

# Yield-Curve Factors as the Instruments of Monetary Policy

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# 1. Introduction

- The modern view of central banking stresses the importance of guiding expectations about future monetary policy actions. It is not just the policy rate today that matters, but the whole sequence of future rates. Monetary policy is a process of shaping or managing the yield curve.
- To a first approximation, fluctuations in the yield curve can be summarized by movements in just a few factors—such as “level” and “slope”.
- This suggests that such yield curve factors could be usefully considered as the instruments of monetary policy. Monetary policy would manage the yield curve by jointly setting each of the yield-curve factors.
- We provide empirically-based guidance for the optimal setting of two yield-curve factors—level and slope—in an estimated macro-finance model that links these two factors to inflation and output.

## 2. A Factor Representation of the Yield Curve

Variation in term structure well captured by a few underlying factors

Variety of dimension-reduction techniques are available

(for example, Nelson and Siegel, 1987; Knez, Litterman, and Scheinkman, 1994; Dai and Singleton, 2000; and Diebold and Li, 2002).

Data:

Quarterly observations on  $i_{m,t}$ ,

Yield to maturity at time  $t$  of an  $m$ -month nominal zero-coupon bond.

U.S. Treasury securities for maturities of 1, 6, 12, 24, 60, and 120 months.

Sample runs from 1960:Q1 to 2004:Q4

First principal component captures 97.2 percent of the variation in the six yields.

First and second principal components capture 99.5 percent of the variation.

Similar results with our preferred Nelson-Siegel (1987) factor representation:

$$i_{m,t} = L_t + S_t \left( \frac{1 - e^{-k_t m}}{k_t m} \right) + C_t \left( \frac{1 - e^{-k_t m}}{k_t m} - e^{-k_t m} \right) + \varepsilon_{m,t} , \quad (1)$$

where  $L_t$ ,  $S_t$ ,  $C_t$ , and  $k_t$  are parameters of a cross-sectional regression.

Diebold and Li (2005) interpret this as a three-factor model.

**Table**  
**Measure of fit for Nelson-Siegel representation**

Median	Alternative representations	
	3-Factor ( $L, S, C$ )	2-Factor ( $L, S$ )
Standard Error (in basis points)	9	12

Note: The table provides the median standard error across all quarters.

### 3. Monetary Policy and Yield-Curve Factors

Decompose long rate into expected future short rate and term premium:

$$i_{m,t} = (1/m) \sum_{j=0}^{m-1} E_t(i_{t+j}) + \psi_{m,t}$$

We assume:

Fed controls short rate ( $i_t$ ) at each point in time.

Fed controls expectations of future short rate at each point in time.

Fed controls yield curve by altering expected future path for the short rate.

We don't assume that the Fed controls risk premium; however,

it can offset changes in  $\psi_{m,t}$  with changes in expected future short rate.

For parsimony, rather than specifying the short rate at each point in the future, we instead use policy rules to specify the dynamics of the few yield-curve factors. That is, level and slope will be considered the policy instruments.

## 4. Yield-Curve Factors and the Macroeconomy

Our model is an unrestricted VAR with five variables,  $y_t$ ,  $\pi_t$ ,  $\pi_t^e$ , *level*, and *slope*, where

$y_t$  is the percentage gap between actual GDP ( $q_t$ ) and potential GDP ( $q_t^*$ ) as estimated by the Congressional Budget Office, so  $y_t = 100(q_t - q_t^*)/q_t^*$ ;

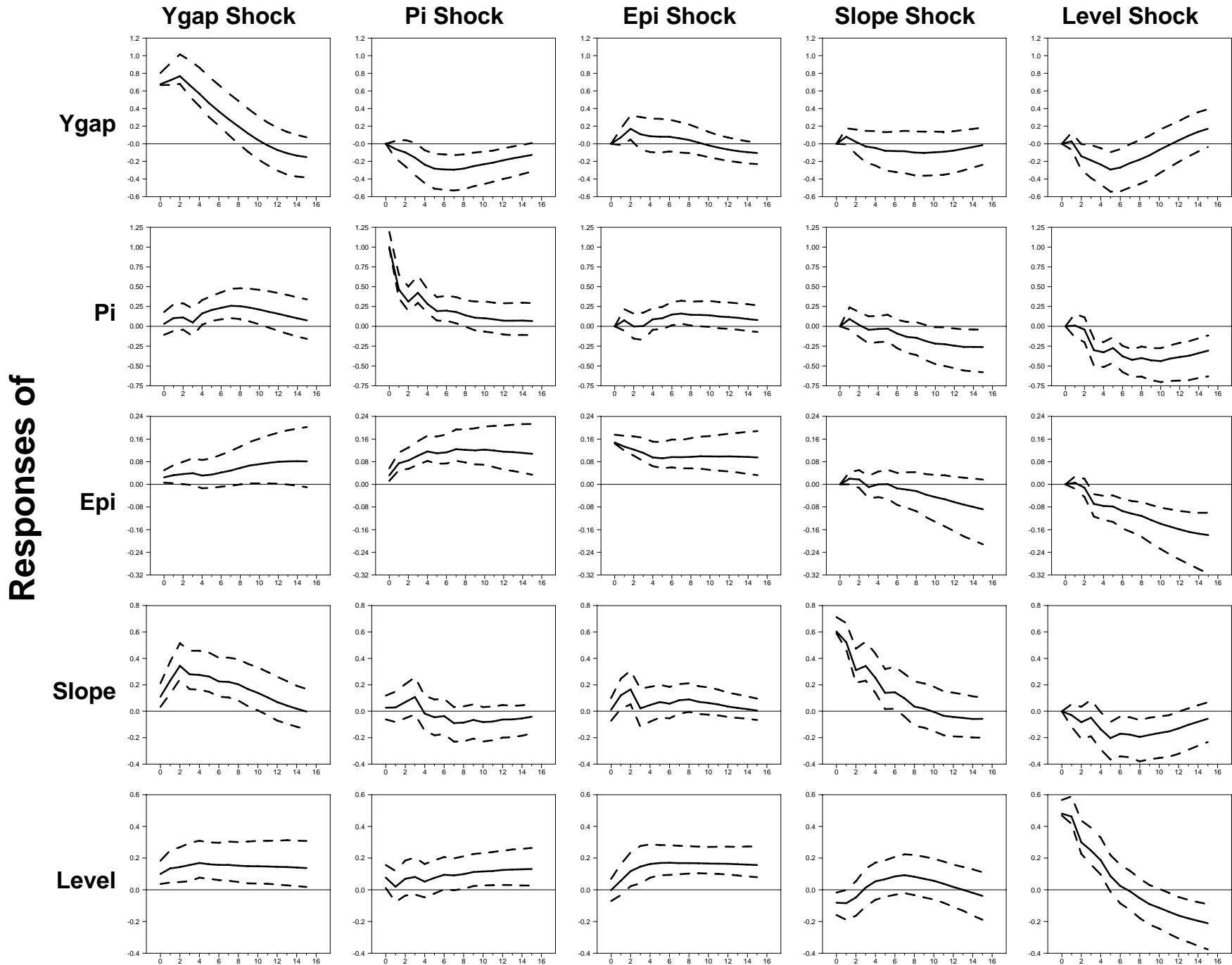
$\pi_t$  is quarterly inflation in the PCE chain-weighted price index ( $p_t$ ) in percent at annual rate, that is,  $400(\ln(p_t) - \ln(p_{t-1}))$ ;

$\pi_t^e$  is long-run (10-year) inflation expectations based on survey measures;

*level* and *slope* are Nelson-Siegel yield curve factors.

Our sample period runs from 1960:Q1 to 2004:Q4.

# Impulse responses



## 5. Optimal Monetary Policy with Yield Curve Factors

### 5.1 Loss Function

Policymaker's objective is to minimize a weighted average of the unconditional variances of the deviation of the inflation rate from a fixed target level,  $\pi^*$ , the output gap, the change in the nominal short-term interest rate, and the change in the long-term nominal interest rate:

$$\text{Loss} = \text{VAR}(\pi) + \lambda \text{VAR}(y) + \nu \text{VAR}(\Delta i) + \phi \text{VAR}(\Delta i_{120}),$$

where  $\lambda$ ,  $\nu$ , and  $\phi$  are non-negative parameters.

This specification differs by allowing penalty on long-rate variability.

### 5.2 Optimal Policy with a Short-Rate Instrument

Consider standard monetary policy with short rate is the sole policy instrument (level determined by its estimated equation; slope by inverting for short rate).

As a benchmark, consider fully-optimal policy, denoted *OPT*.

Also, consider a version of the Taylor Rule:  $i_t = \gamma_\pi^i \bar{\pi}_t + \gamma_y^i y_t$



Table 1: Optimal Policy with a Short-Rate Instrument

Preferences		Loss		Coefficients	
$\nu$	$\phi$	$OPT$	$i$	$\gamma_\pi^i$	$\gamma_y^i$
0.5	0	6.4	10.3	1.6	0.6
0.5	1	6.9	10.8	1.5	0.6
0.5	5	9.1	13.0	1.5	0.6
0.5	10	11.7	15.7	1.5	0.5
0.5	20	16.6	20.9	1.4	0.5
1.0	0	6.7	10.8	1.5	0.6
1.0	1	7.2	11.3	1.5	0.5
1.0	5	9.4	13.4	1.5	0.5
1.0	10	11.9	16.1	1.5	0.5
1.0	20	16.8	21.3	1.4	0.5
5.0	0	7.8	13.3	1.3	0.3
5.0	1	8.3	13.8	1.3	0.3
5.0	5	10.4	15.9	1.3	0.3
5.0	10	12.9	18.4	1.3	0.3
5.0	20	17.8	23.5	1.3	0.3

### 5.3 Optimal Policy with a Slope Instrument

Now consider Taylor Rule that targets the slope of the yield curve:

$$s_t = \gamma_\pi^s \bar{\pi}_t + \gamma_y^s y_t,$$

This equation replaces the estimated equation for the slope in the model. We assume that the central bank has perfect control over the slope. The results are reported in Table 2. Note that the results for the fully-optimal rule are the same as before.

The optimal slope Taylor rule policy rule performs somewhat better than the short-rate rule, but still falls well short of the first best under the fully optimal rule. Interestingly, the coefficients on inflation and the output gap generally are close to each other and well below unity. As before, the optimal coefficients are relatively insensitive to the penalty on long-rate variability, but decline modestly as the penalty on short-rate variability increases.

Table 2: Optimal Policy with a Slope Instrument

Preferences		Loss		Coefficients	
$\nu$	$\phi$	$OPT$	$s$	$\gamma_{\pi}^s$	$\gamma_y^s$
0.5	0	6.4	9.4	0.7	0.7
0.5	1	6.9	9.8	0.7	0.7
0.5	5	9.1	11.6	0.7	0.7
0.5	10	11.7	13.8	0.7	0.6
0.5	20	16.6	18.2	0.6	0.6
1.0	0	6.7	9.8	0.7	0.6
1.0	1	7.2	10.3	0.7	0.6
1.0	5	9.4	12.0	0.6	0.6
1.0	10	11.9	14.2	0.6	0.6
1.0	20	16.8	18.6	0.6	0.5
5.0	0	7.8	13.0	0.5	0.4
5.0	1	8.3	13.4	0.5	0.4
5.0	5	10.4	15.2	0.5	0.4
5.0	10	12.9	17.3	0.5	0.4
5.0	20	17.8	21.7	0.5	0.4

## 5.4 Optimal Policy with Slope and Level Instruments

Consider joint Taylor rule control of slope and the level of the yield curve:

$$\begin{aligned}s_t &= \gamma_\pi^s \bar{\pi}_t + \gamma_y^s y_t, \\ l_t &= \gamma_\pi^l \bar{\pi}_t + \gamma_y^l y_t,\end{aligned}$$

The results are reported in Table 3. For comparison, we continue to report the results for the fully-optimal rule where the policymaker has only one instrument.

Only when there is no or little penalty on long-rate variability do these combinations of slope and level rules perform better than the optimized slope rules in Table 2. Evidently, the estimated level rule provides better stabilization of the four target variables than a simple level rule that responds to inflation and the output gap. Interestingly, the optimized slope rule does not respond to inflation at all, and for small penalties on long-rate variability does not respond to the output gap either. The level response to inflation always exceeds unity and is small, the greater the weight on long-rate variability.

Table 3: Optimal Policy with Slope and Level Instruments

Preferences		Loss		Coefficients			
$\nu$	$\phi$	$L_{OPT}$	L	$\gamma_{\pi}^s$	$\gamma_y^s$	$\gamma_{\pi}^l$	$\gamma_y^l$
0.5	0	6.4	8.0	0.0	0.0	1.8	0.6
0.5	1	6.9	9.8	0.0	0.0	1.7	0.5
0.5	5	9.1	15.4	0.0	0.6	1.4	0.1
0.5	10	11.7	21.0	0.0	0.9	1.4	0.0
0.5	20	16.6	31.6	0.0	0.8	1.3	0.0
1.0	0	6.7	9.0	0.0	0.0	1.7	0.6
1.0	1	7.2	10.7	0.0	0.0	1.6	0.5
1.0	5	9.4	16.2	0.0	0.4	1.4	0.2
1.0	10	11.9	21.9	0.0	0.7	1.4	0.0
1.0	20	16.8	32.4	0.0	0.8	1.3	0.0
5.0	0	7.8	15.1	0.0	0.0	1.5	0.4
5.0	1	8.3	16.4	0.0	0.0	1.5	0.3
5.0	5	10.4	21.4	0.0	0.0	1.4	0.3
5.0	10	12.9	27.2	0.0	0.1	1.3	0.2
5.0	20	17.8	37.8	0.0	0.4	1.3	0.0

In light of the relatively poor performance of the combination of level and slope rules that respond to inflation and the output gap, we now consider alternative specifications of these rules. A natural alternative allows the level factor to respond to long-run inflation and the output gap.

So the policymaker follows the joint rules:

$$\begin{aligned}s_t &= \gamma_\pi^s \bar{\pi}_t + \gamma_y^s y_t, \\ l_t &= \gamma_{\pi^e}^l \pi_t^e + \gamma_y^l y_t,\end{aligned}$$

The results for this combination of rules are reported in Table 4. For small to moderate penalties on long-term interest rate variability, these rules yield performance on par, or even better, than the fully-optimal single instrument rule. In those cases, the slope factor responds modestly to inflation and in some cases the output gap. The level factor responds fairly aggressively to long-term inflation and less so to the output gap.

Table 4: Optimal Policy with Slope and Level Instruments

Preferences		Loss		Coefficients			
$\nu$	$\phi$	$L_{OPT}$	L	$\gamma_{\pi}^s$	$\gamma_y^s$	$\gamma_{\pi^e}^l$	$\gamma_y^l$
0.5	0	6.4	5.4	0.2	0.0	2.3	0.7
0.5	1	6.9	6.4	0.2	0.0	2.1	0.6
0.5	5	9.1	9.9	0.4	0.3	1.8	0.4
0.5	10	11.7	13.5	0.5	0.6	1.7	0.2
0.5	20	16.6	20.2	0.7	0.8	1.5	0.1
1.0	0	6.7	5.9	0.1	0.0	2.2	0.6
1.0	1	7.2	6.9	0.1	0.0	2.1	0.6
1.0	5	9.4	10.3	0.3	0.0	1.8	0.4
1.0	10	11.9	14.0	0.4	0.3	1.7	0.2
1.0	20	16.8	20.8	0.6	0.6	1.5	0.1
5.0	0	7.8	9.6	0.0	0.0	2.0	0.4
5.0	1	8.3	10.4	0.0	0.0	1.9	0.4
5.0	5	10.4	13.5	0.1	0.0	1.8	0.4
5.0	10	12.9	17.2	0.1	0.0	1.7	0.3
5.0	20	17.8	24.1	0.2	0.0	1.6	0.2