Multi-Product Firms and Flexible Manufacturing in the Global Economy*

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Abstract

We present a new model of multi-product firms (MPFs) and flexible manufacturing and explore its implications in partial and general equilibrium. International trade integration affects the scale and scope of MPFs through a competition effect and a demand effect. We demonstrate how MPFs adjust in the presence of single-product firms and in heterogeneous industries. Our results are in line with recent empirical evidence and suggest that MPFs in conjunction with flexible manufacturing play an important role in the impact of international trade on diversity.

Keywords: Multi-product Firms, Flexible Manufacturing, General Oligoplistic Equilibrium (GOLE), International Trade, Diversity

JEL Classification: F12, L13

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1 Introduction

Multi-product firms are omnipresent in the modern world economy, especially in technologically advanced countries. Their importance is documented in a recent study of U.S. firms by Bernard, Redding and Schott (2005)).[1] This shows that multi-product firms are present in all industries; they typically coexist with single-product firms, accounting for less than half (41%) of the total number of firms but a much greater fraction (91%) of total output; and they are very active in varying their product mix: 89% of multi-product firms do so on average every five years. Despite this empirical importance, and despite the interest in trade as a source of increased product diversity, multi-product firms have received relatively little attention in the theory of international trade. In this paper we develop a new model of multi-product firms, explore its implications in partial and general equilibrium, and show how the existence of multi-product firms alters the predictions of conventional trade models.

Partial equilibrium models of multi-product firms (Brander and Eaton (1984), Ottaviano and Thisse (1999), Baldwin and Ottaviano (2001), Johnson and Myatt (2003a, 2003b), Baldwin and Gu (2005), Allanson and Montagna (2005)) provide interesting insights into the adjustment processes within multi-product firms. However, by definition they cannot take into account the impact that these adjustment processes have on factor demands.[2] But since multi-product firms are very common, the impact on factor markets through intra-firm adjustments within multi-product firms can be significant. In addition, changes in factor prices can induce further adjustments within multi-product firms that can either reinforce or counteract the initial changes. Hence, a general equilibrium framework is needed to address the role of multi-product firms in a global economy.

General equilibrium models of international trade typically rely on single-product firms only. In these frameworks, intra-firm adjustments are limited to changes in the scale of production. Changes in diversity are linked exclusively to changes in the number of firms. However, intra-firm adjustments within multi-product firms affect the economy through different channels than adjustments via exit and entry. We will show that these intra-firm adjustments imply quite different predictions regarding the impact of international trade on factor prices and diversity.

Our framework features two important differences between multi-product and single-product firms. First, in contrast to single-product firms, multi-product firms internalize demand linkages between varieties. This feature is called the "cannibalization effect". The existence of a cannibalization effect requires that firms are

[1] This uses a longitudinal database derived from the U.S. Census of Manufactures between 1972 and 1997. Over 140,000 surviving firms are present in each census year. In this study a "product" is defined at the five-digit Standard Industry Classification (SIC) level.

[2] Ottaviano and Thisse (1999) allow for labor market equilibrium in their framework, but since they use quasi-linear preferences, they cannot address wage effects.

large in their markets and behave like oligopolists.[3] Second, the varieties within a firm's product line are linked on the cost side through a flexible manufacturing technology (Milgrom and Roberts (1990), Eaton and Schmitt (1994), Norman and Thisse (1999), Grossmann (2003), Eckel (2005)). Flexible manufacturing emphasizes the fact that firms typically possess a "core competence" in the production of a particular variety and that they are less efficient in the production of varieties outside their core competence. In our framework, this inefficiency translates into higher marginal labor requirements. Hence, flexible manufacturing allows firms to expand their product lines, but this expansion is subject to diseconomies of scope. These two features, cannibalization and flexible manufacturing, are the driving forces behind the intra-firm adjustments in our framework.

This paper addresses the role of adjustment processes within multi-product firms and linkages with factor and goods markets in a global economy. In particular, we analyze how multi-product firms react to an increase in the number of countries participating in the world market, how these intra-firm adjustments affect the demand for labor and how induced changes in the wage rate affect the optimal product range. Furthermore, we extend our framework to include heterogeneous industries and illustrate how global shocks can have asymmetric effects on multi-product firms in different industries. Our analysis will provide plausible explanations for observable facts about multi-product firms and present testable propositions with respect to the impact of economy-wide shocks on the scale and scope of multi-product firms.

2 Scale and Scope of Multi-Product Firms

We begin by considering the behaviour of consumers and multi-product firms in a single industry. In Section 4 we will look at the consumers' optimization problem in detail. For now we assume that preferences exhibit symmetric horizontal product differentiation, and give rise to a linear inverse demand function for each good or variety:

$$p_j(i) = a' - b'\left[(1-e)\,x_j(i) + eY\right]. \tag{1}$$

Here, $p_j(i)$ and $x_j(i)$ denote the price of good i and its quantity produced by firm j, and $Y = \int_0^N x(i)\,di$ denotes the output of the entire industry. The total mass of differentiated goods is given by N. The parameters a', b' and e denote the consumers' maximum willingness to pay, the inverse market size and the degree of product differentiation respectively. The primes attached to a' and b' are a reminder that these parameters, taken as given by firms, are endogenous in general equilibrium, as will be explained in Section 4. If $e = 1$, the goods are homogeneous (perfect substitutes) so that demand depends on aggregate output

[3] Thus it cannot arise in models of multi-product firms in "large-group" monopolistic competition such as Allanson and Montagna (2005).

only. On the other hand, $e = 0$ describes the monopoly case where the demand for each good is completely independent of other goods.

Each multi-product firm produces a mass of products which is denoted by δ_j. Profits for a multi-product firm j are then given by

$$\pi_j = \int_0^{\delta_j} \left[p_j(i) - c_j(i)\right] x_j(i)\, di, \tag{2}$$

where $c_j(i)$ denotes the marginal cost of producing good i. This is constant with respect to the quantity produced, but varies between varieties.

As explained in the introduction, the technology of multi-product firms can be characterized by a core competence and flexible manufacturing. We assume that each firm has a core competence in producing a particular variety, which describes the production process at which the firm is most efficient, i.e. where it exhibits the lowest marginal production costs. We set a firm's core competence at $i = 0$ with $c_j(0) = c_j^0$ and $c_j^0 < c_j(i)\ \forall\ i > 0$. In addition to producing its core competence variety, the firm can add new products to its product line via flexible manufacturing. This describes a firm's ability to produce additional varieties with only a minimum of adaptation. However, some adaptation is necessary, so the new products are subject to higher marginal production costs. Each addition to the product line raises marginal production costs for the new products, but leaves marginal production costs of all other products unchanged. Marginal production costs for variety i are therefore an increasing function of the mass of products produced: $\frac{\partial c_j(i)}{\partial i} > 0$. Furthermore, we assume that the increase in marginal production costs is increasing in the length of the product line: $\frac{\partial^2 c_j(i)}{\partial i^2} > 0$.

Firms simultaneously choose the quantity produced of each good and the mass of products produced. The first-order condition with respect to the scale of production of a particular good h is given by

$$\frac{\partial \pi_j}{\partial x_j(h)} = p_j(h) - c_j(h) - b'\left[(1-e)\, x_j(h) + eX_j\right] = 0, \tag{3}$$

where $X_j = \int_0^{\delta_j} x_j(i)\, di$ denotes the firm's aggregate output. The second-order condition is easily verified: $\frac{\partial^2 \pi_j}{\partial x_j(h)^2} = \frac{\partial p_j(h)}{\partial x_j(h)} - b'(1-e) - b'e\frac{\partial X_j}{\partial x_j(h)} < 0$. Eliminating the price from equations (1) and (3) gives the output of a single variety:

$$2b'(1-e)\, x_j(i) = a' - c_j(i) - b'e\left(X_j + Y\right). \tag{4}$$

Equation (4) nicely illustrates a central feature of multi-product firms: the cannibalization effect. This describes a multi-product firm's internalization of the impact of one product's output on the prices of other products within the firm's product line. Because a larger output of one variety tends to lower the demand for all other products, a multi-product firm has an additional incentive to restrict its output beyond the familiar own-price effect. Equation (4) shows that the cannibalization effect is present in our framework because it implies that the output of a single variety is decreasing in the aggregate size of the firm: $\frac{\partial x_j(i)}{\partial X_j} = -b'e < 0$.

The first-order condition with respect to the scale of production is illustrated in figure 1. Because of the cannibalization effect, the marginal revenue curve is lower than it would be for a single-product firm, so the multi-product firm produces less of each good.

Consider next the firm's choice of product line. Multi-product firms add new products as long as marginal profits are positive. The first-order condition with respect to the scope of production is then:

$$\frac{\partial \pi_j}{\partial \delta_j} = [p_j(\delta_j) - c_j(\delta_j)]\, x_j(\delta_j) = 0. \tag{5}$$

As $\frac{\partial c_j(\delta_j)}{\partial \delta_j} > 0$ and, thus, $\frac{\partial x_j(\delta_j)}{\partial \delta_j} = -\frac{1}{2b'(1-e)} \frac{\partial c_j(\delta_j)}{\partial \delta_j} < 0$, the second-order condition is easily verified: $\frac{\partial^2 \pi_j}{\partial \delta_j^2} = [p_j(\delta_j) - c_j(\delta_j)] \frac{\partial x_j(\delta_j)}{\partial \delta_j} < 0$. From (3), $p_j(\delta_j) - c_j(\delta_j)$ cannot be zero. Equation (5) therefore implies that profit-maximizing multi-product firms choose their product range so that the output of the marginal variety is zero: $x_j(\delta_j) = 0$. Hence, given (4), the first-order condition with respect to scope can also be expressed as

$$c_j(\delta_j) = a' - b'e\,(X_j + Y)\,. \tag{6}$$

The determination of the profit-maximizing product range is illustrated in figure 2. The firm's marginal cost of production is lowest for its core competence and rises at an increasing rate as it expands its product line. The firm will add new varieties up to the point where the marginal cost of producing the marginal variety equals the marginal revenue at zero output.

The cannibalization effect not only affects the scale of production, it also influences the scope of production. Total differentiation of (6) shows that $\frac{\partial \delta_j}{\partial X_j} = -\frac{b'e}{\partial c_j(\delta_j)/\partial \delta_j} < 0$. Because firms internalize the impact of one variety's output on the demand for all of their varieties, they not only produce less of each product, they also produce fewer products.

Taken together, the two first-order conditions provide a nice expression for the output of a single variety. Substitute (6) into (4) to obtain:

$$2b'(1-e)\, x_j(i) = c_j(\delta_j) - c_j(i)\,. \tag{7}$$

Equation (7) expresses the output of a single variety in terms of the difference in marginal costs between this variety and the marginal variety. It also provides us with a direct correspondence between a firm's product range and the output of any infra-marginal variety: $\frac{\partial x_j(i)}{\partial \delta_j} = \frac{1}{2b'(1-e)} \frac{\partial c_j(\delta_j)}{\partial \delta_j} > 0$.

Integrating (7) over the entire mass of products produced yields

$$2b'\left(1-e\right)X_j = A_j\left(\delta_j\right), \tag{8}$$

where $A_j\left(\delta_j\right) = \delta_j c_j\left(\delta_j\right) - \int_0^{\delta_j} c_j\left(i\right)di$ and $\frac{\partial A_j(\delta_j)}{\partial \delta_j} = \delta_j \frac{\partial c_j(\delta_j)}{\partial \delta_j} > 0$. $A_j\left(\delta_j\right)$ measures the total cost savings from flexible manufacturing and is represented by the shaded region in Figure 2. Equation (8) provides an expression for the output of firm j as a function of its product range δ_j.

The first-order condition for scope implies, from (6), that higher firm output encourages a fall in product range because of the cannibalization effect. The first-order conditions for scale and scope combined imply, from (8), that an increase in product range encourages an increase in firm output. Taken together, these two equations jointly determine scale and scope, X_j and δ_j, for given industry output Y. They can be combined to yield a single equation that describes the product range setting behavior by multi-product firms:

$$c_j\left(\delta_j\right) + \frac{e}{2\left(1-e\right)}A_j\left(\delta_j\right) = a' - b'eY \tag{9}$$

This implies that $\delta_j = \delta_j\left[a', b', \left\{c_j\left(i\right)\right\}, e, Y\right]$, and, since the left-hand side is increasing in δ_j, it is clear that $\frac{\partial \delta_j}{\partial a'} > 0$, $\frac{\partial \delta_j}{\partial b'} < 0$, $\frac{\partial \delta_j}{\partial e} < 0$, and $\frac{\partial \delta_j}{\partial Y} < 0$. Profit-maximizing multi-product firms broaden their product range if demand for their products increases (a' rises or b' falls) or if competition falls (e or Y falls). In addition, the product range also depends on the exact location and shape of the marginal cost curve as depicted in figure 2. It is immediately obvious that the product range contracts if the core competence marginal production costs $c_j\left(0\right)$ rise (for a given shape of the $c_j\left(i\right)$ curve) or if the $c_j\left(i\right)$ curve becomes more convex (for a given $c_j\left(0\right)$). Lemma 1 summarizes the determinants of the profit maximizing product range:

Lemma 1 *The profit maximizing product range is given by the following:*[4]

$$\delta_j = \delta_j[\underset{+}{a'}, \underset{-}{b'}, \{\underset{-}{c_j\left(i\right)}\}, \underset{-}{e}, \underset{-}{Y}]. \tag{10}$$

While all of these determinants are exogenous to an individual firm, they are affected by changes in the industry or in the economy. In partial equilibrium, industry output is endogenous, and in general equilibrium, a', b' and $\left\{c_j\left(i\right)\right\}$ are also endogenous. In the next section we show how industry output is determined and in the following sections we show how demand and cost parameters are determined in general equilibrium.

[4] The negative sign under the cost terms $\left\{c_j\left(i\right)\right\}$ refers to the effects of unambiguous increases in the marginal costs of producing some or all varieties. More complex shifts in the cost schedule (for example, if more flexible manufacturing requires an increase in the core competence cost) have ambiguous effects on δ_j.

3 Partial Equilibrium

At this point we impose symmetry on all multi-product firms so we can drop the indices j. The market structure in the industry is characterized by a heterogenous Cournot oligopoly where multi-product firms and single-product firms compete side by side. Since we wish to focus on intra-firm adjustments as opposed to adjustments via exit and entry, we assume that both the number of multi-product firms m and the number of single-product firms n are exogenously given. Assuming symmetry within both categories, industry output is then given by

$$Y = mX + nx^s, \tag{11}$$

where x^s is the output of a single-product firm.[5] Single-product firms face the same demand function (1) and are subject to constant marginal production costs c^s. Hence, their output is given by

$$2b'(1-e)x^s = a' - c^s - b'eY. \tag{12}$$

Naturally, there is no cannibalization effect for single-product firms, so equation (12) is independent of X.

By substituting (8) and (12) in (11) we derive a single expression for industry output:

$$Y = \frac{mA(\delta) + n(a' - c^s)}{b'[2(1-e) + ne]}. \tag{13}$$

Equation (13) expresses the industry's output for a given product range δ. Naturally, when the product range rises and multi-product firms become larger, industry output also rises: $\frac{\partial Y}{\partial \delta} = \frac{m\delta}{b'(2(1-e)+ne)} c_\delta(\delta) > 0$, where $c_\delta(\delta) = \frac{\partial c(\delta)}{\partial \delta} > 0$.

Equation (9), which gives the product range of multi-product firms for a given industry output, and equation (13), which gives industry output for a given product range, yield two equations in δ and Y that allow us to solve the partial equilibrium. The equilibrium is illustrated in (δ, Y) space in figure 3. From equation (9), an increase in industry output Y implies an increase in the competition facing each multi-product firm, so product range δ contracts and the curve labeled $Scope|_{MPF}$ is downward-sloping. By contrast, from equation (13), an increase in the product range of every multi-product firm implies an increase in industry output Y, so the curve labeled $IE|_{PE}$ is upward-sloping.

Figure 3 provides some quick comparative static results. Changes in the number of firms (m and n) and changes in the marginal production costs of single-product firms (c^s) shift the $IE|_{PE}$ curve but leave the

[5] It may seem strange to add the output of a finite number of single-product firms to that of the multi-product firms, each of which produces a continuum of products. However, this poses no problems since the total output of each multi-product firm, X, is itself finite. It may be helpful to think of the single-product firms as producing a continuum of identical products along the unit interval.

$Scope|_{MPF}$ curve unaffected. Hence, $\frac{\partial Y}{\partial m}, \frac{\partial Y}{\partial n}, \frac{\partial Y}{-\partial c^s} > 0$ and $\frac{\partial \delta}{\partial m}, \frac{\partial \delta}{\partial n}, \frac{\partial \delta}{-\partial c^s} < 0$. These shocks are pure supply shocks that either increase competition directly via an increase in the number of competitors (m, n rises) or indirectly via an increase in the competitiveness of the competitors (c^s falls).

On the other hand, a change in the market size parameter b' shifts both curves outwards. In fact, the shift is identical for both curves, so that $\frac{\partial Y}{\partial b'} = \frac{Y}{b'} > 0$ and $\frac{\partial \delta}{\partial b'} = 0$. Hence, an increase in the size of the market has no impact on the product range of multi-product firms. Finally, the impact of changes in a' and e on the product range δ are the same as the impacts laid out in lemma 1: $\frac{\partial \delta}{\partial a'} > 0$ and $\frac{\partial \delta}{\partial e} < 0$.

Our analysis provides two important insights that are highlighted in proposition 1:

Proposition 1 *In partial equilibrium, an increase in competition reduces the product range δ and raises industry output Y. An increase in the size of the market also leads to an increase in industry output Y but leaves the product range δ unaffected.*

From a welfare perspective, the impact on the product range of individual firms is not as important as the impact on the overall diversity of products offered. The total number of varieties in the market is given by $N = m\delta + n$. If m and n stay constant, the change in the product range also determines the change in diversity: $dN = md\delta$. However, if the number of firms changes, the impact on diversity consists of two effects: a direct effect through the change in the number of firms and an indirect effect through induced adjustments of the product range. As product range is decreasing in both m and n, $\frac{\partial \delta}{\partial m}, \frac{\partial \delta}{\partial n} < 0$, these two effects work in opposite directions so that the overall impact on diversity is ambiguous.

This is an important observation because it highlights a major difference between our framework and models of international trade with only single-product firms. In the latter case, an increase in the number of firms always increases diversity because, by definition, these models cannot take account of adjustments in the product range. In our framework we see that changes in the product range are an important adjustment process that has a non-trivial impact on diversity.

Given (9) and (13), the impact of a change in m on N is given by

$$\frac{\partial N}{\partial m} = \left[1 - \frac{1}{\Delta'} \frac{2b'(1-e)\, meX}{\phi}\right] \delta, \tag{14}$$

where $\Delta' = me\delta + \left[1 + \frac{e\delta}{2(1-e)}\right] [2(1-e) + ne] > 0$, and $\phi \equiv \delta c_\delta(\delta)$, the semi-elasticity of marginal cost, evaluated at the marginal variety. In the case of a change in n the impact on N is given by

$$\frac{\partial N}{\partial n} = 1 - \frac{1}{\Delta'} \frac{2b'(1-e)\, me\delta x^s}{\phi}. \tag{15}$$

Clearly both derivatives can become negative if ϕ is sufficiently small: $\frac{\partial N}{\partial m} < 0$ if $\phi < 2b'(1-e)\frac{meX}{\Delta'}$ and

$\frac{\partial N}{\partial n} < 0$ if $\phi < 2b' (1-e) \frac{me\delta x^s}{\Delta'}$. Hence, ϕ is an important determinant of the change in diversity. This is not surprising because it measures the degree of flexibility in manufacturing. If ϕ is high, then changes in the product range lead to large cost effects. This corresponds to a less flexible manufacturing technology, so that adjustments take place primarily via adjustments of output levels and less via changes in the product range. Traditional trade models are clearly the extreme case where ϕ is infinite. On the other hand, if ϕ is low, then changes in the product range lead to only small cost effects. This corresponds to a high degree of flexibility in manufacturing. In this case, adjustments take place primarily via changes in the product range. We can state the following proposition:

Proposition 2 *In partial equilibrium, the impact of changes in the number of firms on diversity depends on the degree of flexibility in manufacturing. If flexibility is low, diversity rises when the number of firms increases, otherwise diversity falls.*

4 General Equilibrium

We now turn to the level of the economy as a whole, extending the model of general oligopolistic equilibrium (GOLE) set out in Neary (2002) to allow for multi-product firms. We assume that the economy consists of a continuum of industries, each of which has an oligopolistic market structure. Consumers have identical preferences and maximize a utility function that depends on individual consumption levels $q(i,z)$ of all $N(z)$ goods produced in each industry z, where z varies over the interval $[0,1]$.

The upper tier utility function is an additive function of a continuum of sub-utility functions, each corresponding to one industry:

$$U\left\{u\left[q(0,z), ..., q(N(z),z)\right]\right\} = \int_0^1 u\left[q(0,z), ..., q(N(z),z)\right] dz. \tag{16}$$

Each sub-utility function in turn is quadratic:

$$\begin{aligned} u\left[q(0,z), ..., q(N(z),z)\right] &= a\int_0^{N(z)} q(i,z)\, di \\ &\quad -\frac{1}{2}b(1-e)\int_0^{N(z)} q(i,z)^2\, di - \frac{1}{2}be\left(\int_0^{N(z)} q(i,z)\, di\right)^2. \end{aligned} \tag{17}$$

The utility parameters a, b and e are assumed to be identical for all consumers. Consumers maximize utility subject to the budget constraint

$$\int_0^1 \int_0^{N(z)} p(i,z)\, q(i,z)\, didz \leq I, \tag{18}$$

where I denotes individual income. This leads to the following individual inverse demand functions:

$$\lambda p(i,z) = a - b(1-e)\, q(i,z) - be \int_0^{N(z)} q_j(i,z)\, di. \tag{19}$$

The parameter λ is the Lagrange multiplier, which denotes the consumer's marginal utility of income.

At this point we introduce the international trade component of the model. We assume that there is free trade between the home country, where L consumers are located, and k identical foreign countries each with L^* consumers.[6] In spite of the differences in nationalities, we continue to assume that all consumers (domestic and foreign) have identical preferences. However, as income may differ between countries, they may have different consumption levels and, thus, different marginal utilities of income. We assume that the goods markets of all countries are completely integrated in a single world market, whereas national labor markets are segmented (so there is no international labor mobility). Therefore, the market demand for a particular variety i in industry z, $x(i,z)$, facing a firm in any country consists of demand from domestic consumers, $Lq(i,z)$, plus demand from all foreign consumers, $kL^*q^*(i,z)$. The inverse world market demand function for good i in industry z can then be written exactly as in (1):

$$p(i,z) = a' - b' \left[(1-e)\, x(i,z) + eY(z)\right]. \tag{20}$$

where

$$a' \equiv \frac{a}{\bar{\lambda}}, \qquad b' \equiv \frac{b}{\bar{\lambda}(L+kL^*)} \tag{21}$$

and

$$\bar{\lambda} \equiv \frac{L}{L+kL^*}\lambda + \frac{kL^*}{L+kL^*}\lambda^*, \tag{22}$$

The parameter $\bar{\lambda}$ is a population-weighted average of the home and foreign marginal utilities of income and so can be interpreted as the world marginal utility of income. Because they depend on $\bar{\lambda}$, the parameters a' and b' are endogenously determined in general equilibrium. However, with a continuum of industries they are perceived as exogenous by individual firms. Hence firms are "large" in their own market but "small" in the economy as a whole, which allows a consistent analysis of oligopoly in general equilibrium. (See Neary (2002) for details.)

On the firm side we decompose the marginal production costs $c(i,z)$ of each multi-product firm into marginal labor requirements $\gamma(i,z)$ and the economy-wide wage rate w:

$$c(i,z) = w\gamma(i,z). \tag{23}$$

[6] Foreign variables are denoted by an asterisk throughout.

The flexible manufacturing features of the cost function, a core competence and increasing convex marginal costs of new varieties, are now imposed on the marginal labor requirements, i.e. $\gamma(0,z) = \gamma^0(z)$ and $\frac{\partial\gamma(i,z)}{\partial i}, \frac{\partial^2\gamma(i,z)}{\partial i^2} > 0$. The marginal production costs of single-product firms at home and abroad are simply $c^s(z) = w\gamma^s(z)$ and $c^*(z) = w^*\gamma^*(z)$.

It is convenient to define real wages at home and abroad W and W^* not in units of a particular good or a basket of some kind, but in terms of utils at the margin. Thus, the nominal wage is weighted by the average marginal utility $\bar{\lambda}$:

$$W = w\bar{\lambda} \qquad W^* = w^*\bar{\lambda}. \tag{24}$$

Labor markets are perfectly competitive and fully integrated within each country, so the wage rate is the same for all firms and all industries within each country. The labor demand for multi-product firms in industry z consists of labor requirements for each variety over the interval of the entire product range:

$$l^D_{MPF}(z) = \int_0^{\delta(z)} \gamma(i,z)\, x(i,z)\, di. \tag{25}$$

The labor demand for single-product firms in industry z is simply $l^D_{SPF}(z) = \gamma^s(z)\, x^s(z)$. Labor market equilibrium requires that the entire labor demand over all industries equals the endowment of labor, L:

$$\int_0^1 \left[m(z)\, l^D_{MPF}(z) + n(z)\, l^D_{SPF}(z)\right] dz = L. \tag{26}$$

In principle, the same holds for the foreign labor market. However, we assume that multi-product firms are located in the home country only. First and foremost, this assumption is a simplification that allows us to concentrate on the home country for adjustments within multi-product firms. Once these adjustments are understood, extending multi-product firms to all countries is just a technicality. Secondly, this assumption introduces an asymmetry between countries that allows us to interpret the home country as a fully industrialized country and the foreign countries as developing countries or emerging market economies that are not yet advanced enough to implement flexible manufacturing technologies. Hence, foreign labor market equilibria are given by

$$\int_0^1 n^*(z)\, \gamma^*(z)\, x^*(z)\, dz = L^*. \tag{27}$$

The two labor market clearing conditions complete the system of equations.

We can now set out the full description of an equilibrium in the world economy. Given (21), (23) and (24), the first-order condition for scale, equation (6), and that for scale and scope combined, equation (8), can be rewritten as

$$be\left[X\left(z\right)+Y\left(z\right)\right]=\left[a-W\gamma\left(\delta,z\right)\right]\left(L+kL^{*}\right) \tag{28}$$

and

$$2b\left(1-e\right)X\left(z\right)=W\alpha\left(\delta,z\right)\left(L+kL^{*}\right), \tag{29}$$

where $\alpha\left(\delta,z\right)\equiv\delta\left(z\right)\gamma\left(\delta,z\right)-\int_{0}^{\delta(z)}\gamma\left(i,z\right)di$. The output of domestic and foreign single-product firms can now be expressed as

$$2b\left(1-e\right)x^{s}\left(z\right)=\left[a-W\gamma^{s}\left(z\right)\right]\left(L+kL^{*}\right)-beY\left(z\right) \tag{30}$$

and

$$2b\left(1-e\right)x^{*}\left(z\right)=\left[a-W^{*}\gamma^{*}\left(z\right)\right]\left(L+kL^{*}\right)-beY\left(z\right). \tag{31}$$

The expression for industry output takes into account that there are domestic and foreign single-product firms:

$$Y\left(z\right)=m\left(z\right)X\left(z\right)+n\left(z\right)x^{s}\left(z\right)+kn^{*}\left(z\right)x^{*}\left(z\right). \tag{32}$$

Equations (28) to (32) can be solved for $\delta\left(z\right)$, $X\left(z\right)$, $x^{s}\left(z\right)$, $x^{*}\left(z\right)$ and $Y\left(z\right)$ for each industry z for given values of the two economy-wide real wage rates W and W^{*}. The two labor market clearing conditions (26) and (27) then provide the final two equations.

5 Globalization with Symmetric Industries

We assume that globalization leads to an increase in the number of foreign countries k participating in the world market. In order to solve for explicit solutions we make some further simplifying assumptions. In this section, we assume that all industries are identical, while in the next section we consider the case where industries can be divided into two subgroups.

When all industries are perfectly symmetric, the index z can be omitted. In this case, the full general equilibrium can be described by only four equations. First, equations (28) and (29) can be combined and the output of multi-product firms X eliminated to give:

$$\gamma\left(\delta\right)+\frac{e}{2\left(1-e\right)}\alpha\left(\delta\right)=\frac{1}{W}\left(a-be\frac{Y}{L+kL^{*}}\right) \tag{33}$$

This equation is the general equilibrium equivalent of (9). It determines δ, the product range of a typical multi-product firm, for given Y and W. Next, we can use equations (27), (29) and (30) to eliminate outputs

x^*, X and x^s from the expression for industry output (32):

$$be\zeta Y = [mW\alpha(\delta) + n(a - W\gamma^s)](L + kL^*) + \frac{kL^*}{\gamma^*}, \tag{34}$$

where $\zeta \equiv \frac{2(1-e)}{e} + n$. Equation (34) is the general equilibrium equivalent of (13). It determines industry output Y for a given δ and W.

The remaining two equations give the conditions for labor-market equilibrium at home and abroad. Using equations (7), (25) and (30), the domestic labor market equilibrium (26) can be expressed as

$$[mW\beta(\delta) + n\gamma^s(a - W\gamma^s)](L + kL^*) = n\gamma^s beY + 2b(1-e)L, \tag{35}$$

where $\beta(\delta) \equiv \int_0^\delta \gamma(i)(\gamma(\delta) - \gamma(i))\,di$ measures the average labor requirement of a multi-product firm, corrected for the cost savings from flexible manufacturing: $\beta(\delta) = \alpha(\delta)\frac{l^D_{MPF}}{X}$. Naturally, the domestic labor market clearing condition determines W for a given δ and Y. Finally, the foreign labor market equilibrium condition comes from equations (27) and (31):

$$W^* = \frac{1}{\gamma^*}\left[a - 2b(1-e)\frac{L^*}{n^*\gamma^*} - be\frac{Y}{L + kL^*}\right] \tag{36}$$

This determines the foreign real wage as a function of Y only.

The four equations (33) to (36) determine the equilibrium values of the four key variables δ, Y, W and W^*. Of these, equation (36) determines W^* residually. Hence, we can concentrate on equations (33) to (35) which uniquely determine industry output Y, the product range of multi-product firms δ and the domestic real wage W for a given number of firms (m, n, and n^*) and countries (k).

In order to illustrate the equilibrium diagrammatically, we can reduce the number of equations to two. Figure 4 provides explicit solutions for the two domestic variables W and δ, with implicit solutions for Y and W^*. The IE contour describes the industry equilibrium in $W - \delta$ space. It is derived by solving (33) for Y and substituting into (34):

$$\zeta\left[\gamma(\delta) + \frac{e}{2(1-e)}\alpha(\delta)\right] + m\alpha(\delta) - n\gamma^s = \frac{2(1-e)}{eW}\left(a - \frac{be}{\gamma^*}\frac{kL^*}{L + kL^*}\right), \tag{37}$$

The left-hand side is increasing in δ and so the IE curve has a negative slope. (See the Appendix for a formal proof.). If W rises for a given δ, equation (33) implies that competition (Y) falls. This tends to boost outputs (both X and x^s rise)[7]. In this case, restoring industry equilibrium requires that δ falls, thus the

[7]Note that equations (28) to (30) imply $X = WA(\delta)\frac{L+kL^*}{2b(1-e)}$ and $x^s = W\left[\gamma(\delta) - \gamma^s + \frac{e}{2(1-e)}A(\delta)\right]\frac{L+kL^*}{2b(1-e)}$, so that *for a*

negative slope of the IE curve.

The LL contour describes the labor market equilibrium in $W-\delta$ space. It is derived by substituting Y from (33) into (35):

$$n\gamma^s\left[\gamma(\delta)+\frac{e}{2(1-e)}\alpha(\delta)\right]+m\beta(\delta)-n(\gamma^s)^2=\frac{2b(1-e)L}{W(L+kL^*)}. \tag{38}$$

The slope of the LL curve is also negative. Again, equation (33) implies that if W rises, competition (Y) falls for a given δ. The implicit increase in outputs creates an excess demand for labor. Hence, labor market clearing also requires that δ falls.

We show in the appendix that the LL curve must be steeper than the IE curve. Hence the intersection of the two curves as illustrated in figure 4 determines the domestic real wage W and the product range of multi-product firms δ in a global general equilibrium.

Equations (37) and (38) can be combined in a single equation:

$$\Gamma(\delta)=\frac{1}{\gamma^*}+\frac{L+kL^*}{beL}\left(a-\frac{be}{\gamma^*}\right), \tag{39}$$

where $\Gamma(\delta)\equiv\frac{m\alpha(\delta)-n\gamma^s+\zeta\left[\gamma(\delta)+\frac{e}{2(1-e)}\alpha(\delta)\right]}{m\beta(\delta)-n(\gamma^s)^2+n\gamma^s\left[\gamma(\delta)+\frac{e}{2(1-e)}\alpha(\delta)\right]}>0$. Equation (39) provides an implicit solution for the equilibrium product range δ as a function of demand parameters a, b, e, L, and kL^* as well as of supply parameters m, n, γ^* and γ^s.

Having established the general equilibrium we can now turn to the comparative statics of globalization. We assume that globalization raises the number of countries participating on the world market, so k rises. We obtain the following results for the elasticities of Y, δ and W with respect to k (see the appendix):

$$\begin{aligned}\frac{\partial Y}{\partial k}\frac{k}{Y}=\frac{a\gamma^*}{be}\frac{kn^*x^*}{\Delta Y}&\left[(m\delta)^2\sigma_\gamma^2(\delta)+\frac{be}{a\gamma^*}\frac{2(1-e)}{e}m\delta\mu_\gamma'(\delta)^2+nm\delta\left\{\gamma^s-\mu_\gamma'(\delta)\right\}^2\right.\\&\left.+\varphi\left(nm\delta\sigma_\gamma^2(\delta)+\frac{be}{a\gamma^*}\frac{2(1-e)}{e}\left\{m\delta\sigma_\gamma^2(\delta)+n(\gamma^s)^2\right\}\right)\right]>0,\end{aligned} \tag{40}$$

$$\frac{k}{\delta}\frac{d\delta}{dk}=\frac{1}{\Delta}\left[\frac{2b(1-e)}{W(L+kL^*)}\right]^2\left(1-\frac{a\gamma^*}{be}\right)\frac{kn^*x^*}{\delta\gamma_\delta(\delta)}L\gtreqless 0, \tag{41}$$

$$\frac{k}{W}\frac{dW}{dk}=\frac{1}{\Delta}\frac{2b(1-e)L}{W(L+kL^*)^2}kn^*x^*(m\delta+\varphi\zeta)(\gamma^*-\tilde{\gamma})\gtreqless 0, \tag{42}$$

where $\varphi\equiv 1+\frac{e\delta}{2(1-e)}$ and the determinant of the equation system is denoted by Δ which is unambiguously

given δ, $\frac{\partial X}{\partial W}>0$ and $\frac{\partial x^s}{\partial W}>0$.

positive.[8] Here we have expressed $\alpha(\delta)$ and $\beta(\delta)$ in terms of the first and second moments of the distribution of $\gamma(i)$. Define the first moment about zero (the mean) as $\mu'_\gamma(\delta) \equiv \frac{1}{\delta}\int_0^\delta \gamma(i)\,di$ and the second moment about zero as $\mu''_\gamma(\delta) \equiv \frac{1}{\delta}\int_0^\delta \gamma(i)^2\,di$. Then, $\alpha(\delta) = \delta\left[\gamma(\delta) - \mu'_\gamma(\delta)\right]$ and $\beta(\delta) = \delta\left[\gamma(\delta)\mu'_\gamma(\delta) - \mu''_\gamma(\delta)\right]$. The variance of $\gamma(i)$ is then given by $\sigma^2_\gamma(\delta) = \mu''_\gamma(\delta) - \mu'_\gamma(\delta)^2$. Finally, the variable $\tilde{\gamma}$ can be interpreted as a weighted average of domestic labor requirements in single- and multi-product firms: $\tilde{\gamma} = \frac{\varphi}{m\delta+\varphi\zeta}n\gamma^s + \frac{m\delta}{m\delta+\varphi\zeta}\mu'_\gamma(\delta)$.

The results in equations (40) to (42) show that industry output clearly rises, but the impact on the product range and on the real wage is ambiguous. We can summarise these results as follows:

Proposition 3 *With symmetric industries, an increase in foreign competition raises industry output but has ambiguous effects on the product range of multi-product firms and on the real wage. The product range rises if $a\gamma^* < be$ but falls if $a\gamma^* > be$. The wage rate rises if $\gamma^* > \tilde{\gamma}$ but falls if $\gamma^* < \tilde{\gamma}$.*

The ambiguities are caused by the fact that the increase in k affects the domestic economy through two channels, a competition effect and a demand effect, that have counteracting effects on δ and W:

1. An increase in k increases competition on the product market because the integration of new countries into the world trading system also brings in new firms. The primary effect (before firm adjustments take place) can be derived from equation (32): $\left.\frac{\partial Y}{\partial k}\right|_{\text{Primary}} = n^*x^* > 0$. We will refer to this channel as the *competition effect*.

2. An increase in k also increases demand for all products because the number of consumers rises: $\left.\frac{\partial(L+kL^*)}{\partial k}\right|_{\text{Primary}} = L^* > 0$. We call this channel the *demand effect*.

The competition effect and the demand effect both tend to increase industry output, but they work in different directions with respect to their impact on the domestic real wage W and on the product range δ. An increase in competition reduces the market shares of domestic firms and demand for domestic labor falls. Hence, the competition effect tends to lower the domestic real wage. But an increase in demand from the newly integrated economies raises demand for labor at home, so that the demand effect tends to raise the real wage.

Changes in the wage rate affect the production costs of domestic firms. These cost effects are important in determining the impact on the product range. In partial equilibrium, the demand effect (an increase in the size of the market) has no impact on the product range. But when the wage rate rises endogenously in general equilibrium, the range of products produced by multi-product firms falls. Hence, the demand effect

[8] Δ equals: $(m\delta)^2\sigma^2_\gamma(\delta) + \frac{2(1-e)}{e}m\delta\mu'_\gamma(\delta)^2 + nm\delta\left\{\gamma^s - \mu'_\gamma(\delta)\right\}^2 + \varphi\left(nm\delta\sigma^2_\gamma(\delta) + \frac{2(1-e)}{e}\left\{m\delta\sigma^2_\gamma(\delta) + n(\gamma^s)^2\right\}\right) > 0$

tends to lower the product range δ. In the case of the competition effect, the general equilibrium effect even reverses the partial equilibrium result. As we saw in Section 3, an increase in competition lowers the product range in partial equilibrium. By contrast, in general equilibrium, the competition effect actually leads to an increase in the range of products. This extension of the product range is possible because the wage rate falls, and the wage effect dominates the partial equilibrium competition effect. This is illustrated in figure 5.

Lemma 2 summarizes the mathematical results for the two effects.

Lemma 2 *(i) The competition effect:* $\frac{\partial Y}{\partial k}\frac{k}{Y}\big|_{CE} > 0$, $\frac{\partial \delta}{\partial k}\frac{k}{\delta}\big|_{CE} > 0$ *and* $\frac{\partial W}{\partial k}\frac{k}{W}\big|_{CE} < 0$. *(ii) The demand effect:* $\frac{\partial Y}{\partial k}\frac{k}{Y}\big|_{DE} > 0$, $\frac{\partial \delta}{\partial k}\frac{k}{\delta}\big|_{DE} < 0$ *and* $\frac{\partial W}{\partial k}\frac{k}{W}\big|_{DE} > 0$.

Proof. See appendix. ■

The aggregate impact on the product range depends on whether $a\gamma^* \gtreqless be$. This expression can be interpreted in terms of the impact of the competition effect and the demand effect on the price of the marginal good produced by the multi-product firm $(x(\delta))$. To see this, define $P(\delta) = p(\delta)\bar{\lambda}(L + kL^*)$, where $P(\delta)$ denotes the price of the marginal product in units of world marginal utility. Since the output of the marginal product is zero, $P(\delta)$ can be expressed as $P(\delta) = a(L + kL^*) - beY$. Then, clearly, the impact of an increase in the number of foreign countries in the world market on $P(\delta)$ is $\frac{k}{P(\delta)}\frac{dP(\delta)}{dk} = \frac{1}{P(\delta)}\left[a(L + kL^*)\frac{\partial(L+kL^*)}{\partial k}\frac{k}{L+kL^*} - beY\frac{k}{Y}\frac{dY}{dk}\right]$. This expression shows nicely the two effects, the demand effect, $a(L + kL^*)\frac{\partial(L+kL^*)}{\partial k}\frac{k}{L+kL^*}$, and the competition effect, $-beY\frac{\partial Y}{\partial k}\frac{k}{Y}$. Now note that $\frac{\partial(L+kL^*)}{\partial k}\frac{k}{L+kL^*} = \gamma^*\frac{kn^*x^*}{L+kL^*}$ because $L^* = n^*\gamma^*x^*$, so that $\frac{k}{P(\delta)}\frac{dP(\delta)}{dk} = \frac{kn^*x^*}{P(\delta)}\left(a\gamma^* - be\frac{Y}{kn^*x^*}\frac{k}{Y}\frac{dY}{dk}\right)$. From (40), $\frac{dP(\delta)}{dk}$ is zero if $a\gamma^* = be$, it is positive if $a\gamma^* > be$, and it is negative if $a\gamma^* < be$. Hence, the expression $a\gamma^* - be$ indicates whether the competition effect or the demand effect dominates the change in demand for the marginal product.

As for the aggregate effect on the domestic wage rate, it depends on the relative efficiency of domestic firms vis-à-vis foreign firms. If foreign firms are relatively inefficient $(\gamma^* > \tilde{\gamma})$, domestic firms will gain market shares and labor demand at home will rise $\left(\frac{dW}{dk} > 0\right)$. But if foreign firms are relatively more efficient, so that $\gamma^* < \tilde{\gamma}$, then labor demand at home will fall $\left(\frac{dW}{dk} < 0\right)$.

However, the change in the real wage can also be related to the balance between the demand and the competition effect. Equation (39) shows that there is a relation between the expression $a\gamma^* - be$ and the features of flexible manufacturing inherent in the term $\Gamma(\delta)$. This relation allows us to rewrite equation (42) in terms of $a\gamma^* - be$:

$$\frac{k}{W}\frac{dW}{dk} = -\frac{kL^*}{L + kL^*} + \frac{1}{\Delta}\left(a - \frac{be}{\gamma^*}\right)\frac{1}{W}\frac{2(1-e)}{e}\left[m\delta\mu'_\gamma(\delta) + n\gamma^s\varphi\right]\frac{kL^*}{L + kL^*}. \tag{43}$$

Equations (40), (41) and (43) show that if $a\gamma^* = be$, the results simplify to $\frac{k}{Y}\frac{dY}{dk} = \frac{kn^*x^*}{Y}$, $\frac{k}{\delta}\frac{d\delta}{dk} = 0$ and $\frac{k}{W}\frac{dW}{dk} = -\frac{kL^*}{L+kL^*}$.[9] In this case, there are no adjustments of scope within multi-product firms and the result is identical to the case with single-product firms only. But if $a\gamma^* \neq be$, adjustments of the product range take place, and these adjustments have an impact on the other variables as well. If $a\gamma^* > be$, then $\frac{k}{Y}\frac{dY}{dk} > \frac{kn^*x^*}{Y}$, $\frac{k}{\delta}\frac{d\delta}{dk} < 0$ and $\frac{k}{W}\frac{dW}{dk} > -\frac{kL^*}{L+kL^*}$. If $a\gamma^* < be$, then $\frac{k}{Y}\frac{dY}{dk} < \frac{kn^*x^*}{Y}$, $\frac{k}{\delta}\frac{d\delta}{dk} > 0$ and $\frac{k}{W}\frac{dW}{dk} < -\frac{kL^*}{L+kL^*}$. Note that an increase in the wage rate is a sufficient condition for a fall in the product range, whereas a decrease in the wage rate is only a necessary condition for an increase in the product range.

Our results are illustrated in figure 6. Globalization shifts both curves to the left, since the right-hand sides of (37) and (38) fall when k rises. The extent of these shifts depends on the size of the demand effect vis-à-vis the competition effect, so that we can differentiate between the three cases shown.

Our result with respect to the impact on the product range has significant implications for the welfare effect of globalization. Since utility is clearly increasing in N, consumers value diversity. An increase in diversity raises welfare while a reduction in N lowers welfare. As $N = m\delta + n + kn^*$, the relative impact of a change in k on N is given by $\frac{k}{N}\frac{dN}{dk} = \frac{m\delta}{N}\frac{k}{\delta}\frac{d\delta}{dk} + \frac{kn^*}{N}$. If $\frac{d\delta}{dk} > 0$, then N must unambiguously rise with k. However, if the demand effect dominates and $\frac{d\delta}{dk} < 0$, then N can actually fall. Diversity actually falls (so $\frac{dN}{dk}$ is negative) if

$$\phi < \frac{1}{\Delta}\left(\frac{a\gamma^*}{be} - 1\right)\left[\frac{\alpha(\delta)}{X}\right]^2 m\delta x^* L, \tag{44}$$

where $\phi = \delta\gamma_\delta(\delta)$. Note that there is a striking correspondence to the corresponding partial equilibrium result in proposition 2. Again, the degree of flexibility ϕ is a key determinant of whether overall diversity rises or falls. If flexibility is high (low ϕ), overall diversity can fall, whereas if flexibility is low (high ϕ), overall diversity rises. However, equation (44) also shows that $\frac{dN}{dk} < 0$ is never possible if $a\gamma^* < be$. Hence, a large demand effect is a necessary condition and the combination of a large demand effect and high flexibility is a sufficient condition for a fall in overall diversity.

Proposition 4 *If the demand effect dominates in the impact on the product range, and flexibility in manufacturing is high, overall diversity can fall.*

Proposition 4 presents a result that differs fundamentally from the predictions of standard trade theory. Because conventional workhorse models in international trade theory disregard multi-product firms all together, they cannot take into account how globalization can affect the scope of diversity within firms. With single-product firms only, there is a direct correspondence between the number of firms and diversity. Hence, an increase in the number of firms in the world market raises diversity by assumption. Here, however, we

[9] The first result follows by noting that, when $a\gamma^* = be$, the expression inside the square brackets in (40) reduces to Δ as in footnote 5. The other two are obvious by inspection.

show that an increase in the number of competitors can actually lead to counteracting adjustment processes within firms that can lower overall diversity.

6 High-Tech and Low-Tech Industries

In this section we relax our previous assumption regarding the perfect symmetry of industries. Instead, we assume that the mass of industries can be divided into two groups: high-tech and low-tech industries. The difference between these is that low-tech industries are subject to competition from developing countries whereas high-tech industries are located entirely in the industrialized world. In our two country framework this translates into assuming that the home country possesses both types of industries whereas the foreign country has only access to the low-tech technology and thus hosts only single-product firms in this group of industries. For simplicity we assume that the two groups are of equal size. Let low-tech industries be in the interval $z \in \left(0, \frac{1}{2}\right)$ and high-tech industries in the interval $z \in \left(\frac{1}{2}, 1\right)$. Otherwise, firms and consumers in all industries continue to be symmetric.

With two groups of industries in the home country there must be one set of equations for firm behavior and industry equilibrium for each group. Only the labor market equilibrium is common to both groups. However, we need to adjust the labor market equilibrium for the fact that the demand for labor can differ between firms in high-tech and low-tech industries.

The product range of multi-product firms in low-tech (L) and high-tech (H) industries is determined by:

$$\gamma\left(\delta_L\right) + \frac{e}{2\left(1-e\right)}\alpha\left(\delta_L\right) = \frac{1}{W}\left(a - \frac{beY_L}{L+kL^*}\right) \tag{45}$$

and

$$\gamma\left(\delta_H\right) + \frac{e}{2\left(1-e\right)}\alpha\left(\delta_H\right) = \frac{1}{W}\left(a - \frac{beY_H}{L+kL^*}\right), \tag{46}$$

where δ_L and δ_H denote the product range in low-tech and high-tech industries and Y_L and Y_H denote the respective industry outputs in the two groups of industries. The industry outputs are given by:

$$\zeta_L \frac{beY_L}{L+kL^*} = m_L W\alpha\left(\delta_L\right) + n_L\left(a - W\gamma^s\right) + \frac{1}{\gamma^*}\frac{kL^*}{L+kL^*}, \tag{47}$$

where $\zeta_L = \frac{2(1-e)}{e} + n_L$, and

$$\zeta_H \frac{beY_H}{L+kL^*} = m_H W\alpha\left(\delta_H\right) + n_H\left(a - W\gamma^s\right), \tag{48}$$

where $\zeta_H = \frac{2(1-e)}{e} + n_H$. The parameters m_L, m_H, n_L and n_H denote the number of multi- and single-product firms in both groups of industries. Note that there are no foreign variables in the determination of industry output in high-tech industries, equation (48), reflecting our assumption that high-tech industries are not subject to competition from developing countries.

The domestic and foreign labor market equilibria are given by:

$$\begin{aligned}\frac{2b(1-e)L}{W} &= \frac{1}{2}m_H B(\delta_H) + \frac{1}{2}m_L B(\delta_L) \\ &+ \frac{1}{2}n_H\gamma^s\left[\frac{e}{2(1-e)}A(\delta_L) + \gamma(\delta_L) - \gamma^s\right] \\ &+ \frac{1}{2}n_L\gamma^s\left[\frac{e}{2(1-e)}A(\delta_H) + \gamma(\delta_H) - \gamma^s\right]\end{aligned} \tag{49}$$

and

$$\frac{1}{2}n^*\gamma^* x^* = L^*. \tag{50}$$

Clearly, in this setup the high-tech industries are shielded from direct foreign competition. Hence, there is no direct competition effect. Firms in the high-tech industries are only affected indirectly through changes in the economy wide wage rate W. The product range of multi-product firms in these high-tech industries can be determined via equations (46) and (48):

$$\left[\left\{\frac{e}{2(1-e)}\alpha(\delta_H) + \gamma(\delta_H)\right\}\zeta_H + m_H\alpha(\delta_H) - n_H\gamma^s\right]W = a\frac{2(1-e)}{e} \tag{51}$$

Equation (51) provides a unique relation between the real wage W and the product range δ_H in high-tech industries with $\frac{\partial\delta_H}{\partial W} < 0$. Note that this relation is independent of k, so that changes in δ_H are brought about by changes in the wage rate exclusively. If the wage rate rises, production costs in the high-tech industries increase and firms react to the cost increase by pruning their product range.

In the low-tech industries, this relationship is not independent of k because the low-tech industry is subject to foreign competition. The industry equilibrium can be expressed as a function of both W and k:

$$\left[\left\{\frac{e}{2(1-e)}\alpha(\delta_L) + \gamma(\delta_L)\right\}\zeta_L + m_L\alpha(\delta_L) - n_L\gamma^s\right]W = a\frac{2(1-e)}{e} - \frac{1}{\gamma^*}\frac{2kL^*}{L+kL^*}. \tag{52}$$

We have seen in the previous section that the general equilibrium effects are ambiguous in the case of symmetric industries because they are subject to two counteracting forces. Here, these effects are even more complicated because four different types of firms coexist: 2 types of firms (multi-product and single-product) in 2 groups of industries (low-tech and high-tech). Instead of going into the details of the various ambiguities, we simplify this case a little in order to highlight the role of the wage rate in determining the equilibrium product range in high-tech and low-tech industries.

Assume that all firms in the industrialized home country are multi-product firms, so that $n_L = n_H = 0$. Then, equations (49), (51) and (52) reduce to

$$\frac{1}{2}\left[m_H\beta\left(\delta_H\right) + m_L\beta\left(\delta_L\right)\right] = \frac{2b\left(1-e\right)L}{W\left(L+kL^*\right)}, \tag{53}$$

$$\left[\gamma\left(\delta_H\right) + \frac{e}{2\left(1-e\right)}\left(m_H+1\right)\alpha\left(\delta_H\right)\right]W = a \tag{54}$$

and

$$\left[\gamma\left(\delta_L\right) + \frac{e}{2\left(1-e\right)}\left(m_L+1\right)\alpha\left(\delta_L\right)\right]W = a - \frac{1}{\gamma^*}\frac{e}{2\left(1-e\right)}\frac{kL^*}{L+kL^*}. \tag{55}$$

First of all, equation (55) represents the industry equilibrium in the low-tech industries analogous to (37). We refer to its graphical representation in figure 7 as the IE_{LT} curve. This curve exhibits very similar features to the industry equilibrium curve in the previous section. It is also negatively sloped in $W-\delta_L$ space, and it is shifted downwards on aggregate if k rises. In fact, the shift is also subject to a positive demand effect and a negative competition effect, and the size of these two effects determines the aggregate shift. Finally, the curve is independent of δ_H.

Equation (54) provides the same relation for the high-tech industries. Note that this locus (referred to as the IE_{HT} locus) is also negatively sloped in $W-\delta_H$ space (the left-hand quadrant of figure 7), but it is not affected by changes in k or δ_L.

Finally, equation (53) describes the labor market equilibrium as a function of the wage rate W and the two product ranges δ_H and δ_L. Since this condition depends on the product ranges in both types of industries it could be illustrated in either $W-\delta_H$ or $W-\delta_L$ space. We use a graphical illustration in $W-\delta_L$ space (again referred to as the LL locus) in order to differentiate between how the competition effect and the demand effect influence the equilibrium product range in the low-tech industries and how general equilibrium feedback effects influence the product range in the high-tech industries.

The elasticity of the LL locus is given by

$$\left.\frac{\partial W}{\partial \delta_L}\frac{\delta_L}{W}\right|_{LL} = \frac{\frac{1}{2}m_L\delta_L\mu_\gamma'\left(\delta_L\right)\gamma_\delta\left(\delta_L\right)\delta_L\frac{W}{a}\frac{e}{2(1-e)}}{\frac{1}{2}\frac{e}{2(1-e)}\frac{m_H\delta_H}{\varphi_H}\mu_\gamma'\left(\delta_H\right) - \frac{beL}{a(L+kL^*)}}, \tag{56}$$

where $\varphi_H = 1 + \frac{(m_H+1)e\delta_H}{2(1-e)}$. Equation (56) shows that the elasticity of the LL locus in $W-\delta_L$ space depends also on the labor requirements in the high-tech industries, $\frac{1}{2}\frac{e}{2(1-e)}\frac{m_H\delta_H}{\varphi_H}\mu_\gamma'\left(\delta_H\right)$. In fact, it can even become positive if the labor requirements in the high-tech industries are relatively large. This indicates that labor market clearing is achieved through adjustment processes in both types of industries. For example, an increase in the wage rate W at a given δ_L implies a large decrease in competition (Y falls, see equation (45)) which tends to boost output and raise labor demand. This adjustment process is familiar from our earlier analysis of homogeneous industries. In the case of heterogeneous industries, an increase in W also lowers the product range in the high-tech industries δ_H (see equation (54)) which tends to lower labor demand. Depending on which of the two effects dominates, the LL locus is either upward or downward sloping. Figure 7 illustrates the equilibrium for the case of a downward sloping LL curve.

The comparative statics of an increase in k yield the following results for proportionate changes in δ_L, δ_H, and W:[10]

$$\frac{k}{\delta_L}\frac{d\delta_L}{dk} = \frac{a\vartheta}{\varphi_L \delta_L \gamma_\delta\left(\delta_L\right)}\left[\frac{be}{a} - \frac{1}{2}\frac{e}{2\left(1-e\right)}\frac{m_H\delta_H}{\varphi_H}\mu'_\gamma\left(\delta_H\right) - b\left(1-e\right)\gamma^*\right], \tag{57}$$

$$\frac{k}{\delta_H}\frac{d\delta_H}{dk} = \frac{a\vartheta}{\varphi_H \delta_H \gamma_\delta\left(\delta_H\right)}\left[\frac{1}{2}\frac{e}{2\left(1-e\right)}\frac{m_L\delta_L}{\varphi_L}\mu'_\gamma\left(\delta_L\right) - b\left(1-e\right)\gamma^*\right], \tag{58}$$

$$\frac{k}{W}\frac{dW}{dk} = \vartheta W\left[b\left(1-e\right)\gamma^* - \frac{1}{2}\frac{e}{2\left(1-e\right)}\frac{m_L\delta_L}{\varphi_L}\mu'_\gamma\left(\delta_L\right)\right], \tag{59}$$

where $\varphi_H = 1 + \left(m_H + 1\right)\frac{e\delta_H}{2(1-e)}$ and $\varphi_L = 1 + \left(m_L + 1\right)\frac{e\delta_L}{2(1-e)}$, and ϑ is a composite parameter which is always positive. (The expression for ϑ is given in the appendix.)

First of all, equations (58) and (59) show that changes in the product range in high-tech industries are driven entirely by changes in the wage rate:

$$\varphi_H\delta_H\gamma_\delta\left(\delta_H\right)\frac{k}{\delta_H}\frac{d\delta_H}{dk} = -\frac{a}{W}\frac{k}{W}\frac{dW}{dk} \tag{60}$$

corresponding to movements along the IE_{HT} curve in figure 7. Second, equations (57) to (59) illustrate that there are three possible outcomes:

1. $b\left(1-e\right)\gamma^* < \frac{be}{a} - \frac{1}{2}\frac{e}{2(1-e)}\frac{m_H\delta_H}{\varphi_H}\mu'_\gamma\left(\delta_H\right)$: In this case, foreign firms are very competitive, i.e. their labor requirements are very low vis-à-vis domestic firms. This indicates that the competition effect is very strong and leads to a large reduction in the wage rate. As a consequence, multi-product firms in both types of industries expand their product ranges: $\frac{dW}{dk} < 0$, $\frac{d\delta_L}{dk} > 0$, $\frac{d\delta_H}{dk} > 0$.

2. $b\left(1-e\right)\gamma^* > \frac{1}{2}\frac{e}{2(1-e)}\frac{m_L\delta_L}{\varphi_L}\mu'_\gamma\left(\delta_L\right)$: This case is the exact opposite to case 1. Here, foreign firms have very high labor requirements compared to domestic firms and are, thus, not very competitive on the

[10] The mathematical details are provided in the appendix.

world market. In this case, the demand effect dominates and the wage rate rises a lot. Consequently, multi-product firms in both industries prune their product ranges: $\frac{dW}{dk} > 0$, $\frac{d\delta_L}{dk} < 0$, $\frac{d\delta_H}{dk} < 0$.

3. $\frac{be}{a} - \frac{1}{2}\frac{e}{2(1-e)}\frac{m_H \delta_H}{\varphi_H}\mu'_\gamma(\delta_H) < b(1-e)\gamma^* < \frac{1}{2}\frac{e}{2(1-e)}\frac{m_L \delta_L}{\varphi_L}\mu'_\gamma(\delta_L)$: In this intermediate case, neither the competition effect nor the demand effect clearly dominates. The competition effect still leads to a fall in the wage rate, but this fall is not large enough to reverse the demand effect in its impact on the product range in the low-tech industries. Hence, the product ranges in the low-tech industries contract. However, any fall in the wage rate induces firms in the high-tech industries to expand their product ranges. Therefore, this case describes a scenario where firms in different types of industries react differently to a globalization shock. Firms in high-tech industries expand their product ranges while firms in low-tech industries contract: $\frac{dW}{dk} < 0$, $\frac{d\delta_L}{dk} < 0$, $\frac{d\delta_H}{dk} > 0$.

Our results show that the case of heterogeneous industries is subject to the same forces as the case of homogeneous industries, i.e. a competition effect and a demand effect. But the absence of the competition effect in the high-tech industries drives a wedge between changes in the product ranges in low-tech industries and the corresponding adjustments in the high-tech industries. Hence, it is possible that the two types of industries adjust differently to a globalization shock. This is the case if foreign labor requirements are not too extreme in either direction. This case is illustrated in figure 8.

If the LL curve is upward sloping $\left(\frac{\partial W}{\partial \delta_L}\frac{\delta_L}{W}\Big|_{LL} > 0\right)$, case 1 cannot arise. In this case, the share of labor demand in low-tech industries is relatively small, so the impact of the competition effect on the aggregate labor demand is also relatively small. As a consequence, the fall in the domestic wage rate induced by the increase in the foreign competition is small compared to the demand effect, and the product range in the low-tech industries always falls. To summarise:

Proposition 5 *With heterogeneous industries, globalization can lead to asymmetric product range adjustments between high-tech and low-tech industries, where low-tech industries prune their product ranges due to an increase in competition from abroad whereas high-tech industries profit from a lower domestic wage rate and expand their product ranges.*

7 Conclusion

In this paper we have developed a new model of multi-product firms which highlights the role of flexible manufacturing but which is sufficiently tractable that it can be embedded in a model of general oligopolistic equilibrium. Our analysis shows that the GOLE model provides a coherent framework within which the implications of multi-product firms can be addressed. Our focus is on the intra-firm adjustments within multi-product firms and we find that economy-wide shocks can have a considerable impact on both the scale and scope of multi-product firms. In addition, our analysis shows that the general equilibrium feedback effects, through changes in wages and income, are an important determinant of changes in product ranges.

Our results suggest that adjustment processes within multi-product firms are significantly different from adjustments within industries through exit and entry. Standard trade theory based on single-product firms in monopolistic competition predicts that international market integration raises the real wages of all participating countries and unambiguously increases the choices available to consumers. While this outcome is still possible in our framework, our results show that other outcomes are also possible depending on the competitiveness of foreign firms and on consumer preferences. If the competition effect dominates, diversity rises, but the real wage falls. On the other hand, if the demand effect dominates, the real wage rises, but domestic multi-product firms prune their product ranges which tends to lower the choices available to consumers.

In addition, we illustrate that flexibility in manufacturing plays an important part in determining the extent of product range adjustments. Very flexible technologies tend to raise the magnitude of product range adjustment. Hence, if the demand effect dominates, and if manufacturing technologies are highly flexible, overall diversity in the world market can fall when new countries enter the world market. This result is substantially different from the predictions of standard trade theory even though in both cases the results are driven by the same forces, an increase in the number of firms and an increase in the size of the market. This difference in predictions underlines the importance of intra-firm adjustments.

Our framework can be extended in various directions. We present an extension that analyzes the general equilibrium feedback effects between asymmetric industries. This extension provides insights into how adjustments within multi-product firms can differ between industries. We illustrate that if certain industries are not subject to foreign competition (our high-tech industries), they are still affected by a competition effect through the labor market. Further extensions, to allow for heterogeneous firms within industries, and to consider how firms choose their degree of flexibility, seem well worth exploring in our framework.

Empirical evidence suggests that multi-product firms are an important part of modern industries. Our study shows that adjustment processes within multi-product firms differ substantially from adjustments via exit and entry and that globalization can be a driving force of these adjustment processes.

8 Appendix

8.1 Ranking of Elasticities

The elasticity of the IE curve is given by

$$\left.\frac{\partial W}{\partial \delta}\frac{\delta}{W}\right|_{IE} = -\frac{\left(\zeta\varphi + m\delta\right)\delta\gamma_{\delta}\left(\delta\right)}{2\frac{(1-e)}{e}\gamma\left(\delta\right) + \left(m+1\right)\alpha\left(\delta\right) + n\left[\gamma\left(\delta\right) - \gamma^{s} + \frac{e}{2(1-e)}\alpha\left(\delta\right)\right]} < 0, \tag{61}$$

where $\zeta = \frac{2(1-e)}{e} + n$, $\varphi = 1 + \frac{e\delta}{2(1-e)}$ and $\gamma\left(\delta\right) - \gamma^{s} + \frac{e}{2(1-e)}\alpha\left(\delta\right) = \frac{2b(1-e)x^{s}}{W(L+kL^{*})} > 0$.

The elasticity of the LL curve is given by

$$\left.\frac{\partial W}{\partial \delta}\frac{\delta}{W}\right|_{LL} = -\frac{\left[n\gamma^{s}\varphi + m\delta\mu_{\gamma}'\left(\delta\right)\right]\delta\gamma_{\delta}\left(\delta\right)}{\left[n\gamma^{s}\left\{\gamma\left(\delta\right) - \gamma^{s} + \frac{e}{2(1-e)}\alpha\left(\delta\right)\right\} + m\beta\left(\delta\right)\right]} < 0. \tag{62}$$

Alternatively, the elasticity of the LL curve can be expressed as $\left.\frac{\partial W}{\partial \delta}\frac{\delta}{W}\right|_{LL} = -\left[n\gamma^{s}\varphi + m\delta\mu_{\gamma}'\left(\delta\right)\right]\frac{W(L+kL^{*})}{2b(1-e)L}\delta\gamma_{\delta}\left(\delta\right)$.

Recall that $\alpha\left(\delta\right) = \delta\left[\gamma\left(\delta\right) - \mu_{\gamma}'\left(\delta\right)\right]$ and $\beta\left(\delta\right) = \delta\left[\gamma\left(\delta\right)\mu_{\gamma}'\left(\delta\right) - \mu_{\gamma}'\left(\delta\right)^{2} - \sigma_{\gamma}^{2}\left(\delta\right)\right]$. Subtracting (62) from (61), the LL curve is more steeply sloped (in absolute value) than the IE curve provided that:

$$-nm\delta\left[\mu_{\gamma}'\left(\delta\right) - \gamma^{s}\right]^{2} < \frac{2\left(1-e\right)}{e}\left[m\delta\mu_{\gamma}'\left(\delta\right)^{2} + \varphi n\left(\gamma^{s}\right)^{2}\right] + \left(\zeta\varphi + m\delta\right)m\delta\sigma_{\gamma}^{2}\left(\delta\right), \tag{63}$$

which always holds.

8.2 Comparative Statics with Homogeneous Industries

Taking derivatives of equations (33) to (35) yields the following set of equations:

$$\begin{aligned} &beY\hat{Y} + \left(L + kL^{*}\right)W\varphi\delta\gamma_{\delta}\left(\delta\right)\hat{\delta} + \left[\gamma\left(\delta\right) + \frac{e}{2\left(1-e\right)}\alpha\left(\delta\right)\right]\left(L + kL^{*}\right)W\hat{W} \\ &\quad = be\left[a - W\gamma\left(\delta\right) - \frac{e}{2\left(1-e\right)}W\alpha\left(\delta\right)\right]kL^{*}\hat{k}, \end{aligned} \tag{64}$$

$$\begin{aligned} &\zeta beY\hat{Y} - mW\left(L + kL^{*}\right)\delta\gamma_{\delta}\left(\delta\right)\delta\hat{\delta} - \left[m\alpha\left(\delta\right) - n\gamma^{s}\right]\left(L + kL^{*}\right)W\hat{W} \\ &\quad = \left[mW\alpha\left(\delta\right)L^{*} + n\left(a - W\gamma^{s}\right)L^{*} + 2b\left(1-e\right)\frac{L^{*}}{\gamma^{*}}\right]k\hat{k}, \end{aligned} \tag{65}$$

$$-n\gamma^s beY\hat{Y} + m\left(L + kL^*\right)W\delta\gamma_\delta\left(\delta\right)\mu'_\gamma\left(\delta\right)\delta\hat{\delta} + \left[m\beta\left(\delta\right) - n\gamma^s\gamma^s\right]\left(L + kL^*\right)W\hat{W}$$
$$= -\left[mW\beta\left(\delta\right) + n\gamma^s\left(a - W\gamma^s\right)\right]L^*k\hat{k}. \tag{66}$$

where a circumflex denotes a proportional rate of change (e.g., $\hat{Y} = d\ln Y$). These equations can be written more compactly as follows:

$$\underline{\Delta}\overrightarrow{v} = \overrightarrow{\omega}kL^*\hat{k}, \tag{67}$$

where:

$$\underline{\Delta} = \begin{bmatrix} 1 & \varphi & \gamma\left(\delta\right) + \frac{e}{2(1-e)}\alpha\left(\delta\right) \\ \zeta & -m\delta & -m\alpha\left(\delta\right) + n\gamma^s \\ -n\gamma^s & m\delta\mu'_\gamma\left(\delta\right) & m\beta\left(\delta\right) - n(\gamma^s)^2 \end{bmatrix}, \overrightarrow{v} = \begin{bmatrix} beY\hat{Y} \\ \left(L + kL^*\right)W\gamma_\delta\left(\delta\right)\delta\hat{\delta} \\ \left(L + kL^*\right)W\hat{W} \end{bmatrix} \tag{68}$$

and

$$\overrightarrow{\omega} = \begin{bmatrix} a - W\gamma\left(\delta\right) - \frac{e}{2(1-e)}W\alpha\left(\delta\right) \\ mW\alpha\left(\delta\right) + n\left(a - W\gamma^s\right) + 2b\left(1-e\right)\frac{1}{\gamma^*} \\ -mW\beta\left(\delta\right) - n\gamma^s\left(a - W\gamma^s\right) \end{bmatrix}. \tag{69}$$

The determinant of coefficients $\Delta = |\underline{\Delta}|$ is clearly positive: see the explicit expression in footnote 5. Cramer's rule then provides the results presented in equations (40) to (42).

8.3 Proof of Lemma 2

In order to distinguish between the competition effect and the demand effect, we divide $\overrightarrow{\omega}$ into two vectors. The competition effect is derived by holding $(L + kL^*)$ constant and the demand effect is derived by holding $n^*x^* = \frac{kL^*}{\gamma^*}$ constant:

$$\overrightarrow{\omega} = \underbrace{\begin{bmatrix} 0 \\ 2b\left(1-e\right)\frac{1}{\gamma^*} \\ 0 \end{bmatrix}}_{\text{Competition effect}} + \underbrace{\begin{bmatrix} a - W\gamma\left(\delta\right) - \frac{e}{2(1-e)}WA\left(\delta\right) \\ mWA\left(\delta\right) + n\left(a - W\gamma^s\right) \\ mWB\left(\delta\right) + n\gamma^s\left(a - W\gamma^s\right) \end{bmatrix}}_{\text{Demand effect}} \tag{70}$$

We obtain the following solutions for the competition effect:

$$\left.\frac{\partial Y}{\partial k}\frac{k}{Y}\right|_{CE} = \frac{1}{\Delta}\frac{2\left(1-e\right)}{eY}\frac{kL^*}{\gamma^*}\left[\varphi\left\{m\delta\sigma^2_\gamma\left(\delta\right) + n\left(\gamma^s\right)^2\right\} + m\delta\mu'_\gamma\left(\delta\right)^2\right] > 0, \tag{71}$$

$$\left.\frac{\partial\delta}{\partial k}\frac{k}{\delta}\right|_{CE} = \frac{1}{\Delta}\frac{kn^*x^*}{\delta\gamma_\delta\left(\delta\right)}L\left[\frac{2b\left(1-e\right)}{W\left(L + kL^*\right)}\right]^2 > 0, \tag{72}$$

$$\left.\frac{\partial W}{\partial k}\frac{k}{W}\right|_{CE} = -\frac{1}{\Delta}\frac{2b\left(1-e\right)kL^{*}}{\gamma^{*}\left(L+kL^{*}\right)W}\left[m\delta\mu_{\gamma}'\left(\delta\right)+n\gamma^{s}\varphi\right] < 0. \tag{73}$$

For the demand effect, we obtain

$$\left.\frac{\partial Y}{\partial k}\frac{k}{Y}\right|_{DE} = \frac{1}{\Delta}\frac{a}{be}\frac{kL^{*}}{Y}m\delta\left[\left(m\delta+\varphi n\right)\sigma_{\gamma}^{2}\left(\delta\right)+n\left\{\gamma^{s}-\mu_{\gamma}'\left(\delta\right)\right\}^{2}\right] > 0, \tag{74}$$

$$\left.\frac{\partial \delta}{\partial k}\frac{k}{\delta}\right|_{DE} = -\frac{1}{\Delta}\frac{a\gamma^{*}}{be}\frac{kn^{*}x^{*}}{\delta\gamma_{\delta}\left(\delta\right)}L\left[\frac{2b\left(1-e\right)}{W\left(L+kL^{*}\right)}\right]^{2} < 0, \tag{75}$$

$$\left.\frac{\partial W}{\partial k}\frac{k}{W}\right|_{DE} = \frac{1}{\Delta}\frac{2b\left(1-e\right)kL^{*}}{\gamma^{*}W\left(L+kL^{*}\right)^{2}}\left[\left(m\delta+\varphi\zeta\right)\gamma^{*}L+\left\{\left(m\delta\mu_{\gamma}'\left(\delta\right)+\varphi n\gamma^{s}\right)\right\}kL^{*}\right] > 0. \tag{76}$$

8.4 Comparative Statics with Heterogeneous Industries

By taking derivatives of equations (53) to (55) we obtain the following set of equations:

$$\begin{aligned} & m_{H}\delta_{H}\mu_{\gamma}'\left(\delta_{H}\right)\gamma_{\delta}\left(\delta_{H}\right)W\delta_{H}\hat{\delta}_{H}+m_{L}\delta_{L}\mu_{\gamma}'\left(\delta_{L}\right)\gamma_{\delta}\left(\delta_{L}\right)W\delta_{L}\hat{\delta}_{L} \\ & +\left[m_{H}\beta\left(\delta_{H}\right)+m_{L}\beta\left(\delta_{L}\right)\right]W\hat{W} = -\frac{4b\left(1-e\right)LL^{*}}{\left(L+kL^{*}\right)^{2}}k\hat{k}, \end{aligned} \tag{77}$$

$$\begin{aligned} & \varphi_{L}\gamma_{\delta}\left(\delta_{L}\right)W\delta_{L}\hat{\delta}_{L}+\left[\gamma\left(\delta_{L}\right)+\frac{\left(m_{L}+1\right)e}{2\left(1-e\right)}\alpha\left(\delta_{L}\right)\right]W\hat{W} \\ & = -\frac{1}{\gamma^{*}}\frac{e}{2\left(1-e\right)}\frac{2L^{*}L}{\left(L+kL^{*}\right)^{2}}k\hat{k}, \end{aligned} \tag{78}$$

$$\varphi_{H}\gamma_{\delta}\left(\delta_{H}\right)W\delta_{H}\hat{\delta}_{H}+\left[\gamma\left(\delta_{H}\right)+\frac{\left(m_{H}+1\right)e}{2\left(1-e\right)}\alpha\left(\delta_{H}\right)\right]W\hat{W} = 0, \tag{79}$$

where φ_H and φ_L are defined in the text.

In matrix format, this can be written as:

$$\underline{\underline{\Delta}}\overrightarrow{v} = \overrightarrow{\omega}\frac{2LL^{*}}{\left(L+kL^{*}\right)^{2}}k\hat{k}, \tag{80}$$

where:

$$\underline{\underline{\Delta}} = \begin{bmatrix} m_{H}\delta_{H}\mu_{\gamma}'\left(\delta_{H}\right) & m_{L}\delta_{L}\mu_{\gamma}'\left(\delta_{L}\right) & m_{H}\beta\left(\delta_{H}\right)+m_{L}\beta\left(\delta_{L}\right) \\ 0 & \varphi_{L} & \gamma\left(\delta_{L}\right)+\frac{\left(m_{L}+1\right)e}{2\left(1-e\right)}\alpha\left(\delta_{L}\right) \\ \varphi_{H} & 0 & \gamma\left(\delta_{H}\right)+\frac{\left(m_{H}+1\right)e}{2\left(1-e\right)}\alpha\left(\delta_{H}\right) \end{bmatrix}, \quad \overrightarrow{v} = \begin{bmatrix} \gamma_{\delta}\left(\delta_{H}\right)W\delta_{H}\hat{\delta}_{H} \\ \gamma_{\delta}\left(\delta_{L}\right)W\delta_{L}\hat{\delta}_{L} \\ W\hat{W} \end{bmatrix} \tag{81}$$

and

$$\overrightarrow{\omega} = \begin{bmatrix} -2b\,(1-e) \\ -\frac{1}{\gamma^*}\frac{e}{2(1-e)} \\ 0 \end{bmatrix}. \tag{82}$$

The determinant of coefficients $\Delta = |\underline{\Delta}|$ is clearly positive:

$$\Delta = \frac{m_H \delta_H}{\varphi_H}\left[\mu_\gamma'\left(\delta_H\right)^2 + \sigma_\gamma^2\left(\delta_H\right)\varphi_H\right] + \frac{m_L \delta_L}{\varphi_L}\left[\mu_\gamma'\left(\delta_L\right)^2 + \sigma_\gamma^2\left(\delta_L\right)\varphi_L\right] > 0 \tag{83}$$

Cramer's rule provides the results presented in equations (57) to (59), with $\vartheta \equiv \frac{4LkL^*}{W^2(L+kL^*)^2\gamma^*\Delta} > 0$.

Note that (57) can also be written as

$$\frac{k}{\delta_L}\frac{d\delta_L}{dk} = -\frac{a\vartheta\left[\frac{1}{2}\frac{e}{2(1-e)}\frac{m_H\delta_H}{\varphi_H}\mu_\gamma'\left(\delta_H\right) - \frac{be}{a}\frac{L}{L+kL^*} + b\,(1-e)\,\gamma^* - \frac{be}{a}\frac{kL^*}{L+kL^*}\right]}{\varphi_L\delta_L\gamma_\delta\left(\delta_L\right)}. \tag{84}$$

Since $b\,(1-e)\,\gamma^* - \frac{be}{a}\frac{kL^*}{L+kL^*} > 0$ (from 55), $\frac{d\delta_L}{dk} < 0$ always holds if $\frac{1}{2}\frac{e}{2(1-e)}\frac{m_H\delta_H}{\varphi_H}\mu_\gamma'\left(\delta_H\right) - \frac{be}{a}\frac{L}{L+kL^*} > 0$. This proves that if the LL locus is upward sloping $\left(\frac{\partial W}{\partial\delta_L}\frac{\delta_L}{W}\Big|_{LL} > 0\right)$, case 1 $\left(\frac{d\delta_L}{dk} > 0\right)$ can never hold.

References

[1] Allanson, Paul and Catia Montagna (2005): "Multiproduct firms and market structure: An explorative application to the product life cycle," *International Journal of Industrial Organization*, 23:7-8, September, 587-597.

[2] Baldwin, Richard E. and Gianmarco I.P. Ottaviano (2001): "Multiproduct Multinationals and Reciprocal Dumping," *Journal of International Economics*, 54, 429-448.

[3] Bernard, Andrew B., Stephen Redding and Peter K. Schott (2005): "Multi-product Firms and the Dynamics of Product Mix," mimeo.

[4] Brander, James A. and Jonathan Eaton (1984): "Product Line Rivalry," *American Economic Review*, 74 (3), 323-334.

[5] Eaton, Curtis B. and Nicolas Schmitt (1994): "Flexible Manufacturing and Market Structure," *American Economic Review*, 84 (4), 875-888.

[6] Eckel, Carsten (2005): "International Trade, Flexible Manufacturing and Outsourcing," CeGE Discussion Paper #45, University of Göttingen.

[7] Grossmann, Volker (2003): "Firm Size and Diversification: Asymmetric Multiproduct Firms Under Cournot Competition," CESifo Working Paper No. 1047, Munich.

[8] Johnson, Justin P. and David P. Myatt (2003a): "Multiproduct Cournot Oligopoly," Discussion Paper No. 145, University of Oxford, Department of Economics, Oxford.

[9] Johnson, Justin P. and David P. Myatt (2003b): "Multiproduct Quality Competition: Fighting Brands and Product Line Pruning," *American Economic Review*, 93 (3), 748-774.

[10] Milgrom, Paul and John Roberts (1990): "The Economics of Modern Manufacturing: Technology, Strategy, and Organization," *American Economic Review*, 80 (3), 511-528.

[11] Neary, J. Peter (2002): "International Trade in General Oligopolistic Equilibrium," mimeo., University College Dublin.

[12] Norman, George and Jacques-François Thisse (1999): "Technology Choice and Market Structure: Strategic Aspects of Flexible Manufacturing," *Journal of Industrial Economics*, 47, 345-372.

[13] Ottaviano, Gianmarco I.P. and Jacques-François Thisse (1999): "Monopolistic Competition, Multiproduct Firms and Optimum Product Diversity," CEPR Discussion Paper No. 2151, London.

Figure 1: The Scale of Production and the Cannibalization Effect

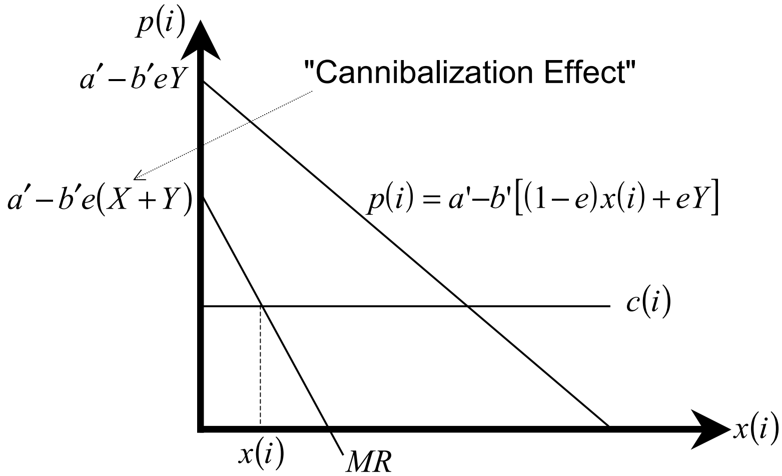


Figure 2: Core Competence and Flexible Manufacturing: The Profit-Maximizing Product Range

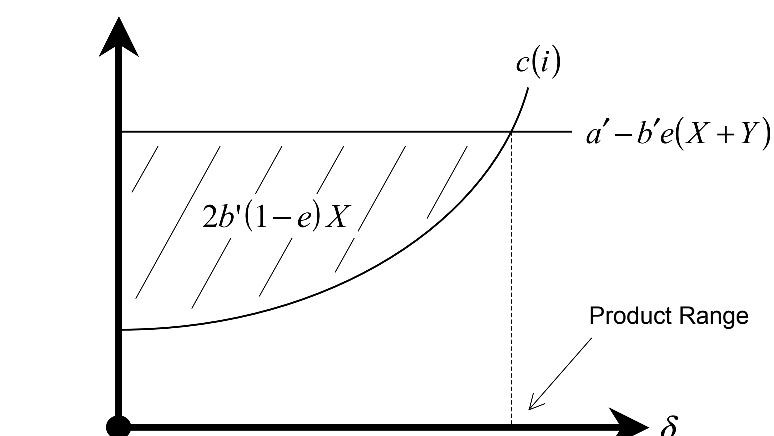


Figure 3: Partial Equilibrium

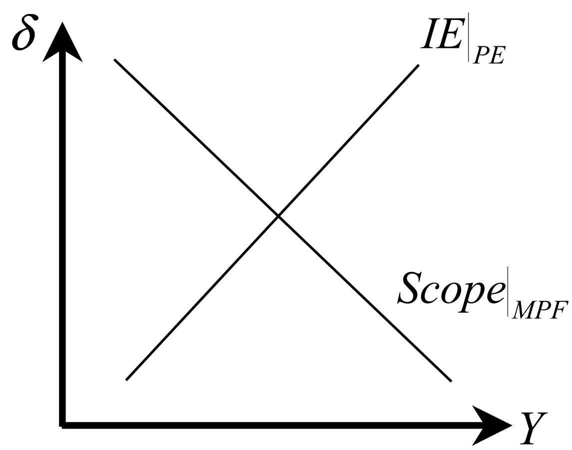


Figure 4: General Equilibrium

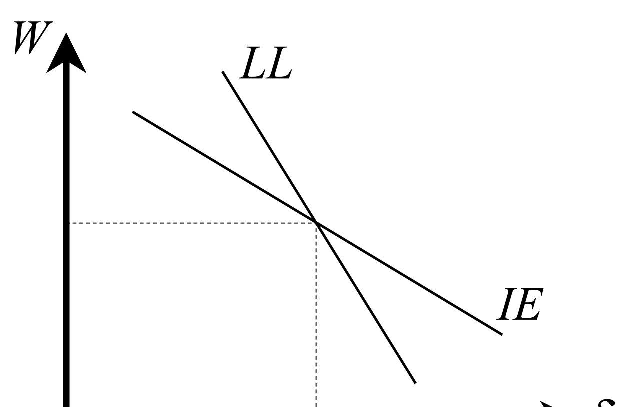


Figure 5: The Competition Effect in General Equilibrium

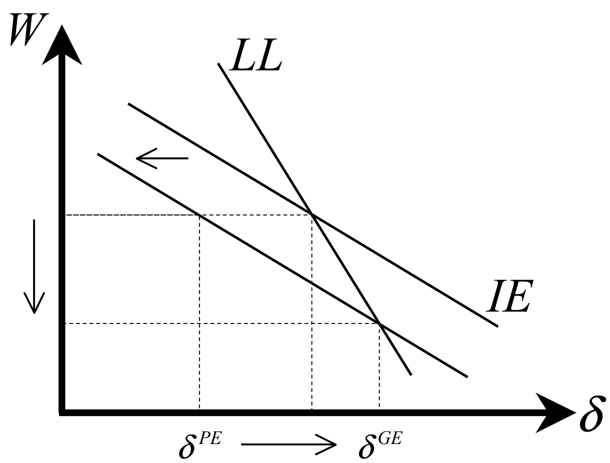
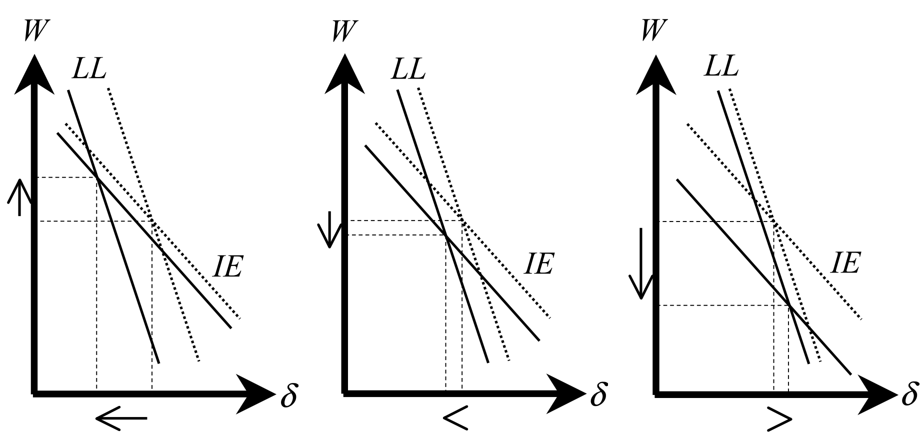


Figure 6: General Equilibrium Results



6.a: Demand Effect dominates

6.b: Intermediate Case

6.c: Competition Effect dominates

Figure 7: Global Equilibrium with High-Tech and Low-Tech Industries

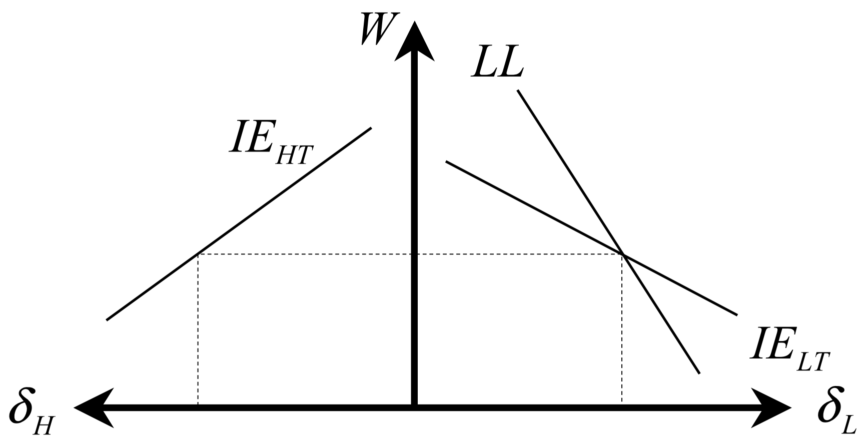


Figure 8: Asymmetric Adjustments in High-Tech and Low-Tech Industries