CARBON SEQUESTRATION OR ABATEMENT?
THE EFFECT OF RISING CARBON PRICES ON THE OPTIMAL PORTFOLIO OF GREENHOUSE-GAS MITIGATION STRATEGIES

Klaas van 't Veld

Department of Economics, University of Michigan

Andrew Plantinga

Department of Agricultural and Resource Economics, Oregon State University

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Abstract

In this paper, we consider socially optimal carbon sequestration and abatement decisions under different expectations about future carbon prices. It is shown that if carbon prices are expected to increase over time—consistent with projections from integrated assessment models under various assumptions about future climate-policy goals—it becomes optimal to delay certain carbon sequestration projects, whereas the optimal timing of abatement projects remains unchanged. Equivalently, the opportunity cost of undertaking these carbon sequestration projects immediately—i.e., of *not* delaying them—increases relative to that of carbon abatement projects. Earlier estimates of the relative costs of carbon sequestration and abatement have, in almost all cases, been based on an assumption that prices are constant over time, in which case there is no reason to delay either type of project. The central implication of this study is that the incentive to delay carbon sequestration projects under increasing prices reduces, relative to the constant-price case, the share of carbon sequestration in an optimal portfolio of greenhouse-gas mitigation strategies, and more so, the more rapidly carbon prices are expected to increase.

This analytical result is of course relevant to climate policy only if it is quantitatively, by some measure, large. Using defensible values for the parameters in our analytical model, we find that a 3% rate of increase in carbon prices results in about a 60% reduction in the optimal share of carbon sequestration relative to constant-price projections. Simulations with our numerical model, based on predicted carbon-price paths from the Nordhaus (2001) RICE01 model for a range of climate-policy goals, indicate quantitatively similar reductions under an economically efficient scenario; somewhat larger initial reductions under a scenario that extends Kyoto-Protocol emissions limits forever; and very large reductions (80–100% for up to a century) under a scenario that aims to limit the atmospheric CO_2 concentration to double its pre-industrial level.

1. Introduction

The United Nations Framework Convention on Climate Change (UNFCCC) was adopted by a majority of the world's nations in response to global concern over human-induced climate change. The UNFCCC provides the institutional structure for efforts, continuing since its adoption in 1992, to develop, adopt, and implement an international agreement on climate change. A central, and often controversial, issue in the negotiations has been the use of terrestrial carbon sinks (e.g., forests, agricultural soils) to reduce net emissions of carbon dioxide (CO_2). The 1997 Kyoto Protocol to the UNFCCC includes provisions for industrialized nations to manage forest carbon sinks in order to meet specified emissions-reduction targets. Subsequent negotiations in The Hague in 2000 were to clarify the permit trading and carbon sink provisions of the Kyoto agreement. These negotiations failed to produce an agreement, in large part because of disagreement over how to assign credits for CO_2 reductions from carbon sinks. Agreement on the carbon sink provisions was reached at subsequent meetings in Bonn and Marrakesh in 2001, though this time without the participation of the United States. The Bush Administration, while withholding support of the Kyoto Protocol, has proposed a series of climate change initiatives that include additional funding for land conservation to enhance carbon storage.

Why have carbon sinks been at the center of international negotiations over a climate treaty? While many factors have come into play, the high level of interest in carbon sinks stems, in part, from a belief that they offer a relatively low-cost means of offsetting CO_2 emissions.[1] In studies appearing first in the late 1980s, costs of sequestering carbon in forests have been estimated for many regions of the world. The basic approach taken in most studies is to estimate the opportunity cost of taking land out of its current use (e.g., agricultural production), compute the additional carbon sequestered if the land is converted to forest, and then combine these figures to yield estimates of the average or marginal cost of sequestering carbon.

In one such study, focusing on the U.S., Stavins (1999) estimates a marginal cost schedule for carbon sequestration and proceeds to compare it with other researchers' estimates of the marginal cost schedule for energy-based carbon abatement strategies. He finds that marginal sequestration costs lie everywhere above marginal abatement costs, but up to a marginal cost level of around

[1] Carbon sink management can also complement other environmental objectives such as rainforest preservation (Pfaff, Kerr, Hughes, Liu, Sanchez-Azofeifa, Schimel, Tosi and Watson (2000)) and reducing agricultural externalities (Plantinga and Wu (2003)).

US$50 per ton of carbon (tC), the difference is quite small. This leads him to conclude that (subject to various caveats) "sequestration ought to be *part* of our overall portfolio of greenhouse strategies in the short term, providing a significant fraction of overall carbon reductions, although less than from conventional abatement activities" (emphasis in the original). A recent review article by Kauppi et al. in IPCC (2001), which summarizes a large number of sequestration cost estimates for various regions of the world, similarly concludes that "most studies, of all methodologies, suggest that there are many opportunities for relatively low-cost carbon sequestration through forestry. Estimates of the private costs of sequestration range from about US$0.10–US$100/tC, which are modest compared with many of the energy alternatives."

An assumption made in almost all of the sequestration cost studies is that the incentives for carbon sequestration are constant through time. The main objective of this paper is to show that this assumption has important implications for the costs of sequestering carbon in forests. In particular, when carbon prices are allowed to rise over time (below, we argue why this is a plausible case to consider), the marginal cost schedule for carbon sequestration shifts up relative to when carbon prices are assumed to remain constant. At first blush, it would seem that rising prices could only enhance incentives for the conversion of land of forest and, thus, cause the carbon sequestration supply curve to shift out. We show that precisely the opposite is true: any expected increase in carbon prices will induce landowners to delay conversion, thereby reducing the short-term supply of carbon sequestration. This implies a smaller contribution from carbon sequestration to an optimal portfolio of greenhouse-gas mitigation strategies relative to the constant-price case.

The intuition for this result is easiest to grasp if one assumes that conversion is always to permanent forest stands, rather than to periodically harvested ones. In this case, the trees in a given stand are young, and therefore sequestering carbon at high rates, only once over the lifetime of a project. When the price of carbon is constant, the fact that trees are young only once does not provide the landowner with any incentive to delay conversion: the landowner would prefer to have the payments for carbon sequestration sooner rather than later. When the price of carbon increases over time, however, the landowner will want to delay conversion for some range of forestry projects, because having young trees at a later date implies that higher-valued units of carbon will be sequestered at a greater rate. This result has a biological origin: it hinges on the fact that rates of carbon sequestration decline as a forest stand ages. In contrast, there is no such incentive to delay a project that offsets emissions at a constant rate over its lifetime, a characteristic of typical energy-based abatement strategies.

In Section 2, below, we provide background for this study by discussing carbon flows in managed forest stands and reviewing results from integrated models of the economy and the climate system (commonly referred to as "integrated assessment models") to support the case for rising carbon prices. Section 3 presents an analytical model of the problem when to optimally undertake a carbon sequestration or abatement project, and Sections 4 and 5 explore the implications of this model for the optimal aggregate supply of both types of projects. There, we establish the paper's key result, namely that rising prices reduce optimal sequestration supply relative to the constant-price case. The analytical model abstracts from many real-world complications, however, most notably by assuming that conversion is always to permanent forests and that the rates at which sequestration declines and carbon prices increase are constant over time. In Section 6, we therefore develop a numerical simulation model that accounts for these complications. The model combines data on actual sequestration profiles for southern pine and spruce-fir forests compiled by Birdsey (1992) with price and discount-rate paths estimated for various scenarios with Nordhaus's (2001) RICE01 integrated assessment model. In addition, the numerical model allows for harvesting of forests at endogenously determined intervals. A final section presents discussion and conclusions.

2. Background

In this section, we discuss carbon flows associated with forest management and review results on the time path of carbon prices from integrated assessment models. This provides background material for the analysis in the remainder of the paper.

Trees and other plants in the forest convert CO_2 to carbon through photosynthesis. Because more carbon is typically stored in forests than in lands used for agriculture, the conversion of agricultural land to forest achieves a net reduction in atmospheric CO_2 concentrations.[2] After the establishment of a forest stand, the total volume of carbon in the forest increases, as carbon is

[2] Other strategies for augmenting carbon sinks include modifying the management of existing forests to increase rates of tree growth and switching to cropland tillage practices that conserve organic matter in the soil. As with permanent forest stands, the rate of carbon accumulation in agricultural soils declines as a maximum level of carbon storage is reached.

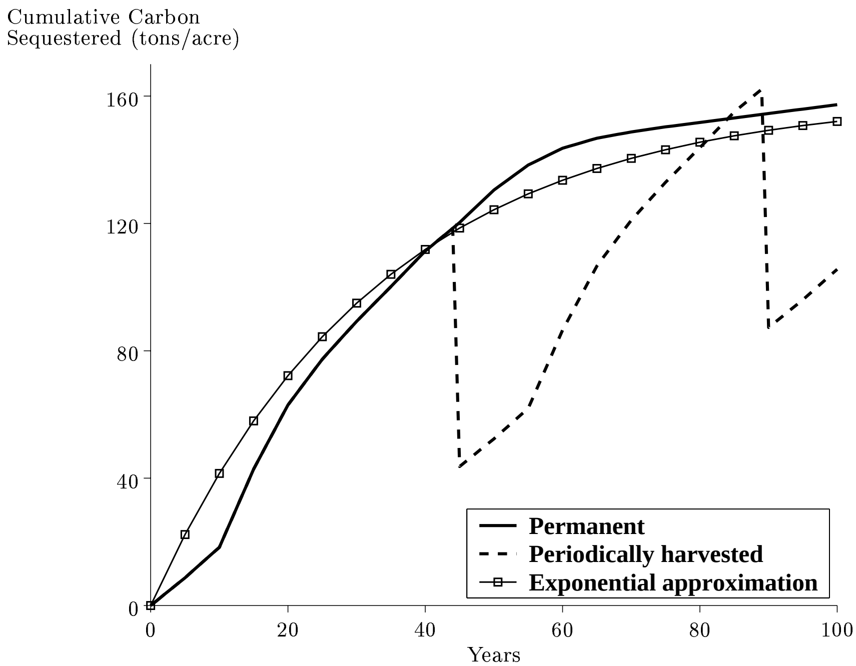

FIGURE 1. Cumulative carbon sequestered by (i) permanent stands of loblolly pine, and (ii) periodically harvested stands. Also shown is a stylized approximation with sequestration declining exponentially at constant rate $\lambda = 0.03$. Based on data from Newell and Stavins (2000).

stored in the biomass of trees and understory plants, and is deposited in forest soils and floor litter through leaf and root abscission. Following the pattern of tree growth, the rate of carbon storage increases in young stands, but then declines as the stand ages. In an old stand, forest carbon is roughly constant over time, as old trees die and provide opportunities for new trees to grow. Cumulative carbon sequestration is shown in Figure 1 (solid line) for a permanent loblolly pine stand in the southeastern U.S. planted on former cropland. For this species, the rate of carbon storage begins to decline at approximately age 20 and is close to zero by age 100.

The harvesting of trees for timber releases carbon back into the atmosphere. Within a decade following a harvest, most of the carbon in the litter, understory vegetation, and nonmerchantable portions of trees (e.g., branches) is converted back to CO_2.[3] For thirty-year old southeastern loblolly pine, this amounts to approximately 40% of the total carbon stored in the stand. Additional carbon is released when the merchantable portion of trees is processed into primary products (e.g., lumber, paper), when primary products are transformed into end-use products (e.g., housing), and when end-use products are disposed of. For sawtimber harvests in the U.S. southeast, approximately 60% of the carbon in the merchantable portion of trees has been released by 50 years after the

[3] Carbon in the soils, however, is largely unaffected by timber harvesting and thus accumulates over successive rotations.

Study	1991–2000	2001–2010	2011–2020	2021–2030
Nordhaus (1994)				
– best guess	5.3	6.8	8.6	10.0
– expected value	12.0	18.0	26.5	n.a.
Cline (1992,1993)	5.8–124	7.6–154	9.8–186	11.8–221
Peck & Teisberg (1992)	10–12	12–14	14–18	18–22
Maddison (1994)	5.9	8.1	11.1	14.7

Source: Pearce et al. (1996)

Table 1. Estimates by various studies of marginal damages from CO_2 emissions (in constant 1990 U.S. dollars/ton of carbon) after discounting to the period of emission.

harvest (Plantinga and Birdsey (1993)). The rest of the carbon can remain in end-use products and landfills for a century or more. Cumulative carbon storage is shown in Figure 1 (dashed line) for a loblolly pine stand that is periodically harvested.

We next turn to the results of integrated assessment models. Table 1 lists a number of well-known studies that have used such models to compute economically efficient policies to mitigate climate change. All these studies find that, rather than adopting the draconian measures that would be needed to stabilize marginal damages form carbon emissions any time soon, marginal damages should be permitted to increase over time, at rates that vary from 1.5% to 4% per year. The implication of these findings is that, if one believes that future climate negotiations will seek to implement an efficient policy, one should expect carbon prices to increase at roughly these rates.

It is important to note, however, that economic efficiency is only one of many goals that future climate negotiations might pursue, and is not necessarily the most likely goal. As discussed at some length by Nordhaus and Boyer (2000), because of concerns about hard-to-predict, catastrophic impacts of climate change (e.g., shifts in ocean currents or monsoons, collapsing Antarctic ice sheets resulting in sudden sea-level increases, melting permafrost resulting in runaway global warming) the public debate on climate policy is more commonly framed in terms of alternative goals, such as limiting the buildup of CO_2 in the atmosphere, or limiting the increase in global temperatures.[4] According to several studies, cost-effective implementation of these alternative goals still requires, however, that carbon prices increase over time; in fact, they should increase even faster than if the goal is economic efficiency. Goulder and Mathai (2000), for example, show that if the goal is to stabilize CO_2 concentrations by some target date in a way that optimally exploits the natural rate

[4] The UNFCCC in fact specifically identifies “stabilization of GHG concentrations in the atmosphere at a level that would prevent dangerous anthropogenic interference with the climate system” as its ultimate objective.

of removal of atmospheric CO_2, then carbon taxes or credit prices should until that date increase at a rate that strictly exceeds the rate of price increase along an economically efficient path. Moreover (and importantly for the analysis of this paper, as will become clear below), this optimal rate of price increase will strictly exceed the discount rate as well. Using an integrated assessment model, Goulder and Mathai estimate that in order to keep the CO_2 concentration below 550 ppmv (double its pre-industrial level) carbon prices should increase at initial rates of close to 6% if the discount rate is 5%. Using a different integrated assessment model, Nordhaus and Boyer (2000), arrive at essentially the same conclusion, and find that very similar initial rates of price increase are optimal if the goal is to keep global temperature increases below 2.5°C.

3. The Model

In this section, we start our analysis of optimal carbon sequestration and abatement decisions by developing an analytical model that incorporates two key, stylized facts discussed in the previous section: (i) carbon sequestration by forests tends to decline over time, and (ii) carbon prices can be expected to increase over time.

Consider then a public or private decisionmaker choosing at what time T (if ever) to undertake a project that either sequesters or abates carbon. Either type of project will involve a one-time investment cost I incurred immediately at T, and thereafter constant flow opportunity costs a. If the project in question is a carbon abatement project—e.g., converting an electric power plant from coal to an alternative, less carbon-intensive energy source—then I may include up-front construction costs, and a may include costs of maintenance and possibly higher fuel expenditures. If the project is a carbon sequestration project—e.g., converting an acre of agricultural land to forest—then I may include costs of planting trees, and a, the opportunity cost of agricultural profits forgone after T.

For analytical convenience, we assume that all abatement projects are infinitely-lived, and reduce carbon emissions by a constant amount q_0^a per period (superscript a for abatement), for which the decisionmaker receives an equivalent amount of carbon credits.[5] Similarly, we assume

[5] Alternatively, we could assume that abatement projects have a fixed, finite lifetime S, after which the decisionmaker has to renew the project by again investing I. We could then treat the infinite sequence of such projects as a single "meta-project"

that all sequestration projects involve conversion to *permanent* forest, so that trees planted at time T are never harvested.[6] Consistent with stylized fact (i) above, we assume that trees sequester carbon at a rate $q^s(t)$ (superscript s for sequestration) that declines over time, at a constant exponential rate $\lambda > 0$. Thus,

$$q^s(t) = q_0^s e^{-\lambda t}, \tag{1}$$

where q_0^s is the rate at which newly planted trees sequester carbon. (The curve labeled "exponential approximation" in Figure 1 is the cumulative of equation (1) for $q_0^s = 4.8$ and $\lambda = 0.03$.) The decisionmaker receives carbon credits for each ton sequestered, as and when that sequestration occurs. That is, at time $T + t$, the decisionmaker receives credits at exactly the rate $q^s(t)$ at which a t-year old forest sequesters carbon.[7]

Consistent with stylized fact (ii) above, we assume that the market price of carbon credits increases over time, at a constant exponential rate $\alpha \geq 0$. The carbon price at any given time $T + t$ is therefore

$$p(T + t) = p(0)e^{\alpha(T+t)},$$

where $p(0)$ is the price at time zero, when the carbon market is first established.[8] Lastly, up until Section 5 below, we assume that $\alpha < r$, where r is the constant rate at which the decisionmaker discounts the future.

Given these assumptions, the decisionmaker's optimization problem is to choose T so as to maximize the net present value of the project. Expressed as a function of T and of the rate of price

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with investment cost

$$\tilde{I} = \sum_{i=0}^{\infty} e^{-irS} I = \frac{I}{1 - e^{-rS}},$$

and the analysis of this section would go through unchanged.

[6] We relax this, and several other assumptions in Section 6.

[7] Alternatively, we could assume that the forest is harvested and re-planted (i.e., "rotated") at fixed intervals S, at which time the decisionmaker receives a stumpage value P for the harvested timber but has to re-incur the planting costs I. We could then treat the infinite sequence of rotations as a single "meta-project" with *net* investment cost

$$\hat{I} = I + \sum_{i=1}^{\infty} \left[e^{-irS}(I - P) \right] = \frac{I - e^{-rS}P}{1 - e^{-rS}}.$$

Again, the analysis of this section would go through unchanged, provided we retain the assumption that sequestration by this sequence of rotations can be roughly approximated by equation (1). (In Figure 1, a curve representing the cumulative of equation (1) for $q_0^s = 2.8$ and $\lambda = 0.016$ would provide a not unreasonable approximation to the cumulative-sequestration profile of periodically harvested stands of loblolly pine.) In Section 6, we introduce a numerical model that allows for harvesting at endogenously determined intervals, and takes into account the actual resulting sequestration profile.

[8] Although we assume for simplicity that the price path is deterministic, our main results do not depend on this assumption. See footnote 19 below for further details.

increase α, this net present value can be written as

$$NPV(T, \alpha) = \int_{T}^{\infty} e^{-rt} p(0) e^{\alpha t} q_0^i e^{-\lambda(t-T)} \, dt - \int_{T}^{\infty} e^{-rt} a \, dt - e^{-rT} I, \tag{2}$$

where for an abatement project $q_0^i = q_0^s$ and $\lambda = 0$, while for a sequestration project $q_0^i = q_0^s$ and $\lambda > 0$. Converting I to its annualized equivalent rI, and letting $c \equiv a + rI$ denote the total annualized costs, we can simplify equation (2) to

$$NPV(T, \alpha) = \left[\frac{p(0) q_0^i}{r - \alpha + \lambda} e^{\alpha T} - \frac{c}{r} \right] e^{-rT}. \tag{3}$$

To analyze the decision whether or not to undertake the project *immediately*, at time $T = 0$, two critical values of c are important. One is the critical cost level up to which immediately undertaking the project is *profitable,* in the sense that $NPV(T, \alpha) \geq 0$ at $T = 0$. Letting $\overline{c}(\alpha)$ denote this critical cost level expressed as a function of α, we have, from solving $NPV(0, \alpha) = 0$ for c,

$$\overline{c}(\alpha) = \frac{r}{r - \alpha + \lambda} p(0) q_0^i. \tag{4}$$

The other critical cost level is that up to which immediately undertaking the project is not just profitable but also *optimal,* in the sense that $NPV(T, \alpha)$ is maximized with respect to T at $T = 0$. Letting $c^*(\alpha)$ denote this critical cost level, we have, from solving $\partial NPV(0, \alpha)/\partial T = 0$ for c,[9]

$$c^*(\alpha) = \frac{r - \alpha}{r - \alpha + \lambda} p(0) q_0^i. \tag{5}$$

Consider now first the case where the decisionmaker has myopic price expectations, i.e., expects the price to remain constant forever at its initial value $p(0)$. Evaluating (4) and (5) at $\alpha = 0$ yields

$$\overline{c}(0) = c^*(0) = \frac{r}{r + \lambda} p(0) q_0^i, \tag{6}$$

showing that the two critical cost levels coincide in this case. The implication is that when $\alpha = 0$, any project with sufficiently low costs to make undertaking it profitable—in the sense of yielding non-negative net present value—should optimally be undertaken immediately in order to maximize that net present value.[10]

[9] It can be shown that $NPV(T, \alpha)$ is strictly quasi-concave in T. The condition $\partial NPV(0, \alpha)/\partial T \leq 0$ is therefore both necessary and sufficient for $T = 0$ to maximize $NPV(T, \alpha)$ subject to $T \geq 0$. Moreover, because $\partial^2 NPV(0, \alpha)/\partial T \partial c > 0$, the largest value of c for which the condition holds is that at which it holds with equality.

Consider next the case where the decisionmaker has forward-looking price expectations, i.e., expects prices to increase over time at rate $\alpha > 0$. In this case, it is immediate from comparing (4) and (5) that the critical cost levels $\overline{c}(\alpha)$ and $c^*(\alpha)$ no longer coincide. More specifically, differentiating $\overline{c}(\alpha)$, we find that

$$\frac{\partial \overline{c}(\alpha)}{\partial \alpha} = \frac{r}{(r-\alpha+\lambda)^2} p(0) q_0^i > 0.$$

This shows that, not surprisingly, an expectation of higher future carbon prices will expand the range of projects that can profitably be undertaken immediately. In contrast, however, differentiating $c^*(\alpha)$ yields

$$\frac{\partial c^*(\alpha)}{\partial \alpha} = \frac{-\lambda}{(r-\alpha+\lambda)^2} p(0) q_0^i \leq 0,$$

where the inequality is strict if $\lambda > 0$. This shows that the range of projects that will *optimally* be undertaken immediately remains unchanged in the case of abatement projects, but actually shrinks in the case of sequestration projects.

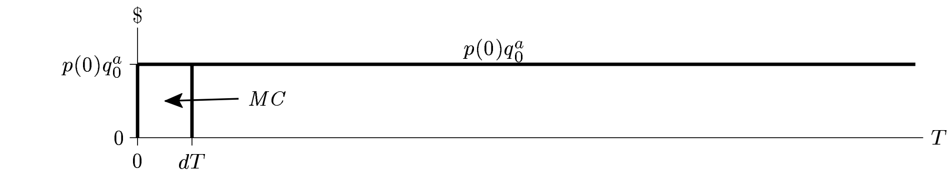
To understand this result—which is the central result of this paper—consider Figure 2. The top four panels of this figure show the revenue streams from undertaking a project either immediately, at time 0, or slightly later, at time dT, under various assumptions about α and λ. The bottom panel (e) shows the associated cost streams, where it is assumed that the project in question has annualized cost $c^*(0)$, implying that it is the marginal project optimally supplied immediately when $\alpha = 0$.

Consider first panel (a) of the figure, which shows the revenue stream of an abatement project ($\lambda = 0$) when the price of carbon is constant ($\alpha = 0$). In this case, the marginal cost of delaying the project until dT (represented by the area marked MC in the figure) is just the immediate revenue forgone as a result, or $p(0)q_0^a \, dT$. Offsetting this marginal cost is a marginal benefit of delay (represented by the area in panel (e) marked MB_1) which consists of the costs avoided, or $c^*(0)\, dT$. The marginal project *optimally* supplied immediately is that for which the marginal

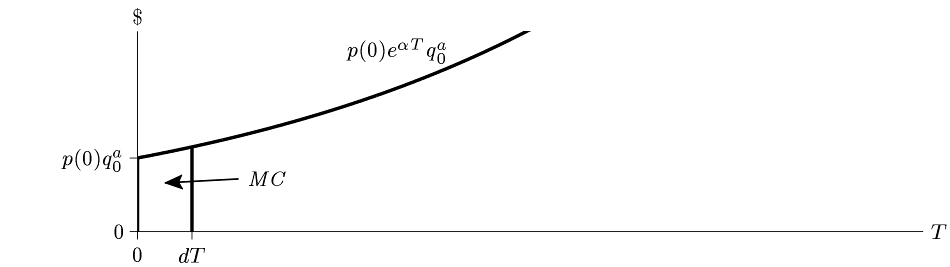
[10] The same implication follows more directly from evaluating equation (3) at $\alpha = 0$, to obtain

$$NPV(T,0) = \left[\frac{p(0)q_0^i}{r+\lambda} - \frac{c}{r}\right] e^{-rT}.$$

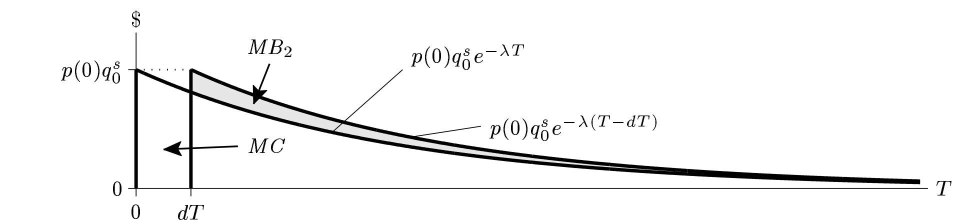
Clearly, for any project with costs c such that $NPV(T,0) \geq 0$, the right-hand side of this expression is decreasing in T (albeit only weakly so at conversion costs $c = \overline{c}(0)$, when $NPV(T,0) = 0$). Again, therefore, any project worth undertaking at all should be undertaken immediately.



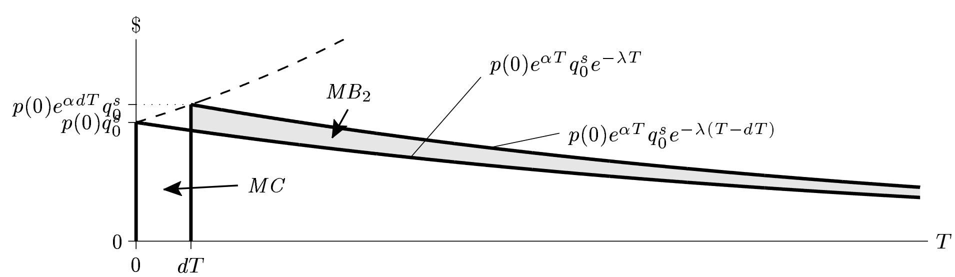
(a) Revenues when $\alpha = \lambda = 0$



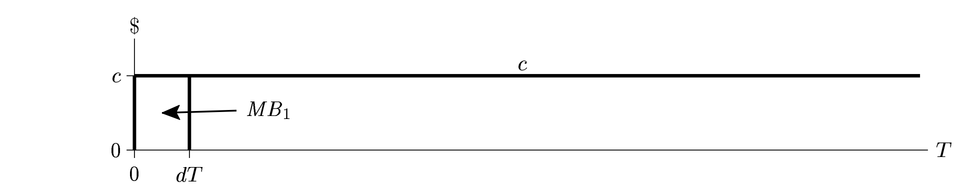
(b) Revenues when $\alpha > 0$, $\lambda = 0$



(c) Revenues when $\alpha = 0$, $\lambda > 0$



(d) Revenues when $\alpha > 0$, $\lambda > 0$



(e) Costs when $c = c^*(0)$

Figure 2. Marginal costs and benefits of delaying conversion from time 0 to time dT, for various values of α and λ.

benefit of delay just equals the marginal cost; that is, it is the project with costs $c^*(0) = p(0)q_0^a$, consistent with equation (6) evaluated at $\lambda = 0$.

The marginal project *profitably* supplied immediately is that for which discounted revenues from time 0 just equal discounted costs, making the project's net present value zero. Because revenues are in this case constant over time, their discounted value is just $p(0)q_0^a/r$. The marginal project profitably supplied immediately is therefore such that $\overline{c}(0)/r = p(0)q_0^a/r$; that is, $\overline{c}(0) = c^*(0)$, again consistent with equation (6).

Consider next what happens to this same abatement project with costs $c^*(0)$ when $\alpha > 0$. As shown in panel (b), the undiscounted revenue streams from undertaking the project at either time 0 or time dT will then be increasing over time. Importantly, however, because the abatement rate q_0^a is constant over time, the revenue streams from undertaking the project at either time 0 or time dT still coincide from dT onwards. The marginal cost of delaying the project until dT therefore still consists (to a first-order approximation) of the immediate revenues $p(0)q_0^a\, dT$ forgone. Because the marginal benefit of delay is unchanged as well, it must still be the case that the project with costs $c^*(0)$ is the marginal project *optimally* supplied immediately; that is, $c^*(\alpha) = c^*(0) = p(0)q_0^a$, consistent with equation (5) evaluated at $\lambda = 0$.

At the same time, discounted revenues from undertaking this project immediately are obviously higher when $\alpha > 0$ than when $\alpha = 0$. Since costs are unchanged, the *net* present value of undertaking the project immediately is therefore now strictly positive. But then the marginal project for which this net present value is zero—i.e., the marginal project *profitably* supplied immediately—must now have higher costs: that is, $\overline{c}(\alpha) > \overline{c}(0) = p(0)q_0^a$, consistent with equation (4) evaluated at $\lambda = 0$. In sum, panels (a) and (b) combined show that when α increases, the range of abatement projects *optimally* supplied immediately is unaffected, even though the range of projects *profitably* supplied increases.

Consider next panel (c). Here we are back to assuming that $\alpha = 0$, but we now have a sequestration project, and therefore a sequestration rate $q^s(t)$ that declines over time. The important difference this makes relative to the case of panel (a) is that the revenue streams from undertaking the project at either time 0 or time dT no longer coincide from dT onwards. More specifically, because the trees will be younger at all times after dT if the project is delayed, they will sequester carbon at higher rates, and undiscounted revenues are therefore higher by an amount corresponding to the area labeled MB_2 in the figure. The present value of these additional revenues represents a second, "future" marginal benefit of delaying conversion, additional to the "immediate" marginal benefit MB_1 of avoided costs. Clearly, then, for the marginal project optimally supplied immediately, the marginal cost of delay $p(0)q_0^s\,dT$ must strictly exceed the immediate marginal benefit $c^*(0)\,dT$ alone. This is consistent with (6) evaluated at $\lambda > 0$, which shows that $c^*(0) = [r/(r+\lambda)]p(0)q_0^s < p(0)q_0^s$.[11]

As for whether this project with costs $c^*(0)$ is also *profitably* supplied immediately, note that because the price of carbon is constant, total undiscounted revenues from the project are unchanged if the project is delayed. Since the same is true of undiscounted costs, it follows that delay reduces discounted revenues and costs, and thereby the project's net present value, by the same factor e^{-rdT}. But then, for the project owner to be indifferent about delay—i.e., for the project to indeed be the marginal one *optimally* supplied immediately—it must be the case that revenues and costs are identical to begin with, i.e., that the project's net present value is zero. If so, then by definition the project is also the marginal project *profitably* supplied immediately; that is, $\bar{c}(0) = c^*(0)$, again consistent with equation (6) evaluated at $\lambda > 0$.

Consider, finally, what happens to this same sequestration project with costs $c^*(0)$ when $\alpha > 0$. As shown in panel (d), the undiscounted revenue streams will then be higher after time 0 than in panel (c), and may or may not increase over time, depending on whether or not α exceeds λ.[12] Such future price increases have no (first-order) effect on either the marginal cost MC or the immediate marginal benefit MB_1 of delay. However, they clearly increase the future marginal benefit MB_2, because the younger trees will now be sequestering *higher-valued* tons of carbon at a higher rate. The *net* benefit of marginally delaying the project is therefore now strictly positive. But then the marginal project for which this net benefit is zero—i.e., the marginal project *optimally* supplied immediately—must have costs that are lower, thereby reducing MB_1 to offset the higher MB_2; that is, it must be the case that $c^*(\alpha) < c^*(0) = p(0)q(0)$, consistent with equation (5) evaluated at $\lambda > 0$.

As was true for the abatement project in panel (b), the increase in α also has the effect of increasing discounted revenues from undertaking the project immediately. This makes the project's

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[11] The discounted increase in revenues from dT onwards can be written as

$$\int_{dT}^{\infty} e^{-rt}p(0)q_0^s\left[e^{-\lambda(t-dT)} - e^{-\lambda t}\right]dt = \frac{\lambda}{r+\lambda}p(0)q_0^s\,dT,$$

which is the difference between $MB_1 = c^*(0)\,dT = [r/(r+\lambda)]p(0)q_0^s\,dT$ and $MC = p(0)q_0^s\,dT$.

[12] The relative magnitude of α and λ is immaterial to our argument here; it matters only that both are positive.

net present value strictly positive, implying that the marginal project *profitably* supplied immediately must have higher costs: $\overline{c}(\alpha) > \overline{c}(0) = p(0)q_0^s$, consistent with equation (4) evaluated at $\lambda > 0$. In sum, panels (c) and (d) combined show that when α increases, the range of sequestration projects *optimally* supplied immediately shrinks, even though the range of projects *profitably* supplied increases.

Our key result that any increase in α reduces the annualized cost of the marginal sequestration project optimally supplied immediately, namely from $c^*(0)$ to $c^*(\alpha)$, can also be viewed in terms of the optimal conversion time of a project with given costs c. Solving for the maximum of $NPV(T, \alpha)$ with respect to T, subject to $T \geq 0$, yields that the optimal time to convert is

$$T^* = \max\left[\frac{1}{\alpha}\log\left\{\frac{(r-\alpha+\lambda)}{r-\alpha}\frac{c}{p(0)q_0^s}\right\}, 0\right]. \tag{7}$$

This time will be positive if the expression in braces is greater than 1, which is the case for any $c > c^*(\alpha)$. For any sequestration project with costs exceeding $c^*(\alpha)$, it is optimal to delay conversion until some time $T^* > 0$, when the marginal cost $p(0)q_0^s e^{\alpha T}\, dT$ of further delay has grown sufficiently to offset the sum of the marginal benefits.

4. Aggregate supply when $\alpha < r$

The previous section established that, when carbon prices are expected to increase over time at some rate $\alpha > 0$ instead of staying constant, the range of carbon abatement projects optimally supplied at any given initial price $p(0)$ stays the same, but the range of sequestration projects optimally supplied shrinks. In particular, we showed that sequestration projects with costs $c \in [0, c^*(0)]$ will optimally be undertaken immediately when $\alpha = 0$, but when $\alpha > 0$, the subset of these projects with costs $c \in (c^*(\alpha), c^*(0)]$ will only be undertaken with delay.

Combined with the evidence presented in Section 2, indicating that carbon prices can indeed be expected to increase over time under a range of climate-policy scenarios, this result clearly has implications for the contribution of carbon sequestration projects in an optimal portfolio of greenhouse-gas mitigation strategies. In particular, relative to existing projections of this contribution—which typically assume that carbon prices will remain constant over time—the optimal contribution will be reduced, both in absolute terms and relative to the projected contribution of carbon abatement projects.

In this section, we make a first attempt at quantifying this reduction, whereby we focus initially on climate-policy scenarios for which $\alpha < r$, leaving the case where $\alpha \geq r$ (for which we have to slightly modify the analytical model) to Section 5. Before we can do so, however, we must introduce a number of additional assumptions.

First, we assume that both the interest rate r and the carbon-price $p(t)$ at any given time t are socially optimal, i.e., equal respectively to the shadow price of capital and the shadow cost of releasing an additional ton of carbon into the atmosphere (which in turn is equal to the present value of additional damages from that ton inflicted over all future time). Given this assumption, and provided decisionmakers correctly anticipate future carbon-price increases, the privately optimal aggregate supply of sequestration and abatement will then also be socially optimal.

Second, we assume that both the interest rate r and the rate of carbon-price increases α can be treated as fixed, even when we consider discrete—and possibly large—changes in the aggregate supply of sequestration. This assumption may appear strong, particularly in light of our earlier discussion of studies suggesting that carbon sequestration should play a significant role in any socially optimal portfolio of greenhouse strategies. Simulations with Nordhaus and Boyer's (2000) DICE99 model and Nordhaus's (2001) RICE01 model suggest, however, that the assumption is in fact quite weak. In the DICE99 model, for example, even halving or doubling[13] marginal abatement costs turns out to have a negligible effect on either the shadow price of capital or the rate at which the shadow cost of carbon increases over time.[14]

[13] Halving is the more relevant experiment. This is because the DICE99 model's mitigation cost function is based on estimates by Nordhaus (1991) that suggest a very limited optimal share of carbon sequestration in overall greenhouse-gas mitigation strategies: on the order of 10% or less. Raising this share to something closer to 50%, consistent with later studies, will therefore tend to *reduce* marginal mitigation costs overall.

[14] More specifically, under the economically efficient price scenario (which is most relevant to the analysis of this section) even carbon-price levels are hardly affected: it is the optimal emissions-control rate that adjusts, roughly doubling for example when marginal costs are halved. The reason why this doubling of control rates does not feed back into significantly lower carbon prices is that emissions-reduction levels under the optimal scenario are quite low to begin with, particularly in comparison to the stock of atmospheric carbon. It is this stock that drives the damages from global warming, and thereby the shadow cost of carbon. In contrast, under the concentration-limit scenario (which is most relevant to the analysis of Section 5), it is the emissions-control rate that is hardly affected, and carbon-price levels that adjust. The reason here is that climate policy under this scenario aims to keep the atmospheric CO_2 concentration below some limit at minimal cost. This goal essentially fixes a path of control rates, and the carbon price simply adjusts to equal the marginal mitigation cost along that path. However, this adjustment takes place in a manner that leaves the carbon-price growth rate essentially the same, and it is this growth rate that matters to our analysis.

Third and last, in order to quantify what the incentives of *individual* landowners to delay conversion imply for the *aggregate* supply of sequestration, we need some estimate of how conversion costs are distributed across acres of non-forested land. Clearly, the optimal reduction in initial acreage converted at $\alpha > 0$ relative to that at $\alpha = 0$ (and thereby the reduction in the initial sequestration supply) will depend on what fraction of those acres have conversion costs $c \in (c^*(\alpha), c^*(0)]$. For a rough estimate of this fraction, we make use of a carbon sequestration study by Newell and Stavins (2000) based on data from 36 counties in the Mississippi Delta region of the U.S. The sequestration supply curve estimated by Newell and Stavins (under an assumption of constant carbon prices) suggests a very close to linear relationship between marginal conversion costs and total acreage converted, up to a point where marginal conversion costs reach a level of about \$80/acre.[15] It is reasonable to assume that most of the variation in opportunity costs of conversion across different acres comes from variation in the agricultural potential of land, i.e., from variation in the component a of $c \equiv a + rC$. Below, we assume that the up-front conversion cost C is constant across all acres.

4.1. *Quantifying Optimal Sequestration Supply for the U.S.*

Given constant up-front conversion costs, we have from substituting equation (5) into the identity $a \equiv c - rC$ that the marginal acre optimally supplied immediately yields agricultural profits (expressed as a function of α) equal to

$$a(\alpha) = \frac{r - \alpha}{r - \alpha + \lambda} p(0) q_0^s - rC \tag{8}$$

up to the time of conversion. A linear relationship between agricultural profits and total acreage converted then implies that the aggregate acreage supply can be written as

$$A(\alpha) = ka(\alpha) = k \left[\frac{r - \alpha}{r - \alpha + \lambda} p(0) q_0^s - rC \right] \tag{9}$$

for some constant $k > 0$. Newell and Stavins' estimates, when extrapolated to the U.S. as a whole in the manner adopted by Stavins (1999),[16] suggest that k is around 2.8 million.

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[15] Expressed, in 1990 US\$, as are all prices hereafter in the paper.
[16] See Stavins' footnote 15.

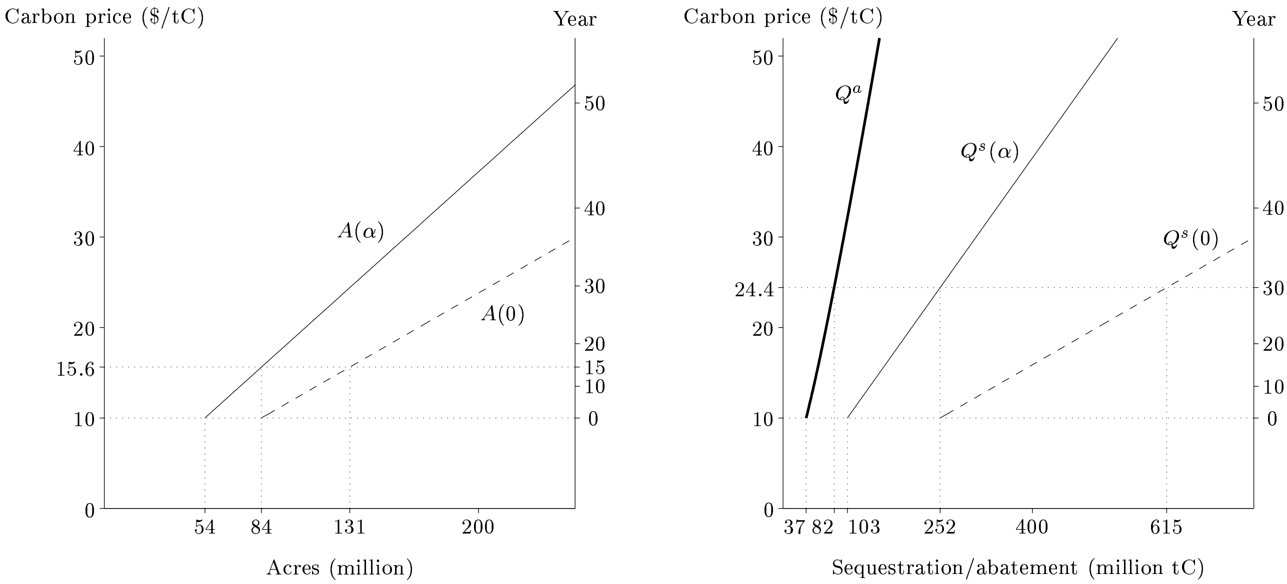


FIGURE 3. Optimal acreage, sequestration, and abatement supply at $\alpha = 0$ and at $\alpha = 0.03$.

With this estimate in hand, and using equation (9), we can quantify the importance of expectations about the rate of future price increases α for the optimal aggregate acreage supply, at any given initial carbon price $p(0)$ and for given up-front conversion cost C. To do this, it will be useful to first consider the special case where C is essentially zero—for example, because forests are not actively planted but simply allowed to regenerate naturally, or because planting costs are offset by stumpage revenues from future harvests (a plausible case that our analytical model cannot capture, but see footnote 7 above). Optimal acreage supply becomes in this case directly proportional to the carbon price.

The dashed curve labeled $A(0)$ in the left-hand panel of Figure 3(a) represents the optimal acreage supply curve when both $\alpha = 0$ and $C = 0$, and when the remaining parameters of our model take on what we shall treat as benchmark values, namely $r = 0.05$, $\lambda = 0.03$, $q_0^s = 4.8$, and $k = 2.8$ million. At these values, equation (9) evaluates to

$$A(0) = k\left[\frac{r q_0^s}{r + \lambda}\right] p(0) \approx 8.4\, p(0). \tag{10}$$

Also shown in the figure, in the right-hand panel, is the corresponding sequestration supply curve, labeled $Q^s(0)$. This curve represents the aggregate quantity of sequestration supplied at any given carbon price $p(0)$, calculated on an annualized basis. That is, rather than considering the actual, time-varying rate $q^s(t)$ at which each of the $A(0)$ acres supplied sequesters carbon, we treat each acre as if it sequesters carbon at the constant rate $rq_0^s/(r+\lambda)$. This rate is the annualized equivalent of $q^s(t)$ in that, over an infinite time horizon, it yields the same present value of tons sequestered:

$$\int_0^\infty e^{-rt} p(0) q^s(t)\, dt = \int_0^\infty e^{-rt} p(0) \frac{rq_0^s}{r+\lambda}.$$

This conversion is made in order to facilitate comparison with the aggregate quantity of carbon abatement supplied at the same price: in a present-value sense, each acre of land converted can be treated as equivalent to $rq_0^s/(r+\lambda)$ tons per year of carbon abatement projects undertaken. At our benchmark parameters, this rate evaluates to 3 tons/year.

Multiplying equation (10) by $rq_0^s/(r+\lambda)$, we have

$$Q^s(0) = A(0)\frac{r}{r+\lambda} q_0^s = k\left[\frac{rq_0^s}{r+\lambda}\right]^2 p(0) \approx 25.2\, p(0). \tag{11}$$

To put this estimate in perspective, the U.S. annual emissions target for the years 2008–2012 originally agreed to by the Clinton administration under the Kyoto Protocol was about 1,500 MtC—93% of total U.S. emissions in 1990—which is about 700 MtC below projections of business-as-usual emissions in the year 2010. Moreover, Nordhaus's (2001) RICE01 model predicts that, if the U.S. were to re-join the Kyoto Protocol at its original target, the initial carbon price would be around \$10 per ton. Figure 3 shows that, at this price, 84 million acres of farmland would optimally be converted, corresponding to about 14.4% of total U.S. farmland. The resulting annualized sequestration supplied would be 252 MtC, or close to 40% of the required reduction.[17]

The central result of this paper, however, is that this supply would be optimal only if the carbon price were to *remain* at \$10 forever, i.e., if $\alpha = 0$. If instead the price increases over time then, from equation (9), the optimal acreage supply will be lower at any given value of the initial carbon price $p(0)$. In particular, as shown in the left-hand panel of Figure 3(a), at our benchmark value of $\alpha = 0.03$, optimal acreage supply is reduced by 36%, to only 54 million acres at $p(0) = 10$.

[17] Note our implicit assumption that both agricultural profits in the Mississippi Delta and sequestration rates of southern pine can be extrapolated to the U.S. as a whole. Stavins (1999) makes the same assumption, but estimates the annualized contribution of carbon sequestration at a constant price of \$10 to be only about a quarter as large as ours. The reason is that his estimate of annualized per-acre sequestration is only half as large as ours—around 1.5 tons per acre—in part because he assumes that forests will be periodically harvested. As a result, his estimate of acreage supply at \$10 is only half as large as well—around 42 million acres—and the resulting total supply of annualized sequestration is only a quarter as large—around 63 MtC. Note from equation (11) that annualized per-acre sequestration enters quadratically in aggregate sequestration supply.

Moreover, because the actual sequestration $q^s(t)$ by each of these acres is weighted towards the present, when carbon prices are relatively low, the annualized equivalent of $q^s(t)$ is reduced as well. Specifically, since

$$\int_0^\infty e^{-rt}p(0)e^{\alpha t}q^s(t)\,dt = \int_0^\infty e^{-rt}p(0)e^{\alpha t}\frac{(r-\alpha)q_0^s}{r-\alpha+\lambda},$$

the "abatement equivalent" value of each acre supplied drops to $(r-\alpha)q_0^s/(r-\alpha+\lambda)$, or only 1.92 tons/year at our benchmark parameter values. As shown in the right-hand panel of Figure 3(a), the compounded effect of both the drop in acreage supply and the drop in annualized sequestration by each acre reduces optimal sequestration supply by 59%, to only 103 million tons at $p(0) = \$10$.

Of course, if the carbon price indeed increases over time, it will be optimal to convert additional acres in subsequent years. The logarithmic time scales on the right of both panels in the figure indicate that it would take 15 years for the optimal acreage supply under forward-looking price expectations to grow from 54 million acres to 84 million acres, i.e., to catch up with the optimal *immediate* supply predicted under a constant-price assumption, and another 15 years for annualized sequestration supply to catch up as well.

However, because by that time the carbon price will have increased to $10e^{0.03\cdot 30} \approx \$24.4$, the optimal supply of carbon abatement, which we showed above is *not* affected by price expectations, will have increased as well. This is illustrated in the right-hand panel of the figure by the curve labeled Q^a, which represents the optimal U.S. supply of carbon abatement according to the RICE01 model, given a carbon price path $p(t) = 10e^{0.03t}$. Over the price range shown in Figure 3, this abatement supply curve is close to linear, implying that the optimal supply of both sequestration and abatement will increase over time at about the same rate. Both increase by a factor of roughly 2.5 over the first 30 years, with abatement growing from 37 MtC initially to 82 MtC.

As a result, even though the optimal *level* of sequestration supply catches up in year 30 with the constant-price projection of supply in year 0, the optimal *share* of carbon sequestration in the overall portfolio of greenhouse strategies may never catch up.[18] Graphically, this follows from

[18] It is worth noting, as an aside, that a 30-year catch-up time for the aggregate supply *level* may itself constitute a significant delay if policies to encourage carbon sequestration are seen as a way of buying time for the development of cheaper carbon abatement technologies. See, for example, the March 2001 testimony at the Senate Committee on Agriculture, Nutrition and Forestry's Hearing on Biomass and Environmental Trading by agricultural economist Bruce McCarl, arguing that carbon sequestration could provide an approximate 20-year bridge for the energy industry to come to terms with its CO_2 emissions and develop a less costly and longer-term system to reduce them. ("Panel eyes emissions trading plan for farmers, foresters," *Environment and Energy Daily,* April 2, 2001).

the fact that when α increases, the entire sequestration supply *curve* pivots inwards relative to the abatement supply *curve*. The reduction in the sequestration share relative to constant-price projections therefore applies not just at the initial price, but over the entire future price path.

4.2. *Effect of Changes in α and λ*

To see how the proportional supply reduction depends on the key model parameters α and λ, let $\Delta Q^s/Q^s$ denote this proportional reduction evaluated at a given level of up-front conversion costs C. Using equation (9), and the fact that $Q^s(\alpha) = A(\alpha)(r-\alpha)q_0^s/(r-\alpha+\lambda)$, we then have

$$\begin{aligned}\Delta Q^s/Q^s &= \frac{Q^s(0) - Q^s(\alpha)}{Q^s(0)} \\ &= \frac{\left[\left\{\dfrac{rq_0^s}{r+\lambda}\right\}^2 - \left\{\dfrac{(r-\alpha)q_0^s}{r-\alpha+\lambda}\right\}^2\right] p(0)}{\left\{\dfrac{rq_0^s}{r+\lambda}\right\}^2 p(0) - rC}\end{aligned} \tag{12}$$

If up-front conversion costs C are zero, this ratio becomes independent of the initial price $p(0)$ and the initial sequestration rate q_0^s, reducing to[19]

$$\Delta Q^s/Q^s = 1 - \left(\frac{r-\alpha}{r-\alpha+\lambda}\frac{r+\lambda}{r}\right)^2. \tag{13}$$

Figure 4 plots $\Delta Q^s/Q^s$ as a function of α for various values of λ, assuming $C = 0$ and $r = 0.05$. Three things are worth noting in the figure. First, the supply reduction is zero when $\lambda = 0$, regardless of the value of α. This follows immediately from our discussion of Figure 2: if sequestration rates do not decline over time, the revenue streams from conversion at either time 0 or time dT coincide from time dT onwards, eliminating any future marginal benefit from delay. As

[19] Since abatement supply Q^a does not depend on α when the price path is deterministic, the expression on the right-hand side of (13) also represents the proportional reduction in the share of sequestration relative to that of abatement in the overall portfolio of mitigation strategies. That is, we can also write

$$\frac{Q^s(0)/Q^a(0) - Q^s(\alpha)/Q^a(\alpha)}{Q^s(0)/Q^a(0)} = 1 - \left(\frac{r-\alpha}{r-\alpha+\lambda}\frac{r+\lambda}{r}\right)^2.$$

It can be shown that this same expression still applies if the price path follows a geometric Brownian motion $dp = \alpha p(t)\,dt + \sigma p(t)\,dz$, provided the marginal cost of abatement is linear in aggregate supply of abatement projects (just as the marginal cost of converting land is assumed to be linear in the aggregate supply of acres). Under these conditions, the relative share $Q^s(\alpha)/Q^a(\alpha)$ is for all values of α independent of σ, and therefore equal to the share in the deterministic case. In other words, uncertainty around the expected price path then affects both abatement and sequestration supply by proportionally the same amount, and therefore has no effect on either the supply ratio or the reduction in that ratio when α increases.

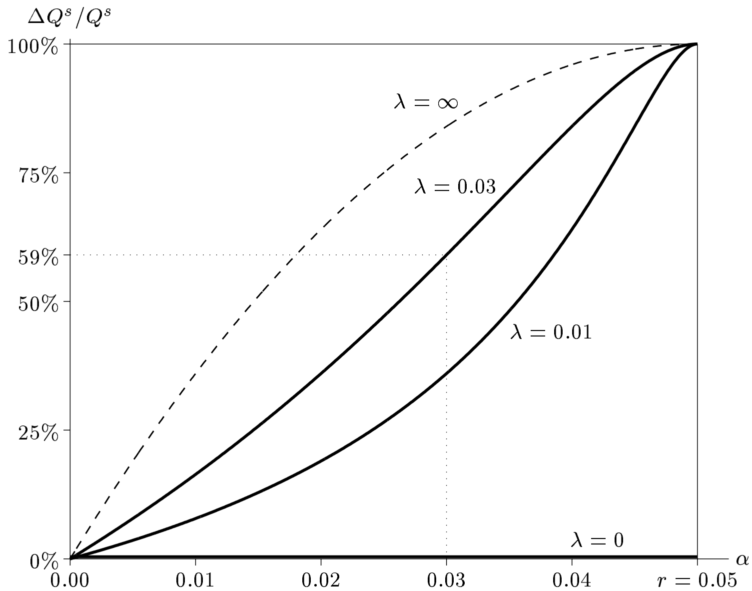


FIGURE 4. Proportional reduction in sequestration supply as a function of α and λ.

a result, the optimal time to convert depends only on the comparison of the marginal cost of delay to the immediate marginal benefit, and future price increases are irrelevant.

Second, when α approaches r, the supply reduction goes to 100%, regardless of the value of λ. The intuition here is that, in the limit where prices increase at rate $\alpha = r$, the present value of sequestration revenues becomes independent of the conversion time: delaying conversion delays the time at which these revenues start coming in, but since their current value grows at rate r, their present value is unaffected. Delaying conversion does, however, reduce the present value of conversion costs, so that it becomes optimal in the limit as $\alpha \to r$ to delay all conversion forever.

Finally, for any given value of α, the supply reduction increases in λ. Any increase in λ in fact reduces the optimal sequestration supply *level* even when prices are constant, because the present value of sequestration revenues is lower if the sequestration rate drops more sharply over time. Because $Q^s(0)$ falls by more than $Q^s(\alpha)$, however, the proportional reduction $\Delta Q^s/Q^s$ is increasing in λ.[20]

Consider next the case where the up-front conversion cost is *non*-negligible. As is clear from equation (12), the terms involving the initial carbon price $p(0)$ and sequestration rate q_0^s then

[20] This result assumes q_0^s is independent of λ. If, however, high-λ tree species tend to mature rapidly, they will tend to have high values of q_0^s as well. In the (unrealistic) case where all tree species eventually attain the same cumulative carbon sequestration level $V \equiv q_0^s/\lambda$, so that we can substitute $V\lambda$ for q_0^s in the expressions for $A(0)$ and $A(\alpha)$, the optimal acreage supplied for rapidly maturing (i.e., high-λ *and* high-q_0^s) species will in fact be higher than that for slowly maturing ones. Because the $V\lambda$ term still cancels out in equation (13), however, the ratio $\Delta Q^s/Q^s$ still increases in λ.

no longer cancel. However, because the right-hand side of (12) is increasing in C, the simpler expression (13) can still serve as a lower bound on the actual supply reduction. Moreover, it can be shown that the comparative-statics results discussed above for the *proportional* supply reduction at $C = 0$ apply qualitatively unchanged to the *absolute* supply reduction

$$\Delta Q^s \equiv Q^s(0) - Q^s(\alpha) = \left[\left\{ \frac{r q_0^s}{r+\lambda} \right\}^2 - \left\{ \frac{(r-\alpha) q_0^s}{r-\alpha+\lambda} \right\}^2 \right] p(0).$$

That is, the absolute reduction in sequestration supply is also increasing in α and decreasing in λ.

5. Aggregate supply when initially $\alpha \geq r$

Although the model developed above constrained α to lie strictly below r, our discussion in the introduction suggested that α may reasonably be expected to equal or exceed r for many decades, namely if future climate-change negotiations continue to focus on keeping atmospheric CO_2 concentrations or global temperature increases below limits considered "safe." Under a concentration-limit scenario considered by Goulder and Mathai (2000), for example, prices optimally increase at rates of about 0.8% above the discount rate for 200 years. Similarly, under concentration- and temperature-limit scenarios considered by Nordhaus and Boyer (2000), prices increase at rates above the discount rate for about 100 years, after which the rate of price increase drops quite abruptly to a level well below the discount rate.

To obtain an analytically tractable approximation to these scenarios, we assume in this section that the price rises at rate $\alpha \geq r$ until some time $\overline{T}$ and then abruptly stop rising, staying constant thereafter at $p(\overline{T}) = p(0)e^{\alpha\overline{T}}$. In this case, the net present value of profits from converting land at any time $T \leq \overline{T}$ becomes

$$\begin{aligned} NPV(T,\alpha) &= \int_T^{\overline{T}} e^{-rt} p(0) e^{\alpha t} q_0^s e^{-\lambda(t-T)}\, dt + \int_{\overline{T}}^{\infty} e^{-rt} p(0) e^{\alpha\overline{T}} q_0^s e^{-\lambda(t-T)}\, dt - \int_T^{\infty} e^{-rt} a\, dt - e^{-rT} C \\ &= \left[\frac{1}{r-\alpha+\lambda} p(0) e^{\alpha T} q_0^s \left(1 - e^{-(r-\alpha+\lambda)(\overline{T}-T)}\right) + \frac{1}{r+\lambda} p(0) e^{\alpha\overline{T}} q_0^s e^{-(r+\lambda)(\overline{T}-T)} - \frac{a}{r} - C \right] e^{-rT}. \end{aligned}$$

Differentiating with respect to T yields

$$\frac{\partial NPV(T,\alpha)}{\partial T} = \Big[-\frac{r-\alpha}{r-\alpha+\lambda} p(0)e^{\alpha T} q_0^s - \left\{\frac{r}{r+\lambda} - \frac{r-\alpha}{r-\alpha+\lambda}\right\} p(0)e^{\alpha T} q_0^s e^{-(r-\alpha+\lambda)(\overline{T}-T)} + a + rC\Big] e^{-rT}. \quad (14)$$

An important difference with the case where $\alpha < r$ is that, if the target time $\overline{T}$ lies sufficiently far into the future, this derivative may be positive at $T = 0$ for *any* positive value of conversion costs $a + rC$. That is, the actual supply curve may have a positive intercept, even when up-front conversion costs C are negligible. Evaluating (14) at $a + rC = 0$ and solving for T, we find that supply becomes positive only after critical time

$$T^{\ell} \equiv \overline{T} - \frac{1}{r-\alpha+\lambda} \log\left(1 - \frac{r-\alpha+\lambda}{r-\alpha}\frac{r}{r+\lambda}\right).$$

At $r = 0.05$, $\alpha = 0.058$, and $\overline{T} = 100$ (values roughly consistent with Nordhaus and Boyer's (2000) $2{\times}CO_2$ scenario), and at our benchmark value of $\lambda = 0.03$, T^{ℓ} evaluates to 55. At these parameter values, therefore, it would be optimal to delay conversion of *any* agricultural land to at least 55 years after the carbon market is first established.

Solving (14) for a shows that, once T exceeds T^{ℓ}, so that supply indeed becomes positive, the marginal acre supplied at price $p(T) = p(0)e^{\alpha}T$ will yield agricultural profits

$$a(\alpha) = \frac{r-\alpha}{r-\alpha+\lambda} p(T)q_0^s + \left\{\frac{r}{r+\lambda} - \frac{(r-\alpha)}{r-\alpha+\lambda}\right\} p(T)q_0^s e^{-r-\alpha+\lambda}(\overline{T}-T) - rC. \quad (15)$$

Using again our assumption that $A = ka$, it is straightforward to derive expressions for the proportional and absolute reduction in sequestration supply relative to projected supply at constant prices. Both measures of the delay effect can be shown to again be increasing in α and λ.[21]

6. Numerical model

In this section, we report simulation results from a numerical model that relaxes many of the simplifying assumptions of the analytical model.

[21] In fact, these comparative statics apply even if the price rises at a rate $\alpha < r$ until some given time $\overline{T}$ and then stays constant. Qualitatively, therefore, none of our results from Section 4 require the carbon price to increase literally forever.

First, rather than assuming that sequestration declines with forest age at a constant exponential rate—which ignores the typical initial increase in sequestration just after planting (i.e., the initially convex portion of the cumulative sequestration curve depicted in Figure 1)—we use data by Birdsey (1992) on the actual time profile of sequestration by a southern pine stand in the Southeast of the U.S., and by a spruce-fir stand in the Lake States region. Southern pine refers to a collection of commercially important pine species found primarily in the southern U.S.; the Southeast region would likely play a central role in a national afforestation policy, given climatic conditions suitable for fast-growing southern-pine plantations and the prevalence of wood-products manufacturing in the region.[22] We also examine spruce-fir, the dominant commercial timber type in the Lake States region, because recent empirical evidence (e.g., Plantinga, Mauldin and Miller (1999)) suggests that sequestration costs are relatively low in this region.

Second, rather than assuming that carbon prices increase at a constant exponential rate and that the discount rate is constant, we use Nordhaus's (2001) RICE01 integrated assessment model to simulate optimal price and discount-rate paths under three different climate-policy scenarios. The "optimal" scenario of the RICE01 model solves for the economically efficient carbon-price path from the year 2005 onwards, by equating the price at each point in time to the present value of all future damages caused by a marginal unit of carbon released. The "Kyoto forever" scenario solves for the carbon-price path that would result if the emissions limits agreed upon under the Kyoto Protocol for just the years 2008–2012 were extended indefinitely (i.e., over the entire time horizon of the model), with the U.S. participating under the terms it originally agreed to. Lastly, the "$2{\times}CO_2$" scenario solves for the carbon-price path that most cost-effectively prevents the atmospheric concentration of CO_2 from ever exceeding 560 parts per million, double its pre-industrial level.

Third, rather than assuming that conversion is always to permanent forest, we allow landowners to generate revenue from timber harvesting, at endogenously determined intervals. A new assumption that then becomes relevant is that landowners internalize the full social cost of all carbon releases resulting from timber harvests. More specifically, we assume that landowners have to buy carbon credits at the prevailing carbon price to cover all resulting emissions, as and when those emissions occur. This assumption abstracts from the many problems associated with implementing such a policy in practice, but allows us to focus on simulating first-best optimal sequestration paths.

[22] The Conservation Reserve Program provides subsidies to farmers to convert cropland to grassland, trees, and other permanent vegetative cover. Compared to other regions, the Southeast region has by far the highest percentage of land enrolled in the tree planting category.

Further detail on the assumptions underlying the simulations and on the solution procedure used is provided in Appendix A.

To show how these modifications alter the landowner's optimization problem, it is useful to introduce some additional notation. First, let

$$\delta(t) \equiv e^{\int_0^t r(x)\,dx}.$$

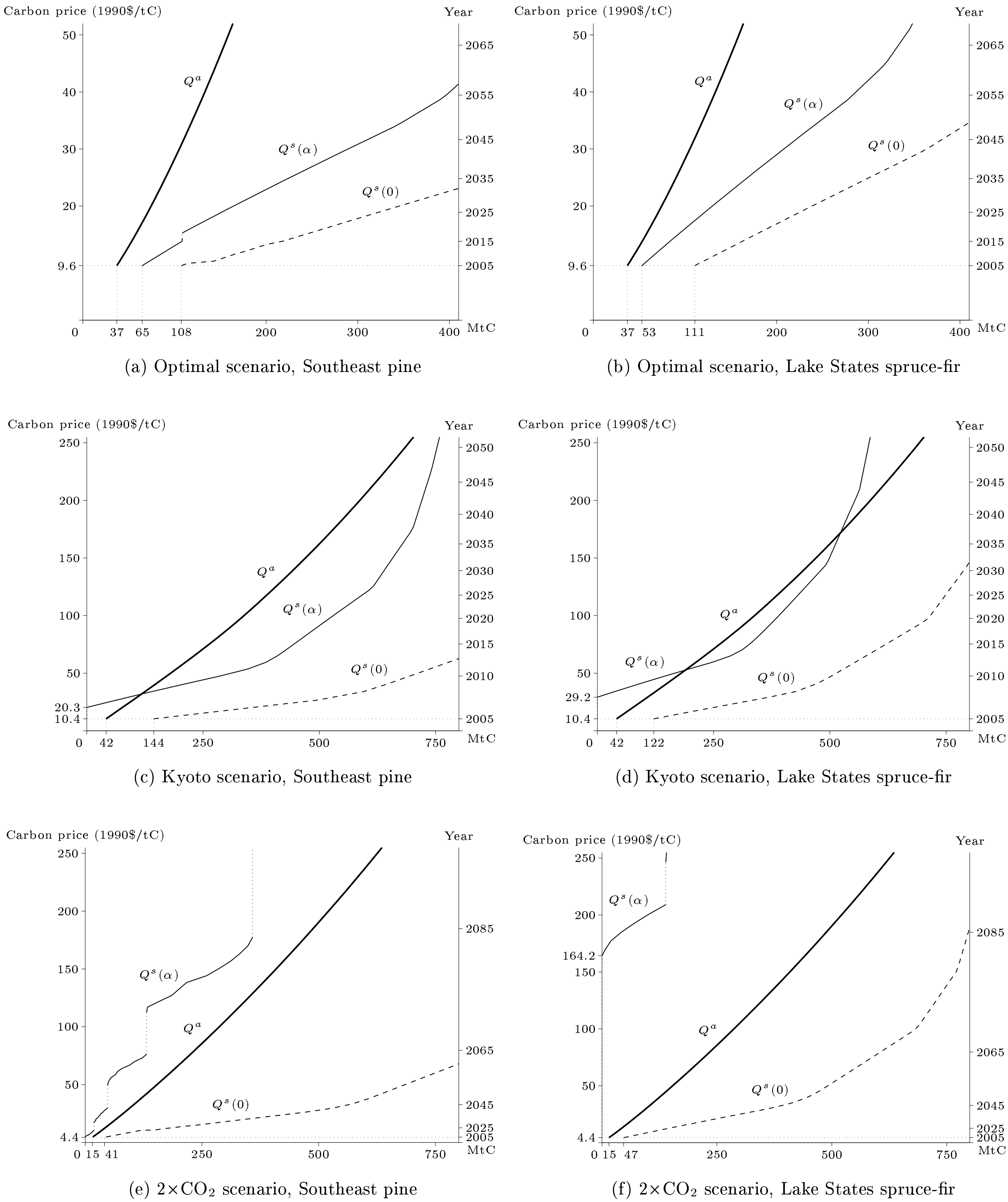
denote the discount factor at time t, given the now variable discount rate function $r(t)$. Also, let T_0 now denote the time of initial conversion to forest, and $T_1, T_2, T_3, \ldots$ the subsequent times at which the forest is rotated, i.e., harvested and replanted. Let

$$Q^s(T_i, T_{i+1}) \equiv \int_{T_i}^{T_{i+1}} q(t - T_i)\,dt$$

denote the cumulative amount of carbon sequestered by a rotation planted at time T_i and harvested at time T_{i+1}, and let $Q^s_m(T_i, T_{i+1})$ denote the carbon sequestered by that same rotation in merchantable wood only. Finally, let γ denote the quantity of carbon contained in a volumetric unit of merchantable wood—the unit in which the stumpage price P is expressed—and let $\ell_m(s)$ denote the fraction of carbon originally sequestered in wood products that is released s years after harvest. The landowner's optimization problem can then be written as

$$\begin{aligned}
\max_{T_i} NPV(T_0, T_1, \ldots) = \sum_{i=0}^{\infty} \Big\{ & -\delta(T_i)C + \int_{T_i}^{T_{i+1}} \delta(t)[p(t)q(t - T_i) - a]\,dt \\
& + \delta(T_{i+1})P\gamma Q^s_m(T_i, T_{i+1}) \\
& - \delta(T_{i+1})p(T_{i+1})\left[Q^s(T_i, T_{i+1}) - Q^s_m(T_i, T_{i+1})\right] \\
& - \int_{T_{i+1}}^{\infty} \delta(t)p(t)Q^s_m(T_i, T_{i+1})\ell_m(t - T_{i+1})\,dt \Big\}
\end{aligned}$$

The first two terms on the right-hand side are familiar from the analytical model: they represent respectively the cost of planting (or for later rotations, re-planting) forest and the gross revenues from sequestration less the flow opportunity costs of forgone agricultural revenues over the course of each rotation. The last three terms are new: they represent respectively the stumpage revenue received at the time of harvest and the social costs of carbon releases at and after the time of harvest.



(a) Optimal scenario, Southeast pine

(b) Optimal scenario, Lake States spruce-fir

(c) Kyoto scenario, Southeast pine

(d) Kyoto scenario, Lake States spruce-fir

(e) 2×CO_2 scenario, Southeast pine

(f) 2×CO_2 scenario, Lake States spruce-fir

FIGURE 5. Simulated optimal acreage supply curves for two region/species combinations and three climate-policy scenarios.

The six panels of Figure 5 show the results of simulating this model for the two types of forest and for the three climate-policy scenarios from the RICE01 model. The interpretation of the axes and the curves labeled Q^a, $Q^s(\alpha)$, and $Q^s(0)$ in each of the panels is identical to that in the right-hand panel of Figure 3, except that the initial year in which the carbon price first becomes positive is taken to be the year 2005 rather than some arbitrary year 0.[23] In general, the simulation results confirm those of the analytical model. Optimal sequestration supply given perfect foresight of the increasing carbon-price path under all three climate-policy scenarios is below projected supply at constant prices, and more so, the more rapidly prices are expected to increase.[24]

Under the optimal RICE01 scenario, the carbon price starts out in the year 2005 at \$9.60/ton and thereafter increases at a rate that is always below the discount rate; the gap between the two (analogous to $r - \alpha$ in the analytical model) is about 1% initially, and widens to 2% by the year 2050.[25] As shown in panels (a) and (b), the optimal initial supply of abatement under this scenario is 37 MtC. For both southern pine and spruce-fir, the projected supply of sequestration assuming the carbon price will stay at \$9.60 forever is about three times larger: 108 MtC for southern pine, and 111 MtC for spruce-fir. The *optimal* supply given perfect foresight about future price increases is less than twice as large as that of abatement, however: only 65 MtC for southern pine and 53 MtC for spruce-fir. Compared to the constant-price estimate of initial sequestration supply, this amounts to respectively a 40% and 52% reduction. By the year 2025, when under this scenario the carbon price has roughly doubled to \$18.90, the optimal sequestration supply has roughly doubled as well, and thereby caught up with the projected initial supply. However, the optimal abatement supply has by then also doubled, implying that the optimal *share* of carbon sequestration relative to that of abatement has caught up at all. This share remains for many decades lower than the initial constant-price estimate.

[23] In particular, the simulations extrapolate regional conditions to the U.S. as a whole in the same manner as that used to generate Figure 3. That is, the distribution of agricultural profits in the Mississippi Delta is extrapolated to all farmland in the U.S., and growth conditions of respectively southern pine forest in the Southeast and spruce-fir forest in the Lake States are similarly applied to the U.S. as a whole.

[24] As a sensitivity check, we have run simulations (not reported here) in which we extrapolate our model of U.S. sequestration supply in a somewhat crude fashion to the world as a whole, and then iteratively update the RICE01 model's price and discount-rate paths to account for feedback effects from sequestration supply to the optimal supply of carbon abatement and optimal investment. Consistent with our experiments with the DICE99 model, discussed in Section 4, these feedback effects turn out to be quite small; the major difference with the simulation results reported here is that the level (but not the rate of increase) of carbon prices is considerably lower in the $2\times CO_2$ scenario when feedback effects are allowed for.

[25] The discount rate itself falls over time, and in fact does so in the same manner under all three scenarios; starting at about 5%, it drops to around 2% in the year 2250. Under the optimal scenario, the rate of price increase drops to 0% around that same year.

The distinction between catch-up in levels vs. catch-up in shares is brought out even more sharply in the case of the Kyoto forever RICE01 scenario. Under this scenario, the carbon price starts out in the year 2005 at $10.40/ton and thereafter increases very rapidly to $75 in the year 2015 and to $97 in the year 2020. By 2020, however, the rate of price increase slows significantly, to below the discount rate, and thereafter the gap between the two rates roughly matches that under the optimal scenario. As shown in panels (c) and (d), the optimal initial supply of abatement under this scenario is 42 MtC, and for both southern pine and spruce-fir, the projected constant-price supply of sequestration is about three times larger: 144 MtC for southern pine, and 122 MtC for spruce-fir. In contrast, the optimal initial sequestration supply under this scenario is zero for both southern pine and spruce-fir, i.e., 100% below projected constant-price supply. The latter supply is in turn about three times larger than optimal initial abatement supply. Just five years later, however, in 2010, the optimal sequestration supply *level* has already caught up with constant-price projections. But precisely because of the very rapid initial rate of price increase, the optimal supply of carbon abatement increases rapidly as well, in fact quadrupling from 42 MtC to 171 MtC. The optimal carbon sequestration *share* remains for several decades thereafter roughly comparable to that of carbon abatement.

Consistent with the analysis of Section 5, the most dramatic effects of differing price expectations arise under the $2{\times}CO_2$ scenario. Under this scenario, the carbon price starts out in the year 2005 at just $4.40/ton and thereafter increases for over a century at a rate 0.8% above the discount rate. After the year 2115, however, when the $2{\times}CO_2$ concentration limit is reached, the rate of price increase quite abruptly drops well below the discount rate. As shown in panel (c), the optimal resulting sequestration supply for southern pine is zero initially, but almost immediately becomes positive. Catch-up with projected constant-price supply of 41 MtC does not occur until almost 40 years later, however, by which time the optimal supply of carbon abatement has increased 6-fold, from 15 MtC to 92 MtC. The optimal supply *share* is therefore still only half that of abatement, and remains at roughly half long after. This amounts to an 80% reduction in share relative to the constant-price estimate. As for spruce-fir, panel (d) shows that the optimal supply for this type of forest remains zero for almost 80 years, and then very rapidly catches up projected constant-price supply of 47 MtC. The optimal supply *share* stays far below that of carbon abatement, however.

The marked difference between southern pine and spruce-fir in the delay until optimal sequestration supply becomes positive under the $2{\times}CO_2$ scenario merits further comment. It is driven by the different importance of soil sequestration for the two forest types, relative to sequestration by the trees, understory, and forest floor. For southern pine, soil sequestration accounts for just 18% of total "gross" sequestration (gross of releases from non-soil components at and after harvesting) and 50% of total net sequestration. For spruce-fir, in contrast, these figures are much higher, at 45% and 90% respectively.

To see why this difference matters, it is useful to think of the value of delaying conversion identified in Section 3 as arising from the value of an option destroyed by planting forest immediately, namely the option of planting forest later. If the forest, once planted, can never be harvested—as the analytical model assumed for simplicity—then this option is destroyed forever. If, on the other hand, periodic harvesting is allowed for—as in the numerical model—then the option is destroyed only temporarily: it becomes available again as soon as the initially planted forest is in fact harvested. At first blush, this would appear to place an upper bound on the time that it can be optimal to delay initial conversion. Specifically, if the minimum initial rotation length that yields positive discounted profits (net of agricultural profits forgone) is R years, then it would appear to be suboptimal to delay conversion by longer than R years: such delay would imply forgoing those positive profits without any offsetting benefits in terms of preserving option value.

This reasoning only applies, however, if planting first-rotation forest after R years of agricultural production yields the *same* profits from that point onwards as does planting second-rotation forest after R years of forest production. In reality, this is not the case. The Birdsey data show that tree-growth and carbon-sequestration paths of second-rotation forest generally differ from those of first-rotation forest, particularly in the soil component. More precisely, as mentioned in footnote 3, sequestration by forest soil after initial conversion from farmland is largely independent of whether and when the forest is subsequently rotated; the stock of carbon contained in the soil grows at a declining rate for about a century, and then reaches a plateau.

It follows that, even with periodic harvesting, the process of sequestration by forest soil conforms quite closely to the assumptions of our analytical model. In particular, immediately converting to forest *does* destroy forever the option of starting the process of soil sequestration later. Because of the much larger importance of soil sequestration for spruce-fir forest (relative to that for southern pine) this one-off option to sequester in soil at high rates is relatively much more valuable, and in effect drives the initial conversion decision. Specifically, because this option increases in *present* value under the $2\times CO_2$ scenario for close to a century, it becomes optimal to delay conversion of even marginal farmland for almost 80 years. In contrast, the decision to convert to southern-pine forest is driven largely by the non-soil component of sequestration, and thereby by the minimum-profitable rotation length.

A final feature of the simulation results that merits comment are the discontinuous upward jumps in the optimal sequestration supply curves, most notably under the $2{\times}CO_2$ scenario. These discontinuities arise for reasons to do with optimal timing of rotations in anticipation of a sudden future drop in the rate of increase of carbon prices, as occurs under the $2{\times}CO_2$ scenario around the year 2115. Because a full explanation of the discontinuities is quite complex and somewhat peripheral to the main point of our paper, we relegate it to Appendix B.

7. Conclusions

In this paper, we consider socially optimal carbon sequestration and abatement decisions under different expectations about future carbon prices. It is shown that if carbon prices are expected to increase over time—consistent with projections from integrated assessment models under various assumptions about future climate-policy goals—it becomes optimal to delay certain carbon sequestration projects, whereas the optimal timing of abatement projects remains unchanged. Equivalently, the social opportunity cost of undertaking these carbon sequestration projects immediately—i.e., of *not* delaying them—increases relative to that of carbon abatement projects. Earlier estimates of the relative costs of carbon sequestration and abatement have, in almost all cases, been based on an assumption that prices are constant over time, in which case there is no reason to delay either type of project. The central implication of this study is that increasing carbon prices reduce, relative to constant-price projections, the share of carbon sequestration in an optimal portfolio of greenhouse-gas mitigation strategies, and more so, the more rapidly carbon prices are expected to increase.

This analytical result is of course relevant to climate policy only if it is quantitatively, by some measure, large. Using defensible values for the parameters in our analytical model, we find that a 3% rate of increase in carbon prices results in about a 60% reduction in the optimal share of carbon sequestration relative to constant-price projections. Simulations with our numerical model, based on predicted carbon-price paths from the Nordhaus (2001) RICE01 model for a range of climate-policy goals, indicate quantitatively similar reductions under an economically efficient scenario; somewhat larger initial reductions under a scenario that extends Kyoto-Protocol emissions limits forever; and very large reductions (80–100% for up to a century) under a scenario that aims to limit the atmospheric CO_2 concentration to double its pre-industrial level.

Although our analysis focuses on carbon sequestration in forests, our key findings should hold also for other approaches to greenhouse-gas mitigation characterized by declining rates of effectiveness over time. Enhancing carbon sequestration in agricultural soils through adoption of conservation tillage and other best-management practices is one such case. Lal and Bruce (1999) estimate the worldwide potential contribution of this approach to be about 450–600 MtC per year, but add that "soil's capacity to sequester C through adoption of BMPs may be filled over a 20–50-year period" and that the rate of sequestration usually peaks within 10–15 years. Our results suggest that, as with sequestration by forests, rising carbon prices will reduce the role of this approach in an optimal portfolio of mitigation strategies as well, though further analysis will be needed to quantify any such effect.

Appendix A.

The following assumptions and solution procedure were used to generate the numerical simulation results of Section 6.

Stumpage price and reforestation costs: Based on published prices, the stumpage price P is set at \$12.60 per cord (1 cord = 128 cu. ft) for southern pine and \$21.95 per cord for spruce-fir. The reforestation cost C is set at \$61 per acre for southern pine (based on a figure given by Moulton and Richards (1990) for conversion of wet cropland in the southeast) and \$20 per acre for spruce-fir (a figure at the low end of tree-establishment share payments under the Conservation Reserve Program).

Tree-growth and carbon-sequestration paths: Estimated forest-growth and carbon-sequestration paths are taken from Birdsey (1992). These estimates distinguish between the first rotation following conversion and subsequent rotations, and distinguish also between sequestration by trees, understory, forest floor, and soil. Both tree growth and sequestration are assumed to cease after 120 years (the upper limit on the data), i.e., both the stock of merchantable wood and cumulative sequestration are assumed to then reach a steady state. If the forest is harvested at any time, it is assumed that all carbon sequestered in the understory, forest floor, and the nonmerchantable portion of trees (which is 59% of the total wood for southern pine and 36% for spruce-fir) is released immediately to the atmosphere. The carbon sequestration in soil is unaffected, however, and continues to build up until it reaches a plateau after 120 years.

Over time, part of the carbon sequestered in merchantable wood is assumed to also be released to the atmosphere, as a result of burning and decay in landfills. The assumed time path of such releases is based on estimates by Plantinga and Birdsey (1993), which imply that cumulatively about 60% of all carbon sequestered in merchantable wood is released over the course of 50 years after harvesting. We assume that landowners pay a harvest penalty for *all* releases as and when they occur, including these delayed releases from merchantable wood.

Carbon-price and discount-rate paths: Carbon-price and discount-rate paths under various policy scenarios are generated from Nordhaus's (2001) RICE01 integrated assessment model.[26] Carbon prices p_t for for the initial decade, from 1995–2004, are constrained to equal zero. At the other end, both the carbon-price and discount-rate paths are truncated at the year 2295 and assumed to thereafter stay constant forever at their year-2295 levels.[27]

Solution procedure: In order to convert the landowner's infinite-horizon problem to a dynamic programming problem we make use of the fact that, if an acre of land has been converted to forest by the year 2295, the landowner's problem of when to optimally rotate the forest from then onwards is recursive—essentially a variant of the standard Faustmann problem. Solving this problem numerically allows us to assign a terminal value $V_{2295}(S)$ to each possible forest age $S \in 1,300$ at the start of year 2295, where S is treated as a state variable. The only additional state that the acre of land might be in at the start of year 2295 is that of agricultural land. This state is labeled $S = 0$ and assigned a terminal value of $V_{2295}(0) = a/r_{2295}$, where a represents agricultural profits for the acre in question.[28] Using these terminal values, we calculate for each year t prior to

[26] Available online at http://www.econ.yale.edu/ nordhaus/homepage/homepage.htm.

[27] Simulations using alternative assumptions about the price- and discount-rate paths after 2295 yielded negligibly different results.

[28] To simplify the exposition, the discussion here glosses over some details of the actual procedure used. In particular, the actual simulations distinguished between pre- and post-conversion agricultural land (to incorporate our assumptions about

a value function

$$V_t(S) = \max_{A_t \in \mathcal{A}(S)} \left[v_t(A_t, S) + \frac{1}{1+r_t} V_{t+1}(S'(A_t, S)) \right],$$

where $\mathcal{A}(S)$ denotes the set of feasible actions conditional on being in state S, $v_t(A_t, S)$ the within-period payoff from each action A_t, and $S'(A_t, S)$ the state transitioned to as a result of that action. For forest of age S, the feasible actions are to

(*i*) leave the forest undisturbed, resulting in a transition to state $S' = S + 1$ and a payoff v_t equal to a year's worth of sequestration revenues

(*ii*) rotate the forest, resulting in a transition to state $S' = 1$ and a payoff equal to stumpage revenues minus forest re-establishment costs and the total harvest penalty for all carbon released into the atmosphere at and after t as a direct consequence of the harvest

(*iii*) revert to agricultural land, resulting in a transition to that state and a payoff equal to a year's worth of agricultural profits plus stumpage revenues, minus the harvest penalty.

For agricultural land, the feasible actions are to

(*iv*) keep the land in agriculture, resulting in a transition to state $S' = 0$ and a payoff equal to a year's worth of agricultural profits

(*v*) convert to forest, resulting in a transition to state $S' = 0$ and a payoff equal to a year's worth of sequestration revenues minus forest establishment costs.

Finally, we calculate *optimal* sequestration supply in any given year t by solving numerically for the agricultural profit level at which a landowner is just indifferent between actions (*iv*) and (*v*), assuming perfect foresight about prices and interest rates. The marginal acre supplied at t will have this agricultural profit level.

Essentially the same procedure is used also to calculate sequestration supply under constant-price expectations. The only difference is that prices and discount rates are held constant at $p(t)$ and $r(t)$ forever.[29]

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soil sequestration) as well as between initial and subsequent forest rotations, making for a total of 602 rather than 301 states.

[29] To isolate the effects of price expectations alone, we have also run simulations that (somewhat schizophrenically) combined myopic price expectations with perfect foresight about discount rates. The results were negligibly different, showing that it is price expectations that drive our results.

Appendix B.

In this appendix, we provide some intuition for why the sequestration supply curve can exhibit discontinuous upwards jumps if landowners foresee a future drop in the rate of increase of carbon prices. In a longer version of this appendix (available from the authors upon request) we prove analytically that such jumps occur in a simplified version of the model of Section 6, which assumes that

(i) up to some given time $\overline{T}$, the price of carbon increases exponentially at a rate equal to the constant discount rate r; after $\overline{T}$, the carbon price stays constant forever at level $p(0)e^{r\overline{T}}$

(ii) the rate of sequestration falls at a constant exponential rate λ

(iii) a constant fraction β of the total carbon sequestered in forest is released at the time of harvest; the remaining fraction $1-\beta$ is sequestered forever in wood products

(iv) the stumpage price is expressed in carbon units, so that $\gamma = 1$

With these assumptions, the landowner's optimization problem becomes

$$\max_{T_0,T_1,\dots} NPV(T_0,T_1,\dots) = -e^{-rT_0}\frac{a}{r} + \sum_{i=0}^{\infty}\Big\{-e^{-rT_i}C + \int_{T_i}^{T_{i+1}} e^{-rt}p(t)q_0^s e^{-\lambda(t-T_i)}\,dt$$
$$+ e^{-rT_{i+1}}P(1-\beta)Q^s(T_i,T_{i+1}) - e^{-rT_{i+1}}p(T_{i+1})\beta Q^s(T_i,T_{i+1})\Big\}, \quad \text{(B1)}$$

where the cumulative amount of carbon sequestered by a rotation planted at time T_i and harvested at time T_{i+1} is now

$$Q^s(T_i,T_{i+1}) = \frac{q_0^s}{\lambda}[1-e^{-\lambda(T_{i+1}-T_i)}],$$

and the carbon price $p(t)$ is equal to $p(0)e^{rt}$ for all $t \leq \overline{T}$ and equal to $p(\overline{T})$ for all $t \geq \overline{T}$.

Solving this problem backwards for the optimal rotation times $T_1^*(T_0), T_2^*(T_0), \dots$ conditional on T_0 and substituting these back into the $NPV(T_0,T_1,\dots)$ function, we obtain a function $NPV^*(T_0)$ representing the net present value of converting at any given time T_0 given optimal subsequent forest management. We then prove the following proposition:

Proposition 1. *The $NPV^*(T_0)$ function is continuous, but for each $n \geq 0$, there exists a time $\hat{T}_n$ at which its first derivative jumps discontinuously upwards, implying that the function itself is locally convex. Moreover, $\hat{T}_n < \hat{T}_{n-1} < \dots < \hat{T}_1 < \overline{T}$.*

The proposition implies that there are time intervals straddling each of the times $\hat{T}_n, \hat{T}_{n-1}, \ldots, \hat{T}_1$ during which no landowner will optimally convert, even though the carbon price continues to increase at rate r over the course of these intervals. This in turn implies that aggregate supply plotted against the carbon price will jump discontinuously upward across some range of prices straddling $p(\hat{T}_n)$, then again across some range of prices straddling $p(\hat{T}_{n-1})$, etc.

In this, shorter version of the appendix, we present only the intuition underlying the proposition, key to which are the following two facts:

Fact 1. *It is never optimal to end a rotation exactly at* $\overline{T}$.

Fact 2. *The first derivative of the* $NPV^*(T_0)$ *function increases in the length* $T_1^*(T_0) - T_0$ *of the first rotation.*

Fact 1 follows from our assumptions about the carbon-price path before and after $\overline{T}$, and the resulting change in value of the "harvest penalty" $\beta p(T_{i+1})Q^s(T_i, T_{i+1})$ levied on landowners whenever the forest is rotated. Specifically, our assumption that the carbon price increases at the discount rate r up to $\overline{T}$ implies that marginally delaying a rotation time T_{i+1} *before* $\overline{T}$ does not reduce a given harvest penalty's present value, because its current value increases to offset the delay. In contrast, delaying a rotation time *after* $\overline{T}$ does reduce the penalty's present value, by our assumption that the carbon price stays constant from $\overline{T}$ onwards.

It follows that whenever it is optimal to increase a rotation time all the way *up to* $\overline{T}$, it will be optimal to increase it to *beyond* $\overline{T}$, because exactly at $\overline{T}$ the additional marginal benefit $r\beta p(\overline{T})Q^s(T_i, T_{i+1})$ of reducing the harvest penalty's present value kicks in. Conversely, whenever it is optimal to reduce a rotation time all the way *down to* $\overline{T}$, it will optimal to reduce it to *before* $\overline{T}$, because exactly at $\overline{T}$ the marginal cost $r\beta p(\overline{T})Q^s(T_i, T_{i+1})$ falls away. The upshot is that it is never optimal to end a rotation exactly at $\overline{T}$.

Fact 2 follows from the envelope theorem, which implies that the total derivative of $NPV^*(T_0)$ is equal to the partial derivative holding all rotation times $T_1^*(T_0), T_2^*(T_0), \ldots$ constant. Using (B1), and using R_1^* as shorthand for $T_1^*(T_0) - T_0$, we can therefore write

$$\frac{dNPV(T_0)}{dT_0} = \frac{\partial NPV(T_0)}{\partial T_0} = e^{-rT_0}[a + rC] - [e^{-r(T_0+R_1^*)}P + p(0)](1-\beta)q_0^s e^{-\lambda R_1^*} \tag{B2}$$

for any $T_0 < \overline{T}$. The first term on the right-hand side represents the marginal benefit of delaying conversion, equal to the additional agricultural profits earned plus the value of delaying plantation costs. The second term represents the marginal cost of delay, equal to the forgone stumpage and sequestration revenues (net of the harvest penalty) from shortening the first rotation. This marginal cost declines in R_1^*: the longer the first rotation is to begin with, the smaller the annual growth increment at the end of the rotation, and therefore the smaller the revenues lost from forgoing that increment.

Consider now all conversion times T_0 conditional on which it is optimal to complete exactly n rotations before $\overline{T}$, and let $R_{1,n}^*(T_0)$ denote the conditionally optimal length of the first rotation. By substituting $R_{1,n}^*(T_0)$ into equation (B2), setting the result equal to zero, and solving for a, we obtain the agricultural profit level at which it is optimal to convert at any given T_0. Moreover, it is easy to show that (since $\partial^2 NPV/\partial T_0 \partial a > 0$), landowners with higher agricultural profits will convert later. As a result, if they continue to complete n rotations before $\overline{T}$, each rotation will tend to be shorter, and so will yield lower sequestration and stumpage revenues to offset the constant replanting costs. Eventually, then, it becomes optimal to "give up" on the n-th rotation, and instead to complete only $n-1$ rotations before $\overline{T}$.

This is were Facts 1 and 2 come into play. Let $\hat{T}_n$ denote the conversion time at which landowners are just indifferent between undertaking either n or $n-1$ rotations before $\overline{T}$. By Fact 1, if a landowner switches from undertaking n rotations before $\overline{T}$ to undertaking only $n-1$, the conditionally optimal n-th rotation time will jump *discretely* from some time strictly before $\overline{T}$ to some time strictly after. It can be shown that, as a result, it becomes optimal to make not just the n-th rotation, but also all $n-1$ rotations before discretely longer. In particular, this is true of the very first rotation. But then, by Fact 2, the first derivative of the $NPV^*(T_0)$ function must jump discretely upwards at $\hat{T}_n$, as asserted in Proposition 1. Moreover, the same will be true (by the same reasoning) at the conversion time $\hat{T}_{n-1}$ at which landowners are indifferent between undertaking either $n-1$ or $n-2$ rotations before $\overline{T}$, etc.

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