Was There a Nasdaq Bubble in the Late 1990s?

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Abstract

Not necessarily. The fundamental value of a firm increases with uncertainty about average future profitability, and this uncertainty was unusually high in the late 1990s. We calibrate a stock valuation model that includes this uncertainty, and show that the uncertainty needed to match the observed Nasdaq valuations at their peak is high but plausible. The high uncertainty might also explain the unusually high return volatility of Nasdaq stocks in the late 1990s. Uncertainty has the biggest effect on stock prices when the equity premium is low.

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"Before we relegate a speculative event to the fundamentally inexplicable or bubble category driven by crowd psychology, however, we should exhaust the reasonable economic explanations... "bubble" characterizations should be a last resort because they are non-explanations of events, merely a name that we attach to a financial phenomenon that we have not invested sufficiently in understanding." Garber (2000, p.124)

1. Introduction

On March 10, 2000, the Nasdaq Composite Index closed at its all-time high of 5,048.62. For comparison, the same index stood at 1,114 in August 1996 as well as in October 2002. The unusual rise and fall in the prices of technology stocks has led many academics and practitioners to describe the event as a stock price "bubble." [1] This label seems appropriate if the term "bubble" is interpreted as an ex post description of an extended rise in prices followed by a sharp fall (e.g., Kindleberger, 1978). However, a more common interpretation is that the prices of technology stocks exceeded their fundamental values in the late 1990s. This paper challenges the notion that technology stocks were overvalued at that time.

To analyze whether stocks are fairly priced, one needs a model of the fundamental value. We argue that the models that have been used to value technology stocks omit an important determinant of the fundamental value, namely the uncertainty about a firm's average future profitability, which can also be thought of as the uncertainty about the average future growth rate of the firm's book value. This uncertainty increases the firm's fundamental value, as shown by Pástor and Veronesi (2003). We argue that the late 1990s witnessed high uncertainty about the average growth rates of technology firms, and that this uncertainty was partly responsible for the high level of technology stock prices.

To illustrate why uncertainty about a firm's growth rate raises the firm's fundamental value, consider the Gordon growth model, $P/D = 1/(r - g)$, where P is the stock price, D is the next period's dividend, r is the discount rate, and g is the dividend growth rate. If g is uncertain, then P/D is equal to the expectation of $1/(r - g)$, under the conditions discussed in the Appendix. This expectation increases with uncertainty about g, because $1/(r - g)$ is convex in g.[2] Loosely speaking, a firm with some probability of failing (a very low g) and some probability of becoming the next Microsoft (a very high g) is very valuable.[3]

Ofek and Richardson (2002) argue that the earnings of Internet firms would have to grow

[1] See, for example, Thaler (1999), Shiller (2000), Ofek and Richardson (2002, 2003), Ritter and Warr (2002), Ritter and Welch (2002), Abreu and Brunnermeier (2003), Brunnermeier and Nagel (2003), Ljungqvist and Wilhelm (2003), and Stein (2004).

[2] As shown in Appendix C, P/D increases with uncertainty about g even if that uncertainty increases r.

[3] Interestingly, Bill Miller, portfolio manager of the Legg Mason Value Trust, used similar logic to justify the valuation of Amazon.com in 1999: "...being wrong isn't very costly, and being right has a high payoff... With Amazon, we believe the payoff for being right is high." Amazon's Allure..., Barron's, 15 Nov 1999.

at implausibly high rates to justify the Internet stock prices in the late 1990s. Their argument implicitly assumes that the earnings growth rate is known. However, when uncertainty about the growth rate is acknowledged, the observed price can be justified with a significantly lower expected growth rate. For example, consider a stock with $r = 20\%$ and P/D of 50. To match the observed P/D in the Gordon formula with a known value of g, the required dividend growth rate is $g = 18\%$. Suppose instead that g is unknown and drawn from a uniform distribution with a standard deviation of 4%. The expected g required to match the P/D of 50 then drops to 13.06%. Mathematically, Jensen's inequality implies that

$$\frac{P}{D} = \mathrm{E}\left(\frac{1}{r-g}\right) > \frac{1}{r - \mathrm{E}(g)}, \tag{1}$$

so simply plugging the expected growth rate, $\mathrm{E}(g)$, into the Gordon formula understates the P/D ratio. This understatement is especially large when uncertainty about g is large.

Although the Gordon model conveys our basic idea, it is not well suited for pricing technology firms, because many of those firms pay no dividends. To avoid this problem, we develop a stock valuation model that focuses on the ratio of the market value to book value of equity (M/B) instead of the price-dividend ratio. In our closed-form pricing formula, M/B is an increasing function of uncertainty about the average growth rate of the firm's book value. The pricing formula can be inverted to compute "implied uncertainty," i.e., the level of uncertainty that sets the firm's model-implied M/B equal to the observed M/B.

We calibrate the valuation model, and compute the implied uncertainty of the Nasdaq-traded firms on March 10, 2000. We argue that this uncertainty is plausible because it implies return volatility that is close to the volatility observed in the data. The Nasdaq stock prices in the late 1990s were not only high but also highly volatile, and both facts are consistent with high uncertainty about average profitability. We conclude that Nasdaq prices at the peak of the "bubble" are justifiable in our rational valuation model.

We also examine the time variation in implied uncertainty. First, we extract the time series of the equity premium from the observed valuations of the NYSE and Amex stocks, assuming that those stocks are fairly priced with no uncertainty. Given the equity premium, we construct the time series of implied uncertainty for the Nasdaq index as a whole. We find that this uncertainty increased dramatically in the late 1990s, and declined thereafter.

Consistent with this pattern in implied uncertainty, there are good reasons to believe that uncertainty about the growth rates of technology firms was unusually high in the late 1990s. The past decade witnessed rapid technological progress, especially in the Internet and telecom industries.[4] Technological revolutions are likely to be accompanied by high uncertainty about future growth. When old paradigms are fading away and a "new era" is being embraced, uncertainty increases because the historical experience is discounted. The popular press contains numerous suggestions that investors appeared to be unusually uncertain about future growth in the late 1990s.[5]

Empirical evidence also indicates high uncertainty about the average growth rates of technology firms at the end of the past decade. First, Nasdaq return volatility increased dramatically in the late 1990s and declined after year 2000, both in absolute terms and relative to the NYSE/Amex volatility. Second, the dispersion of profitability across Nasdaq stocks also increased in the late 1990s and declined afterwards. Third, the stock price reaction to earnings announcements was unusually strong in the late 1990s (e.g., Ahmed, Schneible, and Stevens, 2003, and Landsman and Maydew, 2002), which is consistent with high uncertainty because signals elicit large revisions in beliefs when prior uncertainty is high. Fourth, technology firms went public unusually early in their life-cycles in the late 1990s (e.g., Schultz and Zaman, 2001). Uncertainty about the average growth rates of firms with short earnings track records should be especially high. Finally, Pástor and Veronesi (2004) argue that high uncertainty about the average profitability of new firms attracted many private firms to go public at the end of the past decade.

The turn of the millenium can be characterized not only by high uncertainty but also by a low equity premium. Although the academics do not agree on the exact magnitude of the premium, they tend to agree that the premium was relatively low in the late 1990s (e.g., Welch, 2001). We argue that the low equity premium amplified the effect of uncertainty on stock prices in the late 1990s. In the Gordon formula, the convexity of P/D in g is strongest when r is low, and the same intuition holds in our model. When the discount rate is low, a large fraction of the firm value comes from earnings in the distant future, and those earnings are the most affected by uncertainty about the firm's average future growth rate.

Two recent studies provide different explanations for the high valuations of technology stocks in the late 1990s. According to Ofek and Richardson (2003), these valuations were high partly due to short-sale constraints. According to Cochrane (2002), technology stocks

[4]For example, the number of utility patents (i.e., patents for inventions) granted by the U.S. Patent and Trademark Office in 1999 (153,485) was 51% higher than the number granted in 1995 (101,419), which was 5% higher than the number granted in 1991 (96,513). See www.uspto.gov for more information.

[5]E.g., "...the projections of revenue growth were, by and large, wild guesses." New Economy, Bad Math, ... Avital Louria Hahn, Investment Dealers Digest, 23 October 2000. E.g., "The problem is that since we know so little about where the Net is headed, predicting cash flow so far into the future is largely meaningless... investing in this new technology was a bet..." You Believe? ... Fortune Magazine, June 7, 1999. Trueman (2001) discusses the Internet firms' "highly unpredictable growth rates."

were valued highly because they had high convenience yields. Neither study demonstrates that the magnitudes of these effects could be large enough to justify the observed valuations of Nasdaq firms. In contrast, our calibration shows that the effect of uncertainty can be strong enough to rationalize those valuations. Moreover, neither of the two studies explains why the prices of technology stocks were so volatile at that time. In our model, the high return volatility is a natural consequence of high uncertainty.

In another related study, Schwartz and Moon (2000) argue that the observed valuations of the Internet stocks can be rationalized by revenue growth that is both sufficiently high and sufficiently volatile. They calibrate their model to match the valuation of Amazon.com, but they report that the implied return volatility is too high, and they also find the implied revenue distribution unrealistic. Our model, which is substantially different from theirs, produces distributions of returns and cash flows that seem more realistic.

This paper is also related to the theoretical literature on asset price bubbles, e.g., Allen and Gorton (1993), Santos and Woodford (1997), Abreu and Brunnermeier (2003), Allen, Morris, and Shin (2003), and Scheinkman and Xiong (2003). See Camerer (1989) and Brunnermeier (2001) for literature reviews. Also related is Garber (2000), who proposes rational explanations (unrelated to ours) for the Dutch tulip mania of the 1630s and other historical "bubble" episodes. Donaldson and Kamstra (1996) develop a neural network model for dividends, and use it to dismiss the idea of a stock price bubble in the 1920s.

We don't claim that investor behavior in the late 1990s was fully rational. Good examples of apparent irrationality are presented by Cooper, Dimitrov, and Rau (2001), Lamont and Thaler (2003), and others. Also, we don't attempt to rule out any behavioral explanations for the "bubble." We only argue that such explanations are not necessary, because stock prices in March 2000 are also consistent with a rational model. The notion of a Nasdaq "bubble" caused by investor irrationality should not be held as a self-evident truth.

The paper is organized as follows. Section 2 describes our valuation model. Section 3 calibrates the model. Section 4 computes the implied uncertainty on March 10, 2000 for the Nasdaq index as a whole, as well as for individual firms such as Amazon, Cisco, Ebay, and Yahoo. Section 5 examines the variation in implied uncertainty over time. Section 6 analyzes the cross-section of implied uncertainty. Section 7 discusses some important issues such as the effect of uncertainty on the discount rate. Section 8 concludes.

2. The Model

The stock valuation model developed in this section builds on the models of Pástor and Veronesi (2003, 2004; henceforth PV). Let $\rho_t^i = Y_t^i / B_t^i$ denote firm i's instantaneous profitability at time t, where Y_t^i is the earnings rate and B_t^i is the book value of equity. We assume that profitability follows a mean-reverting process:

$$d\rho_t^i = \phi^i \left(\overline{\rho}_t^i - \rho_t^i \right) dt + \sigma_{i,0} dW_{0,t} + \sigma_{i,i} dW_{i,t}, \quad \phi^i > 0, \ t < T_i, \tag{2}$$

where $W_{0,t}$ and $W_{i,t}$ are uncorrelated Wiener processes that capture the systematic ($W_{0,t}$) and firm-specific ($W_{i,t}$) components of the random shocks that drive the firm's profitability. We also assume that the firm's average profitability, $\overline{\rho}_t^i$, can be decomposed as

$$\overline{\rho}_t^i = \overline{\rho}_t + \overline{\psi}_t^i. \tag{3}$$

The common component, $\overline{\rho}_t$, exhibits mean-reverting variation that reflects business cycles:

$$d\overline{\rho}_t = k_L \left(\overline{\rho}_L - \overline{\rho}_t \right) dt + \sigma_{L,0} dW_{0,t} + \sigma_{L,L} dW_{L,t}, \quad k_L > 0, \tag{4}$$

where $W_{L,t}$ is uncorrelated with both $W_{0,t}$ and $W_{i,t}$. The firm-specific component, $\overline{\psi}_t^i$, which we refer to as the firm's average excess profitability, is assumed to slowly decay to zero:

$$d\overline{\psi}_t^i = -k_\psi \overline{\psi}_t^i dt, \quad k_\psi > 0, \ t < T_i. \tag{5}$$

This assumption is made for analytical convenience. Assuming constant $\overline{\psi}_t^i$ would present technical complications related to the transversality condition, as discussed later. From the economic perspective, the gradual decay in average excess profitability can be interpreted as an outcome of slow-moving competitive market forces.

Competition in the firm's product market can also arrive suddenly, at some random future time T_i. We assume that T_i is exponentially distributed with density $h(T^i; p)$, so that at any point in time, there is probability p that T_i arrives in the next instant. The sudden entry of competition at time T_i eliminates the present value of the firm's future abnormal earnings, defined as earnings in excess of those earned at the rate equal to the cost of capital. As a result, the firm's market value of equity at time T_i equals the book value, $M_{T_i}^i = B_{T_i}^i$. This implication follows from the residual income model (e.g., Ohlson, 1995), in which the market equity equals book equity plus the present value of future abnormal earnings.

We assume that the firm pays out a constant fraction of its book equity in dividends, $D_t^i = c^i B_t^i$, where $c^i \geq 0$. We refer to c^i as the dividend yield, for simplicity. The firm is financed only by equity, and it issues no new equity. These assumptions are made for analytical convenience; relaxing them would add complexity with no obvious new insights. (For example, debt financing has no effect as long as its dynamics do not affect the profitability process.) Given these assumptions, the clean surplus relation implies that book equity grows at the rate equal to the firm's profitability minus the dividend yield:

$$dB_t^i = \left(Y_t^i - D_t^i\right) dt = \left(\rho_t^i - c^i\right) B_t^i dt. \tag{6}$$

The market value of equity is the present value of any dividends plus the final payoff B_{T_i}:

$$M_t^i = E_t \left[\int_t^\infty \left(\int_t^{T^i} \frac{\pi_s}{\pi_t} D_s^i ds + \frac{\pi_{T^i}}{\pi_t} B_{T_i}^i \right) h(T^i; p)\ dT^i \right]. \tag{7}$$

The stochastic discount factor (SDF) π_t is assumed to be given by

$$\pi_t = e^{-\eta t - \gamma(s_t + \varepsilon_t)}, \tag{8}$$

where

$$\begin{aligned} s_t &= a_0 + a_1 y_t + a_2 y_t^2, & (9)\\ dy_t &= k_y \left(\overline{y} - y_t\right) dt + \sigma_y dW_{0,t}, & (10)\\ d\varepsilon_t &= \mu_\varepsilon dt + \sigma_\varepsilon dW_{0,t}. & (11) \end{aligned}$$

PV (2004) derive π_t in equation (8) as an outcome of a habit utility model, in which ε_t is log aggregate consumption, and s_t is the log surplus consumption ratio introduced by Campbell and Cochrane (1999). Similar SDFs have been used in the term structure literature (e.g., Constantinides, 1992). Given this specification, the equity premium varies over time due to the time-varying risk aversion of the representative investor. As shown in the Appendix, high values of y_t imply a low volatility of the SDF, and thus a low equity premium.

2.1. Valuation

The following function is used repeatedly:

$$Z^i \left(y_t, \overline{\rho}_t, \rho_t^i, \overline{\psi}_t^i, s\right) = e^{Q_0(s) + \mathbf{Q}(s)' \cdot \mathbf{N}_t + Q_5(s) y_t^2}, \tag{12}$$

where $\mathbf{N}_t = \left(y_t, \overline{\rho}_t, \rho_t^i, \overline{\psi}_t^i\right)$, and the Q functions are defined in the Appendix.

Proposition 1. Suppose that $\overline{\psi}_t^i$ is known. The firm's M/B ratio is given by

$$\frac{M_t^i}{B_t^i} = G^i \left(y_t, \overline{\rho}_t, \rho_t^i, \overline{\psi}_t^i\right) = \left(c^i + p\right) \int_0^\infty Z^i \left(y_t, \overline{\rho}_t, \rho_t^i, \overline{\psi}_t^i, s\right) ds. \tag{13}$$

This proposition serves as a benchmark for the more interesting case, analyzed next, in which $\overline{\psi}_t^i$ is unobservable. According to Proposition 1, a firm's M/B ratio is high if expected profitability is high and if the discount rate is low. As for profitability, M/B increases with $\overline{\rho}_t$, $\overline{\psi}_t^i$, and ρ_t^i. As for the discount rate, M/B increases with y_t in the plausible parameter range: when y_t is high, the equity premium is low and M/B is high. The dependence of M/B on c^i and p is unclear because Z^i is decreasing in both variables. A higher dividend yield increases near-term cash flow, but it also reduces the growth rate of book value, so the overall effect of c^i on M/B is ambiguous. A higher p brings the terminal cash flow ($B_{T_i}^i$) closer in time, but it also shortens the expected period over which abnormal earnings can be earned, so its effect on M/B depends on the parameter choices as well. For most reasonable parameter values, however, M/B increases when p decreases.

Suppose now that $\overline{\psi}_t^i$ is unknown, and that the investors' beliefs about $\overline{\psi}_t^i$ can be summarized by the probability density function $f_t(\overline{\psi}_t^i)$. The law of iterated expectations implies

$$\frac{M_t^i}{B_t^i} = E_t\left[G^i\left(y_t, \overline{\rho}_t, \rho_t^i, \overline{\psi}_t^i\right)\right] = \int G^i\left(y_t, \overline{\rho}_t, \rho_t^i, \overline{\psi}_t^i\right) f_t\left(\overline{\psi}_t^i\right) d\overline{\psi}_t^i. \tag{14}$$

Since $G^i(y_t, \overline{\rho}_t, \rho_t^i, \overline{\psi}_t^i)$ is a convex function of $\overline{\psi}_t^i$, more uncertainty about $\overline{\psi}_t^i$ (i.e., a mean-preserving spread in $\overline{\psi}_t^i$) implies a higher expected value of $G^i(y_t, \overline{\rho}_t, \rho_t^i, \overline{\psi}_t^i)$, and thus a higher M/B ratio. This relation holds for any distribution $f_t(\overline{\psi}_t^i)$. To obtain a closed-form solution for M/B, we assume that $f_t(\overline{\psi}_t^i)$ is normal.

Proposition 2. Suppose that $\overline{\psi}_t^i$ is unknown, and that the market perceives a normal distribution for $\overline{\psi}_t^i$, $f_t(\overline{\psi}_t^i) = N(\widehat{\psi}_t^i, \widehat{\sigma}_{i,t}^2)$. The firm's M/B ratio is given by

$$\frac{M_t^i}{B_t^i} = (c^i + p)\int_0^\infty Z^i\left(y_t, \overline{\rho}_t, \rho_t^i, \widehat{\psi}_t^i, s\right) e^{\frac{1}{2}Q_4(s)^2\widehat{\sigma}_{i,t}^2} ds. \tag{15}$$

The fact that M/B increases with $\widehat{\sigma}_{i,t}$, uncertainty about $\overline{\psi}_t^i$, is the key relation in the paper. The formulas for the expected return and return volatility of firm i are presented in the Appendix, along with the proofs of Propositions 1 and 2.

3. Calibration

In this section, we calibrate the model to match some key features of the data on asset returns and profitability. The parameters are summarized in Table 1.

We divide firms into two groups, the "new economy" and the "old economy". For simplicity, we assume that the new economy includes firms traded on Nasdaq, and the old economy includes firms traded on the NYSE and Amex. The new economy firms are described in Section 2. The old economy's aggregate profitability is given by $\overline{\rho}_t$ in equation (4). The old economy pays aggregate dividends forever at the rate of $D_t^O = c^O B_t^O$, where B_t^O is the old economy's aggregate book value. We compute $c^O = 5.67\%$ as the time-series average of the old economy's annual dividend yields, each of which is computed as the sum of the current-year dividends across all NYSE/Amex firms, divided by the sum of the book values of equity at the end of the previous year.[6] The old economy's aggregate market value is given by $M_t^O = E_t\left[\int_t^\infty \frac{\pi_s}{\pi_t} D_s^O ds\right]$, and its M/B ratio is determined by the economy's profitability and the market-wide discount rate:

$$M_t^O / B_t^O = \Phi\left(\overline{\rho}_t, y_t\right) \tag{16}$$

An explicit formula for the function Φ is provided in Proposition 3 in the Appendix.

To estimate the process for $\overline{\rho}_t$ in equation (4), we compute the old economy's profitability as the sum of the current-year earnings across all NYSE/Amex stocks, divided by the sum of the book values of equity at the end of the previous year. This time series is adjusted for inflation by using the GDP deflator, obtained from NIPA. Equation (4) implies a normal likelihood function for $\overline{\rho}_t$, as described in Lemma 3 in the Appendix. Maximizing this likelihood yields $k_L = 0.3574$, $\overline{\rho}_L = 12.17\%$ per year, and an estimate of the total volatility, $\sigma_L = \sqrt{\sigma_{L,0}^2 + \sigma_{L,L}^2}$. This estimate is split into its components, $\sigma_{L,0} = 1.47\%$ and $\sigma_{L,L} = 1.31\%$ per year, by using the covariance of $\overline{\rho}_t$ with the SDF, which is discussed next.

We choose the parameters of the SDF to produce reasonable properties for the returns and M/B of the old economy, as well as for the risk-free rate. First, we construct the 1962-2002 annual time series of the old economy's M/B, by computing the ratio of the sums of the market values and the most recent book values of equity across all NYSE/Amex firms. We then invert the pricing formula in equation (16) to obtain the time series of y_t as a function of the old economy's M/B ratio, $\overline{\rho}_t$, and the parameters $\Theta = (\eta, \gamma, \overline{y}, k_y, \sigma_y, \sigma_{L,0}, \sigma_{L,L})$. To help us choose reasonable parameters, we estimate Θ by a simple minimum distance procedure. The moment conditions are constructed from the stationary distribution of $(\overline{\rho}_t, y_t)$, obtained by substituting $(\overline{\rho}_t, y_t)$ for $\mathbf{z}_t$ in Lemma 3 in the Appendix. We also impose additional moment conditions to ensure that the average values of the estimated equity premium $\mu_{R,t}$, market return volatility $\sigma_{R,t}$, and the real interest rate $r_{f,t}$ are close to the values observed

[6]Throughout the paper, firms are defined as ordinary common shares (i.e., CRSP sharecodes 10 and 11). Gebhardt, Lee, and Swaminathan (2001) argue that "In theory, share repurchases and new equity issues that can be anticipated in advance should also be included in the dividend payout estimate. However, we know of no reliable technique for making these forecasts." Like Gebhardt et al., we do not attempt to forecast share repurchases and new equity issues for the purpose of computing the expected dividend yield.

in the data. The estimated parameters, which are listed in Table 1, imply the average equity premium of 5.06% per year, the average old-economy volatility of 14.47%, the average real risk-free rate of 6.25%, and the average risk-free rate volatility of 1.55% (all annual).

In the process for $\overline{\psi}_t^i$ in equation (5), we set $k_\psi = 0.0139$, which implies a half-life of 50 years. We choose such slow mean reversion for $\overline{\psi}_t^i$ so that $\overline{\psi}_t^i$ can be thought of as virtually constant before time T_i; in fact, we drop the t subscript from $\overline{\psi}_t^i$ in the rest of the paper, for simplicity. If we assumed $\overline{\psi}^i$ to be literally constant, we would need to impose a finite upper bound on the range of possible values of $\overline{\psi}^i$ to ensure that prices are well defined. This upper bound would prevent us from using a normal distribution for $\overline{\psi}^i$, which would complicate the analysis and make the pricing formulas less elegant. Nonetheless, we have solved the model with constant $\overline{\psi}^i$ under the assumption of a truncated normal distribution for $\overline{\psi}^i$, and obtained the same conclusions throughout the paper.

4. Matching Nasdaq Prices on March 10, 2000

In this section, we examine the ability of our valuation model to match the prices of Nasdaq stocks on March 10, 2000, the day when the Nasdaq index peaked.

4.1. Matching Nasdaq's Valuation

In this subsection, we view Nasdaq as one large firm, whose profitability ρ_t^N follows the process (2).[7] The parameters of this process are estimated by maximum likelihood, where the likelihood function is obtained by substituting $\left(\rho_t^N, \overline{\rho}_t, y_t\right)$ for $\mathbf{z}_t$ in Lemma 3 in the Appendix. In this substitution, we take as given the parameters of the $\overline{\rho}_t$ and y_t processes described in Section 3., without imposing any restrictions on the covariances between ρ_t^N and $(\overline{\rho}_t, y_t)$. This procedure yields $\phi^N = 0.3667$, $\sigma_{N,0} = 2.46\%$, and $\sigma_{N,N} = 4.90\%$ per year.

For each Nasdaq-traded firm, we compute the market value of equity on March 10, 2000 by multiplying the share price by the number of shares outstanding, both obtained from CRSP. Nasdaq's M/B ratio is the sum of the market values of all Nasdaq firms on March 10, 2000, divided by the sum of the end-of-1999 book values of equity. This ratio is equal to 8.55. Nasdaq's dividend yield, $c = 1.35\%$, is the sum of the dividends of all Nasdaq firms in 1999, divided by the sum of the end-of-1998 book values. We measure profitability as the accounting return on equity (ROE), following the definition of ρ_t^i. Nasdaq's current

[7] Nasdaq's profitability is computed in the same way as the NYSE/Amex profitability in Section 3.

profitability, $\rho_t^N = 9.96\%$ per year, is computed as the 1999Q4 annualized value of Nasdaq's aggregate profitability, i.e., the sum of 1999Q4 earnings across all Nasdaq firms, divided by the sum of the most recent pre-1999Q4 book values of equity. Analogously, $\overline{\rho}_t = 18.62\%$ per year is computed as the 1999Q4 annualized value of NYSE/Amex's aggregate profitability. It is not surprising that this value is higher than $\overline{\rho}_t$'s central tendency of 12.17%, because the U.S. economy was near the peak of a ten-year expansion at the end of 1999.[8]

We assume that the competition that wipes out the present value of Nasdaq's future abnormal earnings can arrive in any instant with probability $p = 1/20$. As a result, the expected time over which Nasdaq can earn abnormal profits ($\overline{\psi}^N$) is $\mathrm{E}(T) = 1/p = 20$ years. Later, we also review the results for $\mathrm{E}(T) = 15$ and 25 years.

Panel A of Table 2 reports the model-implied M/B for Nasdaq on March 10, 2000 under zero uncertainty about $\overline{\psi}^N$, for different values of the equity premium and $\widehat{\psi}^N$. (Recall that $\widehat{\psi}^N$ is Nasdaq's expected profitability in excess of the NYSE/Amex profitability.) The model-implied M/B increases with $\widehat{\psi}^N$ and decreases with the equity premium, as expected. With $\widehat{\psi}^N \leq 3\%$ per year, not even the equity premium of 1% per year can match Nasdaq's M/B of 8.55. With $\widehat{\psi}^N = 4\%$ per year, the premium needed to match Nasdaq's M/B is about 1.4% per year, and with $\widehat{\psi}^N = 5\%$, the required premium is about 2.8% per year.

Panel B of Table 2 reports the model-implied return volatility for Nasdaq on March 10, 2000 under zero uncertainty. This volatility ranges mostly between 20% and 30% per year. For comparison, we compute Nasdaq's actual return volatility in March 2000 as the standard deviation of the daily Nasdaq returns in that month, and obtain 41.49% per year.[9] Since this estimate is noisy, we also compute the average of the monthly volatilities in 2000, all computed from daily returns within the month, and obtain 47.03% per year. Both 41.49% and 47.03% are far above the model-implied volatility values in Panel B. Our model is clearly unable to match Nasdaq's return volatility under the assumption of zero uncertainty.

Next, we recognize that $\overline{\psi}^N$ is unknown. Table 3 is an equivalent of Table 2 under the assumption that the standard deviation of the perceived distribution of $\overline{\psi}^N$ is 3% per year. The M/B ratios in Table 3 are higher than in Table 2, as expected from Proposition 2. For example, with the equity premium of 3% and $\widehat{\psi}^N = 3\%$, the model-implied M/B ratio is

[8]The data are obtained from Compustat. Earnings are computed as income before extraordinary items available for common, plus deferred taxes and investment tax credit. Book equity is stockholders' equity plus deferred taxes and investment tax credit, minus the book value of preferred stock. Throughout the paper, we eliminate the values of market equity and book equity below $1 million, as well as the values of ROE (earnings over book equity) above 1,000% in absolute value.

[9]The annualization is performed by multiplying the daily standard deviation by $\sqrt{252}$.

7.41, whereas the corresponding ratio in Table 2 is only 4.87. The return volatilities are also higher in Table 3: under the same parameters, the model-implied return volatility is 40.37%, compared to 24.52% in Table 2. Acknowledging uncertainty about $\overline{\psi}^N$ leads to values of M/B and volatility that are closer to the values observed in the data.

The differences between the values in Tables 2 and 3 are the biggest for the lowest values of the equity premium. For example, for $\widehat{\psi}^N = 0$ and the equity premium of 1%, M/B in Table 3 is 1.41 times larger than M/B in Table 2, and the return volatility is 1.86 times larger. Under the 8% equity premium, M/B in Table 3 is only 1.08 times larger, and the volatility is only 1.20 times larger. When the equity premium is lower, future cash flow is discounted at a lower rate when valuing a firm. As a result, a bigger fraction of the firm's value comes from earnings in the distant future, which are more affected by uncertainty about $\overline{\psi}^N$ than earnings in the near future, due to compounding. Therefore, uncertainty about $\overline{\psi}^N$ has the biggest effect on prices when the equity premium is low.

Panel A of Table 4 reports implied uncertainty, defined as the uncertainty that equates the model-implied M/B to the observed M/B.[10] Implied uncertainty is listed as zero for all pairs of $\widehat{\psi}^N$ and the equity premium that deliver M/B $\geq$ 8.55 in Panel A. When $\widehat{\psi}^N = 0$ and the equity premium is 3%, matching M/B of 8.55 requires the uncertainty of 5.06% per year. Raising $\widehat{\psi}^N$ to 3% per year, implied uncertainty drops to 3.38%. What values of implied uncertainty are plausible? This question is the subject of the following subsection.

4.1.1. Plausible Values of Implied Uncertainty

To judge the plausibility of a given value of uncertainty, we need to find a measurable quantity that is closely related to uncertainty. One natural candidate is return volatility, which is strongly positively associated with uncertainty in the model. For any value of uncertainty, we can compute the model-implied return volatility (equation (26)). The plausibility of a given value of uncertainty can then be assessed by comparing the corresponding return volatility with the volatility observed in the data. We judge uncertainty to be implausibly high if it produces return volatility that is significantly higher than the observed volatility.

The model-implied return volatilities are reported in Panel B of Table 4. The implied

[10] This calculation is similar in spirit to computing the implied volatility of an option. The idea of backing out the prior uncertainty needed to match the observed evidence is not new. For example, in a mean-variance framework where investors can invest in U.S. as well as non-U.S. stocks, Pástor (2000) computes the prior uncertainty about mispricing that is necessary to explain the observed degree of home bias. Similarly, in a regime-switching model for the drift rate of earnings, David and Veronesi (2002) use options data to back out the implied uncertainty about the future earnings growth of the S&P 500 index.

uncertainty of 3.38%, discussed above, produces return volatility of 46.66% per year. This value is close to Nasdaq's observed volatility computed earlier (about 41.49% to 47.03% per year). This result suggests that the implied uncertainty of 3.38%, obtained in the combination of $\widehat{\psi}^N = 3\%$ and the equity premium of 3%, is plausible.

More generally, Table 4 identifies the pairs of values of the equity premium and $\widehat{\psi}^N$ for which implied uncertainty matches not only Nasdaq's M/B but also its return volatility. One such pair is $\widehat{\psi}^N = 2\%$ and the equity premium of 2%, which leads to implied uncertainty of 3.54%, which then produces return volatility of 47.54%. Another pair, discussed earlier, is $\widehat{\psi}^N = 3\%$ and the equity premium of 3%. Yet another pair is $\widehat{\psi}^N = 4\%$ and the equity premium of 4%, which leads to implied uncertainty of 3.32% and return volatility of 47.81%. These combinations of the equity premium and $\widehat{\psi}^N$ seem plausible, as discussed next.

4.1.2. Plausible Values of the Discount Rate

What are the plausible levels of the equity premium in March 2000? Several recent studies argue that the equity premium was low at that time. According to Cochrane (2002), "The top of the largest economic boom in postwar U.S. history is exactly when you'd expect a risk premium to be low and stock prices to be high." Lettau, Ludvigson, and Wachter (2004) argue that the equity premium declined significantly in the 1990s due to a decline in the volatility of aggregate consumption. Fama and French (2002) estimate the equity premium for 1951-2000 to be 2.6% and 4.3% per year, based on their dividend and earnings growth models, but some of their premium estimates for 1991-2000 are as low as 0.32% per year. Claus and Thomas (2001) use the residual income model and analyst forecasts to estimate the equity premium in 1998 at about 2.5%. Welch (2001) surveys 510 academics in 2001 and reports a median equity premium forecast of 3%. Pástor and Stambaugh (2001) estimate the premium of 4.8% at the end of 1999. Ilmanen (2003) estimates the premium of 2% in March 2000. Based on this evidence, we regard the equity premium values between 1% and 5% per year as the most plausible.

Given its exposure to the SDF, Nasdaq commands a higher risk premium than the old economy. In fact, Nasdaq's expected excess return in our model is more than twice the equity premium. For example, when the equity premium is 3%, Nasdaq's expected excess return equals 6.62% for $\widehat{\psi}^N = 0$, and 6.24% for $\widehat{\psi}^N = 3\%$. Also note that the risk-free rate in Tables 2 through 4 is not specified exogenously. Both the risk-free rate and the equity premium are driven by the same variable, y_t, so by specifying the equity premium, we are implicitly choosing the risk-free rate as well. The implied risk-free rates for Tables 2 through 4 look reasonable. For example, the real rates corresponding to the equity premiums between 1% and 6% are all between 4.79% (for the equity premium of 3%) and 6.32% per year (for the premium of 6%). For comparison, on March 10, 2000, the annual nominal yields on Treasury bonds with maturities between one and 20 years were between 6.21% and 6.55%. With 2% to 3% inflation[11], the real rates in our model slightly exceed the observed real rates, which induces a mild conservative bias in the valuation procedure.

4.1.3. Plausible Values of Expected Profitability

To match Nasdaq's M/B and return volatility in Table 4, we need relatively small positive values of $\widehat{\psi}^N$, such as 2% or 3% per year. The market indeed appears to have expected higher average profitability from Nasdaq firms than from NYSE/Amex firms in March 2000. For example, consider equity analyst forecasts provided by I/B/E/S. For each firm, we compute the average forecast of long-term earnings growth by averaging forecasts across all analysts covering the firm in March 2000. The average of these average forecasts is 15.1% when computed across NYSE/Amex firms, but the same average computed across Nasdaq firms is substantially higher, at 28.8%. When these forecasts are combined with the data on current profitability (9.96% for Nasdaq, 18.62% for NYSE/Amex) and dividend yield (1.35% for Nasdaq, 5.67% for NYSE/Amex), they imply that Nasdaq's profitability should begin exceeding the NYSE/Amex profitability in year 2005. The long-term growth forecasts are generally considered valid for up to five years. Since we do not know of any forecast data with a longer horizon, we project these forecasts further into the future, for illustration. By year 2010, the forecast of Nasdaq's ROE exceeds the forecast of NYSE/Amex's ROE by 3.73%, and in the limit, the ROE difference grows to 5.36%. While this example is only illustrative, it suggests that $\widehat{\psi}^N > 0$ might be reasonable. There is also abundant anecdotal evidence that cash flow expectations for Nasdaq were relatively high at that time.[12]

The assumption $\widehat{\psi}^N > 0$ has some support in historical data as well. Consider two portfolios formed in 1972, one year after Nasdaq was created. The first (second) portfolio includes all Nasdaq (non-Nasdaq) firms with valid book values in Compustat at the end of 1972. We compute the annual profitability (i.e., the sum of earnings divided by the sum of the most recent book values) of these two portfolios going forward, without accounting for

[11]The GDP deflator increased by 2.2% in 2000, and the core inflation rate in 2000 was 2.6%. Both the inflation and interest data are obtained from the website of the Federal Reserve Bank of St. Louis.

[12]E.g., "Applegate: This business cycle is extraordinary... Today tech earnings are growing 24%, which is significantly better than the rest of the market. Their prices are accordingly richer... I'm comfortable right here, sticking with the Ciscos, Microsofts and Intels." David Henry, USA Today, December 16, 1999.

any migration of firms between the exchanges and without including any newly-listed firms. Between 1973 and 1999, the average profitability of the Nasdaq portfolio (14.63%) exceeds the profitability of the non-Nasdaq portfolio (13.28%) by 1.35% per year. In fact, for any portfolio formation year between 1972 and 1982, the average ROE of the Nasdaq portfolio by 1999 exceeds the average ROE of the non-Nasdaq portfolio.

These results may seem surprising, since Nasdaq's ROE has generally been lower than NYSE/Amex's ROE (Figure 6). This apparent contradiction can be explained by a "migration bias" induced by firms that switch exchanges. Since the listing requirements are less strict on Nasdaq than on the other two exchanges, firms that migrate from NYSE/Amex to Nasdaq typically perform poorly (they no longer meet NYSE/Amex's listing requirements), and firms that migrate from Nasdaq to NYSE/Amex typically perform well.[13] Due to the persistence in ROE, this migration reduces the ROE of Nasdaq relative to NYSE/Amex. However, the ROE earned by an investor who buys a portfolio of Nasdaq stocks is obviously unaffected by any post-purchase migration. Therefore, $\widehat{\psi}^N$ should exceed the historical difference between the ROEs on the regularly-rebalanced Nasdaq and NYSE/Amex indexes.

Another reason to be optimistic about Nasdaq's future profitability is the increasing importance of intangible assets in the economy. Intangible assets are often not included in the book value of the firm, but they do contribute to the firm's earnings. As a result, intangible assets increase the earnings-to-book ratio, or ROE. Assuming a continued trend towards a knowledge-based economy, the average ROE should be permanently higher than before, especially for firms with a lot of intangible capital. To the extent that Nasdaq firms have more intangible capital than NYSE/Amex firms, $\widehat{\psi}^N > 0$ seems reasonable.

One example of beliefs that justify Nasdaq's valuation in March 2000 is $\widehat{\psi}^N = 3\%$, the equity premium of 3%, and $\mathrm{E}(T) = 20$ years. To better understand these beliefs, we use equation (2) to compute the distribution of Nasdaq's profitability over the following 20 years. As shown in Panel A of Figure 1, Nasdaq's profitability is expected to improve over the first three years, after which it is expected to decline slowly until time T. Twenty years ahead, in year 2019, the 1st, 50th, and 99th percentiles of the predictive distribution for Nasdaq's ROE (ρ^i_{t+20}) are -3.07%, 14.56%, and 32.20% per year. The same percentiles for the average ROE between 1999 and 2019 are 4.90%, 15.07%, and 25.25%, as shown in Panel B.

[13] We verify the existence of the migration bias empirically. For each firm that switched exchanges between 1973 and 1999, we compute the firm's ROE in the year immediately preceding the year in which the migration took place. We then compute the firm's excess ROE as the firm's ROE minus the aggregate ROE of all firms on the same original exchange (NYSE/Amex or Nasdaq) in the same year. We find that the excess ROE of the firms that migrated from NYSE/Amex to Nasdaq is -42.6% per year, on average. In contrast, the average excess ROE of the firms that migrated from Nasdaq to NYSE/Amex is +4.9% per year.

While an average ROE of 25.25% per year over a 20-year period has not been observed in Nasdaq's short history, such a possibility cannot be dismissed. Certain sectors of the economy have delivered comparable average profitability for much longer periods of time. Suppose that, in 1954, you formed the portfolio of all firms in the pharmaceutical industry that had valid book values in Compustat at the end of 1954. This portfolio earned the average ROE of 25.20% over the 45-year period between 1955 and 1999. A similarly constructed candy-and-soda industry portfolio, formed in 1963, earned the average ROE of 24.34% over the 36-year period between 1964 and 1999. Over the same 36-year period, the tobacco products industry portfolio formed in 1963 earned the average ROE of 22.12%. We do not wish to push this anecdotal evidence too far; after all, it is easier to observe high average ROE at the level of an industry than at the level of an index that includes firms from many different industries. The purpose of our examples is only to illustrate the fact that sustained high profitability is possible, even at the sectoral level.[14]

To provide an additional perspective on the beliefs that justify Nasdaq's M/B in March 2000, we plot in Figure 2 the model-predicted distribution of the future ratio of Nasdaq's book value to the NYSE/Amex/Nasdaq book value. At the end of 1999, the value of this ratio was 0.18. The 1st, 50th, and 99th percentiles for this ratio after 10 years are 0.12, 0.27, and 0.50. The 1st, 5th, 50th, 95th, and 99th percentiles after 20 years are 0.11, 0.17, 0.43, 0.73, and 0.82. These distributions do not seem implausible to us. Note that these are predictions for firms that are trading on Nasdaq vs. NYSE/Amex today (as of March 2000), not for firms that will be trading on these exchanges in the future. Given the migration bias discussed earlier, the future Nasdaq is likely to be smaller than is suggested by Figure 2.

In Table 4, Nasdaq is expected to earn abnormal profits over 20 years after March 2000. Table 5 is an equivalent of Table 4 with $\mathrm{E}(T) = 15$ and 25 years. With $\mathrm{E}(T) = 15$, matching Nasdaq's M/B requires more optimism (i.e., higher $\widehat{\psi}^N$) than with $\mathrm{E}(T) = 20$. For example, with a 3% equity premium, we need $\widehat{\psi}^N = 5\%$ to match Nasdaq's M/B and also obtain return volatility (41.76%) that corresponds to the values estimated from the data. In contrast, matching Nasdaq's M/B is easier with $\mathrm{E}(T) = 25$. With a 3% equity premium, we need $\widehat{\psi}^N$ of only about 2% to match both M/B and return volatility.

To summarize, Nasdaq's M/B on March 10, 2000 seems consistent with reasonable values of the equity premium, expected profitability, and uncertainty about average profitability.

[14] We define industries based on the 48-industry classification scheme from Ken French's website. A portfolio's ROE in a given year is computed as the sum of earnings in that year divided by the sum of book values at the end of the previous year, where both sums are computed across all firms in the original portfolio. That is, the industry portfolios are not rebalanced to include new firms in the industry.

4.2. Matching the Valuations of Individual Firms

In this subsection, we use the model to value 12 high-profile technology firms: Akamai, Amazon, Ciena, Cisco, Dell, Ebay, Immunex, Intel, Microsoft, Priceline, Red Hat, and Yahoo. The parameters for the process governing profitability ρ_t^i in equation (2) are chosen to match the median Nasdaq firm in the data. For each year and each firm, we compute the firm's profitability (ROE) as the ratio of the firm's current-year earnings and its book value of equity at the end of the previous year. For each firm, we construct the longest uninterrupted time series of valid ROEs (i.e., ROEs smaller than 1,000% in absolute value). If this series is at least 10 years long, we estimate an AR(1) model for ROE. The slope coefficients are adjusted for the small-sample bias (e.g., Stambaugh, 1999). The median value of ϕ^i across all Nasdaq firms satisfying $0 < \phi^i < 1$ is 0.3891. The median residual volatility of ROE is 10.46% per year. We decompose this volatility into $\sigma_{i,0} = 6.65\%$ and $\sigma_{i,i} = 8.07\%$ per year, which implies a M/B ratio of 1.7 for a firm with zero uncertainty, zero $\overline{\psi}^i$, and $\rho_t^i = \overline{\rho}_t = \overline{\rho}_L$.

Each firm's M/B ratio is computed by dividing the March 10, 2000 market value of equity by the end-of-1999 book value of equity. Firm profitability, ρ_t^i, is computed as the 1999 earnings divided by the end-of-1998 book equity. If the end-of-1998 book value is not available on Compustat, we replace it by the end-of-1999 book value.[15] The dividend yield, c^i, is computed as the 1999 dividends divided by the end-of-1998 book equity. Only one firm in our set, Intel, paid dividends in 1999.

Table 6 reports the implied uncertainty and the associated return volatility on March 10, 2000 for all 12 firms. Throughout the table, the expected horizon is 15 years, $\mathrm{E}(T) = 15$. If we used the same horizon as for Nasdaq, $\mathrm{E}(T) = 20$, the firms' valuations would be easier to match. However, it seems reasonable to assume a shorter horizon for individual firms than for Nasdaq. Recall that $\mathrm{E}(T)$ reflects the market's expectation of the time when the present value of future abnormal earnings is eliminated by the arrival of competition. For any given firm, competition can arrive in the form of a single firm that develops a superior product. In contrast, competition that wipes out the future abnormal earnings of Nasdaq as a whole is likely to arrive in the form of new technology that the Nasdaq incumbents will fail to implement (e.g., Hobijn and Jovanovic, 2001). Such competition is likely to arrive later for the index as a whole than for any given firm.[16]

[15]This is the case for Akamai, Priceline, and Red Hat. Amazon's book value fluctuates dramatically in 1998 through 2000; in fact, it turns negative in 2000Q2. Since earnings are computed over the course of the whole year 1999, we use book equity halfway through 1999 (i.e., the end-of-1999Q2 value) to compute Amazon's profitability in 1999.

[16]Our choice of a 15-year expected horizon seems fairly conservative. Schwartz and Moon (2000) use a

To simplify the description of our results, we assume the equity premium of 3% per year. However, Table 6 reports the results for all values of the equity premium between 1% and 6%. Naturally, the lower the equity premium, the easier it is for our model to match the observed M/B ratios. Importantly, our results are not overly sensitive to the equity premium, and the discussion below would be very similar under the equity premium of 2% or 4%.

First, we consider some of the biggest technology firms: Microsoft (market capitalization \$516bn on March 10, 2000), Cisco (\$456bn), Intel (\$395bn), and Dell (\$130bn). The M/B ratios of these firms were high on March 10, 2000: 18.79 for Microsoft, 39.02 for Cisco, 11.09 for Intel, and 24.47 for Dell. All of these M/B ratios can be matched with reasonable levels of uncertainty about $\overline{\psi}^i$. As before, we judge the plausibility of a given value of implied uncertainty by comparing the model-implied return volatility with the observed volatility.

Consider $\widehat{\psi}^i = 0$. The implied uncertainty for Microsoft is 3.84%, which implies return volatility of 59.44%. This value is close to Microsoft's actual volatility, which is estimated to be (57.44%, 56.10%), where the first value is based on the March 2000 returns and the second value is based on all returns in 2000, as before. Intel's implied uncertainty is 4.86%, which yields return volatility of 69.90%, which is close to Intel's actual volatility of (45.81%, 68.71%). Dell's implied uncertainty is 3.85%, which leads to return volatility of 59.71%, which is close to Dell's actual volatility of (51.75%, 69.50%). In other words, under the assumption that the average future profitabilities of Microsoft, Intel, and Dell are equal to the profitability of the old economy, the implied uncertainty in our model matches not only the firms' observed M/B ratios but approximately also their return volatilities.

Among our four biggest firms, only Cisco has an M/B and volatility that cannot be matched with $\widehat{\psi}^i = 0$. However, Cisco's implied uncertainty under $\widehat{\psi}^i = 4\%$ is 3.33%, which implies return volatility (59.90%) that is close to Cisco's estimated volatility of (51.75%,69.50%). Assuming that Cisco can deliver average profitability of 4% in excess of the old economy's profitability over the expected horizon of 15 years strikes us as plausible.[17]

The assumption of $\widehat{\psi}^i = 4\%$ can also rationalize Yahoo's M/B of 78.41. Yahoo's implied uncertainty of 4.40% implies return volatility of 81.78%, which is close to Yahoo's observed volatility of (75.41%, 90.61%). Compared to Cisco, Yahoo has a higher implied uncertainty, which seems reasonable because its M/B and return volatility are both higher than Cisco's.

25-year horizon when valuing Amazon. Ofek and Richardson (2002) consider horizons of 10 to 30 years.

[17]For comparison, the average of all of Cisco's valid ROEs at the time (1992 to 1999) was 47.13%, significantly higher than the old economy's average ROE of 13.03% over the same period.

Next, consider the M/B ratios of Ebay (27.87), Red Hat (26.50), and Immunex (105.70). One might expect that matching these high M/Bs requires large values of $\widehat{\psi}^i$, but that is not the case; in fact, the value that works best for all three firms is $\widehat{\psi}^i = -2\%$. This surprising finding is due to the fact that all three firms have highly volatile returns. Under $\widehat{\psi}^i = -2\%$, the firms' implied uncertainties are 5.83%, 6.35%, and 6.04%, respectively, and the model-implied return volatilities are close to the observed volatilities of (129.24%, 113.64%) for Ebay, (121.00%, 122.33%) for Red Hat, and (155.94%, 117.71%) for Immunex. For these firms' stock returns to be so volatile, uncertainty about the firms' growth rates must be so large that the expected growth rates can be below the growth rate of the old economy.

The firms whose M/Bs are the most difficult to match are Amazon (88.07) and Priceline (39.58). This difficulty stems from the firms' extremely poor profitability: Amazon's 1999 ROE is -126%, and Priceline's ROE is -264%. Investors holding Amazon and Priceline must have expected these firms to become highly profitable in the future. If investors expected $\widehat{\psi}^i = 4\%$, which works well for Yahoo and Cisco, the implied uncertainties for Amazon and Priceline would be implausibly large: they would imply return volatilities of 147.12% for Amazon and 191.66% for Priceline, but the actual return volatilities are smaller: (71.67%, 103.33%) for Amazon, and (128.17%, 133.65%) for Priceline. Matching the observed volatilities (as well as M/B) requires $\widehat{\psi}^i$ of about 10% for both Amazon and Priceline.

If $\widehat{\psi}^i = 10\%$ seems large, note that in the absence of uncertainty about $\overline{\psi}^i$, justifying Amazon's M/B ratio would require $\overline{\psi}^i > 16\%$, and justifying Priceline's M/B would require $\overline{\psi}^i > 20\%$. Moreover, the implied return volatilities for both firms would be counterfactually low. Uncertainty about $\overline{\psi}^i$ helps us understand these firms' high valuations and volatilities.

To assess the plausibility of the beliefs that justify Amazon's valuation in March 2000 ($\widehat{\psi}^i = 10\%$ and 3% equity premium), we use equation (2) to compute the distribution of Amazon's future profitability, $\rho^i_{t+\tau}$, over the next 15 years. As shown in Panel A of Figure 3, Amazon's profitability is expected to improve sharply, but it is expected to remain negative for about four years. Median profitability turns positive in 2004, and it reaches 20.30% in 2014.[18] Five years ahead (in 2004), the 1st and 99th percentiles of the predictive distribution for Amazon's ROE are -27.34% and 32.44%. In 2014, the same percentiles are -10.11% and 50.71%. These quantities are well within the range of the ROEs observed in the data.

Panel B of Figure 3 plots the model-predicted distribution of Amazon's future book value as a fraction of the 1999 book value. Due to the large current losses, the median forecast of

[18]In reality, Amazon's earnings turned positive in 2003Q3. Amazon's first profitable year was 2003. Its stock price at the time of this writing (April 2004) is about the same as its stock price in April 2000.

Amazon's book value is below its 1999 value even after 15 years. But M/B depends on the expectation of the future book value, not its median. This expectation exceeds the median, as shown in Figure 3, because the distribution of the future book value is right-skewed. For example, in 2014, the 10th and 90th percentiles of the distribution of B^i_{t+15}/B^i_t are 0.16 and 2.76, and the 1st and 99th percentiles are 0.05 and 8.75. The logic behind Amazon's high M/B ratio in March 2000 then seems clear. High uncertainty about Amazon's future growth rate leads to a right-skewed distribution of Amazon's future book value, which in turn leads to a high expected book value, which then gives Amazon a high M/B ratio.

Figure 4 illustrates the same logic on the example of Yahoo. The distribution of Yahoo's ROE in 2014 has a median of 15.66%, with the 1st and 99th percentiles of -13.44% and 44.77%. Yahoo's book value is expected to grow more quickly than Amazon's, because Yahoo's 1999 ROE (10.52%) is higher than Amazon's. In 2014, the 10th and 90th percentiles of the distribution of Yahoo's B^i_{t+15}/B^i_t are 2.73 and 45.79, and the 1st and 99th percentiles are 0.87 and 144.46. As a result of this large skewness, Yahoo's expected book value in 2014 is more than 20 times its 1999 book value, which implies a large current M/B ratio.

Is this growth rate in Yahoo's book value realistic? Recall that book value grows at the rate equal to profitability, assuming no dividends and no issues or withdrawals of equity, so the average growth rate of book equals average profitability. Yahoo's average profitability over the next 15 years (2000-2014) has a distribution whose 1st, 50th, and 99th percentiles are -0.97%, 16.11%, and 33.18%. This distribution seems plausible. While the 99th percentile of 33.18% is large, it is far from unprecedented. Consider all 2,969 firms whose longest continuous series of valid annual ROEs between 1950 and 2002 are at least 15 years long. In this universe, there are 35 firms (1.2% of the total) whose average ROE over the previous 15 or more years exceeds 33.18%.[19] Microsoft's and Oracle's average annual ROEs between 1988 (first year available) and 1999 are 44.46% and 47.19%, respectively.

Given our assumptions, we can infer the probability that the market assigned at the end of 1999 to the event that "Yahoo will become the next Microsoft." Specifically, we can compute the probability that Yahoo's average ROE over the 12 years after 1999 will exceed Microsoft's average ROE of 44.46% over the previous 12 years. This probability is 0.0066, or one in 152, which does not strike us as implausibly large. Yahoo's valuation in March 2000

[19]These numbers should be viewed as merely illustrative, since they are subject to an obvious survival bias. Also, the cross-sectional standard deviation of average ROE in this universe of firms is 11.28%, which exceeds all values of implied uncertainty in Table 6. Since the cross-sectional dispersion can serve as a standard deviation of economically noninformative prior beliefs about average ROE, it should exceed the market's uncertainty about any given firm's average ROE. It is comforting to see that it does.

seems consistent with plausibly high uncertainty about average excess profitability.

We make some strong simplifying assumptions in this section. For example, we assume that all firms face an expected horizon of 15 years over which abnormal profits can be earned, but firms operating in industries with different barriers to entry are likely to face different horizons. We also assume that the parameters governing the mean reversion and volatility of the firm's ROE (ϕ^i, $\sigma_{i,0}$, and $\sigma_{i,i}$) are equal across firms, but these parameters can vary across firms and industries. All these assumptions are made for simplicity, and they can be relaxed if this model is to be applied in practice.

5. The Time Series of Implied Uncertainty

In Section 4., we use our model to match the M/B ratios of Nasdaq firms on March 10, 2000. In this section, we match the whole time series of the aggregate M/B ratios in the new and old economy. The time series of both M/B ratios are plotted in Figure 5. The new economy's M/B is the ratio of the sums of the market values and the most recent book values of equity across all Nasdaq firms. The old economy's M/B is computed analogously for the NYSE/Amex firms. The M/B of the new economy rises and falls dramatically around year 2000, but the M/B of the old economy exhibits a substantially less pronounced pattern. Based on this figure, we concur with Cochrane (2002), who observes that "if there was a 'bubble,' it was concentrated in Nasdaq stocks."

The purpose of this section is to analyze the time series of the implied uncertainty about Nasdaq's average excess profitability. This time series is obtained in two steps. First, we extract the time series of the equity premium from the observed M/B and profitability of the old economy. Second, we compute the uncertainty that equates the observed M/B of the new economy to its model-implied value, given the new economy's observed profitability and the equity premium computed in the first step.

In the first step, we compute the time series of y_t, which is the key determinant of the equity premium, by inverting the pricing formula in equation (16). In this formula, the M/B ratio of the old economy is a function of y_t and $\overline{\rho}_t$, so y_t can be computed conditional on M/B and $\overline{\rho}_t$. The time series of $\overline{\rho}_t$, whose construction is described in Section 3., is plotted in Figure 6, along with the realized profitability of the Nasdaq index. The old economy's ROE was relatively high in the 1990s, around 15% per year, but it fell to about 10% after year 2000. The new economy's ROE experienced a substantially larger fall, from about 10%

in the 1990s to about -20% per year in 2001.[20] This dramatic fall in profitability must have contributed to the declines in the Nasdaq index in 2000 and 2001.

Figure 7 plots the time series of the implied equity premium. The premium increases from about 5% to about 7.5% per year in the mid-1970s, and then it gradually declines into the late 1990s: to about 5% at the end of 1994, 4% at the end of 1996, and 1% at the end of 1998. The premium then rises to 1.8% at the end of 1999, and 3.2% at the end of 2002.

In the second step, we rely on Proposition 2 to relate Nasdaq's M/B to uncertainty about $\overline{\psi}^N$, as well as to y_t, the old-economy profitability $\overline{\rho}_t$, the new-economy profitability ρ_t^N, and the new-economy expected excess profitability $\widehat{\psi}^N$. For a given value of $\widehat{\psi}^N$, we invert the formula (15) for every t to obtain the time series of implied uncertainty $\{\widehat{\sigma}_{N,t}\}$.

The outcome of this procedure must be interpreted with caution. Given the deterministic process for $\overline{\psi}^N$ in equation (5), $\widehat{\sigma}_{N,t}$ in our model can only go down as more information becomes available, but $\widehat{\sigma}_{N,t}$ inferred from the observed prices may well increase over time. This dynamic inconsistency has an analogy in option valuation, where it is customary to invert the Black-Scholes pricing formula at various points in time to obtain the time series of implied volatility. This time series is generally considered informative about time-varying volatility, even though it is inconsistent with the constant volatility assumption of the Black-Scholes model. The Black-Scholes model can be extended to remove this inconsistency, but only at the cost of added complexity. Similarly, our model can be extended to allow for increases in $\widehat{\sigma}_{N,t}$ by adding random shocks to $\overline{\psi}^N$, and by allowing investors to learn about $\overline{\psi}^N$ from signals with time-varying precision. We have examined such a (significantly more complicated) extension, and found that random fluctuation in $\overline{\psi}^N$ increases prices, for the same reason that uncertainty about $\overline{\psi}^N$ increases prices. Therefore, our simplifying assumption of a deterministic $\overline{\psi}^N$ can be viewed as conservative in that it makes it more difficult for our model to match the observed prices.

Panel A of Figure 8 plots the time series of implied uncertainty, $\widehat{\sigma}_{N,t}$, computed under three different assumptions about $\widehat{\psi}^N$. For $\widehat{\psi}^N = 2\%$, implied uncertainty is zero until 1980, it then rises to about 3% per year in the early 1980s, and then it falls back to zero. The uncertainty rises in the second half of the 1990s, to about 3% at the ends of 1999 and 2000, and then to about 4% at the end of 2001, before falling in 2002. This pattern underlines the message of this paper. We argue that the runup in technology stock prices in the late 1990s was partly due to an increase in uncertainty about average profitability.

[20]Several large Nasdaq firms reported unprecedented losses in 2001. For example, JDS Uniphase lost over $50bn, mostly as a result of write-offs from bad acquisitions.

We also examine the time variation in return volatility implied by this variation in uncertainty. In the model, the new economy's volatility is positively related to uncertainty, but the old economy's volatility is not. Therefore, increases in uncertainty should increase the difference between the model-implied return volatilities in the new and old economies. We compute the year-end time series of this difference, and plot it in Panel B of Figure 8. The difference is high in the early 1980s, but it is especially high in the late 1990s: it ranges from about 20% to about 50% between 1999 and 2001, across three different values of $\widehat{\psi}^N$.

Interestingly, while Nasdaq's implied uncertainty in Panel A of Figure 8 is only slightly higher in the late 1990s than in the early 1980s, the difference between the return volatilities in Panel B is substantially higher in the late 1990s. The reason is that the equity premium in the late 1990s is substantially lower than in the early 1980s (Figure 7). Uncertainty has the biggest effect on prices when the equity premium is low, as argued earlier.

How does the model-implied pattern in Panel B of Figure 8 compare with the data? Panel A of Figure 9 plots the time series of the realized return volatilities of the Nasdaq and NYSE/Amex indexes. Each month, return volatility is computed as the standard deviation of the daily index returns in that month. To plot smoother annual series, we average the monthly values within each year. The Nasdaq volatility increases gradually from 12% per year in 1994 to its peak of 47% in 2000, before it declines to 34% in 2002. The NYSE/Amex volatility increases from 9% per year in 1994 to 18% in 2000, and to 21% in 2002.

Panel B of Figure 9 plots the difference between the Nasdaq and NYSE/Amex return volatilities. This difference rises from 3% per year in 1994 to almost 30% in 2000, after which it falls to 13% in 2002. This pattern is similar to the model-implied pattern in the 1990s in Panel B of Figure 8. This empirical evidence is consistent with an increase in uncertainty about $\overline{\psi}_N$ in the late 1990s, followed by a decline in 2002.

Note that the patterns in Panels B of Figures 7 and 8 are similar mainly in the 1990s. Before 1990, Nasdaq volatility is smaller than the NYSE/Amex volatility, although it should always be bigger according to the model. Moreover, the difference in the observed volatilities before 1990 is almost flat, whereas the model predicts a mild increase in the early 1980s. One likely reason behind these discrepancies is that our designation of Nasdaq as the new economy is less appropriate before 1990. Some of the best known technology firms before the 1990s, such as IBM, have always traded on the NYSE rather than on Nasdaq.

In addition to return volatility, we also analyze cash flow volatility. Uncertainty about average future ROE is likely to be high when the cross-sectional variance of ROE is high. By variance decomposition, this variance is the sum of the cross-sectional variance of expected ROE and the cross-sectional expectation of the variance of ROE, both of which make the average future ROE less certain. We compute the cross-sectional standard deviation of ROE for Nasdaq stocks, as well as for NYSE/Amex stocks. Figure 10 shows that the dispersion in the Nasdaq ROEs increases dramatically relative to the dispersion in the NYSE/Amex ROEs in the late 1990s. The difference between the two dispersions is unusually high between 1995 and 2000, after which it declines sharply. This pattern is consistent with an increase in uncertainty in the late 1990s, followed by a decline.

To summarize, we find that the volatilities of Nasdaq returns and profits increased sharply at the end of the past decade, both in absolute terms and relative to NYSE/Amex. This evidence supports our premise that the uncertainty about the average profitability of Nasdaq firms was unusually high in the late 1990s. The effect of this uncertainty on stock prices was further amplified by a relatively low equity premium at that time.

6. The Cross Section of Implied Uncertainty

We argue that Nasdaq valuations in the late 1990s were high partly due to high uncertainty about average profitability. Under this argument, stocks with the highest uncertainty should have not only some of the highest M/B ratios, but also some of the highest return volatilities. Anecdotal evidence consistent with this argument is provided in Section 4. In this section, we examine the whole cross-section of Nasdaq firms, and we document a strong positive relation between implied uncertainty (computed from M/B) and return volatility.

We compute implied uncertainty on March 10, 2000 for all 2,691 Nasdaq firms with valid M/B and ROE data at the end of 1999. For each firm, we choose $\widehat{\psi}^i = 0$, $\mathrm{E}(T) = 15$ years, and $c = 1.35\%$ (Nasdaq's dividend yield in 1999), for simplicity. The variables y_t and $\overline{\rho}_t$ are the end-of-1999 values computed in Section 5. We find substantial differences in implied uncertainty across firms. For 66.6% of firms, implied uncertainty is zero (i.e., the model-implied M/B matches or exceeds the actual M/B under zero uncertainty). The 90th percentile of the cross-sectional distribution of implied uncertainty is 5.79%, and the 99th percentile is 9.03% per year.[21] The highest implied uncertainty is observed in the Internet, biotechnology, and telecommunications sectors.

According to the model, stocks with high implied uncertainty should have highly volatile

[21] In the subsequent analysis, we winsorize the top 1% of observations of implied uncertainty, i.e, we set their values equal to the 99th percentile of the cross-sectional distribution of implied uncertainty.

returns. Specifically, equation (26) predicts a linear positive relation between squared implied uncertainty and idiosyncratic return volatility. We compute idiosyncratic volatility for a given stock in a given month as the residual volatility from the regression of the stock's daily returns within the month on the contemporaneous and lagged market returns.[22] Idiosyncratic volatility in a given year is computed as the average of the 12 monthly volatilities.

The model's prediction is strongly supported by the data. The cross-sectional correlation between squared implied uncertainty on March 10, 2000 and idiosyncratic return volatility in 2000 is 53%. When the year-2000 values of idiosyncratic volatility are replaced by the noisier March 2000 values, the correlation remains high, at 38%. To assess the significance of the correlation, we regress squared implied uncertainty on March 10, 2000 on idiosyncratic volatility in 2000. Since many observations of implied uncertainty are censored at zero, we estimate a censored regression model, by using the maximum likelihood procedure. The estimated slope coefficient implies that a 10% per year difference in return volatility translates into the difference of 0.00056 in squared implied uncertainty, which is the difference between the implied uncertainties of zero and 2.37% per year, or between 5% and 5.53%. The asymptotic t-statistic for the slope coefficient is 19.69, which indicates a highly significant relation, assuming that the residuals are cross-sectionally independent.

To provide additional evidence on the relation between implied uncertainty and return volatility, we compute implied uncertainty and idiosyncratic volatility for all Nasdaq firms at the end of each year between 1973 and 2002, and we run the same censored cross-sectional regression at each year-end. The slope coefficient is positive in every single year, and the t-statistic computed from the time series of the 30 estimated coefficients is equal to 4.79.[23]

Figure 11 plots the year-end time series of the estimated cross-sectional correlation between squared implied uncertainty and idiosyncratic return volatility. The correlation computed on March 10, 2000 is also shown. The correlation varies from the low of 4% at the end of 1990 to the high of 53% on March 10, 2000, and the time-series average of the year-end values is 25%. High correlations are observed not only in the late 1990s, but also in the early 1980s. The early 1980s witnessed a high-technology boom that has been characterized as a "biotech revolution" (Malkiel, 1999), so it seems plausible for uncertainty to play a role in that period. Note that Panel A of Figure 8 indicates high implied uncertainty in the early

[22]The regression is run only if at least 10 valid returns are available in the given month. Market returns lagged by one and two days are included in the regression to mitigate potential concerns about nonsynchronous trading. Note that the exact definition of idiosyncratic volatility does not appear crucial because all results in this section are highly significant also when residual variance is replaced by total variance.

[23]See Fama and MacBeth (1973). The t-statistic is adjusted for any significant serial correlation in the time series of the estimated coefficients. The t-statistic that assumes serial independence is equal to 8.14.

1980s, without using any information about return volatility.

The above discussion focuses on the univariate relation between implied uncertainty and return volatility, but this relation survives controls for various firm characteristics, such as market capitalization and the dividend yield. PV (2003) report a significant positive cross-sectional relation between M/B (the key determinant of implied uncertainty) and idiosyncratic return volatility, after controlling for a larger set of firm characteristics. To summarize, we find that implied uncertainty is positively cross-sectionally related to idiosyncratic return volatility, as predicted by the model.

7. Discussion

The first part of this section discusses the effect of uncertainty about $\overline{\psi}$ on the discount rate. The second part discusses several issues – the firm's investment and dividend policies, the dynamics of learning, the convergence of market value to book value, uncertainty about T, and employee stock options – that help us understand the extent to which our valuation procedure can be viewed as conservative.

7.1. The effect of uncertainty on the discount rate

Before we discuss the effect of uncertainty about average profitability on the discount rate, we dispel two myths that we have repeatedly encountered when presenting this paper. The first myth is that uncertainty about average profitability always increases the risk premium. In a general equilibrium model in which dividends equal consumption, uncertainty about average dividend/consumption growth can increase or decrease the risk premium, depending on the representative agent's elasticity of intertemporal substitution (Veronesi, 2000). In the presence of uncertainty, observing realized dividends leads investors to revise their expectations of future consumption growth, which leads to intertemporal consumption smoothing, which pushes prices in the direction opposite to that obtained in the absence of uncertainty. In a power utility setting, Veronesi shows that more uncertainty translates into a lower risk premium if the elasticity of intertemporal substitution is less than one. A more general version of the same result is presented in Appendix C.2.

The second myth is that uncertainty about average consumption growth affects the volatility of the SDF in a consumption-based pricing model. Suppose that consumption growth follows an i.i.d. Brownian motion:

$$dc = gdt + \sigma_c dW.$$

If the drift g is unknown, the consumption process can be rewritten as

$$dc = \hat{g}_t dt + \sigma_c d\hat{W},$$

where $\hat{g}_t$ is the current perception of g and $d\hat{W}$ is a Brownian motion. Since the local variance σ_c is the same whether or not g is known, uncertainty about g has no effect on the SDF volatility in the standard power utility framework (see also Appendix C.2). The same result can be obtained in the habit utility framework of Campbell and Cochrane (1999).

Although our SDF in equation (8) is independent of $\overline{\psi}$, it turns out that uncertainty about $\overline{\psi}$ increases the risk premium for any reasonable parameter values. For example, under the beliefs used in Figure 1 ($\hat{\psi}^N = 3\%$ and the equity premium of 3%), Nasdaq's risk premium increases from 5.36% under zero uncertainty to 6.62% under 4% uncertainty.

One might conjecture that uncertainty would have a stronger effect on the risk premium if the SDF were allowed to depend on $\overline{\psi}^N$. To evaluate this conjecture, we consider an extension of our model in which the drift of ϵ_t in equation (11) depends on the central tendency of Nasdaq's profitability process, $\overline{\rho}_t + \overline{\psi}_t^N$:

$$d\varepsilon_t = \left(\alpha_0 + \alpha_1(\overline{\rho}_t + \overline{\psi}_t^N)\right) dt + \sigma_\varepsilon dW_{0,t}. \tag{17}$$

In our model, $\alpha_1 = 0$ in equation (17), but $\alpha_1 > 0$ also seems plausible. In the consumption-based interpretation of equation (11), ϵ_t is log aggregate consumption, and it seems plausible for expected consumption growth to be positively related to Nasdaq's expected profitability.[24] An especially reasonable value of α_1 is the fraction of total consumption that is financed by Nasdaq dividends.[25] In 1999, this fraction was $\alpha_1 = 0.00138$, i.e., Nasdaq dividends accounted for less than one seventh of one percent of total consumption.[26]

[24] To preserve the habit utility interpretation of the SDF, we also need to modify the process for y_t in equation (10), so that $\bar{y}$ is allowed to depend on $\overline{\rho}_t + \overline{\psi}_t^N$. In the habit framework, expected consumption growth feeds back into the surplus consumption ratio, which is driven by y_t (equation (9)). Therefore, in the extended model, we set $\bar{y} = \bar{y}_0 + \bar{y}_1(\overline{\rho}_t + \overline{\psi}_t^N)$. A more detailed explanation is available upon request.

[25] To see this, decompose total consumption into the component financed by Nasdaq dividends and the component financed by other sources: $C^T = D^N + C^O$. The growth rate of C^T is $dC^T/C^T = (D^N/C^T)(dD^N/D^N) + dC^O/C^T$. Since $D^N = c^N B^N$, $dD^N/D^N = dB^N/B^N = (\rho_t^N - c^N)dt$. Therefore, $dC^T/C^T = (D^N/C^T)(\rho_t^N - c^N)dt + dC^O/C^T$, so that expected consumption growth is equal to (D^N/C^T) times Nasdaq's expected profitability plus terms unrelated to ρ_t^N. Hence, $\alpha_1 = D^N/C^T$ seems reasonable.

[26] The dividends paid out by all NYSE/Amex/Nasdaq firms accounted for 2.85% of consumption in 1999.

To assess the effect of uncertainty on the risk premium, consider the difference between the Nasdaq risk premium obtained under 3% uncertainty and the premium obtained under zero uncertainty (using $\hat{\psi}^N = 3\%$ and the equity premium of 3%, as before). This difference is 0.67% per year for $\alpha_1 = 0$. For $\alpha_1 = 0.00138$, the difference grows, but only to 0.73%. A 0.06% per year difference in the risk premium has a small effect on M/B, since it is tiny compared to Nasdaq's expected total real return, which exceeds 11%. We also consider larger values of α_1, because Nasdaq dividends could potentially become larger relative to total consumption in the future. For $\alpha_1 = 5 \times 0.00138$, the risk premium difference grows to 0.84%. For $\alpha_1 = 10 \times 0.00138$, the difference declines to 0.83%, and for $\alpha_1 = 20 \times 0.00138$, the difference declines further to 0.76%. That is, as α_1 increases, the positive effect of uncertainty on the risk premium becomes slightly stronger at first, but then it weakens for larger values of α_1, due to intertemporal consumption smoothing discussed earlier in this section. To summarize, the effect of uncertainty on the risk premium does not change much when we allow Nasdaq's expected profitability to enter the SDF.

7.2. Is Our Valuation Procedure Conservative?

The firm's investment policy. The profitability process in equation (2) is a reduced-form model for the firm's investment policy. We assume that the firm makes optimal investment decisions and that the resulting profitability process is mean-reverting, consistent with empirical evidence (e.g., Beaver, 1970). For tractability, we also assume that the firm cannot shut down its operations, even if its $\overline{\psi}$ turns out to be low. This assumption makes our approach conservative, because the shut-down option would make the firm more valuable.

The firm's dividend policy. The assumption that dividends are a constant fraction of book value also makes our approach conservative. PV (2003) explain that if the firm issues more equity when expected profits are high and pays higher dividends when expected profits are low, then the firm's market value becomes even more convex in $\overline{\psi}$. As a result, uncertainty about $\overline{\psi}$ has an even bigger positive effect on firm value, and the observed valuations can be matched with more conservative beliefs about future profitability.

The dynamics of learning. In our model, investors learn about average excess profitability ($\overline{\psi}$) by observing realized profits. Given the deterministic process for $\overline{\psi}$ in equation (5), Bayesian updating implies that uncertainty about $\overline{\psi}$ declines deterministically over time. (See PV (2003) for a detailed discussion of learning in a closely related model.) As explained in Section 5., extending our model to allow for increases in uncertainty would make it easier to match the observed prices in Section 4., which makes our approach conservative.

The convergence of market value to book value. We assume that a firm's M/B ratio equals one when competition arrives and wipes out the firm's future abnormal profits. This assumption might be too conservative. The absence of intangible assets from the accounting books implies that M/B is likely to exceed one even after profits are competed away, and the convention of conservative accounting (that profits are booked when earned but losses when anticipated) has the same implication. In practice, therefore, it might be reasonable to assume M/B>1 when competition arrives. Of course, such an assumption makes it easier for our model to match the observed Nasdaq prices.

To illustrate this point, suppose that the assumption of M/B $\rightarrow 1$ is replaced by M/B $\rightarrow$ 1.77, which is the average M/B ratio of the old economy (Table 1). Also assume a 3% equity premium. In the equivalent of Table 4, Nasdaq's M/B and volatility can be matched with $\widehat{\psi}^N = 1\%$ instead of $\widehat{\psi}^N = 3\%$. (With $\widehat{\psi}^N = 1\%$, the uncertainty that matches Nasdaq's M/B of 8.55 is 3.67%, which implies return volatility of 46.50%, which is close to the observed volatility.) It seems difficult to argue that $\widehat{\psi}^N = 1\%$ is an overly optimistic belief about Nasdaq's future profitability. As in Figure 2, we compute the model-predicted distribution of the ratio of Nasdaq's book value to the NYSE/Amex/Nasdaq book value after 20 years. The 5th, 50th, and 95th percentiles of this distribution are 0.13, 0.36, and 0.68; this distribution seems even more plausible than the distribution obtained under M/B $\rightarrow 1$. In short, our assumption of M/B $\rightarrow 1$ is conservative, and a realistic modification of this assumption makes the observed Nasdaq valuations easier to match.

Uncertainty about T. Firm i can earn abnormal profits until competition arrives at time T_i (or T, for short). We assume that competition can arrive in any instant with probability p, so that T is exponentially distributed, the expected value of T is $\mathrm{E}(T) = 1/p$, and the median is $-\log(0.5)\mathrm{E}(T) \approx 0.69\mathrm{E}(T)$. For $p = 1/20$ (used for Nasdaq), the median value of T is 13.9 years, and for $p = 1/15$ (used for individual firms), the median T is 10.4 years.

Uncertainty about T increases the firm's M/B ratio. Just like uncertainty about $\overline{\psi}$ increases M/B because M/B is convex in $\overline{\psi}$, uncertainty about T increases M/B because M/B is convex in T. The reason in both cases is compounding; loosely speaking, $\exp(\overline{\psi}T)$ is convex in both $\overline{\psi}$ and T. To illustrate the effect of uncertainty about T on M/B, we reconstruct Table 4 under the assumption that $T = \mathrm{E}(T) = 20$ years with certainty. With 3% equity premium, we need $\widehat{\psi}^N$ of about 5.5% (along with uncertainty of about 5.3%) to match Nasdaq's M/B as well as volatility, as opposed to $\widehat{\psi}^N = 3\%$ when T is uncertain. Of course, T is unknown in practice. Acknowledging uncertainty about T makes it easier for us to match the observed Nasdaq valuations.[27]

Our analysis focuses on the implied distribution of $\overline{\psi}$, holding the distribution of T constant. In principle, we could also reverse the focus and compute the implied distribution of T, holding the distribution of $\overline{\psi}$ constant. To implement such an exercise, it would be useful to replace the single-parameter exponential distribution for T by a two-parameter distribution, so that the mean and variance of T can be governed by different parameters. In the interest of tractability, we leave such an exercise for future research.

Employee stock options. The reported earnings extracted from Compustat are not adjusted for employee stock option expense. If stock options were expensed, the reported earnings of the S&P 500 firms would be reduced by about 8% in 1999, and by almost 10% in 2000 (The Wall Street Journal, July 16, 2002). Botosan and Plumlee (2001) report median reductions between 9.8% and 14% in 1996-1999 based on the sample of 100 fastest-growing firms in the U.S., as identified by Fortune magazine in September 1999. As a simple though imperfect robustness check, we repeat the analysis in this section with all Nasdaq earnings reduced by a seventh, and we find that the observed valuations and volatilities can still be matched with reasonable parameter values. For example, to match Nasdaq's M/B with $\widehat{\psi}^N = 3\%$ and a 3% equity premium, the implied uncertainty increases from 3.38% to 3.47%, which implies return volatility of 48.33%, which is still close to the volatility observed in the data.

Reducing current earnings is an accounting adjustment for employee stock options. This adjustment reduces future book value, but the actual reduction in book value is more complicated, because call options tend to be exercised after good stock performance. The optimal exercise is likely to reduce the skewness of the distribution of the future book equity per share, and this reduction could potentially be larger than the reduction obtained under the accounting approach. Alas, quantifying the effect of the optimal option exercise on future book value seems too difficult to be attempted here. Our accounting adjustment provides only an approximation to the effect of employee options on firm value.

8. Conclusions

Some academics and practitioners hold it to be self-evident that Nasdaq stocks were overvalued in the late 1990s. We argue that the Nasdaq valuations were not necessarily irrational ex

[27]Uncertainty about T should not affect the SDF. The arrival of competition leads to a transfer of wealth between firms, but it should not affect aggregate consumption, because the representative investor owns not only the incumbent firm but also its competitor. Also, consumption does not exhibit jumps in the data.

ante because uncertainty about average future profitability, which increases the fundamental value, was unusually high in the late 1990s. We calibrate a stock valuation model that explicitly incorporates such uncertainty, and show that the Nasdaq valuations observed at the peak of the "bubble" can be rationalized by high but plausible levels of this uncertainty. The high uncertainty seems plausible because it matches not only the high level but also the high volatility of Nasdaq stock prices at that time. Stocks with the highest M/B ratios in the late 1990s also had some of the highest return volatilities, which is consistent with our premise that these stocks had the most uncertain average future growth rates.

The Nasdaq "bubble" was accompanied not only by high return volatility, but also by a high volume of trading. The trading volume for Internet stocks, for example, was three times higher on average than for other stocks (Ofek and Richardson, 2003). The high trading volume is broadly consistent with high uncertainty about the average profitability of technology firms. There is no trading in our single-agent model, but consider an extension in which some agents observe different signals about average future profitability, and some agents trade for liquidity reasons. In this extension, agents will trade because different signals imply different perceptions of the fundamental value. Moreover, the amount of trading is likely to increase with uncertainty about average profitability. When uncertainty is high, signals are drawn from a wider distribution, which implies perceptions of value that are more disperse across agents, and hence more trading. This model can be explored in future work.

Why did the "bubble" burst? Nasdaq's expected excess profitability, $\hat{\psi}^N$, must have been revised downward when Nasdaq's profitability plummetted in 2000 and 2001. This revision is likely to have been substantial, given the high uncertainty at that time. For example, consider the parameters $\hat{\psi}^N = 3\%$, the equity premium of 3%, and the uncertainty of 3.38%, which match Nasdaq's M/B and volatility in Table 4. Under these parameters, the observed decline in Nasdaq's profitability from 9% in 1999 to -3% in 2000 implies a revision in the value of $\hat{\psi}^N$ from 3% at the end of 1999 to 0.9% at the end of 2000. Given this revision, the model implies that Nasdaq's M/B ratio of 6.9 at the end of 1999 should fall to 3.4 at the end of 2000, holding other variables equal. In reality, Nasdaq's M/B ratio fell to 3.5 (see Figure 5), not far from the model's prediction. While this back-of-the-envelope calculation is only illustrative, it suggests that the downward revision in Nasdaq's expected profits might account for most of the dramatic decline in Nasdaq's M/B ratio in 2000.

The effect of uncertainty on stock prices is especially strong when the equity premium is low. For example, our analysis suggests that uncertainty was high not only in the late 1990s but also during the biotech boom in the early 1980s. The M/B ratios were higher in the 1990s because the equity premium declined between the early 1980s and the late 1990s, according to our model. A decline in the equity premium boosts prices in two ways: by reducing the discount rate, and by amplifying the positive effect of uncertainty on prices.

A decline in the equity premium also strengthens the impact of uncertainty on idiosyncratic return volatility. As a result, holding uncertainty constant, idiosyncratic volatility increases in our model when the equity premium declines. Therefore, the gradual increase in average idiosyncratic return volatility, documented by Campbell et al. (2001), might to some extent be due to the apparent gradual decline in the equity premium over the past few decades. This conjecture can be further examined in future work.

Future research can also test our model against alternatives that involve behavioral biases. To allow a fair horserace, it would be useful to develop a behavioral model that can be calibrated to match the observed prices and volatilities of Nasdaq firms in the late 1990s, as our model does. Until our model is rejected in favor of such an alternative, the existence of a Nasdaq "bubble" in the late 1990s should not be taken for granted.

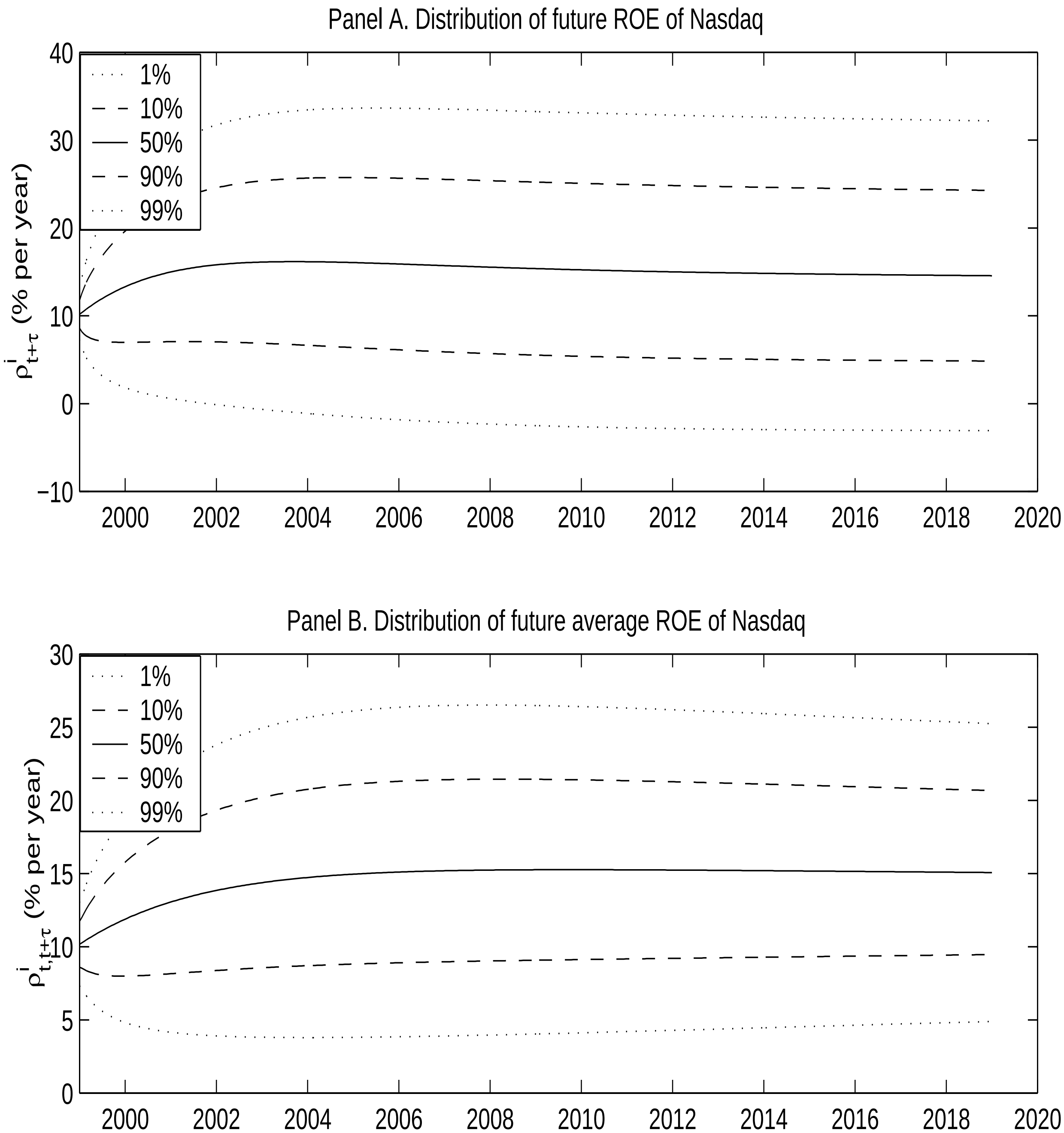

Figure 1. Model-predicted distributions of future profitability and average future profitability for Nasdaq. Panel A plots the selected percentiles of the model-predicted distribution of Nasdaq's future profitability (measured as return on equity, ROE). Panel B plots the selected percentiles of the distribution of Nasdaq's average future profitability, computed by averaging the ROE values between 1999 and the year given on the horizontal axis. In both panels, the market's expectation of Nasdaq's average excess profitability is 3% per year, the associated uncertainty is 3.38%, the equity premium is 3% per year, and the expected horizon is 20 years. Under these assumptions, the model-implied M/B ratio and return volatility correspond to Nasdaq's actual M/B ratio and return volatility observed on March 10, 2000.

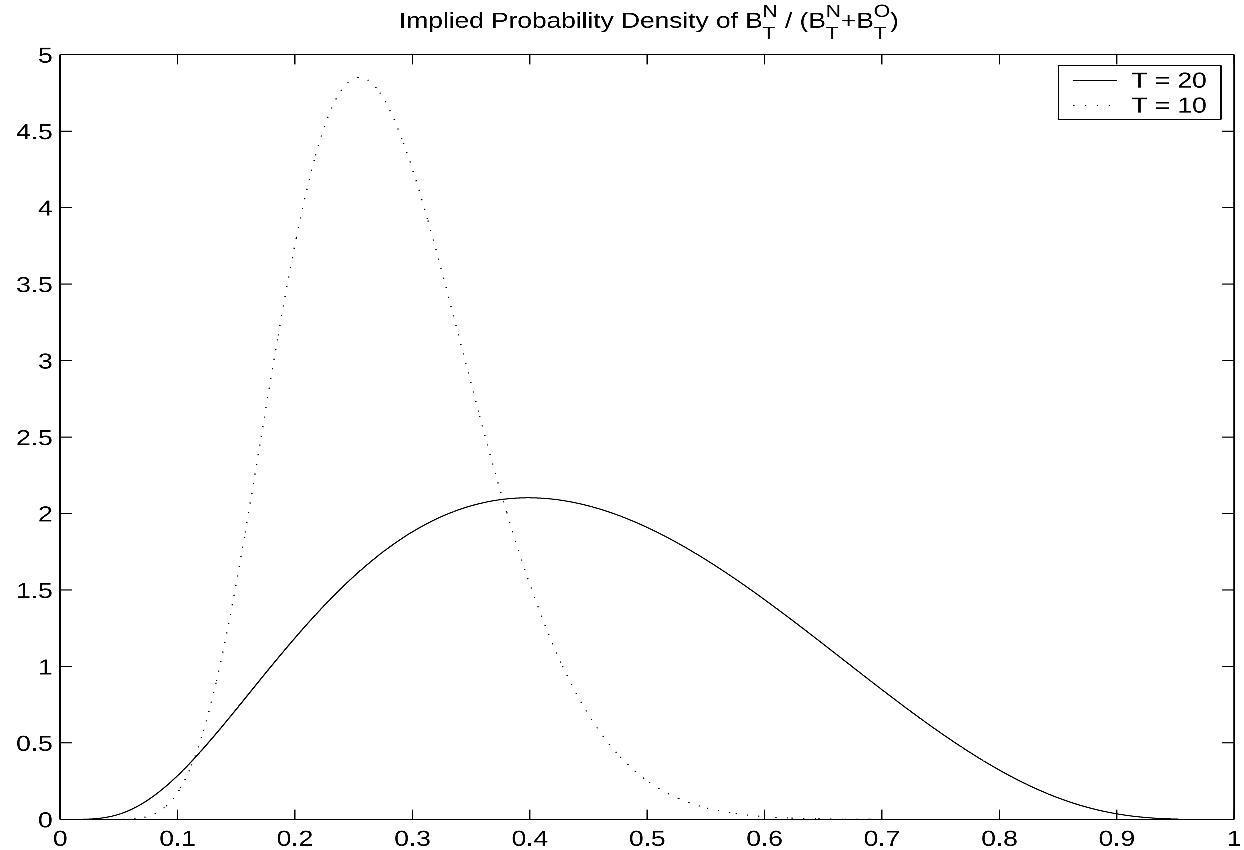

Figure 2. Model-predicted distribution of the future ratio of Nasdaq book value to NYSE/Amex/Nasdaq book value. The figure plots the model-predicted distribution of the ratio of Nasdaq book value to NYSE/Amex/Nasdaq book value $T = 10$ (dotted line) and $T = 20$ (solid line) years in the future. The market's expectation of Nasdaq's average excess profitability is 3% per year, the associated uncertainty is 3.38%, the equity premium is 3% per year, and the expected horizon is 20 years. Under these assumptions, the model-implied M/B ratio and return volatility correspond to Nasdaq's actual M/B ratio and return volatility observed on March 10, 2000.

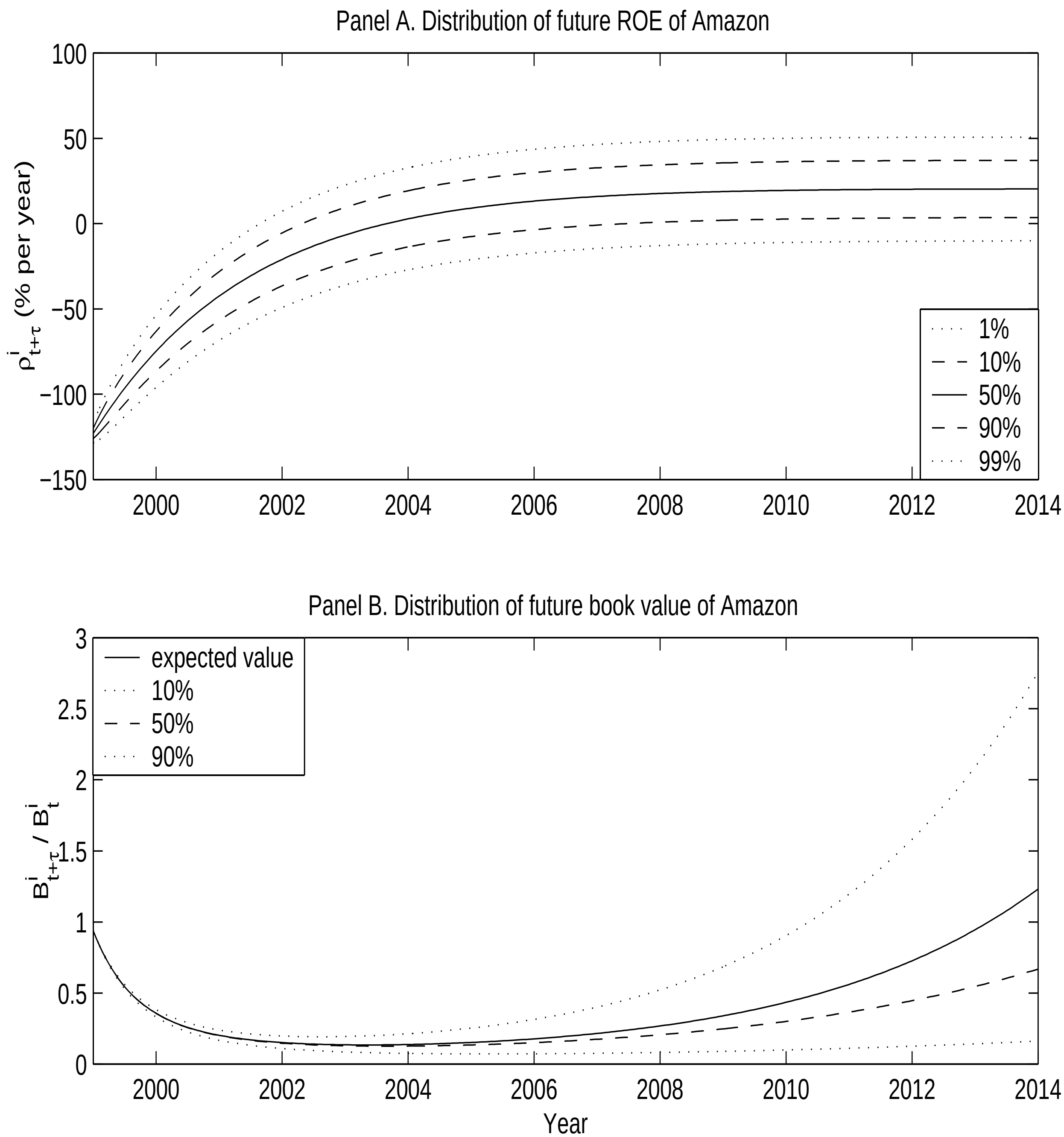

Figure 3. Model-predicted distributions of future profitability and book value for Amazon. Panel A plots the selected percentiles of the model-predicted distribution of Amazon's future profitability (measured as return on equity, ROE). Panel B plots Amazon's expected future book value, along with the selected percentiles of the book value's model-predicted distribution. The future book values are normalized by the 1999 book value. In both panels, the market's expectation of Amazon's average excess profitability is 10% per year, the associated uncertainty is 4.51%, the equity premium is 3% per year, and the expected horizon is 15 years. Under these assumptions, the model-implied M/B ratio and return volatility correspond to Amazon's actual M/B ratio and return volatility observed on March 10, 2000.

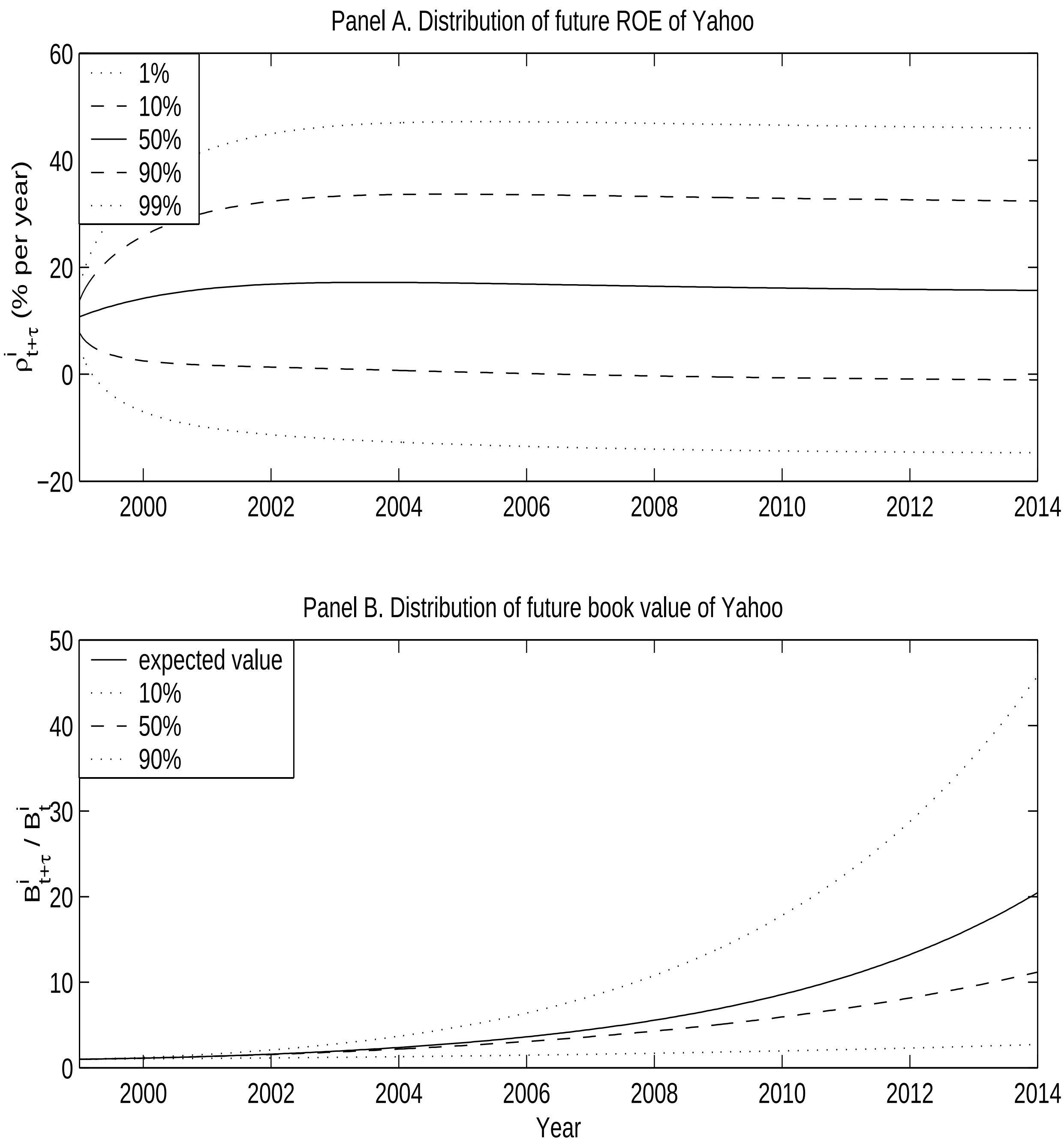

Figure 4. Model-predicted distributions of future profitability and book value for Yahoo. Panel A plots the selected percentiles of the model-predicted distribution of Yahoo's future profitability (measured as return on equity, ROE). Panel B plots Yahoo's expected future book value, along with the selected percentiles of the book value's model-predicted distribution. The future book values are normalized by the 1999 book value. In both panels, the market's expectation of Yahoo's average excess profitability is 4% per year, the associated uncertainty is 4.40%, the equity premium is 3% per year, and the expected horizon is 15 years. Under these assumptions, the model-implied M/B ratio and return volatility correspond to Yahoo's actual M/B ratio and return volatility observed on March 10, 2000.

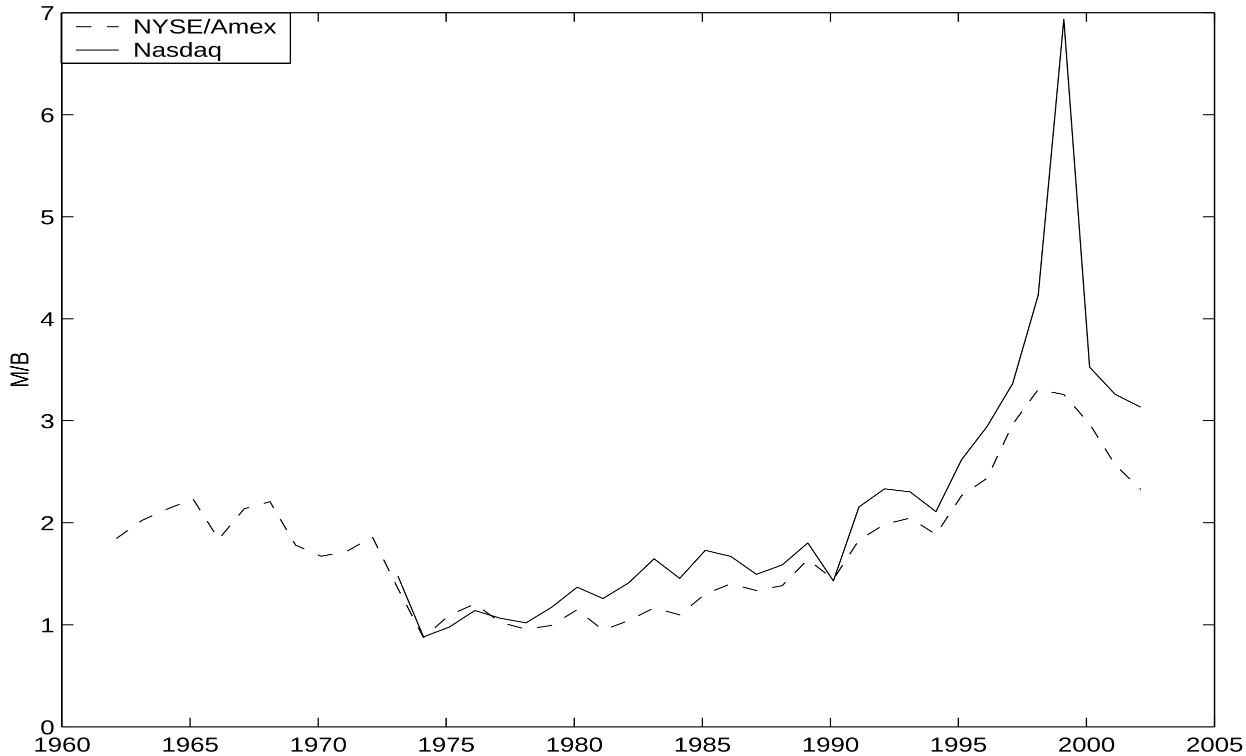

Figure 5. M/B ratios. This figure plots the annual time series of the market-to-book ratios (M/B) of the Nasdaq index and the combined NYSE/Amex index. The M/B ratio of each index is computed as the sum of the market values of equity across firms in the index at the end of the current year, divided by the sum of the book values of equity at the end of the previous year.

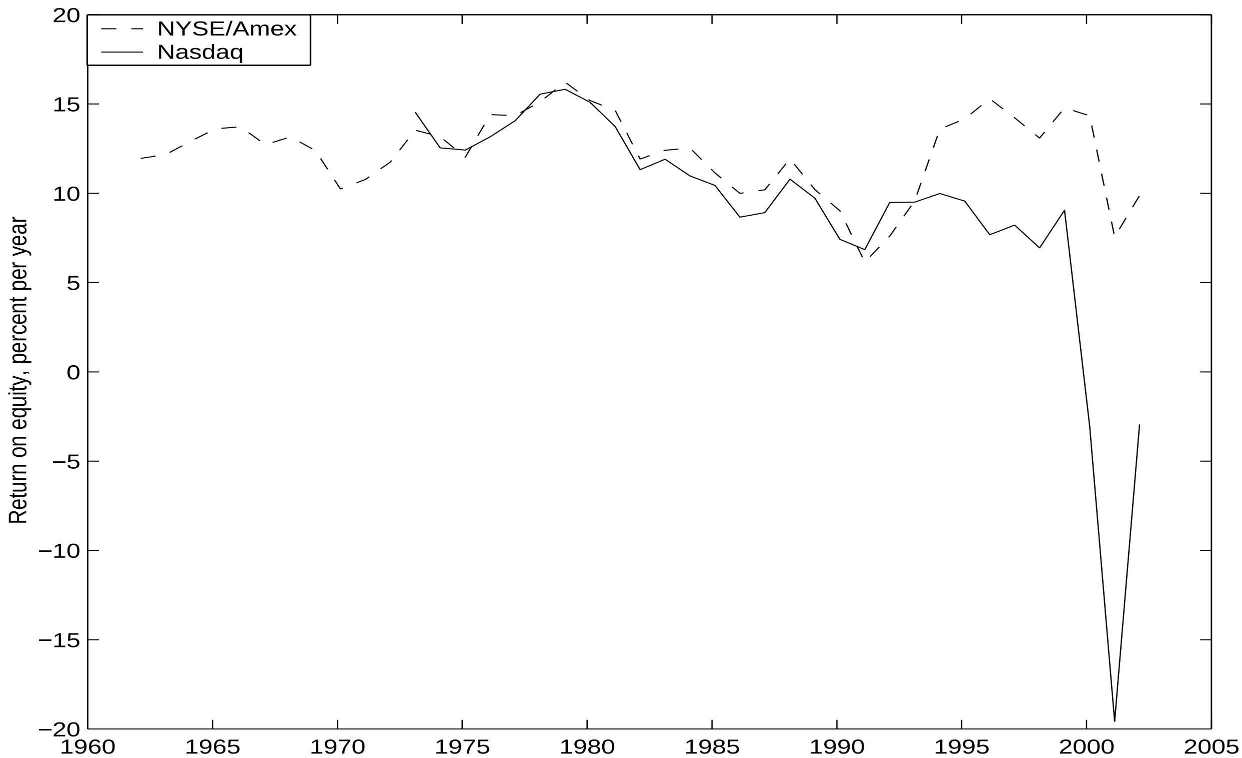

Figure 6. Realized profitability. This figure plots the annual time series of the real realized profitability (return on equity, ROE) of the Nasdaq index and the combined NYSE/Amex index. The ROE of each index is computed as the sum of current-year earnings across firms in the index, divided by the sum of the book values of equity at the end of the previous year.

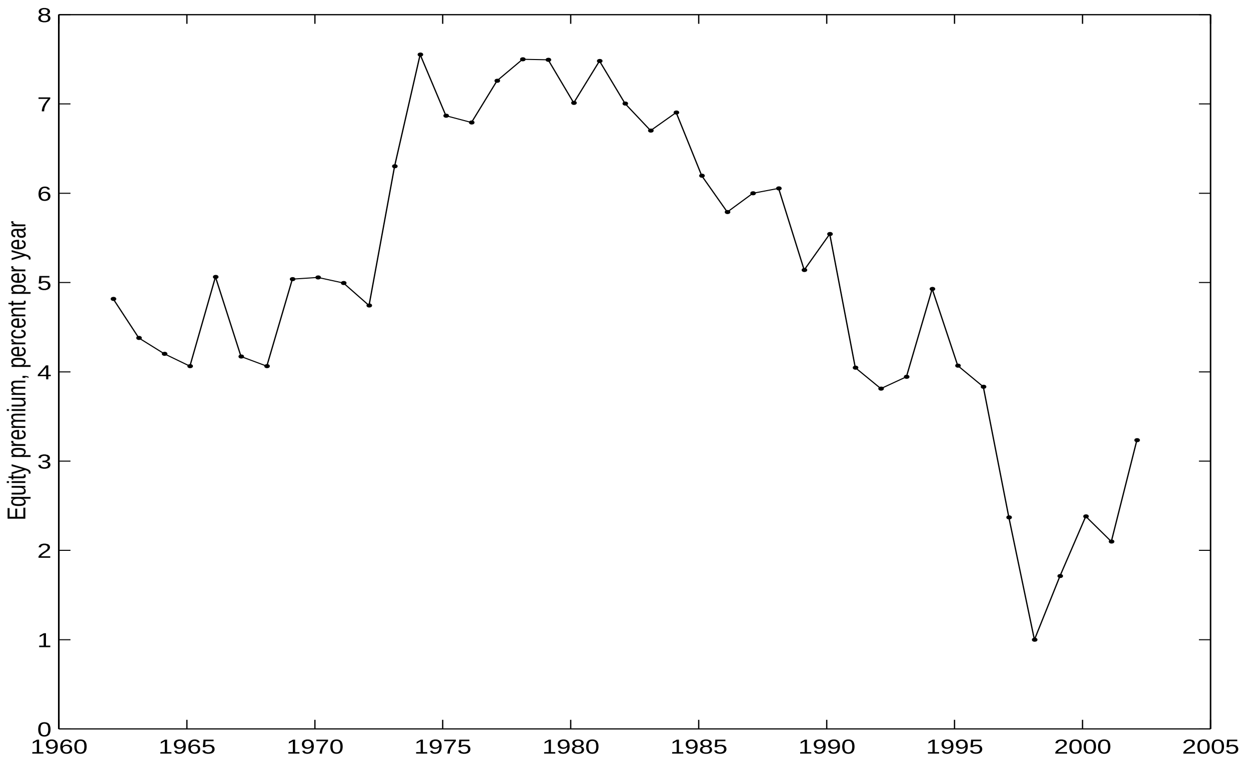

Figure 7. Implied equity premium. This figure plots the annual time series of the equity premium that sets the actual M/B ratio of the NYSE/Amex index at the end of the current year equal to its model-implied value.

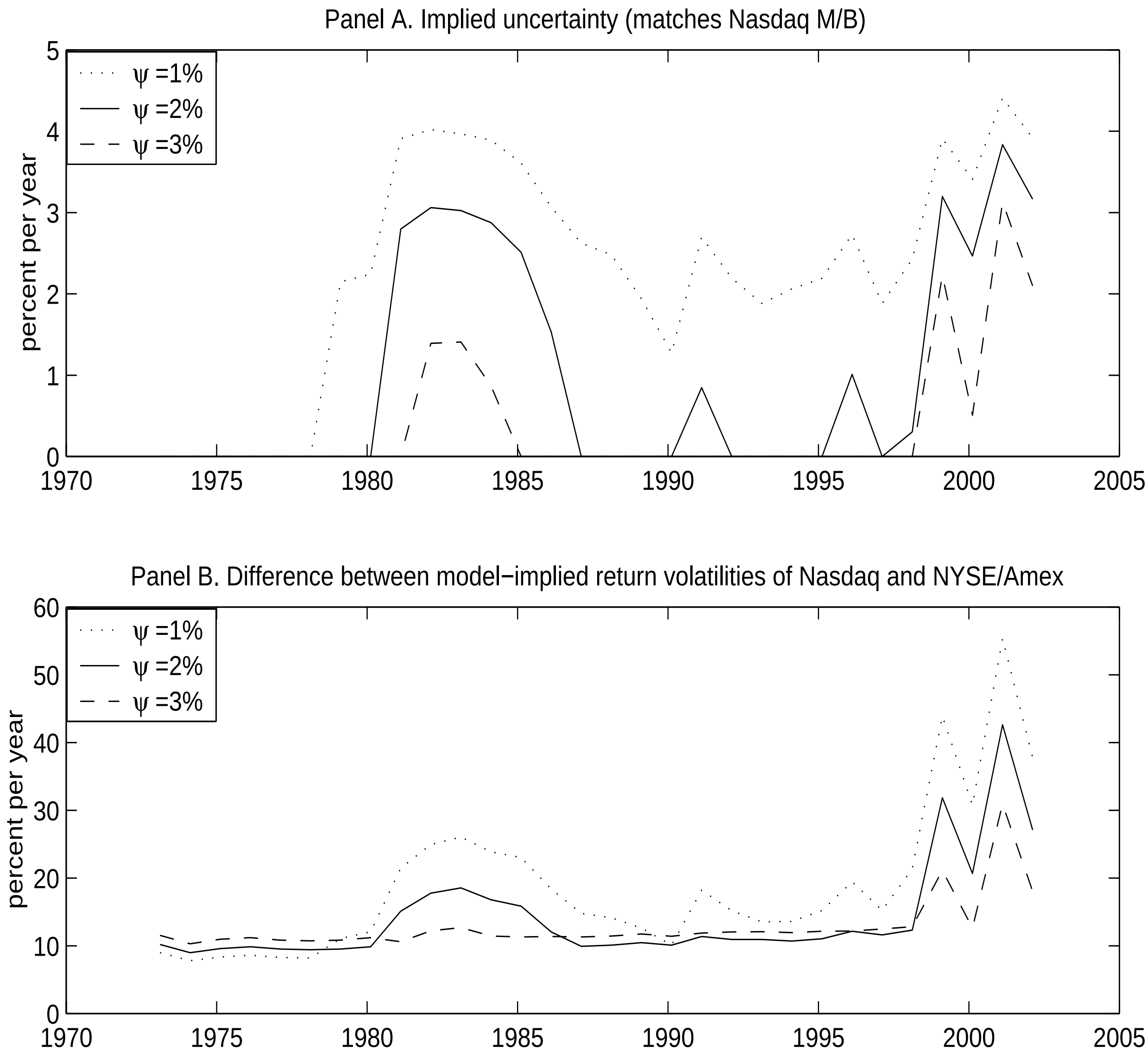

Figure 8. Implied uncertainty. Panel A plots the time series of the implied uncertainty that sets the actual M/B ratio of the Nasdaq index at the end of the current year equal to its model-implied value. Implied uncertainty is plotted for three different values of expected excess profitability. Panel B plots the model-implied difference between the return volatilities of the new and old economies.

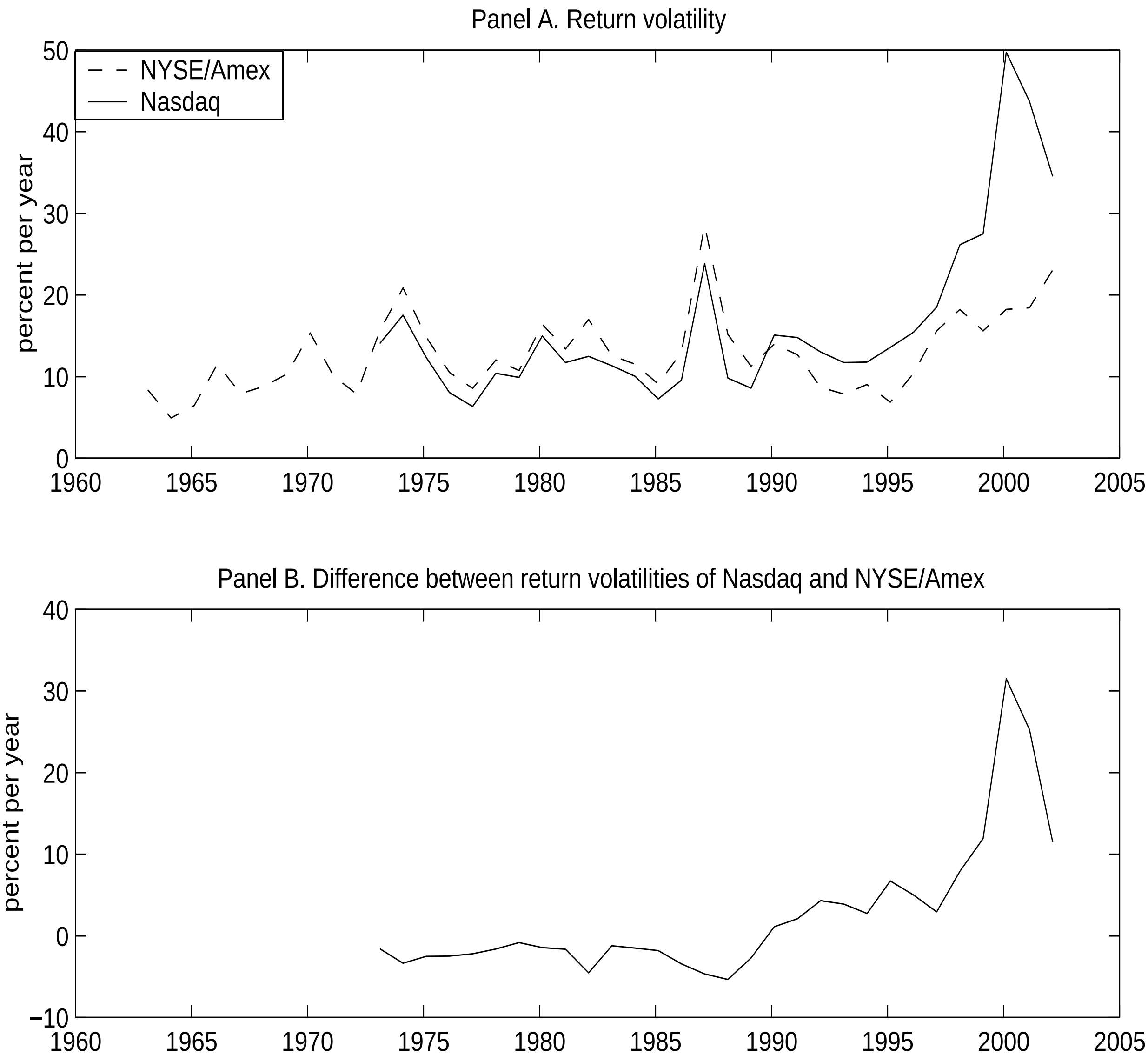

Figure 9. Return volatility. Panel A plots the time series of the return volatility of the Nasdaq index and the NYSE/Amex index. The return volatility in each month is computed as the standard deviation of the daily index returns within the month. The annual volatility values are then computed by averaging the monthly values within the year. Panel B plots the differences between the return volatilities of the Nasdaq and NYSE/Amex indices.

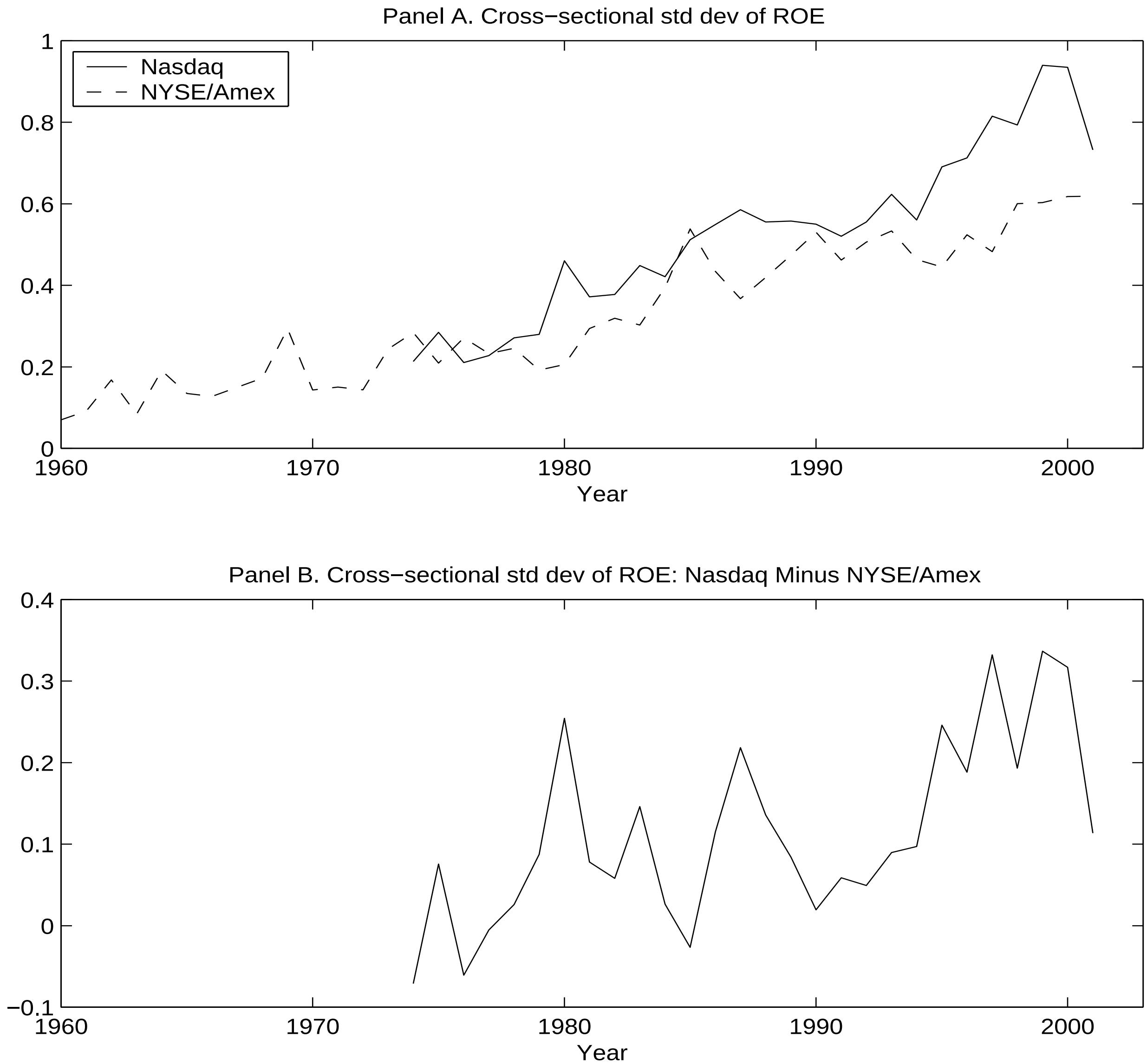

Figure 10. Cross-sectional standard deviation of profitability. Panel A plots the cross-sectional standard deviation of profitability for Nasdaq firms and for NYSE/Amex firms. Profitability (return on equity, ROE) of each firm in each year is computed as the firm's earnings in the given year divided by the firm's book equity at the end of the previous year. ROEs larger than 1,000% per year in absolute value are excluded. Panel B plots the difference between the cross-sectional standard deviations of Nasdaq and NYSE/Amex.

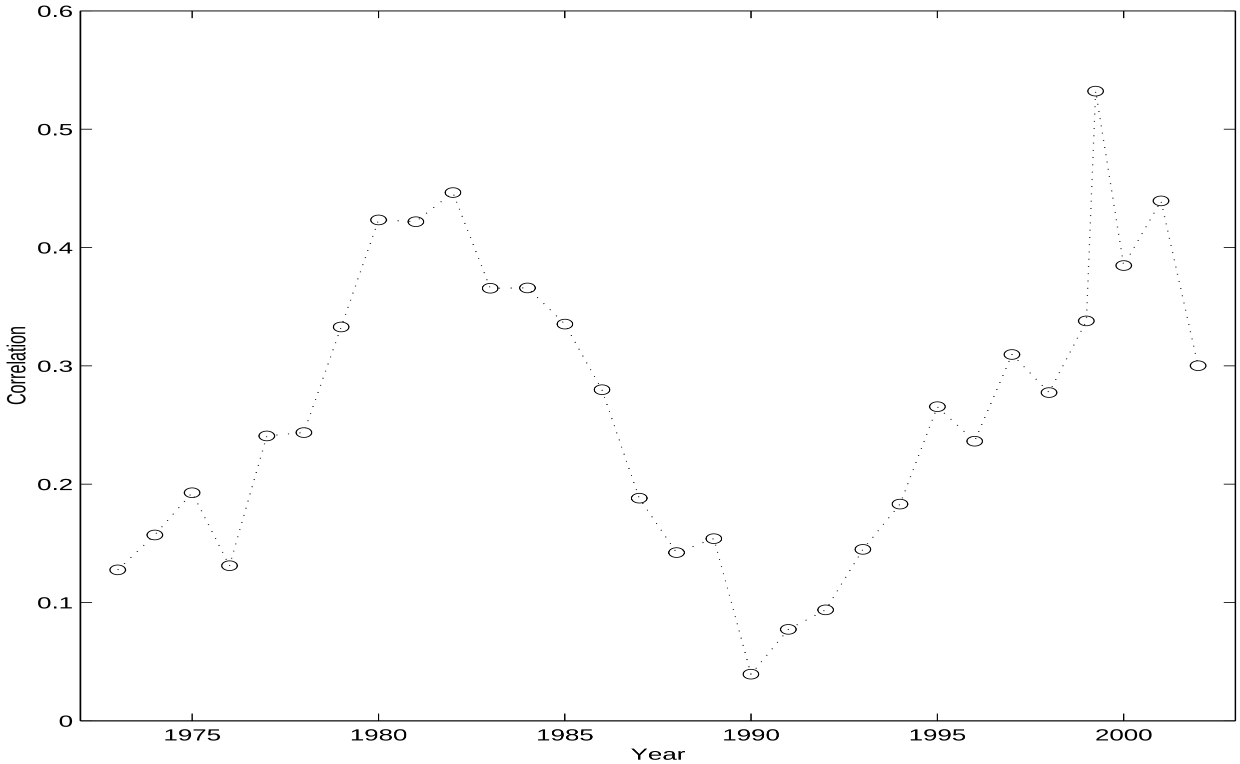

Figure 11. Cross-sectional correlation between implied uncertainty and idiosyncratic return volatility. The figure plots the time series of the correlation between squared implied uncertainty and idiosyncratic return volatility. The correlation values are computed at each year-end, as well as on March 10, 2000, across all Nasdaq firms with valid data.

Table 1
Parameter Values in the Calibrated Model.

The table reports the parameter values used to calibrate our model. The parameters of the processes for the new-economy and old-economy aggregate profitability are estimated by maximum likelihood from the data on the aggregate profitability of Nasdaq and NYSE/Amex firms. The parameters of the individual firm profitability process are calibrated to the median Nasdaq firm in our sample. The utility parameters (η and γ), the parameters defining the log surplus consumption ratio $s(y) = a_0 + a_1 y_t + a_2 y_t^2$, and those characterizing the state variable y_t are calibrated to match the observed levels of the equity premium, market volatility, aggregate M/B, and the interest rate. The means and standard deviations of the fitted quantities are computed from the time series of the fitted values of the old economy's M/B ratio, conditional expected excess return $\mu_{R,t}^{mkt}$, conditional standard deviation of excess returns $\sigma_{R,t}^{mkt}$, and the real risk-free rate $r_{f,t}$ over the period 1962-2002. All entries are annualized.

Old Economy Profitability				New Economy Profitability			Individ. Firm Profitability		
k_L	$\overline{\rho}_L$	$\sigma_{L,0}$	$\sigma_{L,L}$	ϕ^N	$\sigma_{0,N}$	$\sigma_{N,N}$	ϕ^i	$\sigma_{i,0}$	$\sigma_{i,i}$
0.3574	12.17%	1.47%	1.31%	0.3551	2.93%	4.88%	0.3891	6.65%	8.07%
Stochastic Discount Factor									
η	γ	k_y	$\overline{y}$	σ_y	a_0	a_1	a_2	μ_ε	σ_ε
0.0471	3.9474	0.0367	-0.08%	25.30%	-2.8780	0.3084	-0.0413	2%	1%
Means of Fitted Quantities					Std Deviations of Fitted Quantities				
$E[M/B]$	$E\left[\mu_{R,t}^{mkt}\right]$	$E\left[\sigma_{R,t}^{mkt}\right]$	$E[r_{f,t}]$		$\sigma[M/B]$	$\sigma\left[\mu_{R,t}^{mkt}\right]$	$\sigma\left[\sigma_{R,t}^{mkt}\right]$	$\sigma[r_{f,t}]$	
1.77	5.06%	14.47%	6.25%		0.6477	1.72%	2.24%	1.55%	

Table 2
Nasdaq's Valuation on March 10, 2000 Assuming Zero Uncertainty

Panel A reports the model-implied M/B for the Nasdaq Composite Index on March 10, 2000, assuming zero uncertainty about average excess profitability $\overline{\psi}^N$. Panel B reports the model-implied return volatility for Nasdaq under zero uncertainty. The observed M/B for Nasdaq on March 10, 2000 is 8.55. Nasdaq's annualized standard deviation of daily returns in March 2000 is 41.49%, and its average monthly volatility in 2000 is 47.03% per year. Nasdaq's most recent annualized profitability (ROE in 1999Q4) is $\rho_t^N = 9.96\%$ per year, and its most recent dividend yield (dividends over book equity in 1999) is $c = 1.35\%$ per year. The expected time period over which the Nasdaq index can earn abnormal profits is $\mathrm{E}(T) = 20$ years. All variables (equity premium, $\widehat{\psi}^N$, and return volatility) are expressed in percent per year.

Excess ROE	Equity Premium (% per year)							
$\hat{\psi}^N$ (% per year)	1	2	3	4	5	6	7	8
	Panel A: Model-implied M/B with zero uncertainty (Actual M/B: 8.55)							
-5	1.46	1.41	1.30	1.18	1.04	0.89	0.74	0.56
0	3.33	3.02	2.63	2.23	1.84	1.47	1.12	0.76
1	4.15	3.70	3.17	2.64	2.14	1.68	1.25	0.83
2	5.27	4.62	3.89	3.19	2.53	1.95	1.41	0.90
3	6.83	5.89	4.87	3.92	3.05	2.29	1.62	1.00
4	9.06	7.68	6.23	4.92	3.75	2.74	1.88	1.11
5	12.28	10.22	8.15	6.31	4.71	3.36	2.23	1.26
6	17.02	13.92	10.90	8.28	6.04	4.19	2.69	1.45
7	24.09	19.38	14.91	11.12	7.93	5.36	3.32	1.69
8	34.80	27.55	20.85	15.28	10.67	7.02	4.20	2.02
	Panel B: Model-implied return volatility with zero uncertainty (Actual volatility: 41.49% in March 2000, 47.03% in 2000)							
-5	15.47	16.94	18.07	18.90	19.50	19.83	19.83	19.25
0	18.09	20.17	21.76	22.93	23.76	24.18	24.10	23.04
1	18.69	20.93	22.65	23.92	24.83	25.31	25.22	24.05
2	19.31	21.71	23.57	24.97	25.97	26.52	26.46	25.18
3	19.93	22.50	24.52	26.05	27.18	27.83	27.81	26.45
4	20.54	23.30	25.47	27.16	28.44	29.21	29.27	27.85
5	21.14	24.07	26.42	28.27	29.72	30.66	30.85	29.42
6	21.71	24.82	27.34	29.37	31.01	32.15	32.52	31.15
7	22.25	25.53	28.23	30.44	32.28	33.65	34.27	33.05
8	22.76	26.20	29.06	31.45	33.51	35.13	36.05	35.10

Table 3
Nasdaq's Valuation on March 10, 2000 Assuming Uncertainty of 3% Per Year

Panel A reports the model-implied M/B for the Nasdaq Composite Index on March 10, 2000, assuming that uncertainty about average excess profitability $\overline{\psi}^N$ is 3% per year. Panel B reports the model-implied return volatility for Nasdaq under 3% uncertainty. The observed M/B for Nasdaq on March 10, 2000 is 8.55. Nasdaq's annualized standard deviation of daily returns in March 2000 is 41.49%, and its average monthly volatility in 2000 is 47.03% per year. Nasdaq's most recent annualized profitability (ROE in 1999Q4) is $\rho_t^N = 9.96\%$ per year, and its most recent dividend yield (dividends over book equity in 1999) is $c = 1.35\%$ per year. The expected time period over which the Nasdaq index can earn abnormal profits is $\mathrm{E}(T) = 20$ years. All variables (equity premium, $\widehat{\psi}^N$, and return volatility) are expressed in percent per year.

Excess ROE	Equity Premium (% per year)							
$\hat{\psi}^N$ (% per year)	1	2	3	4	5	6	7	8
	Panel A: Model-implied M/B with 3% uncertainty (Actual M/B: 8.55)							
-5	1.70	1.61	1.46	1.30	1.13	0.95	0.78	0.58
0	4.70	4.09	3.43	2.81	2.23	1.72	1.26	0.82
1	6.16	5.27	4.33	3.47	2.69	2.02	1.44	0.90
2	8.29	6.95	5.59	4.39	3.33	2.43	1.67	1.00
3	11.44	9.40	7.41	5.69	4.21	2.99	1.98	1.13
4	16.17	13.03	10.07	7.57	5.46	3.76	2.40	1.30
5	23.39	18.53	14.05	10.35	7.29	4.87	2.99	1.52
6	34.59	26.96	20.10	14.53	9.99	6.49	3.83	1.82
7	52.23	40.10	29.44	20.91	14.08	8.89	5.04	2.24
8	80.36	60.90	44.08	30.83	20.36	12.54	6.86	2.85
	Panel B: Model-implied return volatility with 3% uncertainty (Actual volatility: 41.49% in March 2000, 47.03% in 2000)							
-5	24.90	25.23	25.36	25.29	25.01	24.49	23.60	21.95
0	33.66	33.93	33.97	33.76	33.22	32.25	30.62	27.60
1	35.63	35.96	36.06	35.89	35.37	34.36	32.60	29.23
2	37.59	38.02	38.21	38.12	37.66	36.66	34.81	31.11
3	39.51	40.05	40.37	40.41	40.05	39.13	37.26	33.25
4	41.33	42.03	42.50	42.70	42.51	41.73	39.93	35.70
5	43.05	43.91	44.56	44.94	44.96	44.40	42.77	38.46
6	44.65	45.67	46.50	47.09	47.35	47.07	45.73	41.52
7	46.11	47.29	48.31	49.10	49.63	49.67	48.72	44.85
8	47.45	48.77	49.96	50.96	51.75	52.14	51.65	48.36

Table 4
Matching Nasdaq's Valuation on March 10, 2000

Panel A reports the implied uncertainty for the Nasdaq Composite Index on March 10, 2000, i.e., the uncertainty about average excess profitability $\overline{\psi}^N$ that equates Nasdaq's model-implied M/B to Nasdaq's observed M/B of 8.55. Panel B reports the model-implied return volatility for Nasdaq computed under implied uncertainty. Nasdaq's annualized standard deviation of daily returns in March 2000 is 41.49%, and its average monthly volatility in 2000 is 47.03% per year. Nasdaq's most recent annualized profitability (ROE in 1999Q4) is $\rho_t^N = 9.96\%$ per year, and its most recent dividend yield (dividends over book equity in 1999) is $c = 1.35\%$ per year. The expected time period over which the Nasdaq index can earn abnormal profits is $\mathrm{E}(T) = 20$ years. All variables (equity premium, expected excess profitability $\widehat{\psi}^N$, implied uncertainty, and return volatility) are expressed in percent per year.

Excess ROE	Equity Premium (% per year)							
$\hat{\psi}^N$ (% per year)	1	2	3	4	5	6	7	8
	Panel A. Uncertainty needed to match the observed M/B							
-5	6.41	6.60	6.81	7.04	7.29	7.57	7.89	8.36
0	4.39	4.71	5.06	5.43	5.81	6.22	6.67	7.27
1	3.81	4.17	4.59	5.01	5.44	5.89	6.38	7.03
2	3.08	3.54	4.04	4.53	5.03	5.54	6.08	6.77
3	2.08	2.73	3.38	3.98	4.57	5.15	5.75	6.50
4	0.00	1.45	2.51	3.32	4.04	4.71	5.39	6.22
5	0.00	0.00	0.97	2.43	3.40	4.22	5.00	5.91
6	0.00	0.00	0.00	0.78	2.56	3.63	4.56	5.58
7	0.00	0.00	0.00	0.00	1.18	2.90	4.06	5.23
8	0.00	0.00	0.00	0.00	0.00	1.86	3.47	4.84
	Panel B. Model-implied return volatility under implied uncertainty (Actual volatility: 41.49% in March 2000, 47.03% in 2000)							
-5	141.49	151.69	165.51	182.07	202.27	226.99	258.78	307.51
0	64.69	73.70	85.81	100.56	119.11	142.51	173.80	223.37
1	51.35	60.15	71.80	85.98	103.93	126.70	157.49	206.85
2	38.94	47.54	58.69	72.23	89.41	111.45	141.54	190.50
3	27.79	36.12	46.66	59.43	75.73	96.84	126.12	174.41
4	20.54	26.53	36.07	47.81	63.03	83.03	111.22	158.63
5	21.14	24.07	27.70	37.78	51.53	70.16	96.98	143.20
6	21.71	24.82	27.34	30.14	41.66	58.44	83.59	128.22
7	22.25	25.53	28.23	30.44	34.09	48.24	71.18	113.79
8	22.76	26.20	29.06	31.45	33.51	40.08	60.04	100.05

Table 5
Matching Nasdaq's Valuation on March 10, 2000 With Different Horizons

The table is an equivalent of Table 4 with expected time horizons of $\mathrm{E}(T) = 15$ and 25 years.

Excess ROE	Equity Premium (% per year)						Equity Premium (% per year)					
$\hat{\psi}^i$ (% per year)	1	2	3	4	5	6	1	2	3	4	5	6
	Implied Uncertainty (% per year)						Implied Return Volatility (% per year) (Actual volatility: 41.49% in March 2000, 47.03% in 2000)					
Panel A: E[T] = 15												
-5	7.17	7.35	7.55	7.76	8.00	8.26	168.07	178.26	192.54	209.97	231.48	257.78
0	5.41	5.67	5.97	6.30	6.64	7.01	87.17	96.09	108.72	124.44	144.37	169.68
1	4.93	5.23	5.57	5.93	6.31	6.72	72.82	81.45	93.63	108.81	128.21	152.94
2	4.39	4.73	5.12	5.53	5.96	6.40	59.16	67.59	79.31	93.91	112.62	136.69
3	3.74	4.14	4.61	5.08	5.56	6.06	46.32	54.55	65.80	79.75	97.72	121.02
4	2.92	3.43	4.00	4.56	5.12	5.69	34.43	42.48	53.21	66.46	83.56	105.92
5	1.68	2.48	3.25	3.96	4.63	5.27	23.93	31.70	41.76	54.17	70.31	91.60
6	0.00	0.52	2.22	3.20	4.04	4.81	20.00	23.01	31.87	43.13	58.06	78.11
7	0.00	0.00	0.00	2.15	3.33	4.28	20.56	23.39	25.66	33.81	47.11	65.61
8	0.00	0.00	0.00	0.00	2.37	3.64	21.09	24.09	26.52	28.48	37.85	54.32
Panel B: E[T] = 25												
-5	5.92	6.12	6.34	6.58	6.84	7.12	126.31	136.50	149.96	165.95	185.35	208.90
0	3.70	4.06	4.46	4.86	5.27	5.70	52.11	61.22	72.98	87.11	104.74	126.87
1	3.00	3.44	3.93	4.39	4.87	5.35	39.57	48.47	59.75	73.27	90.21	111.67
2	2.02	2.66	3.28	3.86	4.42	4.96	28.34	36.94	47.60	60.37	76.49	97.12
3	0.00	1.43	2.44	3.21	3.89	4.53	21.07	27.30	36.94	48.67	63.72	83.30
4	0.00	0.00	0.97	2.35	3.27	4.04	21.69	24.75	28.54	38.61	52.20	70.43
5	0.00	0.00	0.00	0.73	2.45	3.47	22.28	25.52	28.15	31.00	42.36	58.76
6	0.00	0.00	0.00	0.00	1.07	2.75	22.83	26.24	29.05	31.37	34.89	48.64
7	0.00	0.00	0.00	0.00	0.00	1.70	23.33	26.90	29.88	32.39	34.59	40.67
8	0.00	0.00	0.00	0.00	0.00	0.00	23.78	27.50	30.64	33.33	35.75	37.81

Table 6
Matching the Valuations of Selected Technology Firms on March 10, 2000

The table reports the implied uncertainty for selected technology firms on March 10, 2000, i.e., the uncertainty about average excess profitability $\overline{\psi}^i$ that equates the firm's model-implied M/B to its observed M/B. The table also reports the model-implied return volatility computed under the corresponding value of implied uncertainty. Each firm's name is accompanied by the firm's market capitalization on March 10, 2000, the firm's observed M/B on the same day, the 1999 values of the firm's realized profitability ρ_t^i and dividend yield c^i, as well as two estimates of the firm's actual return volatility: the standard deviation of the stock's daily returns in March 2000, and the average monthly volatility in 2000, in that order. The expected time period over which the firms can earn abnormal profits is $\mathrm{E}(T) = 15$ years. All variables (equity premium, expected excess profitability $\widehat{\psi}^i$, and implied uncertainty) are expressed in percent per year.

Excess ROE	Equity Premium (% per year)						Equity Premium (% per year)					
$\hat{\psi}^i$ (% per year)	1	2	3	4	5	6	1	2	3	4	5	6
	Implied Uncertainty (% per year)						Implied Return Volatility (% per year)					
	AKAMAI (\$26.15bn): $M/B = 92.92$, $\rho_t = -20.15\%$, $c = 0$, Return volatility: $(88.98\%, 141.91\%)$											
-2	6.39	6.54	6.70	6.87	7.06	7.28	142.70	150.38	158.85	167.95	178.30	190.34
0	5.86	6.02	6.20	6.39	6.60	6.83	122.39	130.18	138.76	147.95	158.42	170.57
2	5.26	5.45	5.65	5.86	6.10	6.36	102.55	110.45	119.09	128.35	138.87	151.09
4	4.57	4.79	5.03	5.27	5.54	5.83	83.44	91.43	100.09	109.33	119.85	132.06
6	3.73	4.01	4.30	4.60	4.91	5.25	65.51	73.54	82.14	91.26	101.64	113.71
8	2.59	3.00	3.40	3.78	4.17	4.58	49.56	57.53	65.86	74.65	84.67	96.40
10	0.00	1.32	2.10	2.70	3.25	3.78	38.18	44.72	52.34	60.40	69.67	80.66
	AMAZON (\$23.45bn): $M/B = 88.07$, $\rho_t = -126.08\%$, $c = 0$, Return volatility: $(71.67\%, 103.33\%)$											
0	6.94	7.07	7.21	7.35	7.52	7.72	171.63	178.75	186.55	194.90	204.44	215.59
4	5.96	6.12	6.28	6.46	6.65	6.88	131.76	139.13	147.12	155.65	165.36	176.67
8	4.77	4.97	5.18	5.40	5.64	5.91	93.21	100.84	109.01	117.66	127.45	138.83
10	4.02	4.26	4.51	4.77	5.05	5.35	74.95	82.72	90.93	99.57	109.32	120.64
12	3.09	3.40	3.72	4.03	4.37	4.72	58.02	65.89	74.04	82.55	92.14	103.25
16	0.00	0.00	0.67	1.74	2.45	3.07	39.16	43.49	47.87	54.99	63.18	72.94
	CIENA (\$23.40bn): $M/B = 41.24$, $\rho_t = -2.25\%$, $c = 0$, Return volatility: $(116.68\%, 121.45\%)$											
-2	5.79	5.97	6.16	6.36	6.57	6.82	115.02	123.03	132.00	141.70	152.78	165.66
0	5.16	5.37	5.58	5.81	6.06	6.33	95.07	103.13	112.13	121.85	132.98	145.92
2	4.42	4.67	4.93	5.20	5.48	5.79	76.12	84.20	93.12	102.76	113.81	126.70
4	3.49	3.82	4.15	4.48	4.82	5.19	58.74	66.76	75.48	84.86	95.64	108.29
6	2.14	2.66	3.14	3.59	4.04	4.49	43.90	51.68	59.93	68.78	79.00	91.11
8	0.00	0.00	1.51	2.34	3.02	3.63	37.53	41.40	47.69	55.51	64.69	75.81
	CISCO (\$455.72bn): $M/B = 39.02$, $\rho_t = 26.58\%$, $c = 0$, Return volatility: $(49.81\%, 68.88\%)$											
0	4.59	4.84	5.10	5.37	5.65	5.97	76.33	84.39	93.52	103.50	115.05	128.59
2	3.67	3.99	4.33	4.66	5.00	5.37	58.84	66.78	75.62	85.30	96.53	109.80
4	2.34	2.85	3.33	3.78	4.23	4.68	43.92	51.59	59.90	68.96	79.57	92.25
6	0.00	0.00	1.77	2.57	3.24	3.85	36.69	40.28	47.41	55.42	64.93	76.56
8	0.00	0.00	0.00	0.00	1.65	2.72	37.57	41.46	44.74	47.57	53.68	63.61
	DELL (\$129.88bn): $M/B = 24.47$, $\rho_t = 58.55\%$, $c = 0$, Return volatility: $(51.75\%, 69.50\%)$											
-4	5.05	5.29	5.55	5.82	6.10	6.41	76.88	84.69	94.10	104.81	117.48	132.54
-2	4.15	4.47	4.80	5.13	5.48	5.84	59.28	66.80	75.76	85.97	98.19	112.93
0	2.90	3.37	3.85	4.30	4.75	5.19	44.30	51.44	59.71	69.14	80.56	94.57
2	0.00	1.42	2.43	3.17	3.82	4.41	34.31	39.50	46.77	55.06	65.25	78.05
4	0.00	0.00	0.00	0.97	2.46	3.40	35.64	38.92	41.52	44.60	53.07	64.11

Table 6 (continued)

Excess ROE $\hat{\psi}^i$ (% per year)	Equity Premium (% per year) 1	2	3	4	5	6	Equity Premium (% per year) 1	2	3	4	5	6
	Implied Uncertainty (% per year)						Implied Return Volatility (% per year)					
	EBAY (\$23.76bn): $M/B = 27.87$, $\rho_t = 7.79\%$, $c = 0$, Return volatility: $(129.24\%, 113.64\%)$											
-4	6.03	6.20	6.38	6.58	6.79	7.03	119.33	127.40	136.60	146.59	158.06	171.40
-2	5.42	5.62	5.83	6.05	6.29	6.56	99.01	107.11	116.29	126.33	137.86	151.29
0	4.71	4.94	5.20	5.45	5.73	6.04	79.70	87.76	96.87	106.81	118.26	131.68
2	3.83	4.13	4.45	4.77	5.10	5.45	61.89	69.87	78.74	88.42	99.62	112.81
4	2.61	3.06	3.51	3.93	4.35	4.78	46.43	54.18	62.59	71.74	82.40	95.09
6	0.00	1.15	2.12	2.80	3.41	3.98	36.63	41.92	49.45	57.65	67.34	79.09
8	0.00	0.00	0.00	0.00	2.00	2.92	37.55	41.43	44.68	47.48	55.45	65.65
	IMMUNEX (\$37.56bn): $M/B = 105.70$, $\rho_t = 17.91\%$, $c = 0$, Return volatility: $(155.94\%, 117.71\%)$											
-4	6.54	6.68	6.84	7.01	7.20	7.42	145.55	153.37	162.07	171.46	182.16	194.58
-2	6.01	6.17	6.35	6.54	6.75	6.98	124.88	132.81	141.62	151.12	161.95	174.52
0	5.42	5.60	5.81	6.02	6.25	6.51	104.71	112.71	121.59	131.15	142.06	154.72
2	4.73	4.96	5.19	5.44	5.70	5.99	85.30	93.36	102.24	111.78	122.68	135.35
4	3.91	4.19	4.48	4.77	5.08	5.42	67.10	75.17	83.94	93.35	104.09	116.64
6	2.82	3.21	3.60	3.98	4.36	4.77	50.84	58.82	67.30	76.36	86.74	98.93
8	0.56	1.68	2.38	2.94	3.47	3.99	38.00	45.52	53.32	61.65	71.28	82.74
10	0.00	0.00	0.00	1.12	2.20	2.99	38.20	42.29	45.81	50.50	58.74	68.90
	INTEL (\$395.49bn): $M/B = 11.09$, $\rho_t = 28.65\%$, $c = 0.0148$, Return volatility: $(45.81\%, 68.71\%)$											
-2	5.11	5.38	5.68	5.98	6.30	6.64	70.70	78.20	87.56	98.52	111.75	127.82
0	4.11	4.47	4.86	5.25	5.64	6.05	54.00	61.14	69.90	80.13	92.66	108.13
2	2.64	3.22	3.80	4.34	4.86	5.37	40.09	46.82	54.75	64.01	75.48	89.94
4	0.00	0.00	2.08	3.05	3.84	4.54	33.53	36.26	42.85	50.81	60.83	73.79
6	0.00	0.00	0.00	0.00	2.27	3.44	34.86	37.98	40.40	42.26	49.42	60.38
	MICROSOFT (\$516.00bn): $M/B = 18.79$, $\rho_t = 48.28\%$, $c = 0$, Return volatility: $(57.44\%, 56.10\%)$											
-4	5.04	5.28	5.55	5.82	6.10	6.41	76.54	84.32	93.70	104.37	117.01	132.05
-2	4.15	4.46	4.80	5.13	5.48	5.84	59.01	66.50	75.42	85.60	97.80	112.50
0	2.89	3.36	3.84	4.30	4.74	5.19	44.10	51.21	59.44	68.84	80.22	94.20
2	0.00	1.40	2.42	3.16	3.81	4.41	34.24	39.33	46.58	54.83	64.97	77.74
4	0.00	0.00	0.00	0.93	2.45	3.39	35.61	38.87	41.44	44.42	52.86	63.86
	PRICELINE (\$15.94bn): $M/B = 39.58$, $\rho_t = -264.12\%$, $c = 0$, Return volatility: $(128.17\%, 133.65\%)$											
0	7.82	7.93	8.05	8.17	8.32	8.49	217.10	223.77	231.08	238.95	247.98	258.58
2	7.42	7.54	7.66	7.80	7.95	8.13	197.18	203.93	211.33	219.25	228.36	239.02
4	7.00	7.12	7.25	7.40	7.56	7.75	177.34	184.19	191.66	199.68	208.84	219.56
6	6.55	6.68	6.82	6.98	7.15	7.35	157.62	164.57	172.13	180.22	189.44	200.25
8	6.06	6.20	6.36	6.52	6.71	6.93	138.07	145.13	152.78	160.94	170.24	181.09
10	5.52	5.68	5.85	6.03	6.24	6.47	118.76	125.96	133.70	141.92	151.27	162.18
12	4.92	5.10	5.29	5.50	5.73	5.98	99.83	107.17	115.01	123.29	132.68	143.59
	RED HAT (\$10.43bn): $M/B = 26.50$, $\rho_t = -10.15\%$, $c = 0$, Return volatility: $(121.00\%, 122.33\%)$											
-4	6.23	6.39	6.57	6.75	6.96	7.18	129.65	137.61	146.60	156.34	167.48	180.41
-2	5.66	5.84	6.04	6.25	6.47	6.73	109.13	117.17	126.23	136.05	147.28	160.34
0	5.00	5.22	5.45	5.69	5.94	6.23	89.38	97.47	106.50	116.31	127.54	140.64
2	4.22	4.49	4.76	5.05	5.35	5.67	70.80	78.86	87.78	97.44	108.56	121.56
4	3.21	3.57	3.94	4.29	4.66	5.05	54.04	61.98	70.61	79.94	90.70	103.38
6	1.57	2.25	2.83	3.34	3.83	4.31	40.23	47.82	55.85	64.51	74.58	86.59
8	0.00	0.00	0.42	1.88	2.70	3.40	37.52	41.38	44.83	52.25	61.08	71.91
	YAHOO (\$98.90bn): $M/B = 78.41$, $\rho_t = 10.52\%$, $c = 0$, Return volatility: $(75.41\%, 90.61\%)$											
-2	5.95	6.12	6.30	6.49	6.70	6.94	122.40	130.35	139.20	148.76	159.64	172.29
0	5.35	5.55	5.75	5.97	6.20	6.46	102.28	110.29	119.20	128.80	139.77	152.50
2	4.66	4.89	5.13	5.38	5.65	5.94	82.96	91.03	99.93	109.50	120.44	133.17
4	3.81	4.10	4.40	4.70	5.02	5.36	64.94	73.01	81.78	91.18	101.95	114.52
6	2.67	3.08	3.49	3.89	4.28	4.70	49.01	56.95	65.39	74.40	84.75	96.93
8	0.00	1.39	2.19	2.81	3.36	3.90	37.55	44.13	51.81	60.03	69.56	80.96
10	0.00	0.00	0.00	0.63	2.02	2.86	38.20	42.29	45.80	49.39	57.44	67.43

9. Appendix

(A) The Stochastic Discount Factor

The properties of the SDF are described in detail in PV (2004). This appendix contains a brief summary. The process in equation (10) implies a normal unconditional distribution for y_t with mean $\overline{y}$ and variance $\sigma_y^2/2k_y$. Let $y_D = \overline{y} - 4\sigma_y/\sqrt{2k_y}$ and $y_U = \overline{y} + 4\sigma_y/\sqrt{2k_y}$ be the boundaries between which y_t lies 99.9% of the time. To ensure that s_t (log surplus) conforms to the economic intuition of a habit formation model, PV (2004) impose the following parametric restrictions: $a_2 < 0$, $a_1 > -2a_2y_U$ and $a_0 < 1/4\left(a_1^2/a_2\right)$. The resulting process for the stochastic discount factor $\pi_t = e^{-\eta t - \gamma(\varepsilon_t + s_t)}$ is given by

$$d\pi_t = -r_{f,t}\pi_t dt - \pi_t \sigma_{\pi,t} dW_{0,t},$$

where

$$r_{f,t} = R_0 + R_1 y_t + R_2 y_t^2,$$

with

$$\begin{aligned}
R_0 &= \eta + \gamma\mu_\varepsilon + \gamma a_1 k_y \overline{y} - \frac{1}{2}\gamma^2\sigma_\varepsilon^2 + (\gamma a_2 - \frac{1}{2}\gamma^2 a_1^2)\sigma_y^2 - \gamma^2 a_1 \sigma_\varepsilon \sigma_y \\
R_1 &= \gamma\left(2a_2 k_y \overline{y} - a_1 k_y - 2a_2\gamma\left(\sigma_\varepsilon\sigma_y + a_1\sigma_y^2\right)\right) \\
R_2 &= 2a_2\gamma\left(-k_y - \gamma a_2 \sigma_y^2\right)
\end{aligned}$$

and

$$\sigma_{\pi,t} = \gamma\left(\sigma_\varepsilon + (a_1 + 2a_2 y_t)\,\sigma_y\right). \tag{18}$$

The parameter restrictions imposed earlier imply that $\sigma_{\pi,t}$ decreases as y_t increases. As a result, expected return and return volatility are low when y_t is high. See PV (2004) for more details.

(B) Proofs

Lemma 1: Let $\widetilde{b}_t$ follow the process

$$d\widetilde{b}_t = \left(\zeta_0\overline{\rho}_t + \zeta_1\rho_t^i - \zeta_2\right)dt,$$

where ρ_t^i and $\overline{\rho}_t$ follow the processes in equations (2) and (4), and ζ_i are constants. Define $\mathbf{Y}_t = \left(v\widetilde{b}_t - \gamma\varepsilon_t, y_t, \overline{\rho}_t, \rho_t^i, \overline{\psi}_t^i\right)'$ and $g\left(\mathbf{Y}_T\right) = e^{Y_{1,T} - \gamma a_1 Y_{2,T} - \gamma a_2 Y_{2,T}^2}$, where v is a constant, $Y_{i,t}$ denotes the i-th element of $\mathbf{Y}_t$, and γ, a_1, and a_2 are taken from equations (8) and (9). Then

$$E_t\left[e^{-\eta(T-t)} g\left(\mathbf{Y}_T\right) | \overline{\psi}_t^i\right] \equiv H\left(\mathbf{Y}_t, t; T\right) = e^{K_0(t;T) + \mathbf{K}(t;T)'\cdot\mathbf{Y}_t + K_6(t;T)Y_{2,t}^2} \tag{19}$$

where $K_0(t;T)$, $\mathbf{K}(t;T) = \left(K_1(t;T), .., K_5(t;T)\right)'$, and $K_6(t;T)$ satisfy a system of ordinary differential equations (ODE)

$$\begin{aligned}
\frac{dK_6(t;T)}{dt} &= -2K_6^2(t;T)\,\sigma_y^2 + 2K_6(t;T)\,k_y && (20)\\
\left(\frac{d\mathbf{K}(t;T)}{dt}\right)' &= -\mathbf{K}(t;T)'\cdot\left[\mathbf{B}_Y + 2K_6(t;T)\left[\mathbf{\Sigma}_Y\mathbf{\Sigma}_Y'\right]_2\mathbf{e}_2\right] - 2K_6(t;T)\,k_y\overline{y}\mathbf{e}_2 && (21)\\
\frac{dK_0(t;T)}{dt} &= \eta - \mathbf{K}(t;T)'\cdot\mathbf{A}_Y - \frac{1}{2}\mathbf{K}(t;T)'\,\mathbf{\Sigma}_Y\mathbf{\Sigma}_Y'\mathbf{K}(t;T) - K_6(t;T)\,\sigma_y^2 && (22)
\end{aligned}$$

subject to the final condition $K_6(T;T) = -\gamma a_2$, $\mathbf{K}(T;T) = (1, -\gamma a_1, 0, 0, 0)$, and $K_0(T;T) = 0$. In the above, $\mathbf{e}_2 = (0, 1, 0, .., 0)$, and

$$\mathbf{A}_Y = \begin{pmatrix} -\gamma\mu_\epsilon - v\zeta_2 \\ k_y\overline{y} \\ k_L\overline{\rho}_L \\ 0 \\ 0 \end{pmatrix}; \mathbf{B}_Y = \begin{pmatrix} 0 & 0 & v\zeta_0 & v\zeta_1 & 0 \\ 0 & -k_y & 0 & 0 & 0 \\ 0 & 0 & -k_L & 0 & 0 \\ 0 & 0 & \phi^i & -\phi^i & \phi^i \\ 0 & 0 & 0 & 0 & -k_\psi \end{pmatrix}; \mathbf{\Sigma}_Y = \begin{pmatrix} -\gamma\sigma_\epsilon & 0 & 0 \\ \sigma_y & 0 & 0 \\ \sigma_{L,0} & \sigma_{L,L} & 0 \\ \sigma_{i,0} & 0 & \sigma_{i,i} \\ 0 & 0 & 0 \end{pmatrix}.$$

Proof of Lemma 1: From the definition of the vector $\mathbf{Y}_t$, we have

$$d\mathbf{Y}_t = (\mathbf{A}_Y + \mathbf{B}_Y\mathbf{Y}_t)\,dt + \mathbf{\Sigma}_Y d\mathbf{W}_t.$$

The Feynman-Kac theorem implies that $H(\mathbf{Y}_t, t)$ from (19) solves the partial differential equation

$$\frac{\partial H}{\partial t} + \sum_{i=1}^{5}\left(\frac{\partial H}{\partial Y_i}\right)[\mathbf{A}_Y + \mathbf{B}_Y\mathbf{Y}_t]_i + \frac{1}{2}\sum_{i=1}^{5}\sum_{j=1}^{5}\frac{\partial^2 H}{\partial Y_i \partial Y_j}\left[\mathbf{\Sigma}_Y\mathbf{\Sigma}_Y'\right]_{ij} = \eta H \tag{23}$$

subject to the boundary condition

$$H(\mathbf{Y}_T, T; T) = g(\mathbf{Y}_T). \tag{24}$$

It can be easily verified that the exponential quadratic function (19) indeed satisfies (23) subject to (24), as long as $K_0(t;T)$, $\mathbf{K}(t;T)$, and $K_6(t;T)$ are the solutions to the system of ODEs in (20) through (22) under the final conditions presented in the claim of the Lemma.■

Lemma 2. If average excess profitability $\overline{\psi}_t^i$ is observable and T_i is known, the firm's ratio of market value to book value of equity is given by

$$\frac{M_t^i}{B_t^i} = c^i\int_0^{T_i - t}\widetilde{Z}^i\left(y_t, \overline{\rho}_t, \rho_t^i, \overline{\psi}_t^i, s\right)ds + \widetilde{Z}^i\left(y_t, \overline{\rho}_t, \rho_t^i, \overline{\psi}_t^i, T_i - t\right),$$

where

$$\widetilde{Z}^i\left(y_t, \overline{\rho}_t, \rho_t^i, \overline{\psi}_t^i, s\right) = e^{\widetilde{Q}_0(s) + \mathbf{Q}(s)'\cdot\mathbf{N}_t + Q_5(s)y_t^2},$$

and where $\mathbf{N}_t = \left(y_t, \overline{\rho}_t, \rho_t^i, \overline{\psi}_t^i\right)$ is the vector of state variables characterizing firm i, $\widetilde{Q}_0(s) = K_0(0;s)$, $Q_i(s) = K_{i+1}(0;s)$ for $i = 2, 3, 4$, $Q_1(s) = K_2(0;s) + \gamma a_1$, and $Q_5(s) = K_6(0;s) + \gamma a_2$, where $K_i(.;s)$ are in Lemma 1, all for the parameterization $\zeta_0 = 0$, $\zeta_1 = v = 1$, and $\zeta_2 = c^i$.

Proof of Lemma 2: Let $t = 0$, for notational simplicity. For given T_i, the pricing formula is

$$M_0^i = E_0\left[\int_0^{T_i}\frac{\pi_s}{\pi_0}D_s^i ds\right] + E_0\left[\frac{\pi_{T_i}}{\pi_0}B_{T_i}\right] = c^i\int_0^{T_i}E_0\left[\frac{\pi_s}{\pi_0}B_s^i\right]ds + E_0\left[\frac{\pi_{T_i}}{\pi_0}B_{T_i}\right].$$

We need to compute the folowing expectation:

$$E_0\left[\frac{\pi_s}{\pi_0}B_s^i\right] = e^{\gamma\varepsilon_0 + \gamma a_1 y_0 + \gamma a_2 y_0^2}E_0\left[e^{-\eta s}e^{b_s^i - \gamma\varepsilon_s - \gamma a_1 y_s - \gamma a_2 y_s^2}\right] = e^{\gamma\varepsilon_0 + \gamma a_1 y_0 + \gamma a_2 y_0^2}H(\mathbf{Y}_0, 0; s),$$

where the H function is given in equation (19). Since $\mathbf{B}_Y$ has only zeros in its first column, we have $\left[\mathbf{K}(t;T_i)' \cdot \mathbf{B}_Y\right]_1 = 0$ in equation (21). This implies $\frac{dK_1(t;T_i)}{dt} = 0$ and thus $K_1(t;T_i) = 1$ for $t \leq T_i$. By substituting in $H(\mathbf{Y}_0, 0; s)$, we obtain

$$E_0\left[\frac{\pi_s}{\pi_0} B_s^i\right] = e^{\gamma \varepsilon_0 + \gamma a_1 y_0 + \gamma a_2 y_0^2} \times H(\mathbf{Y}_0, 0; s) = B_0^i \times e^{\gamma a_1 y_0 + \gamma a_2 y_0^2} \times e^{K_0(0;s) + \sum_{i=2}^5 K_i(0;s) Y_{i,0} + K_6(0;s) Y_{2,0}^2}$$

This expression leads immediately to the claim upon redefinition of the variables. ■

Proof of Proposition 1:

The density of the exponential distribution is $h(s,p) = pe^{-ps}$. We assume throughout that parameters are chosen such that $Q_0(s) = -ps + \widetilde{Q}_0(s) \to -\infty$, and all $Q_i(s)$ for $i \neq 0$ converge to finite numbers, where $\widetilde{Q}_0(s)$ and $Q_i(s)$ are defined in Lemma 2. Such parameters exist, because $\mathbf{B}_Y$ in Lemma 1 has negative eigevalues, and thus the convergence conditions are met for instance if $\mathbf{\Sigma}_Y = \mathbf{0}$. We now prove Proposition 1 under these conditions.

For given T, the expected discounted value of the future cash flow is given in Lemma 2:

$$E_t\left[\int_t^T \frac{\pi_\tau}{\pi_t} D_\tau^i d\tau | T\right] + E_t\left[\frac{\pi_T}{\pi_t} B_T^i | T\right] = B_t^i c^i \int_t^T \widetilde{Z}^i\left(y_t, \overline{\rho}_t, \rho_t^i, \overline{\psi}_t^i, s-t\right) ds + B_t^i \widetilde{Z}\left(y_t, \overline{\rho}_t, \rho_t^i, \overline{\psi}_t^i, T-t\right)$$

Integrating over all possible T's, the value of the stock today is given by

$$M_t^i = B_t^i c^i \int_t^\infty \left(pe^{-p(T-t)}\right) \int_t^T \widetilde{Z}\left(y_t, \overline{\rho}_t, \rho_t^i, \overline{\psi}_t^i, s-t\right) ds\, dT + B_t^i \int_t^\infty pe^{-p(T-t)} \widetilde{Z}\left(y_t, \overline{\rho}_t, \rho_t^i, \overline{\psi}_t^i, T-t\right) dT. \tag{25}$$

Using integration by parts and recalling that $\int pe^{-p(T-t)} dT = \frac{p}{-p} e^{-p(T-t)} = -e^{-p(T-t)}$, we find

$$\begin{aligned}\int_t^\infty \left(pe^{-p(T-t)}\right) \int_t^T \widetilde{Z}\left(y_t, \overline{\rho}_t, \rho_t^i, \overline{\psi}_t^i, s-t\right) ds\, dT &= \left[-e^{-p(T-t)} \int_t^T \widetilde{Z}\left(y_t, \overline{\rho}_t, \rho_t^i, \overline{\psi}_t^i, s-t\right) ds\right]_{T=t}^{T=\infty} \\ &\quad - \int_t^\infty -e^{-p(T-t)} \widetilde{Z}\left(y_t, \overline{\rho}_t, \rho_t^i, \overline{\psi}_t^i, T-t\right) dT.\end{aligned}$$

Under the assumption stated earlier ($Q_0(s) \to -\infty$ and $Q_i(s)$'s converge to finite numbers), we have $e^{-p(T-t)} \int_t^T \widetilde{Z}\left(y_t, \overline{\rho}_t, \rho_t^i, \overline{\psi}_t^i, s-t\right) ds \to 0$ as $T \to \infty$. From equation (22), the leading term in $\tilde{Q}_0(s)$ is linear in s, while the other terms converge to finite numbers. Thus, the properties of the integral $\int_t^T \widetilde{Z}\left(y_t, \overline{\rho}_t, \rho_t^i, \overline{\psi}_t^i, s-t\right) ds$ as $T \to \infty$ are determined by a term of the form $\int_t^T e^{m(s-t)} ds$ for some constant m determined as part of the solution of (22). Under the assumptions stated earlier, $e^{-p(T-t)} \int_t^T e^{m(s-t)} ds = 1/m \left(e^{(-p+m)(T-t)} - e^{-p(T-t)}\right) \to 0$ as $T \to \infty$. Thus,

$$\int_t^\infty \left(pe^{-p(T-t)}\right) \int_t^T \widetilde{Z}\left(y_t, \overline{\rho}_t, \rho_t^i, \overline{\psi}_t^i, s-t\right) ds dT = \int_t^\infty e^{-p(T-t)} \widetilde{Z}\left(y_t, \overline{\rho}_t, \rho_t^i, \overline{\psi}_t^i, T-t\right) dT.$$

Substituting this back into equation (25), we find the relation (13) in Proposition 1. ■

Proof of Proposition 2: By the law of iterated expectations, the pricing function is

$$M_t^i = E_t\left[\int_t^{T_i} \frac{\pi_s}{\pi_t} D_s^i ds + \frac{\pi_{T_i}}{\pi_t} B_{T_i}^i\right] = E_t\left[E_t\left[\int_t^{T_i} \frac{\pi_s}{\pi_t} D_s^i ds + \frac{\pi_{T_i}}{\pi_t} B_{T_i}^i | \overline{\psi}_t^i\right]\right].$$

The inner expectation is computed in Proposition 1. Thus

$$M_t^i = B_t^i \left(c^i + p\right) \times E_t \left[\int_0^\infty e^{Q_0(s)+\mathbf{Q}(s)'\cdot\mathbf{N}_t+Q_5(s)y_t^2} ds\right].$$

Under the assumptions stated in the proof of Proposition 1, the integral exists. The only variable in $\mathbf{N}_t$ that is not known at t is $\overline{\psi}_t$. The claim of Proposition 2 then follows from the rules of the lognormal distribution, as

$$E_t \left[e^{Q_4(s)\overline{\psi}_t^i}\right] = e^{E\left[Q_4(s)\overline{\psi}_t^i\right]+\frac{1}{2}Var\left[Q_4(s)\overline{\psi}_t^i\right]} = e^{Q_4(s)\widehat{\psi}_t^i+\frac{1}{2}Q_4^2(s)\widehat{\sigma}_{i,t}^2}. \quad \blacksquare$$

Expected Return and Volatility. Let $M_t^i/B_t^i \equiv \Phi^i\left(\rho_t^i, \overline{\rho}_t, y_t, \widehat{\psi}_t^i, \widehat{\sigma}_{i,t}^2\right)$. Under the assumptions of Proposition 2, Ito's Lemma implies that firm i's return volatility is given by $\sqrt{\sigma_R^i \sigma_R^{i\prime}}$, where

$$\sigma_R^i = \frac{1}{\Phi^i}\left(\frac{\partial\Phi^i}{\partial y_t}\bar{\sigma}_y + \frac{\partial\Phi^i}{\partial\overline{\rho}_t}\sigma_L + \frac{\partial\Phi^i}{\partial\rho_t^i}\sigma_i + \frac{\partial\Phi^i}{\partial\widehat{\psi}_t^i}\sigma_{\widehat{\psi},t}\right), \tag{26}$$

$\bar{\sigma}_y = (\sigma_y, 0, 0)$, $\sigma_L = (\sigma_{L,0}, \sigma_{L,L}, 0)$, $\sigma_i = (\sigma_{i,0}, 0, \sigma_{i,i})$, and $\sigma_{\widehat{\psi},t} = \left(0, 0, \frac{\phi^i}{\sigma_{i,i}}\widehat{\sigma}_{i,t}^2\right)$. The formula for $\sigma_{\widehat{\psi},t}$, the diffusion vector of the posterior mean $d\widehat{\psi}_t$, follows from the Kalman-Bucy filter (e.g. Liptser and Shiryayev, 1977). Expected excess return is given by

$$E\left[dR_t^i\right] = \sigma_{R,1}^i \sigma_{\pi,t},$$

where $\sigma_{R,1}^i$ is the first element in σ_R^i, and $\sigma_{\pi,t}$ is given in equation (18).

Proposition 3: The M/B value of the old economy is given by

$$M_t^O/B_t^O = \Phi\left(\overline{\rho}_t, y_t\right) = c^O \int_0^\infty Z\left(y_t, \overline{\rho}_t, s\right) ds, \tag{27}$$

where

$$Z\left(y_t, \overline{\rho}_t, s\right) = e^{Q_0^O(s)+Q_1^O(s)y_t+Q_2^O(s)\overline{\rho}_t+Q_3^O(s)y_t^2},$$

and $Q_0^O(s) = K_0(0;s)$, $Q_1^O(s) = K_2(0;s) + \gamma a_1$, $Q_2^O(s) = K_3(0;s)$ and $Q_3^O(s) = K_6(0;s) + \gamma a_2$, where $K_i(.;s)$ are in Lemma 1, all for the parametrization $\zeta_0 = \zeta_2 = v = 1$, and $\zeta_1 = 0$.

Proof of Proposition 3: The claim follows from the same argument as in Proposition 1, but for the parameterization $\zeta_0 = \zeta_2 = v = 1$, and $\zeta_1 = 0$ in Lemma 1. The functions of time $Q_j^O(s)$, $j = 0, .., 3$, are computed as in Proposition 1. $\blacksquare$

Lemma 3. (e.g., Duffie, 1996). For any linear vector process $\mathbf{z}_t$ that satisfies

$$d\mathbf{z}_t = \left(\mathbf{A}_z + \mathbf{B}_z\mathbf{z}_t\right)dt + \mathbf{\Sigma}_z d\mathbf{W}_t, \tag{28}$$

we have

$$\mathbf{z}_{t+\tau}|_{\mathbf{z}_t} \sim N\left(\mu_{\mathbf{z}}\left(\mathbf{z}_t, \tau\right), \mathbf{S}_{\mathbf{z}}\left(\tau\right)\right),$$

where μ_z and $\mathbf{S}_z$ are given by

$$\begin{aligned}
\mu_{\mathbf{z}}\left(\mathbf{z}_t, \tau\right) &= \Psi(\tau)\mathbf{z}_t + \int_0^\tau \Psi(\tau - s)\mathbf{A}_z ds \\
\mathbf{S}_z(\tau) &= \int_0^\tau \Psi(\tau - s)\mathbf{\Sigma}_z\mathbf{\Sigma}_z'\Psi(\tau - s) ds
\end{aligned}$$

and $\Psi(\tau) = \mathbf{U}e^{\mathbf{\Lambda}\tau}\mathbf{U}^{-1}$, where $\mathbf{\Lambda}$ is the diagonal matrix with the eigenvalues of $\mathbf{B}_z$ on its principal diagonal, $\mathbf{U}$ is the matrix collecting the respective eigenvectors on each column, and $e^{\mathbf{\Lambda}\tau}$ is the diagonal matrix with $\mathrm{e}^{\lambda_{ii}\tau}$ on its principal diagonal.

(C) The Gordon growth model with an uncertain growth rate.

This section formalizes the discussion in the third paragraph of the introduction. The Gordon model assumes that the drift rate g of dividends is constant:

$$\frac{dD_t}{D_t} = g dt + \sigma_D dW_D. \tag{29}$$

We consider two different specifications of the stochastic discount factor.

C.1. The Stochastic Discount Factor Independent of the Dividend Process.

Suppose that the SDF is governed by the following process with constant drift and volatility:

$$\frac{d\pi_t}{\pi_t} = -r_f dt - \sigma_\pi dW_\pi.$$

The price of the asset is then given by

$$P_t = E_t\left[\int_t^\infty \frac{\pi_s}{\pi_t} D_s ds\right] = D_t\ E_t\left[\int_t^\infty \frac{\pi_s D_s}{\pi_t D_t} ds\right], \tag{30}$$

assuming that the expectation exists. Let $x_t = \log(\pi_t D_t)$. Ito's lemma implies that

$$dx_t = \left(-r_f + g - \sigma_\pi\sigma_D\rho_{D,\pi} - \frac{1}{2}\left(\sigma_\pi^2 + \sigma_D^2 - 2\sigma_\pi\sigma_D\rho_{D,\pi}\right)\right) dt - \sigma_\pi dW_\pi + \sigma_D dW_D,$$

where $\rho_{D,\pi}$ is the correlation between dW_D and dW_π. Using the properties of the lognormal distribution,

$$E_t\left[\frac{\pi_s D_s}{\pi_t D_t}\right] = E_t\left[e^{(x_s - x_t)}\right] = e^{-\left(r_f + \sigma_\pi\sigma_D\rho_{D,\pi} - g\right)(s-t)}.$$

The price of the asset is then

$$\begin{aligned} P_t &= D_t \int_t^\infty E_t\left[\frac{\pi_s D_s}{\pi_t D_t}\right] ds = D_t \int_t^\infty e^{-\left(r_f + \sigma_\pi\sigma_D\rho_{D,\pi} - g\right)(s-t)} ds = \frac{D_t}{r_f + \sigma_\pi\sigma_D\rho_{D,\pi} - g} \\ &= \frac{D_t}{r - g}, \end{aligned}$$

where $r = r_f + \sigma_\pi\sigma_D\rho_{D,\pi}$ is the sum of the risk-free rate and the risk premium. This is the well-known Gordon growth formula in a continuous-time framework.

When g is unknown, it follows from the law of iterated expectations that

$$P_t = E_t\left[\int_t^\infty \frac{\pi_s D_s}{\pi_t} ds\right] = E_t\left[E_t\left[\int_t^\infty \frac{\pi_s D_s}{\pi_t} ds \mid g\right]\right] = D_t\ E_t\left[\frac{1}{r-g}\right].$$

That is, the P/D ratio is equal to the expectation of the P/D ratio in the case where g is known. (This expectation exists only under the assumption that the distribution of g assigns positive likelihood only to the values of g that satisfy a transversality condition.) Note that the risk premium is unchanged compared to the case of known g. By explicitly modeling the learning process, this fact can be proven directly by adapting the results in Veronesi (2000).

C.2. The Stochastic Discount Factor Dependent on the Dividend Process.

Following Campbell (1986) and Abel (1999), we assume the existence of a representative agent with a CRRA utility over aggregate consumption, which is given by

$$C_t = D_t^{\lambda}.$$

In a dynamic economy, the SDF is given by the marginal utility of consumption

$$\pi_t = e^{-\eta t} C_t^{-\gamma},$$

where γ is the coefficient of risk aversion. The SDF then follows the process

$$\frac{d\pi_t}{\pi_t} = -\left(\eta + \lambda\gamma g - \frac{1}{2}\lambda\gamma(1+\lambda\gamma)\sigma_D^2\right) dt - \lambda\gamma\sigma_D dW_D \tag{31}$$

$$= -\left(\eta + \lambda\gamma E_t\left[g\right] - \frac{1}{2}\lambda\gamma(1+\lambda\gamma)\sigma_D^2\right) dt - \lambda\gamma\sigma_D d\widetilde{W}_D. \tag{32}$$

The process (31) is written with respect to the true Brownian motion W_D from equation (29), whereas the (equivalent) process (32) is written with respect to the Brownian motion $\widetilde{W}_D$ perceived by the agent with incomplete information about g. The equality between the processes in equations (31) and (32) follows from Girsanov's theorem. Importantly, equation (32) shows that uncertainty about g has no impact on the volatility of the SDF.

The price of the asset is given by

$$P_t = E_t\left[\int_t^{\infty} \frac{\pi_s}{\pi_t} D_s ds\right] = D_t\ E_t\left[\int_t^{\infty} e^{-\eta(s-t)} \left(\frac{D_s}{D_t}\right)^{1-\lambda\gamma} ds\right], \tag{33}$$

assuming that the expectation exists.

If g is observable, the same calculation as in Section C.1 shows that

$$\frac{P_t}{D_t} = \int_t^{\infty} e^{-\left(\eta-(1-\lambda\gamma)g+\frac{1}{2}\lambda\gamma(1-\lambda\gamma)\sigma_D^2\right)(s-t)} ds = \frac{1}{\eta-(1-\lambda\gamma)g+\frac{1}{2}\lambda\gamma(1-\lambda\gamma)\sigma_D^2}. \tag{34}$$

Note that as long as $\lambda\gamma \neq 1$, the P/D ratio is convex in g.

If g is unobservable, the law of iterated expectations implies that

$$\frac{P_t}{D_t} = E_t\left[\frac{1}{\eta-(1-\lambda\gamma)g+\frac{1}{2}\lambda\gamma(1-\lambda\gamma)\sigma_D^2}\right].$$

Due to the previously mentioned convexity, an increase in uncertainty about g (i.e., a mean-preserving spread on the density of g) leads to an increase in the P/D ratio.

By following the approach in Veronesi (2000), it is also possible to show that an increase in uncertainty decreases (increases) expected excess return if and only if $\gamma > 1/\lambda$ ($\gamma < 1/\lambda$).

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