Fiscal Rules in a Monetary Union*

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Abstract

This paper studies optimal fiscal policy rules in a monetary union where monetary policy is decided by an independent central bank. We consider a two-country model with trade in goods and assets, augmented with sticky prices, labor income taxes and stochastic government consumption. Optimal fiscal policy is a simple, linear function of last period change in debt and the underlying current shocks to the economy. It is optimal to finance an increase in government spending in part by running deficits and in part by raising income taxes, even though these are distortionary. Real public debt and taxes display random walk behavior. The optimal response of taxes to the change in debt is larger with the level of public debt so that fiscal policy is tighter for countries with higher debt-to-GDP ratios. Optimal monetary policy is less aggressive in response to a government spending shock than the policy implied by an interest rate rule; the welfare cost of monetary policy delegation is high, about 0.29 percent of steady state consumption. Optimal fiscal policy delivers lower variability of the income tax rate than a deficit limit á la Stability and Growth Pact (SGP); however, the welfare cost of the SGP is small (between 0.001 and 0.036 percent of steady state consumption) as the SGP is unlikely to bind.

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1 Introduction

The creation of the Economic and Monetary Union (EMU) in Europe has put much emphasis on the design of monetary and fiscal institutions. The Maastricht Treaty and its subsequent pacts stipulate that the European Central Bank (ECB) should be independent of the fiscal authorities and that it should pursue a 2 percent inflation target. The Stability and Growth Pact (SGP) stipulates that EMU members should not run deficits in excess of 3 percent of GDP, except in deep recessions, that fiscal policy should aim to bring the debt to GDP ratio to about 0.6 and balance the budget over the business cycle after that.

The SGP has been criticized for being too strict and for hampering automatic stabilizers, especially for those countries that joined EMU with high debt levels. On the other hand, many recognize that some form of fiscal restraint may be necessary in a monetary union to avoid excessive deficits that could raise inflation expectations and harm the credibility of the ECB. The SGP was in fact a necessary condition for the participation in EMU of countries with a good reputation in terms of fighting inflation such as Germany.

Early in 2004 the Council decided not to apply the sanctions stipulated by the SGP against Germany and France, which broke the three percent limit in the fiscal year 2003. This decision has raised doubts over the relevance of the SGP and it has opened a debate over whether fiscal limits should be abandoned altogether in EMU or whether they should be replaced and by what.

The goal of this paper is to study optimal fiscal policy in a monetary union. We find the optimal fiscal policy that can be achieved by commitment by solving the Ramsey problem. We consider a two-country model that trade in goods and assets augmented with sticky prices; both countries belong to a monetary union and the common monetary policy is decided by an independent central bank. Government spending as well as technology are stochastic and each government decides fiscal policy in its own country by setting the labor income tax and by deciding how much public debt to issue. Governments can commit to their policies.

We find that optimal fiscal policy is a simple function of the lagged deficit and the current shocks. Hence, optimal fiscal policy can be expressed as a simple and transparent rule. In response to a government spending shock, it is optimal to finance it in part by running a deficit and in part by raising income taxes. The deficit leads to a long-run increase in real public debt that also raises the income tax rate in the long run. Hence, real public debt and taxes display a random walk behavior.

Optimal fiscal policy calls for an increase in the labor income tax in response to a deficit; interestingly, this response gets stronger as the debt to GDP ratio gets higher. This implies that countries starting with higher debt levels follow tighter fiscal policies. We characterize the optimal tax schedule by finding the optimal income tax for various debt levels.

In our model monetary policy is run by an independent central bank that follows an interest rate rule. We compare the optimal fiscal policy in this setup with that of a model where both fiscal and monetary policies are chosen optimally. We find that optimal monetary policy is less aggressive in raising interest rates in response to a government spending shock than the policy that results from a Taylor-type rule. Because optimal monetary policy is less aggressive, fiscal policy is also less aggressive: the tax rate raises less and real public debt raises more in response to a government spending shock.

Because real debt and the tax rate display near-random-walk behavior, our model is non-stationary so that welfare comparison based on the linearization around the steady state are typically incorrect. We evaluate welfare in a model where stationarity is induced via a portfolio adjustment cost. In this model we find that the welfare cost of monetary delegation is equivalent to reducing steady state consumption by 0.29 percent. This is a large number.

Optimal fiscal policy in one country of the monetary union is sensitive to shocks in the other country of the union. For example, the optimal fiscal policy in the home country in response to a government spending shock in the foreign country demands an increase in the home tax rate and the magnitude is sixty percent the increase in response to a government spending shock in the home country. The international dimension, namely the responsiveness of optimal domestic fiscal policy to shocks in other countries of the monetary union, is important even though it depends from the degree of integration among the countries in the trade of goods and assets.

We characterize optimal fiscal policy under deficit limits as dictated by the SGP. We find that the SGP typically delivers a higher volatility of the income tax rate than the optimal (and unrestricted) fiscal policy. The welfare cost of the SGP is relatively small – it is below 0.036 percent for debt-to-GDP ratios below 1. The reason for such small welfare cost is that the SGP is unlikely to bind under realistic calibrations of the model.

This paper builds on the existing literature on optimal monetary and fiscal policy. This literature studies the determination of optimal monetary and fiscal policy when the government problem is to finance an exogenous stream of public consumption by levying taxes and issuing debt and money so as to maximize welfare. Optimal monetary and fiscal policies depend on the environment in which they operate. When prices are flexible, competition is perfect and government debt is state contingent, as in Lucas and Stokey (1983) and Correia and Teles (1996), government debt and tax rates have the same stochastic process of the exogenous shocks to the economy. When government debt is nominally non-state-contingent, as in Chari et al. (1991), the government finds it optimal to use inflation as a lump-sum tax on financial wealth; as a result inflation is highly volatile while the tax rate remains stable over the business cycle. Schmitt-Grohe and Uribe (2003) consider a setting with imperfect competition where the government cannot implement production subsidies to undo the distortions stemming from it; government debt is nominally non-state-contingent. They find that, as in the perfectly competitive case, the labor tax rate is smooth and inflation is highly volatile because the government uses changes in the price level as a lump-sum tax on wealth.

A number of recent papers have focused on studying optimal monetary policy in an environment with nominal rigidities and monopolistic competition. Part of his literature assumes that the government has access to lump-sum taxes to finance its consumption and that it can implement a production subsidy that eliminates the inefficiency stemming from monopolistic competition.[1] As a result, the optimal inflation rate is stable and close to zero. Intuitively, lump-sum taxes are used to finance government spending without creating distortions and prices can remain stable to minimize the costs of inflation in an environment with price rigidities. Schmitt-Grohe and Uribe (2002) depart from this literature by studying optimal monetary and fiscal policy in a model where the government can only resort to distortionary income taxation and it can issue only nominal non-state-contingent bonds. Schmitt-Grohe and Uribe find that optimal inflation volatility is almost zero even if taxation is distortionary and even for low degrees of price stickiness. Moreover, shocks to the economy induce random walk behavior in government debt and tax rates. Correia et al. (2001) enrich the set of fiscal instruments and find that the set of frontier implementable allocations is the same and it does not depend on the degree of price stickiness.

We study optimal fiscal policy when monetary policy is decided by a central bank that follows a Taylor-type rule; similarly to Schmitt-Grohe and Uribe (2002), we consider an environment with monopolistic competition, distortionary income taxation and nominal non-state-contingent government debt; our setting, however, envisions two countries in a monetary union. The random walk behavior of government debt and tax rates in our international setting in response to shocks, both domestic and foreign, is alike that in Schmitt-Grohe and Uribe (2002), confirming that it is the assumption of market incompleteness that lies at the core of such behavior. Recent work by Marcet et al. (2002) also confirms this result, as it finds a near unit-root component in government debt and tax rates in a model that retains the characteristics of Lucas and Stokey's original piece but the bond market, which is assumed to be incomplete in that the government can only buy or sell one period risk-free bonds.

A number of papers analyze the interaction of monetary and fiscal policies to determine the optimal design of monetary and fiscal institutions. Dixit and Lambertini (2003a) and Lambertini (2004) study the interaction between a conservative and independent central bank and a benevolent fiscal authority that maximizes social welfare in models where purpose of fiscal policy is to stabilize prices and output. In such setting, equilibrium outcomes are suboptimal when one or both policies are discretionary. The optimal design of institutions assigns non-conflicting goals to the authorities and, if policies are discretionary, it assigns a price goals that is appropriately conservative. Dixit and Lambertini (2001, 2003b) extend this analysis to a monetary union and find that the spillover of one country's fiscal policy on others exacerbates the suboptimality of the equilibrium. Sibert (1992), Levine and Brociner (1994) and Beetsma and Bovenberg (1998) consider monetary-fiscal interactions in a monetary union where the purpose of fiscal policy is to provide public goods. Chari and Kehoe (2004) conclude that fiscal limits are desirable in a monetary union when the central bank cannot commit in advance. We depart from this literature in two ways. First we assume that government spending is exogenous and stochastic and that the problem faced by the fiscal authority is how to finance such spending stream in the least disruptive way. Second, we assume that the fiscal authority can commit to its policies.

[1]See Erceg et al. (2000), Gali and Monacelli (2000), Khan et a. (2000), Rotemberg and Woodford (1999) and Woodford (2000, 2003).

A vast literature focuses on the Stability Growth Pact. Among these contributions, Eichengreen and Wyplosz (1998) suggest that the Stability Pact is likely to affect the fiscal behavior of the EMU members and that it is likely to partly hamper automatic stabilizers,[2] thereby wondering if the SGP will only be a "minor nuisance". Our numerical analysis shows that, indeed, the SGP is a minor nuisance, if any, because the welfare costs of harmstrung automatic stabilizers have to be weighted against the welfare gains stemming from lower long-run debt levels. Wyplosz

Several proposals to change the SGP or parts of it are being discussed – see, for example, De Grauwe (2002), Wyplosz (200, 2003), Aghion et al. (2003) and Uhlig (2002). Our paper contributes to this literature by making a concrete proposal and evaluating it with respect to the SGP.

2 The Model

We consider a world economy that consists of two countries, country 1 and country 2. These two countries are in a monetary union and therefore share a common currency. They have separate governments that run fiscal policies; monetary policy, on the other hand, is decided by a common and independent central bank. We now proceed to model country 1; country 2 is symmetric.

2.1 Consumers

The representative household in country 1 maximizes the discounted sum of utilities of the form

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} U\left(C_{1,s}, N_{1,s}, m_{1,s}\right) \tag{1}$$

where $C_{1,t}$ is consumption and $m_{1,t} \equiv M_{1,t}/P_t$ are real balances. $0 < \beta < 1$ is a discount factor. $N_{1,t}$ is labor supply

$$N_{1,t} = \int_0^n N_{1,t}(i)di$$

and $N_{1,t}(i)$ is the quantity of labor of type i supplied by the representative individual to domestic firms. It is assumed that each differentiated good uses a specialized labor input in its production and the individual supplies labor input to all domestic firms. This assumption is not necessary but convenient, as households with identical initial assets supply the same quantities of labor and receive the same labor income.

There is a continuum of differentiated goods distributed over the interval $[0,1]$; a fraction n of these goods is produced in country 1 while the fraction $1-n$ is produced in country 2.

[2] Gali and Perotti (2003), however, find no empirical evidence that the SGP has impaired fiscal stabilization.

$C_{1,t}$ is the real consumption index

$$C_{1,t} = \left[\int_0^1 C_{1,t}(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \tag{2}$$

where $C_{1,t}(i)$ is consumption of good i at time t and $\theta > 1$ is the constant elasticity of substitution among the individual goods. The representative household consumes all goods produced in the world economy. The price index P_t corresponding to the consumption index $C_{1,t}$ is

$$P_t = \left[\int_0^n P_{1,t}(i)^{1-\theta} di + \int_n^1 P_{2,t}(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}, \tag{3}$$

which is the minimum cost of a unit of the aggregate consumption good defined by (2), given the individual goods prices $P_{1,t}(i), P_{2,t}(i)$.

The representative household in country 2 has symmetric preferences to those in (1) and (2). All households in country 1 begin with the same amount of financial assets. Hence, they will have the same intertemporal budget constraints and will therefore choose the same sequences of consumption, real balances and efforts. The budget constraint for the representative agent in country 1 is

$$\frac{B^p_{1,t}}{P_t(1+i_t)} + \frac{M_{1,t}}{P_t} + C_{1,t} = \frac{B^p_{1,t-1}}{P_t} + \frac{M_{1,t-1}}{P_t} + \int_0^n \frac{W_{1,t}}{P_t}(1-\tau_{1,t})N_{1,t}(i)di + \int_0^n \Pi_{1,t}(i)di + \tau^m_{1,t}. \tag{4}$$

Here $B_{1,t}$ is the purchase of a riskless, non-contingent nominal bond. This bond is the only asset available for borrowing or lending between the two countries and $1 + i_t$ is the gross nominal interest rate. $W_{1,t}(i)$ is the nominal wage of labor of type i in period t and $\Pi_{1,t}(i)$ are real profits of the country 1 firm producing good i. We assume that each household in country 1 owns an equal share of all the firms in the country, but no shares in the firms in country 2. $\tau^m_{1,t}$ are transfers received from the household in country 1 at time t and $\tau_{1,t}$ is a distortionary tax levied by the government of country 1 at time t on the labor income of citizens of that country.

The budget constraint can be divided by P_t and be written in real terms as

$$\frac{b^p_{1,t}}{1+i_t} + m_{1,t} + C_{1,t} = \frac{b^p_{1,t-1}}{\pi_t} + \frac{m_{1,t-1}}{\pi_t} + \int_0^n w_{1,t}(1-\tau_{1,t})N_{1,t} + \int_0^n \Pi_{1,t}(i)di + \tau^m_{1,t}, \tag{5}$$

where

$$b^p_{1,t} \equiv \frac{B^p_{1,t}}{P_t}, \quad w_{1,t} \equiv \frac{W_{1,t}}{P_t}.$$

In addition, the household is subject to the transversality condition

$$\lim_{T\to\infty} q_{1,t,T} \left[\frac{b^p_{1,T+1}}{1+i_{T+1}} + m_{1,T+1} \right] = 0 \tag{6}$$

on total financial wealth, where $q_{1,t,T}$ is the stochastic discount factor for country 1 at time t with the property that

$$q_{1,t,t} = 1 \qquad \text{and} \qquad q_{1,t,T} = \Pi^T_{v=t+1} q_{v-1,v}.$$

The household solves the problem

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} \left\{ U(C_{1,s}, N_{1,s}, m_{1,s}) - \lambda_{1,s} \left[\frac{b^p_{1,s}}{1+i_s} + m_{1,s} + C_{1,s} - \frac{b^p_{1,s-1}}{\pi_s} - m_{1,s-1} - \right. \right. \tag{7}$$

$$\left. \left. - w_{1,s}(1-\tau_{1,s})N_{1,s} - \int_0^n \Pi_{1,s}(i)di - \tau^m_{1,s} \right] \right\}$$

where $\lambda_{1,t}$ is the lagrangean multiplier at time t on the agent's budget constraint (5), taking $\{i_t, w_t, \tau^m_t, \Pi_t(i), P_t(i), P_t\}$ as given. The agent faces five choices; the first-order conditions are

$$C_{1,t}(i) = \left(\frac{P_{j,t}(i)}{P_t} \right)^{-\theta} C_{1,t}, \tag{8}$$

$$U_c(C_{1,t}, N_{1,t}, m_{1,t}) = \lambda_{1,t}, \tag{9}$$

$$U_m(C_{1,t}, N_{1,t}, m_{1,t}) = \lambda_{1,t} - \beta E_t \frac{\lambda_{1,t+1}}{\pi_{t+1}}, \tag{10}$$

$$\frac{\lambda_{1,t}}{1+i_t} = \beta E_t \frac{\lambda_{1,t+1}}{\pi_{t+1}}, \tag{11}$$

$$-U_N(C_{1,t}, N_{1,t}, m_{1,t}) = \lambda_{1,t} w_{1,t}(1-\tau_{1,t}), \tag{12}$$

First, the agent chooses how to allocate consumption across the differentiated goods. This is described by (8): the optimal consumption of good i produced in country j falls as its relative price rises. The agent also optimizes with respect to $C_{1,t}$, as described in (9). The first-order condition with respect while (10) describes is the demand of real balances $m_{1,t}$, which depends negatively on the nominal interest rate. (11) is the Euler equation that describes the optimal choice of $b^p_{1,t}$ and (12) is the first-order condition with respect to $N_{1,t}(i)$ that describes the household's decision of how much labor to supply to the production of good i. Notice that this equation does not depend on i.

2.2 Firms

Goods are produced making use of labor. The production function for the goods produced in country 1 is given by

$$Y_{1,t}(i) = A_{1,t} N_{1,t}(i), \tag{13}$$

where $A_{1,t}$ is an exogenous stochastic technological factor common to all firms in country 1. Country 2 has a similar production function.

Firms maximize the present discounted value of profits. Firm i in country 1 maximizes

$$E_t \sum_{s=t}^{\infty} q_{1,t,s} P_s \Pi_{1,s}(i), \tag{14}$$

where $q_{1,t,s}$ is the stochastic discount factor for country 1 at time t with the property that

$$q_{1,t,t} = 1, \qquad \frac{1}{1+i_t} = E_t q_{1,t,t+1} \qquad \text{and} \qquad q_{1,t,s} = \Pi^s_{v=t+1} q_{v-1,v}.$$

Real profits are described by

$$\Pi_{1,t}(i) = \frac{P_{1,t}(i)}{P_t} Y_{1,t}(i) - w_{1,t} N_{1,t}(i). \tag{15}$$

Real profits are the revenues from selling the goods minus the cost of producing them, which is the real wage bill for the employed labor. The firm takes the real wage as given.

The demand faced by $i-$th producer in country 1 is

$$Y_{1,t}(i)^d = \left[\frac{P_{1,t}(i)}{P_t}\right]^{-\theta} (C_t + G_t) \tag{16}$$

where $C_t \equiv nC_{1,t}+(1-n)C_{2,t}$ is union-wide private consumption and $G_t \equiv nG_{1,t}+(1-n)G_{2,t}$ is union-wide public consumption that will be decribed in detail later.

If prices are flexible, firm i chooses its relative price every period to maximize (14). Hence, the firm chooses the current relative price so as to maximize current profits. The first-order condition implies that

$$p_{1,t} = \frac{\theta}{\theta - 1} \frac{w_{1,t}}{A_{1,t}}, \tag{17}$$

where $p_{1,t} = P_{1,t}/P_t$ is the relative price of country 1. The optimal relative prices is the markup $\theta/(\theta-1)$ over the marginal cost. The markup falls as θ, the monopolistic power of the firm, becomes smaller. Notice also that the optimal relative price does not depend on i, which implies that all firms that can set new prices choose the same price.

We assume that prices are sticky à la Calvo (1983). Every period, a fraction $\phi \in [0,1)$ of randomly chosen firms is not allowed to change the nominal prices of the goods they produce. The remaining fraction $1-\phi$ of firms set their prices optimally so as to maximize the expected present discounted value of real profits. At time t, all firms that have the opportunity of changing the price maximize

$$E_t \sum_{s=t}^{\infty} \phi^{s-t} q_{1,t,s} P_s \left[\frac{P_{1,t}(i)}{P_s} Y_{1,s}(i) - w_{1,s} \frac{Y_{1,s}(i)}{A_s}\right]. \tag{18}$$

and the associated first-order condition with respect to $P_{1,t}(i)$ is

$$\theta E_t \sum_{s=t}^{\infty} q_{1,t,s} Y_s \frac{w_{1,s}}{A_{1,s}} \left(\frac{P_{1,t}(i)}{P_s}\right)^{-\theta-1} = (\theta-1) E_t \sum_{s=t}^{\infty} q_{1,t,s} Y_s \left(\frac{P_{1,t}(i)}{P_s}\right)^{-\theta}, \tag{19}$$

where $Y_t = C_t + G_t$ is aggregate demand, which the firm takes as given. The optimal price is an average of current and expected future marginal cost and revenues. Let $\tilde{P}_{1,t}(i)$ be the optimal price level chosen at t by firm i, if such firm has the opportunity to change its price. Since all firms face exactly the same problem, they all choose the same price level and $\tilde{P}_{1,t}(i) = \tilde{P}_{1,t}$. Solving (19) for $\tilde{P}_{1,t}$, we obtain

$$\tilde{P}_{1,t} = \frac{\theta E_t \sum_{s=t}^{\infty} q_{1,t,s} Y_s \frac{w_{1,s}}{A_{1,s}} P_s^{\theta+1}}{(\theta-1) E_t \sum_{s=t}^{\infty} q_{1,t,s} Y_s P_s^{\theta}}.$$

Let $\tilde{p}_{1,t} \equiv \tilde{P}_{1,t}/P_t$. Using the expression above, we obtain that

$$\tilde{p}_{1,t} = \frac{\theta E_t \sum_{s=t}^{\infty} q_{1,t,s} Y_s \frac{w_{1,s}}{A_{1,s}} \Pi_{v=t+1}^{s} \pi_s^{\theta+1}}{(\theta-1) E_t \sum_{s=t}^{\infty} q_{1,t,s} Y_s \Pi_{v=t+1}^{s} \pi_s^{\theta}}, \tag{20}$$

where $\pi_t = P_t/P_{t-1}$ is the gross inflation rate. Under the Calvo formulation, the dynamics of prices is

$$1 = \phi \pi_t^{\theta-1} + (1-\phi)\left[n\tilde{p}_{1,t}^{1-\theta} + (1-n)\tilde{p}_{2,t}^{1-\theta}\right]. \tag{21}$$

2.3 Policymakers

There is a common central bank that runs monetary policy for the monetary union; in addition, there is a government that decides fiscal policy in each country. The central bank is instrument-independent in the sense that it chooses monetary policy freely and it does not share the government budget constraints. We assume that the central bank follows the interest rate rule

$$i_t = \bar{i} + \phi_y \left(\frac{Y_t - Y}{Y}\right) + \phi_\pi \left(\frac{\pi_t - \pi}{\pi}\right), \tag{22}$$

where $\bar{i}$ is the steady-state value of the nominal interest rate. (22) describes a Taylor rule whereby the central bank sets the nominal rate as a function of the deviations of monetary union-wide output and inflation from their steady state values. It is typically assumed that the coefficient on inflation ϕ_y is greater than one, which implies that the central bank raises the nominal interest rate in response to an increase in inflation.

The budget constraint for the central bank is

$$n\tau_{1,t}^{m} + (1-n)\tau_{2,t}^{m} = \frac{M_t - M_{t-1}}{P_t}. \tag{23}$$

The central bank rebates seignorage back to households in the two countries. Most importantly, the central bank does not share the budget constraint of the fiscal authorities.

The government in each country decides how to finance an exogenous and stochastic stream of public consumption. Government spending $G_{1,t}$ is stochastic and takes the form

$$G_{1,t} = \left[\int_0^1 G_{1,t}(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}, \tag{24}$$

and similarly for country 2, and the government demand for good i is therefore

$$G_{1,t}(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} G_{1,t}, \tag{25}$$

for all $i \in [0,1]$.

The budget constraint for the government is

$$\frac{B_{1,t}^{g}}{1+i_t} = B_{1,t-1}^{g} + P_t G_{1,t} - W_{1,t} N_{1,t} \tau_{1,t},$$

where $B^g_{1,t}$ is country 1's government nominal debt issued in period t and coming to maturity in period $t+1$. We can express the government budget constraint in real terms as follows:

$$\frac{b^g_{1,t}}{1+i_t} = \frac{b^g_{1,t-1}}{\pi_t} + G_{1,t} - w_{1,t}N_{1,t}\tau_{1,t}, \tag{26}$$

where $b^g_{1,t}$ is government real debt.

2.4 Equilibrium

In equilibrium, firms meet demand. Aggregate output in country 1 is

$$Y_{1,t} \equiv \int_0^n p_{1,t}(i)Y_{1,t}(i)\,di, \tag{27}$$

and similarly in country 2. The real net asset position of country 1 and 2 are

$$b_{1,t} = b^p_{1,t} - b^g_{1,t}, \qquad b_{2,t} = b^p_{2,t} - b^g_{2,t}. \tag{28}$$

Clearing on the bond market requires

$$nb_{1,t} + (1-n)b_{2,t} = 0, \qquad \forall t. \tag{29}$$

This implies clearing in the goods market

$$C_t + G_t = Y_t, \tag{30}$$

where

$$Y_t = nY_{1,t} + (1-n)Y_{2,t}.$$

3 The Ramsey Problem

Each period the government decides how to finance the exogenous stream of public spending. The optimal fiscal policy is therefore the sequence of tax rates $\{\tau_t\}$ associated with the equilibrium described above that maximizes the utility of the representative agent.

Formally, the lagrangean of the Ramsey problem is

$$\mathcal{L}_t = E_t \sum_{s=t}^{\infty} \beta^{s-t} \sum_{i=1}^{2} \Big\{ U(C_{i,s}, N_{i,s}, m_{i,s}) + \lambda^c_{i,s}\left[U_{c,i}(s) - \lambda_{i,s}\right] + \tag{31}$$

$$\lambda^m_{i,s}\left[\frac{b_{i,s}}{1+i_s} + w_{i,s}N_{i,s}\left(1 + \frac{U_{N,i}(s)}{U_{c,i}(s)w_{i,s}}\right) - \frac{b_{i,s-1}}{\pi_s} - G_{i,s}\right] + \lambda^p_{i,s}\left[-\tilde{p}_{i,t} + \right.$$

$$\left. + \frac{\theta E_t \sum_{s=t}^{\infty} q_{i,t,s} Y_s \frac{w_{i,s}}{A_{i,s}} \Pi_{v=t+1}^{s} \pi_s^{\theta+1}}{(\theta-1)E_t \sum_{s=t}^{\infty} q_{i,t,s} Y_s \Pi_{v=t+1}^{s} \pi_s^{\theta}}\right] + \lambda^{int}_{i,s}\left[i_s - \frac{U_{m,i}(s)}{\lambda_{i,s} - U_{m,i}(s)}\right] + \lambda^r_s\left[C_s + G_s - Y_s\right] +$$

$$+\lambda_s^\pi \left[\phi \pi_s^{\theta-1} + (1-\phi)\left(n\tilde{p}_{1,s}^{1-\theta} + (1-n)\tilde{p}_{2,s}^{1-\theta}\right) - 1\right] +$$
$$+\lambda_s^b \left[-i_s + \frac{\pi}{\beta} - 1 + \phi_y \left(\frac{Y_s - Y}{Y}\right) + \phi_\pi \left(\frac{\pi_s - \pi}{\pi}\right)\right]\Bigg\}$$

given $B_{1,t-1}, B_{2,t-1}, M_{1,t-1}, M_{2,t-1}, P_{1,t-1}, P_{2,t-1}$. The first-order conditions of the Ramsey problem are spelled out in Appendix A.

In an environment with flexible prices, it is optimal for the government to inflate the nominal debt away in the first period by choosing an infinite price level. This amounts to a lump-sum on financial wealth that is preferable to the use of distortionary taxation. To avoid this unrealistic policy, the literature on optimal policy typically assumes that the initial price level is given. To maintain comparability with the existing literature, we also assume that the initial price level is given and that inflation remains equal to zero in the first period. Notice, however, that in this setting the government would not find it optimal to choose an infinite price level in the first period: with price stickyness, a large increase in P_t generates persistent price dispersion, which reduces welfare.

The Ramsey's policy functions form a dynamic system that cannot be solved analytically. We linearize the model around the non-stochastic steady state and present quantitative results.

4 Calibration

Table 4 summarizes the parameter values used in our simulation. The time unit is meant to be a quarter. We assume that, in period 0, the monetary union is at the non-stochastic steady state associated with the equilibrium described in 2.4. Hence, the monetary union is at an equilibrium with constant consumption, output, taxes, government spending and inflation rate. In our setup, the optimal inflation rate is zero: inflation creates price dispersion that harms social welfare. We assume that the central bank's inflation goal is indeed zero. The debt-to-GDP ratio in country 1 is assumed to be 0.4 in the benchmark simulation; however, we are going to consider and simulate our economies for different debt-to-GDP ratios. Government consumption is assumed to be 20% of GDP in the steady state in both countries, as consistent with post-war Germany. We set the discount factor β to 0.99, which is consistent with a steady-state real rate of return of 4.1 percent a year.

We assume a period utility function

$$U(C, N, m) = \log C + d \log(1 - N) + \chi \log m.$$

We set the parameter χ in the utility function so that real balances are 5 percent of consumption in the steady state. The parameter d is set equal to 2, which implies that the representative agent works one third of her total time in the steady state. Sbordone (2002) and Galí and Gertler (1999) suggest that the parameter ϕ that summarizes the degree of price staggering is set equal to 2/3; this implies that firms on average change prices every three quarters; Bils and Klenow (2004), on the other hand, suggest a much shorter average

Parameter	Value	Description
β	0.99	Subjective discount factor
π	1.0	Gross inflation rate
d	2	Calibrated to match $N = 0.3$
χ	0.001	Calibrated to match $m = 0.05C$
g/Y	0.2	Government consumption to GDP ratio
b/Y	0.4	Debt to GDP ratio
θ	11	Calibrated to match 1.1 gross value-added markup
ϕ	1/3	Degree of price stickiness
n	0.5	Size of country 1
ϕ_y	0.5	Coefficient on output
ϕ_π	1.5	Coefficient on inflation
ρ_g	0.88	Serial correlation of $\ln g_t$
σ_g	0.02	Standard deviation of innovation to $\ln g_t$
ρ_a	0.95	Serial correlation of $\ln a_t$
σ_a	0.01	Standard deviation of innovation to $\ln a_t$

lifespan of prices – about 4.3 months. I therefore set the parameter ϕ equal to 1/3, that implies an average life span of prices a bit higher than one quarter. The mark-up parameter θ is set equal to 11, so that steady-state mark-up is 10 percent, as consistent with the work of Basu and Fernald (1997) and as used in other works that assume price staggering (see for example Gali (2001)).

Government spending is assumed to follow the stochastic process

$$\ln G_{i,t} = (1 - \rho_g) \ln G_i + \rho_g \ln G_{i,t-1} + \epsilon^g_{i,t}, \quad i = 1, 2. \tag{32}$$

$\epsilon^g_{i,t}$ are i.i.d with normal distribution with mean 0 and standard deviation 0.02; we assume that $\rho_g = 0.88$. This calibration is consistent with the government consumption process of post-war Germany. Technology is assumed to follow the process

$$\ln A_{i,t} = (1 - \rho_a) \ln A_i + \rho_a \ln A_{i,t-1} + \epsilon^a_{i,t}, \quad i = 1, 2. \tag{33}$$

$\epsilon^a_{i,t}$ are i.i.d with normal distribution with mean 0 and standard deviation 0.01; ρ_a is set equal to 0.95. The central bank is assumed to follow a Taylor rule; the parameter ϕ_y is set equal to 0.5 and the parameter ϕ_π is set equal to 1.5. These are the values suggested by Taylor that have been shown to represent well monetary policy in the U.S. as well as other industrialized countries – see Clarida, Gali and Gertler (1998). Table 4 summarizes our choices for the parameters of the model.

5 Results

Figure 1 shows the impulse response function for country 1 in response to a one standard deviation percentage increase in government consumption in country 1. Remember that, in period 0, we constrain prices and therefore inflation and the nominal interest rate not to change in response to a government consumption shock. This explains why there is no response of inflation and the nominal interest rate to the government spending shock at time 0. If we were to eliminate this restriction, the nominal interest rate and inflation would increase at time 0 without changing much the impulse response function of the other variables.

It is optimal for the government to finance an increase in government consumption in partartially by running deficits and partially by raising the labor income tax. The intuition is simple. A sharp increase in the tax rate to balance the government budget in response to a spending shock would reduce labor. On the other hand, an increase in public debt does mitigates the need for a sharp increase in taxation and allows the government to spread the tax distortions over time.

The household anticipates that public debt and the tax rate will be higher in the new steady state. As a result, consumption falls on impact while labor increases, even though current taxes are higher than future ones. The increase in government spending boosts demand which, in turn, raises output in country 1 as well as in country 2. In this setup, a number of producers are unable to adjust their prices in response to a demand shock but instead increase production to meet demand at the established price. Hence, government spending is expansionary here.

Following an increase in government consumption in country 1, monetary-union wide output and inflation raise above their steady state levels. The central bank tightens moneary policy by raising the nominal interest rate.

In Chari et al. (1991), where prices are fully flexible, the government finds it optimal to respond to a government consumption shock by changing prices and making inflation volatile but keeping labor income taxes remarkably constant. There two important difference between our environment and the one in Chari et al.İn our model monetary policy is run by a central bank that follows an interest rate rule. Second, volatile inflation is bad here because price changes are persistent and distort consumption choices for the household and labor choices for the firms.

Schmitt-Grohe and Uribe (2003) also find that it is optimal for the government to run deficits and raise labor tax rates. Our model, however, is different from theirs in a number of dimensions. We have price staggering while they have a price adjustment cost and our model has a different production structure. Moreover, in our model monetary policy is run by a central bank that follows an interest rate rule.

In our model real public debt and taxes display random walk behavior. Following a shock to government consumption, real public debt increases to a new steady state level; as a result, the steady-state tax rate must also increase, thereby affecting labor, consumption and prices. In fact, labor increases while consumption falls in the new steady state. This implies that our local approximation technique becomes more inaccurate the longer the horizon of our simulations.

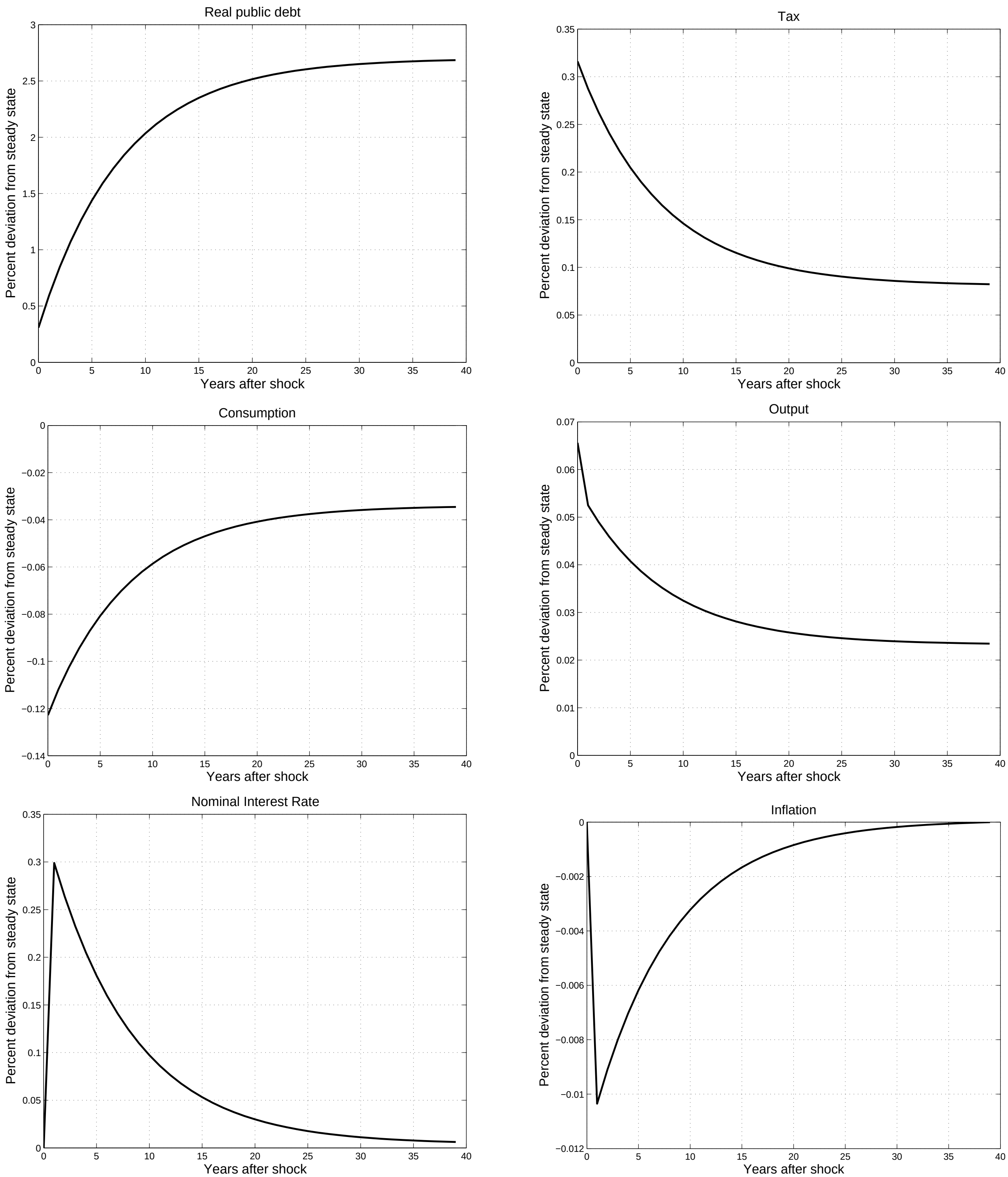

Figure 1: Impulse response to a government consumption shock

6 Welfare Comparisons

We wish to compare welfare under alternative policy specifications. To do so, we assume that the economy is at a steady state at time t. Let the sequence of consumption, labor and real balances under the benchmark regime labeled as c_t^b, N_t^b, m_t^b, respectively. Welfare under the benchmark regime is therefore

$$W_t^b \equiv E_t \sum_{s=t}^{\infty} \beta^{s-t} U(c_s^b, N_s^b, m_s^b). \tag{34}$$

This is the utility of the representative agent from time$t$ on under the benchmark regime. Assuming the economy starts at the same steady state, welfare under policy regime a is

$$W_t^a \equiv E_t \sum_{s=t}^{\infty} \beta^{s-t} U(c_s^a, N_s^a, m_s^a). \tag{35}$$

Let Ψ denote the welfare cost of adopting policy a instead of sticking to the benchmark. We measure Ψ as the percent reduction in steady-state consumption under the benchmark regime that makes the agent as well off as under policy a. Formally, this implies

$$W_t^a = E_t \sum_{s=t}^{\infty} \beta^{s-t} U((1-\Psi)c_s^b, N_s^b, m_s^b).$$

With an additive period utility function and constant elasticity of substitution of consumption specification of the following type

$$U(c, N, m) = \frac{c^{1-\sigma} - 1}{1-\sigma} + v(N) + g(m),$$

we can easily solve for Ψ and obtain the following expression for the welfare cost associated with policy a relative to the benchmark policy:

$$\Psi = \left\{ 1 - \left[\frac{(1-\beta)(1-\sigma)(W_t^a - W_t^b)}{c^{1-\sigma}} + 1 \right]^{\frac{1}{1-\sigma}} \right\} \times 100, \tag{36}$$

where c is steady-state consumption under the benchmark policy. When $\sigma = 1$ and the utility from consumption has log form, the expression (36) simplifies to

$$\Psi = \left[1 - \exp^{(1-\beta)(W_t^a - W_t^b)} \right] \times 100.$$

7 Monetary Policy Delegation

It is natural to ask how different monetary and fiscal policy would be if the benevolent government were in charge not only of fiscal but also of monetary policy. Under this scenario,

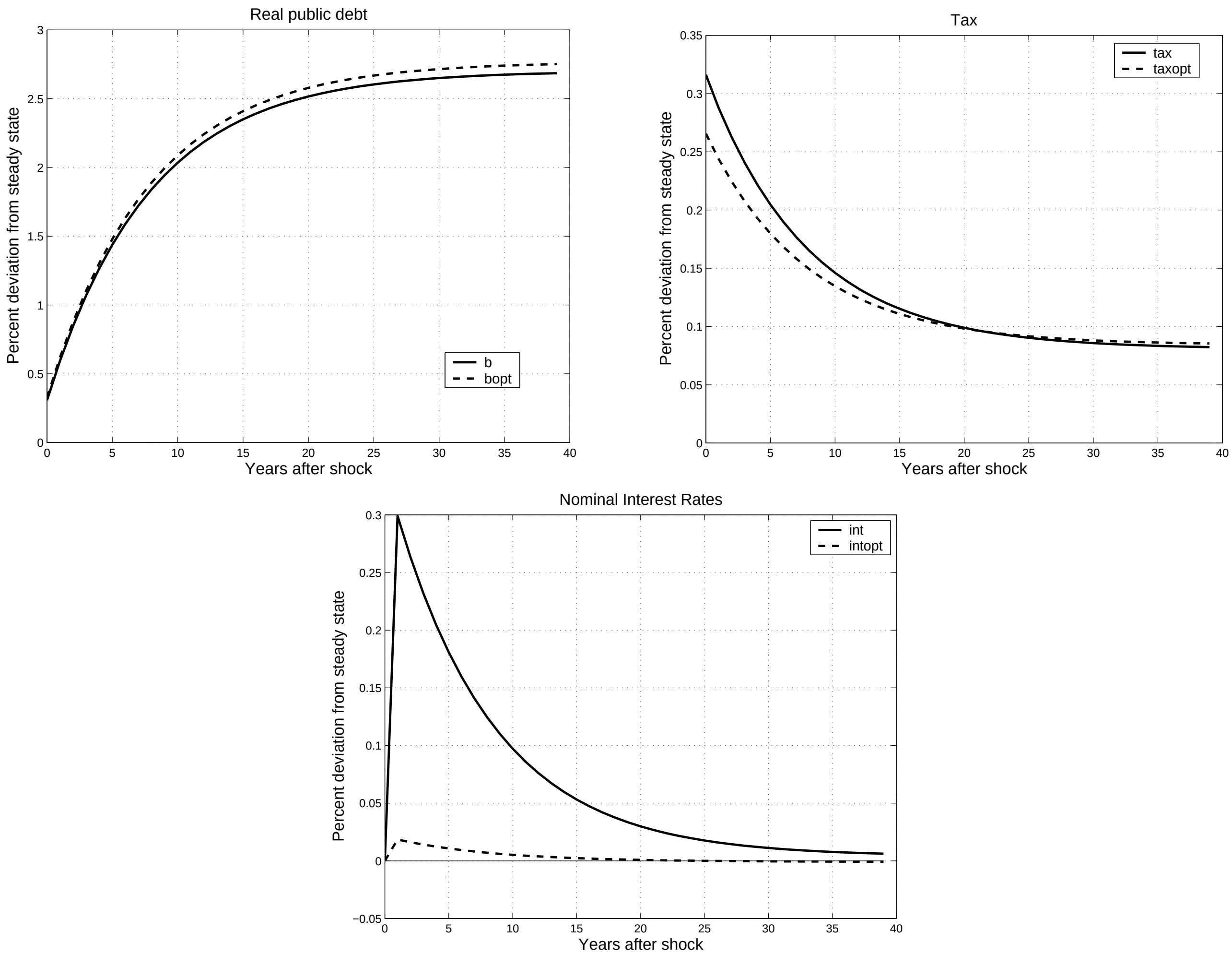

Figure 2: Impulse response to a government consumption shock

the Taylor rule of (22) does not hold, the government chooses directly monetary policy and seignorage becomes part of the revenues of the government.

Figure 2 compares the impulse response functions to a government spending shock in country 1 when the government chooses monetary as well as fiscal policy and when government chooses only fiscal policy. We assume that the economy is at the same non-stochastic steady state of section 4; the parameters are also assumed to be those of table 4.

If the government can choose monetary policy, it is more aggressive in raising inflation and therefore the nominal interest rate in response to a government spending shock than a central bank following a Taylor rule. On the fiscal side, the government reduces the tax rate on impact so that output (not plotted) increases in response to a government spending shock; as a result, real public debt increases more than under the scenario of section 4.

8 Fiscal Policy Rules

The optimal fiscal policy is a function of the state variables of the economy; when we log-linearize our model, optimal fiscal policy is a linear function of the state variables. The optimal tax rate can be written as

$$\tau_t = \Delta x_{t-1} + \Omega z_t, \tag{37}$$

where x_{t-1} is a column vector of the state variables, lagged once, of the system and ω is a column vector of the exogenous stochastic variables. For the benchmark economy of table 4,

$$x_{t-1} = [b_{t-1}]$$

and

$$z_t^{'} = [a_{1,t}, g_{1,t}, a_{2,t}, g_{2,t}].$$

The row vector Δ summarizes the impact of the state variables on the current tax rate and the row vector Ω describes the impact of the exogenous stochastic variables on the current tax rate. For our economy:

$$\tau_t = \Delta b_{t-1} + \Omega z_t, \tag{38}$$

where, for the benchmark economy of table 4:

$$\Delta = 0.029 \qquad \Omega^{'} = [2.17, 0.32, -0.33, 0.21]$$

In words, the optimal tax increases 0.029 percent with respect to its steady state value in response to a one percent deviation of lagged real public debt from its steady state value and 0.32 percent in response to a government spending shock in country 1. The other elements of the matrix Ω have a similar interpretation.

The solution to the Ramsey problem gives the optimal fiscal policy associated with a given initial steady state of the economy. The fiscal rule discussed just above is associated with an initial debt-to-GDP ratio of 40 percent for country 1. To check the robustness of our results, we simulate our model for different initial debt levels and characterize the optimal fiscal policies associated with them.

Figure 3 shows the debt and tax rate response to a government spending shock associated with four different initial levels of the debt to GDP ratio: 0.40, 0.60, 0.80 and 1. The lines plotted in figure 3 show the percent deviations from steady state. In all cases optimal fiscal policy requires an increase of the tax rate and issuing debt. Interestingly, the percent response of the debt is higher for lower debt to GDP ratios. Intuitively, countries with lower debt to GDP ratios have more breathing space so their debts can increase more (in percent terms) in response to a government spending shock. As for the tax rate, countries with lower debt to GDP ratios display a stronger short-run response of the tax rate; in the long run, however, the tax rate of high debt countries remains higher. Countries with lower debt levels haver lower tax rates in the steady state and can therefore afford to raise tax rates more aggressively in the short run; countries with higher debt level, however, cannot do so because

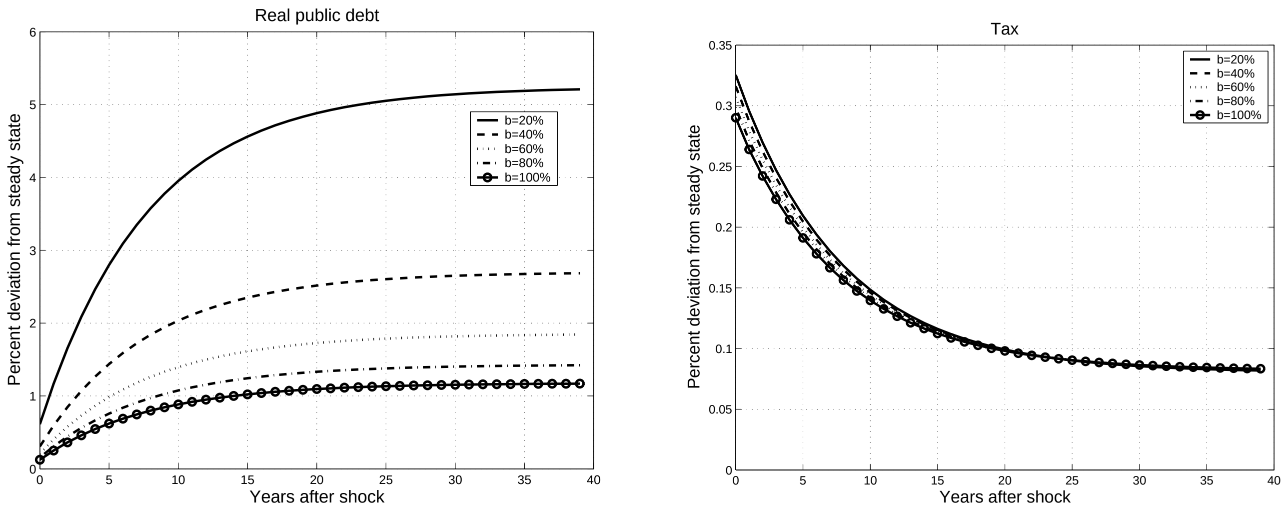

Figure 3: Impulse response to a government consumption shock: real public debt

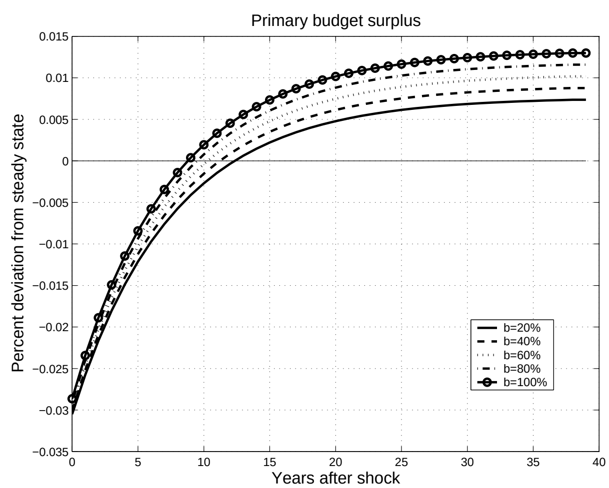

Figure 4: Impulse responseof primary surplus to a government consumption shock

this would excessively reduce labor supply. Figure 4 shows the primary surplus respons to a shock in government spending for initial debt levels; the lines here are *not* percent deviation from steady state but levels. It shows that countries with higher debt levels run a tighter fiscal policies in response to a shock

Let $\Omega' = [\Omega_1, \Omega_2, \Omega_3, \Omega_4]$. Table 8 shows the parameters of the fiscal rule for different steady state levels of the debt to GDP ratio.

The optimal tax responds positively to an increase in real public debt and this response becomes stronger as the steady-state-debt to GDP ratio is higher. A one percent deviation of last period real public debt from its steady state value triggers a 0.016 percent deviation of the tax rate if debt is 20 percent of GDP, a 0.029 percent deviation if debt is 40 percent

Debt to GDP ratio	Δ	Ω_1	Ω_2	Ω_3	Ω_4
0.2	0.016	2.24	0.32	-0.33	0.21
0.4	0.029	2.17	0.32	-0.33	0.21
0.6	0.044	2.10	0.32	-0.33	0.20
0.8	0.058	2.03	0.30	-0.34	0.19
1	0.070	1.98	0.29	-0.34	0.18

of GDP, a 0.044 percent if debt is 60 percent of GDP, 0.058 percent if debt is 80 percent of GDP and 0.07 percent if debt is 100 percent of GDP. Hence, the optimal tax response increases more than proportionally to an increase in real public debt.

The optimal tax policy also responds contemporaneously to the exogenous shocks in the economy. An increase in government spending at home calls for an increase in the tax rate because it is optimal to finance it in part by raising tax revenues and in part by running budget deficits. An unanticipated technological improvement at home also requires an increase in domestic labor income taxes. Intuitively, labor supply raises at home in response to a temporary technological shock that raises productivity and wages in the short run while consumption and leisure fall; as a result, the optimal labor tax increases leading to budget surpluses and an improved net asset position for the country. In fact, the optimal tax typically moves in the same direction as wages in our setting: when the real wage improves, it is optimal to raise the tax rate.

9 The International Dimension

Optimal fiscal policy responds to shocks at home as well as in the other member country in the monetary union. The interest rale we have postulated for the economy responds to output and inflation movements in the monetary union; by its own design, the interest response to idiosyncratic shocks in a monetary union is smaller than it would arise if each country had its own independent monetary policy; at the same time, the interest rate response to a shock in the other country in the union is typically larger than it would be if country 1 had its own and independent monetary policy.

Table 8 shows that the optimal response to a technological improvement in country 2 requires a contemporaneous reduction in the labor tax rate in country 1; as debt in country 1 grows, the tax rate will increase to reach a new and higher steady state level. In fact, a technological improvement in country 2 while technology is unchanged in country 1 raises the relative price of country 1 goods, thereby curtailing the demand for goods produced in it. Since labor demand falls, real wage also falls in country 1. A reduction in the labor tax rate raises the incentive to work and smoothes the labor supply response and, as a result, the output response. However, public finances deteriorate in this process so that the tax rate must be higher in the new steady state.

Similarly, a government spending increase in country 2 is optimally matched by an increase in the tax rate in country 1. Country 1 experiences an increase in the demand of its goods, which in turn raises output, labor and wages and makes it optimal to raise taxes in the short run. The government in country 1 runs budget surpluses that improve the country's long run net asset position, thereby leading to higher private consumption, lower labor supply and lower income tax rates at the new steady state.

The two countries of our model interact on the asset and on the good markets; a current account surplus in country 1 must be necessarily matched by a current account deficit of the same size in country 2. Extending the model to more countries is likely to reduce the responsiveness of optimal policy in a country to idiosyncratic shocks occurring in the other countries, even if these countries are all members of a monetary union. Nevertheless, we believe that the qualitative features of our analysis will remain in a more general framework.

10 A model with portfolio adjustment costs

Real public debt and tax rates display a near-random walk behavior when they are optimally chosen. This implies that temporary shocks have long-run effects on the economy. As a result, the unconditional variance of variables such as consumption and output is infinite. More generally, endogenous variables move in an infinitely large region in response to bounded shocks. This has important implications on the validity of the computed paths, which are attained through a linearization around a steady state and are therefore valid locally around a given stationary path.

The most important implications of the random walk property of our model's dynamics is that it makes welfare calculations less reliable. In stationary problems, our computation techniques can easily be used to calculate the welfare implications of certain shocks or policies. In a non-stationary problem, however, the economy does not return to the same steady state after a temporary shock. Because our techniques are valid locally, the welfare numbers we obtain are simply not reliable.

A number of modifications to the standard model have been suggested with the purpose of inducing stationarity of the equilibrium dynamics. Such modifications include assuming an endogenous discount factor, allowing for a debt-elastic interest-rate premium or for convex portfolio adjustment costs, or modelling complete markets. Schmitt-Grohe and Uribe (2003) analyze the extent to which these modifications affect the equilibrium dynamics at business-cycle frequencies in a small open economy. They find that all modifications deliver virtually identical dynamics, with the only noticeable difference being that complete markets induce smoother consumption paths.

In this section, we therefore induce stationarity of the equilibrium dynamics by allowing for convex portfolio adjustment costs. The dynamics of real public debt and tax rates are obviously going to be different – and stationary; as a result, other endogenous variables' dynamics is also going to be stationary. Nevertheless, the short-run response of the endogenous variables is very similar to the one studied in the non-stationary model.

We assume agents face convex costs of holding assets in quantities different from a specified long-run level. More precisely, the government faces costs

$$IC^g_{1,t} = \frac{I_1}{2}\left(b^g_{1,t} - b^g_1\right)^2, \tag{39}$$

and private agents face costs

$$IC^p_{1,t} = \frac{I_1}{2}\left(b^p_{1,t} - b^p_1\right)^2, \tag{40}$$

where $I_1 > 0$, for holding assets at a level different from the initial steady state. The revenues from the portfolio costs in (39) and (40) are rebated back to consumers in country 1 in a lump-sum manner. Similar expressions hold for country 2. The government budget constraint with convex portfolio adjustment costs is

$$\frac{b^g_{1,t}}{1+i_t} = \frac{b^g_{1,t-1}}{\pi_t} + G_{1,t} - w_{1,t}N_{1,t}\tau_{1,t} + IC^g_{1,t}, \tag{41}$$

which replaces (26); the budget constraint for private agents is

$$\frac{b^p_{1,t}}{1+i_t} + m_{1,t} + C_{1,t} + +IC^p_{1,t} = \frac{b^p_{1,t-1}}{\pi_t} + \frac{m_{1,t-1}}{\pi_t} + \int_0^n w_{1,t}(1-\tau_{1,t})N_{1,t} + \int_0^n \Pi_{1,t}(i)di + \tau^m_{1,t}. \tag{42}$$

Figure 5 shows the impulse response of real public debt, tax, output, consumption, nominal interest rate and inflation with portfolio costs for the benchmark model.[3] The impulse response functions look extremely similar to those without portfolio adjustment costs shown in figure 1 except for the long-run behavior.

Figure 6 shows the impulse response function of real public debt, tax and nominal interest rate to a government consumption shock in country 1 when all policies are chosen optimally. Once again, the responses look qualititatively similar to those in figure 2, which were obtained without portfolio adjustment costs. When monetary policy is chosen optimally, nominal interest rates react less aggressively to an increase in government consumption; as a result optimal fiscal policy relies more on budget deficits and less on raising the income tax rate, which makes the economy expand more and consumption contract less relative to the case where monetary policy follows an interest rate rule. We can now compare welfare under the alternative regimes where all policies versus fiscal policies only are chosen optimally. We run 1000 simulations of the model with debt equal to 40 percent of GDP in the steady state under the assumption that both monetary and fiscal policies are chosen optimally; using the same underlying shocks, we then simulate the model under the benchmark regime. We find that the welfare cost associated with delegating monetary policy is equal to 0.29 percent of steady state consumption. This means that households in country 1 are willing to cut 0.29 percent of their steady state consumption to avoid switching to a regime where both monetary policy is delegated to a common central bank that follows an interest rate rule.

[3] We assign the value of 0.1 to I_1 and I_2, which implies that a one percent deviation of real public debt from its steady state value generates a portfolio cost equal to 0.001 percent of steady-state real public debt.

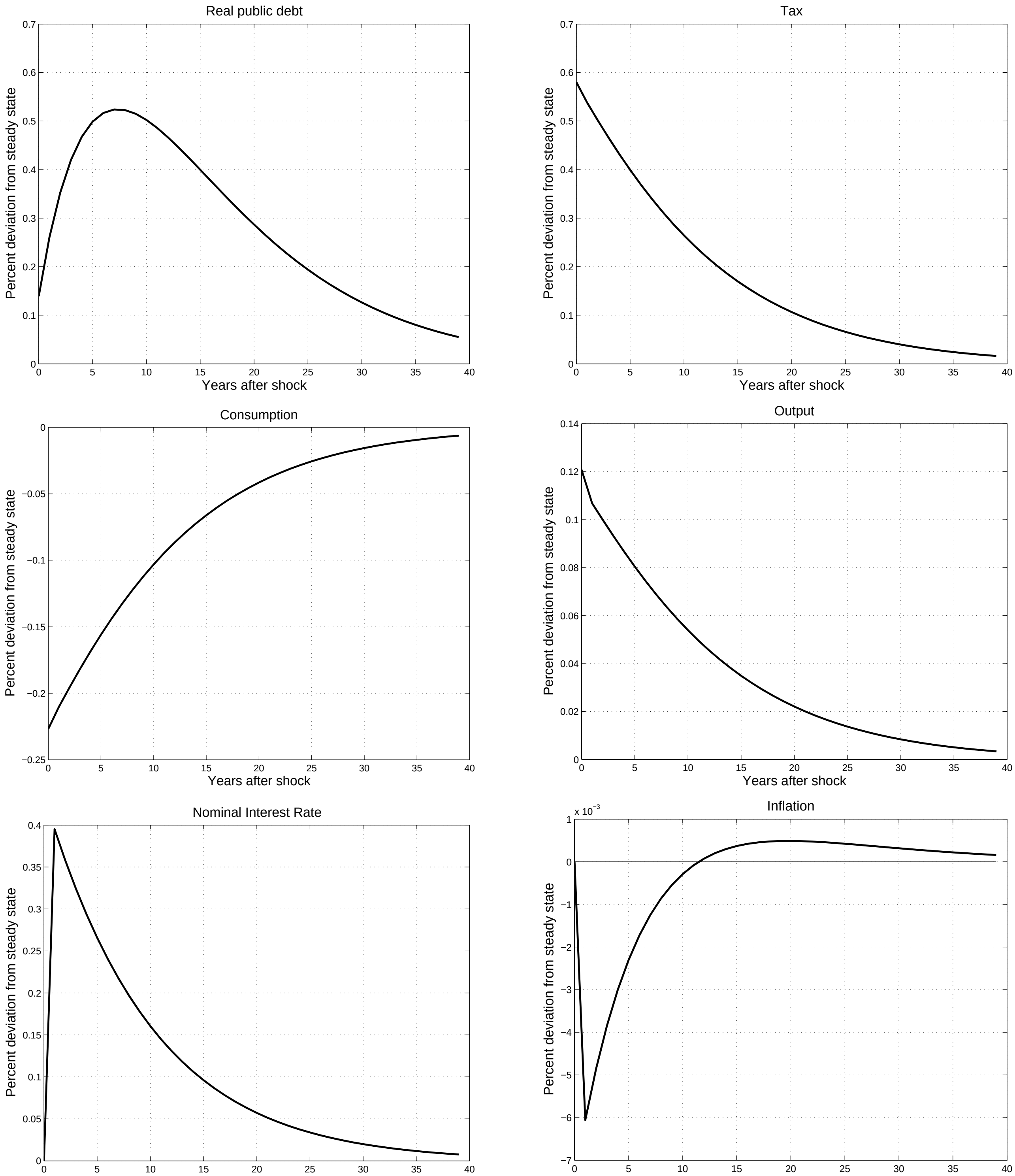

Figure 5: Impulse response to a government consumption shock with portfolio adjustment costs

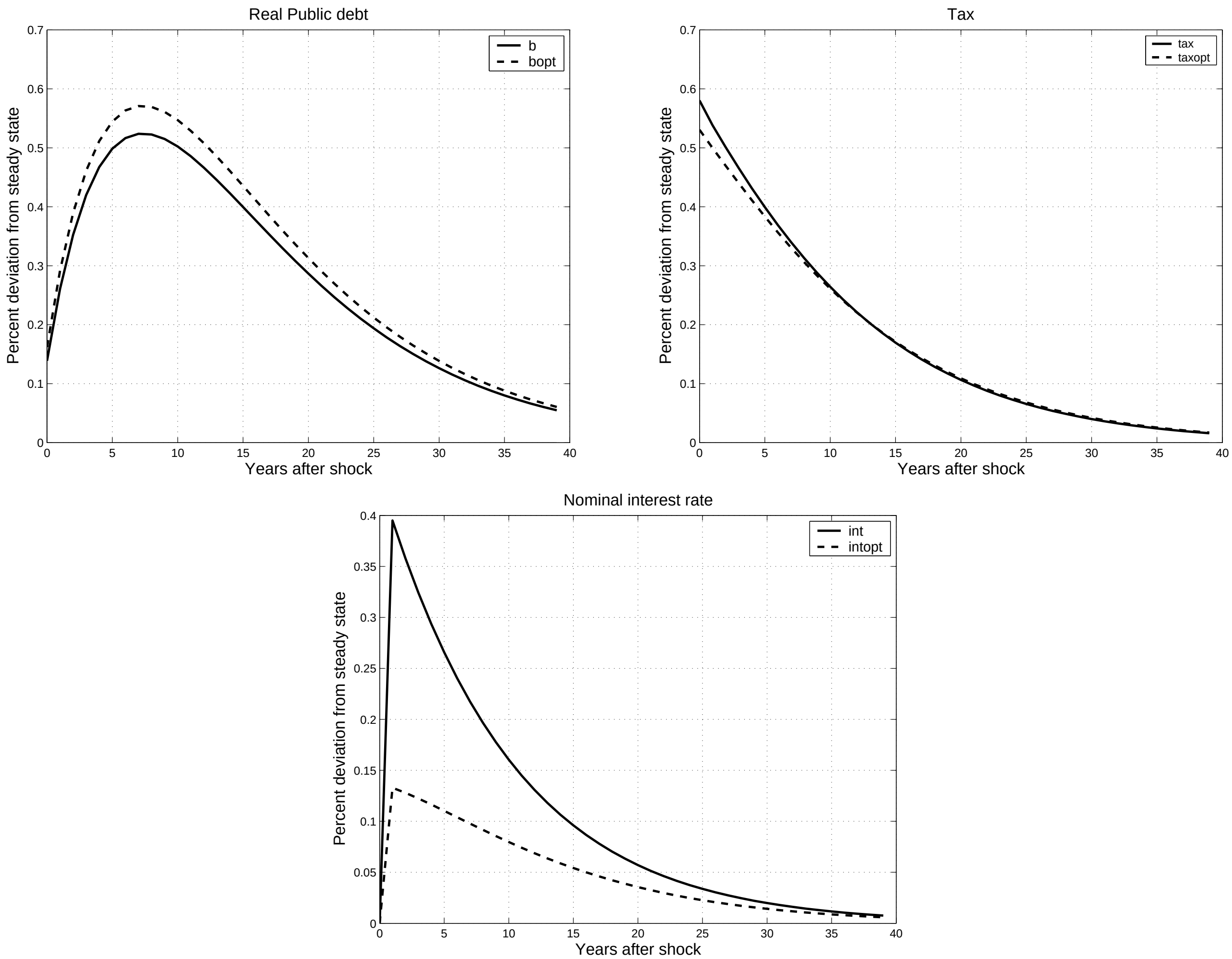

Figure 6: Impulse response to a government consumption shock with portfolio adjustment costs

11 The Stability and Growth Pact

The Maastricht Treaty established that the members of EMU should not have deficits in excess of 3 percent of GDP except during sharp recessions. More precisely, the Stability and Growth Pact (SGP) dictated that deficits in excess of 3 percent of GDP could be run only in years when GDP growth is - 2 percent or lower;[4] failure to do so would result in sanctions equal to 0.2 percent of GDP, which would then be turned into fines if the excessive deficit has in the view of the Council not been corrected.

The SGP has been criticized for being too strict and forcing member countries to run primary surpluses even during recessions. A Country with a high debt-to-GDP ratios must commit a larger component of its budget for interest payment on the debt, thereby leaving

[4]Or -0.75 percent with the concurrence of the Council.

less or no room at all for primary deficits. This was the case for Italy, which run primary surpluses between 3 and 6 percent of GDP between 1999 and 2003 even though real GDP grew very little, in the order of 0.3 percent in the years 2002 and 2003. On the other hand, the SGP has been welcomed by some as a mean to obtain fiscal discipline in the Eurozone and enhance the credibility of the ECB. At the end of 2003 the SGP has been effectively suspended through the reluctance of France and Germany to accept the recommendationis of the European Central Bank and the Commission to manage their budget deficits to below 3 percent limit.

This section studies the welfare implications of fiscal limits as stipulated by the SGP. We characterize the economy with fiscal limits, simulate it and then compare social welfare with the SGP and without it.

The limit imposed by the SGP for country 1 can be written formally as:

$$B_{1,t} - B_{1,t-3} \leq 0.03 \times Y_{1,t} p_{1,t} P_t \qquad \text{if} \qquad \frac{Y_{1,t} p_{1,t} P_t}{Y_{1,t-3} p_{1,t-3} P_{t-3}} - 1 \geq 0.02, \tag{43}$$

for t that coincides with the fourth quarter in a calendar year; for all other quarters, the SGP does not impose any constraint. In words, the SGP imposes a deficit limit for each calendar year. Hence, the deficit limit (43) binds only in the fourth quarters of each year. Notice also that the deficit limit imposed by the SGP needs to be specified in quarterly terms, which is the time unit of our data and simulations. We have assumed that the overall annual deficit cannot exceed 3 percent of current nominal GDP; we experimented by letting the relevant GDP concept be the quarterly average in the fiscal year and it barely changed the quantitative results. Finally, we have assumed that the tax change necessary to satisfy the deficit limit in (43) is carried out entirely in the fourth quarter. In other words, if constraint (43) is binding (and it can only bind in the fourth quarter of a calendar year), the current tax is adjusted so as to bring the deficit in line with the SGP requirement. One could argue that if governments anticipate with some probability that the SGP will be binding, they will find it optimal to engage in precautionary saving and run higher surpluses early in the year, which allows to keep the labor income profile smooth. However, we believe it is realistic to consider the case where the fiscal adjustment is carried out only in the fourth quarter and we therefore abstract from precautionary saving in what follows.

In real terms, the left-hand side of (43) can be rewritten as

$$b_{1,t} - \frac{b_{1,t-3}}{\Pi_{s=0}^{3} \pi_{t-s}} \leq 0.03 Y_{1,t} p_{1,t}. \tag{44}$$

The SGP adds the non-negativity constraint (44) to the Ramsey problem only for the end-of-the-year quarters; for such quarters, the Ramsey problem becomes

$$\mathcal{L}_t = E_0 \sum_{t=0}^{\infty} \beta^t \sum_{i=1}^{2} \Big\{ U(C_{i,t}, N_{i,t}, m_{i,t}) + \lambda_{i,t}^c \left[U_{c,i}(t) - \lambda_{i,t} \right] +$$

$$\lambda_{i,t}^m \left[\frac{b_{i,t}}{1+i_t} + w_{i,t} N_{i,t} \left(1 + \frac{U_{N,i}(t)}{U_{c,i}(t) w_{i,t}} \right) - \frac{b_{i,t-1}}{\pi_t} - G_{i,t} \right] +$$

$$
\begin{aligned}
&\lambda_{i,t}^{p}\left[p_{i,t}-\tilde{p}_{i,t}\right]+\lambda_{i,t}^{int}\left[i_t+\frac{U_{m,i}(t)}{\lambda_{i,t}-U_{m,i}(t)}\right]+\lambda_t^{r}\left[C_t+G_t-Y_t\right] \\
&+\lambda_t^{\pi}\left[\phi\pi_t^{\theta-1}+(1-\phi)\left(n\tilde{p}_{1,t}^{1-\theta}+(1-n)\tilde{p}_{2,t}^{1-\theta}-1\right)\right]+ \\
&\lambda_t^{b}\left[-i_t+\frac{\pi}{\beta}-1+\phi_y\left(\frac{Y_t-Y}{Y}\right)+\phi_\pi\left(\frac{\pi_t-\pi}{\pi}\right)\right]+ \\
&\lambda_{i,t}^{sgp}\left[-b_{i,t}+\frac{b_{1,t-3}}{\Pi_{s=0}^{3}\pi_{t-s}}+0.03Y_{i,t}p_{i,t}\right]\Bigg\},
\end{aligned}
$$

where the last constraint is binding if and only if the right-hand side of (43) is satisfied, i.e. if GDP growth rate is below 2 percent. This problem can be solved applying the Kuhn-Tucker theorem, which adds condition (44) and

$$\lambda_{i,t}^{sgp}\geq 0 \tag{45}$$

for all $i=1,2$. Intuitively, the lagrangean multiplier $\lambda_{i,t}^{sgp}$ is positive in period t if the constraint is binding for country i and is equal to zero otherwise. Hence, the optimal tax rate increases when the SGP is binding, thereby reducing the budget deficit enough to satisfy constraint (44); when the SGP is not binding, the optimal tax rate is equivalent to the unconstrained one.

This model allows us to analyze the welfare consequences of a fiscal limit as in (44); we will refer to such limit as the SGP. First of all, we can ask how often would the SGP bind in our model where fiscal policy is chosen optimally and we can determine if the SGP binds more often for countries with a higher debt-to-GDP ratio in the steady state. Second, we can measure the welfare cost of the SGP.

We run 1000 simulations for the economy with a steady-state debt to GDP ratio of 0.2. Each simulation has 140 periods (quarters); in the first 100 periods the economies are hit by random technology and government shocks with processes as in (32) and (33); the last 40 periods there are no shocks and the economy goes back to the steady state. First, we run our simulations for the unconstrained economy, i.e. for the economy *without* the SGP, and we calculate the change in welfare associated with the sequence of shocks. We then run our simulations with the *same* sequence of technology and government shocks for the economy with the SGP. For each simulation, we keep track of how often the SGP binds; every time the SGP is binding, the equilibrium dynamics is dictated by the appropriate constrained system and then we calculate welfare under the SGP. Finally, we measure the welfare cost of the SGP using (36). We repeat this exercise for the economies with a steady-state debt to GDP ratio of 0.4, 0.6, 0.8 and 1.

Table 11 summarizes our findings. The first column is the debt-to-GDP ratio of country 1 at the initial steady state; the second column of table 11 shows how often the SGP binds for country 1. This is measured as the average number of years the SGP binds for country 1 expressed in percent terms. The figure 1.6 in the first row means that the SGP binds on average 1.6 percent of the years for a steady state level of debt of 20 percent of GDP, i.e. over the 25 years of our simulations, on average the SGP bound 0.4 times. The last column of

Debt-to-GDP	% binding	Ψ_1
0.2	1.6	0.0011
0.4	2.4	0.0052
0.6	3.4	0.011
0.8	4.4	0.018
1	5.4	0.036

Table 1: The welfare cost of the Stability Growth Pact

table 11 is the welfare cost of the SGP measured as the average Ψ over the 1000 simulations for country 1.

The SGP is more likely to bind at higher debt-to-GDP ratios. As the public debt ratio goes from 20 to 100 percent of GDP, the SGP is almost four times more likely to bind. In other words, the SGP binds on average 0.4 years with a 0.2 debt-to-GDP ratio and 1.4 years with a debt-to-GDP ratio equal to one in the 25 years of our simulations. On one hand, this result is surprising because optimal fiscal policy becomes tighter as the debt level goes up in the steady state. On the other hand, the SGP sets a limit to the overall, not the primary, deficit; since interest payments increase with the debt level, it is not totally surprising that the SGP also binds more frequently with higher debt levels.

The welfare cost of the SGP increases with the debt level. For our benchmark specification (where debt-to-GDP is equal to 0.4 in the steady state), households in country 1 are willing to permanently cut their consumption by 0.0052 percent to remain in a regime without the SGP. This figure goes up to 0.036 percent for a debt-to-GDP ratio of 100, but it is still a relatively small number. The reason for such small welfare cost is that the SGP seldom binds in our model.

When the limit on the deficit is binding, the tax rate jumps up and consumption falls. Figure 7 plots real public debt and taxes in country 1 for the unconstrained economy and for the economy with the SGP for one of our simulations. In this simulation, the SGP binds three times, in quarters 32, 52 and 56. The labor income tax jumps up and real public debt falls in those three instances; after that, the dynamics is remarkably similar to that of the economy without SGP.

With the SGP, the volatility of the tax rate, consumption and other endogenous variables is higher than in the absence of it. This can be clearly seen in figure 7, where the smooth behavior of the tax rate and real public debt without the SGP stands in sharp contrast with the saw-toothed behavior of the same variables under the SGP. This may explain the lack of popularity of the SGP and little political support for it. A government facing re-election *and* with a binding SGP is unlikely to raise taxes because this may jeopardize its chances of retaining power.

How does the SGP affect country 2? In our two-country model, a deficit cut in country 1 implies a deficit boost in country 2 through the equilibrium in the asset market. As a result, when country 1 raises its taxes to comply with the deficit limit, country 2 reduces its taxes! Country 2 is on average better off under the SGP. There are two reasons why

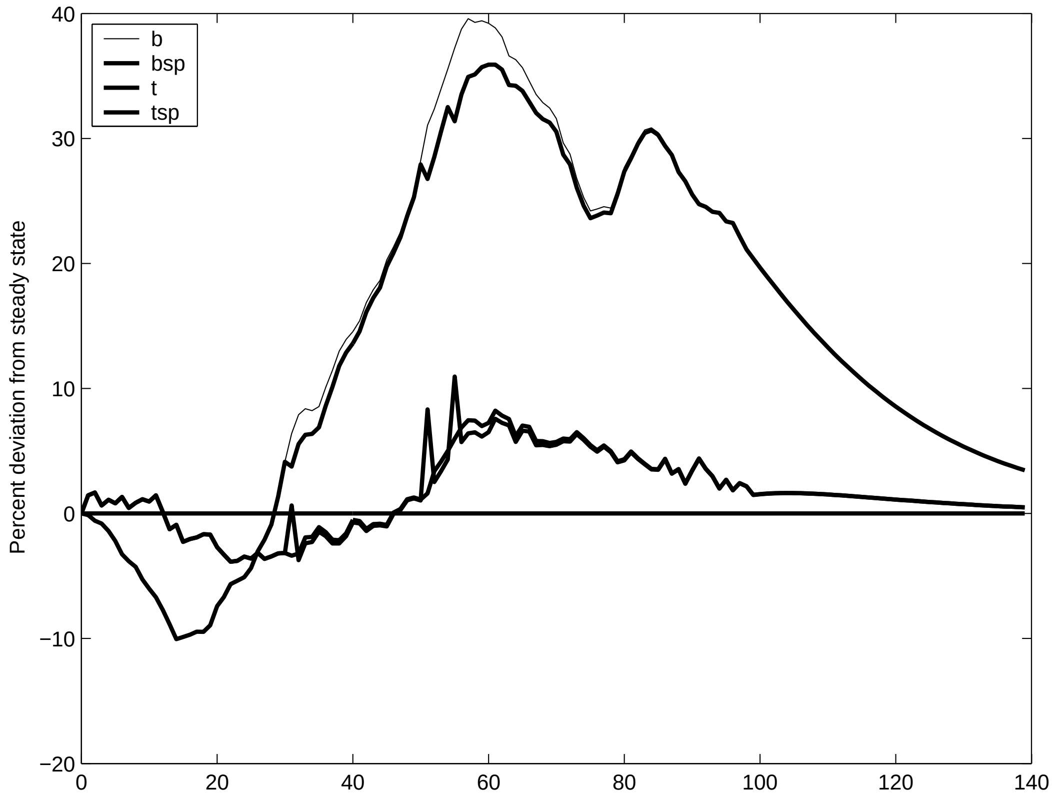

Figure 7: Real public debt and tax rate with and without the SGP

this happens. First, as just mentioned, every time the SGP binds and tax rates increase in country 1, country 2 experiences a symmetric reduction of its tax rates that is completely induced by the asset market equilibrium condition. We do not regard this result as realistic. Even the larger countries in EMU are relatively small in the international capital market, so if the SGP binds for France, Germany is not going to be significantly affected *via the asset market*. Second, our model solves for optimal fiscal policies in the monetary union. The SGP is welfare-reducing for the monetary union overall. In the absence of transfers among countries, however, removing deficit limits can make one country worse off.

12 Conclusions

We have studied optimal fiscal policy in an economy with sticky prices that consists of two countries belonging to a monetary union. The main findings of our work can be summarized as follows. First, in response to a government spending shock, it is optimal to raise taxes and run budget deficits in the country where the shock originates; the other country finds it optimal to also raise tax rates that lead to budget surpluses and an improved long run equilibrium. Second, real public debt and taxes display random walk behavior. Following a government shock, for example, the optimal fiscal policy implies an increase in real debt and therefore a worsening of the net asset position of the country. Third, the optimal fiscal policy changes with the level of debt. Optimal fiscal policy becomes tighter as the steady state debt-to-GDP ratio increases, which means that primary budget deficits gets smaller in response to shocks.

We then consider an economy with portfolio adjustment costs that induce stationarity in the fiscal variables. The short-run responses of fiscal policy to shocks is qualitatively very close to that in the non-stationary model. If monetary policy is also chosen optimally, it responds less aggressively to a shock in government spending. The welfare cost of delegating monetary policy to a central bank that implements an interest rate rule is equal to 0.29 percent of steady state consumption in our benchmark model.

Fiscal limits as dictated by the SGP have small welfare effects. The SGP imposes short-run costs stemming from higher variability of taxes; however, for reasonable values of the volatility of the underlying shocks, the SGP rarely binds.

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Appendix

A The Ramsey Problem

The Ramsey problem is given in (31). The first-order conditions related to (31) for country $i, i = 1, 2$ are spelled below.

FOC $\lambda_{i,t}^c$:

$$U_{c,i}(t) - \lambda_{i,t} = 0 \tag{A.1}$$

FOC $\lambda_{i,t}^m$:

$$\frac{b_{i,t}}{1+i_t} + w_{i,t}N_{i,t}\left(1 + \frac{U_{N,i}(t)}{U_{c,i}(t)w_{i,t}}\right) - \frac{b_{i,t-1}}{\pi_t} - G_{i,t} = 0 \tag{A.2}$$

FOC $\lambda_{i,t}^p$:

$$-\tilde{p}_{1,t} + \frac{\theta E_t \sum_{s=t}^{\infty} q_{1,t,s} Y_s \frac{w_{1,s}}{A_{1,s}} \Pi_{v=t+1}^{s} \pi_s^{\theta+1}}{(\theta-1) E_t \sum_{s=t}^{\infty} q_{1,t,s} Y_s \Pi_{v=t+1}^{s} \pi_s^{\theta}} = 0 \tag{A.3}$$

FOC λ_t^{int}:

$$i_t - \frac{U_{m,i}(t)}{\lambda_{i,t} - U_{m,i}(t)} = 0 \tag{A.4}$$

FOC λ_t^r:

$$C_t + G_t - Y_t = 0 \tag{A.5}$$

FOC λ_t^π:

$$\phi \pi_t^{\theta-1} + (1-\phi)\left(n\tilde{p}_{1,t}^{1-\theta} + (1-n)\tilde{p}_{2,t}^{1-\theta}\right) - 1 = 0 \tag{A.6}$$

FOC λ_t^b:

$$-i_t + \frac{\pi}{\beta} - 1 + \phi_y\left(\frac{Y_t - Y}{Y}\right) + \phi_\pi\left(\frac{\pi_t - \pi}{\pi}\right) = 0 \tag{A.7}$$

FOC $C_{i,t}$:

$$U_{c,i}(t) + \lambda_{i,t}^c U_{cc,i}(t) - \lambda_{i,t}^m \frac{U_{cc,i}(t)N_{i,t}U_{N,i}(t)}{U_{c,i}(t)^2} + \lambda_t^r = 0 \tag{A.8}$$

FOC $m_{i,t}$:

$$u_{m,i}(t) - \lambda_{i,t}^m \frac{b_{i,t}}{\lambda_{i,t}} u_{mm,i}(t) + \lambda_{i,t}^m + \beta E_t \frac{\lambda_{i,t+1}^m}{\pi_{t+1}} = 0 \tag{A.9}$$

FOC $N_{i,t}$:

$$U_{N,i}(t) + \lambda_{i,t}^m\left(\frac{U_{N,i}(t)}{U_{c,i}(t)} + \frac{N_{i,t}}{U_{c,i}(t)} U_{NN,i}(t) + w_{i,t}\right) - \lambda_t^r A_{i,t} p_{i,t} = 0 \tag{A.10}$$

FOC $\lambda_{i,t}$:

$$-\lambda_{i,t}^c + \lambda_{i,t}^m b_{i,t} \frac{U_{m,i}(t)}{\lambda_{i,t}^2} = 0 \tag{A.11}$$

FOC $b_{i,t}$:

$$\lambda_{i,t}^m\left(1 - \frac{U_{m,i}(t)}{\lambda_{i,t}}\right) - \beta E_t \frac{\lambda_{i,t+1}^m}{\pi_{t+1}} = 0 \tag{A.12}$$

FOC $\tilde{p}_{i,t}$:

$$\lambda_t^\pi (1-\phi)(1-\theta) p_{i,t}^{-\theta} - \lambda_{i,t}^p = 0 \tag{A.13}$$

FOC π_t:

$$\frac{\lambda_{i,t}^m}{\pi_t^2} \left(b_{i,t-1} + m_{i,t-1}\right) + \lambda_t^\pi \phi (\theta - 1) \pi_t^{\theta-2} = 0 \tag{A.14}$$

FOC $w_{i,t}$:

$$\lambda_{i,t}^m N_{i,t} + \lambda_{i,t}^p \frac{\theta Y_t / A_{1,t}}{(\theta-1) E_t \sum_{s=t}^{\infty} q_{1,t,s} Y_s \Pi_{v=t+1}^s \pi_s^\theta} = 0 \tag{A.15}$$

FOC i_t:

$$-\frac{\lambda_{i,t}^m b_{i,t}}{(1+i_t)^2} - \lambda_t^b + \lambda_t^{int} = 0 \tag{A.16}$$