firms with greater organizational capabilities. Paradoxically, more efficient firms optimally increase their scope to such an extent that their unit costs are higher than those of less efficient firms. Our model thus explains the empirical puzzle that there is a negative relationship between firm size and Tobin's Q. Positive industry shocks – such as those caused by trade liberalization – induce a merger wave that alters the intra-industry dispersion of observed productivities as high-Q firms buy or sell product lines with low-Q firms.

1 Introduction

It is well recognized that at any given point in time there is a lot of heterogeneity amongst firms within the same industry, both in the value of inputs employed and in the value of outputs produced. As recently documented in Bernard, Redding, and Schott (2005), differences in the value of outputs across firms not only reflect differences in per-product output levels but also large differences in the number of product lines managed by different firms. The conventional wisdom is that large firms have become large because they have been more productive. But this wisdom contradicts the little-known fact that larger firms have lower Tobin's Q than smaller firms (Eeckhout and Jovanovic, 2002).

It is well recognized that there is a lot of reallocation of resources across firms within the same industry – either due to idiosyncratic or aggregate shocks. This shows up in the data through large magnitudes of job reallocation (Davis and Haltiwanger, 1999), simultaneous firm entry and exit (Dunne, Roberts, and Samuelson, 1988), and product switching (Bernard, Redding, and Schott, 2005). Some of this within-industry reallocation occurs through mergers and acquisitions: firms with high values of Tobin's Q typically purchase firms with low values

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of Tobin's Q (Andrade, Mitchell, and Stafford, 2001). A considerable fraction of these mergers and acquisitions take the form of divestitures and acquisitions of product lines rather than entire companies (Maksimovic and Phillips, 2001).

In an increasingly globalized world, trade liberalization and market integration represent important aggregate shocks affecting the allocation of resources both across and within industries. Indeed, following trade liberalization and market integration, many plants and divisions experience a productivity improvement (Trefler, 2004; Pavcnik, 2002), and large companies downsize by divesting product lines. There is also evidence showing that trade liberalization induces waves of mergers and acquisitions (Breinlich, 2005).

Much of the economics literature posits that observable firm heterogeneity is due to intrinsic (unobservable) differences in firms' "efficiency" or "organizational capability". As Lucas (1978) notes, the following question then arises: why does the most efficient firm not manage all resources? In the context of his single-product model, the answer given by Lucas is that there are diminishing returns to the *span of control*: holding fixed the firm's intrinsic efficiency, its marginal cost is increasing with the level of output. In the present paper, we assume instead that there are constant returns at the product level but diminishing returns to the span of control at the firm level: the more product lines a firm manages, the less good it is at managing each one of them. We posit that (i) holding fixed the number of product lines, firms with greater organizational capability have (weakly) lower unit costs, and that (ii) the greater is a firm's organizational capability, the less responsive are unit costs to an increase in the number of product lines.

In this paper, we develop a model where firms differ in their organizational capabilities. We analyze the relationship between a firm's organizational capability and its equilibrium number of product lines. We show that firms with greater organizational capability choose to manage a (much) larger number of product lines. Paradoxically, this choice will result in these firms exhibiting higher unit costs of output than firms with lesser organizational capability. Our paper thus resolves the empirical puzzle that larger firms have lower values of Tobin's Q. Hence, what might appear to be a large unproductive conglomerate may in fact be a highly efficient firm. This observation has important implications for productivity measurement. Since a firm's unit cost of output is endogenous (and may be inversely related to the firm's organizational capability), estimating a firm's intrinsic efficiency requires correcting for the endogenous number of product lines.

In an extension of the model to two countries, we investigate the effects of two types of trade liberalization on firms' choice of product lines: a symmmetric trade liberalization, where both countries reduce their tariff barriers, and a unilateral trade liberalization, where only one country reduces its tariffs. For each type of trade liberalization, we distinguish between the short-run and the long run effects. In the short run, the number of product lines in the industry is fixed but firms can trade product lines on a competitive merger market. In the long run, however, both the number of product lines and the number of firms are endogenous.

Holding fixed the market price of product lines, a symmetric trade liberalization implies that all firms want to add product lines, but this incentive is greater for smaller firms (i.e., for firms with lesser organizational capability and lower observed unit costs) than for larger firms. In the short run, the market price of product lines therefore increases, and large firms sell product lines to small firms. Our model thus generates a number of empirical implications. First, trade liberalization leads to a merger wave. Second, trade liberalization leads to a "flattening" of the size distribution of firms: large firms downsize while small firms become larger. Third, trade liberalization leads to a reallocation of assets (product lines) from low-Q firms with high unit costs to high-Q firms with low unit costs, and reduces the "weighted average" unit costs in the industry.

A unilateral trade liberalization in country i has an asymmetric effect on the reallocation of product lines in the two countries: in the liberalizing country i, small firms will sell product lines to large firms, while the opposite occurs in country $j \neq i$, which now faces lower foreign tariffs. The long run effects of a symmetric or unilateral trade liberalization on the size distribution of firms are qualitatively similar to the short run effects.

Related Literature. This paper contributes to the literature on firm heterogeneity that spans many fields (Jovanovic, 1982; Hopenhayn, 1992; Melitz, 2003; Bernard, Eaton, Jensen, and Kortum, 2003). In this literature, each firm is assumed to produce only a single product, and firms with higher efficiency levels have lower unit costs and produce more output. The efficiency advantage of larger firms results in a positive relationship between firm size and Tobin's Q, thus contradicting the empirical evidence. In contrast, in the present paper firms can choose to manage multiple products, and firm heterogeneity takes a different form. Were all firms to produce a single product, they would have the same unit costs. However, firms differ in the extent to which an increase in the number of product lines raises unit costs. Since the number of product lines managed by each firm is a choice variable, each firm's unit cost is an endogenous outcome. In equilibrium, there is an inverse relationship between a firm's organizational capability and its unit cost of production.

Within the literature on firm heterogeneity and trade, Melitz (2003) is also concerned with the productivity effects of trade liberalization and market integration. In his model, the aggregate productivity effects are the result of changes in the composition of production across firms. By construction, there are no productivity effects at the firm level. Hence, in contrast to the present paper, Melitz (2003) is unable to explain Trefler's (2004) finding that trade liberalization improves labor productivity of plants in large firms.

There are two recent papers that analyze the effects of trade liberalization with multi-product firms. Neither paper is able to explain the negative relationship between firm size and Tobin's Q found in the data. Baldwin and Gu (2005) consider an oligopoly model where firms can choose to manage multiple products. Firms with higher efficiency levels are assumed to have lower unit costs. As in Melitz (2003), trade liberalization has no effect on firm-level productivity, and there are no mergers and acquisitions. In contrast, trade liberalization does affect firm-level (but not product-level) productivity in Eckel and Neary (2005). However, in their paper, there is no firm heterogeneity, and therefore no mergers and acquisitions.

Plan of the Paper. In the next section, we describe and analyze a model of a monopolist who can choose the number of its product lines. We show that, under general conditions on demand, the monopolist's choice will result in a negative relationship between the monopolist's organizational capability on the one hand and Tobin's Q and marginal cost on the other. In section 3, we embed this model in a monopolistic competition setting and analyze the effects of industry shocks. In section 4, we analyze the effects of both a symmetric and a unilateral trade liberalization in a two-country version of the model. In section 5, we test two key predictions of our model using firm-level data from the United States. In the final section, we discuss our results and conclude.

2 Organizational Capability and the Optimal Scope of a Multiproduct Firm

In this section, we develop a simple model of a firm that can choose how many product lines to manage. We analyze how firms with different organizational capabilities solve the fundamental trade off between firm scope and productivity.

2.1 The Model

A firm with organizational capability $\theta > 0$ can manage any number $n \geq 1$ of product lines. (For simplicity, we will treat n as a continuous variable.) We assume that the firm has constant returns at the product level but decreasing returns to the span of control at the firm level: the more products a firm manages, the higher are its unit costs for each product line. In this model, we abstract from firm heterogeneity amongst single-product firms that lies at the heart of the recent literature on firm heterogeneity and trade. Instead, we focus on firm heterogeneity in the trade off between firm scope and productivity.

The firm faces two types of costs. First, there is a fixed cost r per product line. This can be thought of as either a cost of inventing a product or as a cost of purchasing an existing product line. Second, there is a constant marginal cost associated with the production of each unit of output. The marginal cost of a firm with organizational capability θ that manages n product lines is given by

$$c(n;\theta) = c_0 n^{1/\theta}. \tag{1}$$

This formulation of marginal cost captures in a simple way the following properties. First, organizational capability does not affect production costs of single-product firms: $c(1;\theta) = c_0$. Second, the greater is a firm's organizational capability θ, the lower are the marginal costs of a multiproduct firm, $dc(n;\theta)/d\theta < 0$ for $n > 1$. Third, an increase in the number of product lines increases the firm's marginal cost, $dc(n;\theta)/dn > 0$, but at a slower rate for firms with greater organizational capability, $d^2c(n;\theta)/dnd\theta < 0$. In fact, in our formulation, organizational capability θ is the inverse of the elasticity of marginal cost with respect to the number of product lines.

On the demand side, product lines are symmetric, and there are no demand linkages. For each product line, the firm faces market demand $Q(p)$, where p is the price of that product. We assume that demand is downward-sloping, $Q'(p) < 0$ for all p such that $Q(p) > 0$. Further, we impose a mild regularity condition on the demand function which requires that demand is not too convex, $Q'(p) + [p - c_0]Q''(p) \leq 0$ for all $p \geq c_0$.[1]

The firm's optimization problem consists in choosing the number of product lines, n, and the price for each product line i, p_i, so as to maximize its profit.

[1] In fact, a weaker condition would suffice for our main result. Let $p(c)$ denote the profit-maximizing price of

2.2 Equilibrium Analysis

We consider first the firm's price-setting problem for any given number of product lines. Since the firm has the same (constant) marginal cost for each product line and the demand function is the same for each product line, the firm will optimally charge the same price for each product line. Let $p(c(n;\theta))$ denote the profit-maximizing price of a firm with organizational capability θ that manages n product lines. Since there are no demand linkages between product lines, the firm's price setting problem can be analyzed separately for each product line. Hence,

$$p(c(n;\theta)) \equiv \arg\max_p [p - c(n;\theta)]Q(p).$$

The first-order condition is given by

$$Q(p(c(n;\theta))) + [p(c(n;\theta)) - c(n;\theta)]\, Q'(p(c(n;\theta))) = 0. \tag{2}$$

We consider now the firm's optimal choice of the number of product lines. Given the optimal pricing policy, the profit of a firm with organizational capability θ that manages n product lines is given by

$$n\left[\pi(c(n;\theta)) - r\right],$$

where

$$\pi(c(n;\theta)) \equiv [p(c(n;\theta)) - c(n;\theta)]\, Q(p(c(n;\theta))) \tag{3}$$

is the firm's gross profit per product line. From the envelope theorem, $\pi'(c(n(\theta);\theta)) = -Q(p(c(n(\theta);\theta)))$, and so the first-order condition for the optimal choice of the number of product lines, $n(\theta)$, can be written as

$$[\pi(c(n(\theta);\theta)) - r] - n(\theta)Q(p(c(n(\theta);\theta)))\frac{\partial c(n(\theta);\theta)}{\partial n} = 0. \tag{4}$$

The impact of an additional product line on the firm's profit can be decomposed into two effects. The first term on the l.h.s. of equation (4) is the net profit of the marginal product line. The second term summarizes the negative externality that the marginal product line imposes on the $n(\theta)$ inframarginal product lines: the production cost of each product line increases by $Q(p(c(n(\theta);\theta)))\partial c(n(\theta);\theta)/\partial n$ since the firm is now less good at managing each one of them.

>From the cost function (1), $n(\theta)\partial c(n(\theta);\theta)/\partial n = (1/\theta)c(n(\theta);\theta)$. Hence, the optimal choice of the number of product lines, $n(\theta)$, enters the first-order condition (4) only through the induced marginal cost $c(n(\theta);\theta)$. This means that the firm's problem can equivalently be viewed as one of choosing c rather than n. Indeed, using the gross profit function (3), the first-order condition can be rewritten as

$$\Psi(c(\theta);\theta) \equiv \{[p(c(\theta)) - c(\theta)]\, Q(p(c(\theta)) - r\} - \frac{c(\theta)}{\theta}Q(p(c(\theta))) = 0, \tag{5}$$

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a firm with marginal cost c. Then, we assume that

$$\left[1 + \frac{p(c) - c}{p(c)}\right] Q'(p(c)) + [p(c) - c]\, Q''(p(c)) \leq 0$$

for all $c \geq c_0$.

where $c(\theta) \equiv c(n(\theta);\theta)$.

Henceforth, we will assume that the fixed cost r is not too large so that the firm can make a strictly positive profit by managing a single product line, i.e.,

$$\pi(c_0) = [p(c_0) - c_0]\, Q(p(c_0)) > r.$$

We are now in the position to state our central result on the relationship between a firm's organizational capability and its observed productivity.

Proposition 1 *The optimal choice of product lines is such that the induced marginal cost $c(\theta)$ is weakly increasing in the firm's organizational capability θ. Specifically, there exists a unique cutoff $\widetilde{\theta}$ given by*

$$\widetilde{\theta} \equiv \frac{c_0}{[p(c_0) - c_0]\, Q(p(c_0)) - r}$$

such that $c(\theta) = c_0$ for all $\theta \leq \widetilde{\theta}$, and $c(\theta)$ is strictly increasing in θ for all $\theta \geq \widetilde{\theta}$.

Proof. See appendix. ■

For a given number n of product lines, the negative externality that the marginal product line exerts on the inframarginal product lines is the smaller, the greater is the firm's organizational capability. Not surprisingly then, firms with greater organizational capability will optimally choose a weakly larger number of product lines than firms with inferior organizational capability: $n(\theta) = 1$ for $\theta \leq \widetilde{\theta}$, and $n(\theta)$ is strictly increasing in θ for $\theta \geq \widetilde{\theta}$. Perhaps paradoxically, however, for $\theta \geq \widetilde{\theta}$, $n(\theta)$ is increasing so fast with θ that firms with greater organizational capability will, in fact, exhibit higher unit costs. To see this, consider two firms, firm 1 and firm 2, with organizational capability $\theta_1 \geq \widetilde{\theta}$ and $\theta_2 > \theta_1$, respectively. From the first-order condition (5), firm 1 will optimally choose $n(\theta_1)$ such that its marginal cost $c(\theta_1)$ satisfies $\Psi(c(\theta_1);\theta_1) = 0$. Suppose now firm 2 were to choose the number of product lines such that its induced marginal cost is also equal to $c(\theta_1)$. If so, the two firms would charge the same price $p(c(\theta_1))$ and sell the same quantity $Q(p(c(\theta_1)))$ per product line. Hence, the net profit of the marginal product line, $[p(c(\theta)) - c(\theta)]\, Q(p(c(\theta)) - r$, would be the same for the two firms. However, as can be seen from equation (5), the absolute value of the negative externality that the marginal product line imposes on the inframarginal product lines, $\chi(c(\theta);\theta) \equiv (1/\theta)c(\theta)Q(p(c(\theta))$, is smaller for the firm with the greater organizational capability, and so $\Psi(c(\theta_1);\theta_2) > 0$. Hence, firm 2 can increase its profit by further adding product lines, even though this implies higher unit costs, $c(\theta_2) > c(\theta_1)$. This is illustrated graphically in figure **??**.

Proposition 1 shows that observed unit cost is *inversely* related to the firm's intrinsic efficiency (its organizational capability θ). This raises a potentially important conceptual issue for empirical work that attempts to identify a firm's intrinsic efficiency from its costs. Our model shows that even if unit costs are observable such an exercise is valid only if one corrects for the number of product lines:

$$\theta = \frac{\ln(n)}{\ln\left(\frac{c}{c_0}\right)}.$$

In practice, it is often hard to measure costs correctly. A popular alternative measure of firm efficiency is Tobin's Q, the market-to-book ratio

$$T(\theta) \equiv \frac{M(\theta)}{B(\theta)},$$

where

$$M(\theta) \equiv n(\theta)Q(p(c(\theta)))\,[p(c(\theta)) - c(\theta)]$$

is the market value of a firm (including its assets) or, equivalently, its gross profits, and

$$B(\theta) \equiv n(\theta)r$$

is the firm's book value, i.e., the value of its assets (product lines). The next lemma shows that this often-used measure of firm efficiency is also negatively related to a firm's intrinsic efficiency.

Lemma 1 *A firm's market-to-book ratio (Tobin's Q), $T(\theta)$, is decreasing in the firm's organizational capability θ.*

Proof. Tobin's Q is given by

$$\begin{aligned} T(\theta) &\equiv \frac{M(\theta)}{B(\theta)} = \frac{n(\theta)Q(p(c(\theta)))\,[p(c(\theta)) - c(\theta)]}{n(\theta)r} \\ &= \frac{Q(p(c(\theta)))\,[p(c(\theta)) - c(\theta)]}{r}, \end{aligned}$$

and so equal to a firm's gross profit per product line. Clearly, this profit is lower the larger are the firm's unit costs. Since unit cost $c(\theta)$ is increasing in θ, it follows that $T(\theta)$ is decreasing in θ. ■

Our model predicts a relationship between organizational capability θ and various measures of firm size. Let

$$S(\theta) \equiv n(\theta)Q(p(c(\theta)))p(c(\theta))$$

denote the sales of a firm with organizational capability θ.

Lemma 2 *A firm's sales $S(\theta)$, book value $B(\theta)$, and market value $M(\theta)$ are increasing in the firm's organizational capability θ.*

Proof. See appendix. ■

Lemma 1 establishes a relationship between Tobin's Q and organizational capability, while lemma 2 establishes a relationship between firm size and organizational capability. We thus obtain the following result.

Proposition 2 *A firm's market-to-book ratio (Tobin's Q), $T(\theta)$, is inversely related to various measures of firm size: sales $S(\theta)$, book value $B(\theta)$, and market value $M(\theta)$.*

Proof. This follows immediately from lemmas 1 and 2. ■

This prediction is consistent with the empirical evidence provided in Eeckhout and Jovanovic (2002). Using Compustat data, they show that (i) Tobin's Q is lower for firms with higher book value, and that (ii) Tobin's Q is lower for firms with larger sales.

While consistent with our model, the empirical evidence on the relationship between market-to-book ratio and firm size contradicts the predictions of standard models of firm heterogeneity, including Jovanovic (1982), Hopenhayn (1992), Melitz (2003), and Asplund and Nocke (2006). In these models, firms produce a single product, incur a setup cost r, and differ in their efficiency levels. Let $\pi(\varphi)$ denote the gross profit of a firm with efficiency φ. Then, the market-to-book ratio is given by $\pi(\varphi)/r$, which is strictly increasing in φ since more efficient firms make larger gross profits.

How can we reconcile the fact that larger firms have lower market-to-book ratios than smaller firms with the finding by Bernard, Redding, and Schott (2005) that larger firms exhibit seemingly higher total factor productivity than smaller firms? Suppose that unit costs increase with the number of product lines because as the firm adds product lines it needs to change the composition of its workforce to include more highly talented workers to oversee and coordinate the different product lines. This hypothesis is indeed consistent with the well-known empirical regularity that larger firms pay higher wages. However, the data used in Bernard, Redding, and Schott (2005) do not allow the authors to account for variation in factor quality across firms. To the extent that larger firms use more talented workers, Bernard, Redding, and Schott overestimate the total factor productivity of large firms.

In this section, we have assumed that the firm acts as a monopolist for each one of its product lines. Alternatively, we could have assumed that there is a continuum of monopolistically competitive firms which differ in their organizational capabilities. If the residual demand curve that firms face for each product line satisfies the mild regularity condition we imposed on $Q(\cdot)$, proposition 1 carries over to this setting: firms with greater organizational capability have higher unit costs than firms with inferior organizational capability. In the next section, we will turn to the effects of industry shocks in a monopolistically competitive setting.

3 Firm Scope and Product Market Competition: The Effects of Industry Shocks

In this section, we embed our theory of organizational capability and firm scope into a model of monopolistic competition. Within this model, we consider the effects of an industry-wide shock to either productivity or demand. In our analysis, we distinguish between the short-run and the long run effects. In the short run, the number of firms and product lines is fixed but product lines can be traded between firms. In the long run, both the number of firms and the number of product lines are endogenous.

3.1 The Model

Consider a monopolistically competitive industry with a mass M of firms that differ in their organizational capabilities. Let $G(\cdot)$ denote the c.d.f. of organizational capability $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$ in the population of firms. Each firm can choose how many product lines to manage. For each product line that a firm operates, it needs to incur a cost of r. The marginal cost of a firm with organizational capability θ that manages n product lines consists of two components:

$$c(n;\theta)+t,$$

where the first component, $c(n;\theta)$, depends, as before, on the firm's organizational capability θ and the number n of its product lines,

$$c(n;\theta)=c_0 n^{1/\theta},$$

while the second component, t, is common to all firms and can be thought of as transport or distribution costs, or more generally as the price of an intermediate input that enters the product function in a Leontief fashion.

There is a mass S of identical consumers with the following linear-quadratic utility function:

$$U=\alpha\int x(i)di-\int [x(i)]^2\,di-2\sigma\left[\int x(i)di\right]^2+H,$$

where $x(i)$ is consumption of product line i, H is consumption of the Hicksian composite commodity, and $\alpha,\sigma>0$ are parameters. Assuming that consumer income is sufficiently large, each consumer's inverse demand for product line i is then given by

$$p(i)=\alpha-2x(i)-4\sigma\int x(j)dj.$$

3.2 Equilibrium

We begin our equilibrium analysis with a discussion of a firm's optimal output decision.[2] Since each product line is of measure zero, a firm's choice of output for one product line does not affect its choice of output for another product line. Note also that a firm's output for a given product line is S times each consumer's consumption of that product. Consider a firm with marginal cost $c+t$. It chooses output per product line, $q(c+t)$, so as to maximize its profit per product line:

$$q(c+t)=\arg\max_q q\left[\alpha-2\frac{q}{S}-4\sigma\frac{\int q(c+t)\mu(dc)}{S}-c-t\right],$$

where μ is a Borel measure summarizing the (endogenous) distribution of marginal costs of product lines. That is, for any interval A, $\mu(A)$ gives the mass of product lines with marginal costs in A.

It is straightforward to verify that, in equilibrium, each firm faces the residual demand curve

$$\frac{S}{2}\left(a-p\right),$$

[2] Since there is a continuum of firms (and product lines), quantity setting and price setting yield identical equilibrium allocations.

where the endogenous demand intercept a is given by

$$a = \frac{1 + \sigma \int (c+t)\mu(dc)}{1 + \sigma \int \mu(dc)}. \tag{6}$$

Equilibrium output, price, and gross profit per product line of a firm with marginal cost $c+t$ are

$$\begin{aligned} q(c+t) &= \frac{S}{4}\left(a - c - t\right), \\ p(c+t) &= \frac{1}{2}\left(a + c + t\right), \end{aligned}$$

and

$$\pi(c+t) = \frac{S}{8}\left(a - c - t\right)^2.$$

To simplify notation, we will henceforth normalize market size $S \equiv 8$.

We now turn to a firm's choice of the number of product lines. Each firm faces the same linear residual demand curve for each product line, and so we can apply the first-order condition (4) that we derived in section 2:

$$\left[(a - t - c(\theta))^2 - r\right] - 2n(\theta)\left(a - t - c(\theta)\right)\frac{\partial\left[c(n(\theta);\theta) + t\right]}{\partial n} = 0.$$

Since $\partial\left[c(n(\theta);\theta) + t\right]/\partial n = (1/\theta)c(n(\theta);\theta)/n(\theta)$, the first-order condition simplifies to

$$\Phi(c(\theta);\theta;t) \equiv \left[(a - t - c(\theta))^2 - r\right] - \frac{2c(\theta)}{\theta}\left(a - t - c(\theta)\right) = 0, \tag{7}$$

where, as before, $c(\theta) \equiv c(n(\theta);\theta)$. The term in brackets on the left-hand side of the equation is the net profit per product line, while the second term is the negative externality that the marginal product line imposes on profits. For convenience, we will henceforth assume that $[\underline{\theta}, \overline{\theta}]$ is such that for any firm with organizational capability $\theta \in [\underline{\theta}, \overline{\theta}]$ the choice of $c(\theta) + t$ is given by the solution to the first-order condition $\Phi(c(\theta);\theta;t) = 0$.

Note that the equilibrium profit of a firm with marginal cost $c+t$ that faces the residual demand curve $S(a-p)/2$ is the same as the equilibrium profit of a firm with marginal cost c that faces the residual demand curve $S(a-t-p)/2$. Recasting each firm's problem that way, shows that our regularity condition on demand that we imposed in section 2 continues to hold in the case of monopolistic competition with linear demand. Hence, proposition 1 carries over to the current setting: a firm's marginal cost $c(\theta) + t$ is (strictly) increasing with organization capability θ.

$>$From equation (6), the equilibrium value of the demand intercept a is given by

$$a = \frac{1 + \sigma M \int n(\theta)c(n(\theta);\theta)dG(\theta) + \sigma N t}{1 + \sigma N}, \tag{8}$$

where $n(\theta) = \left[c(\theta)/c_0\right]^{\theta}$ is the optimal choice of the number of product lines by a firm with organizational capability θ, and the mass N of product lines satisfies

$$N = M\int_{\underline{\theta}}^{\overline{\theta}} n(\theta)dG(\theta). \tag{9}$$

In the following, we will analyze the effects of an industry shock to the cost parameter t, which equivalently can be thought of as a common shock to preferences. We will first consider the short-run effects before turning to the long-run effects.

3.3 The Short-Run Effects of Industry Shocks

We assume that, in the short, the mass M of firms and the mass $N > M$ of product lines is fixed. We may think of M and N being in pre-shock long-run equilibrium. While the mass N of product lines is fixed in the short-run, we assume that firms can trade product lines at an endogenous market price r. Trade in product lines correspond to partial acquisitions and divestitures, which are about half of all M&A activity in the US (Maksimovic and Phillips, 2001).

Definition 1 *A short-run equilibrium is the collection* $\{n(\cdot), c(\cdot), r, a\}$ *satisfying the cost function (1), the first-order condition (7), the equation for the endogenous demand intercept* a*, (8), and the merger market condition (9).*

The industry shock will lead to a "reshuffling" of product lines across firms in the short run – i.e., a change in $n(\theta)$ – and thus alter the endogenous demand intercept a. The following lemma shows how a changes in response to high-θ firms selling product lines to low-θ firms.

Lemma 3 *Suppose there exists a marginal type* $\widehat{\theta}$ *such that all firms with organizational capability* $\theta > \widehat{\theta}$ *divest product lines,* $\Delta n(\theta) < 0$ *for* $\theta > \widehat{\theta}$*, while all other firms add product lines,* $\Delta n(\theta) > 0$ *for* $\theta < \widehat{\theta}$*, holding the total mass of product lines fixed,* $\int \Delta n(\theta) dG(\theta) = 0$*. Then, the weighted average (by the number of product lines) marginal costs in the industry decrease:*

$$\int \frac{d}{dn} \left[nc(n;\theta) \right] \Big|_{n=n(\theta)} \Delta n(\theta) dG(\theta) < 0.$$

Hence, the endogenous demand intercept a *decreases,* $\Delta a < 0$*.*

Proof. See appendix. ■

The following proposition summarizes the short-run effects of a decrease in the cost parameter t.

Proposition 3 *Consider a small positive industry shock, i.e., a small decrease in the cost parameter* t*,* $dt < 0$*. There exists a marginal type* $\widehat{\theta} \in (\underline{\theta}, \overline{\theta})$ *such that all firms with organizational capability* $\theta > \widehat{\theta}$ *respond by divesting product lines, while all firms with organizational capability* $\theta < \widehat{\theta}$ *respond by purchasing additional product lines.*

Proof. We need to show that $dc(\theta)/dt$ is positive for high-θ (i.e., high-c) firms and negative for low-θ (i.e., low-c) firms. Applying the implicit function theorem to the first-order condition (7), we obtain

$$\frac{dc(\theta)}{dt} = -\frac{\Phi_t(c(\theta);\theta;t)}{\Phi_c(c(\theta);\theta;t)},$$

where the subscript $s \in \{t, c\}$ indicates the partial derivative with respect to variable s. Note that $\Phi_c(c(\theta); \theta; t) < 0$ since $\Phi(c(\theta); \theta; t) = 0$ is a profit maximum. Consequently, the sign of $dc(\theta)/dt$ is equal to the sign of $\Phi_t(c(\theta); \theta; t)$. Market clearing for product lines requires that some firms sell product lines while others purchase product lines, and so the sign of $\Phi_t(c(\theta); \theta; t)$ will vary with θ. In the following, we will show that $d\Phi_t(c(\theta); \theta; t)/d\theta > 0$.

Taking the partial derivative of $\Phi(c(\theta); \theta; t)$, as defined by equation (7), with respect to the cost parameter t, yields

$$\Phi_t(c(\theta); \theta; t) = \left\{ -2\left(a - t - c(\theta)\right) + \frac{2c(\theta)}{\theta} \right\} \left[1 - \frac{da}{dt} \right] - \frac{dr}{dt}. \tag{10}$$

>From the first-order condition (7),

$$\frac{2c(\theta)}{\theta} = \frac{[a - t - c(\theta)]^2 - r}{[a - t - c(\theta)]}.$$

Inserting this expression into equation (10) and simplifying, we obtain

$$\Phi_t(c(\theta); \theta; t) = \left\{ - \frac{[a - t - c(\theta)]^2 + r}{[a - t - c(\theta)]} \right\} \left[1 - \frac{da}{dt} \right] - \frac{dr}{dt}.$$

Observe that θ enters this equation only through the endogenous marginal cost $c(\theta)$. Hence,

$$\begin{aligned} \frac{d\Phi_t(c(\theta); \theta; t)}{d\theta} &= \frac{d}{dc} \left\{ - \frac{[a - t - c(\theta)]^2 + r}{[a - t - c(\theta)]} \right\} \left[1 - \frac{da}{dt} \right] \frac{dc(\theta)}{d\theta} \\ &= \left\{ \frac{[a - t - c(\theta)]^2 - r}{[a - t - c(\theta)]^2} \right\} \left[1 - \frac{da}{dt} \right] \frac{dc(\theta)}{d\theta}. \end{aligned}$$

>From the first-order condition (7), the expression in curly brackets is strictly positive. Since $dc(\theta)/d\theta > 0$, the sign of $d\Phi_t(c(\theta); \theta; t)/d\theta$ is thus equal to the sign of $[1 - da/dt]$.

We claim that $da/dt < 1$. To see this, suppose first that $da/dt = 1$. Then, $d\Phi_t(c(\theta); \theta; t)/d\theta = 0$, and so three cases may arise: (i) $dc(\theta)/dt > 0$ for all θ, (ii) $dc(\theta)/dt < 0$ for all θ, or else (iii) $dc(\theta)/dt = 0$ for all θ. But cases (i) and (ii) cannot occur since there is a fixed number of product lines. Hence, we must have $dc(\theta)/dt = 0$ for all θ; that is, there is no trade in product lines. But then, from equation (8), $da/dt < 1$. A contradiction. Next, suppose that $da/dt > 1$. Then, $d\Phi_t(c(\theta); \theta; t)/d\theta < 0$. Hence, there exists a threshold type $\widehat{\theta} \in (\underline{\theta}, \overline{\theta})$ such that – following a small *increase* in t – all firms with $\theta < \widehat{\theta}$ purchase product lines (and so $dc(\theta)/d\theta < 0$) while all firms with $\theta > \widehat{\theta}$ sell product lines (and so $dc(\theta)/d\theta > 0$). From lemma 3, it follows that this "reshuffling" of product lines reduces the endogenous demand intercept a. From (8), the direct effect of an increase in t on a, holding $n(\theta)$ fixed, satisfies $\partial a/\partial t < 1$. Hence, the total effect of a small increase in t on a satisfies $da/dt < 1$. A contradiction. We have thus shown that $da/dt < 1$, and so there exists a treshold type $\widehat{\theta}$, such that – in response to a small increase in t – all firms with $\theta < \widehat{\theta}$ sell product lines while all firms with $\theta > \widehat{\theta}$ acquire product lines. The reverse conclusion holds if $dt < 0$.

■

Corollary 1 *Consider a positive industry shock, i.e., a decrease in t. Then, firms with large market-to-book ratios $T(\theta)$ purchase product lines from firms with small market-to-book ratios.*

To the extent that much of the merger and acquisition activity is due to positive productivity shocks, as argued by Jovanovic and Rousseau (2002), our model thus predicts that firms with high values of Tobin's Q buy assets from firms with low Tobin's Q. This is indeed consistent with the empirical evidence summarized by Andrade, Mitchell, and Stafford (2001).

3.4 The Long-Run Effects of Industry Shocks

We assume that, in the long run, the mass M of firms and the mass N of product lines are endogenous. Specifically, there is a sufficiently large mass of ex ante identical potential entrants. If a firm decides to enter, it has to pay a fixed entry cost ϕ; if it decides not to enter, it obtains a payoff normalized to zero. After paying the entry cost, a firm a receives a draw of its organizational capability θ from the c.d.f. $G(\cdot)$. A firm then decides on the number of its product lines; the fixed development cost per product line is r.

We assume that the life span of each product line is limited, which implies that, in long-run equilibrium, the market price of each product line is equal to the exogenous development cost r, and the merger market does not play any allocative role. Since potential entrants are ex ante identical, the expected net profit of each entrant must be equal to zero in long-run equilibrium:

$$\int_{\underline{\theta}}^{\overline{\theta}} n(\theta) \left\{ [a - t - c(\theta)]^2 - r \right\} dG(\theta) - \phi = 0. \tag{11}$$

Definition 2 *A long-run equilibrium is a collection $\{n(\cdot), c(\cdot), a, M, N\}$ satisfying the cost function (1), the first-order condition (7), the equation for the endogenous demand intercept a, (8), the adding-up condition (9), and the free-entry condition (11).*

The following result is concerned with long-run effects of a positive industry shock.

Proposition 4 *Consider a positive industry shock, i.e., a decrease in the cost parameter t, $dt < 0$. This will induce an increase in the mass of active firms, $dM > 0$, and in the mass of product lines in the industry, $dN > 0$, but does not affect the mapping from organizational capability to the number of product lines, $dn(\theta) = 0$, and hence $dc(\theta) = 0$.*

Proof. We first show that $da/dt = 1$ so that, following a trade liberalization, $d(a - t) = 0$. Suppose otherwise that $da/dt > 1$ so that, following a reduction in t, $d(a - t) < 0$. Whatever marginal cost a firm chooses in response to the trade liberalization, it could have chosen before the trade liberalization. However, since $a - t$ was larger prior to the trade liberalization, the firm must have made a larger net profit before the trade liberalization. Since this holds for all firms, independently of θ, the expected of an entrant must fall in response to the trade liberalization. But this contradicts the free entry condition (11). A similar argument can be used to show that $da/dt < 1$ is also inconsistent with free entry. Hence, $da/dt = 1$.

Since $a - t$ remains constant, it follows from the first-order condition (7) that the functions $c(\cdot)$ and $n(\cdot)$ remain unchanged. This, in turn, implies that the average number of product lines per firm, N/M, remains unchanged as well. From (8), $\partial a/\partial t \in (0,1)$ and $(\partial a/\partial M)|_{N/M=const.} > 0$, it then follows that the number of firms must increase in reponse to the trade liberalization, $dM > 0$, so that $da/dt = 1$. ■

An immediate consequence of the proposition is that the size distribution of firms is unchanged in the long run.

4 The Effects of Trade Liberalization on Firm Scope

In this section, we turn to the effects of trade liberalization and market integration on firm scope. To this end, we extend the model of section 3. Specifically, we assume that there are two countries, $i = 1, 2$. The cost parameter t is now indexed by a country pair (i, j): t_{ij} is the transport cost or tariff per unit of output from country i to country j. We assume that transport costs have to be incurred only for exports from one country to the other, and so $t_{11} = t_{22} = 0$, $t_{12} > 0$, and $t_{21} > 0$. We assume that countries differ only in their tariffs.

As before, we normalize market size $S_i \equiv 8$ for notational simplicity Consider a firm from country i with marginal production c. Its output $q_{ij}(c)$ and gross profit per product line $\pi_{ij}(c)$ from sales in country j are then given by

$$q_{ij}(c) = 2(a_j - t_{ij} - c),\ i, j = 1, 2,$$

and

$$\pi_{ij}(c) = (a_j - t_{ij} - c)^2,\ i, j = 1, 2,$$

where a_i is the endogenous residual demand intercept in country i.

Lemma 4 *Suppose the two countries impose identical tariffs, $t_{12} = t_{21} = t$, so that the demand intercept is the same in both countries, $a_1 = a_2 = a$. Then, if the common tariff t is sufficiently small, all firms will choose to sell in both countries.*

Proof. Suppose a firm with organizational capability θ chose not to sell in the foreign market. Then, from the first-order condition (7), its induced marginal cost $c(\theta)$ would satisfy $c(\theta) \leq a\theta/(2+\theta) \leq a\overline{\theta}/(2+\overline{\theta})$, where the first inequality is strict if $r > 0$. Hence, if $t < 2a/(2+\overline{\theta})$, the firm could increase its profit by selling in the foreign market, even without changing its number of product lines. ■

Henceforth, we will assume that all firms sell in both countries. The first-order condition (7) then becomes

$$\begin{aligned}\Omega^i(c_i(\theta);\theta;t_{12},t_{21}) \quad &\equiv \quad \left\{(a_i - c_i(\theta))^2 + (a_j - t_{ij} - c_i(\theta))^2 - r_i\right\} \\ &\quad -\frac{2c_i(\theta)}{\theta}\left\{(a_i - c_i(\theta)) + (a_j - t_{ij} - c_i(\theta))\right\} \\ &= \quad 0, \end{aligned} \qquad (12)$$

where $c_i(\theta) = c_0\,[n_i(\theta)]^{1/\theta}$ is the implicit choice of marginal cost by a firm with organizational capability θ based in country i, and r_i the fixed cost per product line in country i. The demand intercept in country i can now be written as

$$a_i = \frac{1+\sigma\int\left[M_i n_i(\theta)c_i(\theta)+M_j n_j(\theta)c_j(\theta)\right]dG(\theta)+\sigma N_j t_{ji}}{1+\sigma(N_1+N_2)},\ i\neq j,\ i=1,2, \tag{13}$$

where M_i is the mass of firms in country i, and N_i the mass of product lines managed by firms from country i, which is given by

$$N_i = M_i \int_{\underline{\theta}}^{\overline{\theta}} n_i(\theta)dG(\theta),\ i=1,2. \tag{14}$$

Implicit in this formulation is the assumption that each firm can produce only in its country of origin. The long-run free-entry condition for country i is

$$\int_{\underline{\theta}}^{\overline{\theta}} n_i(\theta)\left\{[a_i-c_i(\theta)]^2+[a_j-t_{ij}-c_i(\theta)]^2-r_i\right\}dG(\theta)-\phi=0,\ i=1,2. \tag{15}$$

The following result is an extension of lemma 3 to two countries.

Lemma 5 *Suppose there exist marginal types $\widehat{\theta}_1$ and $\widehat{\theta}_2$ such that all firms in country $i\in\{1,2\}$ with organizational capability $\theta>\widehat{\theta}_i$ divest product lines, $\Delta n_i(\theta)<0$ for $\theta>\widehat{\theta}_i$, while all other firms in country i add product lines, $\Delta n_i(\theta)>0$ for $\theta<\widehat{\theta}_i$, holding the total mass of product lines in each country i fixed, $\int \Delta n_i(\theta)dG(\theta)=0$. Then, the weighted average (by the number of product lines) marginal costs of firms producing in country i decreases:*

$$\int \frac{d}{dn}\left[nc_i(n;\theta)\right]\bigg|_{n=n_i(\theta)} \Delta n_i(\theta)dG(\theta)<0.$$

Hence, the endogenous demand intercept a_i decreases, $\Delta a_i<0$.

Proof. See appendix. ■

In short-run equilibrium, we assume that the location of production of a product line is fixed, which implies that the endogenous (short-run) market price of a product line, r_i, may differ across countries. We can then define a *short-run equilibrium* as a collection $\{c_i(\cdot),n_i(\cdot),a_i,r_i\}_{i=1}^2$ satisfying the cost equation (1), the first-order condition (12), the equation for the endogenous demand intercept, (13), and the merger market condition (14).

In long-run equilibrium, we assume that the (exogenous) development cost per product r is the same across countries. We can then define a *long-run equilibrium* as a collection $\{c_i(\cdot),n_i(\cdot),a_i,N_i,M_i\}_{i=1}^2$ satisfying the cost equation (1), the first-order condition (12), the equation for the endogenous demand intercept, (13), the adding-up condition (14), and the free-entry condition (15).

4.1 Symmetric Trade Liberalization

We assume that, initially, the two countries are identical: $N_1=N_2=N$, $M_1=M_2=M$, and $t_{12}=t_{21}=t$. We consider a small symmetric reduction in the common tariff t. We first analyze the short-run effects of such a symmetric trade liberalization.

Proposition 5 *Suppose that the countries impose identical tariffs, $t_{12} = t_{21} = t$, and consider the short-run effects of a small symmetric trade liberalization, $dt < 0$. There exists a marginal type $\widehat{\theta} \in (\underline{\theta}, \overline{\theta})$ such that all firms with organizational capability $\theta > \widehat{\theta}$ respond by divesting product lines, while all firms with organizational capability $\theta < \widehat{\theta}$ respond by purchasing additional product lines.*

Proof. See appendix. ■

An immediate implication of the proposition is that, following a symmetric trade liberalization, large firms decide to downsize by divesting product lines. If the market price of a product line were unchanged, all firms would actually want to purchase product lines. However, the number of product lines is fixed, and so the price per product line r increases in response to a symmetric trade liberalization. Given this price increase, only the firms with the lowest marginal costs (i.e., the firms with inferior organizational capability) find it optimal to add product lines as they

We now turn to the long-run effects of a symmetric trade liberalization.

Proposition 6 *Suppose that the countries impose identical tariffs, $t_{12} = t_{21} = t$, and consider the long-run effects of a small symmetric trade liberalization, $dt < 0$. There exists a marginal type $\widetilde{\theta} \in [\underline{\theta}, \overline{\theta}]$ such that all firms with organizational capability $\theta > \widetilde{\theta}$ have a reduced number of product lines, $dn(\theta) < 0$, while all firms with organizational capability $\theta < \widetilde{\theta}$ have an increased number of product lines, $dn(\theta) > 0$.*

Proof. See appendix. ■

Qualitatively, the long-run effects of a trade liberalization are similar to the short-run effects: there is a tendency for small firms with inferior organizational capability (but low marginal cost) to increase the number of product lines, while the reverse tends to hold for large firms with superior organizational capability (but high marginal cost). In contrast to the short run, however, it is conceivable that $n(\theta)$ moves in the same direction for all firms, namely when $\widetilde{\theta} = \underline{\theta}$ or $\widetilde{\theta} = \overline{\theta}$.

4.2 Unilateral Trade Liberalization

As before, we assume that, initially, the two countries are identical: $N_1 = N_2 = N$, $M_1 = M_2 = M$, and $t_{12} = t_{21} = t$. But we now consider a small unilateral reduction in the tariff t_{21}. We first analyze the short-run effects of such a unilateral trade liberalization by country 1.

Proposition 7 *Suppose that the countries initially impose identical tariffs, $t_{12} = t_{21} = t$, and consider the short-run effects of a small unilateral trade liberalization by country 1, $dt_{21} < 0$. In the liberalizing country 1, there exists a marginal type $\widehat{\theta}_1 \in (\underline{\theta}, \overline{\theta})$ such that all firms with organizational capability $\theta > \widehat{\theta}_1$ respond by purchasing additional product lines, while all firms with organizational capability $\theta < \widehat{\theta}_1$ respond by divesting product lines. In contrast, in country 2, there exists a marginal type $\widehat{\theta}_2 \in (\underline{\theta}, \overline{\theta})$ such that all firms with organizational capability $\theta > \widehat{\theta}_2$ respond by divesting product lines, while all firms with organizational capability $\theta < \widehat{\theta}_2$ respond by purchasing additional product lines.*

Proof. See appendix. ■

The short-run effects of a unilateral trade liberalization are very different than those of a symmetric trade liberalization. In the liberalizing country 1, increased competition with foreign firms induces the largest firms to add product lines while the smallest firms become even smaller as they divest product lines. Hence, a country that unilaterally reduces its trade barriers with the rest of the world will experience a steepening of the size distribution of its firms. The improved access of country 2 firms to country 1's market has the opposite impact on firms in that country: the size distribution becomes flatter as large firms contract and small firms expand.

We now turn to the long-run effects of a unilateral trade liberalization by country 1.

Proposition 8 *Suppose that the countries initially impose identical tariffs,* $t_{12} = t_{21} = t$*, and consider the long-run effects of a small unilateral trade liberalization by country 1,* $dt_{21} < 0$*. In the liberalizing country 1, there exists a marginal type* $\widehat{\theta}_1 \in [\underline{\theta}, \overline{\theta}]$ *such that all firms with organizational capability* $\theta > \widehat{\theta}_1$ *have an increased number of product lines,* $dn_1(\theta) > 0$*, while all firms with organizational capability* $\theta < \widehat{\theta}_1$ *have a reduced number of product lines,* $dn_2(\theta) < 0$*. In contrast, in country 2, there exists a marginal type* $\widehat{\theta}_2 \in [\underline{\theta}, \overline{\theta}]$ *such that all firms with organizational capability* $\theta > \widehat{\theta}_2$ *have a reduced number of product lines,* $dn_2(\theta) < 0$*, while all firms with organizational capability* $\theta < \widehat{\theta}_2$ *have an increased number of product lines,* $dn_2(\theta) > 0$*.*

Proof. See appendix. ■

The long-term implications of a unilateral trade liberalization for the size distribution of firms are similar to those of the short-run. In the liberalizing country, production becomes more concentrated in the largest firms while production becomes less concentrated in the other country. As was the case in the symmetric liberalization, it is possible that all firms within a country contract or expand.

5 Empirics

In this section, we use firm-level panel data to test two key predictions of the model. First, we investigate the relationship between a firm's size as measured by its sales and that firm's market-to-book ratio. Next, we investigate the effect of an international shock on the size distribution of firms within an industry.

5.1 Size and Tobin's Q

Proposition 2 generates a key prediction of the model: the larger is a firm's sales the lower is its value of Tobin's Q. A literature within the field of corporate finance avers that this relationship is indeed negative, but tends to explain this phenomenon as the outcome of incentive problems within the firm.[3] To confirm this result, we collected a sample of firms from the *Compustat*

[3] For an exception, see Santalo (2002).

database over the period 1977-1997. The panel nature of the database allows us to isolate within industry variation in firm characteristics. Let MV_{jit} be firm j's market value at time t in industry i, while BV_{jit} is the same firm's book value. Tobin's Q for a given firm is simply the ratio of that firm's MV_{jit} to its BV_{jit} [4]

We begin by regressing the logarithm of a firm's market value on the logarithm of its book value. Our sample includes over 10 thousand firms and nearly fifty-thousand observations. Our specification includes a full set of three-digit industry-year fixed effects. The coefficient estimate on $\log BV_{jit}$ is 0.884 with a robust standard error that allows for clustering by firm is 0.005. The R-squared is 0.82. The results confirm several existing studies that find evidence that the growth of a firm's book value leads to a smaller growth in a firm's market value, i.e. there exists a size discount.

Our second specification involves a regression of the logarithm of a firm's Q-value on the logarithm of the size of its corporate sales controlling for a full set of three-digit SIC industry-year effects. Our coefficient estimate on the logarithm of a firm's sales is -0.045 and its robust standard error that allows for clustering by firm is 0.003. The R-squared is 0.017. As predicted by the model, the larger is a firm as measured by its sales, the smaller its Q-value. Large firms appear to be less efficient than small firms.

5.2 The Size Distribution of Firms and the Real Exchange Rate

In this section we test our model's prediction that changes in the international trading environment alter the size distribution of firms. In particular, we test the implications of proposition 7. A shock to home's trading regime that lowers foreign firms' cost of selling in home induces large, high-θ firms in that country to add product lines and the small, low-θ firms to drop product lines, thereby causing production in home to become even more concentrated in the largest firms. The opposite prediction holds for the concentration of production in foreign where firms have improved access to home's market. Shocks that are consistent with the spirit of the proposition are changes in tariffs or movements in the real exchange rate.

Our empirical analysis investigates the link between the U.S. real exchange rate and the degree of concentration of U.S. production in its largest firms. To measure the degree of dispersion we consider the shape of the size distribution of U.S. firms – i.e. the relationship between the logarithm of an individual firm's domestic sales and the logarithm of its rank within the industry in terms of its sales. To assess this prediction that real exchange rate appreciations (depreciations) induce an increase (decrease) in industrial concentration, we consider versions of the following specification:

$$\log Sales_{jit} = \alpha_{jt} + \beta_0 \log Rank_{jit} + \beta_1 (\log Rank_{jit})^2 + \beta_2 \log RER_t \log Rank_{jit} + \varepsilon_{jit}, \quad (16)$$

where $Sales_{jit}$ is the sales of firm j in industry i at time t, $Rank_{jit}$ is the rank of this firm in the size distribution (the largest firm has $Rank_{jit} = 1$), RER_t is the U.S. real exchange rate at time t (an increase is an appreciation), α_{jt} is an industry-time fixed effect, and ε_{jit} are

[4] We follow Jovanovic and Rousseau (2002) in calculating market value as the value of common equity (product of items 24 and 35), plus the book value of preferred shares (item 130) and short- and long-term debt (items 34 and 9). Book value is computed similarly but uses instead the book value of common equity.

unobserved determinates of a firm's sales. We allow for non-linearities in the relation between size and rank by including $(\log Rank_{jit})^2$, and we allow for the intercept (α_{jt}) to vary across within a year across industries and to vary within an industry across years.

The gradient of $\log Sales_{jit}$ with respect to $\log Rank_{jit}$ (which is negative by construction) summarizes the size distribution of firms:

$$\frac{\partial \log Sales_{jit}}{\partial \log Rank_{jit}} = \beta_0 + \beta_1 \log Rank_{jit} + \beta_2 \log RER_t.$$

As the gradient becomes steeper (negative, but with greater absolute value), a larger share of production is concentrated in the relatively larger firms. Our model predicts that an appreciation of the real exchange rate (increase in RER_t) should be associated with a steeper gradient. Hence, the model predicts $\beta_2 < 0$.

To estimate (16) we require only firm-level sales data ($Sales_{jit}$) and the real exchange rate (RER_t). Our firm level data was collected from the *Compustat* database. From this database, we obtained an unbalanced panel of 6,730 firms in 108 three-digit manufacturing SIC industries over the years 1977-1997. We observe each firm's sales in the U.S. market (exports and any other sales in foreign markets are removed). A firm's rank in the size distribution (at the three-digit industry level) was then computed. Our measure of the real exchange rate was taken from the *International Financial Statistics* of the International Monetary Fund. Descriptive Statistics are shown in Table 1.

Table 1	*mean*	*stdev*	min	max
$\log Rank$	2.98	1.38	0	5.99
$\log Sale$	3.78	2.50	−7.12	11.95
$\log RER$	4.61	0.17	4.39	4.96
$\log GAP$	−0.03	0.40	−0.73	4.32
$RINT$	4.84	3.07	−3.67	9.99

The results of estimating equation (16) are shown in Table 2. Note that all-specifications include industry-year fixed effects and that the standard errors (shown in parentheses) allow for both heteroskedasticity and clustering by firm. The baseline results are shown in column 1. The negative and statistically significant coefficient on the quadratic term ($(\log Rank_{jit})^2$) indicate that the size-distribution is not well described by a Pareto distribution. Critically, the coefficient on $\log RER_t$ is negative and very statistically significant: in years of a highly appreciated dollar the domestic sales of U.S. firms are more highly concentrated in the largest firms.[5]

While we do allow for a full set of industry-year fixed effects, the potential for spurious correlation needs to be addressed. The real exchange rate could be correlated with other macroeconomic variables that in turn are related to the size distribution of firms. In particular, real interest parity conditions suggest that movements in the real exchange rate are determined by international differences in real interest rates. Since movements in real interest rates might have an asymmetric effect on firms, we include as a control the interaction between the real interest rate $RINT_t$ and $\log Rank_{jit}$. Our measure of the real interest rate is the

[5] All data is initially demeaned and then the regression is run. Thus, the very large R-squared suggests a very tight relationship.

difference between the nominal interest rate charged to low-risk corporate borrowers and the contemporaneous rate of inflation. To control for the possibility that changes in credit market conditions might make credit constraints facing small firms relatively more severe, we include the interaction between logarithm of the difference between the nominal interest rates charged to high and low risk borrowers $INTGAP_t$ and $\log Rank_{jit}$. Both measures were collected from the *Economic Report of the President*.

Table 2	1	2	3
			non−
	traded	*traded*	*traded*
$\log Rank$	0.25	−0.26	−0.63
	(0.29)	(0.30)	(0.55)
$(\log Rank)^2$	−0.19	−0.19	−0.25
	(0.01)	(0.01)	(0.02)
$\log RER \log Rank$	−0.31	−0.20	−0.08
	(0.06)	(0.002)	(0.12)
$\log GAP \log Rank$		−0.10	−0.12
		(0.01)	(0.03)
$RINT \log Rank$		0.00	−0.01
		(0.002)	(0.12)
N	60,436	60,436	18,670
$R-Sq$	0.78	0.78	0.75

The results of estimating the extended specification are shown in column 2. The coefficient on $\log Rank_{jit} \log RER_t$ continues to be negative and statitistically significant at any standard level, although its magnitude is smaller. Movements in the real interest rate do not have an important impact on the size distribution, but a growing gap in the interest rate charged to high and low risk borrowers is associated with a steeper gradient on the size distribution of firms. This result suggests that time-varying credit constraints do have an impact on the degree of concentration within an industry.

Finally, we estimate equation (16) on a sample of firms in non-financial, service industries that are largely non-traded.[6] Since the output of these industries are non-traded, there should be no relationship between movements in the real exchange rate and the size distribution of firms. A finding of a negative and statistically-significant relationship would suggest that the results shown in columns 1 and 2 are spurious. The results are shown in the third column of Table 2. Of particular interest is the coefficient on $\log RER_t \log Rank_{jit}$ which, although negative, is small relative to the coefficient in column 2 and is not statistically significant. This result suggests that the negative coefficient obtained in the traded good sample is not suprious for if it were driven by correlation with omitted variables, we would expect the same result in the non-trade service industries.

A number of other robustness checks were considered. In the interest of conserving space, we

[6] It is difficult to identify a purely non-traded industry. For instance, most business services have large international components. We are relatively conservative in our definition of a non-traded service. These industries are all wholesale and retail industries (all 3-digit SIC in SIC 50), Repair industries (SIC 75 and 76), and the amusement industry (79).

simply describe these alternative specifications. First, since the appropriate level of industrial aggregation in a multiproduct setting is not obvious, we also constructed the size distribution of firms at the two-digit SIC level. The coefficient estimates for equation (16) obtained using this alternate sample were nearly identical to those reported in Table 2. Second, to allow for unobserved firm heterogeneity, we also experimented with specifications that included firm-level fixed effects. The results were even stronger than those reported in Table 2. We conclude that the size distribution of firms is altered by changes in the real exchange rate in a manner that is consistent with the predictions of our model.

6 Conclusion

To be written....

7 Appendix

Proof of proposition 1. Recall that

$$\Psi(c;\theta) \equiv Q(p(c)\left\{p(c) - (1+1/\theta)\,c\right\} - r.$$

The first-order condition (5) then states that $\Psi(c(\theta);\theta) = 0$. We proceed in several steps.

Step 1. We show that $\Psi(c;\theta)$ is strictly decreasing in c whenever $\Psi(c;\theta) \geq 0$. Taking the derivative with respect to c, we obtain

$$\begin{aligned}
\Psi_c(c;\theta) &= -(1+1/\theta)Q(p(c) + \left[Q'(p(c)\left\{p(c) - (1+1/\theta)\,c\right\} + Q(p(c)\right]\frac{dp(c)}{dc} \\
&= -(1+1/\theta)Q(p(c)) - (1/\theta)Q'(p(c))c\frac{dp(c))}{dc} \\
&= -Q(p(c)) - \frac{1}{\theta}\left\{Q(p(c)) + cQ'(p(c))dp(c)/dc\right\},
\end{aligned}$$

where the second equality follows from using the first-order condition for pricing, equation (2). Suppose the expression in curly brackets is nonnegative. Then, $\Psi_c(c;\theta) < 0$. Suppose now that the expression in curly brackets is negative. Since $\Psi(c;\theta) \geq 0$ implies that

$$\frac{1}{\theta} < \frac{p(c) - c}{c},$$

we then obtain

$$\Psi_c(c;\theta) < -Q(p(c)) - \frac{p(c) - c}{c}\left\{Q(p(c)) + cQ'(p(c))dp(c)/dc\right\} \equiv \delta.$$

We will now show that $\delta \leq 0$, and so $\Psi_c(c;\theta) < 0$. Using the first-order condition for pricing, (2), this bound can be rewritten as

$$\begin{aligned}
\delta &= -Q(p(c)) - \frac{p(c) - c}{c}\left\{Q(p(c)) - c\frac{Q(p(c))}{[p(c) - c]}dp(c)/dc\right\} \\
&= -\frac{Q(p(c))}{c}\left\{p(c) - cdp(c)/dc\right\}.
\end{aligned}$$

Applying the implicit-function theorem to the first-order condition for pricing, (2), yields

$$\frac{dp(c)}{dc} = \frac{Q'(p(c))}{2Q'(p(c)) + [p(c) - c]Q''(p(c))} > 0.$$

Rewriting the expression for δ, we obtain

$$\delta = -\frac{Q(p(c))}{c}\left\{p(c) - \frac{cQ'(p(c))}{2Q'(p(c)) + [p(c) - c]Q''(p(c))}\right\}.$$

Hence, $\delta \leq 0$ if and only if

$$\left[2Q'(p(c)) + [p(c) - c]Q''(p(c))\right] p(c) \leq cQ'(p(c)),$$

or

$$\left[1 + \frac{p(c) - c}{p(c)}\right] Q'(p(c)) + [p(c) - c]Q''(p(c)) \leq 0,$$

which holds by assumption. We have thus shown that $\Psi_c(c;\theta) < 0$ whenever $\Psi(c;\theta) \geq 0$. In particular, $\Psi_c(c(\theta);\theta) < 0$ for any $\theta > 0$.

Step 2. It can easily be verified that

$$\Psi_\theta(c;\theta) = \frac{cQ(p(c))}{\theta^2} > 0.$$

Step 3. We now show that $c(\theta) = c_0$ if and only if $\theta \leq \widetilde{\theta}$. It is straightforward to check that $\widetilde{\theta}$ is the unique solution to $\Psi(c_0;\theta) = 0$. Since $\Psi_\theta(c;\theta) > 0$, it follows that $\Psi(c_0;\theta) \leq 0$ for all $\theta \leq \widetilde{\theta}$, and $\Psi(c_0;\theta) > 0$ for all $\theta > \widetilde{\theta}$. Moreover, since $\Psi_c(c;\theta) < 0$, it follows that $\Psi(c;\theta) < 0$ for all $\theta \leq \widetilde{\theta}$ and all $c > c_0$. Hence, the corner solution $c(\theta) = c_0$ obtains for all $\theta \leq \widetilde{\theta}$. In contrast, for all $\theta > \widetilde{\theta}$, $c(\theta)$ is given by the first-order condition $\Psi(c(\theta);\theta) = 0$.

Step 4. We finally show that $c(\theta)$ is strictly increasing in θ for all $\theta \geq \widetilde{\theta}$. Using the implicit function theorem, we have

$$\frac{dc(\theta)}{d\theta} = -\frac{\Psi_\theta(c(\theta);\theta)}{\Psi_c(c(\theta);\theta)} > 0,$$

where the inequality follows from $\Psi_\theta(c(\theta);\theta) > 0$ and $\Psi_c(c(\theta);\theta) < 0$. ■

Proof of lemma 2. >From equation (1), a firm's unit cost $c(n;\theta)$ is strictly increasing in n, and strictly decreasing in θ for $n > 1$. >From proposition 1, $c(n(\theta);\theta)$ is increasing in θ. Hence, $n(\theta)$ must be increasing in θ. It follows that a firm's book value $B(\theta) = n(\theta)r$ is increasing in θ.

We now claim that a firm's market value $M(\theta)$ is strictly increasing in θ. To see this, note first that since a high-θ can always replicate the choice of product lines by a small-θ firm, but at lower unit costs, a firm's net profit is increasing in θ. Next, the firm's market value is equal to the sum of the firm's net profit and book value. Hence, the firm's market value is strictly increasing in θ.

Finally, we show that a firm's sales are increasing in θ. To see this, note that

$$S(\theta) = n(\theta)Q(p(c(\theta)))p(c(\theta)) = \left(\frac{c(\theta)}{c_0}\right)^\theta Q(p(c(\theta)))p(c(\theta)).$$

Taking the derivative with respect to θ, we obtain

$$\left(\frac{c(\theta)}{c_0}\right)^{\theta} Q(p(c(\theta))) \ln\left(\frac{c(\theta)}{c_0}\right) p(c(\theta)) + \frac{\theta}{c_0}\left(\frac{c(\theta)}{c_0}\right)^{\theta-1} Q(p(c(\theta)))c'(\theta)p(c(\theta))$$
$$+\left(\frac{c(\theta)}{c_0}\right)^{\theta} p'(c(\theta))c'(\theta)\left[Q'(p(c(\theta)))p(c(\theta)) + Q(p(c(\theta)))\right].$$

Observe that each of the three terms is strictly positive for $\theta > \widetilde{\theta}$ (and equal to zero for $\theta < \widetilde{\theta}$). Hence, $S(\theta)$ is increasing in θ. ■

Proof of lemma 3. The first step consists in showing that $\frac{d}{dn}\left[nc(n;\theta)\right]\big|_{n=n(\theta)}$ is positive and strictly increasing in θ. To see this, note that

$$\begin{aligned}
\frac{d}{dn} n(\theta)c(n(\theta);\theta)|_{n=n(\theta)} &= \frac{d}{dn} c_0 \left[n\right]^{(1+\theta)/\theta}\Big|_{n=n(\theta)} \\
&= \left(\frac{1+\theta}{\theta}\right) c_0 \left[n(\theta)\right]^{1/\theta} \\
&= \left(\frac{1+\theta}{\theta}\right) c(\theta) \\
&> 0.
\end{aligned}$$

>From the first-order condition (7),

$$\frac{dc(\theta)}{d\theta} = \frac{c(\theta)\left[a - t - c(\theta)\right]}{\theta^2(a-t-c(\theta)) + \theta(a-t-2c(\theta))}.$$

Observe that this expression is strictly positive since, from (7), $\theta(a - t - c(\theta)) - 2c(\theta) \geq 0$, and $a - t > 0$. Hence,

$$\begin{aligned}
\frac{d}{d\theta}\left(\frac{1+\theta}{\theta}\right) c(\theta) &= \left(\frac{1+\theta}{\theta}\right)\frac{dc(\theta)}{d\theta} - \frac{c(\theta)}{\theta^2} \\
&= \frac{1}{\theta^2}\left\{\frac{\left[c(\theta)\right]^2}{\theta(a-t-c(\theta)) + (a-t-2c(\theta))}\right\} \\
&> 0.
\end{aligned}$$

We have shown that $\frac{d}{dn}\left[nc(n;\theta)\right]\big|_{n=n(\theta)}$ is positive and strictly increasing in θ.

The next step consists in showing that $\int \frac{d}{dn}\left[nc(n;\theta)\right]\big|_{n=n(\theta)} \Delta n(\theta) dG(\theta) < 0$. But this follows immediately from the following observations: (i) $\frac{d}{dn}\left[nc(n;\theta)\right]\big|_{n=n(\theta)}$ is positive and strictly increasing in θ, (ii) $\Delta n(\theta) > 0$ for $\theta < \widehat{\theta}$ and $\Delta n(\theta) < 0$ for $\theta > \widehat{\theta}$, and (iii) $\int \Delta n(\theta) dG(\theta) = 0$.

The final step consists in showing that $\Delta a < 0$. But this follows immediately from the previous results and the equilibrium condition for a, equation (8). ■

Proof of lemma 5. The proof proceeds analogously to that of lemma 3. As shown there,

$$\frac{d}{dn} nc_i(n;\theta)|_{n=n_i(\theta)} = \left(\frac{1+\theta}{\theta}\right) c_i(\theta) > 0.$$

Since $dc_i(\theta)/d\theta > 0$, it follows that (as shown in lemma 3), $d\{(1+\theta)c(\theta)/\theta\}/d\theta > 0$. Using the same arguments as before,

$$\int \frac{d}{dn}\left[nc_i(n;\theta)\right]\Big|_{n=n_i(\theta)} \Delta n_i(\theta)dG(\theta) < 0.$$

>From the equilibrium condition for a_i, equation (13), it then follows that $\Delta a_i < 0$ for each country i. ■

Proof of proposition 5. The proof proceeds along the same lines as the proof of proposition 3. We need to show that $dc(\theta)/dt$ is positive for high-θ (i.e., high-c) firms and negative for low-θ (i.e., low-c) firms. Under symmetric tariffs, the first-order condition (12) can be rewritten as

$$\begin{aligned}\Omega(c(\theta);\theta;t) &\equiv \left\{(a-c(\theta))^2+(a-t-c(\theta))^2-r\right\} \\ &\quad -\frac{2c(\theta)}{\theta}\left\{(a-c(\theta))+(a-t-c(\theta))\right\} \\ &= 0, \end{aligned} \tag{17}$$

Applying the implicit function theorem to this equation, we obtain

$$\frac{dc(\theta)}{dt} = -\frac{\Omega_t(c(\theta);\theta;t)}{\Omega_c(c(\theta);\theta;t)},$$

where the subscript $s \in \{t,c\}$ indicates the partial derivative with respect to variable s. Note that $\Omega_c(c(\theta);\theta;t) < 0$ since $\Omega(c(\theta);\theta;t) = 0$ is a profit maximum. Consequently, the sign of $dc(\theta)/dt$ is equal to the sign of $\Omega_t(c(\theta);\theta;t)$. Market clearing for product lines requires that some firms sell product lines while others purchase product lines, and so the sign of $\Omega_t(c(\theta);\theta;t)$ will vary with θ. In the following, we will show that $d\Omega_t(c(\theta);\theta;t)/d\theta > 0$.

Taking the partial derivative of $\Omega(c(\theta);\theta;t)$, as defined by equation (17), with respect to the cost parameter t, yields

$$\begin{aligned}\Omega_t(c(\theta);\theta;t) &= 2\left\{(a-c(\theta))+(a-t-c(\theta))-\frac{2c(\theta)}{\theta}\right\}\frac{da}{dt} \\ &\quad -2(a-t-c(\theta))+\frac{2c(\theta)}{\theta}-\frac{dr}{dt}.\end{aligned} \tag{18}$$

>From the first-order condition (17),

$$\frac{2c(\theta)}{\theta} = \frac{(a-c(\theta))^2+(a-t-c(\theta))^2-r}{(a-c(\theta))+(a-t-c(\theta))}.$$

Inserting this expression into equation (18) and simplifying, we obtain

$$\Omega_t(c(\theta);\theta;t) = \left\{\frac{2(a-c(\theta))(a-t-c(\theta))+r}{(a-c(\theta))+(a-t-c(\theta))}\right\}\left[2\frac{da}{dt}-1\right]+t-\frac{dr}{dt}.$$

Observe that θ enters this equation only through the endogenous marginal cost $c(\theta)$. Hence,

$$\begin{aligned}\frac{d\Omega_t(c(\theta);\theta;t)}{d\theta} &= \frac{d}{dc}\left\{\frac{2(a-c(\theta))(a-t-c(\theta))+r}{(a-c(\theta))+(a-t-c(\theta))}\right\}\left[2\frac{da}{dt}-1\right]\frac{dc(\theta)}{d\theta} \\ &= -2\left\{\frac{\left[(a-c(\theta))^2+(a-t-c(\theta))^2\right]-r}{\left[(a-c(\theta))+(a-t-c(\theta))\right]^2}\right\}\left[2\frac{da}{dt}-1\right]\frac{dc(\theta)}{d\theta}.\end{aligned}$$

>From the first-order condition (17), the expression in curly brackets is strictly positive. Since $dc(\theta)/d\theta > 0$, the sign of $d\Omega_t(c(\theta);\theta;t)/d\theta$ is thus equal to the sign of $[1-2da/dt]$.

We claim that $da/dt < 1/2$. To see this, suppose first that $da/dt = 1/2$. Then, $d\Omega_t(c(\theta);\theta;t)/d\theta = 0$, and so three cases may arise: (i) $dc(\theta)/dt > 0$ for all θ, (ii) $dc(\theta)/dt < 0$ for all θ, or else (iii) $dc(\theta)/dt = 0$ for all θ. But cases (i) and (ii) cannot occur since there is a fixed number of product lines. Hence, we case (iii) must apply: $dc(\theta)/dt = 0$ for all θ; that is, there is no trade in product lines. But then, from equation (13), $da/dt = \sigma N/[1+2\sigma N] < 1/2$. A contradiction. Next, suppose that $da/dt > 1/2$. Then, $d\Omega_t(c(\theta);\theta;t)/d\theta < 0$. Hence, there exists a threshold type $\widehat{\theta} \in (\underline{\theta},\overline{\theta})$ such that – following a small *increase* in t – all firms with $\theta < \widehat{\theta}$ purchase product lines (and so $dc(\theta)/d\theta < 0$) while all firms with $\theta > \widehat{\theta}$ sell product lines (and so $dc(\theta)/d\theta > 0$). From lemma 5, it follows that this "reshuffling" of product lines reduces the endogenous demand intercept a. From (13), the direct effect of an increase in t on a, holding $n(\theta)$ fixed, satisfies $\partial a/\partial t < 1/2$. Hence, the total effect of a small increase in t on a satisfies $da/dt < 1/2$. A contradiction. We have thus shown that $da/dt < 1/2$, and so there exists a treshold type $\widehat{\theta}$, such that – in response to a small increase in t – all firms with $\theta < \widehat{\theta}$ sell product lines while all firms with $\theta > \widehat{\theta}$ acquire product lines. The reverse conclusion holds if $dt < 0$. ■

Proof of proposition 6. We need to show that there exists a $\widetilde{\theta} \in [\underline{\theta},\overline{\theta}]$ such that $dc(\theta)/dt$ is positive for $\theta > \widetilde{\theta}$ and negative for $\theta < \widetilde{\theta}$. As shown in the proof of proposition 5, the sign of $dc(\theta)/dt$ is equal to the sign of $\Omega_t(c(\theta);\theta;t)$, where

$$\begin{aligned}\Omega_t(c(\theta);\theta;t) &= 2\left\{(a-c(\theta))+(a-t-c(\theta))-\frac{2c(\theta)}{\theta}\right\}\frac{da}{dt} \\ &\quad -2(a-t-c(\theta))+\frac{2c(\theta)}{\theta}\end{aligned}$$

since $dr/dt = 0$ in the long run. Using the same steps as in the proof of proposition 5,

$$\Omega_t(c(\theta);\theta;t) = \left\{\frac{2(a-c(\theta))(a-t-c(\theta))+r}{(a-c(\theta))+(a-t-c(\theta))}\right\}\left[2\frac{da}{dt}-1\right]+t,$$

and

$$\frac{d\Omega_t(c(\theta);\theta;t)}{d\theta} = -2\left\{\frac{\left[(a-c(\theta))^2+(a-t-c(\theta))^2\right]-r}{\left[(a-c(\theta))+(a-t-c(\theta))\right]^2}\right\}\left[2\frac{da}{dt}-1\right]\frac{dc(\theta)}{d\theta}. \qquad (19)$$

We now claim that $da/dt < 1/2$ in the long run. To see this, suppose otherwise that $da/dt \geq 1/2$. Then, the profit of each firm of type θ would strictly increase following a small increase in t, even holding fixed the choice of the number of product lines, $n(\theta)$:

$$\frac{d}{dt}\left\{(a-c(\theta))^2+(a-t-c(\theta))^2\right\}>0 \text{ for all } \theta.$$

But this is inconsistent with free entry.

Since $da/dt<1/2$, equation (19) implies that $d\Omega_t(c(\theta);\theta;t)/d\theta>0$. Hence, the assertion of the proposition follows. ■

Proof of proposition 7. We need to show that $dc_1(\theta)/dt_{21}$ is negative for high-θ (i.e., high-c) firms and positive for low-θ (i.e., low-c) firms, while the opposite holds for $dc_2(\theta)/dt_{21}$. From the first-order condition (12), $\Omega^i(c_i(\theta);\theta;t_{12},t_{21})=0$, and so

$$\frac{2c_i(\theta)}{\theta}=\frac{(a_i-c_i(\theta))^2+(a_j-t_{ij}-c_i(\theta))^2-r_i}{(a_i-c_i(\theta))+(a_j-t_{ij}-c_i(\theta))}. \tag{20}$$

Applying the implicit function theorem to the first-order condition, we obtain

$$\frac{dc_i(\theta)}{dt_{21}}=-\frac{\Omega^i_{t_{21}}(c_i(\theta);\theta;t_{12},t_{21})}{\Omega^i_c(c_i(\theta);\theta;t_{12},t_{21})},$$

where the subscript $s\in\{t,c\}$ indicates the partial derivative with respect to variable s. Note that $\Omega^i_c(c_i(\theta);\theta;t_{12},t_{21})<0$ since $\Omega^i(c_i(\theta);\theta;t_{12},t_{21})=0$ is a profit maximum. Consequently, the sign of $dc_i(\theta)/dt_{21}$ is equal to the sign of $\Omega^i_{t_{21}}(c_i(\theta);\theta;t_{12},t_{21})$. Market clearing for product lines requires that some firms sell product lines while others purchase product lines, and so the sign of $\Omega^i_{t_{21}}(c_i(\theta);\theta;t_{12},t_{21})$ will vary with θ. In the following, we will show that $d\Omega^1_{t_{21}}(c_i(\theta);\theta;t_{12},t_{21})/d\theta<0$ and $d\Omega^2_{t_{21}}(c_i(\theta);\theta;t_{12},t_{21})/d\theta>0$.

Consider first country 1. Using the first-order condition (12) and initial symmetry between countries, we obtain

$$\begin{aligned}\Omega^1_{t_{21}}(c(\theta);\theta;t_{12},t_{21}) &= \left[2(a-c(\theta))-\frac{2c(\theta)}{\theta}\right]\left[\frac{da_1}{dt_{21}}+\frac{da_2}{dt_{21}}\right]-2t\frac{da_2}{dt_{21}}-\frac{dr_1}{dt_{21}}\\ &= \left[2(a-c(\theta))-\frac{(a-c(\theta))^2+(a-t-c(\theta))^2}{(a-c(\theta))+(a-t-c(\theta))}\right]\left[\frac{da_1}{dt_{21}}+\frac{da_2}{dt_{21}}\right]\\ &\quad -2t\frac{da_2}{dt_{21}}-\frac{dr_1}{dt_{21}},\end{aligned}$$

where the second equality follows from equation (20). Taking the partial derivative of this expression with respect to c, yields

$$\frac{d\Omega^1_{t_{21}}(c(\theta);\theta;t_{12},t_{21})}{d\theta}=-2\left\{\frac{(a-c(\theta))^2+(a-t-c(\theta))^2-r}{[(a-c(\theta))+(a-t-c(\theta))]^2}\right\}\left[\frac{da_1}{dt_{21}}+\frac{da_2}{dt_{21}}\right]\frac{dc(\theta)}{d\theta}.$$

>From the first-order condition, the expression in curly brackets is strictly positive. Since $dc(\theta)/d\theta>0$, the sign of $d\Omega^1_{t_{21}}(c(\theta);\theta;t_{12},t_{21})/d\theta$ is thus equal to the sign of $-[da_1/dt_{21}+da_2/dt_{21}]$.

Consider now country 2. We have

$$\begin{aligned}
\Omega^2_{t_{21}}(c(\theta);\theta;t_{12},t_{21}) &= \left[2(a-c(\theta))-\frac{2c(\theta)}{\theta}\right]\left[\frac{da_1}{dt_{21}}+\frac{da_2}{dt_{21}}-1\right]+2t\left[1-\frac{da_1}{dt_{21}}\right]-\frac{dr_2}{dt_{21}} \\
&= \left[2(a-c(\theta))-\frac{(a-c(\theta))^2+(a-t-c(\theta))^2}{(a-c(\theta))+(a-t-c(\theta))}\right]\left[\frac{da_1}{dt_{21}}+\frac{da_2}{dt_{21}}-1\right] \\
&\quad +2t\left[1-\frac{da_1}{dt_{21}}\right]-\frac{dr_2}{dt_{21}},
\end{aligned}$$

where the second equality follows again from equation (20). Taking the partial derivative of this expression with respect to c, yields

$$\frac{d\Omega^2_{t_{21}}(c(\theta);\theta;t_{12},t_{21})}{d\theta} = -2\left\{\frac{(a-c(\theta))^2+(a-t-c(\theta))^2-r}{[(a-c(\theta))+(a-t-c(\theta))]^2}\right\}\left[\frac{da_1}{dt_{21}}+\frac{da_2}{dt_{21}}-1\right]\frac{dc(\theta)}{d\theta}.$$

>From the first-order condition, the expression in curly brackets is strictly positive. Since $dc(\theta)/d\theta > 0$, the sign of $d\Omega^2_{t_{21}}(c(\theta);\theta;t_{12},t_{21})/d\theta$ is thus equal to the sign of $[1 - da_1/dt_{21} - da_2/dt_{21}]$.

We claim that $0 < da_1/dt_{21} + da_2/dt_{21} < 1$. To see this, suppose first that $da_1/dt_{21} + da_2/dt_{21} \geq 1$. Then, $d\Omega^1_{t_{21}}(c(\theta);\theta;t_{12},t_{21})/d\theta < 0$ and $d\Omega^1_{t_{21}}(c(\theta);\theta;t_{12},t_{21})/d\theta \leq 0$. Hence, there exists a threshold type $\widehat{\theta}_1 \in (\underline{\theta},\overline{\theta})$ in country 1 such that firms of type $\theta > \widehat{\theta}_1$ in country 1 will sell product lines to firms of type $\theta < \widehat{\theta}_1$. In country 2, either $n_2(\theta)$ remains unchanged, namely if $da_1/dt_{21}+da_2/dt_{21} = 1$, or else there also exists a threshold type $\widehat{\theta}_2 \in (\underline{\theta},\overline{\theta})$ such that firms of type $\theta > \widehat{\theta}_2$ in country 2 will sell product lines to firms of type $\theta < \widehat{\theta}_2$. From lemma 5, it follows that this "reshuffling" of product lines reduces the endogenous demand intercepts a_1 and a_2. Moreover, from (13), the "direct" effect of an increase in t_{21} on the demand intercepts satisfies $\partial a_1/\partial t_{21} < 1/2$ and $\partial a_2/\partial t_{21} = 0$. It follows that the total effect of a small increase in t_{21} on the demand intercepts satisfies $da_1/dt_{21} + da_2/dt_{21} < 1$. A contradiction. A similar argument can be used to show that $da_1/dt_{21} + da_2/dt_{21} \leq 0$ leads to a contradiction. ■

Proof of proposition 8. We need to show that there exist thresholds $\widetilde{\theta}_1 \in [\underline{\theta},\overline{\theta}]$ and $\widetilde{\theta}_2 \in [\underline{\theta},\overline{\theta}]$ such that $dc_1(\theta)/dt_{21}$ is negative for $\theta > \widetilde{\theta}_1$ and positive for $\theta < \widetilde{\theta}_1$, while the opposite holds for $dc_2(\theta)/dt_{21}$. As shown in the proof of proposition 7, the sign of $dc_i(\theta)/dt_{21}$ is equal to the sign of $\Omega^i_{t_{21}}(c_i(\theta);\theta;t_{12},t_{21})$, where

$$\begin{aligned}
\Omega^1_{t_{21}}(c(\theta);\theta;t_{12},t_{21}) &= \left[2(a-c(\theta))-\frac{(a-c(\theta))^2+(a-t-c(\theta))^2}{(a-c(\theta))+(a-t-c(\theta))}\right]\left[\frac{da_1}{dt_{21}}+\frac{da_2}{dt_{21}}\right] \\
&\quad -2t\frac{da_2}{dt_{21}},
\end{aligned}$$

and

$$\begin{aligned}
\Omega^2_{t_{21}}(c(\theta);\theta;t_{12},t_{21}) &= \left[2(a-c(\theta))-\frac{(a-c(\theta))^2+(a-t-c(\theta))^2}{(a-c(\theta))+(a-t-c(\theta))}\right]\left[\frac{da_1}{dt_{21}}+\frac{da_2}{dt_{21}}-1\right] \\
&\quad +2t\left[1-\frac{da_1}{dt_{21}}\right],
\end{aligned}$$

since r is fixed in the long run. As we have shown in the proof of proposition 7,

$$\frac{d\Omega^1_{t_{21}}(c(\theta);\theta;t_{12},t_{21})}{d\theta}=-2\left\{\frac{(a-c(\theta))^2+(a-t-c(\theta))^2-r}{\left[(a-c(\theta))+(a-t-c(\theta))\right]^2}\right\}\left[\frac{da_1}{dt_{21}}+\frac{da_2}{dt_{21}}\right]\frac{dc(\theta)}{d\theta}.$$

and

$$\frac{d\Omega^2_{t_{21}}(c(\theta);\theta;t_{12},t_{21})}{d\theta}=-2\left\{\frac{(a-c(\theta))^2+(a-t-c(\theta))^2-r}{\left[(a-c(\theta))+(a-t-c(\theta))\right]^2}\right\}\left[\frac{da_1}{dt_{21}}+\frac{da_2}{dt_{21}}-1\right]\frac{dc(\theta)}{d\theta}.$$

We now claim that $da_1/dt_{21}+da_2/dt_{21}<1$ in the long run. To see this, suppose otherwise that $da_1/dt_{21}+da_2/dt_{21}\geq 1$. Consider the change in the profit per product line of a country-1 firm with marginal cost $c(\theta)$:

$$\frac{d\left[\pi_{11}(c(\theta))+\pi_{12}(c(\theta))\right]}{dt_{21}}=2(a-c(\theta))\left[\frac{da_1}{dt_{21}}+\frac{da_2}{dt_{21}}\right]-2t\frac{da_2}{dt_{21}}.$$

Free entry implies that this expression cannot be strictly positive for all values of $c(\theta)$. Hence, $da_2/dt_{21}>0$. Consider now change in the profit per product line of a country-2 firm with marginal cost $c(\theta)$:

$$\frac{d\left[\pi_{22}(c(\theta))+\pi_{21}(c(\theta))\right]}{dt_{21}}=2(a-t-c(\theta))\left[\frac{da_1}{dt_{21}}+\frac{da_2}{dt_{21}}-1\right]+2t\frac{da_2}{dt_{21}}.$$

Free entry implies that this expression cannot be strictly positive for all values of $c(\theta)\leq a-t$ (which holds by assumption). Hence, $da_2/dt_{21}\leq 0$. A contradiction.

We now claim that $da_1/dt_{21}+da_2/dt_{21}>0$ in the long run. To see this, suppose otherwise that $da_1/dt_{21}+da_2/dt_{21}\leq 0$. Free entry implies that $d\left[\pi_{11}(c(\theta))+\pi_{12}(c(\theta))\right]/dt_{21}$ cannot be strictly negative for all values of $c(\theta)$. Hence, $da_2/dt_{21}\leq 0$. Free entry also implies that $d\left[\pi_{22}(c(\theta))+\pi_{21}(c(\theta))\right]/dt_{21}$ cannot be strictly negative for all values of $c(\theta)$. Hence, $da_2/dt_{21}>0$. A contradiction.

Since $0<da_1/dt_{21}+da_2/dt_{21}<1$, it then follows that $d\Omega^1_{t_{21}}(c(\theta);\theta;t_{12},t_{21})/d\theta<0<d\Omega^2_{t_{21}}(c(\theta);\theta;t_{12},t_{21})/d\theta$. ■