Procyclicality, collateral values, and financial stability

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Contents

Abstract 3

Summary 4

1 Introduction 6

2 The structure of the model 9

3 Complete financial markets and the first best 12

4 Constraints on financial claims 19

5 Welfare and policy implications 25

6 Final remarks 27

Appendix 1: The other constrained cases 29

Appendix 2: Under-insurance in both states of the world 34

References 35

Abstract

This paper analyses how the risk-sharing capacity of the financial system varies over the business cycle, leading to procyclical fragility. We show how financial imperfections contribute to under-insurance by entrepreneurs, generating a pecuniary externality that leads to the build-up of systematic risk during upturns. Increased asset price uncertainty emerges as a symptom of the sectoral concentration that builds up during booms. The liquidity of the collateral asset is shown to play a key role in amplifying the financial cycle. The welfare costs of financial stability, in terms of the efficiency costs due to financial frictions and the volatility costs due to amplification, are also illustrated.

Summary

A financial system plays a highly beneficial role in an economy by helping to transfer resources to sectors where they can be used most productively, with transfers taking place both across time and states of the world. In principle, a perfect financial system could insure the constituent sectors of an economy from the idiosyncratic risks that they face, so that fluctuations in economic activity at the macroeconomic level would reflect only systematic shocks. But financial systems operate under frictions such as asymmetric information, which make financial contracts costly to monitor and enforce. A practical view that appears to be becoming more widespread suggests that when financial systems operate with frictions, economic shocks can be amplified and propagated, exaggerating economic upturns and prolonging the severity of economic downturns, and leave economies more vulnerable to such shocks during expansionary phases of the business cycle.

This paper outlines a model that analyses both how macroeconomic shocks can be amplified and how pro-cyclical macroeconomic risk can be generated endogenously within a macro-financial system. The model is constructed so that shocks that boost the productivity of one sector adversely affect the productivity of the other sectors, and *vice versa*. Thus, a series of shocks that raise the output of one sector, such as a clustering of technological innovations, will cause the aggregate economy to grow (as the effects of previous shocks cumulate) but also to become more concentrated. And, as the economy becomes more concentrated, it becomes more vulnerable to the dominant sector being hit by an adverse shock at some point in the future.

The financial system in the model allows entrepreneurs in the economy to insulate their balance sheets from the effects of shocks. But financial contracts must be supported by collateral, such as real estate, to solve ex ante commitment problems. If the collateral asset is also used in production, a feedback loop between aggregate output and the value of collateral emerges. A key contribution of the paper is to show how such feedback loops are maintained in the presence of insurance markets. An initial decline in aggregate output reduces entrepreneurs' net worth and, hence, the price of the collateral asset, as demand for the asset for use in future production declines. The decline in the value of the collateral asset implies that producers are unable to obtain sufficient insurance, exposing balance sheets to shocks. Since entrepreneurs are risk averse, their response to additional balance-sheet uncertainty is to reduce the scale of production. This leads to subsequent declines in the price of the collateral asset, completing the feedback loop. And since

the collateral asset is held by both sectors, any decline in its value as a result of incomplete insurance by one sector leads to inadequate insurance by other sectors. This externality increases the level of systemic risk in the economy. Systemic risk imposes welfare costs on the economy as it leads to inefficient production and results in balance-sheet uncertainty. Both aspects are captured by the model.

1 Introduction

In recent years, there has been growing concern that financial systems in developed countries may have an excessively *procyclical* influence on the wider economy – exaggerating economic upturns and increasing the severity and length of downturns. It has focused attention amongst policymakers on the welfare costs of financial stability arising from the combined effect of financial frictions and macroeconomic shocks. Increasingly, a '*new view*' of financial stability is beginning to emerge – one emphasising the distortions affecting inter-temporal savings–investment decisions that appear as financial imbalances, such as an excessive build-up of credit or increased asset price volatility.(1) The 'new view' also lays stress on the endogeneity of risk over the business cycle as the capacity of the financial system to share risks fluctuates with shocks.

The presence of financial frictions, such as limited commitment to financial contracts, forces agents to confront a basic trade-off between sub-optimal levels of debt and insurance. Sub-optimal borrowing imparts inefficiencies via its impact on capital allocation in the economy. And inadequate insurance imparts an amplification mechanism, whereby shocks to the economy are magnified and propagated via the financial system, giving rise to excessive fluctuations in economic activity. The welfare costs of financial frictions in a macroeconomic setting are, thus, comprised of an *efficiency* effect and a *volatility* effect.(2)

Kiyotaki & Moore (1997) and Bernanke *et al* (1999) show how amplification can arise when financial frictions limit the ability of agents to commit to financial contracts or to properly monitor investment projects.(3) But an unsatisfactory aspect of these models is that they do not permit a role for insurance markets that might allow agents to hedge themselves against shocks. In the presence of financial frictions, inappropriate insurance by any one agent imposes risks on others in the financial system, principally through exposure to a set of common factors. This *pecuniary externality* makes for inappropriate risk sharing by other agents, who end up not being properly diversified over the normal course of the business cycle, generating a build up of systemic risk. Inappropriate risk bearing by agents in the financial system, therefore, plays a central role in the

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(1) See, for example, Borio *et al* (2001), Borio *et al* (2003), and Haldane *et al* (2004).
(2) Haldane *et al* (2004) discuss this issue in greater detail.
(3) A partial list of models that also embed financial frictions in a macroeconomic setting includes Bernanke & Gertler (1989), Carlstrom & Fuerst (1997), Holmstrom & Tirole (1997), Chen (2001), and Lorenzoni (2003).

amplification of shocks and financial instability.

Recent work by Krishnamurthy (2003) demonstrates how the introduction of hedging eliminates the amplification channel in the Kiyotaki & Moore framework.[(4)] Amplication of the effects of productivity shocks on output and prices only re-emerges when collateral constraints are applied to the suppliers of insurance. But these constraints only bind occasionally and an implication of the model is that the welfare costs of amplification arise only in extreme events, such as during a financial crisis. As such, it fails to adequately characterise procyclicality and sits uneasily with the intuition of the 'new view'. Agents may not be fully diversified over the normal course of the business cycle, and are usually over-exposed to risk during the upturn.

In this paper, we develop a framework to explore how the risk-sharing capacity of the economy varies over the cycle and generates an amplification effect that leads to procyclical behaviour. The economy is treated as a composite of risky industries, each of which is subject to a sector-specific shock. Complete financial markets allow firms to invest in each other, enabling diversification of risk. Our model, thus, departs from Kiyotaki & Moore (1997) by allowing for such insurance. Like them, however, we introduce financial frictions by assuming an absence of trust so that entrepreneurs are forced to use collateral to back financial securities. Collateral, moreover, serves a dual role and is used by entrepreneurs in production.

The presence of collateral constraints means that agents are limited in the extent to which they can smooth consumption over time and states of the world, leaving them under-insured against risks from their own sectors. When shocks hit the economy, lucky sectors grow while those less fortunate shrink. As lucky sectors dominate, their influence on aggregate activity grows, so the business cycle becomes driven mainly by shocks to the dominant sector and shocks to the smaller sector become relatively immaterial. And since the collateral asset links the sectors, there is a common exposure to a particular risk factor. Aggregate risk is, thus, procyclical since risk becomes more systematic as the upturn begins. In this respect, our model formalises the intuition of Borio *et al* (2001) and others of the 'new view' school.

Our paper also shows that asset price uncertainty is a *symptom* of incomplete diversification. When sectors are under-insured, the rise of a dominant sector means that sector-specific shocks

(4) Suarez & Sussman (1997) make a similar point, although in their model indexation of financial contracts assumes centre stage.

are not absorbed as well as as might be the case if sectors are similarly sized. The price of the collateral asset is subject to greater variation because fewer firms are able to stand on the other side of the market (eg to buy, when the dominant sector sells).

Although the presence of state-contingent contracts allows agents to potentially insure away any fluctuations in the price of the collateral asset, practices concerning the valuation of collateral also have implications for the credit cycle.[(5)] We show that if there are margin requirements on the value of collateral holdings, or if collateral is relatively illiquid, collateral values cannot be fully insured and the *amplification effect* re-emerges. Since entrepreneurs insure less, a negative shock to the economy results in less wealth being transfered to the next period. Since collateral also serves in production, the reduced demand for the asset in the later period drives down its price. But this, in turn, feeds back on to the collateral constraint tightening it further and lowering the extent of insurance even more. The amplification effect exacerbates the efficiency costs of the financial friction as well as the uncertainty surrounding asset prices in the economy.

Our model extends and develops the Krishnamurthy (2003) framework in a number of respects. First, the Krishnamurthy model has only two sectors and generates amplification by imposing aggregate collateral constraints, i.e requiring that both sectors – real and financial – be financially constrained, posting collateral to back financial securities. By contrast, we introduce an additional sector and impose financial frictions only on the entrepreneurs, leaving the financial sector unconstrained throughout. Second, our amplification mechanism arises from the relative illiquidity of the collateral asset rather than via a premium on insurance due to limits in the supply of finance. The limited insurance of the individual entrepreneur places a pecuniary externality on the allocations of other agents via the price of the collateral asset. And third, our focus on risk-sharing over the cycle means that entrepreneurs in our model are risk averse (instead of risk neutral), while the financial sector is risk-neutral and willing to accomodate any level of financial claims at the actuarial price.

Several other points of contact with the literature are worthy of note. Schnabel & Shin (2004) and Cifuentes *et al* (2005) formally analyse how common exposures to illiquid assets on the balance sheet are a key source of systemic risk and can leave agents vulnerable to damaging fluctuations in asset prices. Saint-Paul (1992) studies the interaction between financial markets and risky

(5) Borio *et al* (2001) note that banks in Europe limit loan-to-value ratios to 60-85% of the value of residential property, while in Hong Kong a “recommended” maximum loan-to-value ratio of 70% was in place during the 1990s.

technologies and shows how risk-diversification can lead to greater specialisation in production. Without financial markets, agents limit risk by opting for less specialised (and less productive) technologies. Acemoglu & Zilibotti (1997) show how productivity endogenously increases as diversification improves. In the early stages of develepment, agents seek insurance by investing in safe and unproductive assets. Since insurance opportunities are limited, development is highly random and only 'lucky' economies grow and benefit from better diversification and productivity. Finally, Koren and Tenreyro (2004) examine how sectoral diversification affects the volatility of the business cycle.

The paper is organised as follows. Section 2 presents the basic structure of the model and introduces the sequence of events. Section 3 analyses the benchmark case of unrestricted and complete financial markets, comparing the outcome with a first-best world. Section 4 introduces financial frictions into the model and describes the amplification mechanism induced by the illiquidity of collateral. Section 5 discusses some implications for welfare and policy. A final section concludes.

2 The structure of the model

Consider an economy in which a durable asset – land – serves as collateral for financial contracts and as a factor of production. Land is in fixed total supply, K, and is used by each of three sectors to produce a single, perishable, consumption good. Two of these sectors are farms, $f = m, r$, that will be subject to collateral constraints. The remaining sector, which is unconstrained throughout, acts as a buffer that provides an alternative use for the collateralised asset. To fix ideas, we label the sectors as mango farms (m), rice farms (r), and banks (b) respectively. There are two periods, and sectors begin with endowments of wealth that are potentially different, w_1^m, w_1^r, and w_1^b. There is a common discount rate, β, for all sectors.

Farmers and bankers are distinguished by their preferences over consumption (c_t) and by their production technologies. Specifically, farmers are risk averse and dislike the uncertaintly that productivity shocks bring to output. Their utility functions take the form

$$u_t^f = \ln(c_t^f),$$

while they produce the consumption good with a linear technology

$$y_t^f = s_t^f a k_t^f,$$

where a is the marginal product of land in farming in the absence of shocks, and s_t^f is a stochastic productivity shock. Shocks to productivity only occur in the first period and depend on two states of the world that occur with equal probability – a *rainy* state and a *sunny* state. Since mango farmers benefit from sun and are hindered by rain, while rice farmers benefit from rain and hindered by sun, we assume that the productivity shocks take the values

$$\left.\begin{array}{l} s_1^m = 1 - z \quad \text{and} \quad s_1^r = 1 + z \quad \text{if rain} \\ s_1^m = 1 + z \quad \text{and} \quad s_1^r = 1 - z \quad \text{if sun} \end{array}\right\}$$

where $z > 0$, i.e the productivity shocks to the two farming sectors are perfectly negatively correlated. Second period output is unaffected by productivity shocks, so $s_2^m = s_2^r = 1$.

Bankers, by contrast, are risk neutral with utility

$$u_t^b = c_t^b,$$

and produce the consumption good with diminishing returns. Their production function is not subject to productivity shocks and takes a quadratic form

$$\begin{aligned} y_t^b &= g(k_t^b) \\ &= k_t^b(A - k_t^b) \end{aligned}$$

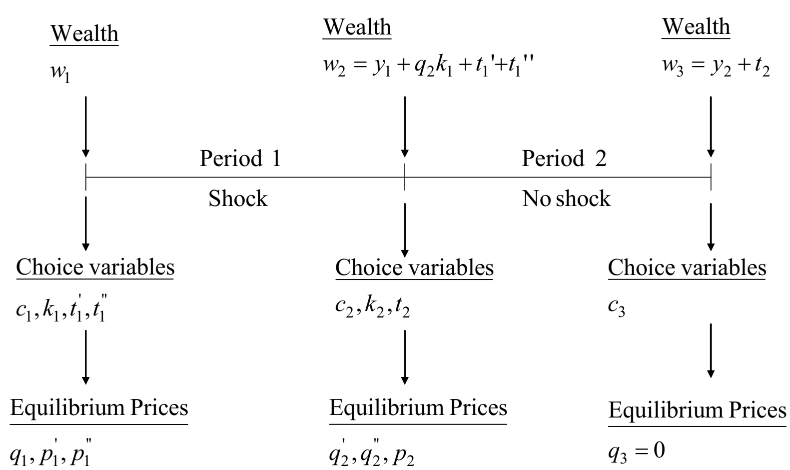
We suppose that $2K \geqslant A - a \geqslant 0$, which ensures that both farmers and bankers hold a positive quantity of land in equilibrium. Following Krishnamurthy (2003), bankers are assumed to have ample endowments of the consumption good in each period.

Farmers trade one-period state-contingent financial contracts with bankers for insurance across states and to help transfer consumption through time. These Arrow-Debreu securities are purchased/sold at the start of period t, and pay one unit of the consumption good at the end of that period if the agreed state of the world materialises. The payoff of the security is zero otherwise. Let $t_1^{f\prime}$and $t_1^{f\prime\prime}$ denote the quantity of financial claims held by farmers that pay out at the end of period 1 in the rainy and sunny states respectively. With no production uncertainty in period 2, there is only one Arrow-Debreu security, t_2^f. The common component within the quantities of the state-contingent contracts $t_1^{f\prime}$and $t_1^{f\prime\prime}$ reflects borrowing/lending, while differences in the quantities reflect the insurance component.[6]

The sequence of events is as follows. At the start of period 1, farmers and bankers allocate their initial wealth, w_1, to purchases of land, financial contracts, and current consumption. We do not presume that $w_1^m = w_1^r$.[7] The strength of demand for land relative to the fixed supply determines the first period land price, q_1, while financial claims that deliver in the rainy and sunny states trade for p_1' and p_1'' respectively. The state of the world is realised during the period, so the resulting start-of-period 2 wealth, w_2, will vary across states. This consists of output, plus any payoffs from Arrow-Debreu securities, and the value of land holdings at the end of period 1.

In period 2, wealth is again allocated between consumption, purchases of land for production, and purchases of financial contracts at price p_2. The demand for land determines the equilibrium price at the start of the period, q_2. Importantly, q_2 depends on the state of the world in period 1 – rain or sun. Production then takes place and is supplemented by payoffs from any Arrow-Debreu securities. With no further production periods, land holdings cease to have value at the end of the second period. Chart 1 illustrates the timing of the model.

Chart 1: Timing of the model



(6) For example, a mango farmer who borrows 5 units from the bank and takes out 2 units of insurance pays out 5 units of the consumption good at the end of the period as repayment, plus a further 2 if it has been sunny or, if it has been rainy, receives 2 units. This is equivalent to selling 3 Arrow-Debreu securities that pay out in the rainy state and 7 that pay out in the sunny state.
(7) Indeed, we assume that w_1^m is generally different to w_1^r. Otherwise productivity shocks have no aggregate consequences, as the effects of a positive shock to one farmer will be perfectly offset by the effects of the negative shock to the other farmer.

3 Complete financial markets and the first best

We first consider the benchmark case of complete financial markets. Since farmers know that their sectors are subject to idiosyncratic shocks, they are motivated to hold Arrow-Debreu securities, which replicate claims on sectors other than their own, in order to share risk. With complete financial markets, Arrow-Debreu securities exist for all states and can be traded in potentially unlimited quantities.

Bankers

The model is solved by working backwards from period 2. We begin by considering the banking sector, since the risk-neutrality of bankers pins down the prices of Arrow-Debreu securities in the model. Bankers aim to maximise start-of-period 2 consumption, c_2^b, plus discounted end-of-game consumption (output in the final period) minus transfers made to mango and rice farmers when the single Arrow-Debreu security matures (t_2^m, t_2^r). They face a budget constraint, namely that output at the start of period 2, together with the value of land and income from securities sold to farmers, must equal consumption, investment in land holdings, plus payments owed to farmers on financial claims maturing from the previous period ($t_1^m + t_1^r$). The banker's problem can be expressed as

$$\max_{k_2^b, c_2^b, t_2^m, t_2^r} c_2^b + \beta[g(k_2^b) - (t_2^m + t_2^r)] \quad \textbf{(1)}$$

$$\begin{aligned} s.t.\ c_2^b + q_2' k_2^b + t_1^{m\prime} + t_1^{r\prime} &= g(k_1^b) + q_2 k_1^b + p_2 t_2^m + p_2 t_2^r \quad \text{if rain} \\ c_2^b + q_2'' k_2^b + t_1^{m\prime\prime} + t_1^{r\prime\prime} &= g(k_1^b) + q_2 k_1^b + p_2 t_2^m + p_2 t_2^r \quad \text{if sun} \end{aligned}$$

with first-order conditions

$$\begin{aligned} q_2 &= \beta g'(k_2^b) \quad \text{if rain} \\ q_2 &= \beta g'(k_2^b) \quad \text{if sun} \\ \text{and} \quad p_2 &= \beta, \end{aligned} \quad \textbf{(2)}$$

In other words, bankers equate their discounted marginal product of land to the land price, and the price of the Arrow-Debreu security equals the inter-temporal rate of substitution, β.

At the start of the first period, bankers still aim to maximise consumption at the start of period 1, as well as expected consumption at the start of period 2 and at the end of the game. Start-of-period 2 consumption is period 1 output plus the value of land holdings, plus income from sales of the period 2 Arrow-Debreu security, minus the costs of period 2 land investment and payments to farmers who bought period-1 Arrow-Debreu securities. The budget constraint again requires that output, the value of land, and receipts from sales of financial claims equal spending on consumption, land purchases and Arrow-Debreu payments. We thus have

$$\begin{aligned} & \max_{c_1^b, k_1^b, t_1^m, t_1^r} c_1^b + \beta E\left[g(k_1^b) + q_2 k_1^b + p_2 t_2^m + p_2 t_2^r - q_2 k_2^b - t_1^{m\prime} - t_1^{m\prime\prime} - t_1^{r\prime} - t_1^{r\prime\prime}\right] \\ & +\beta^2 E[g(k_2^b) - t_2^m - t_2^r)] \qquad \textbf{(3)} \\ s.t.\ c_1^b + q_1 k_1^b &= w_1^b + p_1'(t_1^{m\prime} + t_1^{r\prime}) + p_1''(t_1^{m\prime\prime} + t_1^{r\prime\prime}) \end{aligned}$$

with first order conditions

$$\begin{aligned} q_1 &= \beta E\left[g'(k_1^b) + q_2\right] \qquad \textbf{(4)} \\ \text{and } p_1' &= p_1'' = \frac{1}{2}\beta. \end{aligned}$$

These conditions state that the period 1 price of land equals the expected discounted value of holding on to it. A unit of land is worth q_2 in period 2 and also generates an output $g'(k_1^b) = A - 2k_1^b$, at the margin. The price of a financial claim is equal to its discounted actuarial value – the probability of a state occuring multiplied by the discount factor. The risk neutrality of bankers means that no risk premium is introduced into the price of insurance and that they are, thus, able to absorb any amount of Arrow-Debreu securities at the actuarial price.

Farmers

In similar fashion, the second period problem of the two farmers can be represented as

$$\max_{c_2^f, k_2^f, t_2^f} \ln c_2^f + \beta \ln(ak_2^f + t_2^f) \quad \textbf{(5)}$$

$$\begin{aligned} s.t.\ \ c_2^f + q_2' k_2^f + \beta t_2^f &= s_1^f a k_1^f + q_2' k_1^f + t_1^{f\prime} \quad \text{if rainy} \\ c_2^f + q_2'' k_2^f + \beta t_2^f &= s_1^f a k_1^f + q_2'' k_1^f + t_1^{f\prime\prime} \quad \text{if sunny} \end{aligned}$$

where s_1^f is state dependent and varies across farmers. Note the reversal of signs on the t^f terms in the farmers' problem, compared with t^m and t^r in the banker's problem. This is because Arrow-Debreu securities *bought* by farmers are *sold* by bankers.

Farmer's wealth at the end of the game comprises non-stochastic output and receipts from financial claims purchased at the start of period 2. Again, the budget constraint requires that consumption, purchases of land for second period investment, and purchases of the period 2 financial claim, priced by bankers at β per unit, come from wealth. This wealth is comprised of period 1 output, the value of land at the start of period 2, and the payoffs from Arrow-Debreu securities at the end of period 1.

Define start-of period 2 wealth as $w_2^f = s_2^f a k_1^f + q_2 k_1^f + t_1^{f\prime} + t_1^{f\prime\prime}$, which is state dependent. The first order conditions are

$$\frac{q_2}{w_2^f - q_2 k_2^f - \beta t_2^f} = \beta \frac{a}{ak_2^f + t_2^f}, \quad \textbf{(6)}$$

and

$$\frac{\beta}{w_2^f - q_2 k_2^f - \beta t_2^f} = \beta \frac{1}{ak_2^f + t_2^f}. \quad \textbf{(7)}$$

From (6) and (7), we find that $q_2 = a\beta$, regardless of the state that occurred in period 1, i.e. $q_2' = q_2'' = a\beta$. Note that this is also true regardless of the distribution of farmers' initial wealth, (w_1^m, w_1^r).

The solution pair (k_2^f, t_2^f) therefore satisfies:

$$\frac{w_2^f}{(1+\beta)} = ak_2^f + t_2^f = c_2^f. \quad \textbf{(8)}$$

As there is no uncertainty in period 2, financial claims and the purchase of land for production both serve as vehicles to facilitate the transfer of consumption from the start of period 2 to the end of the game. Farmers' wealth at period 2 is used to finance current consumption, c_2^f, and consumption at the end of the game, c_3^f. The optimal way to allocate this wealth is in the ratio $1 : \beta$, where β is invested, via land or securities, for end-game consumption.

In equilibrium, farmers are indifferent between (a) selling the good for a claim that pays one unit for every β purchased at the start of period 2; or (b) investing in production, which yields a for each unit of land purchased at price q_2. It follows, therefore, that $q_2 = a\beta$. And since both methods deliver a unit of consumption at the end of the game for each β invested, $c_2^f = c_3^f = \frac{w_2^f}{(1+\beta)} = ak_2^f + t_2^f$.

The period 1 problem for the farmers is to maximise their expected discounted consumption stream

$$\max_{c_1^f, k_1^f, t_1^{f\prime}, t_1^{f\prime\prime}} \ln c_1^f + \beta E\left[\ln c_2^f\right] + \beta^2 E[\ln c_3^f] \tag{9}$$

$$s.t. w_1^f = c_1^f + k_1^f q_1 + p_1' t_1^{f\prime} + p_1'' t_1^{f\prime\prime}$$

where, again, wealth is allocated to consumption, investment in land, and purchases of Arrow-Debreu securities that pay off in either state. Using the fact that $p_1' = p_1'' = \frac{1}{2}\beta$ and $c_2^f = c_3^f$, allows the maximisation problem to be simplified to

$$\max_{c_1^f, k_1^f, t_1^{f\prime}, t_1^{f\prime\prime}} \ln c_1^f + (\beta + \beta^2) E\left[\ln \frac{w_2^f}{(1+\beta)}\right] \tag{10}$$

$$s.t. w_1^f = c_1^f + k_1^f q_1 + \frac{\beta}{2}(t_1^{f\prime} + t_1^{f\prime\prime}).$$

Now, however, start-of-period 2 wealth level, w_2^f, depends on the period 1 shock and may take one of the following values:

$$\left.\begin{aligned} w_2^{m\prime} &= (1-z)ak_1^m + q_2' k_1^m + t_1^{m\prime} \\ w_2^{m\prime\prime} &= (1+z)ak_1^m + q_2'' k_1^m + t_1^{m\prime\prime} \\ w_2^{r\prime} &= (1+z)ak_1^r + q_2' k_1^r + t_1^{r\prime} \\ w_2^{r\prime\prime} &= (1-z)ak_1^r + q_2'' k_1^r + t_1^{r\prime\prime}. \end{aligned}\right\} \tag{11}$$

In the case of the mango farmer, the problem becomes

$$\max_{c_1^m, k_1^m, t_1^{m\prime}, t_1^{m\prime\prime}} \ln c_1^m + \frac{\beta+\beta^2}{2}\left[\ln \frac{(1-z)ak_1^m + q_2' k_1^m + t_1^{m\prime}}{(1+\beta)} + \ln \frac{(1+z)ak_1^m + q_2'' k_1^m + t_1^{m\prime\prime}}{(1+\beta)}\right] \tag{12}$$

$$s.t. w_1^m = c_1^m + k_1^m q_1 + \frac{\beta}{2}(t_1^{m\prime} + t_1^{m\prime\prime})$$

The solution to equation (12) can be made tractable by obtaining the first-order conditions for $t_1^{m\prime}$and $t_1^{m\prime\prime}$, and then substituting the optimal values for state-claims into the farmer's problem, before solving for k_1^m. The first-order conditions for holding Arrow-Debreu securities imply

$$t_1^{m\prime\prime} + 2zak_1^m + (q_2'' - q_2')k_1^m = t_1^{m\prime} \tag{13}$$

In other words, mango farmers buy more financial securities that pay out in the rainy state than in the sunny state to compensate for output being relatively low. Moreover, they buy more financial securities that pay out in the rainy state if the price of land is lower in the rainy state than in the sunny state. The difference in holdings of state-claims is also proportional to the scale of production.

The optimal holding of Arrow-Debreu securities by the mango farmer in the two states is given by

$$t_1^{m\prime} = zak_1^m + [E(q_2) - q_2']k_1^m + \frac{(\beta+1)(w_1^m - q_1 k_1^m) - ak_1^m - E(q_2)k_1^m}{(1+\beta+\beta^2)} \tag{14}$$

and

$$t_1^{m\prime\prime} = - zak_1^m + [E(q_2) - q_2'']k_1^m + \frac{(\beta+1)(w_1^m - q_1 k_1^m) - ak_1^m - E(q_2)k_1^m}{(1+\beta+\beta^2)}. \tag{15}$$

Similar expressions can be derived for the rice farmer. As equations (14) and (15) make clear, the first two terms are equal in magnitude, but opposite in sign, to (a) the productivity shock on wealth; and (b) the loss relative to expectations in the value of land holdings. So farmers use financial claims to fully insure themselves against productivity shocks and fluctuations in land prices.

Farmers also transfer consumption across time with loans (the third term, which is the same across states). Since $E(q_2) = a\beta$, we observe that

$$t_1^{m\prime} = zak_1^m + [E(q_2) - q_2']k_1^m + \frac{(\beta+1)(w_1^m - q_1 k_1^m - ak_1^m)}{(1+\beta+\beta^2)} \quad \textbf{(16)}$$

If immediate consumption in the absence of Arrow-Debreu securities $(w_1^m - q_1 k_1^m)$ falls short of end-period consumption (ak_1^m), then the mango farmer can raise utility by borrowing to bring some consumption forward. The third term in equation (**(16)**) represents borrowing, which will be the case when w_1^m and w_1^r are relatively low. If, however, $w_1^m - q_1 k_1^m - ak_1^m > 0$, farmers are net lenders in the economy.

We now substitute the optimal financial claim into the start-of-period 1 problem for the farmer. With full insurance, wealth is constant so the demand for land is the same in both states and there can only be one land price at the start of period 2, i.e $E(q_2) = q_2$. This gives the first-order condition for k_1^m

$$(\beta a + \beta q_2 - q_1)\left(\frac{1 + (\beta + \beta^2)}{w_1 + k_1(\beta a + \beta q_2 - q_1)}\right) = 0 \quad \textbf{(17)}$$

which, in turn, implies

$$q_1 = a\beta + \beta q_2 \quad \textbf{(18)}$$

From the banker's problem, period 1 land prices are $q_1 = \beta E\left[g'(k_1^b) + q_2\right]$. It follows, therefore, that $a = g'(k_1^b)$, i.e the marginal product of the banking sector is equated with the marginal product of (aggregate) farming. Similarly, in the second period, $q_2 = \beta g'(k_2^b) = a\beta$, so $a = g'(k_2^b)$.

Finally, the market for land must clear in equilibrium. From banker's period 2 problem we know $q_2 = a\beta$ and from banker's period 1 problem, we can infer that $q_1 = a\beta(1+\beta)$. The aggregate farming demand curve is not inconsistent with this. Farmers demand any quantity of land available at $q_1 \leq a\beta(1+\beta)$ and $q_2 \leq a\beta$ and zero otherwise.

Comparison with the first-best

The decentralised equilibrium with complete financial markets replicates the first-best allocation of land chosen by a central planner. A planner who prefers aggregate output to be as high as possible in expectation, but as low as possible in variation might choose to maximise

$$\underset{k_t^b, k_t^m, k_t^r}{Max}\ E\left[g(k_t^b) + s_t^m a k_t^m + s_t^r a k_t^r\right] - \frac{\rho}{2} var\left[g(k_t^b) + s_t^m a k_t^m + s_t^r a k_t^r\right] \qquad \textbf{(19)}$$

where $\rho/2$ is an arbitrary weight reflecting the planner's dislike of output variability.

Making use of the fact that $k_t^b = K - K_t^f$, $E[s_t^m] = E[s_t^r] = 1$, and $var[s_t^m] = var[s_t^r] = z^2$, gives the following first-order conditions with respect to k_t^m and K_t^f

$$\begin{aligned} -g'(K - K_t^f) + a - \frac{\rho}{2}\left[2a^2(K_t^f - k_t^m)z^2 - 2a^2 k_t^m z^2\right] &= 0 \\ \text{and } \ 2a^2 k_t^m z^2 - 2a^2(K_t^f - k_t^m)z^2 - 2a^2 K_t^f z^2 + 4a^2 k_t^m z^2 &= 0 \end{aligned}$$

The second of these implies that $k_t^m = k_t^r = \frac{K_t^f}{2}$, i.e. land for farming should be split equally between the two farming sectors. Substituting this into the first condition yields:

$$g'(K - K_t^f) = A - 2(K - K_t^f) = a$$

which is the same as in the decentralised equilibrium. The optimal strategy for the planner is to allocate land to the banking sector until its marginal product declines to the expected marginal product of the farmers. Subsequent land is then allocated 50:50 between the rice and mango farmers.

The 50:50 division ensures that uncertainty about the marginal product of (aggregate) farming is eliminated – favourable shocks to one farm are completely offset by the unfavourable shock to the other. With complete financial markets, farmers are able to replicate the first-best by selling state-claims to each other via the bank, which serves as an intermediary. The decentralised and planning solutions both ensure a non-stochastic aggregate outcome – the *certain* marginal product of banking is equated with the *certain* marginal product of aggregate farming in both periods.

4 Constraints on financial claims

We now introduce financial frictions into the model. In the spirit of Kiyotaki & Moore (1997), we continue to assume that the banking sector is unconstrained, but that the absence of trust means farmers cannot credibly promise to pay out on financial claims unless land is used as collateral to back the claim. Importantly, however, the amount a farmer can credibly commit is only a fraction $0 \leq \phi \leq 1$, of the future value of land holdings. This could reflect different loan-to-value ratios across countries (as in footnote 5) or the fact that land is a relatively illiquid form of collateral which is difficult to dispose of when obligations arise[8]. More generally, the fraction $1 - \phi$ captures costs built into the specifics of collateral agreements, such as dispute resolution procedures, that distinguish more broadly-based collateral assets from highly liquid forms such as cash ($\phi = 1$).

There are three instances under which the financial constraint may bind for either farmer. It could bind in the sunny state, the rainy state, or in both states. Since our focus is on insurance and the amplification effects of collateral values, we emphasise the situation where the (mango) farmer is unable to meet his promises to pay out. We, therefore, highlight the case where the mango farmer is constrained only in the sunny (favourable) state. Appendix 1 summarises the results when the mango farmer is constrained in the other two cases. Recall that we have do not assume a symmetric distribution of initial wealth amongst farmers, so that $w_1^m \neq w_1^r$.

Bankers

Since bankers are unconstrained, their period 1 and 2 problems are unchanged. The previous solutions for prices in each period therefore apply, namely

$$
\begin{aligned}
q_2 &= \beta g'(K_2^b);\ p_2 = \beta; \\
q_1 &= \beta g'(k_1^b) + E(q_2); \quad p_1' = p_1'' = \frac{\beta}{2}.
\end{aligned}
$$

Farmers

(8) The notion of illiquidity that we have in mind is one of markets being unable to transact high volumes at a particular instant. This should not be confused with trades having a high price impact, while possible being conducted in large volumes, which is another notion of illiquidity.

Farmers are only able to purchase financial claims to the extent that these securities are backed by the future value of land holdings. Land is worthless at the end of the game, so farmers cannot sell Arrow-Debreu securities to the bank at the start of period 2, as they would have no collateral to support the potential pay out. As a result, they are limited to transfering consumption from the start of period 2 to the end of the game via production.

The second period problem of the farmers is

$$\max_{k_2^f, c_2^f, c_3^f} \ln c_2^f + \beta \ln c_3^f \tag{20}$$

$$\begin{aligned} st \ \ c_2^f &= w_2^f - q_2 k_2^f \\ c_3^f &= a k_2^f. \end{aligned}$$

And the first-order condition is

$$\frac{-q_2}{w_2^f - q_2 k_2^f} + \beta \frac{a}{a k_2^f} = 0. \tag{21}$$

So the optimal start of second period investment is

$$k_2^f = \frac{\beta w_2^f}{(\beta + 1) q_2}. \tag{22}$$

Moreover, c_2^f and c_3^f can be written in terms of w_2^f as

$$c_2^f = w_2^f - q_2 k_2^f = w_2^f - q_2 \frac{\beta w_2^f}{(\beta + 1) q_2} = \frac{w_2^f}{(\beta + 1)} \tag{23}$$

$$c_3^f = a k_2^f = \frac{a \beta w_2^f}{(\beta + 1) q_2}. \tag{24}$$

We now turn to the farmers' first period problem. A mango farmer would like to sell securities that pay out at the end of period 1 if it has been sunny. This is because he is able to meet these payments from output, which will have been higher due to the positive productivity shock. He is constrained, however, in the magnitude of these sales, so the constraint $-t_1^{m''} = \phi q_2'' k_1^m$ applies, whilst the other potential collateral constraint for the rainy state, $-t_1^{m'} \leq \phi q_2' k_1^m$, remains slack.

Since $p_1' = p_1'' = \beta/2$, the farmers' problem can be written as

$$\begin{aligned}
&\max \ln c_1^m && \textbf{(25)}\\
&+\frac{\beta}{2}\left[\ln\frac{(1-z)ak_1^m + q_2' k_1^m + t_1^{m\prime}}{1+\beta} + \ln\frac{(1+z)ak_1^m + q_2'' k_1^m + t_1^{m\prime\prime}}{1+\beta}\right]\\
&+\frac{\beta^2}{2}\left[\ln\frac{a\beta\left[(1-z)ak_1^m + q_2' k_1^m + t_1^{m\prime}\right]}{(1+\beta)\,q_2'} + \ln\frac{a\beta\left[(1+z)ak_1^m + q_2'' k_1^m + t_1^{m\prime\prime}\right]}{(1+\beta)\,q_2''}\right]
\end{aligned}$$

$$\begin{aligned}
\text{s.t.}\quad c_1^m &= w_1^m - k_1^m q_1 - \frac{\beta}{2}t_1^{m\prime} - \frac{\beta}{2}t_1^{m\prime\prime}\\
-t_1^{m\prime\prime} &= \phi q_2'' k_1^m.
\end{aligned}$$

The optimal holding of Arrow-Debreu securities by the mango farmer can now be readily contrasted with his holdings under complete financial markets (equations 14, 15). Specifically, the first-order conditions to the maximisation problem in (25) imply

$$\begin{aligned}
t_1^{m\prime} &= \frac{2}{2+\beta+\beta^2}azk_1^m + \left(1 - \frac{(\beta+\beta^2)}{2+\beta+\beta^2}(1-\phi)\right)\left(E(q_2) - q_2'\right)k_1^m && \textbf{(26)}\\
&+\frac{(1+\beta)\left(2w_1^m - 2k_1^m q_1 + \beta\phi E(q_2)k_1^m\right) - 2\left[ak_1^m + E(q_2)k_1^m\right]}{2+\beta+\beta^2}
\end{aligned}$$

and

$$t_1^{m\prime\prime} = -\ \phi q_2'' k_1^m. \qquad \textbf{(27)}$$

Equation (**(26)**) also indicates that mango farmers are unable to fully insure their wealth against variations in output. With complete financial markets, farmers were able to fully absorb production risk by purchasing azk_1^m units of insurance. In the presence of the collateral constraint, however, they only purchase $\frac{2}{2+\beta+\beta^2}azk_1^m$ units.

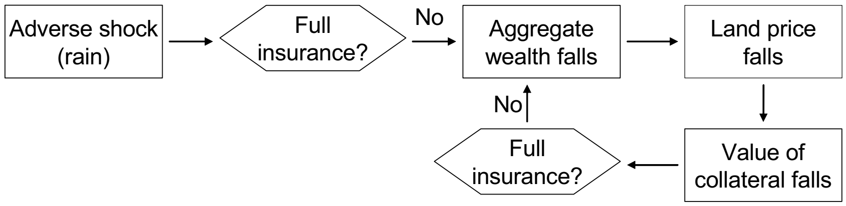
The second term in (**(26)**) shows the extent to which the mango farmer insulates his wealth from any fluctuations in land prices. If $\phi = 1$, trading in Arrow-Debreu securities may be supported up to the full value of collateral. As a result, wealth uncertainty due to fluctuations in the value of collateral can be fully insured. This is the same as in the complete financial markets case. But if $\phi < 1$, insufficient Arrow Debreu securities can be traded to fully insulate wealth from fluctuations in the value of collateral. This is due to the limited pledgeability of land as collateral.

The dual role of land as a factor of production and as a collateral asset implies that land contributes to the transfer of wealth across time in two ways. It generates consumption goods during a production period and it maintains value as a tradable asset for use in future production periods. Both of these contributions to wealth are uncertain, but can be insured by unconstrained trading of Arrow-Debreu securities. The introduction of collateral constraints, however, implies that production risk can not be fully insured, while insurance of collateral values is also partial if Arrow-Debreu trading is less than fully collateralised ($\phi < 1$). In the first case, fluctuations in wealth and, hence, equilibrium land holdings and prices reflect only the fundamental productivity shocks. But when collateral values can not be fully insured, the effects of productivity shocks are *amplified*.

The amplification effect in our model arises as follows. A negative shock to the mango farmer leads to a decline in the demand for land by the mango farmer as his wealth declines, since he is unable to fully insure production risk and collateral values. If the mango farm is assumed to be the dominant sector, this leads to a fall in the price of land despite the positive effect of the shock on the rice farmer. As the future value of collateral declines for both types of farmer, the initial collateral constraint binds tighter, so that fewer Arrow-Debreu securities can be traded in the first instance. This further reduces the degree of under-insurance and heightens the pecuniary externality imposed by the mango farmer on the rice farmer via the land price. As a result, the initial productivity shock is amplified and propagated, with significant effects on equilibrium land holdings and prices. Note that excess (over fundamental) asset price uncertainty emerges as a *symptom* of the lack of collateral. Chart 2 illustrates the logic of the amplification mechanism.

In our model, asset price uncertainty and amplification only occur when there is an asymmetry between the two farming sectors. If the two farmers had the same initial wealth and both were subject to collateral constraints in their respective favourable states of the world, then aggregate outcomes would be independent of the state. Although mango and rice farmers would make different contributions to aggregate wealth and the demand for land depending on whether it

Chart 2: Amplification when mango farmer dominates farming wealth



rained or was sunny, these variables and, hence, the price of land, would all be certain in this special case. In this situation, there is no asset price uncertainty and, therefore, no amplification.

As before, we now substitute the farmers' optimal holdings of Arrow-Debreu securities - equations (26) and (27) - into the maximisation problem. This gives

$$\max \ln\left[\frac{2w_1^m+\left[(1-z)a\beta+\beta q_2'+\beta\phi q_2''-2q_1\right]k_1^m}{2+\beta+\beta^2}\right] \quad \textbf{(28)}$$
$$+\frac{\beta}{2}\left[\ln\frac{2w_1^m+\left[(1-z)\beta a+\beta q_2'+\beta\phi q_2''-2q_1\right]k_1^m}{2+\beta+\beta^2}+\ln\frac{\left[(1+z)a+(1-\phi)q_2''\right]k_1^m}{1+\beta}\right]$$
$$+\frac{\beta^2}{2}\left[\ln\frac{a\beta\left[2w_1^m+\left((1-z)\beta a+\beta q_2'+\beta\phi q_2''-2q_1\right)k_1^m\right]}{\left(2+\beta+\beta^2\right)q_2'}+\ln\frac{a\beta\left[(1+z)a+(1-\phi)q_2''\right]k_1^m}{(1+\beta)\,q_2'}\right]$$

and the first order condition yields

$$k_1^m=\frac{\left(\beta+\beta^2\right)}{\left(1+\beta+\beta^2\right)}\frac{w_1^m}{\left(2q_1-\beta q_2'-\beta\phi q_2''-\beta a(1-z)\right)}.$$

The demand schedule is now downwards sloping, rather than perfectly elastic. Without complete insurance, additional land purchases bring additional production and collateral value risk. This requires compensation in the form of a lower price.

Making use of the relationship for investment in land in period 2 (equation 25), the mango farmer's demand for land in period 2 following rain is

$$
\begin{aligned}
k_2^{m\prime} &= \frac{\beta w_2'}{(1+\beta)\, q_2'} = \frac{\beta\left[(1-z)\, a k_1^m + q_2' k_1^m + t_1^{m\prime}\right]}{(1+\beta)\, q_2'}, \\
\text{i.e. } k_2^{m\prime} &= \frac{2 w_1^m \beta}{\left(2+\beta+\beta^2\right)\left(1+\beta+\beta^2\right) q_2'}.
\end{aligned}
\tag{29}
$$

And the demand for land in period 2 following sun is

$$
\begin{aligned}
k_2^{m\prime\prime} &= \frac{\beta w_2''}{(1+\beta)\, q_2''} = \frac{\beta\left[(1+z)\, a + (1-\phi)\, q_2''\right] k_1^m}{(1+\beta)\, q_2''}, \\
\text{i.e } k_2^{m\prime\prime} &= \frac{\left[(1+z)\, \beta a + (1-\phi)\, \beta q_2''\right] \beta w_1^m}{\left(2q_1 - \beta q_2' - \beta\phi q_2'' - \beta a(1-z)\right)\left(1+\beta+\beta^2\right) q_2''}.
\end{aligned}
\tag{30}
$$

Similar expressions can be obtained for the rice farmer.

To solve for land market equilibrium, we again sum the individual land demands in either period. So equilibrium land prices in period 2, q_2' and q_2'', are given by the solutions to

$$k_2^{m\prime} + k_2^{r\prime} + k_2^{b\prime} = K \tag{31}$$

and

$$k_2^{m\prime\prime} + k_2^{r\prime\prime} + k_2^{b\prime\prime} = K. \tag{32}$$

While the equilibrium land price in period 1, q_1, is given by

$$k_1^m + k_1^r + k_1^b = K \tag{33}$$

Equations (**(31)**) – (**(33)**) are cubic in nature and not amenable to closed form solutions. But land prices will depend on the deep parameters of the model: the degree of liquidity of the collateral asset, the distribution of wealth between the farmers, the banks' ability to absorb the collateral asset, the size of the shock, and the common discount factor. Changes in these parameters have implications for welfare, as we show below.

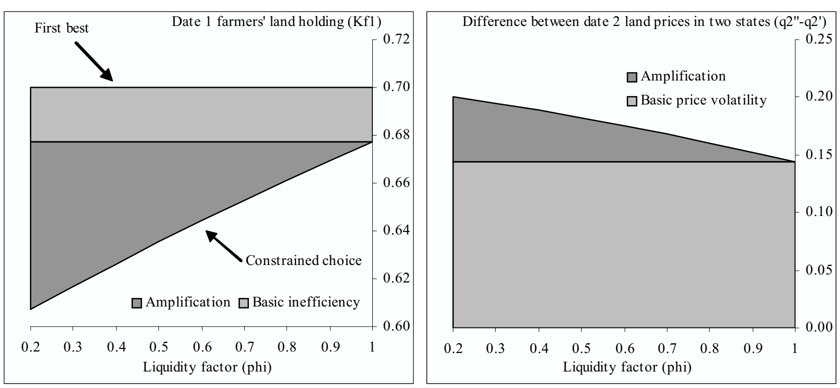
5 Welfare and policy implications

The welfare implications of financial frictions can be characterised by decomposing the effects on equilibrium quantities and asset prices induced by the amplification mechanism. Production inefficiency can be measured by comparing farmers' period 1 land holdings with the first best, $K_1^{f*} - K_1^f$. And asset price uncertainty can be gauged by the absolute difference between the potential post-shock land prices, $|q_2' - q_2''|$. There is no inefficiency or asset price uncertainty in the first best or in the case where collateral constraints do not bind.

Liquidity of the collateral asset

Chart 3 (left) and 3 (right), which are drawn for a set of baseline parameters, illustrate how the optimal allocation of land and asset price uncertainty vary with the liquidity of the collateral asset.[(9)] The dark-shaded area in both figures illustrates the impact of amplification in the model. The lighter-shaded regions indicate the 'basic' impact of financial frictions on inefficiency and asset price uncertainty (i.e. when $\phi = 1$).

Chart 3: Baseline inefficiency and asset price uncertainty



As Chart 3 (left) shows, lower values of ϕ tighten the constraint facing the farmer. The reduced trading of Arrow-Debreu securities that this implies makes wealth more uncertain for a given scale of production. The downward sloping line is a reflection of the fact that farmers are forced to lower production further and further as the constraint tightens. The inability to fully insure wealth also leads to increasing asset price uncertainty - Chart 3 (right) - as discussed in Section 4.

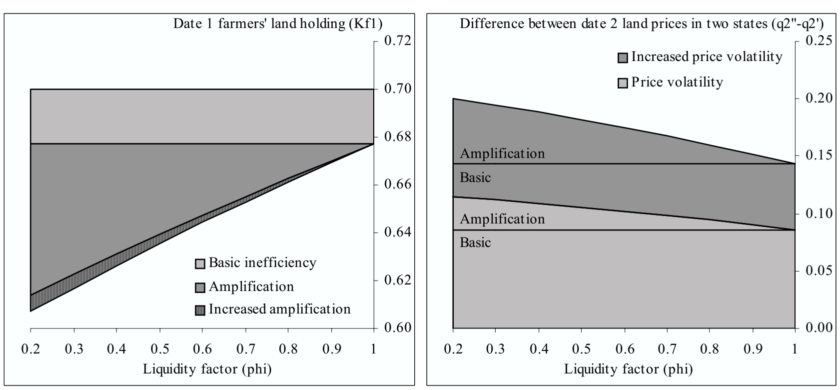
(9) Appendix 1 provides details of the parameter restrictions that must apply for the constraint to bind in each of the three cases. The baseline parameters chosen for our numerical solutions are: $K = 1$, $A = 1.2$, $a = 0.8$, $z = 0.5$, $\beta = 0.9$, $w^m = 0.6$, $w^r = 0.1$.

Changes in wealth distribution

In our model, proportional productivity shocks are perfectly negatively correlated. Thus, when wealth is evenly distributed between the two farmers, a positive shock to one sector (which boosts aggregate wealth) is completely offset by the negative effects of the shock to the other sector. But should one sector receive a string of positive shocks, perhaps as a result of a clustering of technological innovations as in Borio *et al* (2001), it starts to dominate the economy and aggregate wealth rises with each shock. This is because although each shock is as good for one sector as it is bad for the other in proportionate terms, it affects the wealth of the exapanding sector by more than that of the shrinking sector in absolute terms. Aggregate wealth is, therefore, procyclical.

Changes in wealth distribution also have implications for the efficiency and volatility costs of financial frictions. With uneven wealth, a positive shock to the dominant sector in period 1 generates a higher overall demand for land relative to a situation where the positive shock is received by the smaller sector. As a result, the demand for land and land prices become 'less similar' across states. The lower is ϕ, the lower is the extent to which farmers' wealth is insured. So the amplification mechanism creates a divergence in the effects of each sector on aggregate farming wealth. Chart 4 (right) shows how asset price uncertainty and the amplification effect increase as wealth distribution becomes more uneven.

Chart 4: Inefficiency and asset price uncertainty following increase in wealth inequality



The amplification effect also affects the allocation of land. If $\phi = 1$, the value of collateral is fully insured and faremers only face uncertain output. In response to this uncertainty, farmers scale back production relative to the first best – as Chart 4 (left) shows. But if $\phi < 1$, farmers are unable to fully insure collateral values, so wealth becomes more variable for a given scale of production.

If the distribution of wealth is uneven, increased asset price uncertainty and the amplification effect tighten constraints, forcing land holdings and production to be curtailed further.

Public supply of liquidity

The model suggests that there is scope for policymakers to promote the liquidity of financial markets – to limit the welfare costs of financial frictions and damaging fluctuations in asset prices, and facilitate the transfer of wealth from one period to another. One approach, discussed by Borio et al (2001), may be to pursue a discretionary policy towards collateral valuation practices in anticipation of the economy being hit by negative shocks. Thus, authorities may lower margin requirements and relax lending limits placed on assets serving as collateral, and relate provisions to the loan-to-value ratio.[10] More generally, policies that induce market participants to hold liquidity cushions at business cycle frequencies – building liquidity up during booms and drawing it down during recessions – may help curb the pro-cyclicality of the financial system and improve welfare.

An alternative approach to alleviating the welfare costs of financial frictions may be for the central bank to act as a liquidity provider, converting the illiquid asset into something more liquid. Kiyotaki & Moore (2001) suggest that the central bank could purchase the collateral asset in an open market operation and use the stream of income to 'retire' money, i.e offer a dividend on money. Central bank collateral policy may also contribute towards a reduction in the illiquidity of collateral. By accepting a broad range of assets as collateral, such as commercial paper issued by companies, the central bank may influence the preferences of the private sector, reducing the doubts and costs associated with a collateral pledge.

6 Final remarks

This paper has sought to formalise the intuition of the 'new view' of financial stability which lays stress on the role played by distortions to savings-investment decisions and the endogeneity of risk in driving the procyclicality of the economy. We demonstrate how the presence of financial frictions – in the form of collateral constraints – can generate under-insurance, leading to less

(10) For example, supervisory authorities in Japan lowered margin requirements and relaxed lending limits on collateral assets in order to alleviate liquidity constaints and contain distress selling during the stock market crash of 1987.

diversification and the build-up of systematic risk during upturns. A central aspect of amplification in the economic and financial cycle is the role played by liquidity of available instruments (financial and non-financial) that are used as collateral.

Our results point to the possibility that policymakers may be able to limit the welfare costs of financial stability by strengthening the macroprudential focus of financial regulation. Discretionary regulatory instruments, such as the size of haircuts to collateral values could, for example, respond to evolving views of risk. Central bank collateral policy might also accomodate a wider range of collateral assets, reducing the costs implicit in collateral pledges. And, to the extent that economic policies have an impact on the distribution of wealth in the short-run, there may be a role for intervention in preventing the over-extension of some sectors in the economy.

An important limitation of our model is the limited role of financial intermediaries. The banking sector is passive, merely serving as a channel through which agents in the real sector insure each other. Clearly, frictions in financial intermediation are a key aspect of any analysis of financial stability. Existing models that consider the macroeconomic consequences of these frictions largely eschew insurance markets and do not allow intermediaries to hedge shocks. Developing richer models of risk-sharing capacity that incorporate capital markets and frictions in intermediation is an important next step for future research.

Appendix 1: The other constrained cases

Farmers' constraints bind in bad states only

When the mango farmer, for example, is constrained in the rainy state, but is unconstrained when it is sunny, his date 1 problem is

$$
\begin{aligned}
& \max \ln c_1^m + \frac{\beta}{2}\left[\ln \frac{(1-z)ak_1^m + q_2'k_1^m + t_1^{m\prime}}{1+\beta} + \ln \frac{(1+z)ak_1^m + q_2''k_1^m + t_1^{m\prime\prime}}{1+\beta}\right] \\
& + \frac{\beta^2}{2}\left[\ln \frac{a\beta\left[(1-z)ak_1^m + q_2'k_1^m + t_1^{m\prime}\right]}{(1+\beta)\,q_2'} + \ln \frac{a\beta\left[(1+z)ak_1^m + q_2''k_1^m + t_1^{m\prime\prime}\right]}{(1+\beta)\,q_2''}\right] \\
\text{s.t.} \quad c_1^m &= w_1^m - k_1^m q_1 - \frac{\beta}{2}t_1^{m\prime} - \frac{\beta}{2}t_1^{m\prime\prime} \\
\text{s.t.} \quad -t_1^{m\prime} &= \phi q_2' k_1^m
\end{aligned}
$$

The first-order condition with respect to $t_1^{m\prime\prime}$ is

$$
\frac{-\frac{\beta}{2}}{w_1^m - k_1^m q_1 + \frac{\beta}{2}\phi q_2' k_1^m - \frac{\beta}{2}t_1^{m\prime\prime}} + \frac{\beta+\beta^2}{2}\frac{1}{(1+z)ak_1^m + q_2''k_1^m + t_1^{m\prime\prime}} = 0
$$

Rearranging gives the optimal quantity of Arrow-Debreu securities

$$
t_1^{m\prime\prime} = \frac{(1+\beta)\left(2w_1^m - 2k_1^m q_1 + \beta\phi q_2' k_1^m\right) - 2\left[(1+z)ak_1^m + q_2''k_1^m\right]}{2+\beta+\beta^2}, \qquad \textbf{(A-1)}
$$

and can be re-written as

$$
\begin{aligned}
t_1^{m\prime\prime} =& - \frac{2}{2+\beta+\beta^2}azk_1^m - \left(1 - \frac{(\beta+\beta^2)}{2+\beta+\beta^2}(1-\phi)\right)[q_2'' - E(q_2)]k_1^m \\
&+ \frac{(1+\beta)\left(2w_1^m - 2k_1^m q_1 + \beta\phi E(q_2)k_1^m\right) - 2\left[ak_1^m + E(q_2)k_1^m\right]}{2+\beta+\beta^2}
\end{aligned}
$$

The first component of the above expression is an insurance payment in the event of a positive productivity shock. The second component is an additional insurance payment in the event of higher-than-expected land values. This coefficient is less than unity when there are liquidity problems affecting the saleability of land ($\phi < 1$), so this source of wealth uncertainty is no longer completely eliminated, whereas when $\phi = 1$, this coefficient equals unity and there is full insurance of wealth against land price fluctuations. When there is full insurance of the second component, there is no amplification. Hence, when $\phi < 1$, we have an amplified price difference across the two states.

Substituting the optimal Arrow-Debreu holdings (equation **(A-1)**) into the mango farmer's optimisation problem as before gives

$$\max \ln\left[\frac{2w_1^m + \left[(1+z)a\beta + \beta\phi q_2' + \beta q_2'' - 2q_1\right]k_1^m}{2+\beta+\beta^2}\right]$$
$$+\frac{\beta}{2}\left[\ln\frac{\left[(1-z)a + (1-\phi)q_2'\right]k_1^m}{1+\beta} + \ln\frac{2w_1^m + \left[(1+z)\beta a + \beta\phi q_2' + \beta q_2'' - 2q_1\right]k_1^m}{2+\beta+\beta^2}\right]$$
$$+\frac{\beta^2}{2}\left[\ln\frac{a\beta\left[(1-z)a + (1-\phi)q_2'\right]k_1^m}{(1+\beta)\,q_2'} + \ln\frac{a\beta\left[2w_1^m + \left((1+z)\,\beta a + \beta\phi q_2' + \beta q_2'' - 2q_1\right)k_1^m\right]}{\left(2+\beta+\beta^2\right)q_2'}\right]$$

Solving for k_1^m gives

$$\frac{\left(\beta+\beta^2\right)}{\left(1+\beta+\beta^2\right)}\frac{w_1^m}{\left(2q_1 - \beta\phi q_2' - \beta q_2'' - \beta a(1+z)\right)} = k_1^m$$

There is a similar expression for k_1^r

$$\frac{\left(\beta+\beta^2\right)}{\left(1+\beta+\beta^2\right)}\frac{w_1^r}{\left(2q_1 - \beta q_2' - \beta\phi q_2'' - \beta a(1+z)\right)} = k_1^r$$

k_1^m is now used to solve for $k_2^{m\prime}$ and $k_2^{m\prime\prime}$.

$$k_2^{m\prime} = \frac{\left[(1-z)\,\beta a + (1-\phi)\,\beta q_2'\right]\beta w_1^m}{\left(2q_1 - \beta\phi q_2' - \beta q_2'' - \beta a(1-z)\right)\left(1+\beta+\beta^2\right)q_2'}$$

$$k_2^{m\prime\prime} = \frac{2w_1^m\beta}{\left(2+\beta+\beta^2\right)\left(1+\beta+\beta^2\right)q_2''}$$

There are similar expressions for rice farmers,

$$\begin{aligned} k_2^{r\prime} &= \frac{2w_1^r\beta}{\left(2+\beta+\beta^2\right)\left(1+\beta+\beta^2\right)q_2'} \\ k_2^{r\prime\prime} &= \frac{\left[(1-z)\,\beta a+(1-\phi)\,\beta q_2''\right]\beta w_1^r}{\left(2q_1-\beta q_2'-\beta\phi q_2''-\beta a(1-z)\right)\left(1+\beta+\beta^2\right)q_2''} \end{aligned}$$

Period 2 equilibrium in state 1 is given by

$$k_2^{m\prime}+k_2^{r\prime}+k_2^{b\prime}=K \qquad \textbf{(A-2)}$$

Period 2 equilibrium in state 1 is given by

$$k_2^{m\prime\prime}+k_2^{r\prime\prime}+k_2^{b\prime\prime}=K \qquad \textbf{(A-3)}$$

Period 1 equilibrium is given by

$$k_1^m+k_1^r+k_1^b=K \qquad \textbf{(A-4)}$$

Equations (**(A-2)**)-(**(A-4)**) are three (cubic) equations in three unknown prices, q_1, q_2' and q_2''. We use numerical techniques to solve for q_1, q_2' and q_2'' given values of the deep parameters, w_1^m,.w_1^r, a, A, β, z and K. Since the equations arecubic in nature, there is a possibility of multiple positive equilibrium values of q_1, q_2' and q_2''. For the deep parameter values chosen for our experiment, however, we find only unique solutions. In all such cases we can show that $\left|q_2'-q_2''\right|$ increases as ϕ falls, i.e. price uncertainty increases as liquidity falls.

Parameter restrictions

If the constraint binds in the bad state only then, for the mango farmer: $-t_1^{m\prime\prime} <\phi\ k_1^m q_2''$, i.e.

$0 <\phi\ k_1^m q_2'' + t_1^{m''}$. Thus

$$\begin{aligned}
0 & <\phi \quad k_1^m q_2'' + \frac{(1+\beta)\left(2w_1^m - 2k_1^m q_1 + \beta\phi q_2' k_1^m\right) - 2\left[(1+z)ak_1^m + q_2'' k_1^m\right]}{2+\beta+\beta^2} \\
\text{i.e. } 0 & < \frac{\left(2+\beta+\beta^2\right)\left(2q_1 - \beta\phi q_2' - \beta q_2'' - 2\beta a(1+z)\right) - 2\left(1-\phi\right)\beta q_2''}{\left(2q_1 - \beta\phi q_2' - \beta q_2'' - \beta a(1+z)\right)} \qquad \textbf{(A-5)}
\end{aligned}$$

And for the rice farmer: $-t_1^{r\prime} <\phi\ k_1^r q_2'$, i.e. $0 <\phi\ k_1^r q_2' + t_1^{r\prime}$. So

$$0 < \frac{\left(2+\beta+\beta^2\right)\left(2q_1 - \beta q_2' - \beta\phi q_2'' - 2\beta a(1+z)\right) - 2\left(1-\phi\right)\beta q_2'}{\left(2q_1 - \beta q_2' - \beta\phi q_2'' - \beta a(1+z)\right)} \qquad \textbf{(A-6)}$$

The deep parameters must satisfy restrictions (**(A-5)**) and (**(A-6)**) for equilibrium prices, if both farmers are to be constrained only in their respective bad states of the world.

Constraint binds in both states

When the mango farmer is constrained in both states of the world, i.e. $-t_1^{m\prime} = \phi q_2' k_1^m$ and $-t_1^{m''} = \phi q_2'' k_1^m$, his problem is

$$\begin{aligned}
& \max_{k_1^m} \ln c_1^m + \frac{\beta}{2}\left[\ln\frac{(1-z)ak_1^m + q_2' k_1^m + t_1^{m\prime}}{1+\beta} + \ln\frac{(1+z)ak_1^m + q_2'' k_1^m + t_1^{m''}}{1+\beta}\right] \\
& +\frac{\beta^2}{2}\left[\ln\frac{a\beta\left[(1-z)ak_1^m + q_2' k_1^m + t_1^{m\prime}\right]}{(1+\beta)\,q_2'} + \ln\frac{a\beta\left[(1+z)ak_1^m + q_2'' k_1^m + t_1^{m''}\right]}{(1+\beta)\,q_2''}\right] \\
\text{s.t. } \quad c_1^m & = w_1^m - k_1^m q_1 - \frac{\beta}{2}t_1^{m\prime} - \frac{\beta}{2}t_1^{m''} \\
\text{s.t. } \quad -t_1^{m\prime} & = \phi q_2' k_1^m \text{ and } -t_1^{m''} = \phi q_2'' k_1^m
\end{aligned}$$

The solution to this problem gives the optimal value of k_1^m when both constraints bind. Using this and an equivalent expression for k_1^m in the market clearing condition, $k_1^m + k_1^r + k_1^b = K$, gives one (cubic) equation in terms of the prices q_1, q_2' and q_2''.

Given k_1^m, $k_2^{m\prime}$ and $k_2^{m\prime\prime}$ can be computed using

$$k_2^{m\prime} = \frac{\beta w_2'}{(1+\beta)\, q_2'} = \frac{\beta \left[(1-z)\, a k_1^m + (1-\phi) q_2' k_1^m\right]}{(1+\beta)\, q_2'}$$

$$\text{and } k_2^{m\prime\prime} = \frac{\beta w_2''}{(1+\beta)\, q_2''} = \frac{\beta \left[(1+z)\, a + (1-\phi)\, q_2''\right] k_1^m}{(1+\beta)\, q_2''}$$

Again, there are similar expressions for rice farmers. Two more market clearing conditions, namely $k_2^{m\prime} + k_2^{r\prime} + k_2^{b\prime} = K$ and $k_2^{m\prime\prime} + k_2^{r\prime\prime} + k_2^{b\prime\prime} = K$ produce a second and third (cubic) equation in terms of q_1, q_2' and q_2''. All three equations can be solved simultaneously, using numerical methods to obtain the equilibrium values of q_1, q_2' and q_2''. We can then solve for equilibrium quantities k_1, k_2' and k_2'' for each player.

Parameter constraints

Both constraints bind for all valid parameter combinations that have not already been associated with no constraints binding or the constraints binding only in the good state.

Appendix 2: Under-insurance in both states of the world

The first-order condition that gives the optimal amount of state-1 Arrow-Debreu security to hold in an unconstrained (first-best) world is

$$\begin{aligned}&\frac{-\frac{\beta}{2}}{w_1^m - k_1^m q_1 - \frac{\beta}{2}t_1^{m\prime} - \frac{\beta}{2}t_1^{m\prime\prime}} + \frac{\beta+\beta^2}{2}\left[\frac{1}{(1-z)ak_1^m + q_2' k_1^m + t_1^{m\prime}}\right]\\ =\; & F(t_1^{m\prime}, t_1^{m\prime\prime}) = 0\end{aligned}$$

The implicit function theorem states that

$$\frac{\partial t_1^{m\prime\prime}}{\partial t_1^{m\prime}} = -\frac{\partial F}{\partial t_1^{m\prime}} / \frac{\partial F}{\partial t_1^{m\prime\prime}} \qquad \textbf{(B-1)}$$

$\frac{\partial F}{\partial t_1^{m\prime\prime}}$ is the second-order condition of the farmer's problem in which he must decide how much of the state-2 Arrow-Debreu security to hold. As we are looking for a value that maximises the farmer's objective function, this must be negative. If $\frac{\partial F}{\partial t_1^{m\prime}}$ is also negative, it must be the case - from equation **(B-1)** - that $\frac{\partial t_1^{m\prime\prime}}{\partial t_1^{m\prime}}$ is positive, i.e. should $t_1^{m\prime\prime}$ fall below the first-best, then $t_1^{m\prime}$ must also fall below the first best. This is indeed the case since

$$\frac{\partial F}{\partial t_1^{m\prime}} = -\frac{\beta^2}{4}\frac{1}{(w_1^m - k_1^m q_1 - \frac{\beta}{2}t_1^{m\prime} - \frac{\beta}{2}t_1^{m\prime\prime})^2} < 0.$$

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