Antitrust in Innovative Industries

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PRELIMINARY AND INCOMPLETE

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1. Introduction

Much has been written about the increasing importance of intellectual property, and industries based upon it, in the U.S. economy. Even ignoring the recent "dot-com bubble," the list of the largest firms in the U.S. economy is increasingly dominated by firms operating in industries in which innovation is central to a firm's success. Competition in such "new economy" industries is often said to possess characteristics that are quite different from those of traditional "old economy" industries: First, although the technological leaders in such industries often possess dominant market shares, they may nevertheless be rapidly replaced when a competitor succeeds in producing the next significant innovation, a process reminiscent of Schumpeter's gale of creative destruction (Evans and Schmalensee [2002]). Second, the (temporary) market power of technological leaders can be essential for stimulating the innovative activity that drives welfare improvements in these industries. As Evans and Schmalensee [2002] succinctly put it, "firms engage in dynamic competition for the market — usually through research-and development (R&D) to develop the 'killer' product, service, or feature that will confer market leadership and thus dimish or eliminate actual or potential rivals. Static price/output competition on the margin in the market is less important."

In the wake of these changes, and sparked by the recent Microsoft case, a number of commentators have expressed the concern that traditional antitrust analysis of such industries — which has typically ignored almost entirely issues of innovation — might be poorly suited to maximizing welfare.[1] In the Microsoft

[1] For an example of such an argument, see again Evans and Schmalensee [2002]. Issues of innovation have been considered when discussing "innovation markets" in some horizontal

case, for example, arguably the most significant issue in evaluating Microsoft's allegedly anticompetitive practices from a welfare standpoint was the effect that they had on innovation in the industry. In essence, Microsoft argued that while a technological leader like Microsoft may possess a good deal of static market power, this is merely the fuel for stimulating dynamic competition, a process that works well in this industry. The government, in contrast, argued that Microsoft's practices prevented entry of new firms and products, and therefore would both raise prices and retard innovation. (For further discussion, see e.g., Evans and Schmalensee [2002] and Whinston [2001].) How to reconcile these two views, however, was never fully clear in the discussion surrounding the case. For example, if profits are necessary for spurring innovation as Microsoft argued, does this mean that practices that enhance a dominant firm's ability to protect its monopoly position will spur innovation?[2] This lack of clarity was perhaps not surprising, since the issues have seen little or no attention in the industrial organization literature.

In this paper, we study the role of antitrust policy in innovative ("dynamically competitive") industries. We do so using models in which innovation is a continual process, with new innovators replacing current incumbents, and holding dominant market positions until they are themselves replaced. Although a great

merger cases, where there was a concern that a merger might reduce R&D competition. See, e.g., Gilbert and Sunshine [1995].

[2] Note that there is a potentially important distinction here between a policy that restricts Microsoft's behavior and a policy that restricts the behavior of all dominant software producers. The former restrictions are sure to increase the likelihood of success of today's potential entrants. However, the relevant question concerns the latter restrictions, which may not increase innovative activity, because today's potential entrants are spurred precisely by the hope of becoming the next Microsoft.

deal of formal modeling of R&D races has occurred in the industrial organization literature (beginning with the work of Loury [1979] and Lee and Wilde [1980]; see Reinganum [1989] for a survey), this work has typically analyzed a single, or at most a finite sequence, of innovative races. Instead, our models are closer to those that have received attention in the recent literature on growth (e.g., Grossman and Helpman [1991], Aghion and Howitt [1992], Aghion et. al [2001]). The primary distinction between our analysis and the analysis in this growth literature lies in our explicit focus on how antitrust policies affect equilibrium in such industries.[3]

The paper is organized as follows. In Section 2, we introduce and analyze a simple stylized model of antitrust in an innovative industry. This simple model, in which only potential entrants conduct R&D, captures antitrust policy as affecting the profit flows that an incumbent and a new entrant can earn in competition with each other, as well as the profits of an uncontested incumbent. Using the model, we develop some general insights into the effect of antitrust policies on the rate of innovation. We show that a more protective antitrust policy (one that increases a new entrant's profits at the expense of the incumbent) "front-loads" an innovative new entrant's profit stream, and that this feature tends to increase the level of innovative activity by potential entrants to the industry. Indeed, as long as a more protective policy dissipates neither the joint profit of the incumbent and entrant upon entry nor uncontested incumbent profits, it will increase the level

[3] This literature often considers how changes in various parameters will affect the rate of innovation, sometimes even calling such parameters measures of the degree of antitrust policy (e.g., Aghion et al. [2001] refer to the elasticity of substitution as such a measure). We feel, however, that it is important to more explicitly model what antitrust policies do in order to reach proper conclusions on their effects. For example, results that show that more inelastic demand functions lead to more R&D (e.g., Aghion and Howitt [1992]), produce conclusions quite different from those we derive below.

of R&D. We also explore extensions of the model to situations of free entry, to growing markets, and to predatory activities that affect an entrant's probability of survival.

With the stylized model of Section 2 in hand, in Section 3 we develop applications to specific antitrust polices. First, we study a model of long-term (exclusive) contracts and show that a more protective antitrust policy necessarily stimulates innovation and raises both aggregate and consumer welfare. Next, we study a model of predatory pricing. Once again, a more protective policy necessarily stimulates R&D in our model, although we show that the welfare implications are in general ambiguous. We also briefly discuss voluntary deals betwwen the incumbent and entrant, such as buy-outs or licensing agreements, which can be seen to necessarily increase the rate of innovation. All three of these applications have the feature that the joint profit of the incumbent and entrant upon entry, as well as the profit of an uncontested incumbent, are weakly increased by a more protective policy. We conclude Section 3 by discussing an extension of our long-term contracting model to the case of uncertain innovation size with a fixed cost of implementing new innovations. We show that in this situation, a more protective policy may reduce joint profits upon entry and thereby retard innovation.

The analysis of Sections 2 and 3 makes the strong assumption that only potential entrants do R&D. While useful for gaining understanding, this assumption is rarely descriptive of reality. In Section 4 (still incomplete), we turn our attention to models in which both incumbents and potential entrants conduct R&D. Introducing incumbent investment has the potential to substantially complicate our analysis by making equilibrium behavior depend on the level of the incumbent's lead over other firms. We study two models in which we can avoid this state dependence. In one model, the previous leading technology is assumed to enter the public domain whenever the incumbent innovates. In this model, the incumbent does R&D solely to avoid displacement by a rival. In our second model, the profit improvement from a larger lead is assumed to be linear in the size of the lead and potential entrants are assumed to win all "ties," which again leads the incumbent's optimal R&D level to be stationary. In this model, the incumbent does R&D to improve its profit flows until the time that it is displaced by a rival. Interestingly, we show that in both models there are a wide range of circumstances in which a more protective policy can increase the innovation incentives of both the incumbent and potential entrants.

Finally, Section 5 (to be added) concludes.

2. A Simple Model of Antitrust in Innovative Industries

We begin by considering a simple stylized model of continuing innovation. Our aim in this section is to develop a model that yields some general insights into the effect of antitrust policies on the rate of innovation, and that we can apply to a number of different antitrust policies in the remainder of the paper. The model has discrete time and an infinite horizon. There are $N + 1$ firms who discount future profits at rate $\delta \in (0, 1)$. In each period, one of the firms is the "incumbent" I and the others are "potential entrants," denoted collectively by E. In the beginning of each period, each potential entrant i independently chooses its R&D rate, $\phi_i \in [0, 1]$, at a cost $c(\phi_i)$. (Note that in this simple model only the potential entrants may do R&D; we relax this assumption to consider incumbent investment in Section 4). The R&D of a given potential entrant i yields an innovation — which we interpret to be a particular improvement in the quality of the product — with probability ϕ_i. We shall focus on symmetric equilibria, in which all potential entrants choose the same equilibrium level of R&D, denoted by ϕ. In this case, the likelihood that at least one firm among the N potential entrants innovates is given by $s(\phi, N) \equiv \left[1 - (1-\phi)^N\right]$. Among the potential entrants who discover the innovation, only one may receive the patent for that innovation. Given that all other potential entrants are doing R&D at level ϕ, we denote by $r(\phi, N)$ the probability that a given potential entrant receives a patent, conditional on it making a discovery.[4] A potential entrant who is successful at receiving a patent enters and competes with the incumbent in the present period, and then becomes the incumbent in the next period, while the previous incumbent then becomes a potential entrant. In this sense, this is a model of "winner-take-all" competition. While the patent provides perfect protection (forever) to the innovation itself, others may overtake the patent holder by developing subsequent innovations.

We will be interested in the effects of an antitrust policy α that affects the incumbent's competition with an entrant who has just received a patent. To this

[4] When the patent is awarded randomly to one of the successful innovators, we have

$$r(\phi, N) = \sum_{k=0}^{N-1} \left(\frac{1}{k+1}\right) \left(\begin{array}{c} N-1 \\ k \end{array}\right) \phi^k (1-\phi)^{N-1-k}.$$

It should be noted, however, that the results of this section hold for any functions $r(\phi, N)$ and $s(\phi, N)$; for example, there may be some probability that none of the firms that have made discoveries are successful in commericalizing its product.

end, we denote the incumbent's profit in competition with a new entrant by $\pi_I(\alpha)$, and the profit of the entrant by $\pi_E(\alpha)$. We let $\pi'_E(\alpha) > 0$, so that a higher α represents a policy that is more "protective" of the entrant. Let $\pi_m(\alpha)$ denote the per period profit of an incumbent who faces no competition. (In Section 3, when we consider specific applications, we show how these values can be derived from an underlying model of the product market.)

We examine stationary Markov perfect equilibria of the infinite-horizon game using the dynamic programming approach. Let V_I denote the expected present discounted profits of an incumbent, and V_E those of a potential entrant (both evaluated in the beginning of a period). Then, since innovation occurs with probability ϕ, these values should satisfy

$$V_I = \pi_m(\alpha) + \delta V_I + s(\phi, N)\left[\pi_I(\alpha) - \pi_m(\alpha) + \delta(V_E - V_I)\right], \tag{VI}$$

$$V_E = \delta V_E + \phi r(\phi, N)\left[\pi_E(\alpha) + \delta(V_I - V_E)\right] - c(\phi). \tag{VE}$$

Also, since a potential entrant's choice of ϕ should maximize its expected discounted value given that all other potential entrants are choosing R&D level ϕ,

$$\phi \in \arg\max_{\psi\in[0,1]} \left\{\psi r(\phi, N)\left[\pi_E(\alpha) + \delta(V_I - V_E)\right] - c(\psi)\right\}.$$

Letting $W \equiv r(\phi, N)[\pi_E(\alpha) + \delta(V_I - V_E)]$ denote the expected benefit from successful innovation — what we shall call the **innovation prize** — this equation can be rewritten as

$$\phi \in \arg\max_{\psi\in[0,1]} \left\{\psi W - c(\psi)\right\}. \tag{IS}$$

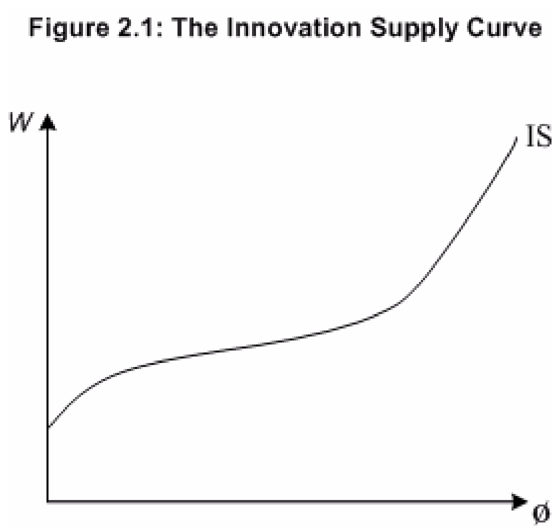

Figure 2.1:

This equation defines the "Innovation Supply" curve—the optimal innovation choice as a function of W. Note that this curve, which we depict in Figure 2.1, is upward sloping.

On the other hand, since $W = r(\phi, N)[\pi_E(\alpha) + \delta(V_I - V_E)]$ and (by subtracting (VE) from (VI)):

$$(V_I - V_E) = \frac{s\pi_I(\alpha) + (1-s)\pi_m(\alpha) - \phi r \pi_E(\alpha) + c(\phi)}{1 - \delta + \delta(s + \phi r)}, \tag{2.1}$$

we can solve for the equilibrium value of the innovation prize W (to simplify notation, we suppress the arguments of $s(\phi, N)$ and $r(\phi, N)$):

$$W = r\left\{\pi_E(\alpha) + \delta\left[\frac{s\pi_I(\alpha) + (1-s)\pi_m(\alpha) - \phi r \pi_E(\alpha) + c(\phi)}{1 - \delta + \delta(s + \phi r)}\right]\right\}$$

Figure 2.2: Equilibrium and Comparative Statics

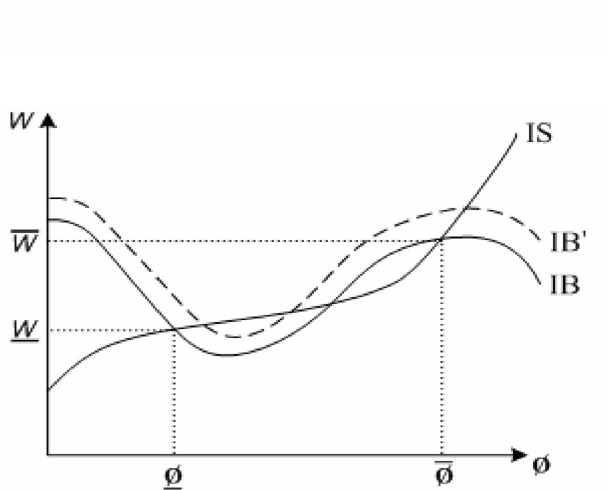

Figure 2.2:

$$
\begin{aligned}
&= r \left(\frac{\pi_E(\alpha)\{1-\delta+\delta[s+\phi r]\}+\delta[s\pi_I(\alpha)+(1-s)\pi_m(\alpha)-\phi r\pi_E(\alpha)+c(\phi)]}{1-\delta+\delta(s+\phi r)} \right) \\
&= r \left(\frac{\pi_E(\alpha)(1-\delta)+\delta\{s[\pi_I(\alpha)+\pi_E(\alpha)]+[1-s]\pi_m(\alpha)+c(\phi)\}}{1-\delta+\delta(s+\phi r)} \right). \qquad \text{(IB)}
\end{aligned}
$$

This equation defines the "Innovation Benefit" curve — the value of the innovation prize as a function of the per firm innovation rate ϕ. An equilibrium pair (W, ϕ) must lie at an intersection of (IS) and (IB), as shown in Figure 2.2 where there are three equilibria.

This representation of equilibrium allows us to examine the comparative statics of innovation with respect to the parameter α in a way that is independent of the properties of the innovation cost function $c(\cdot)$. Note that the (IS) curve does not depend on α at all. By Theorem 1 of Milgrom and Roberts [1994], if α shifts the (IB) curve up or down at all values of ϕ, then it increases or reduces the equilibrium innovation rate in the "largest" and "smallest" equilibria (denoted by $\underline{\phi}$ and $\overline{\phi}$ respectively in Figure 2.2). This can be seen in Figure 2.2, where the dashed curve represents an upward shift of the IB curve. As is also evident in the figure, the same can be shown (using the Implicit Function Theorem) of any "stable" equilibrium if the IB function is shifted up or down in a neighborhood of the equilibrium.[5]

Differentiating (IB) with respect to α then yields the following result:

Proposition 2.1. *An increase in α, the protectiveness of antitrust policy, increases the rate of innovation in the equilibria with the highest and lowest innovation rates if and only if*

$$\left[1+\frac{1-\delta}{\delta s}\right]+\left(\frac{1-s}{s}\right)\left(\frac{\pi'_m(\alpha)}{\pi'_E(\alpha)}\right)\geq -\frac{\pi'_I(\alpha)}{\pi'_E(\alpha)} \tag{2.2}$$

at all s. Moreover, the change in a stable equilibrium's innovation rate in response to a local change in α is positive if and only if (2.2) holds.

A useful alternative way to state inequality (2.2) is

$$\pi'_E(\alpha)+\delta\left[\frac{(1-s)\pi'_m(\alpha)+s\pi'_I(\alpha)}{1-\delta(1-s)}\right]\geq 0. \tag{2.3}$$

This expression makes clear that a change in policy encourages (discourages) innovation precisely when it raises (reduces) the incremental expected discounted

[5] When there is a unique equilibrium, this result then implies determinate comparative statics. Equilibrium can be shown to be unique, for example, when $N=1$, $c'(\phi)\geq 0$, and $c''(\phi)>0$.

profits over an innovation's lifetime: The first term on the left side of (2.3) is the change in an entrant's profit in the period of entry due to the policy change, while the second term is precisely the discounted value of the change in the entrant's profit once it is established as the new incumbent in the following period (the numerator is the derivative of the flow of expected profits in each period of incumbency conditional on still being an incumbent; the denominator captures the "effective" discount rate, which includes the probability of displacement).

To understand the implications of condition (2.2), it is useful to focus first on the case in which antitrust policy affects only the profits of the incumbent and entrant when they compete with one another, so that the monopoly profit π_m is independent of α (hence, $\pi_m'(\alpha) = 0$). Observe, first, that since $\delta < 1$, the left side of (2.2) is then necessarily larger than 1. Hence, a more protective antitrust policy raises innovation whenever $\pi_I'(\alpha) + \pi_E'(\alpha) \geq 0$; that is, provided that an increase in α does not lower the joint profits of the entrant and the incumbent in the period of entry. Second, the smallest level of $-\frac{\pi_I'(\alpha)}{\pi_E'(\alpha)}$ for which increasing antitrust protection lowers innovation is decreasing in s. Thus, other things equal, protecting new entrants is more likely to lower the rate of innovation in industries in which the innovation rate is high. Similarly, this cutoff level of $-\frac{\pi_I'(\alpha)}{\pi_E'(\alpha)}$ falls when δ increases.

In the case where $\pi_m'(\alpha) = 0$, the intuition behind condition (2.2) is simple: As we have noted, a change in policy encourages (discourages) innovation when it raises (reduces) the incremental expected discounted profits over an innovation's lifetime. A successful innovator earns $\pi_E(\alpha)$ when he enters, and earns $\pi_I(\alpha)$ when he is displaced. A more protective antitrust policy that raises π_E and lowers π_I shifts profits forward in time. Since the later profits π_I are discounted by potential entrants (by the probability of a subsequent innovation s as well as the time discount factor δ) this "front loading" of profits necessarily increases the innovation prize provided that the joint profit $\pi_I + \pi_E$ does not decrease. The sizes of δ and s determine how much of this joint profit can be dissipated in this forward shift while still increasing the innovation prize. The larger is δ or s, the more important is the dissipation effect: For s, this is so because larger s moves forward the expected date when the entrant will itself be replaced. For δ, this is so because with larger δ the discounted value of the profits in the period in which the entrant is replaced are greater. In the limit, as $\delta \to 1$, the amount by which this joint profit can be dissipated converges to zero: in this limiting case the cost of a one dollar reduction in the value π_I that the entrant will receive when he is ultimately displaced is exactly equal to the gain from receiving a dollar more in the period in which he enters.

More generally, when $\pi'_m(\alpha) \neq 0$, we also have the extra term $\left(\frac{1-s}{s}\right)\left(\frac{\pi'_m(\alpha)}{\pi'_E(\alpha)}\right)$, whose sign captures the effect that the policy has on incumbent profits when no entry occurs. When this term is non-negative (as will be the case in our models of long-term exclusive contracts and predatory pricing in Sections 3.2 and 3.3), it continues to be the case that the policy encourages innovation as long as it does not lower the joint profit upon entry, $\pi_E + \pi_I$. When this effect is negative, however, by reducing π^m a more protective policy may reduce the innovation prize even when $\pi_E + \pi_I$ increases .

2.1. More general profit functions

In general, all three of the profits $\pi_I, \pi_E,$ and π_m may be affected as well by the rate of innovation s (this is true, for example, in the model of long-term contracts in Section 3.2). Denoting these profits by $\pi_I(\alpha, s)$, $\pi_E(\alpha, s)$, and $\pi_m(\alpha, s)$, we see that the derivation leading to Proposition 2.1 continues to hold in this case, since it involves asking when the IB curve is shifted upward at a particular value of s. Thus, we need only reinterpret the derivatives in (2.2) as being partial derivatives with respect to α holding s fixed.

2.2. Free entry

Above we have taken the number of firms as fixed. In some circumstances, however, it may be more appropriate to assume that there is free entry into R&D competition.[6] To do so, we posit that there is a fixed cost $F > 0$ of doing R&D in each period. The number of firms N is now endogeneous, and we impose the equilibrium condition that $V_E = F$ (we ignore integer constraints on N). Setting $V_E = 0$ in (2.1) we have that

$$V_I = \frac{s\pi_E(\alpha) + (1-s)\pi_m(\alpha)}{1-\delta+\delta s}.$$

[6] The fixed N model is the appropriate model when there are a limited number of firms with the capability of doing R&D in an industry (perhaps because of some complementary assets they possess due to participation in related industries).

In addition, letting the innovation prize again be W, here we have

$$\begin{aligned} W &= r\left\{\pi_E(\alpha) + \delta V_I\right\} \\ &= r\left(\frac{\pi_E(\alpha)(1-\delta) + \delta\{s[\pi_I(\alpha) + \pi_E(\alpha)] + (1-s)\pi_m(\alpha)]\}}{1-\delta+\delta s}\right). \end{aligned} \tag{2.4}$$

Now, in a free entry equilibrium we must have

$$0 = V_E - F = \max_{\psi \in [0,1]} \psi W - c(\psi). \tag{2.5}$$

Observe that (2.5) pins down the value of W as well as the equilibrium level of ϕ in any free entry equilibrium, and that these equilibrium values (W^*, ϕ^*) are independent of α. Thus, in response to a change in α, all adjustment comes in N, which adjusts to keep the level of W unchanged (by altering r and s). This is depicted in Figure 2.3, which depicts the free entry IS and IB curves in (W, N)-space. Note that with free entry, the IS curve is horizontal at W^*.

Differentiating expression (2.4) with respect to α, we see that an increase in α increases the aggregate success rate s (and hence N) if and only if (2.2) holds; that is, under exactly the same conditions as when N is fixed.

2.3. Market growth

The results above rested on the idea that a "front loading" of the profits from successful innovation caused by a more protective antitrust policy raises the innovation prize, and hence the equilibrium rate of innovation. This same logic

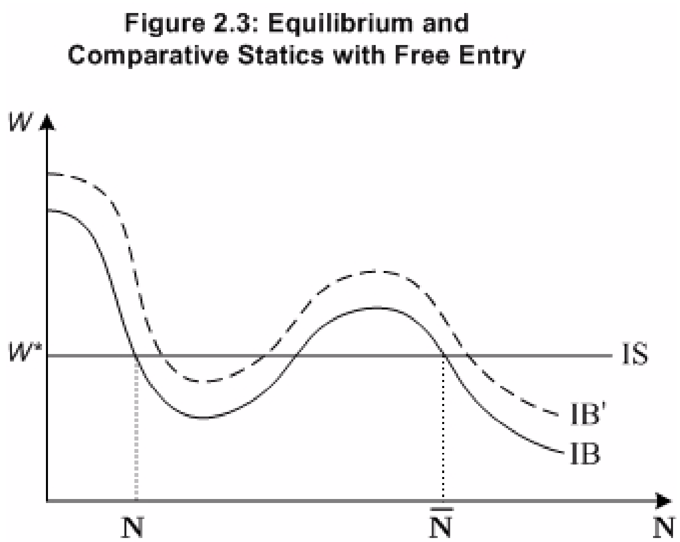

Figure 2.3:

suggests, however, that in situations in which market size is growing rapidly — so that the future looms large relative to the present — such front-loading may no longer encourage innovation. To see this point, consider the simplest possible case of market growth, in which the profit functions in period 1 are $\beta\pi_E(\alpha)$, $\beta\pi_I(\alpha)$, and $\beta\pi_m$, but are $\pi_E(\alpha)$, $\pi_I(\alpha)$, and $\pi_m(\alpha)$ beginning in period 2. The market is initially growing if $\beta < 1$.

Starting in period 2, the market is stationary, and the equilibrium values and innovation rate are exactly those derived above. Denote these, as before, as V_E, V_I, W, and ϕ. Denoting the R&D level of each potential entrant in period 1 by ϕ_1 and letting $r_1 \equiv r(\phi_1, N)$, the period 1 innovation prize W_1 is given by

$$W_1 = r_1[\beta\pi_E(\alpha) + \delta(V_1 - V_0)].$$

By analogy with condition (2.3), the policy change increases the rate of innovation in period 1 if and only if

$$\beta\pi_E'(\alpha) + \delta\left[\frac{(1-s)\pi_m'(\alpha) + s\pi_I'(\alpha)}{1-\delta(1-s)}\right] \geq 0,$$

since only the profits in period 1 are affected by the market growth term β.[7] Rearranging this expression, we see that the equilibrium level of ϕ_1 increases with α if and only if

$$\left(\beta\left[1+\frac{1-\delta}{\delta s}\right] + \left(\frac{1-s}{s}\right)\frac{\pi_m'(\alpha)}{\pi_E'(\alpha)}\right) \geq -\frac{\pi_I'(\alpha)}{\pi_E'(\alpha)}. \tag{2.6}$$

Thus, with market growth ($\beta < 1$), an increase in α may lower the rate of innovation during the growth phase, even if it has no effect on π_m [so that $\pi_m'(\alpha) = 0$] and raises the joint profit $\pi_I + \pi_E$ [so that $-\pi_I'(\alpha)/\pi_E'(\alpha) < 1$].[8]

2.4. Predatory activities

In the analysis to this point, antitrust policy altered the profits earned by the incumbent and the entrant when entry occurs, and possibly uncontested incumbent profits. In some situations, antitrust may affect as well the entrant's probability of survival. Here we focus solely on this effect. Specifically, we take π_I, π_E, and π_m as fixed and suppose that a new entrant's probability of survival following its

[7] This expression can alternatively be derived by solving explicitly for W_1.

[8] Note that we have focused to this point on stationary antitrust policies. The present result suggests that it may be of interest to consider policies that vary over time with market conditions, such as the current market size or growth rate.

entry is $\lambda(\alpha)$ where λ is increasing in α. As before, we focus here on the case in which the number of firms N is fixed.[9]

Now the innovation prize is

$$W = r[\pi^E + \delta\lambda(V_I - V_E)] \tag{2.7}$$

where,

$$V_I = \pi_m + \delta V_I + s(\phi, N)\left[\pi_I - \pi_m + \delta\lambda(\alpha)\left(V_E - V_I\right)\right], \tag{VI-2}$$

$$V_E = \delta V_E + \phi r(\phi, N)\left[\pi_E + \delta\lambda(\alpha)\left(V_I - V_E\right)\right] - c\left(\phi\right). \tag{VE-2}$$

Hence,

$$\begin{aligned} W &= r\left(\pi_E + \delta\lambda(\alpha)\frac{s\pi_I + (1-s)\,\pi_m - \phi r\pi_E + c(\phi)}{1-\delta+\delta\lambda(s+\phi r)}\right) \\ &= r\left(\frac{\pi_E(1-\delta) + \delta\lambda[s(\pi_I+\pi_E) + (1-s)\,\pi_m + c(\phi)]}{1-\delta+\delta\lambda(s+\phi r)}\right). \end{aligned} \tag{2.8}$$

Differentiating (2.8) with respect to ψ we see that

$$sgn\left(\frac{\partial W}{\partial \psi}\right) = sgn\{(1-\delta)[s(\pi_I+\pi_E) + (1-s)\,\pi_m]\},$$

which is strictly positive provided that the profit flows π_I, π_E, and π_m are non-

[9] As before, the exact conditions we derive below apply as well in the case of free entry.

negative.

Hence, a more protective antitrust policy that raises the likelihood of entrant survival necessarily increases the innovation prize. Intuitively, for a fixed s, an increase in λ is a one-to-one transfer of continuation value from the incumbent to entrant. As can be seen in (2.7), this change would clearly increase W if we were to hold $(V_I - V_E)$ fixed.[10] Yet, even though $(V_I - V_E)$ is affected by the change (larger α lowers the difference in values between an incumbent and a potential entrant), the net effect is still necessarily positive.

3. Applications

In this section, we study several models of antitrust policy toward specific practices as an application of the results of Section 1. The models are all versions of the "quality ladder" models introduced in the recent literature on economic growth (e.g., Aghion and Howitt [1992]; Grossman and Helpman [1991]). Before turning to these applications, we first introduce a basic quality ladder model (in which antitrust policy plays no role) to serve as a benchmark.

3.1. A quality ladder model

There are $N + 1$ firms and a continuum of infinitely-lived consumers of measure 1 who may consume a nonstorable and nondurable good. R&D may improve the quality of this good and consumers value "generation j" of the good at $v_j =$

[10] As one implication of this observation, note that the effect of an increase in α on first period innovation in a model of market growth would still necessarily be positive, unlike in our previous model.

$v + j \cdot \Delta$. At any time t, one firm — the current "incumbent" — possesses a perfectly effective and infinitely-lived patent on the latest generation product j_t. Likewise, at time t there is a patentholder for each of the previous generations of the product $(j_t - 1, j_t - 2, ...)$. We assume, as in Section 2, that at time t only firms other than the incumbent in the leading technology — the potential entrants — can invest in developing the generation $j_t + 1$ product. One implication of this assumption is that in each period t the holder of the patent on generation $j_t - 1$ is a firm other than the current incumbent, who holds the patent on the current leading generation j_t. We assume that at time t, the firms engage in Bertrand competition to make sales. Thus, $\pi_E = \pi_m = \Delta$ and $\pi_I = 0$.[11] Specializing (2.2) to this case we have

$$
\begin{aligned}
W &= r \left[\frac{\Delta(1-\delta) + \delta[s\Delta + (1-s)\Delta + c(\phi)]}{1 - \delta + \delta(s + \phi r)} \right] \\
&= r \left[\frac{\Delta + \delta c(\phi)}{1 - \delta + \delta(s + \phi r)} \right]. \qquad (3.1)
\end{aligned}
$$

As before, the equilibrium innovation rate ϕ satisfies $\phi \in \arg\max_{\psi \in [0,1]} \{\psi W - c(\psi)\}$.

Since we now have a fully-specified consumer side (unlike in Section 2), we can compare the equilibrium innovation rate to the rate that maximizes aggregate welfare. To this end, observe that a technological advancement in period t raises gross consumer surplus in every subsequent period by Δ. The present discounted value of this change is $\left(\frac{\Delta}{1-\delta}\right)$. A firm who innovates is critical for advancing the technology in that period if and only if no other firm has successfully

[11] We focus here on the undominated equilibrium in which the incumbent (who makes no sales) charges a price equal to cost and the firm with technology $j_t - 1$ charges a price of Δ.

innovated. Thus, if the socially optimal (symmetric) innovation rate is ϕ°, the "Social Innovation Prize" is given by

$$W_S = (1 - \phi^\circ)^{N-1} \left(\frac{\Delta}{1-\delta} \right), \tag{3.2}$$

and defines a downward-sloping Social Innovation Benefit Curve. The socially optimal innovation rate ϕ° must lie at an intersection of this Social Innovation Benefit curve and the Innovation Supply Curve depicted in Figure 2.1. Since the Innovation Supply Curve is (weakly) upward sloping, there is at most a single intersection. Thus, the relation between ϕ° and ϕ can be determined by the relation between W and W_S.

In general, the equilibrium level of innovation may be either higher or lower than the level that maximizes social surplus. This is due to two distortions: First, there is a "Schumpeterian effect" because an innovator is eventually replaced even though his innovation raises surplus indefinitely. To see this effect, it is useful to define the value of a new patent to be

$$W_P \equiv \left[\frac{\Delta + \delta c(\phi)}{1 - \delta + \delta(s + \phi r)} \right].$$

Doing so, we see that $W = rW_P$. Now note that $V_E \geq 0$ if and only if $\phi r W_P - c(\phi) \geq 0$, which implies using (3.1) that

$$W_P \leq \frac{\Delta}{1 - \delta + \delta s};$$

in any equilibrium with $V_E \geq 0$. Thus, the value of a new patent never exceeds the social value of a technological advancement, $\frac{\Delta}{1-\delta}$. On the other hand, a "business stealing effect" is also present, since a potential entrant is sure to get a patent when all other firms have failed, but also gets the patent in some cases when another firm has succeeded; only in the latter case, however, has the innovation contributed to social surplus. This effect is captured by the fact that, when the patent is awarded randomly to one of the innovators, $r \geq (1-\phi)^{N-1}$.[12] Note that when $N = 1$ only the former effect is present, in which case the equilibrium rate of R&D is less than the level that maximizes social surplus. Likewise, as $\delta \to 1$, the socially optimal R&D rate $\phi^\circ \to 1$, while the equilibrium level is bounded below this level provided that $\lim_{\phi \to 1} c'(\phi) = +\infty$. On the other hand, if $\delta \to 0$ and $N > 1$, the equilibrium rate will exceed ϕ°.

One can also ask how the equilibrium innovation rate compares to the innovation rate that maximizes consumer surplus. Consumers are clearly better off the higher the rate of R&D. Thus, to ask this question sensibly, we should think about maximizing consumer surplus subject to the constraint that $V_E \geq 0$. Note, first, that whenever there are multiple equilibria, equilibria with higher levels of ϕ dominate those with lower levels (both consumer surplus and V_E are higher, the latter because W is). Focusing on the highest equilibrium, if we force the firms to slightly increase their levels of ϕ, this will lower V_E if and only if the IB curve is downward sloping at the equilibrium. If it is, then an improvement in consumer

[12] In Aghion and Howitt [1992], four distortions are present. In our model, however, there is no "appropriability effect" (an incumbent monopolist captures his full incremental contribution to social surplus in a period) nor is there any "monopoly distortion" effect (the incumbent produces the socially optimal quantity in each period).

surplus is possible if $V_E > 0$ at the equilibrium, but not if $V_E = 0$. If the IB curve is instead upward sloping at the equilibrim, then an improvement is possible in either case.

3.2. Long-term (exclusive) contracts

We now consider a model in which the incumbent can sign consumers to long-term contracts. We normalize the total number of consumers in each period to 1. Suppose that in each period t, the incumbent can offer long-term contracts to a share β_{t+1} of period $t+1$ consumers. The contracts specify a sale in period $t+1$ at a price q_t to be paid upon delivery. (In our simple model, this is equivalent to an exclusive contract that prevents the consumer from buying from the entrant, subject to some irrelevant issues with the timing of payments.) The antitrust policy restricts the proportion of customers that can be offered long-term contracts: $\beta_{t+1} \leq 1 - \alpha$. We assume that $c > \Delta$, so that an entrant cannot profitably make a sale to a customer who is bound to a long-term contract.

The timing in period t is:

- Each potential entrant i chooses innovation rate ϕ_{it}. Then innovation success is realized.
- Firms name prices p_t^i to free period t consumers
- Free period t consumers accept/reject these offers.
- The firm with the leading technology chooses to offer to a share $\beta_{t+1} \leq 1-\alpha$ of period $t+1$ consumers a period $t+1$ sales contract at price q_t to be paid upon delivery.

- These consumers accept/reject the contract offer (they assume that they have no effect on the likelihood of future entry).

We look at Markov equilibria, with the payoff relevant state for innovation choices in period t being the share β_t of captive customers in period t, and the payoff relevant state in period t for pricing choices being β_t and the technological levels of the firms. It is immediate that in any such equilibrium, the prices offered to free customers in any period t are $c+\Delta$ by the firm with the leading technology j_t, who wins the sale, and c by the firm with technology $j_t - 1$. If in period t the expected innovation rate in period $t+1$ is s_{t+1}, a period $t+1$ consumer who rejects the leading firm's advance sale offer anticipates getting the period t surplus level $\overline{v} + (j_t - 1)\,\Delta - c$ plus an expected gain in surplus of $s_{t+1}\Delta$ due to the possibility of technological advancement. Thus, he will accept the contract if and only if the price q_{t+1} satisfies $\overline{v} + j_t\Delta - q_{t+1} \geq \overline{v} + (j_t - 1 + s_{t+1})\,\Delta - c$. Hence, the maximum price the incumbent can receive in a long-term contract is $q_{t+1} = c + (1 - s_{t+1})\Delta$.

How many consumers will the leading firm sign up in period t? Observe first that if the aggregate innovation rate s_{t+1} were independent of β_{t+1}, then the leading firm would be indifferent about signing up an extra consumer: its period t expectation of the profit from a free consumer in period $t+1$ is $(1-s_{t+1})\Delta$, which exactly equals its maximal expected profit from a long-term contract. However, the entrants' optimal innovation choice ϕ_{t+1} is decreasing in β_{t+1}, because it reduces the profits a successful entrant can collect in period $t+1$.[13] Therefore, the

[13] Formally, the equilibrium innovation rate of a potential entrant in period $t+1$ satisfies $\phi_{t+1} \in \arg\max_{\psi \in [0,1]} \psi r\left(\phi_{t+1}, N\right) \left[\beta_{t+1}\Delta + \delta \left(V_I^{t+2} - V_E^{t+2}\right)\right]$, where V_I^{t+2}, V_E^{t+2} are the continuation values in period $t+2$ and are independent of β_{t+1}. Since $r\left(\phi_{t+1}, N\right)$ is decreasing in ϕ_{t+1}, there is a unique solution to this fixed-point problem, and by Theorem 1 of Milgrom and Roberts

incumbent will sign up as many long-term customers as the antitrust constraint allows, i.e., $\beta_{t+1} = 1 - \alpha$ in every period. This implies, in particular, that the equilibrium innovation rate s_t is also stationary. We can therefore fit this model into our basic model by taking

$$\begin{aligned} \pi_m(\alpha, s) &= \alpha\Delta + (1-\alpha)(1-s)\Delta \\ \pi_I(\alpha, s) &= (1-\alpha)(1-s)\Delta \\ \pi_E(\alpha, s) &= \alpha\Delta. \end{aligned} \tag{3.3}$$

We see that $\pi'_m/\pi'_E = s$, and $\pi'_I/\pi'_E = -(1-s)$. Thus, condition (2.2) is satisfied, and so we have:

Proposition 3.1. *In our basic model of long-term (exclusive) contracts, restricting the use of long-term contracts encourages innovation.*

The conclusion obtains because long-term contracts reduce both the joint profit of an incumbent and an entrant upon entry and the profit of an uncontested incumbent. Intuitively, the joint profit is lower because the incumbent must offer (in expectation) as much consumer surplus for the old product as would the entrant with an improved product. The uncontested profit is lower simply because the incumbent has had to offer a discount to induce customers to sign due to the possibility of entry, a threat that has turned out not to materialize. Since a larger α also front loads the profit stream over the lifetime of an innovation, it raises the expected discounted return to innovation, thereby encouraging it.

[1994], this solution is decreasing in β_{t+1}.

Two assumptions may matter for the conclusions of Proposition 3.1: first, that only potential entrants can engage in R&D, and second, that the innovation size is constant. We explore the effects of relaxing these assumptions in later sections.

We now consider the welfare effects of a once-and-for-all increase in the policy α. Note first that the increase raises consumer surplus: consumers are indifferent about signing exclusives when the innovation rate is held fixed, but an increase in the innovation rate delivers to them higher-quality goods at the same prices. The current incumbent is hurt by the change: it would not be affected if the innovation rate were held fixed, but it is hurt by the increase in the innovation rate. But we know that the sum of consumer surplus and current incumbent profits goes up when s increases: An innovation in period t reallocates surplus $\alpha\Delta$ from the incumbent to period t consumers. However, in subsequent periods the innovation confers an expected benefit Δ to consumers but at an expected cost to the incumbent that is less than Δ as long as the probability of future displacement is positive (i.e., $s > 0$).

Finally, we consider the effects on the entrants. Since we are staying on the upward-sloping IS curve, the increase in ϕ caused by the increase in α increases W. This implies that each potential entrant becomes better off.[14] Therefore, α raises aggregate welfare. To summarize:

Proposition 3.2. *A once-and-for-all restriction on the share of long-term (exclusive) contracts raises consumer surplus, the profits of potential entrants, and aggregate welfare. It reduces the current incumbent's profits.*

[14] Since, using (VE) and (IS), $(1-\delta)\,V_E = \max_{\psi\in[0,1]}\left\{\psi r(\phi, N)W - c(\psi)\right\}.$

Note that this conclusion is consistent with the possibility of overinvestment due to business stealing as discussed in Subsection 3.1 above. The effect of an exclusive contract is to reduce the prize (value of winning) in the patent race. This reduction is a pure waste of surplus (for a given innovation rate), occuring by preventing some consumers from using the latest technology, and making all contestants worse off. Thus, even though exclusivity may bring a socially excessive innovation rate closer to the first-best level, the waste effect dominates and aggregate welfare is reduced.

3.3. Predatory pricing

We next consider a model of predatory pricing, in which the entrant's probability of survival after its first production period is an increasing function $\lambda(\pi_E)$ of its first-period profit. (This could be due to the entrant's financial constraints in an imperfect credit market, as in Bolton and Scharfstein [1990].) In this situation, the incumbent will be willing to price below c in the period following entry to increase the likelihood of forcing the entrant out of the market. To see this, consider first what the pricing equilibrium would be absent any antitrust constraint. We wish to focus on equilibria in which the entrant still wins, but at a lower price than absent the possibility of predation. In such an equilibrium, the incumbent will charge price p and the entrant will charge price $p + \Delta$. For the incumbent not to deviate to $\hat{p} = p - \varepsilon$, we must have

$$p - c \leq [\lambda(0) - \lambda(p + \Delta - c)] (V_I - V_E) .$$

The highest possible equilibrium price p^* absent an antitrust constraint will have this hold with equality, i.e., the incumbent is just indifferent about undercutting to win the sale, which would yield the incumbent a short-term loss but hurt the entrant's chances of survival. For the entrant to be willing to win, we must also have

$$[\lambda(0) - \lambda(p^* + \Delta - c)]\,(V_I - V_E) \leq (p^* + \Delta - c).$$

Note that the price will satisfy $p^* - c < 0 < p^* + \Delta - c$.

Now let the antitrust constraint be $p \geq \alpha$, and suppose that it is binding, i.e., $p^* < \alpha$. In this case, $\pi_E(\alpha) = \alpha + \Delta - c$, $\pi_I(\alpha) = 0$, and $\pi_m(\alpha) = \Delta$: thus, a higher α raises $\pi_E(\alpha)$ upon entry, does not affect $\pi_I(\alpha)$ or $\pi_m(\alpha)$, and raises $\lambda(\alpha)$. If the policy only had an effect on π_E but not on λ, then, by (2.2), the policy would stimulate innovation. However, the policy also increases the entrant's probability of survival λ. Recall from the argument Subsection 2.4 that this effect also stimulates innovation. Thus, we conclude that a restriction on predatory pricing will stimulate innovation.[15]

The effect of an increase in α on welfare seems ambiguous, because innovation may be either socially insufficient or excessive (due to business stealing). The effect on consumer surplus is also ambiguous: on the one hand, reducing predation in period t increases the price in period t, which hurts that period's consumers. On the other hand, this results in a greater likelihood of entrant survival and

[15] In a more general model with differentiated products, predation would make both Entrant and Incumbent lose money. Thus, increasing α would raise both firms' profits as well as the entrant's probability of survival, and so would again increase innovation.

a higher innovation rate, both of which increase the quality of goods available to future consumers. Intuitively, however, when δ is close to 1, the long-term positive effect of improved technology overwhelms the short-term negative effect of higher prices, and so a policy that reduces predation benefits both consumers and aggregate welfare.

3.4. Voluntary deals

A simple implication of (2.2) is that deals between the entrant and incumbent that are advantageous to both firms are good for innovation. An example would be the incumbent's buyout or licensing of the entrant's technology (provided that the deal is not reached under the threat of anticompetitive action against the entrant, such as predation, which may reduce the entrant's payoff π_E). The effect of these deals on aggregate welfare is ambiguous, according to our analysis in Section 3.1. In addition, even when these deals increase aggregate welfare, they may well reduce consumer surplus by reducing pricing competition.

3.5. Uncertain innovation size

So far we have assumed that the innovation size Δ is constant and known in advance. To add realism, we explore an extension of the model in which Δ is an i.i.d. random variable realized after innovation success. The innovators cannot affect the distribution Δ, and in this sense this is a model of "nontargeted" innovation.

The profits π_E, π_I, and π_m realized in a period may now depend on the current leading generation's innovation size Δ. (Note that these profits will not depend on the sizes of past innovations.) Since the potential entrants do not know their innovation size when they choose innovation rate ϕ_i, the analysis of Subsections 2.1 and 2.2 goes through in terms of the expected values and profit functions, and so (2.2) generalizes as follows:

$$\left(\left[1+\frac{1-\delta}{\delta s}\right]+\left(\frac{1-s}{s}\right)\frac{E\pi_m'(\alpha,s,\Delta)}{E\pi_E'(\alpha,s,\Delta)}\right)\geq -\frac{E\pi_I'(\alpha,s,\Delta)}{E\pi_E'(\alpha,s,\Delta)}$$

We can conclude immediately that in a model in which all the profit functions are linear in Δ, such as the the long-term contracting model of Subsection 3.2, uncertainty about Δ would not affect any of the model's predictions. In particular, since an increase in α results in an increase in both the expected joint profit upon entry, as well as the expected profit of an uncontested incumbent, it necessarily increases the level of R&D. But in models in which Δ affect profits nonlinearly, uncertainty gives rise to new effects. To take a simple example, consider a modification of the long-term contracting model in which a successful innovator, upon observing Δ, must invest a fixed cost f to implement the innovation in the present period, or can sit out of the market for one period (foregoing π_E) and implement the innovation for free in the next period (becoming the incumbent and receiving V_I). If the entrant makes the decision optimally, its profit in the entry period is $\pi_E(\alpha,\Delta)=\max\{\alpha\Delta-f,0\}$, and so is concave in Δ. Assume for simplicity that $\Delta\in\left\{\underline{\Delta},\overline{\Delta}\right\}$, with $0<\alpha\underline{\Delta}<f<\alpha\overline{\Delta}$, and $\Pr\left\{\Delta=\overline{\Delta}\right\}=\gamma\in(0,1)$. Then the entrant operates in the entry period when $\Delta=\overline{\Delta}$ but not when $\Delta=\underline{\Delta}$, and so $E\pi_E'(\alpha,\Delta)=\gamma\overline{\Delta}<E(\Delta)$. As for the profit functions π_m and π_I, they can

be written as though the probability of an entrant operating in the first period following its receiving a patent is γs instead of s. Thus, we can write the profit functions as[16]

$$\begin{aligned} \pi_m(\alpha, s) &= \alpha\Delta + (1-\alpha)(1-\gamma s)\Delta \\ \pi_I(\alpha, s) &= (1-\alpha)(1-\gamma s)\Delta \\ \pi_E(\alpha, \Delta) &= \max\{\alpha\Delta - f, 0\}. \end{aligned}$$

The above inequality can then be rewritten as

$$\left[1 + \frac{1-\delta}{\delta s}\right] + \left(\frac{1-s}{s}\right)\frac{\gamma s E(\Delta)}{\gamma\overline{\Delta}} \geq \frac{(1-\gamma s)E(\Delta)}{\gamma\overline{\Delta}},$$

or

$$\left[1 + \frac{1-\delta}{\delta s}\right] + \left(1 - \frac{1}{\gamma}\right)\frac{E(\Delta)}{\overline{\Delta}} \geq 0.$$

Note that this inequality is less likely to hold when $\gamma < 1$ than when $\gamma = 1$ (i.e., certain innovation size $\overline{\Delta}$). Thus, uncertainty about γ in this case makes a more protective antitrust policy less advantageous to innovation. Mathematically, due to the convexity of π_E, a spread in the uncertainty about Δ holding the expectation $E(\Delta)$ fixed raises π_E while holding π_I fixed, and so may raise the expected

[16] Alternatively, suppose that at the start of period t the current incumbent's product has a value to the consumer of v_t and had an innovation of size Δ_t. Then a consumer who refuses to sign a long-tern contract expects a surplus of $(v_t - \Delta_t)$ if there is no entry in the next period, and of v_t if there is entry. Hence, the consumer will sign a long-term contract for price q_{t+1} as long as $v_t - q_t \geq v_t - (1-\gamma s)\Delta_t$, or $q_t \leq (1-\gamma s)\Delta_t$. This gives the profit from a captive consumer in the expressions for π_I and π_m. The profit from a free consumer when no entry occurs (the other term in π_m) is, as before, Δ_t.

joint profit upon entry. Specifically, observe that when a small innovation ($\underline{\Delta}$) displaces a large one ($\overline{\Delta}$), the joint profit is reduced. Since long-term contracts delay the implementation of small innovations without hurting large ones they may increase the expected joint profits, hence the expected innovation prize W, and thereby equilibrium innovation. A similar benefit may be ascribed to other anticompetitive measures, such as refusals to cooperate on essential facilities, liquidated damages, or predation, which may harm small innovations more than big ones. The result is related to O'Donohue et al.'s [1998] motivation for leading patent breadth.[17] Of course, if Δ were verifiable by courts, then the first-best patent mechanism would condition displacement on it (Llobet et al. [2000] allow for indirect verification of Δ through a message by the innovator). However, when Δ is not verifiable by courts, allowing some degree of long-tern contracting could be a reasonable second-best way to encourage innovation.

4. Incumbent Investment [incomplete]

The analysis above imposed the strong restriction that only potential entrants engaged in R&D. Although useful for gaining insight, this assumption is clearly not respresentative of most settings of interest. In this section, we explore how our conclusions are affected when incumbent firms may also engage in R&D.

Allowing incumbent firms to engage in R&D has the potential to considerably complicate the analysis. In particular, once we allow for incumbent investment,

[17] But their motivation is different because with their R&D process more leading breadth in the absence of licensing would necessarily reduce innovation.

we need in general to introduce a state space to keep track of the incumbent's current lead over the potential entrants. In general, the rates of R&D investment by the incumbent and its challengers may be state dependent (see, for example, Aghion et. al. [2001]).

To date we have focused on two special cases in which R&D strategies are nonetheless stationary. Although clearly restrictive, these two models do have the virtue of capturing two distinct motives for incumbent R&D: (i) preventing displacement by an entrant, and (ii) increasing the flow of profits until displacement by increasing the lead over the previous incumbent.

4.1. R&D to prevent displacement

As earlier, we focus on the case of a fixed number of firms N. The only change from the model of Section 2 is that the incumbent may now do R&D. We denote the levels of R&D for the incumbent and a potential entrant by ϕ_I and ϕ_E respectively, and the respective cost functions by $c_I(\phi_I)$ and $c_E(\phi_E)$ (we allow for the fact that the cost of achieving a discovery may differ between the incumbent and the potential entrants). In this first model, we assume that if the leading quality level in period t is j_t, then quality level $j_t - 1$ is freely available to all potential producers. That is, it enters the public domain. Thus, the incumbent never has a lead greater than one step on the ladder. Thus, the only reason for an incumbent to do R&D is to try to get the patent on the next innovation in cases where at least one potential entrant has made a discovery – that is, to prevent its displacement. To capture this in the simplest possible way, we assume that the incumbent gets the patent whenever it makes the discovery; that is, that the incumbent wins all "ties".[18] With these assumptions, we need not keep track of any states, and there is a stationary equilibrium.

Denoting by V_I and V_E the values of the incumbent and a potential entrant, we now have:

$$V_I = \pi_m(\alpha) + \delta V_I + s(\phi_E, N)(1-\phi_I)\{\pi_I(\alpha) - \pi_m(\alpha) + \delta(V_I - V_E)\} - c_I(\phi_I) \quad (4.1)$$

$$V_E = \delta V_E + \phi_E r(\phi_E, N)(1-\phi_I)\{\pi_E(\alpha) + \delta(V_I - V_E)\} - c_E(\phi_E). \quad (4.2)$$

Letting

$$W_I \equiv s(\phi_E, N)\,[\pi_m(\alpha) - \pi_I(\alpha) + \delta(V_I - V_E)] \quad (4.3)$$

and

$$W_E \equiv r(\phi_E, N)(1-\phi_I)\{\pi_E(\alpha) + \delta(V_I - V_E)\}, \quad (4.4)$$

we have $\phi_i \in \arg\max_{\psi_i \in [0,1]} \psi_i W_i - c_i(\psi_i)$ for $i = I, E$.. Solving (4.1) and (4.2) for

[18] In the usual sort of (Poisson) continuous-time model considered in the R&D literature (see, e.g., Lee and Wilde [1980], Reinganum [1989], and Grossman and Helpman [1991]), the probability of ties is zero, and so one might worry that our formulation here is dependent on a merely technical feature of the discrete-time set-up. Indeed, in such a model, the incumbent would do no R&D here. However, the usual continuous-time model relies on the implicit assumption that following an innovation, all firms reorient their R&D activity immediately to the next technology level. If we were to instead use a continuous-time model in which there is a fixed time period after a rival's success before which R&D for the next technology level cannot be successful, then we would get effects that parallel those in our discrete-time model (where the discount factor δ reflects how quickly R&D activity can be reoriented to the next technology level.) Thus, our discrete-time formulation captures an arguably realistic feature of the economics of R&D.

$(V_I - V_E)$ and substituting we get (suppressing arguments of functions)

$$
\begin{aligned}
W_I &= \left(\frac{s}{D}\right)\{\pi_m - (1-\delta)\pi_I + \delta(1-\phi_I)r\phi_E(\pi_m - \pi_I - \pi_E) + \delta(c_E - c_I)\} \\
W_E &= \left(\frac{r(1-\phi_I)}{D}\right)\{\delta\pi_m + (1-\delta)\pi_E - \delta(1-\phi_I)s(\pi_m - \pi_I - \pi_E) + \delta(c_E - c_I)\},
\end{aligned}
$$

where $D \equiv 1 - \delta + \delta[1 - \phi_I(s + r\phi_E)]$.

In this setting where both the incumbent and potential entrants can do R&D we can distinguish between the direct effects of a change in the policy α and the indirect effects. For the incumbent, the former captures the change in its R&D incentives holding fixed the R&D of potential entrants ϕ_E, and has the same sign as the change in W_I caused by the change in α holding (ϕ_I, ϕ_E) fixed. Similarly, the direct effect for the potential entrants has the same sign as the change in W_E caused by the change in α holding (ϕ_I, ϕ_E) fixed.

Proposition 4.1. *In the model of incumbent R&D to prevent displacement, the direct effect of a more protective antitrust policy (an increase in α) on incumbent R&D is positive if and only if*

$$
-\frac{\pi_I'(\alpha)}{\pi_E'(\alpha)} \geq \frac{\delta(1-\phi_I)r\phi_E}{(1-\delta)+\delta(1-\phi_I)r\phi_E} - \left(\frac{\pi_m'(\alpha)}{\pi_E'(\alpha)}\right)\left(\frac{1+\delta(1-\phi_I)r\phi_E}{(1-\delta)+\delta(1-\phi_I)r\phi_E}\right) \tag{4.5}
$$

and is positive on potential entrant R&D if and only if

$$
\left[1 + \frac{1-\delta}{\delta s(1-\phi_I)}\right] + \left(\frac{1-s(1-\phi_I)}{s(1-\phi_I)}\right)\left(\frac{\pi_m'(\alpha)}{\pi_E'(\alpha)}\right) \geq -\frac{\pi_I'(\alpha)}{\pi_E'(\alpha)}. \tag{4.6}
$$

A few observations can be made about Proposition 4.1. First, note that the direct effects of a more protective antitrust policy on incumbent and potential entrant innovation can *never* both be negative provided that $\pi_m'(\alpha) \geq 0$, since in that case the righthand side of (4.5) is less than 1, while the lefthand side of (4.6) is greater than 1. More strikingly, when $\pi_m'(\alpha) \geq 0$ and $-\frac{\pi_I'(\alpha)}{\pi_E'(\alpha)} \approx 1$ *both* direct effects are positive. Intuitively, when $\pi_I'(\alpha) \leq 0$, a more protective antitrust policy can encourage incumbent innovation when innovation is done to avoid displacement because it reduces the incumbent's profits when entry occurs, thus making avoiding that outcome all the more desirable for the incumbent [the other effect, which leads to the ambiguity in (4.5) in general, is that it also reduces the value of $(V_I - V_E)$]. Similarly, when $\pi_m'(\alpha) \geq 0$ and $\pi_I'(\alpha) \leq 0$, if $\delta(1-\phi_I)\phi_E \approx 0$ (e.g., if the rate of either incumbent innovation or time discount is very high or the rate of entrant innovation is very low) then the incumbent's direct effect is necessarily positive.

The direct effects are not determinative, however, of the overall change in equilibrium innovation rates, because there are interactions between the R&D levels of the incumbent and potential entrants since the level of ϕ_i in general affects the value W_j ($i \neq j$; $j = I, E$). It can be seen from (4.3) and (4.4) that when $(\pi_m - \pi_I - \pi_E) \approx 0$, the level of incumbent innovation increases in ϕ_E, and the level of potential entrant innovation decreases in ϕ_I. When this is so, and the direct effects are both positive, we know that the incumbent's innovation rate ϕ_I must increase with an increase in α.[19]

[19] More generally, when $(\pi_m - \pi_I - \pi_E) \geq 0$ the incumbent's R&D level is increasing in ϕ_I, although the direction of the indirect effect on potential entrants' R&D is in this case ambiguous.

4.2. R&D to increase profit flows

We next consider a model in which rivals do not get access to the second best technology when the incumbent innovates. Thus, the incumbent can increase its flow of profits by innovating, until the time when it is displaced. Specifically, let k denote the number of steps that the incumbent is ahead of its nearest rival (this is our state variable). The variable k affects the incumbent's profit flow when entry does not occur, which we now denote by $\pi_m(k, \alpha)$ (it does not affect either π_I or π_E). We now make two assumptions that will imply that there is an equilibrium in which the R&D levels of the incumbent and potential entrants do not depend upon k. Specifically, we assume that $\pi_m(k, \alpha) = k\pi_m(k)$ and that an entrant gets the patent whenever at least one entrant has made a discovery.

It is clear that there is a solution in which potential entrant R&D ϕ_E and value V_E are stationary. To begin, we allow that the incumbent's R&D and value functions may depend on k: ϕ_I^k and V_I^k. In this case, we can write the value equations as

$$\begin{aligned} V_I^k &= k\pi_m + \delta V_I^k + s(\phi_E, N)\{(\pi_I - k\pi_m) + \delta[V_E - V_I^k]\} \\ &\quad +\phi_I^k[1 - s(\phi_E, N)]\{[(k+1)\pi_m - k\pi_m) + \delta[V_I^{k+1} - V_I^k]\} - c_I(\phi_I^k), \end{aligned} \tag{4.7}$$

for $k \geq 1$, and

$$V_E = \delta V_E + \phi_E r(\phi_E, N)\left[\pi_E + \delta\left(V_I(1) - V_E\right)\right] - c_E\left(\phi_E\right), \tag{4.8}$$

while the equilibrium innovation rates satisfy

$$\phi_I^k \in \arg\max_{\psi_I^k \in [0,1]} \psi_I^k [1 - s(\phi_E, N)]\{[(k+1)\pi_m - k\pi_m) + \delta[V_I(k+1) - V_I(k)]\} - c_I(\psi_I^k), \tag{4.9}$$

$$\phi_E \in \arg\max_{\psi_E \in [0,1]} \psi_E r(\phi_E, N)\left[\pi_E + \delta\left(V_I(1) - V_E\right)\right] - c_E\left(\psi_E\right). \tag{4.10}$$

Now observe from (4.9) that ϕ_I^k will be independent of k if the difference $V_I(k+1) - V_I(k)$ is. Using (4.7) for k and $k+1$ we see that

$$\begin{aligned}(1-\delta)\left[V_I^{k+1} - V_I^k\right] &= \pi_m - s(\phi_E, N)\{\pi_m + \delta[V_I^{k+1} - V_I^k]\} \\ &\quad + [1 - s(\phi_E, N)]\left[\phi_I^{k+1}\{\pi_m + \delta[V_I^{k+2} - V_I^{k+1}]\} - \phi_I^k\{\pi_m + \delta[V_I^{k+1} - V_I^k]\}\right] \\ &\quad - c_I(\phi_I^{k+1}) + c_I(\phi_I^k).\end{aligned}$$

Hence, if ϕ_I^k is independent of k, we have

$$\left[V_I^{k+1} - V_I^k\right] = \frac{\pi_m(1-s)}{1-\delta+\delta s},$$

which is indepent of k. Hence, there exists an equilibrium in which the incumbent's innovation rate is independent of k, $\phi_I \equiv \phi_I^k$, and [specializing (4.9) satisfies

$$\phi_I \in \arg\max_{\psi_I \in [0,1]} \psi_I (1-s)\left[\pi_m + \delta\left(\frac{\pi_m(1-s)}{1-\delta+\delta s}\right)\right] - c_I(\psi_I) = \arg\max_{\psi_I \in [0,1]} \psi_I \pi_m \left(\frac{1-s}{1-\delta+\delta s}\right) - c_I(\psi_I). \tag{4.11}$$

Thus,

$$W_I = \pi_m \left(\frac{1-s}{1-\delta+\delta s}\right). \tag{4.12}$$

We next solve for W_E. Subtracting the expression for V_E from that for V_I^1 we have (omitting arguments of functions for notational simplicity)

$$[V_I^1 - V_E][1-\delta+\delta(s+\phi_E r)] = \pi_m + s(\pi_I - \pi_m) + \phi_I \pi_m \left(\frac{1-s}{1-\delta+\delta s} \right) - \phi_E r \pi_E - (c_I - c_E).$$

Thus,

$$W_E = \left[\frac{r}{1-\delta+\delta(s+\phi_E r)} \right] \left\{ \pi_E[1-\delta+\delta s] + \delta s \pi_I + \delta(1-s) \left[\frac{2-\delta(1-s)}{1-\delta(1-s)} \right] \pi_m - \delta(c_I - c_E) \right\}. \tag{4.13}$$

Examing (4.12) and (4.13) we have

Proposition 4.2. *In the model of incumbent R&D to increase profit flows, the direct effect on incumbent R&D of a more protective antitrust policy (an increase in α) has the same sign as $\pi_m'(\alpha)$, while the direct effect on potential entrant R&D is positive if and only if*

$$\left[1 + \frac{1-\delta}{\delta s} \right] + \left(\frac{1-s}{s} \right) \left(\frac{\pi_m'(\alpha)}{\pi_E'(\alpha)} \right) \geq -\frac{\pi_I'(\alpha)}{\pi_E'(\alpha)}. \tag{4.14}$$

Once again we can get both direct effects to be positive; indeed, this is certain to be the case if the increase in α raises bot hthe monopoly profit π_m and the joint profit upon entry $\pi_I + \pi_E$. Considering now the indirect effects, we see from (4.13) that the level of incumbent R&D has no indirect effect on ϕ_E, while from (4.12) increases in the level of potential entrant R&D (and, hence, s) reduce the level of ϕ_I.

5. Conclusion

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