A Dynamic Model of Entrepreneurship with Borrowing Constraints: Theory and Evidence

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Abstract

This paper studies the interaction between individual's savings and the decision to become an entrepreneur in a multi-period model with borrowing constraints. The model has a simple threshold property: able individuals who start with wealth above a threshold purposely save to become entrepreneurs, while those who start below this threshold fall into a 'poverty trap' and remain wage earners forever. The model also generates a well-defined transition of individuals from wage earners to entrepreneurs - a major focus of recent empirical work. Although the probability of becoming an entrepreneur as a function of wealth is increasing for low wealth levels - as predicted by standard static models - it is decreasing for higher wealth levels. The Model is use to address two quantitative questions. (1) Are poverty traps of quantitative importance in models where agents can overcome them by savings? (2) How costly are borrowing constraints for small businesses in the US? Provided there are not strong decreasing returns to scale, poverty traps remain substantial. To answer the second question, the dynamic model is estimated using US data. Welfare cost are found to be significant, around 6% of lifetime consumption, but poverty traps turn out to be unimportant for the US economy.

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1 Introduction

In the recent development literature occupational choice is at the center of the development process (e.g. Banerjee and Newman (1993), Galor and Zeira (1993)). In these models, borrowing constraints or other financial frictions affect productivity and the distribution of income by restricting agents from profitable occupations that require capital, such as entrepreneurship. Able individuals who start poor are doomed to remain poor.

To make these models tractable often the analysis is carried out under strong assumptions. In particular, generations are assumed to live for a single time period and the evolution of wealth is determined by a warm-glow bequest motive that is not forward-looking. These assumptions have proven useful for understanding the rich dynamics of the wealth distribution across generations and for studying general equilibrium effects. However, these assumptions have made quantitative understanding of these models' implications elusive. Indeed, the models rule out one potentially important and interesting mechanism with which able individuals may overcome borrowing constraints and thus escape poverty traps: life-cycle savings.

As a backdrop, note that borrowing constraints and other financial frictions have played an important role in intertemporal consumption models where individuals experience income shocks. One insight from this literature (e.g. Deaton (1992), Aiyagari (1994), Carroll (1997)) is that individuals facing borrowing constraints will make an effort to overcome them by saving and, as a result, may be able to accomplish a significant amount of consumption smoothing by accumulating and de-accumulating assets. Whether the same logic can be extended to models of occupational choice and whether welfare cost of borrowing constraints to entrepreneurship remain large once individuals can save to overcome them remains an open question.

One purpose of this paper is to construct a tractable dynamic model of occupational choice with life-cycle savings. The model is then used to understand the importance of poverty traps and the welfare cost of borrowing constraints.

Having a tractable model of the life-cycle occupational choice transitions is also important because they are a major focus of a large and recent empirical literature (e.g. Evans and Jovanovic (1989), Holtz-Eakin et. al. (1994), Paulson and Townsend (2002), Hurst and Lusardi (2004)). Most of the empirical literature on entrepreneurship and borrowing constraints had focused on measuring the effect of wealth on the likelihood that an individual becomes an entrepreneur. A positive relationship is then seen as evidence in favor of the direct effects of borrowing constraints. Understandably, many researchers have expressed the concern that wealth and ability, which is unobservable, may then be positively correlated making such an interpretation difficult. A better understanding of models in which wealth and entrepreneurship are jointly determined is thus needed to understand the data. This is a second objective of this paper.

There are four main ingredients to the model: (1) a discrete occupational choice between being a wage earner and an entrepreneur; (2) borrowing constraints; (3) standard intertemporal preferences; (4) heterogeneity in wealth and talent.

The discrete occupational choice introduces a non-convexity into the dynamic problem of households. Fortunately, despite this non-convexity, the continuous time framework allows a simple characterization of the savings problem. The optimal decision to save has a simple threshold property. Able individuals who start with wealth above the threshold but below that needed to start a profitable business save to become entrepreneurs; able individuals who start with wealth below the threshold have low savings and will remain workers. In this sense, individuals with wealth below the threshold are in a poverty trap.

The model is used to investigate the importance of poverty traps as a function of several parameters of interest. More able entrepreneurs are less likely to be in a poverty trap. Indeed, there exists a threshold ability level such that individuals above this threshold are never in a poverty trap regardless of their wealth. Poverty traps are more likely if the intertemporal elasticity of substitution is lower and the discount rate is higher, since these two parameters affect the costs of a high savings plan. Interestingly, the effect of the interest rate is ambiguous. A higher interest rate makes it easier for able individuals to save the capital needed to start a profitable business, but, at the same time, a higher interest rate raises the cost of being in business and makes the option of being a wage earner relatively more attractive.

Turning to quantitative implications, I find significant poverty traps and welfare costs of borrowing constraints for a large set of parameter values consistent with those used in the literature provided returns to scales are not too decreasing. For example, individuals who could earn up to 35% higher income as entrepreneurs will remain workers if they start with zero wealth. Even for workers who do not find themselves in a poverty trap welfare is reduced for two reasons. First, although they eventually become entrepreneurs their entry into entrepreneurship is delayed relative to the case without borrowing constraints. Second, their need to save to overcome the borrowing constraint is costly in terms of utility because it requires an uneven consumption path and individuals prefer smooth consumption. This suggests potentially large welfare cost.

From these quantitative exercises I conclude that, borrowing constraints can remain important in models where agents can potentially overcome them by saving. I then explore the extent by which borrowing constraints are important for small businesses in the US economy by testing the main predictions and estimating the dynamic model.

The model has strong predictions that can be compared to the data: (1) individuals who expect to become entrepreneurs have higher savings rates than individuals who expect to remain workers; (2) the growth rate of consumption of newer entrepreneurs is higher than that of workers and older entrepreneurs; (3) the relationship between the probability of entry into entrepreneurship and wealth is not monotone.

The first prediction summarizes nicely the essential ingredient of the model: that workers can save up to overcome their borrowing constraints to become entrepreneurs. As discussed above, depending on their relative skills some workers will choose to never become entrepreneurs, either because it is not productively efficient or because the savings required to overcome the constraints requires too large a sacrifice. Others find that becoming an entrepreneur will have a large enough surplus and consequently save more. Intuitively, saving is valued because it reduces the time at which they obtain the surplus from entrepreneurship. Thus, workers who expect to become entrepreneurs will save more than those that expect to remain workers.

The second prediction follows from the fact that new entrepreneurs are still constrained with respect to their capital level and consequently have a higher marginal product of capital than older entrepreneurs and higher than the market interest rate.

The last prediction follows from the fact that wealth and entrepreneurship are jointly determined in the model. For low wealth levels, entry into entrepreneurship increases with wealth because it relaxes the borrowing constraint; this is just as predicted by standard static models. But, for high wealth levels, entry into entrepreneurship and wealth become negatively related. This negative relationship reflects the fact that over time individuals with high entrepreneurial skills are selected out of the pool of workers and that this selection effect increases with wealth. Intuitively, if an individual is rich and still works for a wage then it is unlikely that he has a high entrepreneurial skill and thus it is most likely to be a bad entrepreneur and thus to remain a worker.

Moreover, the implications for savings rates, consumption growth and the transition into entrepreneurship as a function of wealth are used to estimate the dynamic model. In particular, the parameters of the model are estimated by minimizing the distance between the statistics describing the behavior of saving rates of entrepreneurs and the dynamics of entrepreneurship in the US data and the same statistics from the simulated model. In other words, the model is estimated by an Indirect Inference procedure as described by Gourieroux and Monfort (1996).

To test and to estimate the model I use data from the Panel Study of Income Dynamics (PSID) and the Consumers Expenditure Survey (CES). This datasets provide rich and complementary information on wealth, income, occupational choice, business ownership and consumption histories of US households.

The data provide a limited support for the theory. As predicted by the model, future entrepreneurs save more than individuals remaining workers. The consumption growth of individuals becoming entrepreneurs is higher than that of workers, while the consumption growth of entrepreneurs is not particularly high. Finally, the relationship between entry into entrepreneurship and wealth is found to be hump-shaped. Nevertheless, the magnitude of the savings differential of entrepreneurs in the data turn out to be too small compared with the ones implied by the model.

2 Related Literature

There is a large and recent literature studying models of occupational choice and credit market frictions to explore the interactions between economic development and inequality. Papers in this literature include Banerjee and Newman (1993), Galor and Zeira (1993), Aghion and Bolton (1997), Lloyd-Ellis and Bernhardt (2000), Caselli and Gennaioli (2002), Matsuyama (2000, 2003), Mokeerjee and Ray (2002, 2003), Galor and Moav (forthcoming). The main objective of these papers is to shed light on complex general equilibrium interactions that arise between wealth distribution and the development process. To make these models tractable often the analysis is either carried out under strong assumptions: generations are assumed to live for a single time period and the evolution of wealth is determined by a warm-glow bequest motive. I complement this literature by studying how forward looking savings interact with occupational choices and, in particular, by studying the extent that financial friction may affect occupational choices in the long run.

This paper follows the original contributions by Skiba (1978) and Brock and Dechert (1983), who study optimal control problems with non-convex technology sets, to study the dynamics of wealth and occupational choices. Majumdar and Mitra (1982) and Dechert and Nishimura (1983) study the discrete time version of the optimal growth problem with convex-concave production function. Related, Matsuyama and Ciccone (1999) study an economy where an aggregate convex-concave production function arises endogenously from demand complementarities at individual level. Part of the contribution of this paper is to show how the problem of an individual facing a discrete occupational choice and an interteporal consumption choice maps into a growth model with a convex-concave production function, extending the application of the methods developed by these authors.

Steps toward understanding the quantitative implications of models of occupational choice with borrowing constraints have recently been taken by Quadrini (1999), Li (2002) and Cagetti and De Nardi (2003) for the US and Gine and Townsend (forthcoming) and Jeong and Townsend (2003) for Thailand. Quadrini (1999) and Cagetti and De Nardi (2003) find that a model with entrepreneurs and borrowing constraints does a good job in fitting the right tale of the US wealth distribution, while Li (2002) quantitatively studies the effects of government credit subsidies in the US. These papers do not attempt to quantify the welfare cost of borrowing constraints. Gine and Townsend (forthcoming) study the role of financial liberalization in explaining the growth and the evolution of inequality in Thailand. Jeong and Townsend (2003) go one more step further and compare the performance of alternative models of growth and inequality: the model of occupational choice of Lloyd-Ellis and Bernhardt (2000) and the model of financial deepening by Greenwood and Jovanovic (1990). As is often done in the literature studying the interaction of growth and inequality, when modelling the interactions between occupational choices and credit frictions they assume warm-glow bequest motives and abstract from life-cycle savings.

A large empirical literature studies the degree to which wealth constrains occupational choices. Evans and Jovanovic (1989) for the US and Paulson and Townsend (forthcoming, 2002) for Thailand are among the few examples that estimate a structural model of occupational choices and financial frictions. This paper extends their work by estimating a fully dynamic version of their model.

Another strand of the empirical literature performs reduced form test of borrowing constraints. In particular, they study the causal effect of wealth on the likelihood of becoming an entrepreneur. See Hurst and Lusardi (2004) for a recent contribution and a critical discussion of the literature. Unfortunately, this literature often times relies on the implications static models of occupational choice limiting the potential information that can be used. I constribute to this literature by deriving the implications of a dynamic model, understanding the bias that arises when drawing inferences from the correlation between wealth and entry into entrepreneurship and by estimating the structural parameters of the model allowing for a welfare analysis of the effects of borrowing constraints.

3 The Model Economy

Time is continuous. Households are endowed with entrepreneurial ability, e, and initial wealth, a_0. In each instant of their life, they have the option to work for a wage, w, and invest their wealth at a constant interest rate, r, or to work and invest in an individual specific technology with productivity e, i.e. to become entrepreneurs. If households decide to be entrepreneurs they must devote all their labor endowment to run their businesses, i.e. occupations are indivisible. This capture a fundamental non-convexity: households are more productive by specializing in one activity. Households face borrowing constraints.

3.1 Preferences

Agents' preferences over consumption profiles are represented by the time separable utility function

$$U\left(c\right)=\int_{0}^{\infty}e^{-\rho t}u\left(c\left(t\right)\right)da \tag{1}$$

where t is the age of the individual and ρ the rate of time preferences. The utility function over consumption $u\left(c\right)$ is strictly increasing and strictly concave.[1]

3.2 Resource Constraints and Technologies

Agents start their lives with wealth a_0. At any time $t\geq 0$, their wealth, $a\left(t\right)$, evolves according to the following law of motion

$$\dot{a}\left(t\right)=y\left(a\left(t\right)\right)-c\left(t\right)\quad t\geq 0, \tag{2}$$

where $y\left(a\left(t\right)\right)$ is the income of the agent with wealth $a\left(t\right)$, and $\dot{a}$ refers to $\frac{\partial a(t)}{\partial t}$.[2] The shape of the income function depends on occupational choices as follows.

[1] In a life-cycle interpretation of the model t is the age of the agents and $\rho=\rho^{*}+p$, where ρ^{*} is the rate of time preferences and p is the constant probability of dying. This interpretation of the model is used when studying the quantitative implications of the theory.

[2] For simplicity of exposition, I drop the time as an argument of the different functions.

3.2.1 Wage Earner

If agents choose to be wage earners, they will sell their labor endowment and invest their wealth for a wage w and a return r. In this case, their income $y(a)$ is

$$y^w(a) = w + ra, \tag{3}$$

where ra is the return on their wealth a. I refer to w as *the wage*, but it should be understood that wages are individual-specific.[3]

3.2.2 Entrepreneur

If individuals run a business they must devote their entire labor endowment to operate the business. Their revenue is given by the function, $f(e,k)$, where e is the agent specific ability and k is the amount of capital invested in the business. [4] $f(e,k)$ is assumed to be strictly increasing and homogeneous of degree 1 in both arguments and strictly concave in capital, $f_e(e,k) > 0$, $f_k(e,k) > 0$, $f_{kk}(e,k) < 0$, and Inada conditions are assumed to hold, $\lim_{k\to 0} f_k(e,k) = \infty$ and $\lim_{k\to\infty} f_k(e,k) = 0$. A higher entrepreneurial ability is associated with a higher marginal product of capital, $f_{ek}(e,k) > 0$, also $f(0,k) = 0$ and $\lim_{e\to\infty} f(e,k) = \infty$.

The amount of capital that agents can invest in their businesses is constrained by their wealth. To focus the analysis on the interaction between the individual savings and the occupational choice I choose a simple specification of borrowing constraints without endorsing a particular market friction generating it. In particular, I assume that the value of individuals business assets, k, must be less than or equal to the value of their wealth, $k \leq \lambda a$.[5] If wealth exceeds the value of desired business assets, the remaining wealth is invested at the rate r. I set $\lambda = 1$ in the theoretical analysis to economize notation.

Therefore, the income of an entrepreneur solves the following profit maximization problem,

$$y^e(e,a) = \max_{k\leq a} \{f(e,k) - r(k-a)\}. \tag{4}$$

Note that the scale of the business equals the individual wealth a as long as wealth is lower than the unconstrained scale of the business, $k_u(e)$.[6]

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[3] Formally, $w = \bar{w}l$, where l are the efficiency units that an individual can supply and $\bar{w}$ is the price of an efficiency unit of labor.

[4] The production function should be interpreted as the reduced form of a more general technology requiring capital and labor,

$$f(e,k) = \max_n \tilde{f}(e,k,n) - \bar{w}n,$$

where n are the efficiency units of labor employed and $\bar{w}$ is the price of an efficiency unit of labor. When calibrating the model and when discussing the predictions of the model for technologies with different capital intensities the more general notation will be used.

[5] A borrowing constraint of the type $k \leq \lambda a$ can be derived from a one shot relationship between a lender and an entrepreneur when the entrepreneur cannot commit to repay the loan and the lender can only seize the capital stock of the entrepreneur.

[6] The unconstrained scale is the solution to the unconstrained profit maximization problem,

3.3 Consumer's Problem

Agents choose profiles for consumption, $c(t)$, wealth, $a(t)$, occupational choice, and business assets, $k(t)$, to solve

$$\max_{c(t),a(t),k(t)\geq 0} \int_0^\infty e^{-\rho t} u(c(t))\, dt$$
$$s.t.$$
$$\dot{a}(t) = y(a(t)) - c(t)$$
$$y(e, a(t)) = \max\{y^e(e, a(t)), y^w(a(t))\}.$$

As is implicitly recognized in the statement of the problem, the occupational decision is a static one. That is, given current wealth, a, agents choose to be entrepreneurs if their income as entrepreneurs, $y^e(e,a)$, exceeds their income as wage earners, $y^w(a)$, i.e. $y^e(e,a) \geq y^w(a)$.

This can be expressed as a simple policy function. Define $\underline{e}$ to be the ability at which individuals are just indifferent between being wage earners and being entrepreneurs if they can borrow at the interest rate r.[7] Under the alternative formulation, able individuals (individuals with ability above $\underline{e}$) decide to be entrepreneurs if their current wealth is higher than the threshold wealth $\underline{a}(e)$, $a \geq \underline{a}(e)$,where $\underline{a}(e)$ solves

$$f(e, \underline{a}(e)) = w + r\underline{a}(e).$$

Intuitively, agents of a given ability choose to become entrepreneurs if they are wealthy enough to run their businesses at a profitable scale. Alternatively, agents of a given wealth a choose to become entrepreneurs if their ability is high enough. Both ability and resources determine the occupational decision.

Given the optimal static decision, the dynamic program is equivalent to a standard capital accumulation problem subject to a production function of the form

$$y(e,a) = \begin{cases} w + ra & \text{if } a \in [0, \underline{a}(e)) \\ f(e,a) & \text{if } a \in [\underline{a}(e), k_u(e)) \\ f(e, k_u(e)) + r(a - k_u(e)) & \text{if } a \in [k_u(e), \infty) \end{cases}.$$

This technology is given by the upper envelope of the "wage earner technology," $y^w(a)$, and the "entrepreneurial technology," $y^e(e,a)$. Figure 1 describes these technologies. Notice that this production function is not concave. The return to capital increases if individuals invest more than $\underline{a}(e)$.

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i.e.

$$k_u(e) = \arg\max_k \{f(e,k) - rk\}.$$

This function is strictly increasing. Inada conditions are necessary to guarantee that this function is well defined for all e.

[7] $\underline{e}$ solve

$$\max_k f(\underline{e}, k) - rk = w.$$

The left hand side of this equation is well define, increasing, continuous and take the value zero for $e = 0$ and goes to infinity as e goes to infinity.

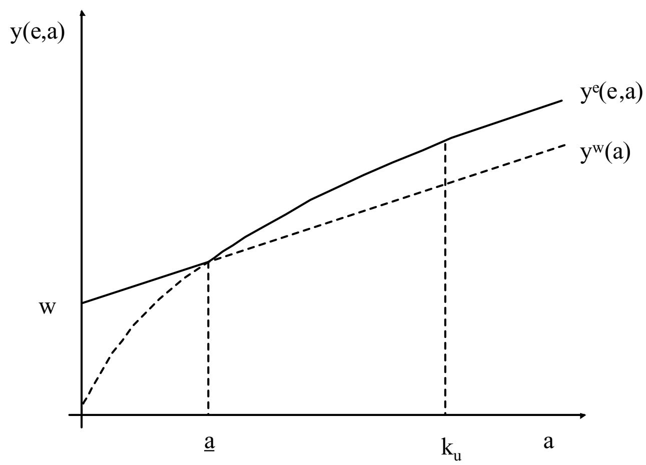

Figure 1: Technologies Available to Households

Necessary conditions for the wealth accumulation problem are given by the Euler equation,

$$\frac{u''(c)\,c}{u'(c)}\frac{\dot{c}}{c} = \begin{cases} r-\rho & \text{if } a \in [0, \underline{a}(e)) \\ f_k(e,a)-\rho & \text{if } a \in [\underline{a}(e), k_u(e)) \\ r-\rho & \text{if } a \in [k_u(e), \infty) \end{cases}, \tag{5}$$

stating that the marginal rate of substitution should equal the marginal rate of transformation; the law of motion for wealth,

$$\dot{a} = y(e,a) - c, \tag{6}$$

describing the evolution of the individual wealth; and the tranversality condition,

$$\lim_{t\to\infty} e^{-\rho t} u'(c(t))\,a(t) = 0, \tag{7}$$

stating that value of wealth should converge to zero.

In the present case, these conditions are only necessary. As in any non-convex problem, there are solutions to the foc's that correspond to global minima or local maxima that are not global maxima. Also, there can be multiple maxima. In section 4 analyzes the optimal accumulation path under this technology.

I finish this section by noting that the model is homogeneous of degree 1 with respect to w, e, and a.

Remark 1: The model is homogeneous of degree 1 in (a, w, e).

It would therefore be useful to normalize all the variables in the model by the wage. This property of the model suggests to use wealth to wage ratios are the relevant measure of resources available to individuals when studying the behavior of entrepreneurs in the data.

4 The Evolution of Individual Wealth

This section characterizes the evolution of individual wealth. The main results are: (a) There exists a threshold wealth level, $a_s(e)$, such that individuals with initial wealth below the threshold, $a_0 < a_s(e)$, follow a path associated with decreasing wealth converging to a zero-wealth steady state where they are wage-earners. Meanwhile, households with initial wealth above the threshold, $a_0 \geq a_s(e)$, save to become entrepreneurs and converge to a high wealth entrepreneurial steady state. (b) The function $a_s(e)$ is strictly decreasing in the entrepreneurial ability and there exists a minimum ability, e_{high}, such that individuals with ability above e_{high} save to become entrepreneurs regardless of their initial wealth. (c) The threshold $a_s(e)$ is increasing in the discount rate and decreasing in the intertemporal elasticity of substitution.

4.1 Analysis of the Phase Diagram

As is standard in the analysis of dynamic models in continuous time, this section proceeds by analyzing the phase diagram of the system of ordinary differential equations (5) and (6) given the agent's focs. The analysis is restricted to the case of $r < \rho$[8].

By setting $\dot{c} = 0$ in (5) an equation that describes the set of wealth-consumption pairs, (a, c), for which consumption is constant over time is obtained. Under the assumption that $f_k(e, \underline{a}) > \rho$ there exists a (unique) solution to this equation. In particular, there exists a unique value of wealth, $a_{ss} \in (\underline{a}, k_u)$, solving the equation

$$f_k(e, a) - \rho = 0.$$

This gives the rightmost vertical curve in the (a, c) space. I label it $(\dot{c} = 0)_1$ in Figure 2. This corresponds to the $\dot{c} = 0$ curve in the standard neoclassical growth model. To the left of this curve the return to capital exceeds the rate of time preferences, therefore consumption increases over time. To the right of this curve the opposite is true.

Additionally, the locus $a = \underline{a}$ divides the space between points for which consumption decreases (to the left) and points for which consumption increases (to the right). At $\underline{a}$ agents switch occupations. Thus, the relevant return to their wealth changes from being low, $r < \rho$, to being high, $f_k(e, \underline{a}) > \rho$ (to the right of $\underline{a}$). This curve is labeled $(\dot{c} = 0)_2$ in Figure 2.

Similarly, by setting $\dot{a} = 0$ in equation (6) I obtain an equation that describes the locus of points where wealth is constant. These correspond to points

[8] Otherwise the wealth of all agents will drift upward.

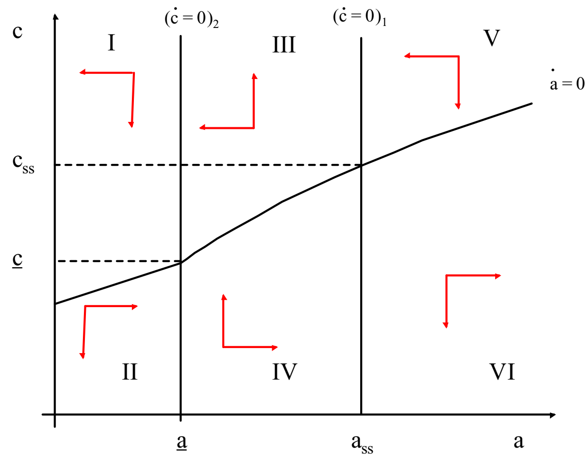

Figure 2: Phase Diagram of Households' Problem ($r < \rho$)

for which consumption equals income. Above this curve consumption exceeds income and therefore wealth decreases over time. Below, consumption is less than income thus wealth decreases over time. This curve is labeled as $\dot{a} = 0$ in Figure 2.

Trajectories in regions III, IV, V and VI behave as in the phase diagram of a standard neoclassical growth model. Trajectories in Region III move to the northwest, those in Region IV move to the northeast, in Region V they move to the southwest, while those in Region VI travel to the southeast.

Trajectories in regions I and II behave as in the phase diagram of a savings problem of a worker facing an interest rate lower than the rate of time preference, $r < \rho$. Combination of wealth and consumption (a, c) in Region I will follow trajectories going to the southwest. These are points for which consumption exceeds income, and wealth is not high enough for the entrepreneurial technology to be profitable, therefore the relevant return to savings is given by the interest rate that is lower than the rate of time preference. Wealth and consumption pairs in Region II also correspond to pairs with low wealth level, and low return to savings, therefore consumption will tend to decrease. But for pairs in Region II, since consumption is lower than income, wealth increases over time.

Of all these trajectories, only the ones converging to the points (a_{ss}, c_{ss}) and $(0, w)$ satisfy the transversality condition and do not exhibit jumps in finite time. For example, trajectories in region I above the one converging to the point $(0, w)$ will eventually hit the y axis above w and will be associated with a discontinuous consumption path in finite time.

To identify the trajectories that converge to the two steady states, it is helpful to view the phase diagram of this model as the combination of the phase diagram of the standard neoclassical growth model (regions III, IV, V and VI) and the phase diagram of the saving problem of a worker with no borrowing constraints and $r < \rho$ (regions I and II).

$>$From the analysis of the standard growth model it is known that there exists a single trajectory converging to the (a_{ss}, c_{ss}) steady state (the stable path). In a similar fashion, from the savings problem with $r < \rho$, it is known that there exists a single trajectory passing through the point $(0, w)$.

Since the problem is not concave, there is not a unique path starting from a given level of initial wealth that satisfies the necessary conditions. Also, the necessary conditions are not sufficient: there may be many trajectories that start from a given level of initial wealth and

satisfy the necessary conditions. But most of them are not optimal.

For instance, for levels of initial wealth close to $\underline{a}$ there exist at least two initial levels of consumption associated with trajectories that satisfy the transversality condition and do not exhibit jumps in finite time. The one with high consumption will lead in finite time to the low wealth steady state. The trajectory associated with low initial consumption will eventually lead to the high wealth entrepreneurial steady state.

Moreover, for $a_0 = \underline{a}$ there exist many other initial consumption levels that eventually converge to one of these steady states after cycling around the point $(\underline{a}, \underline{c})$. Figure 3 illustrates the trajectories satisfying the necessary conditions for the case of intermediate ability (see Proposition 1).

The next result characterizes the possible configurations of the trajectories in the phase diagram: for agents with high entrepreneurial ability only the trajectory converging to the low wealth worker steady state cycles around the point $(\underline{a}, \underline{c})$; for agents with low entrepreneurial ability only the trajectory converging to the high wealth steady state cycles around the point $(\underline{a}, \underline{c})$; for individuals with intermediate entrepreneurial ability Figure 3 is the relevant case.[9]

Proposition 1: *There are three possible configurations of the trajectories satisfying the necessary conditions:*

1. *Only the trajectory converging to the* $(0, w)$ *steady state cycle around the point* $(\underline{a}, \underline{c})$.
2. *Both, the trajectory converging to the* $(0, w)$ *steady state and the trajectory converging to the* (a_{ss}, c_{ss}) *steady state cycle around the point* $(\underline{\text{b}}, \underline{\text{c}})$ (intermediate case).
3. *Only the trajectory converging to the* (a_{ss}, c_{ss}) *steady state cycles around the point* $(\underline{a}, \underline{c})$.

The next step is to discriminate among all of the paths satisfying the necessary conditions for optima. In order to do this, I use results from Skiba (1978)

[9] All the proofs are in the appendix.

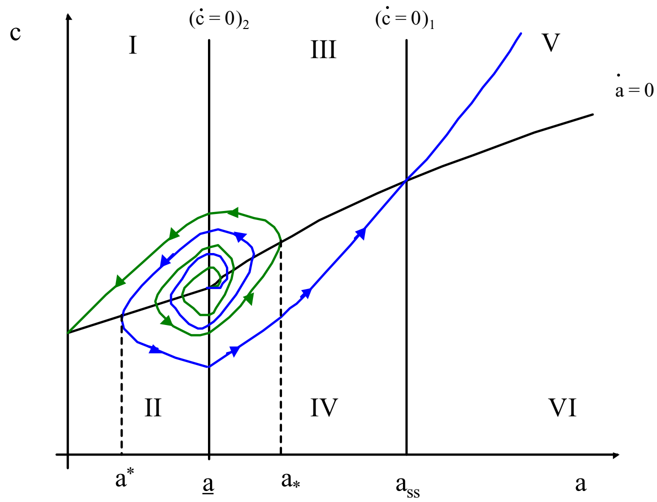

Figure 3: Trajectories Satisfying the Necessary Conditions (Intermediate Ability)

and Brock and Dechert (1983) developed to analyze optimal control problems when the Hamiltonian function is not concave in the state variable (wealth a in the present context). An executive summary of their results is the following: (a) whenever there are multiple candidate paths the ones with extreme initial value for the co-state (or consumption in our application) are optimal; (b) if the optimal co-state as a function of the state jumps (exhibits discontinuities), then it will jump down.

Applying Skiba (1978) and Brock and Dechert (1983) I prove the main result of this section: given an ability level e, households with low initial wealth will follow a path converging to a zero wealth worker steady state, and households with high initial wealth will follow a path converging to a high wealth entrepreneurial steady state. Intuitively, households with low initial wealth require a larger investment in terms of forgone consumption to save up toward the efficient scale. Thus, they prefer to have a lower but smoother consumption profile as wage earners. Figure 4 illustrates the optimal trajectories in the intermediate ability case (for e low enough $a_s > \underline{a}$, implying that there are individuals that start as entrepreneurs, but choose to eat their wealth an eventually become workers).

Proposition 2*: Depending on the configuration of the trajectories satisfying the necessary conditions (see Proposition 1), the optimal trajectories are:*

1. *In the first case of Proposition 1, for all levels of initial wealth it is optimal*

for agents to follow the trajectory converging to the $(0, w)$ *steady state.*

2. *In the intermediate case of Proposition 1, there will a single initial wealth,* $a_s(e)$*, such that individuals starting with wealth level,* $a_s(e)$*, will be indifferent between following the trajectory converging to the* $(0, w)$ *steady state or the trajectory converging to the* (a_{ss}, c_{ss}) *steady state. Agents with initial wealth to the left of* $a_s(e)$ *prefer to follow the trajectory converging to the* $(0, w)$ *steady state. The converse holds for agents starting with wealth to the right of* $a_s(e)$*.*

3. *In the third case of Proposition 1, for all levels of initial wealth it is optimal for agents to follow the trajectory converging to the* (a_{ss}, c_{ss}) *steady state.*

This proposition tells us that the typical policy function for consumption will be discontinuous. For agents with low initial wealth, it is optimal to start with relatively high consumption, but which is decreasing. For agents with high initial wealth it is optimal to start with a relatively low consumption, but which is increasing. Moreover, there is a unique threshold on initial wealth that divides individuals in these two groups. I refer to this threshold as the poverty trap threshold. The poverty trap threshold is a function of entrepreneurial ability.

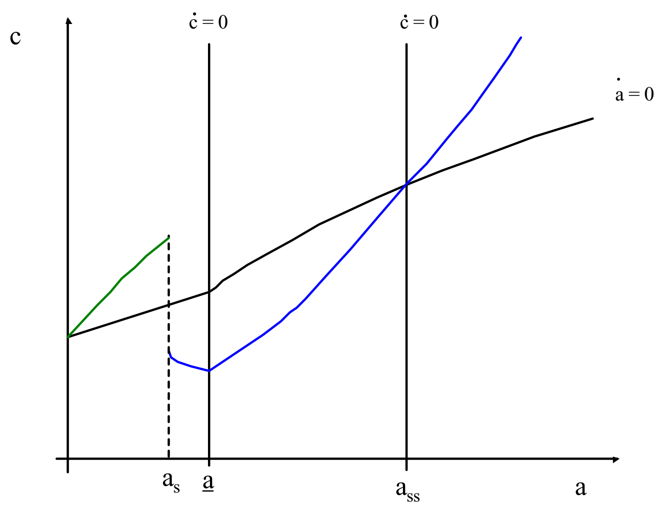

Figure 4: Optimal Trajectories (Intermediate Ability)

This characterization implies the following corollary.

Corollary to Proposition 2: (a) *The savings rates of individuals who eventually become entrepreneurs is higher than the savings rates of individuals who* *remain wage earners. (b) The growth rate of consumption increases after individuals become entrepreneurs.*

This suggests two obvious tests for the model that are performed in section 7.

The next result states that the threshold $a_s(e)$ is decreasing in the agent's entrepreneurial ability. It also tells us that there is a minimum ability e_{low} and a maximum ability e_{high} such that nobody with ability lower than e_{low} is an entrepreneur in the long run, and, everybody with ability higher than e_{high} is an entrepreneur in the long run.

Proposition 3*: If* $a_s(e) \leq \underline{a}(e)$*, the poverty trap,* $a_s(e)$*, is strictly decreasing in the agent's ability, i.e.*

$$\frac{\partial a_s(e)}{\partial e} < 0.$$

Moreover, there exits a strictly positive ability level, e_{low}*, such that*

$$V^w(a, e_{low}) \geq V^e(a, e_{low}) \text{ for all } a$$

and a finite ability level, e_{high}, such that

$$V^e(a, e_{high}) \geq V^w(a, e_{high}) \text{ for all } a,$$

where $V^w(a, e)$ $(V^e(a, e))$ is the value of the problem if the trajectory with high (low) initial consumption is selected.

The intuition of this result is straightfoward. For individuals that are still workers, entrepreneurial ability only affects those who plan to follow the entrepreneurial path. In other words, the more profitable it is to be an entrepreneur, the less likely an individual will be stuck in a poverty trap. The ability to save gives an upper bound on the importance of borrowing constraints.

Next, the relationship between various parameters of the model and the poverty traps threshold is discussed.

Proposition 4*: The following is true in a neighbourhood of* $r = \rho$ *such that* $r < \rho$*: the poverty trap,* a_s*, is increasing in the discount rate,* ρ*, and decreasing in the intertemporal elasticity of substitution,* $\frac{1}{\sigma}$*. Moreover, for e close to* e_{low}*, the poverty trap,* a_s*, increases with the interest rate, while for e close to* e_{high}*, it decreases with the interest rate.*

Poverty traps are more likely if the intertemporal elasticity of substitution is lower and the discount rate is higher, since these two parameters affect the costs of a high savings plan. Interestingly, the effects of the interest rate and a given worker's wage rate are ambiguous. A higher interest rate and a higher wage make it easier for able individuals to save the capital needed to start a profitable business, but, at the same time, a higher interest rate and higher wage raise the costs of being in business and make the option of being a wage earner relatively more attractive.

In the next section, I give a quantitative assessment of the ability of savings to overcome borrowing constraints to entrepreneurship.

5 A Numerical Example

This section studies a calibrated version of the dynamic model to understand the potential of borrowing constraints to generate significant poverty traps and welfare cost when foward-looking saving decisions are incorporated into the analysis. An economy with no credit is chosen as the benchmark, i.e. $\lambda = 1$.

5.1 Parametrization

To parametrize the individual's problem I need to specify preferences and entrepreneurial technologies.

I choose standard CES preferences

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

where σ is the reciprocal of the intertemporal elasticity of substitution.

Following Lucas (1978), I propose a constant return to scale technology on the entrepreneurial ability, capital and labor,

$$\tilde{f}(\tilde{e}, k, n) = \tilde{e}^{\nu} \left(k^{\tilde{\alpha}} n^{1-\tilde{\alpha}}\right)^{1-\nu}$$

where α represents the share of payments going to the variable factors (capital and labor) that are paid to capital, and, ν the share to the entrepreneur (it is also referred to as the span of control parameter (Lucas 1978) or the returns to scale of the variable factors). Note that I only need to specify the reduced form for output net of labor cost as a function of capital and entrepreneurial ability, once labor is maxed- out,

$$\begin{aligned} f(\tilde{e}, k) &= \max_{l} \left\{ \tilde{e}^{\nu} \left(k^{\tilde{\alpha}} n^{1-\tilde{\alpha}}\right)^{1-\nu} - \bar{w} n \right\} \\ &= A \tilde{e}^{1-\alpha} k^{\alpha}, \end{aligned}$$

where $\bar{w}$ is the price of an efficiency unit of labor, $A = \left[\frac{(1-\nu)(1-\tilde{\alpha})}{w}\right]^{\frac{(1-\nu)(1-\tilde{\alpha})}{1-(1-\nu)(1-\tilde{\alpha})}}$, $\alpha = \frac{(1-\nu)\tilde{\alpha}}{1-(1-\nu)(1-\tilde{\alpha})}$.

It convenient to normalized the ability index to correspond to the profits an entrepreneur would make if unconstrained, $e = \max_k \left\{A\tilde{e}^{1-\alpha}k^{\alpha} - rk\right\}$. Note that when holding fix the unconstrained profits, e, and the level of capital, k, the profits are decreasing in the parameter α. The larger the share of capital share the lower the profits of an entrepreneur holding constant the scale and the unconstrained surplus. This implies that the larger the parameter α the larger will be the initial wealth required by an individual of ability e to prefer to save in order to become an entrepreneur, $\frac{\partial a_s(e;\alpha)}{\partial \alpha} > 0$. Also, the scale at which an individuals will find it profitable to start a business will be larger, $\frac{\partial \underline{a}(e;\alpha)}{\partial \alpha} > 0$. The bite of borrowing constraints will be larger the larger is α (see Figure 6 for an illustration of this result).

Finally, I assume that time is discrete and the life of an agent is finite. In particular, I let individuals live and work for 40 periods.

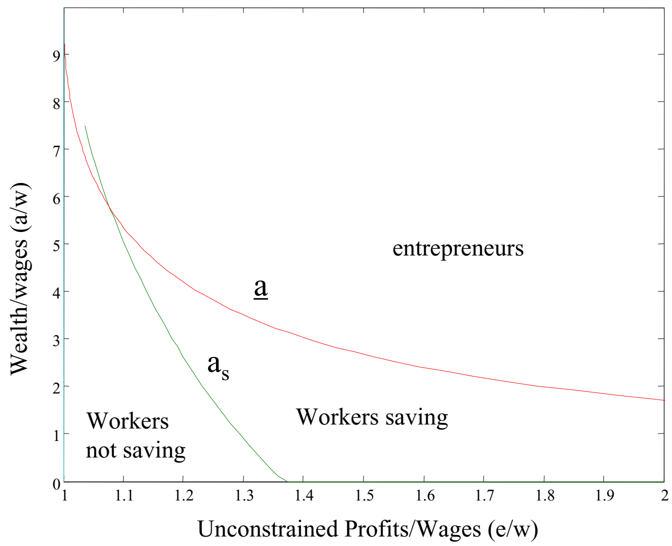

Figure 5: Poverty Trap, $\alpha = 0.5$.

5.2 Simulated Poverty Traps and Welfare Cost of Borrowing Constraints

I need to specify five parameters, σ, ρ, r, δ and α. The first four parameters can be set with little controversy. I choose $\sigma = 1.5$, a reasonable value for the reciprocal of the intertemporal elasticity of substitution. I set the time period to be 1 year, and, correspondingly, I let $\rho = r = 0.02$ to reflect the average market return to wealth. The depreciation of business assets, δ, is set to 0.08.

It is less obvious how to choose a "reasonable value" for the curvature of the entrepreneurial technology, α. On the one hand, the recent evidence from the micro data suggests to use a relatively low value for this parameter: Evans and Jovanovic (1989) estimate a static version of the model in this paper using household data on self-employment an found $\alpha = 0.39$; Cooper and Haltiwanger (2000) estimate $\alpha = 0.51$ using a panel of plants from the Longitudinal Research Dataset. On the other hand, recent numerical exercises evaluating the effect of tax policies using models of entrepreneurs have calibrated their model using values for α above 0.8 (e.g. Cagetti and De Nardi (2003), Li (2002)). I set $\alpha = 0.5$ as the benchmark case.

Figure 5 illustrates the poverty trap threshold, $a_s(e)$, as a function of the ability of the entrepreneur. In the horizontal axis, I measure ability as the profits an individual would make if operating at the unconstrained scale relative to her wage, a measure of the returns to entrepreneurship. In the vertical axis, I measure wealth relative to the wage.[10] Individuals with ability and initial wealth

[10] As discussed earlier, this is a natural normalization of the data since the parametrized

to the southwest of this curve, but with relative ability higher than one, are in a poverty trap. Eventhough they could be profitable businesses if operating at a unconstrained scale, the cost in terms of an uneven consumption profile is too large. I also plot the current wealth and returns combinations such that individuals are today indifferent between starting a business and working for a wage, i.e. the function $\underline{a}(e)$. This curve would be the poverty trap threshold if individuals were not allowed to save. The difference between the two curves gives a measure of the role of savings. Initial wealth no longer fully determines whether individuals become entrepreneurs. Rather, even individuals who start with low wealth could save to become entrepreneurs. Nevertheless, individuals that could earn up to 35% more as unconstrained entrepreneurs remain workers if they start with zero wealth.

In figure A.1.a I plot the time required to start a business starting from the poverty trap threshold, a_s, as a function of ability. There I also plot the time required to earn half the unconstrained returns starting from the poverty trap threshold, a_s. Even people that could earn 100% more income as unconstrained entrepreneurs require five years to save the capital required to become entrepreneurs and almost ten years to operate at a scale at which they make half the unconstrained returns. The delay to entry and to operate at a profitable scale suggest important welfare losses.

These welfare losses are illustrated in Figure A.1.b. There I plot the fraction by which the path of consumption must be increased to make an individual of a given ability indifferent between living in the economy with no credit and in the economy with perfect capital markets if she starts with zero wealth. The lower curve are the welfare costs associated with an economy where individuals are allowed to save, while the upper ones are the welfare costs of a static model. As the time to entry suggests (Figure A.1.a), there are potential enormous welfare losses due to borrowing constraints. Interestingly enough, the ability of individuals to cope with financial frictions by saving is less effective than their ability to ameliorate the effect of missing insurance markets. Aiyagari (1994) found that the welfare cost of missing insurance markets are reduced by half when agents can save to smooth consumption. In the case of financial frictions, not even for extremely productive entrepreneurs (those earning over 100% more income as entrepreneurs) is it the case that the welfare costs of borrowing constraints are reduce by at least 50 % when they are able to save in anticipation of their future business opportunity.

As I mentioned when discussing the choice of parameter values, there is no consensus regarding the value for α. It is therefore important to understand the sensitivity of this numerical exercise to the choice of α. This is done in 6.a (poverty traps) and 6.b (welfare costs). The size of poverty traps and welfare cost of borrowing constraints increase dramatically if we choose value of α above 0.6 while they tend to be unimportant if the entrepreneurial technology is characterized by strong decreasing returns to scales, α below 0.4. This ambiguity

model is homogeneous in the opportunity cost, w, the wealth, a, and the entrepreneurial ability.

motivates the study of US data on savings rates and entrepreneurial dynamics to sharpen the understanding of underlying parameters of the model that is done in section 7[11].

Before turning to study of data on savings and entrepreneurial dynamics, next section explores the predictions of the model for the aggregate relationship between entry into entrepreneurship and wealth, the focus of a large empirical literature that study the importance of borrowing constraints to entrepreneurship.

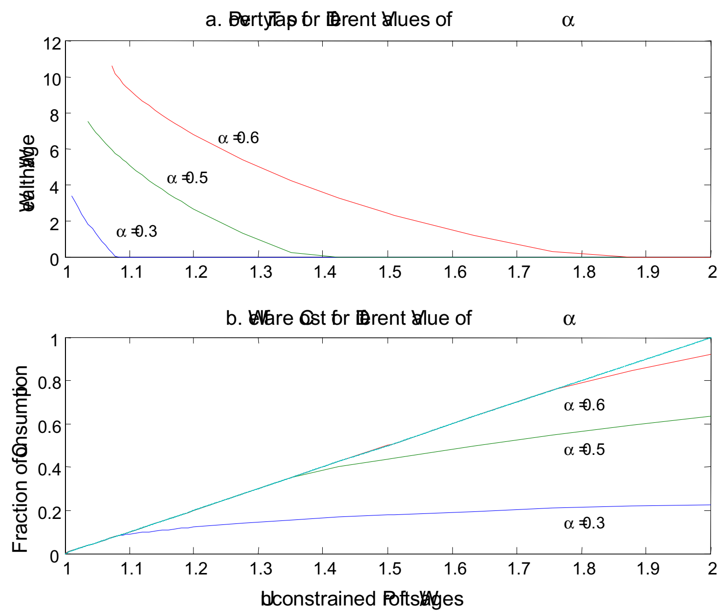

Figure 6: Sensitivity to α

6 Entry into Entrepreneurship and Wealth

Most of the empirical literature on entrepreneurship and borrowing constraints had' focused on measuring the effect of wealth on the likelihood that an individual becomes an entrepreneur. A positive relationship is often seen as evidence in favor of the direct effects of borrowing constraints. Understandably, many researchers have expressed the concern that wealth and ability, which is unobservable, may them be positively correlated making such interpretation problematic. In this section I address both issues.

[11] For similar conclusions regarding the sensitivity of results in heterogeneous agents models to the values of the decreasing returns to the variable factors see Atkeson, Kahn and Ohanian (1996), Kocherlakota (1998) and Cordoba and Ripoll (2004).

The likelihood that an individual become an entrepreneur as a function of current wealth, conditional on age, $P(transition|a,t)$, is a non-monotonic function of wealth. The fraction of individuals that enter entrepreneurship is an increasing function of wealth for low wealth levels, as in a static model (e.g. Evans and Jovanovic (1989)), and a decreasing function of wealth for high wealth levels. Moreover, with time the transition probability is compressed from below and above. The intuition of this result is extremely simple. When looking at transitions we are considering the agents that are working today. But this is a selected sample. In particular, these are the individuals that did not find it profitable to start a business by the current period, i.e. they are relatively low e individuals. Moreover, this selection increases with the wealth of the agents. If somebody is rich and hasn't started a business yet, then he must be a bad entrepreneur. Next, I give a formal statement of this result.

Proposition 5: *There exists an age $\bar{t}$, an increasing function $a_{low}(t)$ and a decreasing function $a_{high}(t)$, satisfying $a_{low}(t) \leq a_{high}(t)$, a positive constant $\xi > 0$, and neighborhoods $\mathcal{N}(a_{high}(t), \xi)$ and $\mathcal{N}(a_{low}(t), \xi)$ such that:*

1. *for all $t < \bar{t}$*

$$P(transition|a,t) = 0 \quad \forall a \in [0, a_{low}(t)] \cup [a_{high}(t), \infty);$$

   and

$$P(transition|a,t) > 0 \quad \forall a \in \mathcal{N}(a_{high}(t), \xi) \text{ and } a \leq a_{high}(t)$$
$$P(transition|a,t) > 0 \quad \forall a \in \mathcal{N}(a_{low}(t), \xi) \text{ and } a \geq a_{low}(t)$$

2. *For all $t \geq \bar{t}$*

$$P(transition|a,t) = 0 \quad \text{all } a.$$

Figure 7 illustrates this result.

This proposition suggests a way to obtain information about the importance of borrowing constraints. In particular, it suggests a way to obtain a lower bound on the unconstrained scale of businesses, i.e. the unconstrained scale of the least efficient business for a fixed wage and borrowing constraints coefficient, λ. Indeed, if we were to observed that the decreasing portion of the transition into entrepreneurship occurs at very high wealth levels we would infer that the entrepreneurial technology are close to linear (or that borrowing constraints are very tight).

Remark 2: *The minimum wealth level such that nobody with wealth higher than this makes the transition to entrepreneurship is a lower bound on the unconstrained scale of the least efficient business in operation, i.e. $a_{high}(t) \leq a_u(\underline{e})$.*[12]

Next, I consider a simple implication of this result.

An important concern of the empirical literature on entrepreneurship and borrowing constraints is to measure the causal effect of wealth on the likelihood

[12] In the case $\lambda > 1$ (i.e., we allow borrowing), this inequality becomes $\lambda a_{high}(t) \leq k_u(\underline{e})$. A large $a_{high}(t)$ can be rationalized by a large scale of businesses (large α) or by a tight borrowing constraint (low λ).

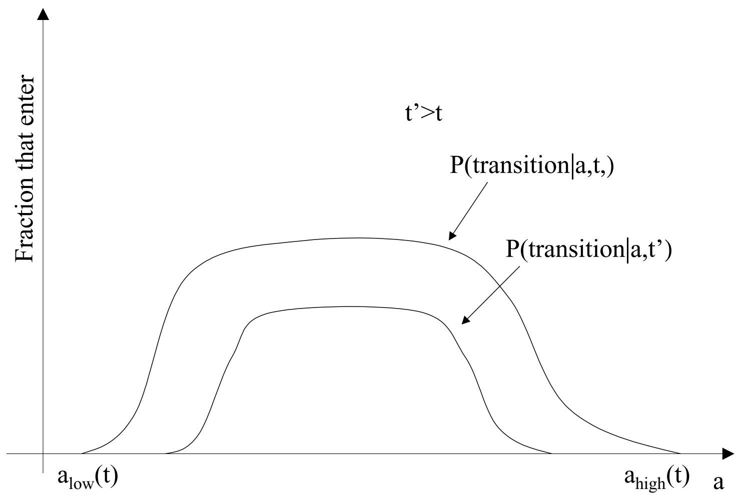

Figure 7: Transition into Entrepreneurship as a Function of Wealth and Age

an individual start a business. It is often believe that the actual correlation between entry into entrepreneurship and wealth overestimates the causal effect (e.g. Holtz-Eakin et. al. (1994)). It turns out that in the dynamic model the opposite is true. From proposition 5 we know that for high wealth level, wealth close to a_{high}, the transition to entrepreneurship as a function of wealth is decreasing. But the effect of a wealth transfer is still strictly positive. Thus, the observed effect of wealth on entrepreneurship underestimates the effect of a wealth transfer. This is the content of the next remark.

Remark 3: *For* $0 < t < \bar{t}$

$$\frac{\partial P\left(transition|a, t, w, \tilde{a}, \Delta\right)}{\partial a} < \frac{\partial P\left(transition|a, t, w, \tilde{a}, \Delta\right)}{\partial \tilde{a}}$$
$$\forall a \in N\left(a_{high}\left(t\right), \delta\right) \text{ and } a \leq a_{high}\left(t\right),$$

where $\tilde{a}$ corresponds to an exogenous component of wealth at time t that individuals expect to be zero with certainty.

7 Taking the Dynamic Model to the Data

In this section, US data on savings rates and consumption growth of entrepreneurs and the relationship between entry and wealth is used evaluate the predictions of and to estimate the dynamic model by Indirect Inference as describe in Gourieroux and Monfort (2002). The idea is to choose the parameters of the dynamic model to minimize the distance between a set of statistics describing the behavior of savings rates and entrepreneurial dynamics in the US data and those same statistics calculated from the simulated model. I begin by studying discussing the main features of the savings rates of entrepreneurs and the dynamics of entrepreneurship in two US datasets.

7.1 Data

I use a yearly panel for the period 1984–1995 from the Panel Study of Income Dynamics (PSID) with rich information on occupational choices, ownership of businesses and the wealth of US households; and a quarterly rotating panel (1984–1999) from the Consumer Expenditure Survey (CEX) providing consumption data and information on occupational choices. Since the model provides a theory of the initial transition into entrepreneurship, I estimated the model with data on the savings behavior and the dynamics of entrepreneurship for young households (those that are up to 31 years old). Therefore, unless otherwise notice, all statistics are calculated for household that are up to 31 years old in the initial period. The data used is described in the data appendix.

Following the recent literature (see Gentry and Hubbard (2001) and Hurst and Lusardi (2004)) an entrepreneur in the data is identified as someone who reports to own a business. Unfortunately, this information is not available for the CEX. In the case of the CEX, an entrepreneur is identified as someone who reports to be self-employed. Whenever it is possible, results are shown for both definitions.

The first and second column of Table 1 reports the main facts regarding the behavior of savings rates and consumption growth of entrepreneurs as measured by the ownership of a business and the self-employed status of the head of the household[13]. Here I sketch a summary of the data.

- **Individuals save more prior to starting a business.** Among the young households (age≤ 31), those that became business owners between periods t and $t+1$ save 7% more in the previous 5 years (between $t-5$ and t) than households that neither owned a business in t nor in $t+1$ (see first row of Table 1). Related, individuals becoming business owners between $t-5$ and t save 26% more between these years than those that do not own businesses in $t-5$ and t. This is refer to as the savings rate differential during entry in Table 2[14].

- **Business owners have higher saving rates than non-business owners, and their saving rates decrease sharply with age**. Household that own a business in $t-5$ and t and are up to 31 years old in period $t-5$ save 26% more than households that neither own a business in $t-5$ nor in t. Among mature households (those that are between 32 and 41 years old in $t-5$), business owners just save 10% more than non-business owners.

[13] All the moments are calculated using individuals characteristics as controls (sex, marital status and education of the head). The value of the different moments should therefore be interpreted as the one for a single white male with college education.

[14] Although the first is a more appropiate measure of savings in anticipation of entry, it cannot always be calculated in the simulated model since for some parameter values all entrepreneurs enter in less than 5 years.

- **Individuals becoming self-employed have higher consumption growth, both relative to workers and individuals that are already self-employed.** The growth in the consumption between $t-1$ and t of individuals becoming self-employed between $t-1$ and t is 7% higher than that of those who are workers in both periods. The consumption growth of households that are already self-employed is not particularly high.

Moments	Data		Model
	Business	Self-employed	
Savings rate differentials of entrepreneurs vis-a-vis workers			
Prior to entry	0.07	-0.06	0.30
	(0.05)	(0.05)	
During entry	0.26	0.19	0.34
	(0.04)	(0.04)	
After entry			
young	0.26	0.30	0.25
	(0.08)	(0.05)	
mature	0.10	0.11	0.11
	(0.05)	(0.06)	
Consumption growth differential entrepreneurs vis-a-vis workers			
During entry	–	0.09	0.24
		(0.04)	
After entry	–	0.01	0.09
		(0.03)	

Table 1: Data and Simulated Moments

The following fact is illustrated in Figure 8.

- **Poor individuals and extremely rich individuals are less likely to start businesses.** Among the young, the probability that a household that is not a business owner in period $t-5$ starts a business in period t is mostly constant (around 10%) for the first 3 wealth to wage ratio quartiles, increases sharply to 20% for wealth to wage ratios in the 90th to 95th percentiles, and then decreases for wealth to wage ratios in the top 5th percentile.

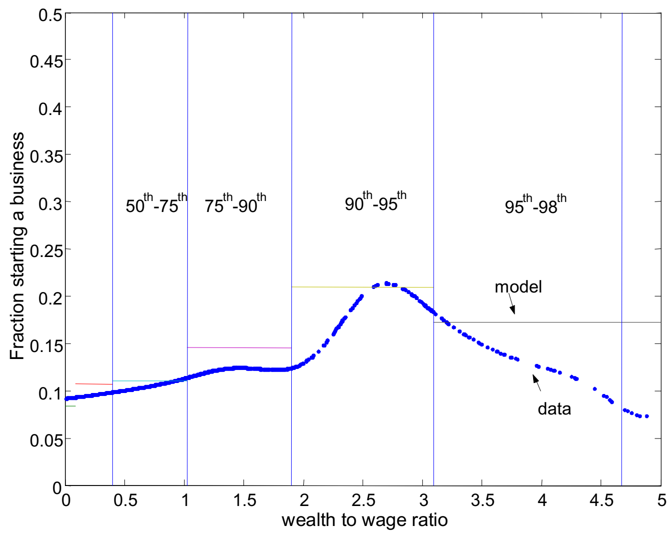

Figure 8: Wealth to wage ratios and the Transition into Business Ownership, PSID and Estimated Model

Related datasets have being studied by other authors. Using the 1983-1989 panel from the Survey of Consumer Finances, Gentry and Hubbard (2000) also report business owners having larger savings rates than non-business owners, and this effect being stronger for younger households as oppose to middle age households. They also find that the households save more while becoming business owners. In their sample, the median household that become a business owner between $t-6$ and t save 30% more between $t-5$ and t than households that are neither business owners in $t-6$ nor in t. Unfortunately, given the short nature of SCF panel they cannot study the savings behavior in anticipation of the entry decision as is done in the current paper. More in line with my findings on relatively low savings rates in anticipation of entry, Hurst and Lusardi (2004) also using the PSID find that previous wealth changes are weakly (and even negatively) correlated with future entry decision. They as well study the relationship between wealth and the likelihood of becoming business owner and also find that the positive effect of wealth is stronger for the top percentiles of the wealth distribution. However, they do not study the relationship between entry into entrepreneurship and wealth to wage ratios as suggested by the model in this paper.

7.2 Structural Estimation

The structure of the model is characterize by two preferences parameters, σ and ρ, one technological parameter, α, the borrowing constraint coefficient, λ, the distribution of ability, $g(e)$. To close the model, the interest rate, r, and initial distribution of wealth, $h(a_0)$, also need to be specified. The parameters are chosen using a combination of calibration and structural estimation. As in Section 5, I choose values for the preferences parameters, the interest rate and the depreciation rate following the standard practices: $\sigma = 1.5$, $\rho = r = 0.02$ and $\delta = 0.08$. The distribution of initial wealth is chosen to correspond to the distribution of wealth to income ratios of non-business owners that are up to 26 years old in the PSID. This leaves the technology parameter α, the borrowing constraint coefficient λ and the distribution of ability to be jointly estimated by minimizing the distance between the moments in the data and the moments in the model. The details of the algorithm are described in the appendix.

Table 2 reports the parameter estimates and their standard errors. An economy where entrepreneurs face strong decreasing returns to scale, $\alpha = 0.36$, and tight borrowing constraints, $\lambda = 1.01$, best fits the data. The distribution of ability turned out to be bimodal with 40% of the profitable entrepreneurs (those with relative ability $\frac{e}{w} > 1$) having low returns (they could earn 5% more income as unconstrained entrepreneurs) and 30% facing large returns to entrepreneurship (they could earn three times as much income as unconstrained entrepreneurs)[15].

Parameters	Estimate
Technology, α	0.36
	(0.01)
Borrowing Constraint, λ	1.01
	(0.03)
Probability (e | e>1)	
e = 1.20	0.39
1.05	0.02
1.50	0.23
3.00	0.32
5.00	0.05

Table 2: Parameter Estimates

Welfare cost of borrowing constraints are large (see Table 3). For the me-

[15] For the estimated economy, since all transitions intro entrepreneurship are permanent, 46% of individuals are entrepreneurs and the probability of $e = 1$ is 0.24 . This is an artifact of not modelling exit and reentry of businesses.

dian among the productive entrepreneurs (individuals with $e > 1$) welfare costs amounts to 22% of lifetime consumption. The average welfare cost for the economy are 6% of lifetime consumption[16]. But, because there are strong decreasing returns to scale, individuals can undo most of the welfare cost of borrowing constraints by internally financing their businesses. This is specially true for very able individuals. In the same direction, the size and economic significance of poverty traps are low. Only individuals that would increase their income by just 5% decide not to save to start a business (12% of the population fall in this category).

Ability	Wealth			
	zero wealth	50th perc. wealth/ wage	75th perc. wealth/ wage	90th perc. wealth/ wage
1.05	5.00	4.94	4.86	4.71
1.20	16.12	14.99	13.35	9.81
1.50	24.48	22.07	18.62	12.11
3.00	40.96	34.42	26.20	18.62
5.00	52.08	39.10	31.02	23.52
Mean Welfare Costs = 6.24				

Table 3: Welfare Costs by Ability (rows) and Initial Wealth (columns) Measured as a Fraction of Lifetime Consumption (%)

The third column of Table 1 and Figure 4 reports the moments from the simulated model at the estimated parameters. The model does a good job in fitting the relationship between the transition into business ownership and wealth, specially the fact that their substantial entry even for large wealth levels (the hump appears at large wealth levels). Savings rates of entrepreneurs, both young and mature, are matched but the model substantially overpredicts the savings rates prior to entry. The same is true for the growth rate of consumption of entrepreneurs. One the one hand, the behavior of savings rates and consumption growth suggest that borrowing constraints are not very important. But the relationship between wealth and the transition into entrepreneurship suggests the opposite conclusion. The estimation is the result of a compromise between these sets of moments.

[16] To calculate the average welfare cost, I force the distribution of ability to match the fraction of entrepreneurs in the data. This is done by shifting the mass of the ability distribution from ability values with $e > 1$ to values with $e = 1$, in a proportional way so that the distribution of ability conditional on $e > 1$ is not affected (and therefore not affecting the shape of the transition function (see Figure 8)).

8 Conclusions

The motivation of this paper can be summarize by two questions. (1) Are borrowing constraints limiting entrepreneurship of any significance once we take into account that individuals can save to overcome them? (2) Is the interpretation of the evidence on the importance of borrowing constraints to entrepreneurs affected by modelling the dynamic aspects of the problem? The answer to both question is "yes". Individuals facing large returns to entrepreneurship will remain workers in the long run provided returns to scale are not too decreasing. While by incorporating the dynamics aspects of the entrepreneurial decision problem we affect substantially the interpretation of the evidence on the effect of borrowing constraints to entrepreneurship.

This is an important first step. Nevertheless, the model abstract from potentially relevant dimensions that require further research.

For instance, while the initial wealth of individuals was allowed to be correlated with the ability of entrepreneurs, it was assumed that the ratio of the wealth to the labor income is uncorrelated with the ratio of the entrepreneurial ability relative to the ability as a worker. In other words, it was not allowed that individuals that are relatively able entrepreneurs are also wealthy relative to their opportunity cost. If able individuals also happen to start rich the cost of imperfect credit markets will tend to be smaller.

As well, in this paper the uncertainty faced by entrepreneurs is not modeled. As suggested by the large amount of exit in the data this is an important dimension. In this lines, it would be extremely interesting to measure the role of uncertainty relative to initial resources in determining entrepreneurship, and the welfare cost associated with each potential market failure. These important questions are left for future research.

A Data

I use a yearly panel for the period 1984–1995 from the Panel Study of Income Dynamics (PSID) with rich information on occupational choices, ownership of businesses and the wealth of US households; and a quarterly rotating panel (1984–1999) from the Consumer Expenditure Survey (CEX) providing consumption data and information on occupational choices.

In the case of the PSID, I create a 7 years panel pooling the households in the 1984-1990 and 1989-1995 samples. This gives a panel with two observations for wealth (1984 and 1989 in the 1984-1990 subsample, 1989 and 1994 in the 1989-1995 subsample), and yearly observations on the ownership of businesses and income. Using the pooled panel, I construct savings rates between the first and the fifth years, $savings_{1-5} = \frac{wealth_5 - wealth_1}{\sum_{t=sd1}^{5} income_t}$, wealth to income ratios (the relevant measure of wealth in the model) and business ownership histories. "Wealth" corresponds the sum of net equity in a main home, other real estate, vehicles, farm/business, stocks, savings accounts and other assets, less debt; "income" equals total family money income plus food stamps minus federal income taxes paid; and business ownership status is determined by the question "Do you (or anyone in your family living there) own part or all of a farm or business?" For comparison purposes, I also use information about the self-employ status of the head of the household.

I restrict the sample to the households that are at least 22 and at most 61 years old in the first period, that are working in the 1, 2, 6 and 7th periods (these are the periods for which business ownership information is used) and I drop the observations with savings rates below and above the 1st and 99th percentiles respectively. These restrictions leaves 5354 observations that are used to calculate the moments reported in Table 1.

In the case of the CEX I use the quarterly Interview component for the 1984-1999 period. From this dataset I use information on non-durable consumption, self-employed status of the head of the household and demographic characteristics. After applying similar restrictions to the PSID sample, 5545 observations of household that are up to 31 years old are left to calculate the moments reported in Table 1.

B Algorithm

Given values for the preferences and technology parameters $(\sigma,\ \rho,\ \alpha, \delta)$, the borrowing constraint coefficient (λ), the interest rate (r), the individual decision problem is solved for each ability level in the ability grid,
$e = \{1, 1.05, 1.1, 1.2, 1.5, 2, 3, 5, 7, 10\}$, by backward induction from the last period, $T = 40$. Given the policy functions and 10000 values for the initial wealth drawn from the distribution of wealth to income ratio of non-business owners that are up to 26 years old in the PSID, 10000 histories are generated for each value of ability, $\{x_t(e)\}_{t=1}^{T}$. Where the initial wealth to income ratio takes values on the grid $a_0 = \{0, 0.17, 0.49, 1, 1.84, 2.87, 8.03\}$ with probabili-

ties $h(a_0) = \{0.25, 0.25, 0.25, 0.15, 0.05, 0.03, 0.02\}$. Then, by stacking the data for individuals at different ages I obtained a simulated (unbalanced) panel of 400000 observations (10000 forty years observations, $\{x_t(e)\}_{t=1}^{T}$, 10000 thirty-nine years observations, $\{x_t(e)\}_{t=2}^{T}$, ...) that is used to calculate the statistics for each value of ability. Given the distribution of ability, a vector of dimension 10, the statistics from the model are calculated. The algorithm then search over values of the returns to scale parameter, α, the borrowing constraint coefficient, λ, and the distribution of ability to minimized the weighted distance between the statistics calculated using the PSID data and the data from the simulated model. The weights are given by the inverse of the covariance matrix of the moments from the PSID data.

A important simplification is introduced by the fact that ability is fixed over the life of an individual, implying that the individual decision problem is independent of the ability distribution. For given values of α and λ, a gradient based method (matlab routine fmincon.m) is used to minimize over the distribution of ability subject to the constraint that at most six ability values get to be assigned a positive probability. Then, grid search is used to minimized over α and λ.

C Proof of the Results in the Paper

Proof of Proposition 1:. For sufficiently high e the first case arises since the trajectories to the left of $\underline{a}$ are not affected by ability, e. Similarly for sufficiently low e (e.g. $e = \underline{e}$) we have the third case. For intermediate value of ability the intermediate case arises. The only thing that need to be proved is that a case where neither the trajectory converging to the $(0, w)$ nor the trajectory converging to the (a_{ss}, c_{ss}) cycle around the point $(\underline{a}, \underline{c})$ is not possible. This is proven by contradiction.

Assume that a case where neither the trajectory converging to the $(0, w)$ nor the trajectory converging to the (a_{ss}, c_{ss}) cycle around the point $(\underline{a}, \underline{c})$ is possible. Then for values of initial wealth close to $a_0 = 0$ the trajectory converging to the (a_{ss}, c_{ss}) steady state is optimal since for $a_0 = 0$ the plan that states forever at the point $(0, w)$ corresponds to a zero of the Hamiltonian function therefore the plan starting with lower consumption and converging eventually to the (a_{ss}, c_{ss}) steady state is preferred.[17] Similarly, for values of initial wealth close to $a_0 = a_{ss}$ the trajectory converging to the $(0, w)$ steady state is optimal . But this contradicts Theorem 2 in Brock and Dechert (1983). ■

Proof of Proposition 2:. The first step is to rule out the trajectories that circle around $(\underline{a}, \underline{c})$. That these trajectories are not optimal follows from the

[17] The Hamiltonian function of this problem is

$$H(a_0, \lambda_0) = \max_c \{u(c) + \lambda_0 (y(e, a_0) - c)\}.$$

This function gives the value of following a path that satisfies the necessary conditions (See Skiba (1978) and Brock and Dechert (1983)). Note that the Hamiltonian is a strictly convex function of λ_0. Thus $\frac{\partial H}{\partial \lambda_0} = \dot{a} = 0$ correspond to a global minima.

Hamiltonian function being strictly convex in the initial Lagrange multiplier, i.e. optimal trajectories are among the trajectories that start with extreme values for Lagrange multiplier and therefore consumption. Let a_* (a^*) be the first point at which the upper (lower) trajectory crosses the $\dot{a} = 0$ locus. Then, the result is a direct application of Theorem 2 in Brock and Dechert (1983) and the fact that at point a_* the trajectory converging to the $(0, w)$ steady state is optimal and at the point a^* the trajectory converging to the (a_{ss}, c_{ss}) steady state is the optimal one. This last observation follow from the fact that at the point a_* the trajectory converging to the (a_{ss}, c_{ss}) steady state path through the $\dot{a} = 0$ curve, a zero of the Hamiltonian function with respect to the Lagrange multiplier. A similar argument applies for $a = a^*$. ■

Proof of Proposition 3. The threshold is implicitly defined by the following equation

$$V^e(a_s, e) - V^w(a_s, e) = 0$$

where V^w is the value of following the upper trajectory and V^e is the value associated with the lower trajectory in Figure 3. For $a_s < \underline{a}$ V^w does not depend on e while V^e is a strictly increasing function of e therefore a_s is strictly decreasing in e. Clearly, for $e < \underline{e}$ $V^w(a, e) > V^e(a, e)$ for all a. Therefore, there is an infimum ability $e_{low} \geq \underline{e}$ such that $V^w(a, e) \geq V^e(a, e)$ for all a. Similarly, since $V^w(0, e)$ is independent of e, as long as $u(c)$ is not bounded, there exist a supremum (finite) ability e_{high} such that $V^e(0, e_{high}) > V^w(0, e_{high})$, and from Theorem 2 in Brock and Dechert (1983), $V^e(a, e_{high}) > V^w(a, e_{high})$ for all a. ■

Proof of Proposition 4. If $r = \rho$, the poverty traps threshold solve $w + ra_s = c^e(\underline{a})$, where $c^e(.)$ is the policy function for consumption associated with the stable path of the entrepreneurial problem. Both is ρ or σ increases then $c^e(\underline{a})$ increases and therefore a_s locally increases. If e is close to e_{high}, a decrease in r has no effect on $V^w(e)$ while it decreases the value of $V^e(e)$ since individuals are saving to become entrepreneurs. For e close to e_{low} the opposite is true. ■

The following assumption is needed for the next result.

Assumption A.1: The policy function, $a(a_0, t, e)$, is strictly increasing and continuous in the entrepreneurial ability, e, for $a_0 \geq a_s(e)$.

This is a natural assumption. It requires that present consumption does not increase too much when ability increases. In terms of assumptions about preferences, it needs that the preferences between consumption today and tomorrow are not too bias toward present consumption. For example, it will true in a two period model if preferences are homothetic.

Proof of Proposition 5. The transition probability conditional on wealth, a, and age, t, is given by the integral over all agents with ability high enough such that they find it profitable to start a business by $t + \Delta$, $e > \underline{\underline{a}}^{-1}(a + \tilde{a}, \Delta)$, but not so high for them to be already entrepreneurs, $e < \underline{a}^{-1}(a)$. Where

$\underline{\underline{a}}(e,\Delta)$ solves the equation $a\left(\underline{\underline{a}}(e,\Delta),\Delta,e\right)=\underline{a}(e)$. Formally

$$P\left(transition|a,t,\tilde{a},\Delta\right)=\frac{\int\limits_{e\in\mathcal{E}^{w}(a,t)\,,\,e>\underline{\underline{a}}^{-1}(a+\tilde{a},\Delta)} g\left(e\right)h\left(a_{0}\left(a,t,e\right)\right)de}{1-P\left(entrepreneur|a,t\right)} \tag{8}$$

where $\mathcal{E}^{w}(a,t)$ is the support of ability conditional on wealth a, age t and currently being a worker of those individuals that will eventually become entrepreneurs, $\mathcal{E}^{w}(a,t)=\{e\in R_{+}:\exists a_{0}$ for which $a(a_{0},t,e)=a$, $e<\underline{a}^{-1}(a)$ and $e\geq a_{s}^{-1}(a)\}$. Then, to proof this result we need to characterize the lowest and highest wealth levels, $a_{low}(t)$ and $a_{high}(t)$, such that this set is non-empty.

That there exist an increasing function $a_{low}(t)$ such that for $a<a_{low}(t)$ the set $\mathcal{E}^{w}(a,t)$ is empty follows trivially from the fact that the wealth of individuals that will eventually become entrepreneurs increases overtime and that $a_{0}\geq 0$.

The function $a_{high}(t)=\underline{a}(e_{\min}(t))$ where $e_{\min}(t)=\inf\{e\in[e_{low},\infty]:a(a_{s}(e),t,e)=\underline{a}(e)\}$. Note that $e_{\min}(t)$ is a strictly increasing function of t since for e close to e_{low} $(a_{s}(e_{low})=\underline{a}(e_{low}))$ we know that $a(a_{s}(e),t,e)>\underline{a}(e)$ and, by continuity, we know that for the minimum root of the equation $a(a_{s}(e),t,e)=\underline{a}(e)$ the function $a(a_{s}(e),t,e)$ is decreasing and crosses the function $a_{s}(e)$ from below. Thus, the function $a_{high}(t)$ is then decreasing since it is the composition of a strictly decreasing function with an increasing function.

The maximum age such that the set $\mathcal{E}^{w}(a,t)$ is non-empty for some wealth level is define by

$$\bar{t}=\inf\left\{t\in R_{++}:a\left(a_{s}\left(e\right),t,e\right)\geq\underline{a}\left(e\right)\quad\forall e\geq e_{\min}\right\}$$

Age $\bar{t}$ is finite since $\underline{a}(e)<a_{ss}(e)$ $\forall e\geq e_{\min}$. ■

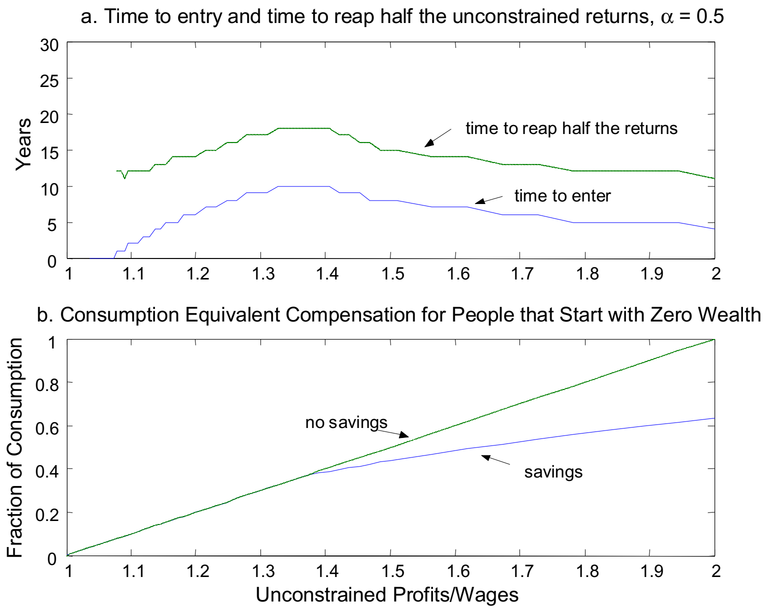

Figure A.1: Time to Enter and Wealfare Cost, $\alpha = 0.5$

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