# A New Micro Model of Exchange Rate Dynamics

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**Abstract**

This paper bridges the microstructure and new macro approaches to exchange rates by addressing how currency markets aggregate information in general equilibrium. Two key features distinguish our model from new macro models: the presence of dispersed information and participation of financial intermediaries who act as market-makers. We depart from microstructure modeling in that real economic activities where dispersed information originates are explicitly identified, as well as the technology by which this information is subsequently aggregated and impounded in price. Financial intermediaries in the model are consolidated with consumers, in the spirit of the "yeoman farmer" consolidation of production and consumption in new macro models. A new forcing variable arises from intermediation that affects both the exchange rate and the currency risk premium (because intermediation involves risks that cannot be fully hedged). The model is structured to permit analysis of order flow, a market-based measure that is much in focus in the empirical literature. In particular, we examine the origins of the positive covariance between exchange rate changes and order flow found in the data. We also identify the specific conditions for using order flow to discriminate transitory from persistent macro shocks. Finally, the model demonstrates why at higher frequencies order flow explains exchange rates better than macro variables, whereas at lower frequencies macro variables predominate.

Keywords: Exchange Rate Dynamics, Dispersed Information, FX Trading.

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## Introduction

This paper addresses a new disconnect puzzle: the distressing disconnect between the two micro-founded approaches to exchange rates that emerged in the 1990s. These are the new open-economy macro approach (henceforth "new macro") and the microstructure approach. New macro modeling is general equilibrium, rich in welfare analysis, but thin on the microeconomics of financial markets and the information environments in which they operate. The microstructure approach, in contrast, has the microeconomics of financial markets at its center, at the cost of relying on partial equilibrium (and rather stylized) analysis. This paper seeks to integrate the microstructure and new macro approaches into what we term a "new micro" approach.[2] Specifically, the model embeds the micro-foundations of currency-relevant information in a dynamic general-equilibrium (GE) setting.

The macro features of our model are standard. There are two countries populated by consumers who have utility defined over a basket of home and foreign goods. Consumers have access to two financial assets, home and foreign currency deposits, which pay interest monthly and can be used to purchase consumption goods in the same currency. Consumers also control a domestic production process subject to exogenous productivity shocks (which differ, home versus foreign). We introduce the international aspect of the model via the information structure. Specifically, the information available to individual agents leads their foreign exchange trades to be more highly correlated with home productivity shocks than with foreign. At the country level, then, agents' trades convey superior information about domestic productivity shocks. It is this information structure that differentiates the macro side of our model from the new macro literature.

The micro features of the model are closely related to microstructure models of asset trade in which financial intermediaries act as market-makers who provide liquidity in the form of two-way prices. We introduce this provision of liquidity by assuming that all agents engage in both consumption and market-making.[3] This consolidates the activities of households with that of financial institutions in a way similar in spirit to the "yeoman farmer" consolidation of production and consumption decisions in new macro models. The consolidation greatly facilitates integration of elements from the microstructure approach into a dynamic GE setting. In particular, it ensures that the objectives of financial-market participants are exactly aligned with those of consumers. All trading in the model is therefore consistent with expected utility maximization; noise traders, behavioral traders, and other non-rational agent types are absent.

This paper belongs to a theoretical literature that emerged recently to address why exchange rate changes are so well explained empirically by signed transaction flows (e.g., $\mathrm{R}^2$ statistics in the 60-80 percent range for a host of major currencies; see Evans and Lyons 2002b). For example, the model of Hau and Rey (2002) addresses the empirical significance of transaction flows by introducing two key elements: a central role for cross-border equity flows and price-elastic excess supply of foreign exchange. The latter means that cross-border equity flows affect exchange rates via induced currency transactions. In a nutshell, their

[2] Though analysis in new micro relies heavily on the theory of microstructure finance, it does not draw uniformly from the modeling approaches within microstructure, nor does it address the same questions, hence the need for a different label. The modeling approach in microstructure finance that does play a central role in new micro is the information approach (versus the inventory and industrial organization approaches). For questions, new micro is oriented toward macro phenomena, whereas microstructure finance is oriented toward micro phenomena (such as institution design, regulation, individual behavior, and partial-equilibrium price determination).

[3] Note the emphasis on liquidity provision that is private, in contrast to the public liquidity provision in the form of central banks at the center of the monetary approach to exchange rates.

focus for understanding currency-market developments is on innovations in equity markets, a substantial departure from the traditional asset approach which emphasizes instead the importance of bond markets. Their focus is not on information aggregation as ours is here (no information aggregation takes place in their model). A second paper along this theoretical line is Bacchetta and van Wincoop (2002), which does explicitly address how transaction flows relate to information aggregation. Their trading model is a rational-expectations model (in the spirit of Grossman and Stiglitz 1980). An important finding in that paper is that greater dispersion of information across agents can lead to greater price impact from non-fundamental trades (resulting from rational confusion of non-fundamental trades for fundamental trades). Our modeling departs from theirs in two main ways. First, our GE setting extends "upstream" in the information process in that it specifies the structural source of the information that currency markets need to aggregate (i.e., the underlying economic activities that produced it.) Second, financial intermediation in our model aligns closely with actual institutions, producing implications that map directly into transactions data. A third recent paper, Devereux and Engel (2002), shares both our GE approach and a role for market-makers. Market-makers in their model are explicitly non-rational, however, so the reason their trades affect price is quite different than in our model.[4]

From the above it is clear that the information processing ability of currency markets is a central theme so let us address it in a bit more detail. The type of information we have in mind is information that is dispersed throughout the economy and aggregated by markets, as opposed to official institutions. Examples include the heterogeneous micro-level activity that, when aggregated, produces measures like output, money demand, inflation, consumption preferences, and risk preferences. For some of these measures official aggregations exist, but publication trails the underlying activity by 1-4 months (not to mention noise as reflected in subsequent revisions), leaving much room for market-based aggregation to precede publication. For other key macro variables, such as realized risk preferences and money demands, official aggregations of the underlying micro-level activity do not exist, leaving the full task to market-based aggregation. In traditional macro modeling of exchange rates, information that needs to be aggregated by markets is not admitted. Instead, relevant information is either symmetric economy-wide, or, in some models, asymmetrically assigned to a single agent—the central bank. As an empirical matter, however, most information that exchange rates need to impound is clearly originating as dispersed, micro-level bits. In addition, there is now strong empirical evidence that this dispersed information is indeed being impounded in exchange rates before ever being symmetrized through official aggregation.[5] Understanding the nature of this information problem and how it is solved remains a significant challenge.

So what do we learn from modeling currency trade in a dynamic GE setting? The question is important: the shift from partial to GE modeling involves much technical complexity and, for most people, a drop in

[4] For example, in a risk neutral setting the trades of marketmakers in the Devereux and Engel (2002) model would not affect price since they can do so only by affecting expected returns (risk premia; see also Jeanne and Rose 2002). In contrast, trades of marketmakers in our model would still affect price under risk neutrality because they do so by affecting expected cash flows.

[5] See, e.g., Lyons (1995), Payne (1999), Rime (2001), Evans (2002), Covrig and Melvin (2002), Froot and Ramadorai (2002), and Evans and Lyons (2002a,b). Among other things, these papers show that actual flows of signed transactions (order flow) and "excess demand" are not the same: that order flow includes an information dimension beyond the pure quantity concept of excess demand is clear from, for example: (1) findings that order flow has different effects on price, dollar for dollar, depending on the institution type behind it and (2) findings that order flow in one currency market has effects on price in other currency markets, despite not occurring in those markets. As a theoretical matter, the two are obviously distinct: excess demand moves price without transactions needing to occur, whereas order flow is a measure of transactions (specifically, signed transaction flow, where the sign is determined from the direction of the non-quoting counterparty).

economic transparency. The following three paragraphs address this question in more detail.

A first lesson from modeling currency trade in a GE setting is that the information problem noted above is a good deal more nuanced than suggested by past partial-equilibrium analysis. For example, the model clarifies that even if the timing of individuals' receipt of information is exogenous, the timing of impounding that information in price is endogenous. This is because the signals within the market that lead to that impounding correspond to the equilibrium actions of participants. Naturally, then, the model is able to characterize when order flow should be especially informative, dollar for dollar, and when less so. In effect, the dynamics of the model create a constant tension between strong and semi-strong form efficiency. (Strong form efficiency means that prices impound all information public and dispersed, whereas semi-strong form efficiency means that prices only impound public information.) This tension is the difference between the union of all individuals' information sets and the smaller set that includes public information only. Finally, relative to partial-equilibrium models, the information structure of the GE model provides needed clarity on whether transaction-flow effects on exchange rates should persist, and, importantly, whether that persistence applies to real exchange rates or only to nominal rates.

A second lesson from GE modeling of price discovery in currency markets is that real decisions are affected in ways not considered in either new macro or microstructure models. For example, our model clarifies the channels through which financial intermediation in currency markets affects consumption and intertemporal consumption hedging. The basic intuition for why these channels are operative is that innovations in agents' learning from currency-market activity are correlated with other things they care about (e.g., real output). Decision-making about real choices is conditioned on information that is generally less that the union of individuals' information sets, which naturally leads to effects on real allocations. Yet, this is optimal: there is no benefit to waiting for resolution of uncertainty because informationally the economy is always in transition (i.e., there is always some dispersed information that remains unaggregated).

A third lesson from GE modeling of price discovery in currency markets is that it uncovers a new type of risk premium that does not arise in new macro models, nor in microstructure models. In particular, co-variation between order flow and the spot rate drives a wedge between the marginal utilities of wealth and consumption, introducing a source of risk that affects asset allocation. The empirical importance of the new hedging terms that arise depends on the manner in which dispersed information is embedded into spot rates rather than on the form of the utility function. The size of these hedging terms depends on the extent to which dispersed information is present in order flow, and the speed at which aggregation of dispersed information takes place.[6] These should vary with the state of the economy, pointing to an unexplored source of risk premia variation.

The GE environment we study has a number of complicating features. It includes a large number of risk averse consumers with heterogenous information who make optimal consumption, investment and trading decisions with incomplete markets. Furthermore, information about the complete state of the economy is only learned by consumers endogenously as they trade. To analyze the model we therefore need to solve the combined inference and decision-making problem facing each consumer. For this purpose, we extend the

[6] With respect to the information conveyed by flows, it is important to distinguish order flows from portfolio flows. Order flows–by tracking the initiating side of transactions–are a theoretically sound way to distinguish shifts in demand curves from movements along demand curves. Informationally, the two are different: there is news in curve shifts but no news in price-induced movements along known curves (the latter representing a type of feedback trading). For portfolio flows, theory provides little guidance on which flows in the aggregate mix reflect the news, i.e., the demand-curve movements.

log-linear approximation techniques developed by Campbell and Viceira (2002) to our GE setting. These techniques allow us to derived analytic approximations for the optimal consumption, investment and trading decisions of consumers at a point in time *given* a conjecture about (i) the equilibrium exchange and interest rate processes (which are generally not i.i.d.), and (ii) the information available to each consumer. The complicated part of solving the model arises from the need to show that aggregate implications of these decisions are consistent with market clearing, and that the conjectured information available to each consumer is supported by an inference problem based on their observations of market activity. One important aspect of this solution procedure is that it accounts for consumer's risk aversion when characterizing their optimal decisions. As a result, risks associated with incomplete knowledge about the state of the economy influence the consumption, investment and trading decisions, which, in turn, affect the inferences consumers draw from their observations of the market. To the best of our knowledge, this is the first paper to solve a GE model with this combination of risk-averse decision-making, heterogeneous information and endogenous learning.

The remainder of the paper is organized as follows. Section 1 discusses the overall structure of the model with particular focus on the consolidation of households with financial institutions. The complete model is laid out formally in Section 2. Section 3 describes how we find the equilibrium. To analyze how dispersed information becomes aggregated into asset prices, Section 4 studies the equilibrium for three different specifications of the productivity process. Section 5 concludes.

# 1 Theoretical Overview

Before presenting specifics of the model, there are three overarching characteristics that warrant attention. The first of these is the above-noted consolidation of consumers and financial intermediaries into households. Whereas a focus of new macro models is richer micro-foundations on the supply side of the economy, hence the consolidation in those models of consumers with producers, our focus is instead richer micro-foundations in the area of financial intermediation. In particular, we focus on how financial markets achieve economy-wide risk sharing in a setting of heterogeneous information. In actual markets this process takes time and involves financial institutions in a non-trivial way, hence the value of embedding the process in a rich and dynamic setting. The consolidation recognizes that consumption depends both on learned information about future consumption opportunities and on the evolution of consumption risks (the latter being affected within the process of market-wide risk sharing). In effect, our specification of financial intermediation represents a risk-sharing and learning "technology." In so doing the two are intimately linked, and also intimately linked to consumption decisions. Households recognize that economy-wide sharing of concentrated risks is not instantaneous and not costless.

The second overarching characteristic of the model is its "simultaneous trade" design (see, e.g., Lyons 1997). The simultaneous trade design itself embeds two important and distinct features. The first is simultaneous actions, in the sense that trading at any point in time occurs simultaneously throughout the economy (in the spirit of simultaneous-move games in game theory). In essence, this assumption imposes a constraint on the information available for making trading decisions because simultaneous moves cannot be conditioned on one another. More concretely, one cannot condition on concurrent trading intentions of other agents in the economy at the time one chooses to trade. We find this an inherently realistic assumption relative to that made in, for example, Walrasian models of trade in which all concurrent trades are conditioned on the information conveyed by all other trades. In effect, the Walrasian clearing mechanism makes heroic assumptions about the transparency of market activity and the information available to price setters.[7] Our simultaneous-move framework is a convenient way to relax the polar extreme represented by the Walrasian framework.[8]

Another important feature embedded in the simultaneous trade design is that quoted prices are single prices, not bid-ask spreads. For the type of macro-level analysis we are doing here, bid-ask spreads enter as a nuisance parameter, hence our choice to omit them. One objection to this suppression of spreads is that intermediaries no longer have an incentive to quote two-way prices (the spread being their compensation). As a matter of modeling, there is a simple fix to restore this incentive that involves spreads but does not alter the basic process of learning from trades. That fix is quite general in the sense that it allows intermediaries to quote a separate bid-ask spread for every possible trade quantity (i.e., a schedule relating every possible trade quantity from minus infinity–customer sale–to plus infinity–customer buy–to a specific price). Each intermediary's schedule would reflect an upward sloping willingness to supply foreign exchange as a function of the single incoming trade. This would not alter the basic process of learning from trades because each individual intermediary's quoted schedule would be conditioned only on the single incoming trade, i.e., there is no feasible way to condition the transaction price on the realization of *all* other concurrently realized trades. Under this specification too, then, individual transaction prices would not embed the Walrasian level of economy-wide information (and, as in the specification we do adopt, prices may not embed this level of information even with long lags).

The third overarching characteristic of the model is the long-run real exchange rate, specifically, the channels through which the long-run real rate is affected by order flow (i.e., information conveyed by trading). One important channel is via household investment decisions. Investment decisions are affected by productive capabilities, which are in turn time-varying and correlated with realized transaction flows in foreign exchange. If production technology is non-linear, then there is a strong channel through which trading information affects the aggregate capital stock and thereby the long-run real exchange rate. In the simple model we present here, production technology is linear, which, among other things, implies that returns to real capital are exogenous. This simplifying assumption limits the degree to which investment decisions affect long run exchange rate dynamics. We make this technology assumption for tractability, not because we are convinced this channel is inoperative or unimportant. There is a second channel through which order flow can affect the long-run exchange rate that we do not address in this paper, but which may be fruitful in future analysis. Specifically, if exchange rates "shocks" that arise in learning from order flow are accommodated by a monetary authority, then such shocks can feed into aggregate price levels. Note that this channel pertains more to the long-run nominal exchange rate than to the long-run real rate.

To summarize, the model is designed to focus on persistent effects from information on price, not on

[7] Another unfortunate feature of Walrasian mechanisms is that agents never take positions that they intend in the future to liquidate (because all trades are conditioned on all concurrent trading information). Among other things, this produces counterfactual predictions about how liquidity is provided in financial markets: transitory position-taking is a deep property of liquidity provision, and is important for understanding how trade quantities (i.e., realized order flow) maps into price changes.

[8] One could also take an intermediate road and assume that financial transactions at any "point" in time are executed sequentially. In this case, early trades would share the feature of all trades in our set-up in that they could not condition on information revealed in later trades (whereas later trades under the sequential set-up could condition on early trades). This alternative would produce the same qualitative constraint on the information set available for setting prices, but in a more awkward way.

so-called microstructure effects, which we take to mean transitory price effects from market-maker inventory management and bouncing between bid and ask prices. These microstructure effects are second-order from a macro perspective. Moreover, our focus is on clarifying the transmission mechanism–the GE process by which information is impounded in price–not on a particular structural interpretation of the driving fundamentals (in our model, productivity). For example, we could just as easily set up the model with a different real shock as the fundamental driver, or with a nominal shock as the fundamental driver (e.g., assuming that individual agents' foreign exchange trades are more highly correlated with realized shocks to home money demand). Finally, for those interested in integrating firms' pricing in the goods markets and addressing longer horizon real exchange rates, this could fit into the model in the usual open-macro model way. We chose the most streamlined structure possible to highlight the new information dimension we are addressing.

# 2 The Model

## 2.1 Environment

The world is populated by a continuum of infinitely-lived consumers indexed by $z \in [0,1]$ who are evenly split between the home country (i.e., for $z \in [0,1/2)$) and foreign country ($z \in [1/2,1]$). For concreteness we shall refer to the home country as the US and the foreign country as the UK. Preferences for the $z$'th consumer are given by:

$$\mathbb{U}_{t,z} = \mathbb{E}_{t,z} \sum_{i=0}^{\infty} \beta^i U(C_{t+i,z}, \hat{C}_{t+i,z}) \tag{1}$$

where $1 > \beta > 0$ is the subjective discount factor, and $U(.)$ is a concave sub-utility function. All consumers have identical preferences over the consumption of US goods $C_{t,z}$ and UK goods $\hat{C}_{t,z}$. $\mathbb{E}_{t,z}$ denotes expectations conditioned on consumer $z's$ information set at time $t$, $\Omega_{t,z}$. Expectations conditioned on a common time $t$ information set (i.e., $\Omega_t \equiv \cap_{z\in[0,1]}\Omega_{t,z}$) will be denoted by $\mathbb{E}_t$.

Decision-making in the model takes place at two frequencies. Consumption-savings decisions take place at a lower frequency than financial decision-making (like the setting of asset prices and the reallocation of portfolios via trading). To implement this idea we split each "month" $t$ into four periods. Consumption-savings decisions are made "monthly" while financial decisions are made periodically within the month. As will become clear, the use of the term "month" is nothing more than a convenient label. The economic intuition developed by the model is exactly the same if we replaced "month" $t$ by some other consumption-relevant period. That said, let us now describe the structure of the model by considering the "monthly" sequence of events.

**Period 1**: Consumers begin the month with their holdings of US and UK currency deposits, $B^1_{t,z}$ and $\hat{B}^1_{t,z}$ and domestic capital: $K_{t,z}$ for US consumers, and $\hat{K}_{t,z}$ for UK consumers. Each consumer then quotes a spot price (\$/£) $S^1_{t,z}$ at which he is willing to buy or sell any amount of foreign currency (£s). These quotes are observable to all consumers.[9]

[9] It will be clear below that consumers in this model have both speculative and non-speculative motives for trading (the non-speculative motive arising from the need to facilitate periodic consumption and investment). That these motives are not purely speculative obviates concern about so-called "no trade" results (i.e., the theorem proposed by Milgrom and Stokey 1982,

**Period 2**: Each consumer $z$ chooses the amount of foreign currency, $T^2_{t,z}$, he wishes to purchase (negative values for sales) by initiating a trade with other consumers (the sum of which constitutes order flow for the period). Trading is simultaneous, trading with multiple partners is feasible, and trades are divided equally among agents offering the same quote. Once these transactions have taken place, consumer $z$'s deposits at the start of period 3 are given by

$$\begin{aligned} B^3_{t,z} &= B^1_{t,z} + S^1_t T^2_{t,z*} - S^1_t T^2_{t,z}, \\ \hat{B}^3_{t,z} &= \hat{B}^1_{t,z} + T^2_{t,z} - T^2_{t,z*}, \end{aligned}$$

where $T^2_{t,z*}$ denotes the foreign currency orders from other consumers trading with $z$. $S^1_t$ is the period-1 spot rate quote at which $z$ purchases pounds. In equilibrium, this will be the spot rate quoted by all consumers (i.e., $S^1_t = S^1_{t,z}$ ) for reasons we explain below. Notice that period-3 currency holdings depend not only on the transactions initiated by $z$, (i.e., $T^2_{t,z}$) but also on the transactions initiated by other consumers $T^2_{t,z*}$. An important assumption of our model is that the choice of $T^2_{t,z}$ by consumer $z$, cannot be conditioned on $T^2_{t,z*}$ because period-2 trading takes place *simultaneously.* Consequently, consumers may to end up with an unanticipated distribution of their wealth between dollar and pound assets due to the arrival of unexpected orders from others.

**Period 3**: All consumers again quote a spot price and also a pair of one-month interest rates for dollar and pound deposits. The spot quote, $S^3_{t,z}$, is good for a purchase or sale of any amount of pounds, while the interest rates, $R_{t,z}$ and $\hat{R}_{t,z}$ indicate the rates at which the consumer is willing to borrow or lend one-month in dollars and pounds respectively. As in period 1, all quotes are publicly observable.

**Period 4**: In period 4, consumers choose their consumption of US and UK goods, their foreign currency purchases, and their investment expenditures. After US consumers $z$ have chosen their consumption of US and UK goods, $C_{t,z}$ and $\hat{C}_{t,z}$, their foreign currency purchases $T^4_{t,z}$, and their level of investment $I_{t,z}$, the resulting capital and deposit holdings in period 1 of month $t+1$ are

$$\begin{aligned} B^1_{t+1,z} &= R_t(B^3_{t,z} + S^3_t T^4_{t,z*} - S^3_t T^4_{t,z} + C_{t,z*} - I_{t,z}), \\ \hat{B}^1_{t+1,z} &= \hat{R}_t(\hat{B}^3_{t,z} + T^4_{t,z} - T^4_{t,z*} - \hat{C}_{t,z}), \\ K_{t+1,z} &= R^k_{t+1}\left(K_{t,z} - C_{t,z} - C_{t,z*} + I_{t,z}\right) \end{aligned}$$

where $R_t$ and $\hat{R}_t$ are the dollar and pound interest rates that are quoted by all consumers in equilibrium (i.e., $R_{t,z} = R_t$, and $\hat{R}_{t,z} = \hat{R}_t$ for all $z$, as shown below). $R^k_t$ is the one-month return on capital. At the end of period-4 trading, the US capital stock is equal to $K_{t,z} - C_{t,z} - C_{t,z*} + I_{t,z}$. We assume that this capital stock is augmented by monthly production, $Y_{t+1,z}$, according to a linear production technology:

$$Y_{t+1,z} = A_{t+1}\left(K_{t,z} - C_{t,z} - C_{t,z*} + I_{t,z}\right)$$

that if I know that your only motive for trade with me is superior information, then I would never want to trade with you at any price at which you want to trade).

where $A_{t+1}$ is a productivity shock. For simplicity we ignore depreciation so the one-month return on capital is $R^k_{t+1} = (1 + A_{t+1})$. As in period 2 trading, actual bond holdings also depend on the actions of other consumers. In particular, orders for foreign currency and US goods received from other consumers , (i.e., , $T^4_{t,z*}$ and $C_{t,z*}$; more on the latter later) affect the stocks of bonds and capital available next month. The dynamics of the bond holdings and capital held by UK consumers is similarly determined by

$$\begin{aligned}
B^1_{t+1,z} &= R_t(B^3_{t,z} + S^3_t T^4_{t,z*} - S^3_t T^4_{t,z} - C_{t,z}), \\
\hat{B}^1_{t+1,z} &= \hat{R}_t(\hat{B}^3_{t,z} + T^4_{t,z} - T^4_{t,z*} + \hat{C}_{t,z*} - \hat{I}_{t,z}), \\
\hat{K}_{t+1,z} &= \hat{R}^k_{t+1}\left(\hat{K}_{t,z} - \hat{C}_{t,z} - \hat{C}_{t,z*} + \hat{I}_{t,z}\right)
\end{aligned}$$

The overnight return on UK capital is $\hat{R}^k_{t+1} = 1 + \hat{A}_{t+1}$ where $\hat{A}_{t+1}$ denotes UK productivity (i.e., $\hat{Y}_{t+1,z} = \hat{A}_{t+1}(\hat{K}_{t,z} - \hat{C}_{t,z} - \hat{C}_{t,z*} + \hat{I}_{t,z})$). As in period 2, trading is simultaneous and independent so UK consumers cannot condition their consumption, investment or currency orders on the decisions of US consumers, and vice versa.

## 2.2 Decision-Making

Consumers make two types of decisions: consumption-savings allocation decisions and financial pricing (quoting) decisions. The former are familiar from standard macro models, but the latter are new. By quoting spot prices and interest rates at which they stand ready to trade, consumers are taking on the liquidity-providing role of financial intermediaries. Specifically, the quote problem facing consumers in periods 1 and 3 is identical to that facing a dealer in a simultaneous trading model (see, for example, Lyons 1997, Rime 2001, Evans and Lyons 2002a, ). We therefore draw on this literature to determine how quotes are set.

Equilibrium quotes have two properties: (i) they must be consistent with market clearing, and (ii) they are a function of public information only. The latter property is important to the information transmission role of flow so let us address it more fully. With this property, unanticipated flow can only be impounded into price when it is realized and publicly observed. This lies at the opposite pole of the information assumptions underlying Walrasian (or Rational Expectations) mechanisms in which the market price at a given time impounds information in *every* trade occurring at that time. The Walrasian mechanism is akin to assuming that all trades are publicly observable and conditioned on one another, which is obviously counter-factual in most markets, including FX. (In the jargon of microstructure, the FX market is not a centralized auction with full transparency, but is instead a decentralized dealer market that is relatively opaque.) As noted in the previous section, what is really necessary for the intertemporal transmission role of flow is that at least some flow information is not impounded in price at the time of execution. That quotes are conditioned only on public information insures this, and goes a bit further to simplify the analytics.

We should add, though, that quotes being conditioned only on public information is not an assumption, but a result. Put differently, we make other assumptions that are sufficient for this outcome (drawing from the simultaneous-trade references above). Those assumptions are (1) that actions within any given quoting or trading period are simultaneous and independent, (2) that quotes are a single price good for any size, and (3) that trading with multiple market-makers is feasible.[10] The resulting solution to the quote problem facing consumer $z$ in periods $j = \{1, 3\}$ will be a quote $S^j_{t,z} = S^j_t$ where $S^j_t$ is a function of public information $\Omega^j_t$ (determined below). Similarly, the period-3 interest rate quotes are given by $R_{t,z} = R_t$ and $\hat{R}_{t,z} = \hat{R}_t$ where $R_t$ and $\hat{R}_t$ are functions of $\Omega^3_t$. To understand why these quotes represent a Nash equilibrium, consider a market-maker who is pondering whether to depart from this public-information price by quoting a weighted average of public information and his own private information. Any price that deviates from other prices would attract unbounded arbitrage trade flows, and therefore could not possibly represent an equilibrium. Instead, it is optimal for market-makers to quote the same price as others (which means the price is necessarily conditioned on public information), and then exploit their private information by initiating trades at other market-makers' prices. (In some models, market-makers can only establish desired positions by setting price to attract incoming trades, which is not the case here since they always have the option of initiating outgoing trades.)

Next we turn to the consumption and portfolio choices made in periods 2 and 4. Let $W^j_{t,z}$ denote the wealth of individual $z$ at the beginning of period $j$ in month $t$. This comprises the value of home and foreign bond holdings and domestic capital:

$$\begin{aligned} W^2_{t,z} &\equiv B^1_{t,z} + S^1_t \hat{B}^1_{t,z} + K_{t,z} + S^1_t \hat{K}_{t,z} \\ W^4_{t,z} &\equiv B^3_{t,z} + S^3_t \hat{B}^3_{t,z} + K_{t,z} + S^3_t \hat{K}_{t,z} \end{aligned}$$

Notice that wealth is valued in dollars using the equilibrium spot rate quoted in the period before trading takes place.

In period 2 consumers initiate transactions, (i.e., choose $T^2_{t,z}$) to achieve an optimal allocation of their wealth between dollar and pound assets. Because trading takes place simultaneously, the choice of $T^2_{t,z}$ cannot be conditioned on the orders they receive from others, $T^2_{t,z*}$. Instead, consumers must choose $T^2_{t,z}$ based on the expected order flow, $\mathbb{E}^2_{t,z} T^2_{t,z*}$. (Hereafter we use $\mathbb{E}^j_{t,z}$ to denote expectations conditioned on information available to individual $z$ at the beginning of period $j$ in month $t$). We formalize this idea by assuming that $T^2_{t,z}$ is chosen to achieve a desired portfolio allocation at the end of period-2 trading conditioned on $\mathbb{E}^2_{t,z} T^2_{t,z*}$.

Let $J^2_z(W^2_{t,z})$ and $J^4_z(W^4_{t,z})$ denote the value functions for consumer $z$ at the beginning of periods 2 and 4. $T^2_{t,z}$ is determined as the solution to the dynamic programming problem

$$J^2_z(W^2_{t,z}) = \max_{\lambda_{t,z}} \mathbb{E}^2_{t,z} \left[ J^4_z(W^4_{t,z}) \right], \tag{2}$$

s.t.

$$W^4_{t,z} = H^3_{t,z} W^2_{t,z}, \tag{3}$$

[10] As noted, it is also true that the assumption of no spreads is not necessary, though it greatly facilitates the analytics. Specifically, each trader-consumer's quote could be a schedule of prices, one for each incoming order quantity from minus infinity to plus infinity, as long as that schedule is conditioned only on the incoming order, as opposed to the realization of all other orders in the market (i.e., the quoting trader can protect against information contained in the single incoming trade).

where

$$
\begin{aligned}
H_{t,z}^3 &\equiv \left(1+\left(\frac{S_t^3}{S_t^1}-1\right)(\lambda_{t,z}-\xi_t)\right), \\
\lambda_{t,z} &\equiv \frac{S_t^1\left(\hat{B}_{t,z}^1+\hat{K}_{t,z}+T_{t,z}^2-\mathbb{E}_{t,z}^2T_{t,z*}^2\right)}{W_{t,z}^2}, \\
\xi_t &\equiv \frac{S_t^1(T_{t,z*}^2-\mathbb{E}_{t,z}^2T_{t,z*}^2)}{W_{t,z}^2}.
\end{aligned}
$$

$\lambda_{t,z}$ identifies the target fraction of wealth consumers wish to hold in pounds given their expectations about the foreign currency orders they will receive during trading, $\mathbb{E}_{t,z}^2T_{t,z*}^2$. (Actual orders, $T_{t,z}^2$, are determined from the optimal value of $\lambda_{t,z}$ given $\mathbb{E}_{t,z}^2T_{t,z*}^2$, $\hat{B}_{t,z}^1+\hat{Y}_{t,z}+\hat{K}_{t,z}$ and $W_{t,z}^2$). $H_{t,z}^3$ identifies the within-month return on wealth (i.e., between periods 1 and 3). This depends on the rate of appreciation in the pound and the actual faction of wealth held in foreign deposits at the end of period-2 trading. The latter term is $\lambda_{t,z}-\xi_t$ where $\xi_t$ represents the effect of filling unexpected pound orders from other consumers (a shock). This means that the return on wealth, $H_{t,z}^3$, is subject to two sources of uncertainty: uncertainty about the future spot rate $S_t^3$, and uncertainty about order flow in the form of trades initiated by other consumers.

In period 4, consumers choose consumption of US and UK goods, foreign currency orders and investment expenditures. Let $\alpha_{t,z}$ and $\gamma_{t,z}$ denote the desired fractions of wealth held in pounds and domestic capital respectively:

$$
\begin{aligned}
\alpha_{t,z} &\equiv \frac{S_t^3\hat{K}_{t,z}+S_t^3\hat{B}_{t,z}^3+S_t^3\left(T_{t,z}^4-\mathbb{E}_{t,z}^4T_{t,z*}^4\right)-S_t^3\hat{C}_{t,z}}{W_{t,z}^4}, \\
\gamma_{t,z} &\equiv \begin{cases} \dfrac{K_{t,z}+I_{t,z}-C_{t,z}-\mathbb{E}_{t,z}^4C_{t,z*}}{W_{t,z}^4} & z<1/2, \\ \dfrac{\hat{K}_{t,z}+\hat{I}_{t,z}-\hat{C}_{t,z}-\mathbb{E}_{t,z}^4\hat{C}_{t,z*}}{W_{t,z}^4} & z\geq 1/2, \end{cases}
\end{aligned}
$$

The period-4 problem can now be written as

$$
J_z^4(W_{t,z}^4)=\max_{\{C_{t,z},\hat{C}_{t,z},\alpha_{t,z},\gamma_{t,z}\}}\left\{U(\hat{C}_{t,z},C_{t,z})+\beta\mathbb{E}_{t,z}^4\left[J_z^2(W_{t+1,z}^2)\right]\right\}, \tag{4}
$$

s.t.

$$
W_{t+1,z}^2=R_tH_{t+1,z}^1W_{t,z}^4-R_t\left(C_{t,z}+S_t^3\hat{C}_{t,z}\right), \tag{5}
$$

where

$$
H_{t+1,z}^1=\begin{cases} 1+\left(\frac{S_{t+1}^1\hat{R}_t}{S_t^3R_t}-1\right)(\alpha_{t,z}-\varsigma_t)+\left(\frac{R_{t+1}^k}{R_t}-1\right)(\gamma_{t,z}-\zeta_t) & z<1/2 \\ \\ 1+\left(\frac{S_{t+1}^1\hat{R}_t}{S_t^3R_t}-1\right)(\alpha_{t,z}-\varsigma_t)+\left(\frac{S_{t+1}^1\hat{R}_{t+1}^k}{S_t^3R_t}-\frac{S_{t+1}^1\hat{R}_t}{S_t^3R_t}\right)\left(\gamma_{t,z}-\hat{\zeta}_t\right) & z\geq 1/2 \end{cases}.
$$

with $R_{t+1}^k\equiv 1+A_{t+1}$, and $\hat{R}_{t+1}^k=1+\hat{A}_{t+1}$.

$H_{t+1,z}^1$ is the excess return on wealth (measured relative to the dollar one-month rate $R_t$). As above, realized returns depend on the actual faction of wealth held in pounds $\alpha_{t,z} - \varsigma_{t,z}$, where $\varsigma_t \equiv S_t^3(T_{t,z*}^4 - \mathbb{E}_{t,z}^4 T_{t,z*}^4)/W_{t,z}^4$ represents the effects of unexpected currency orders. Monthly returns also depend on the fraction of wealth held in the form of capital. For the US case this is given by $\gamma_{t,z} - \zeta_{t,z}$, where $\zeta_{t,z} \equiv \left(C_{t,z*} - \mathbb{E}_{t,z}^4 C_{t,z*}\right)/W_{t,z}^4$ identifies the effects of unexpected demand for US goods (i.e. US exports).[11] In the UK case, the fraction is $\gamma_{t,z} - \hat{\zeta}_{t,z}$, where $\hat{\zeta}_{t,z} \equiv \left(\hat{C}_{t,z*} - \mathbb{E}_{t,z}^4 \hat{C}_{t,z*}\right)/W_{t,z}^4$. Monthly returns are therefore subject to four sources of uncertainty: uncertainty about future spot rates (i.e., $S_{t+1}^1$) that affects bond returns; uncertainty about future productivity that affects the return on capital; uncertainty about currency orders; and uncertainty about export demand.

The first order conditions governing consumption and portfolio choice (i.e., $C_{t,z}, \hat{C}_{t,z}, \lambda_{t,z}, \alpha_{t,z}$) take the same form for both US and UK consumers

$$\hat{C}_{t,z} \quad : \quad U_{\hat{c}}(\hat{C}_{t,z}, C_{t,z}) = \beta R_t S_t^3 \mathbb{E}_{t,z}^4 \left[ V_{t+1,z} H_{t+1,z}^3 \right], \tag{6}$$

$$C_{t,z} \quad : \quad U_c(\hat{C}_{t,z}, C_{t,z}) = \beta R_t \mathbb{E}_{t,z}^4 \left[ V_{t+1,z} H_{t+1,z}^3 \right], \tag{7}$$

$$\lambda_{t,z} \quad : \quad 0 = \mathbb{E}_{t,z}^2 \left[ V_{t,z} \left( \tfrac{S_t^3}{S_t^1} - 1 \right) \right], \tag{8}$$

$$\alpha_{t,z} \quad : \quad 0 = \mathbb{E}_{t,z}^4 \left[ V_{t+1,z} H_{t+1,z}^3 \left( \tfrac{S_{t+1}^1 R}{S_t^3 R_t} - 1 \right) \right], \tag{9}$$

where $V_{t,z} \equiv dJ_z^4(W_{t,z}^4)/dW_{t,z}^4$ is the marginal utility of wealth. The first order conditions governing real investment (i.e. $\gamma_{t,z}$) differ between US and UK consumers and are given by

$$\gamma_{t,z<1/2} \quad : \quad 0 = \mathbb{E}_{t,z}^4 \left[ V_{t+1,z} H_{t+1,z}^3 \left( \frac{R_{t+1}^k}{R_t} - 1 \right) \right], \tag{10}$$

$$\gamma_{t,z\geq 1/2} \quad : \quad 0 = \mathbb{E}_{t,z}^4 \left[ V_{t+1,z} H_{t+1,z}^3 \left( \frac{S_{t+1}^1 \hat{R}_{t+1}^k}{S_t^3 R_t} - 1 \right) \right]. \tag{11}$$

To further characterize the form of optimal consumption, portfolio and investment decisions, we need to identify the marginal utility of wealth. This is implicitly defined by the recursion

$$V_{t,z} = \beta R_t \mathbb{E}_{t,z}^4 \left[ V_{t+1,z} H_{t+1,z}^3 H_{t+1,z}^1 \right]. \tag{12}$$

In a standard macro model where consumers provide no liquidity provision, equations (8) - (12) together imply that $V_{t,z} = U_c(\hat{C}_{t,z}, C_{t,z})$. The first order conditions can then be rewritten in familiar form using the marginal rate of substitution. This is not generally the case in our model. As we shall show, $V_{t,z}$ can diverge from the marginal utility of consumption because unexpected currency and export orders affect portfolio returns.

[11] When superior information about home-country income is not symmetrized by month's end, the residual uncertainty is manifested as a shock to export demand.

### 2.3 Market Clearing

Market clearing in the currency market requires that the dollar value of pound orders initiated equals the dollar value of pound orders received:

$$\int T^j_{t,z} dz = \int T^j_{t,z^*} dz^*,$$

for $j = \{2, 4\}$.

We assume that dollar and pound deposits are in zero net supply so that aggregate deposit holdings at the start of periods 3 and 1 are given by

$$\int B^3_{t,z} dz = 0, \qquad \int \hat{B}^3_{t,z} dz = 0, \tag{13}$$

$$\int B^1_{t+1,z} dz = 0, \qquad \int \hat{B}^1_{t+1,z} dz = 0. \tag{14}$$

Combining these conditions with the budget constraints for dollar and pound deposits implies that both US and UK investment expenditures must equal zero if the bond and goods markets are to clear.[12] The reason is that both currency and goods market transactions only affect the distribution of deposits not their aggregate level. This means that any investment expenditures must be financed by an increase aggregate deposit holdings, an implication that is inconsistent with market clearing. The implications of market clearing for the dynamics of capital are therefore represented by

$$K_{t+1,z} = R^k_t K_{t,z} - \int C_{t,z} dz, \tag{15}$$

$$\hat{K}_{t+1,z} = \hat{R}^k_t \hat{K}_{t,z} - \int \hat{C}_{t,z} dz. \tag{16}$$

# 3 Equilibrium

An equilibrium in this model is described by: (i) a set of quote functions (that define the relationship between public information and both spot rates and interest rates) that clear markets given the consumption, investment and portfolio choices of consumers; and (ii) a set of consumption, investment and portfolio decision rules that maximize expected utility given the spot and interest rates and the exogenous productivity processes. In this section we describe how the equilibrium is constructed given particular specifications for utility and the productivity processes.

### 3.1 Utility and Productivity

We assume that the sub-utility function of both US and UK consumers takes the log form:

$$U(\hat{C}_{t,z}, C_{t,z}) = \tfrac{1}{2} \ln \hat{C}_{t,z} + \tfrac{1}{2} \ln C_{t,z}.$$

[12] Though this feature of the model appears rather special, it is not driving our results.

This assumption simplifies consumers decision-making and allows us to focus more easily on the novel aspects of the model.

The international aspect of our model becomes apparent with the specification of the productivity processes. In particular, the key feature that differentiates US from UK consumers in our model is the comparative advantage they have in acquiring information on local productivity. This information advantage creates an environment where dispersed information exists about the current and future returns to capital across the world. We examine below how this dispersed information becomes aggregated into exchange rates and interest rates via trading. Thus, our focus will be on information transmission process rather than the underlying source of the dispersed information. In a more general model, a comparative local advantage in information acquisition could also apply to monetary policy (in the form of superior local information about the path for future interest rates), or fiscal policy (in the form of superior tax rate forecasts). Our analysis could be readily extended to an environment where dispersed information originates from productivity and other sources.

To describe the information advantage, we characterize the exogenous productivity processes in terms of their implications for the log returns on capital:

$$\ln R_t^k \equiv r_t^k = r + \eta_t + u_t + \Delta e_t \tag{17}$$

$$\ln \hat{R}_t^k \equiv \hat{r}_t^k = r - \eta_t + \hat{u}_t + \Delta \hat{e}_t \tag{18}$$

with $\eta_t \sim i.i.d.N(0, \sigma_\eta^2)$, $u_t \sim i.i.d.N(0, \sigma_u^2)$, $\hat{u}_t \sim i.i.d.N(0, \sigma_u^2)$, $e_t \sim i.i.d.N(0, \sigma_e^2)$. Log returns comprise three random components: $\eta_t$ is a temporary effect common to both processes (with opposite sign), $u_t$ ($\hat{u}_t$) is a temporary effect idiosyncratic to the US (UK) process, and $e_t$ ($\hat{e}_t$) is an intertemporal effect idiosyncratic to the US (UK) process. We should emphasize that these return specifications are not meant to serve as accurate time series representations. Rather they are simple processes that allow the structure of dispersed information to be easily identified from primitive assumptions concerning the information each consumer receives about the return components. For example, below we examine the information structure that arises when all US (UK) consumers learn the values of $r_t^k$ ($\hat{r}_t^k$) and $\eta_t$ in period 1 but do not directly observe $\hat{r}_t^k$ ($r_t^k$). In this example, all consumers within a country have the same information in equilibrium so that dispersed information exists internationally rather than intranationally. This will be a common feature of the equilibria we study below. Allowing information to be dispersed across consumers at both the national and international level would be an interesting but extremely complex undertaking that we leave for the future. Similarly, our analysis could also be extended to deal with general moving average processes for the components of the returns. We chose the specifications in (17) and (18) so that the theoretical consequences of dispersed information can be examined in the simplest possible way.

## 3.2 Log Approximations

To facilitate finding the optimal consumption, investment and currency trading decisions of US and UK consumers we make use of log linear approximations to the budget constraints and first order conditions.[13]

[13] Complete derivations are contained in the appendix.

Combining (3) and (5) the monthly budget constraint is approximated by

$$\Delta w_{t+1,z}^4 \cong r_t + h_{t+1,z}^3 + \ln(1-\mu) + \frac{1}{1-\mu} h_{t+1,z}^1 - \frac{\mu}{1-\mu}\delta_{t,z}, \tag{19}$$

where lowercase letters denote natural logs and $\delta_{t,z} \equiv c_{t,z} - w_{t,z}^4 - \ln(\mu/2)$ is the log consumption wealth ratio. $\mu$ is a positive constant equal to the steady state value of $2C_{t,z}/W_{t,z}^4$. $h_{t,z}^1$ and $h_{t,z}^3$ are the log excess returns on the wealth of consumer $z$ realized respectively in periods 1 and 3 in month $t$. Using the definitions of $H_{t,z}^1$ and $H_{t,z}^3$ represented above, we approximate the within-month returns by

$$h_{t,z}^3 \cong \lambda_{t,z}\left(s_t^3 - s_t^1\right) + \tfrac{1}{2}\lambda_{t,z}\left(1-\lambda_{t,z}\right)\mathbb{V}_{t,z}^2\left(s_t^3\right) - \mathbb{CV}_{t,z}^2\left(s_t^3, \xi_t\right), \tag{20}$$

where $\mathbb{V}_{t,z}^j$ and $\mathbb{CV}_{t,z}^j$ denote the variance and covariance conditioned on consumer $z's$ information at the start of period $j$ in month $t$. This approximation is similar to those adopted by Campbell and Viceira (2002) and is based on a second order approximation that holds exactly in continuous time when the change in spot rates and unexpected order flow follow Wiener processes. Monthly returns are approximated in a similar fashion. For US consumers (i.e. $z < 1/2$) we use

$$\begin{aligned} h_{t+1,z}^1 \cong\; & \alpha_{t,z}\left(s_{t+1}^1 - s_t^3 + \hat{r}_t - r_t\right) + \gamma_{t,z}\left(r_{t+1}^k - r_t\right) + \tfrac{1}{2}\alpha_{t,z}\left(1-\alpha_{t,z}\right)\mathbb{V}_{t,z}^4\left(s_{t+1}^1\right) \\ & + \tfrac{1}{2}\gamma_{t,z}\left(1-\gamma_{t,z}\right)\mathbb{V}_{t,z}^4\left(r_{t+1}^k\right) - \alpha_{t,z}\gamma_{t,z}\mathbb{CV}_{t,z}^4\left(s_{t+1}^1, r_{t+1}^k\right) \\ & - \mathbb{CV}_{t,z}^4\left(s_{t+1}^1, \varsigma_t\right) - \mathbb{CV}_{t,z}^4\left(r_{t+1}^k, \zeta_t\right), \end{aligned} \tag{21}$$

and for UK consumers ($z \geq 1/2$)

$$\begin{aligned} h_{t+1,z}^1 \cong\; & \alpha_{t,z}\left(s_{t+1}^1 - s_t^3 + \hat{r}_t - r_t\right) + \gamma_{t,z}\left(\hat{r}_{t+1}^k - \hat{r}_t\right) + \tfrac{1}{2}\left(\alpha_{t,z} - \gamma_{t,z}\right)\left(1 - \left(\alpha_{t,z} - \gamma_{t,z}\right)\right)\mathbb{V}_{t,z}^4\left(s_{t+1}^1\right) \\ & + \tfrac{1}{2}\gamma_{t,z}\left(1-\gamma_{t,z}\right)\mathbb{V}_{t,z}^4\left(\hat{r}_{t+1}^k + s_{t+1}^1\right) - \left(\alpha_{t,z} - \gamma_{t,z}\right)\gamma_{t,z}\mathbb{CV}_{t,z}^4\left(s_{t+1}^1, \hat{r}_{t+1}^k + s_{t+1}^1\right) \\ & - \mathbb{CV}_{t,z}^4\left(s_{t+1}^1, \varsigma_t\right) - \mathbb{CV}_{t,z}^4\left(r_{t+1}^k, \hat{\zeta}_t\right). \end{aligned} \tag{22}$$

Notice that unexpected order flows and export demand affect returns through the last covariance terms shown in each equation. These terms represent the effects of non-diversifiable risk that arises from liquidity provision. Unexpected currency orders and export orders during period 2 and 4 trading represent a source of risk that consumers cannot fully hedge.

To derive our log approximations to the first order conditions, we combine the log linearized versions of equations (6) - (12) and our assumption of log utility to obtain

$$v_{t,z} = -c_{t,z} - \phi_{t,z}, \tag{23}$$

where $\phi_z \equiv \mathbb{CV}_{t,z}^4\left(s_{t+1}^1, \varsigma_t\right) + \mathbb{CV}_{t,z}^4\left(r_{t+1}^k, \zeta_t\right)$ for $z < 1/2$ (US consumers), and $\phi_z \equiv \mathbb{CV}_{t,z}^4\left(s_{t+1}^1, \varsigma_t\right)$ $+\mathbb{CV}_{t,z}^4\left(\hat{r}_{t+1}^k, \hat{\zeta}_t\right)$ for $z \geq 1/2$ (UK consumers). In the absence of unexpected period-4 currency orders and export demand, the shocks $\varsigma_t$, $\zeta_t$ and $\hat{\zeta}_t$ are zero and the (log) marginal utility of wealth equals the marginal utility of consumption. When these shocks are present and correlated with the future spot rate, and/or returns on capital, the return on wealth is exposed to these sources of systematic risk that may push up or down the log return on wealth according to the sign of the covariance terms. As we shall see, the covariance between currency orders and the future spot rate, $\mathbb{CV}_{t,z}^{4}\left(s_{t+1}^{1},\varsigma_{t,z}\right)$, will differ from zero when period-4 currency trading provides information relevant to the setting of future spot rates. Thus, the transmission of price-relevant information via trading can push a wedge, $\phi_{t,z}$, between the marginal utilities of wealth and consumption.

Substituting for $v_{t,z}$ in the log linearized versions of (6) - (11) gives the following linearized first order conditions:

$$\begin{aligned}
\lambda_{t,z} \quad &: \quad \mathbb{E}_{t,z}^{2}s_{t}^{3}-s_{t}^{1}+\tfrac{1}{2}\mathbb{V}_{t,z}^{2}\left(s_{t}^{3}\right)=\mathbb{CV}_{t,z}^{2}\left(c_{t,z}+\phi_{t,z},s_{t}^{3}\right), && (24)\\
\alpha_{t,z} \quad &: \quad \mathbb{E}_{t,z}^{4}\left[s_{t+1}^{1}-s_{t}^{3}+\hat{r}_{t}-r_{t}\right]+\tfrac{1}{2}\mathbb{V}_{t,z}^{4}\left(s_{t+1}^{1}\right)=\mathbb{CV}_{t,z}^{4}\left(c_{t+1,z}+\phi_{t+1,z}-h_{t+1,z}^{3},s_{t+1}^{1}\right), && (25)\\
c_{t,z} \quad &: \quad \ln\beta+r_{t}=\mathbb{E}_{t,z}^{4}\left[\Delta c_{t+1,z}+\phi_{t+1,z}-h_{t+1,z}^{3}\right]-\tfrac{1}{2}\mathbb{V}_{t,z}^{4}\left(c_{t+1,z}+\phi_{t+1,z}-h_{t+1,z}^{3}\right), && (26)\\
\hat{c}_{t,z} \quad &: \quad c_{t,z}=s_{t}^{3}+\hat{c}_{t,z}, && (27)
\end{aligned}$$

for both US and UK consumers. The linearized versions of (10) and (11) are

$$\begin{aligned}
\gamma_{t,z<1/2} \quad &: \quad \mathbb{E}_{t,z}^{4}\left[r_{t+1}^{k}-r_{t}\right]+\frac{1}{2}\mathbb{V}_{t,z}^{4}\left(r_{t+1}^{k}\right)=\mathbb{CV}_{t,z}^{4}\left(c_{t+1,z}+\phi_{t+1,z}-h_{t+1,z}^{3},r_{t+1}^{k}\right), && (28)\\
\gamma_{t,z\geq 1/2} \quad &: \quad \mathbb{E}_{t,z}^{4}\left[\hat{r}_{t+1}^{k}+s_{t+1}^{1}-s_{t}^{3}-r_{t}\right]+\frac{1}{2}\mathbb{V}_{t,z}^{4}\left(\hat{r}_{t+1}^{k}+s_{t+1}^{1}\right)= \\
&\qquad \mathbb{CV}_{t,z}^{4}\left(c_{t+1,z}+\phi_{t+1,z}-h_{t+1,z}^{3},\hat{r}_{t+1}^{k}+s_{t+1}^{1}\right). && (29)
\end{aligned}$$

Notice that presence of liquidity provision in the model only affect the first order conditions characterizing consumer behavior through the $\phi_{t,z}$ terms. When combined with the linearized budget constraint, these equations allow us to find analytic approximations for the solution to the optimizations problems facing consumers at the beginning of period 2 and 4 (i.e. expressions for $\lambda_{t,z}$, $\alpha_{t,z}$, $\gamma_{t,z}, c_{t,z}$ and $\hat{c}_{t,z}$) given the $r_{t}^{k}$ and $r_{t}^{k}$ processes, and the equilibrium dynamics of spot exchange rates and interest rates (determined below).

We also utilized log linear approximations to the capital stock dynamics implied by market clearing in (15) and (16):

$$\begin{aligned}
k_{t+1}-k_{t} \quad &\cong \quad r_{t+1}^{k}+\ln\left(1-\mu\right)-\frac{\mu}{2(1-\mu)}\left(s_{t}^{3}+\hat{k}_{t}-k_{t}+\int\delta_{t,z}dz\right), && (30)\\
\hat{k}_{t+1}-\hat{k}_{t} \quad &\cong \quad \hat{r}_{t+1}^{k}+\ln\left(1-\mu\right)-\frac{\mu}{2(1-\mu)}\left(k_{t}-s_{t}^{3}-\hat{k}_{t}+\int\delta_{t,z}dz\right). && (31)
\end{aligned}$$

In deriving these equations we have assumed that deposit holdings always represent a small fraction of consumer wealth. This condition is met trivially in the steady state because both US and UK consumers hold all their wealth in the form of domestic capital. The accuracy of these approximations will deteriorate if consumers accumulate substantial financial assets/liabilities relative to their capital holdings when away from the steady state (see Appendix for a further discussion).

### 3.3 Solution Method

We find the equilibrium for the model using a guess and verify method with the following steps:

1. We make a conjecture about the information available to consumers at each point in time. This involves specify what information consumers receive directly and what they learn from observing trading.

2. Based on this information structure, we then guess the form of equilibrium quote functions for spot rates and interest rates in periods 1 and 3 noting that quotes can only be a function of common information.

3. We use the log linearized first order conditions and budget constraint to approximate consumers' optimal consumption, investment and currency choices given the spot and interest rates found in step 2.

4. We check that consumer choices for consumption, investment and currency holdings clear markets.

5. We verify that the conjectured information structure (from step 1) can be supported by an inference problem based on exogenous information available to each consumer, and their observations of quotes and trading activity.

## 4 Dispersed Information Results

To analyze how dispersed information is impounded in asset prices via trading, we study three versions of the model distinguished from one another by their information structures. First we consider an i.i.d. environment in which consumers receive dispersed information about productivity that is readily aggregated and impounded into the exchange rate. Although this version clarifies how information aggregation takes place through trading, the i.i.d. environment rules out other potentially important effects. The second version relaxes the i.i.d. assumption, which allows us to examine how the aggregation of information affects the joint dynamics of exchange rates and interest rates. In the third version the equilibrium is one in which dispersed information cannot be quickly aggregated. This equilibrium is much more complex than the others and displays several features not found in standard models.

### 4.1 Version 1: I.I.D. Capital Returns

We assume that the random returns processes for US and UK capital take the form

$$r_t^k = r + \eta_t + u_t, \tag{32a}$$

$$\hat{r}_t^k = r - \eta_t + \hat{u}_t, \tag{32b}$$

where as before $\eta_t$, $u_t$ and $\hat{u}_t$ are mean zero, mutually independent normally distributed shocks (see equations 17 and 18). We further assume that consumers observe the composition of the return on domestic capital at the start of period 1 each month. Returns on foreign capital are not directly observed. More formally, let $\Omega^j_{t,\mathbb{US}}$ and $\Omega^j_{t,\mathbb{UK}}$ denote the information set available at the start of period $j$ in month $t$, to US and UK consumers respectively. We represent these information assumptions as

$$\Omega^1_{t,\mathbb{US}} = \left\{u_t, \eta_t, \Omega^4_{t-1,\mathbb{US}}\right\}, \quad \Omega^1_{t,\mathbb{UK}} = \left\{\hat{u}_t, \eta_t, \Omega^4_{t-1,\mathbb{UK}}\right\}. \tag{33}$$

Equations (32) and (33) represent the exogenous information structure used to solve the model. In the appendix we derive the solution in detail following the steps described in the last section. This gives us the following equations for the equilibrium spot and interest rates:

$$\begin{aligned} s^1_t &= s^3_{t-1} + 2\eta_t, && \text{(34a)}\\ s^3_t &= s^1_t + u_t - \hat{u}_t, && \text{(34b)}\\ r_t &= r + \theta, && \text{(34c)}\\ \hat{r}_t &= r + \hat{\theta}. && \text{(34d)} \end{aligned}$$

To understand the economics behind these equations, it is useful to consider how the information available to individual consumers evolves in periods 2 - 4 of each month.

$$\begin{array}{ll} \Omega^2_{t,\mathbb{US}} = \left\{S^1_t, \Omega^1_{t,\mathbb{US}}\right\}, & \Omega^2_{t,\mathbb{UK}} = \left\{S^1_t, \Omega^1_{t,\mathbb{UK}}\right\}, \\ \Omega^3_{t,\mathbb{US}} = \left\{\hat{u}_t, \Omega^2_{t,\mathbb{US}}\right\}, & \Omega^3_{t,\mathbb{UK}} = \left\{u_t, \Omega^2_{t,\mathbb{UK}}\right\}, \\ \Omega^4_{t,\mathbb{US}} = \left\{S^3_t, \Omega^3_{t,\mathbb{US}}\right\}, & \Omega^4_{t,\mathbb{UK}} = \left\{S^3_t, \Omega^3_{t,\mathbb{UK}}\right\}, \end{array}$$

Points to note:

- Equations (34a) and (34b) together with our specification for capital returns implies that

  $$s^3_t - s^3_{t-1} = r^k_t - \hat{r}^k_t.$$

  This equation implies that period-3 spot rates are set to equalize the returns on US and UK capital measured in terms of a common currency. To see why this is consistent with market clearing, we combine (30) and (31) to give

  $$k_t - s^3_t - \hat{k}_t = (1-\mu)\left(\Delta s^3_{t+1} + \hat{r}^k_{t+1} - r^k_{t+1}\right) + (1-\mu)\left(k_{t+1} - s^3_{t+1} - \hat{k}_{t+1}\right).$$

  This equation imposes the implications of market clearing on the dynamics of period-3 spot rates. To see this clearly, we iterate forward (with $\lim_{i\to\infty}(1-\mu)^i(k_{t+i} - s^3_{t+i} - \hat{k}_{t+i}) = 0$) and taking expectations conditioned on common period-3 information to give

  $$s^3_t = \mathbb{E}^3_t\left[k_t - \hat{k}_t\right] + \mathbb{E}^3_t \sum_{i=1}^{\infty} (1-\mu)^i \left(r^k_{t+i} - \Delta s^3_{t+i} - \hat{r}^k_{t+i}\right). \tag{35}$$

  This equation determines the value of the log period-3 spot rates consistent with market clearing based on common period-3 information. In equilibrium, the values of $k_t$ and $\hat{k}_t$ are in the period-3 common information set, so the solution of the above equation is $s^3_t = k_t - \hat{k}_t$. [It is straightforward to check that $\mathbb{E}_t^3 \left( r_{t+i}^k - \Delta k_{t+i} + \Delta \hat{k}_{t+i} - \hat{r}_{t+i}^k \right) = 0$ for $i > 0$.) Combining this result with the log dynamics of US and UK capital in (30) and (31) implies that $s_t^3 - s_{t-1}^3 = r_t^k - \hat{r}_t^k$.

- Equation (33) implies that $\eta_t$ shocks become part of the common information set (i.e., $\Omega_t^j \equiv \Omega_{t,\mathbb{US}}^j \cap \Omega_{t,\mathbb{UK}}^j$) in period 1. Because these shocks are price-relevant they are immediately embedded in the period-1 spot rates.

- $u_t$ and $\hat{u}_t$ shocks are not in $\Omega_t^1$ and are not impounded in period-1 spot rate quotes (though they would be impounded if publicly known).

- US (UK) consumers learn about the value of $u_t$ ($\hat{u}_t$) shocks from the unexpected currency orders they receive in period-2 trading. Thus, by period 3, $u_t$ and $\hat{u}_t$ are in the common information set, and can be impounded in the spot price, $S_t^3$.

- Period-2 order flow only provides information about the $u_t(\hat{u}_t)$ shocks because the demand for foreign currency by US (UK) consumers depends on the private information they have about $u_t(\hat{u}_t)$. This information is valuable because it enables consumers to forecast within-month currency returns with greater accuracy than would be possible using only common information.

- By period 3, all information regarding the structure of US and UK capital returns has been aggregated into the common information set. Thus, the dispersed information regarding returns that existed in period-1 has been completely aggregated by the time period-2 trading is complete.

- No information aggregation takes place during period-4 trading because the demand for currency and goods by both US and UK consumers do not depend upon any information not yet aggregated (i.e. information in $\Omega_{t,\mathbb{US}}^4$ or $\Omega_{t,\mathbb{UK}}^4$ but not in $\Omega_t^4$).

- Order flows from period-4 trading are completely predictable and contain no information relevant for the setting of period-1 spot rates the next month. Order flow will not be correlated with monthly innovations in spot rates. Thus, there is no wedge between the marginal utilities of wealth and consumption. This implies that foreign exchange risk premia take the standard form (under log utility).

- Order flow from period-2 trading contain information that is relevant for the setting of period-3 spot rates, so the within-month innovation in exchange rates will be correlated with unexpected order flow.

- Interest rates are constant in this version of the model because the expected return on both US and UK capital (based on period-4 information) are also constant. $\theta$ and $\hat{\theta}$ represent the risk premia on UK and US capital (equations 34c and 34d).

## 4.2 Version 2: Non-I.I.D. Capital Returns

We now introduce serial correlation into the capital return processes:

$$r_t^k = r + \eta_t + \Delta e_t \tag{36a}$$
$$\hat{r}_t^k = r - \eta_t + \Delta \hat{e}_t \tag{36b}$$

where $\eta_t$, $e_t$ and $\hat{e}_t$ are mean zero, mutually independent normally distributed shocks. As above, we assume that consumers observe the composition of the return on domestic capital at the start of period 1 each month. Returns on foreign capital are not directly observed. Formally,

$$\Omega^1_{t,\mathrm{US}} = \left\{e_t, \eta_t, \Omega^4_{t-1,\mathrm{US}}\right\}, \quad \Omega^1_{t,\mathrm{UK}} = \left\{\hat{e}_t, \eta_t, \Omega^4_{t-1,\mathrm{UK}}\right\}. \tag{37}$$

The equilibrium dynamics for spot and interest rates are

$$\begin{aligned} s^1_t &= s^3_{t-1} + 2\eta_t + \hat{e}_{t-1} - e_{t-1} && \text{(38a)} \\ s^3_t &= s^1_t + e_t - \hat{e}_t && \text{(38b)} \\ r_t &= r + \theta - e_t, && \text{(38c)} \\ \hat{r}_t &= r + \hat{\theta} - \hat{e}_t && \text{(38d)} \end{aligned}$$

and the evolution of information is

$$\begin{array}{ll} \Omega^2_{t,\mathrm{US}} = \left\{S^1_t, \Omega^1_{t,\mathrm{US}}\right\}, & \Omega^2_{t,\mathrm{UK}} = \left\{S^1_t, \Omega^1_{t,\mathrm{UK}}\right\}, \\ \Omega^3_{t,\mathrm{US}} = \left\{\hat{e}_t, \Omega^2_{t,\mathrm{US}}\right\}, & \Omega^3_{t,\mathrm{UK}} = \left\{e_t, \Omega^2_{t,\mathrm{UK}}\right\}, \\ \Omega^4_{t,\mathrm{US}} = \left\{S^3_t, r_t, \hat{r}_t, \Omega^3_{t,\mathrm{US}}\right\}, & \Omega^4_{t,\mathrm{UK}} = \left\{S^3_t, r_t, \hat{r}_t, \Omega^3_{t,\mathrm{UK}}\right\}, \end{array}$$

This equilibrium is similar to version 1 in several key respects:

- Period-2 trading conveys information on the values of $e_t$ and $\hat{e}_t$ just as it did on $u_t$ and $\hat{u}_t$ in version 1.
- $e_t$ and $\hat{e}_t$ are not impounded in spot rates until period 3 because that is the first point when they become common information.
- Period-4 trading conveys no information because the structure of returns is common knowledge in period 3. As a result, there is no wedge between the marginal utilities of wealth and consumption.
- The consumption-wealth ratio remains constant because the effect of variations in the expected capital return are exactly offset by changes in interest rates (as is standard under log utility).

Among the features of the equilibrium that are different are:

- Interest rates are no long constant because the expected return on US and UK capital varies respectively with $e_t$ and $\hat{e}_t$.
- Changes in monthly spot rates reflect the arrival of new information regarding returns (in the form of $\eta_t$ shocks), and information that was already embedded in the common information set $(\hat{e}_{t-1} - e_{t-1})$. The latter terms are new to this version of the model. They are present to insure that the expected excess return on foreign currency (conditioned on common information) remains constant: $E^4_t\left[s^1_{t+1} - s^3_t + \hat{r}_t - r_t\right] = \hat{\theta} - \theta$. This is a necessary condition for clearing the deposit markets across months (recall that net supply in these markets is zero).

### 4.3 Version 3: Distinguishing Persistent from Transitory Capital Return Shocks

We now consider the equilibrium in which the inference problem facing individual consumers is too complex for a single trading period to aggregate dispersed information completely. This means that the period 3 and 4 decisions made by consumers are made in an environment where dispersed information still persists. As we shall show, this persistence has important implications.

In this version of the model capital returns are given by

$$\ln R_t^k \equiv r_t^k = r + \eta_t + u_t + \Delta e_t \tag{39a}$$

$$\ln \hat{R}_t^k \equiv \hat{r}_t^k = r - \eta_t + \hat{u}_t + \Delta \hat{e}_t \tag{39b}$$

As in Versions 1 and 2, we do assume that consumers observe domestic capital returns at the start of each month. We do not, however, provide consumers with complete information on the composition of returns. Specifically, we assume that US (UK) consumers observe the values of $r_t^k$ and $\eta_t$ ($\hat{r}_t^k$ and $\eta_t$) in period 1, but not the values of $u_t$ and $e_t$ (or $\hat{u}_t$ and $\hat{e}_t$). Rather the value of $u_t$ ($\hat{u}_t$) is only learned by US (UK) consumers at the start of period 4. Thus, in period 1 consumers know the international component of returns (i.e.,$\eta_t$) but face uncertainty about the persistence of domestic returns that is only resolved at the start of period 4.

The equilibrium dynamics for spot and interest rates are

$$s_t^3 = s_t^1 + e_t + u_t - \hat{e}_t - \hat{u}_t \tag{40a}$$

$$s_{t+1}^1 = s_t^3 + 2\eta_{t+1} + \hat{e}_t - e_t \tag{40b}$$

$$\Delta s_t^3 = 2\eta_t + u_t - \hat{u}_t + \Delta e_{t-1} - \Delta \hat{e}_{t-1} \tag{40c}$$

$$r_t = r + \theta - \kappa(e_t + u_t), \tag{40d}$$

$$\hat{r}_t = r + \hat{\theta} - \kappa(\hat{e}_t + \hat{u}_t) \tag{40e}$$

where $\kappa$ is a signal extraction coefficient $\kappa = \sigma_e^2/(\sigma_e^2 + \sigma_u^2)$ and the evolution of information is

$$\begin{array}{ll}
\Omega_{t,\mathrm{US}}^1 = \left\{\eta_t, e_t + u_t, \hat{e}_{t-1}, \Omega_{t-1,\mathrm{US}}^4\right\}, & \Omega_{t,\mathrm{UK}}^1 = \left\{\eta_t, \hat{e}_t + \hat{u}_t, e_{t-1}, \Omega_{t-1,\mathrm{UK}}^4\right\}, \\
\Omega_{t,\mathrm{US}}^2 = \left\{S_t^1, \Omega_{t,\mathrm{US}}^1\right\}, & \Omega_{t,\mathrm{UK}}^2 = \left\{S_t^1, \Omega_{t,\mathrm{UK}}^1\right\}, \\
\Omega_{t,\mathrm{US}}^3 = \left\{\hat{e}_t + \hat{u}_t, \Omega_{t,\mathrm{US}}^2\right\}, & \Omega_{t,\mathrm{UK}}^3 = \left\{e_t + u_t, \Omega_{t,\mathrm{UK}}^2\right\}, \\
\Omega_{t,\mathrm{US}}^4 = \left\{e_{t,}, S_t^3, \Omega_{t,\mathrm{US}}^3\right\}, & \Omega_{t,\mathrm{UK}}^4 = \left\{\hat{e}_{t,}, S_t^3, \Omega_{t,\mathrm{UK}}^3\right\},
\end{array}$$

This equilibrium has several important new features:

- Period-2 order flow conveys information on the values of the sum $\hat{e}_t + \hat{u}_t$ to US consumers and the values of the sum $e_t + u_t$ to UK consumers. Thus, by period 3, the values of $r_t^k$ and $\hat{r}_t^k$ are common knowledge and $s_t^3 - s_{t-1}^3 = r_t^k - \hat{r}_t^k$ solves (35). However, the complete state of the economy (i.e. the current and past values of $\eta_t$ $\hat{e}_t, \hat{u}_t, e_t$ and $u_t$ individually) is not in the period-3 common knowledge information set, $\Omega_t^3$.

- Order flow from period-4 trading conveys information about the value of the individual shocks $\hat{e}_t$ and ($e_t$) to US (UK) consumers because the demand currency by US (UK) consumers are functions of $e_t$ and $u_t$ ($\hat{e}_t$ and $\hat{u}_t$). As a result, order flows are correlated with currency returns within and across months (across because the information from period-4 trading affects price in period-1 of the next month).

- Unexpected export orders also convey information. The demand for US goods by UK consumers is a function of $\hat{e}_t$ and $\hat{u}_t$ while the demand for UK goods by US consumers is a function of $e_t$ and $u_t$.

- The correlation between monthly order flows and currency returns introduces a wedge between the marginal utilities of wealth and consumption that affects consumption, investment and portfolio decisions.

- Interest rates under-react to persistent shocks in the capital returns process and over-react to transitory shocks. Recall from version 1 that equilibrium interest rates were not affected by $u_t$ and $\hat{u}_t$ shocks because they had no effect on expected capital returns. Similarly, in version 2 $e_t$ and $\hat{e}_t$ shocks were reflected in interest rates so as to keep the expected excess return on capital constant. In this equilibrium interest rates under-react to $e_t$ and $\hat{e}_t$ shocks and over-react to $u_t$ and $\hat{u}_t$ shocks. As above, interest rate are set so that the expected excess return on capital is constant. But, because the expected return on US and UK capital based on common period-3 information is $\mathbb{E}_t^3 r_{t+1}^k = r - \kappa(e_t + u_t)$ and $\mathbb{E}_t^3 \hat{r}_{t+1}^k = r - \kappa(\hat{e}_t + \hat{u}_t)$, where $\kappa = \sigma_e^2/(\sigma_e^2 + \sigma_\eta^2) < 1$, $e_t$ and $\hat{e}_t$ have a less that one-to-one effect on interest rates, and the impact of $u_t$ and $\hat{u}_t$ shocks is greater than zero. In sum interest rates behave differently in this model because common knowledge about the state of the economy is less than complete by period-3 (in the spirit of noisy rational expectations equilibria).

Finally, note that if this model were to include dispersed information about productivity (or other fundamentals) in future months, then exchange rates would impound information about these future paths before their realization, leading to the result that at higher frequencies order flow would explain exchange-rate changes better than macro variables, whereas at lower frequencies macro variables would predominate (a consequence of order flow anticipating long-horizon macro paths that are, on average, realized).

# 5 Conclusion

This paper is certainly not the last word on bridging the gap between the new macro and microstructure approaches. Other structural assumptions can be made (e.g., allowing learning to extend over many "months"). Different questions can be addressed. With respect to exchange rates, it remains clear that new macro models need to find more traction in the data. At the same time, microstructure modeling needs a richer placement within the underlying real economy if it is to realize its potential in addressing macro phenomena. It is precisely this joint need that motivates us to write a paper like this, one which (we hope) helps establish a dialogue.

What have we learned? A first lesson from modeling currency trade in a GE setting is that the information problem faced by the foreign exchange market is more nuanced than suggested by past microstructure analysis. Even if individuals receive information via an exogenous process, the timing of when that information is impounded in price is endogenous because signals correspond to the equilibrium actions of participants. The dynamics of the model create a constant tension between strong and semi-strong form efficiency. Finally, relative to microstructure models, the information structure of the GE model provides needed clarity on whether transaction-flow effects on exchange rates should persist, and, importantly, whether that persistence applies to real exchange rates or only to nominal rates.

A second lesson from GE modeling currency price discovery is that it affects real decisions in ways not considered in either new macro or microstructure models. For example, our model clarifies the channels through which financial intermediation in currency markets affects consumption and intertemporal consumption hedging. As we show, innovations in agents' learning from currency-market activity are correlated with other things they care about (e.g., real output). Decision-making about real choices is conditioned on information that is generally less that the union of individuals' information sets, which naturally leads to effects on real allocations.

A third lesson from GE modeling of currency price discovery is that it uncovers a new type of risk premium that does not arise in new macro models, nor in microstructure models. In particular, co-variation between order flow and the spot rate drives a wedge between the marginal utilities of wealth and consumption, introducing a source of risk that affects asset allocation. The empirical importance of the new hedging terms that arise depends on the manner in which dispersed information is embedded into spot rates rather than on the form of the utility function. The size of these hedging terms depends on the extent to which dispersed information is present in order flow, and the speed at which aggregation of dispersed information takes place. These should vary with the state of the economy, pointing to an unexplored source of risk premia variation.

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# A Appendix

## A.1 Optimization Problems

To derived the budget constraint in (3), we use the definitions of $\lambda_{t,z}$ and $\xi_t$ together with the intraday dynamics of US and UK bonds to obtain

$$\begin{aligned} S_t^3\left(\hat{B}_t^3 + \hat{K}_{t,z}\right) &= \tfrac{S_t^3}{S_t^1}(\lambda_{t,z} - \xi_t)W_{t,z}^2, \\ B_t^3 &= \left[1 - (\lambda_{t,z} - \xi_t)\right] W_t^2 - K_{t,z}. \end{aligned}$$

(Note that consumers only hold domestic capital so that $K_{t,z} = 0$ for $z \geq 1/2$, and $\hat{K}_{t,z} = 0$ for $z < 1/2$.) Substituting these expressions into the definition of $W_{t,z}^4$, gives (3):

$$W_{t,z}^4 = \left(1 + \left(\frac{S_{t+1}^3}{S_t^1} - 1\right)(\lambda_{t,z} - \xi_t)\right) W_{t,z}^2.$$

Let $\varsigma_t \equiv S_t^3(T_{t,z*}^4 - \mathbb{E}_{t,z}^4 T_{t,z*}^4)/W_{t,z}^4$, $\zeta_t \equiv \left(C_{t,z*} - \mathbb{E}_{t,z}^4 C_{t,z*}\right)/W_{t,z}^4$ and $\hat{\zeta}_t \equiv \left(\hat{C}_{t,z*} - \mathbb{E}_{t,z}^4 \hat{C}_{t,z*}\right)/W_{t,z}^4$ respectively denote unexpected order flow, US export demand, and UK export demand measured relative to period-4 wealth. Then using the definitions of $\alpha_{t,z}$, and $\gamma_{t,z}$ together with the overnight dynamics of bonds and capital for US consumers we obtain:

$$\begin{aligned} S_{t+1}\hat{B}_{t+1}^1 &= \frac{S_{t+1}^1 \hat{R}_t}{S_t^3}(\alpha_{t,z} - \varsigma_t) W_t^4, \\ B_{t+1}^1 &= R_t\left[1 - (\alpha_{t,z} - \varsigma_t)\right] W_{t,z}^4 - R_t\left(C_{t,z} + S_t^3\hat{C}_{t,z}\right) - R_t(\gamma_{t,z} - \zeta_{t,z})W_{t,z}^4, \\ K_{t+1,z} &= R_{t+1}^k(\gamma_{t,z} - \zeta_{t,z})W_{t,z}^4. \end{aligned}$$

Substituting these expressions into the definition of $W_{t+1,z}^2$, gives the US version of (5):

$$W_{t+1,z}^2 = R_t\left(1 + \left(\frac{S_{t+1}^1\hat{R}_t}{S_t^3 R_t} - 1\right)(\alpha_{t,z} - \varsigma_t) + \left(\frac{R_{t+1}^k}{R_t} - 1\right)(\gamma_{t,z} - \zeta_t)\right) W_{t,z}^4 - R_t\left(C_{t,z} + S_t^3\hat{C}_{t,z}\right).$$

In the case of UK consumers, we have

$$\begin{aligned} S_{t+1}\hat{B}_{t+1}^1 &= \frac{S_{t+1}^1\hat{R}_t}{S_t^3}\left[(\alpha_{t,z} - \varsigma_t) - (\gamma_{t,z} - \hat{\zeta}_{t,z})\right] W_t^4, \\ B_{t+1}^1 &= R_t\left[1 - (\alpha_{t,z} - \varsigma_t)\right] W_{t,z}^4 - R_t\left(C_{t,z} + S_t^3\hat{C}_{t,z}\right), \\ S_t^3\hat{K}_{t+1,z} &= \hat{R}_{t+1}^k(\gamma_{t,z} - \zeta_{t,z})W_{t,z}^4. \end{aligned}$$

Substituting these expressions into the definition of $W_{t+1,z}^2$, gives

$$W_{t+1,z}^2 = R_t\left(1 + \left(\frac{S_{t+1}^1\hat{R}_t}{S_t^3 R_t} - 1\right)(\alpha_{t,z} - \varsigma_t) + \left(\frac{S_{t+1}^1\hat{R}_{t+1}^k}{S_t^3 R_t} - \frac{S_{t+1}^1\hat{R}_t}{S_t^3 R_t}\right)\left(\gamma_{t,z} - \hat{\zeta}_t\right)\right) W_{t,z}^4 - R_t\left(C_{t,z} + S_t^3\hat{C}_{t,z}\right),$$

which is the UK version of (5).

The first order and envelope conditions from the period-2 optimization problem are

$$0 = \mathbb{E}^2_{t,z}\left[\mathcal{D}J^4_z\left(W^4_{t,z}\right)\left(\frac{S^3_t}{S^1_t}-1\right)\right], \tag{A1}$$

$$\mathcal{D}J^2_z(W^2_{t,z}) = \mathbb{E}^2_{t,z}\left[\mathcal{D}J^4_z(W^4_{t,z})H^3_t\right], \tag{A2}$$

where $\mathcal{D}J_z(.)$ denotes the derivative of $J_z(.)$. The first order conditions for $C_{t,z}, \hat{C}_{t,z}$, and $\lambda_{t,z}$ in the period-4 problem take the same form for US and UK consumers:

$$\lambda_{t,z} \quad : \quad 0 = \mathbb{E}^4_{t,z}\left[\mathcal{D}J^2_z(W^2_{t+1,z})\left(\frac{S^1_{t+1}\hat{R}_t}{S^3_t R_t}-1\right)\right], \tag{A3}$$

$$C_{t,z} \quad : \quad U_c(\hat{C}_t, C_t) = R_t\beta\mathbb{E}^4_{t,z}\left[\mathcal{D}J^2_z(W^2_{t+1,z})\right], \tag{A4}$$

$$\hat{C}_{t,z} \quad : \quad U_{\hat{c}}(\hat{C}_t, C_t) = R_t\beta S^3_t\mathbb{E}^4_{t,z}\left[\mathcal{D}J^2_z(W^2_{t+1,z})\right]. \tag{A5}$$

The first order conditions for $\gamma_{t,z}$ differ:

$$\gamma_{t,z<1/2} \quad : \quad 0 = \mathbb{E}^4_{t,z}\left[\mathcal{D}J^2_z(W^2_{t+1,z})\left(\frac{R^k_{t+1}}{R_t}-1\right)\right], \tag{A6}$$

$$\gamma_{t,z\geq 1/2} \quad : \quad 0 = \mathbb{E}^4_{t,z}\left[\mathcal{D}J^2_z(W^2_{t+1,z})R_t\left(\frac{S^1_{t+1}\hat{R}^k_{t+1}}{S^3_t R_t}-\frac{S^1_{t+1}\hat{R}_t}{S^3_t R_t}\right)\right]. \tag{A7}$$

The envelope condition for US and UK consumers is

$$\mathcal{D}J^4_z(W^4_{t,z}) = \beta R_t\mathbb{E}^4_{t,z}\left[\mathcal{D}J^2_z(W^2_{t+1,z})H^1_{t+1,z}\right]. \tag{A8}$$

Equations (6) - (12) are obtained by combining (A1) - (A8) with $V_{t,z} \equiv \mathcal{D}J^4_z(W^4_{t,z})$.

## A.2 Market Clearing Conditions

For any variable $X$, let $X_{t,\mathbb{US}}$ denote $X_{t,z}$ for $z < 1/2$, and $X_{t,\mathbb{UK}} = X_{t,z}$ for $z \geq 1/2$. Market clearing in US bonds in period 1 of day $t+1$ implies that

$$(B^3_{t,\mathbb{US}} + S^3_t T^4_{t,z*} - S^3_t T^4_{t,\mathbb{US}} + C_{t,\mathbb{UK}} - I_{t,\mathbb{US}}) + (B^3_{t,\mathbb{UK}} + S^3_t T^4_{t,z*} - S^3_t T^4_{t,\mathbb{UK}} - C_{t,\mathbb{UK}}) = 0.$$

With bond market clearing in period 3, this condition further simplifies to

$$S^3_t T^4_{t,z*} - S^3_t T^4_{t,\mathbb{US}} + S^3_t T^4_{t,z*} - S^3_t T^4_{t,\mathbb{UK}} - I_{t,\mathbb{US}} = 0.$$

Since market clearing in currency markets implies that $\int T^j_{t,z}dz = \int T^j_{t,z^*}dz^*$, this condition implies that $I_{t,\mathbb{US}} = 0$. Imposing this restriction on the overnight dynamics of US capital gives (15). Similarly, market clearing in the UK bond markets implies that

$$
\begin{aligned}
0 &= (\hat{B}^1_{t,\mathbb{US}} + T^4_{t,\mathbb{US}} - T^4_{t,z*} - \hat{C}_{t,\mathbb{US}}) + (\hat{B}^1_{t,\mathbf{UK}} + T^4_{t,\mathbb{UK}} - T^4_{t,z*} + \hat{C}_{t,\mathbb{US}} - \hat{I}_{t,\mathbb{US}}) \\
&= T^4_{t,\mathbb{US}} - T^4_{t,z*} + T^4_{t,\mathbb{UK}} - T^4_{t,z*} - \hat{I}_{t,\mathbb{US}} \\
&= -\hat{I}_{t,\mathbb{US}}
\end{aligned}
$$

Imposing $\hat{I}_{t,\mathbb{UK}} = 0$ on the overnight dynamics of UK capital gives (16).

## A.3 Log Approximations

To approximate log portfolio returns we make use of a second order approximation similar to one employed by Campbell and Viceira (2002). Both $h^1_{t,z} \equiv \ln H^1_{t,z}$ and $h^3_{t,z} \equiv \ln H^3_{t,z}$ can be expressed as

$$h^j_{t,z} = \ln\left(1 + (e^x - 1)(a - u) + (e^y - 1)(b - w)\right)$$

where $x, y, u$ and $w$ are random variables that are zero in the steady state. Taking a second order Taylor series approximation to $h^j_{t,z}$ around this point gives

$$h^j_{t,z} \cong ax + by + \frac{1}{2}\left(a - a^2\right)x^2 + \frac{1}{2}(b - b^2)y^2 - abxy - xu - yw.$$

The final step is to replace $x^2, y^2$, $xy$, $xu$ and $yw$ by their respective moments:

$$h^j_{t,z} \cong ax + by + \frac{1}{2}\left(a - a^2\right)\mathbb{V}(x) + \frac{1}{2}(b - b^2)\mathbb{V}(y) - ab\mathbb{CV}(x, y) - \mathbb{CV}(x, u) - \mathbb{CV}(y, w)$$

Campbell and Viceira (2002) show that the approximation error associated with this expression disappears in the limit when $x, y, u$ and $w$ represent realizations of continuous time diffusion processes.

Applying this approximation to the definitions of $\ln H^1_{t+1,z}$ and $\ln H^3_{t,z}$ yields equations (20), (21) and (22). In deriving the solution of the model it is useful to write the latter two equations as:

$$h^1_{t+1,z} = \omega'_{t,z}x_{t+1,z} + \frac{1}{2}\omega'_{t,z}\Lambda_z - \frac{1}{2}\omega'_{t,z}\Sigma_z\omega_{t,z} - \phi_{t,z}, \tag{A9}$$

where $\Sigma_z \equiv \mathbb{V}^4_{t,z}\left(x_{t+1,z}\right)$, and $\Lambda_z \equiv \mathrm{diag}(\Sigma_z)$ with

$$
\begin{aligned}
\omega'_{t,z} &\equiv \begin{bmatrix} \alpha_{t,z} & \gamma_{t,z} \end{bmatrix}, \\
x_{t+1,z} &\equiv \begin{bmatrix} s^1_{t+1} - s^3_t + \hat{r}_t - r_t & r^k_{t+1} - r_t \end{bmatrix}, \\
\phi_{t,z} &\equiv \mathbb{CV}^4_{t,\mathbb{US}}\left(s^1_{t+1}, \varsigma_{t,z}\right) + \mathbb{CV}^4_{t,\mathbb{US}}\left(r^k_{t+1}, \zeta_t\right),
\end{aligned}
$$

for $z < 1/2$ (i.e. US consumers), and

$$\begin{aligned} \omega'_{t,z} &\equiv \begin{bmatrix} \alpha_{t,z} - \gamma_{t,z} & \gamma_{t,z} \end{bmatrix}, \\ x_{t+1,z} &\equiv \begin{bmatrix} s^1_{t+1} - s^3_t + \hat{r}_t - r_t & \hat{r}^k_{t+1} + s^1_{t+1} - s^3_t - r_t \end{bmatrix}, \\ \phi_{t,z} &\equiv \mathbb{CV}^4_{t,z}\left(s^1_{t+1}, \varsigma_{t,z}\right) + \mathbb{CV}^4_{t,z}\left(\hat{r}^k_{t+1}, \hat{\zeta}_t\right), \end{aligned}$$

for $z \geq 1/2$.

## A.4 Marginal Utility of Wealth

To derive the relationship between the marginal utility of wealth and the marginal utility of consumption for US consumers, we first combine (A2)- (A4) and (A8):

$$\begin{aligned} 0 &= \mathbb{E}^4_{t,z}\left[V_{t+1,z}H^3_{t+1,z}\left(\frac{S^1_{t+1}\hat{R}_t}{S^3_t R_t} - 1\right)\right] \\ 0 &= \mathbb{E}^4_{t,z}\left[V_{t+1,z}H^3_{t+1,z}\left(\frac{R^k_{t+1}}{R_t} - 1\right)\right] \\ U_c(\hat{C}_{t,z}, C_{t,z}) &= \beta R_t \mathbb{E}^4_{t,z}\left[V_{t+1,z}H^3_{t+1,z}\right] \\ V_{t,z} &= \beta R_t \mathbb{E}^4_{t,z}\left[V_{t+1,z}H^3_{t+1,z}H^1_{t+1,z}\right] \end{aligned}$$

Log linearizing these equations, with $U_c(\hat{C}_{t,z}, C_{t,z}) = \frac{1}{2}C^{-1}_{t,z}$ we find

$$\mathbb{E}^4_{t,z}\left[s^1_{t+1} - s^3_t + \hat{r}_t - r_t\right] = -\mathbb{CV}^4_{t,z}\left(v_{t+1} + h^3_{t+1,z}, s^1_{t+1}\right) - \tfrac{1}{2}\mathbb{V}^4_{t,z}\left(s^1_{t+1}\right), \tag{A10}$$

$$\mathbb{E}^4_{t,z}\left[r^k_{t+1} - r_t\right] = -\mathbb{CV}^4_{t,z}\left(v_{t+1} + h^3_{t+1,z}, r^k_{t+1}\right) - \tfrac{1}{2}\mathbb{V}^4_{t,z}\left(r^k_{t+1}\right), \tag{A11}$$

$$c_t + \ln\beta + r_t = -\mathbb{E}^4_{t,z}\left[v_{t+1,z} + h^3_{t+1,z}\right] - \tfrac{1}{2}\mathbb{V}^4_{t,z}\left(v_{t+1,z} + h^3_{t+1,z}\right), \tag{A12}$$

$$v_{t,z} - \ln\beta - r_t = \mathbb{E}^4_{t,z}\left[v_{t+1,z} + h^3_{t+1,z} + h^1_{t+1}\right] + \tfrac{1}{2}\mathbb{V}^4_{t,z}\left(v_{t+1,z} + h^3_{t+1,z} + h^1_{t+1}\right). \tag{A13}$$

Stacking (A10) and (A11), and combining (A12) and (A13) and substituting for $h^1_{t+1}$ gives

$$\mathbb{E}^4_{t,z}\left[x_{t+1,z}\right] + \frac{1}{2}\Lambda_z = -\mathbb{CV}^4_{t,z}\left(x_{t+1,z}, v_{t+1,z} + h^3_{t+1,z}\right), \tag{A14}$$

$$v_{t,z} + c_t + \phi_{t,z} = \omega'_{t,z}\mathbb{E}^4_{t,z}\left[x_{t+1,z}\right] + \frac{1}{2}\omega'_{t,z}\Lambda_z + \omega'_{t,z}\mathbb{CV}^4_{t,z}\left(x_{t+1,z}, v_{t+1,z} + h^3_{t+1,z}\right). \tag{A15}$$

Combining these expressions we obtain equation (23). In the case of UK consumers, we work with log linearized versions of (A2), (A3), (A5) and (A8):

$$\begin{aligned} \mathbb{E}^4_{t,z}\left[s^1_{t+1} - s^3_t + \hat{r}_t - r_t\right] &= -\mathbb{CV}^4_{t,z}\left(v_{t+1} + h^3_{t+1,z}, s^1_{t+1}\right) - \tfrac{1}{2}\mathbb{V}^4_{t,z}\left(s^1_{t+1}\right), \\ \mathbb{E}^4_{t,z}\left[\hat{r}^k_{t+1} + s^1_{t+1} - s^3_t - r_t\right] &= -\mathbb{CV}^4_{t,z}\left(v_{t+1} + h^3_{t+1,z}, r^k_{t+1}\right) - \tfrac{1}{2}\mathbb{V}^4_{t,z}\left(\hat{r}^k_{t+1} + s^1_{t+1}\right), \\ c_t + \ln\beta + r_t &= -\mathbb{E}^4_{t,z}\left[v_{t+1,z} + h^3_{t+1,z}\right] - \tfrac{1}{2}\mathbb{V}^4_{t,z}\left(v_{t+1,z} + h^3_{t+1,z}\right), \\ v_{t,z} - \ln\beta - r_t &= \mathbb{E}^4_{t,z}\left[v_{t+1,z} + h^3_{t+1,z} + h^1_{t+1}\right] + \tfrac{1}{2}\mathbb{V}^4_{t,z}\left(v_{t+1,z} + h^3_{t+1,z} + h^1_{t+1}\right). \end{aligned}$$

Proceeding as before with our approximation for $h_{t+1,z}$ for $z \geq 1/2$, gives (A14) and (A15). Hence, equation (23) holds for UK consumers.

## A.5 Dynamics of Capital

The dynamics of US capital can be written as

$$\frac{K_{t+1}}{K_t} = R^k_{t+1}\left(1 - \frac{C_{t,\mathbb{US}}W^4_{t,\mathbb{US}}}{W^4_{t,\mathbb{US}}K_t} - \frac{C_{t,\mathbb{UK}}W^4_{t,\mathbb{UK}}}{W^4_{t,\mathbb{UK}}K_t}\right).$$

Log linearizing the this equation gives

$$k_{t+1} - k_t \cong r^k_{t+1} + \ln(1-\mu) - \frac{\mu}{2(1-\mu)}\left(w^4_{t,\mathbb{US}} - k_t + \delta_{t,\mathbb{US}}\right) - \frac{\mu}{2(1-\mu)}\left(w^4_{t,\mathbb{UK}} - k_t + \delta_{t,\mathbb{UK}}\right).$$

Now bond market clearing implies that $K_t + S^3_t\hat{K}_t = W^4_{t,\mathbb{US}} + W^4_{t,\mathbb{UK}}$ so

$$w^4_{t,\mathbb{US}} - k_t = \ln\left(1 + \frac{S^3_t\hat{K}_t}{K_t} - \frac{W^4_{t,\mathbb{UK}}}{K_t}\right) \cong s^3_t + \hat{k}_t - k_t - (w^4_{t,\mathbb{UK}} - k_t).$$

Combining these equations gives (30). The approximate dynamics of UK capital in a similar manner. Bond market clearing implies that

$$\begin{aligned}\frac{\hat{K}_{t+1}}{\hat{K}_t} &= \hat{R}^k_{t+1}\left(1 - \frac{\hat{C}_{t,\mathbb{US}}W^4_{t,\mathbb{US}}}{W^4_{t,\mathbb{US}}\hat{K}_t} - \frac{\hat{C}_{t,\mathbb{UK}}W^4_{t,\mathbb{UK}}}{W^4_{t,\mathbb{UK}}\hat{K}_t}\right) \\ &= \hat{R}^k_{t+1}\left(1 - \frac{C_{t,\mathbb{US}}W^4_{t,us}}{W^4_{t,\mathbb{US}}S^3_t\hat{K}_t} - \frac{C_{t,\mathbb{UK}}W^4_{t,UK}}{W^4_{t,\mathbb{UK}}S^3_t\hat{K}^3_t}\right)\end{aligned}$$

where the second line follows from the fact that the first order conditions for consumption imply that $C_{t,z} = S^3_t\hat{C}_{t,z}$ for all $z$. Log linearizing this equation gives (31).

## A.6 Solving the Model

We focus on solving the most general form of the model, version 3. Recall that in this version capital returns are assumed to follow

$$\begin{aligned} r^k_t &\equiv r + \eta_t + u_t + \Delta e_t \\ \hat{r}^k_t &\equiv r - \eta_t + \hat{u}_t + \Delta\hat{e}_t \end{aligned}$$

with $\eta_t \sim N(0,\sigma^2_\eta), u_t \sim N(0,\sigma^2_u), \hat{u}_t \sim N(0,\sigma^2_u), e_t \sim N(0,\sigma^2_e)$ and $\hat{e}_t \sim N(0,\sigma^2_e)$.

To find the solution, we first establish that the optimal consumption, portfolio allocation and investment choices for US and UK consumers take the following form (given the quote equations in (40) and information structure in (**??**)):

$$\begin{array}{ll} \text{US: } z < \frac{1}{2} & \text{UK: } z \geq \frac{1}{2} \\ \delta_{t,z} = \delta + \delta_1 e_t + \delta_2 u_t & \delta_{t,z} = \hat{\delta} + \hat{\delta}_1 \hat{e}_t + \hat{\delta}_2 \hat{u} \\ \alpha_{t,z} = \alpha + \alpha_1 e_t + \alpha_2 u_t & \alpha_{t,z} = \hat{\alpha} + \hat{a}_1 \hat{e}_t + \hat{a}_2 \hat{u}_t \\ \gamma_{t,z} = \gamma + \gamma_1 e_t + \gamma_2 u_t & \gamma^4_{t,z} = \hat{\gamma} + \hat{\gamma}_1 \hat{e}_t + \hat{\gamma}_2 \hat{u}_t \\ \lambda_{t,z} = \lambda + \lambda_1 (e_t + u_t) & \lambda_{t,z} = \hat{\lambda} + \hat{\lambda}_1 (\hat{e}_t + \hat{u}_t) \end{array}$$

We begin with the period-4 portfolio choices for US consumers. First we substitute for $v_{t+1,z}$ in (A10) and (A11)to obtain

$$\mathbb{E}^4_{t,z}\left[s^1_{t+1} - s^3_t + \hat{r}_t - r_t\right] = \mathbb{CV}^4_{t,z}\left(\delta_{t+1} + \phi_{t+1,z} + w_{t+1,z} - h^3_{t+1,z}, s^1_{t+1}\right) - \tfrac{1}{2}\mathbb{V}^4_{t,z}\left(s^1_{t+1}\right),$$
$$\mathbb{E}^4_{t,z}\left[r^k_{t+1} - r_t\right] = \mathbb{CV}^4_{t,z}\left(\delta_{t+1} + \phi_{t+1,z} + w_{t+1,z} - h^3_{t+1,z}, r^k_{t+1}\right) - \tfrac{1}{2}\mathbb{V}^4_{t,z}\left(r^k_{t+1}\right).$$

Substituting for $w_{t+1,z} - h^3_{t+1,z}$ with the linearized budget constraint give

$$\mathbb{E}^4_{t,z}\left[s^1_{t+1} - s^3_t + \hat{r}_t - r_t\right] = \mathbb{CV}^4_{t,z}\left(\delta_{t+1} + \phi_{t+1,z}, s^1_{t+1}\right) + \frac{1}{1-\mu}\mathbb{CV}^4_{t,z}\left(h_{t+1,z}, s^1_{t+1}\right) - \tfrac{1}{2}\mathbb{V}^4_{t,z}\left(s^1_{t+1}\right),$$
$$\mathbb{E}^4_{t,z}\left[r^k_{t+1} - r_t\right] = \mathbb{CV}^4_{t,z}\left(\delta_{t+1} + \phi_{t+1,z}, r^k_{t+1}\right) + \frac{1}{1-\mu}\mathbb{CV}^4_{t,z}\left(h_{t+1,z}, r^k_{t+1}\right) - \tfrac{1}{2}\mathbb{V}^4_{t,z}\left(r^k_{t+1}\right).$$

Given the process for $r^k_{t+1}$, the quote equation (40a) and the hypothesized form of the optimal consumption wealth ratio, $\mathbb{CV}^4_{t,z}\left(\delta_{t+1}, s^1_{t+1}\right) = 0$, and $\mathbb{CV}^4_{t,z}\left(\delta_{t+1}, r^k_{t+1}\right) = \delta_1\sigma^2_e + \delta_2\sigma^2_u$. We establish below that $\phi_{t+1,z}$ is a constant, so the equations above can be conveniently rewritten as

$$\mathbb{E}^4_{t,\mathbb{US}} x_{t+1,z} + \frac{1}{2}\Lambda = \Psi + \frac{1}{1-\mu}\Sigma\omega_t, \tag{A16}$$

where $\Psi \equiv \mathbb{CV}^4_{t,z}\left(x_{t+1,z}, \delta_{t+1,z}\right) = \left[\begin{array}{cc} 0 & \delta_1\sigma^2_e + \delta_2\sigma^2_u \end{array}\right]'$. Recall that $\Sigma$ is the covariance of $x_{t+1}$ conditioned on information $\Omega^4_{t,\mathbb{US}}$. Under our conjectures for the equilibrium quote processes and information structure,

$$\begin{aligned} \mathbb{E}^4_{t,\mathbb{US}} r^k_{t+1} - r_t &= \mathbb{E}^4_{t,\mathbb{US}}\left[\eta_{t+1} + u_{t+1} + e_{t+1}\right] - \theta + \kappa(e_t + u_t) - e_t \\ &= -\theta + \kappa(e_t + u_t) - e_t, \end{aligned}$$

and

$$\begin{aligned} \mathbb{E}^4_{t,\mathbb{US}}\left[s^1_{t+1} - s^3_t + \hat{r}_t - r_t\right] &= \hat{\theta} - \psi - \theta + \kappa(e_t + u_t) + \mathbb{E}^4_{t,\mathbb{US}}\left[2\eta_{t+1} + \hat{e}_t\right] - \kappa(\hat{e}_t + \hat{u}_t) - e_t \\ &= \hat{\theta} - \psi - \theta + \kappa(e_t + u_t) - e_t, \end{aligned}$$

because $\mathbb{E}^4_{t,\mathbb{US}}\hat{e}_t = \kappa(\hat{e}_t + \hat{u}_t)$. Hence,

$$\mathbb{E}^4_{t,\mathbb{US}} x_{t+1} = \left[\begin{array}{c} \hat{\theta} - \psi - \theta + \kappa(e_t + u_t) - e_t \\ -\theta + \kappa(e_t + u_t) - e_t \end{array}\right] \text{ and } \Sigma = \left[\begin{array}{cc} 4\sigma^2_\eta + (1-\kappa)\sigma^2_e & 2\sigma^2_\eta \\ 2\sigma^2_\eta & \sigma^2_\eta + \sigma^2_u + \sigma^2_e \end{array}\right].$$

The period-4 portfolio choices can therefore be written as

$$\omega_t = (1-\mu)\Sigma^{-1}\left(\mathbb{E}^4_{t,\mathbb{US}}x_{t+1} + \frac{1}{2}\Lambda - \Psi\right).$$

This equation implies that $\alpha_{t,z}$ and $\gamma_{t,z}$ are linear functions of $e_t$ and $u_t$, as assumed above.

In the UK case, the first order conditions for $\alpha_{t,z}$ and $\gamma_{t,z}$ can be written as

$$\mathbb{E}^4_{t,\mathbb{UK}}\hat{x}_{t+1} + \frac{1}{2}\hat{\Lambda} = \hat{\Psi} + \frac{1}{1-\mu}\hat{\Sigma}\hat{\omega}_t \tag{A17}$$

where $\hat{\Psi} \equiv \mathbb{CV}^4_{t,\mathbb{UK}}\left(\hat{x}_{t+1,z}, \delta_{t+1,z}\right) = \begin{bmatrix} 0 & \hat{\delta}_1\sigma_e^2 + \hat{\delta}_2\sigma_u^2 \end{bmatrix}'$. Using the conjectured information structure for UK consumers,

$$\begin{aligned}
\mathbb{E}^4_{t,\mathbb{UK}}\left[\hat{r}^k_{t+1} + s^1_{t+1} - s^3_t - r_t\right] &= \mathbb{E}^4_{t,\mathbb{UK}}\left[\eta_{t+1} + \hat{u}_{t+1} + \hat{e}_{t+1} - e_t\right] - \theta - \psi + \kappa(e_t + u_t) \\
&= -\theta - \psi
\end{aligned}$$

and target

$$\begin{aligned}
\mathbb{E}^4_{t,\mathbb{UK}}\left[s^1_{t+1} - s^3_t + \hat{r}_t - r_t\right] &= \hat{\theta} - \psi - \theta + \kappa(e_t + u_t) + \mathbb{E}^4_{t,\mathbb{UK}}\left[2\eta_{t+1} - e_t\right] + \hat{e}_t - \kappa(\hat{e}_t + \hat{u}_t) \\
&= \hat{\theta} - \psi - \theta + \hat{e}_t - \kappa(\hat{e}_t + \hat{u}_t),
\end{aligned}$$

because $\mathbb{E}^4_{t,\mathbb{UK}}e_t = \kappa(e_t + u_t)$. Hence,

$$\mathbb{E}^4_{t,\mathbb{UK}}x_{t+1} = \begin{bmatrix} \hat{\theta} - \psi - \theta + \hat{e}_t - \kappa(\hat{e}_t + \hat{u}_t) \\ -\theta - \psi \end{bmatrix} \text{ and } \hat{\Sigma} = \begin{bmatrix} 4\sigma_\eta^2 + (1-\kappa)\,\sigma_e^2 & 2\sigma_\eta^2 + (1-\kappa)\sigma_e^2 \\ 2\sigma_\eta^2 + (1-\kappa)\sigma_e^2 & \sigma_\eta^2 + \sigma_u^2 + \sigma_e^2 + (1-\kappa)\,\sigma_e^2 \end{bmatrix}.$$

The period-4 portfolio choices can therefore be written as

$$\hat{\omega}_t = (1-\mu)\hat{\Sigma}^{-1}\left(\mathbb{E}^4_{t,\mathbb{UK}}x_{t+1} + \frac{1}{2}\hat{\Lambda} - \hat{\Psi}\right).$$

This equation implies that $\alpha_{t,z}$ and $\gamma_{t,z}$ are linear functions of $\hat{e}_t$ and $\hat{u}_t$, as assumed above.

Next, we consider the portfolio choice problem in period 2. This is easy solve from the first order condition for $\lambda_{t,z}$ (with $\phi_{t,z} = \phi_z$) :

$$\mathbb{E}^2_{t,z}s^3_t - s^1_t + \tfrac{1}{2}\mathbb{V}^2_{t,z}\left(s^3_t\right) = \mathbb{CV}^2_{t,z}\left(\delta_{t,z} + w^4_{t,z}, s^3_t\right).$$

Given the assumed process for spot rates the conjectured information structure,

$$\mathbb{E}^2_{t,z}s^3_t - s^1_t = \begin{cases} \psi + e_t + u_t & \text{US:}z < 1/2 \\ \psi - \hat{e}_t - \hat{u}_t & \text{UK:}z \geq 1/2 \end{cases},$$

so solving for $\lambda_{t,z}$ gives

$$\lambda_{t,z}=\begin{cases}\frac{1}{2}+\frac{\left(\psi-\delta_1\sigma_e^2-\delta_2\sigma_u^2\right)}{\left(\sigma_e^2+\sigma_u^2\right)}+\frac{1}{\left(\sigma_e^2+\sigma_u^2\right)}\left(e_t+u_t\right) & \text{US:}z<1/2\\ \frac{1}{2}+\frac{\left(\psi-\delta_1\sigma_e^2-\delta_2\sigma_u^2\right)}{\left(\sigma_e^2+\sigma_u^2\right)}-\frac{1}{\left(\sigma_e^2+\sigma_u^2\right)}\left(\hat{e}_t+\hat{u}_t\right) & \text{UK:}z\geq 1/2\end{cases}.$$

This equation is in the same form as we assumed above.

Finally, we consider the form of the optimal consumption decision. First we write the first order condition for US consumption (with $\phi_{t,z}=\phi_z$) as

$$0=\ln\beta+r_t+\mathbb{E}_{t,z}^4\left[h_{t+1,\mathbb{US}}^3-\Delta w_{t+1,\mathbb{US}}^4-\phi_z-\Delta\delta_{t+1,\mathbb{US}}\right]+\tfrac{1}{2}\mathbb{V}_{t,z}^4\left(h_{t+1,\mathbb{US}}^3-w_{t+1,\mathbb{US}}^4-\delta_{t+1,\mathbb{US}}\right).$$

Using the linearized budget constraint and the assumed form for $\delta_{t,z}$, this equation simplifies to

$$\mathbb{E}_{t,\mathbb{US}}^4h_{t+1,\mathbb{US}}^1-\tfrac{1}{2}\left(\frac{1}{1-\mu}\right)\mathbb{V}_{t,\mathbb{US}}^4\left(h_{t+1,\mathbb{US}}^1\right)=\tfrac{1}{2}(1-\mu)\left(\delta_1^2\sigma_e^2+\delta_2^2\sigma_u^2\right)+\mu\delta+(1-\mu)\phi_z+\delta_1e_t+\delta_2u_t$$

Under an optimally chosen portfolio, the LHS of this equation is

$$\begin{aligned}&\omega_t'\left(\mathbb{E}_{t,\mathbb{US}}^4x_{t+1}+\frac{1}{2}\Lambda-\frac{1}{2}\Sigma\omega_t\right)-\phi-\tfrac{1}{2}\frac{1}{1-\mu}\omega_t'\Sigma\omega_t\\ =\;&\frac{1}{2}\mu\left(1-\mu\right)\left(\mathbb{E}_{t,\mathbb{US}}^4x_{t+1}+\frac{1}{2}\Lambda-\Psi\right)'\Sigma^{-1}\left(\mathbb{E}_{t,\mathbb{US}}^4x_{t+1}+\frac{1}{2}\Lambda+\frac{(2-\mu)}{\mu}\Psi\right)-\phi_z\end{aligned}$$

Taking a second order approximation to this expression around $x\equiv\mathbb{E}x_{t+1}$ gives

$$\begin{aligned}&\frac{1}{2}\mu\left(1-\mu\right)\left\{\left(x+\frac{1}{2}\Lambda-\Psi\right)'\Sigma^{-1}\left(x+\frac{1}{2}\Lambda+\frac{(2-\mu)}{\mu}\Psi\right)+\operatorname{tr}\left(\Sigma^{-1}\Gamma\right)\right\}-\phi_z\\ &+\mu\left(1-\mu\right)\left(x+\frac{1}{2}\Lambda-\tfrac{2(1-\mu)}{\mu}\Psi\right)'\Sigma^{-1}\left(\mathbb{E}_{t,\mathbb{US}}^4x_{t+1}-x\right)\end{aligned}$$

where $\Gamma\equiv\mathbb{V}\left(\mathbb{E}_{t,\mathbb{US}}^4x_{t+1}\right)$. Combining this with the FOC above and equating coefficients gives

$$\begin{aligned}\delta\;&=\;\frac{1}{2}\mu\left(1-\mu\right)\left\{\left(x+\frac{1}{2}\Lambda-\Psi\right)'\Sigma^{-1}\left(x+\frac{1}{2}\Lambda+\frac{(2-\mu)}{\mu}\Psi\right)+\operatorname{tr}\left(\Sigma^{-1}\Gamma\right)\right\}\\ &\quad-(2-\mu)\phi-\tfrac{1}{2}(1-\mu)\left(\delta_1^2\sigma_e^2+\delta_2^2\sigma_u^2\right)-(1-\mu)\phi_z,\\ \delta_1\;&=\;\mu\left(1-\mu\right)\begin{bmatrix}\hat{\theta}-\theta-\psi+\frac{4\sigma_\eta^2+(1-\kappa)\sigma_e^2}{2}\\ \frac{\sigma_\eta^2+\sigma_u^2+\sigma_e^2}{2}-\theta-\frac{2(1-\mu)\left(\delta_1\sigma_e^2+\delta_2\sigma_u^2\right)}{\mu}\end{bmatrix}'\Sigma^{-1}\begin{bmatrix}\kappa-1\\ \kappa-1\end{bmatrix},\\ \delta_2\;&=\;\mu\left(1-\mu\right)\begin{bmatrix}\hat{\theta}-\theta-\psi+\frac{4\sigma_\eta^2+(1-\kappa)\sigma_e^2}{2}\\ \frac{\sigma_\eta^2+\sigma_u^2+\sigma_e^2}{2}-\theta-\frac{2(1-\mu)\left(\delta_1\sigma_e^2+\delta_2\sigma_u^2\right)}{\mu}\end{bmatrix}'\Sigma^{-1}\begin{bmatrix}\kappa\\ \kappa\end{bmatrix}.\end{aligned}$$

In the UK case, the first order condition for consumption is

$$\mathbb{E}^4_{t,\mathbb{UK}} h^1_{t+1,\mathbb{UK}} - \tfrac{1}{2}\left(\frac{1}{1-\mu}\right)\mathbb{V}^4_{t,\mathbb{UK}}\left(h^1_{t+1,\mathbb{UK}}\right) = \tfrac{1}{2}(1-\mu)\left(\hat{\delta}_1^2\sigma_e^2 + \hat{\delta}_2^2\sigma_u^2\right) + \mu\hat{\delta} + (1-\mu)\hat{\phi}_z + \hat{\delta}_1\hat{e}_t + \hat{\delta}_2\hat{u}_t.$$

Proceeding as above, we obtain

$$\begin{aligned}
\hat{\delta} &= \frac{1}{2}\mu(1-\mu)\left\{\left(\hat{x} + \frac{1}{2}\hat{\Lambda} - \hat{\Psi}\right)'\hat{\Sigma}^{-1}\left(\hat{x} + \frac{1}{2}\hat{\Lambda} + \frac{(2-\mu)}{\mu}\hat{\Psi}\right) + \operatorname{tr}\left(\hat{\Sigma}^{-1}\hat{\Gamma}\right)\right\} \\
&\quad -(2-\mu)\hat{\phi} - \tfrac{1}{2}(1-\mu)\left(\hat{\delta}_1^2\sigma_e^2 + \hat{\delta}_2^2\sigma_u^2\right) - (1-\mu)\hat{\phi} \\
\hat{\delta}_1 &= \mu(1-\mu)\left(\hat{x} + \frac{1}{2}\hat{\Lambda} - \tfrac{2(1-\mu)}{\mu}\hat{\Psi}\right)'\hat{\Sigma}^{-1}\begin{bmatrix} 1-\kappa \\ 0 \end{bmatrix} \\
\hat{\delta}_2 &= \mu(1-\mu)\left(\hat{x} + \frac{1}{2}\hat{\Lambda} - \tfrac{2(1-\mu)}{\mu}\hat{\Psi}\right)'\hat{\Sigma}^{-1}\begin{bmatrix} -\kappa \\ 0 \end{bmatrix}
\end{aligned}$$

We have now verified the forms of the consumption, portfolio and investment decisions.

Next, we turn to the quote equations. We established above that the approximate equilibrium dynamics of US and UK capital are given by

$$\begin{aligned}
k_{t+1} - k_t &\cong r^k_{t+1} + \ln(1-\mu) - \frac{\mu}{2(1-\mu)}\left(s^3_t + \hat{k}_t - k_t + \delta_{t,\mathbb{US}} + \delta_{t,\mathbb{UK}}\right) \\
\hat{k}_{t+1} - \hat{k}_t &\cong \hat{r}^k_{t+1} + \ln(1-\mu) - \frac{\mu}{2(1-\mu)}\left(k_t - s^3_t - \hat{k}_t + \delta_{t,\mathbb{US}} + \delta_{t,\mathbb{UK}}\right).
\end{aligned}$$

Combining these equations we have

$$k_t - s^3_t - \hat{k}_t = (1-\mu)\left(\Delta s^3_{t+1} + \hat{r}^k_{t+1} - r^k_{t+1}\right) + (1-\mu)\left(k_{t+1} - s^3_{t+1} - \hat{k}_{t+1}\right),$$

or, after iterating forward (with $\lim_{i\to\infty}(1-\mu)^i(k_{t+i} - s^3_{t+i} - \hat{k}_{t+i}) = 0$)

$$s^3_t = k_t - \hat{k}_t - \sum_{i=1}^{\infty}(1-\mu)^i\left(\Delta s^3_{t+i} + \hat{r}^k_{t+i} - r^k_{t+i}\right).$$

This equation must hold ex ante and ex post as a consequence of market clearing. Under our assumption that period-3 spot rates are set consistent with market clearing, i.e.

$$s^3_t = \mathbb{E}^3_t\left[k_t - \hat{k}_t\right] + \mathbb{E}^3_t\sum_{i=1}^{\infty}(1-\mu)^i\left(r^k_{t+i} - \Delta s^3_{t+i} - \hat{r}^k_{t+i}\right).$$

Given the conjecture information structure, the values of $k_t$ and $\hat{k}_t$ are in the period-3 common information set, so the solution of the above equation is

$$s^3_t = k_t - \hat{k}_t.$$

(It is straightforward to check that $\mathbb{E}_t^3\left(r_{t+i}^k - \Delta k_{t+i} + \Delta\hat{k}_{t+i} - \hat{r}_{t+i}^k\right) = 0$ for $i > 0$.) From this it follows that

$$s_{t+1}^3 - s_t^3 = r_{t+1}^k - \hat{r}_{t+1}^k = 2\eta_{t+1} + u_{t+1} - \hat{u}_{t+1} + \Delta e_{t+1} - \Delta\hat{e}_{t+1}.$$

Period period-1 quotes are given by

$$\begin{aligned} s_t^1 &= \mathbb{E}_t^3 s_{t+1}^3 - \psi \\ &= s_t^3 + 2\eta_{t+1} + \hat{e}_t - e_t - \psi \end{aligned}$$

where $\psi$ is an intraday risk premium determined below. Hence

$$s_t^3 = s_t^1 + e_t + u_t - \hat{e}_t - \hat{u}_t + \psi$$

as conjectured in our solution.

All that now remains is to determine the three risk premia, $\psi$, $\theta$ and $\hat{\theta}$. Market market clearing in the foreign bond, and goods markets implies that

$$\begin{aligned} (\alpha_{t,\mathbb{US}} - \varsigma_t)\, W_{t,\mathbb{US}}^4 + \left[(\alpha_{t,\mathbb{UK}} - \varsigma_t) - (\gamma_t - \zeta_t)\right] W_{t,\mathbb{UK}}^4 &= 0, \\ 1 - \frac{C_{t,\mathbb{US}}}{W_{t,\mathbb{US}}^4} - \frac{C_{t,\mathbb{UK}} W_{t,\mathbb{UK}}^4}{W_{t,\mathbb{UK}}^4 W_{t,\mathbb{US}}^4} &= \gamma_{t,\mathbb{US}} - \zeta_t, \\ 1 - \frac{C_{t,\mathbb{US}} W_{t,\mathbb{US}}^4}{W_{t,\mathbb{US}}^4 S_t^3 \hat{K}_t} - \frac{C_{t,\mathbb{UK}} W_{t,\mathbb{UK}}^4}{W_{t,\mathbb{UK}}^4 S_t^3 \hat{K}_t^3} &= \gamma_{t,\mathbb{UK}} - \hat{\zeta}_t, \end{aligned}$$

In the steady state, these conditions imply that

$$\begin{aligned} \alpha + (\hat{\alpha} - \hat{\gamma}) &= 0, \\ \gamma &= 1 - \mu, \\ \hat{\gamma} &= 1 - \mu. \end{aligned}$$

Taking unconditional expectations on both sides of (A16) and (A17) gives

$$\begin{aligned} x + \frac{1}{2}\Lambda &= \Psi + \frac{1}{1-\mu}\Sigma\omega \\ \hat{x} + \frac{1}{2}\hat{\Lambda} &= \hat{\Psi} + \frac{1}{1-\mu}\hat{\Sigma}\hat{\omega} \end{aligned}$$

where $\omega' = \left[\begin{array}{cc} \alpha & \gamma \end{array}\right]$ and $\hat{\omega}' = \left[\begin{array}{cc} \hat{\alpha}-\hat{\gamma} & \hat{\gamma} \end{array}\right]$. Solving these equations $\alpha$, $\theta, \hat{\theta}$ and $\psi$, gives

$$
\begin{aligned}
\psi &= \hat{\theta}-\theta = \left(\delta_1-\hat{\delta}_1\right)\sigma_e^2+\left(\delta_2-\hat{\delta}_2\right)\sigma_u^2 \\
\alpha &= \frac{(1-\mu)(1-\kappa)\sigma_e^2}{2\left(4\sigma_\eta^2+(1-\kappa)\sigma_e^2\right)}, \\
\theta &= -\delta_1\sigma_e^2-\delta_2\sigma_u^2-\frac{(1-\kappa)\sigma_e^2\sigma_\eta^2}{4\sigma_\eta^2+(1-\kappa)\sigma_e^2}-\frac{1}{2}\left(\sigma_\eta^2+\sigma_u^2+\sigma_e^2\right), \\
\hat{\theta} &= -\hat{\delta}_1\sigma_e^2-\hat{\delta}_2\sigma_u^2-\frac{(1-\kappa)\sigma_e^2\sigma_\eta^2}{4\sigma_\eta^2+(1-\kappa)\sigma_e^2}-\frac{1}{2}\left(\sigma_\eta^2+\sigma_u^2+\sigma_e^2\right).
\end{aligned}
$$

Now from the equations for $\delta_1$ and $\delta_2$ , we have

$$
\begin{aligned}
\delta_1\sigma_e^2+\delta_2\sigma_u^2 &= \Xi\left[\begin{array}{c} \kappa-1 \\ \kappa-1 \end{array}\right]\sigma_e^2+\Xi\left[\begin{array}{c} \kappa \\ \kappa \end{array}\right]\sigma_u^2 \\
&= \Xi\left[\begin{array}{c} \kappa \\ \kappa \end{array}\right]\left(\sigma_e^2+\sigma_u^2\right)-\Xi\left[\begin{array}{c} \sigma_e^2 \\ \sigma_e^2 \end{array}\right] \\
&= 0
\end{aligned}
$$

where $\Xi \equiv \mu(1-\mu)\left(x+\frac{1}{2}\Lambda-\frac{2(1-\mu)}{\mu}\Psi\right)'\Sigma^{-1}$, and similarly,

$$
\begin{aligned}
\hat{\delta}_1\sigma_e^2+\hat{\delta}_2\sigma_u^2 &= \hat{\Xi}\left[\begin{array}{c} 1-\kappa \\ 0 \end{array}\right]\sigma_e^2+\hat{\Xi}\left[\begin{array}{c} -\kappa \\ 0 \end{array}\right]\sigma_u^2 \\
&= \hat{\Xi}\left[\begin{array}{c} -\kappa \\ 0 \end{array}\right]\left(\sigma_e^2+\sigma_u^2\right)+\hat{\Xi}\left[\begin{array}{c} \sigma_e^2 \\ 0 \end{array}\right] \\
&= 0
\end{aligned}
$$

where $\hat{\Xi} \equiv \mu(1-\mu)\left(\hat{x}+\frac{1}{2}\hat{\Lambda}-\frac{2(1-\mu)}{\mu}\hat{\Psi}\right)'\hat{\Sigma}^{-1}$. Hence,

$$
\begin{aligned}
\psi &= \hat{\theta}-\theta = 0 \\
\alpha &= \frac{(1-\mu)(1-\kappa)\sigma_e^2}{2\left(4\sigma_\eta^2+(1-\kappa)\sigma_e^2\right)} \geq 0, \\
\theta &= \hat{\theta} = -\frac{(1-\kappa)\sigma_e^2\sigma_\eta^2}{4\sigma_\eta^2+(1-\kappa)\sigma_e^2}-\frac{1}{2}\left(\sigma_\eta^2+\sigma_u^2+\sigma_e^2\right),
\end{aligned}
$$

and

$$
\begin{aligned}
\delta_1 &= \mu(1-\mu)\left[\begin{array}{c} \frac{1}{2}\sigma_s^2 \\ \sigma_k^2+\frac{(1-\kappa)\sigma_e^2\sigma_\eta^2}{\sigma_s^2} \end{array}\right]'\left[\begin{array}{cc} \sigma_s^2 & \sigma_{sk} \\ \sigma_{sk} & \sigma_k^2 \end{array}\right]^{-1}\left[\begin{array}{c} \kappa-1 \\ \kappa-1 \end{array}\right] \\
\hat{\delta}_1 &= \mu(1-\mu)\left[\begin{array}{c} \frac{1}{2}\sigma_s^2 \\ \sigma_k^2+\frac{(1-\kappa)\sigma_e^2\sigma_\eta^2}{\sigma_s^2}+\frac{(1-\kappa)\sigma_e^2}{2} \end{array}\right]'\left[\begin{array}{cc} \sigma_s^2 & \sigma_{sk}+(1-\kappa)\sigma_e^2 \\ \sigma_{sk}+(1-\kappa)\sigma_e^2 & \sigma_k^2+(1-\kappa)\sigma_e^2 \end{array}\right]^{-1}\left[\begin{array}{c} 1-\kappa \\ 0 \end{array}\right]
\end{aligned}
$$

All that now remains is to show that the conjectured information structure can be supported by an inference problem based on exogenous information available to each consumer, and their observations of quotes and trading activity. For this purpose, we begin by considering the information revealed by period-2 trading. Market clearing in the UK bond market requires that

$$\begin{aligned} S_t^1 \hat{K}_t &= \int (\lambda_{t,z} - \xi_t) W_{t,z}^2 dz \\ &\Rightarrow \quad \xi_t = \int \lambda_{t,z} \frac{W_{t,z}^2}{W_t^2} dz - \frac{S_t^1 \hat{K}_t}{W_t^2} \end{aligned} \tag{A18}$$

where $W_t^2 = \int W_{t,z}^2 dz$ is world wealth, equal to $S_t^1 \hat{K}_t + K_t$ (by bond market clearing). We can write this expression in two ways:

$$\begin{aligned} \xi_t &= \lambda_{t,\mathbb{UK}} \left( 1 - \frac{W_{t,\mathbb{US}}^2}{W_t^2} \right) + \lambda_{t,\mathbb{US}} \frac{W_{t,\mathbb{US}}^2}{W_t^2} - \frac{S_t^1 \hat{K}_t}{W_t^2}, \\ \xi_t &= \lambda_{t,\mathbb{US}} \left( 1 - \frac{W_{t,\mathbb{UK}}^2}{W_t^2} \right) + \lambda_{t,\mathbb{UK}} \frac{W_{t,\mathbb{UK}}^2}{W_t^2} - \frac{S_t^1 \hat{K}_t}{W_t^2} \end{aligned}$$

Since period-3 spot rate quotes are given by $S_t^3 = K_t / \hat{K}_t$, we can use the identity $S_t^1 \hat{K}_t = \left( S_t^1 / S_t^3 \right) S_t^3 \hat{K}_t$ to write $S_t^1 \hat{K}_t = \exp\left( s_t^1 - s_t^3 \right) K_t$. Substituting this condition in the equations above gives

$$\begin{aligned} \xi_t &= \lambda_{t,\mathbb{UK}} \left( 1 - \frac{W_{t,\mathbb{US}}^2}{\left( \exp\left( s_t^1 - s_t^3 \right) + 1 \right) K_t} \right) \\ &\quad + \lambda_{t,\mathbb{US}} \frac{W_{t,\mathbb{US}}^2}{\left( \exp\left( s_t^1 - s_t^3 \right) + 1 \right) K_t} - \frac{1}{\left( \exp\left( s_t^3 - s_t^1 \right) + 1 \right)} \end{aligned} \tag{A19}$$

$$\begin{aligned} \xi_t &= \lambda_{t,\mathbb{US}} \left( 1 - \frac{W_{t,\mathbb{UK}}^2}{\left( \exp\left( s_t^3 - s_t^1 \right) + 1 \right) S_t^1 \hat{K}_t} \right) \\ &\quad + \lambda_{t,\mathbb{UK}} \frac{W_{t,\mathbb{UK}}^2}{\left( \exp\left( s_t^3 - s_t^1 \right) + 1 \right) S_t^1 \hat{K}_t} - \frac{1}{\left( \exp\left( s_t^3 - s_t^1 \right) + 1 \right)} \end{aligned} \tag{A20}$$

Substituting our solutions for $\lambda_{t,\mathbb{US}}, \lambda_{t,\mathbb{UK}}$ and $s_t^3 - s_t^1$ into (??) shows that unexpected period-2 order flow can be written as a function of $\hat{e}_t + \hat{u}_t$ and elements of US consumers information, $\Omega_{t,\mathbb{US}}^2$. Similarly, equation (??) shows that unexpected order flow can also be written as a function of $e_t + u_t$ and elements of $\Omega_{t,\mathbb{UK}}^2$. Hence, based on their observations of period-2 order flow, the information available to US and UK consumers at the start of period 3 is respectively $\Omega_{t,\mathbb{US}}^3 = \left\{ \hat{e}_t + \hat{u}_t, \Omega_{t,\mathbb{US}}^2 \right\}$ and $\Omega_{t,\mathbb{UK}}^3 = \left\{ e_t + u_t, \Omega_{t,\mathbb{UK}}^2 \right\}$ as we conjectured when specifying the information structure.

Finally, we consider the information conveyed by period-4 trading. Unlike period 2, information come from two sources: from the currency market in the form of unexpected order flow, and from the goods market in the form of unexpected foreign demand for domestically produced goods. We consider the goods market first. US consumers receive information in the form of unexpected UK demand for UK goods, $C_{t,\mathbb{UK}} - E_{t,,\mathbb{US}}^4 C_{t,\mathbb{UK}}$, while UK consumers receive information from unexpected US demand for UK goods, $\hat{C}_{t,\mathbb{US}} - E_{t,,\mathbb{UK}}^4 \hat{C}_{t,\mathbb{US}}$. Using the identity $C_{t,z} \equiv \frac{\mu}{2} \exp\left( \delta_{t,z} \right) W_{t,z}$, and the equilibrium condition $S_t^3 \hat{C}_{t,z} = C_{t,z}$, we can write

$$
\begin{aligned}
C_{t,\mathbb{UK}} - E^4_{t,\mathbb{US}} C_{t,\mathbb{UK}} &= \frac{\mu}{2}\left(2K_t - W^4_{t,\mathbb{US}}\right)\left\{\exp\left(\delta_{t,\mathbb{UK}}\right) - E^4_{t,\mathbb{US}} \exp\left(\delta_{t,\mathbb{UK}}\right)\right\} \\
\hat{C}_{t,\mathbb{US}} - E^4_{t,,\mathbb{UK}} \hat{C}_{t,\mathbb{US}} &= \frac{\mu}{2S^3_t}\left(2S^3_t \hat{K}_t - W^4_{t,\mathbb{UK}}\right)\left\{\exp\left(\delta_{t,\mathbb{US}}\right) - E^4_{t,\mathbb{UK}} \exp\left(\delta_{t,\mathbb{US}}\right)\right\}
\end{aligned}
$$

Now recall that $\delta_{t,\mathbb{UK}} = \hat{\delta} + \hat{\delta}_1 \hat{e}_t + \hat{\delta}_2 \hat{u}_t$ and $\delta_{t,\mathbb{US}} = \delta + \delta_1 e_t + \delta_2 u_t$. Making these substitutions in the expressions above allows us to write $C_{t,\mathbb{UK}} - E^4_{t,\mathbb{US}} C_{t,\mathbb{UK}}$ as a function of $\hat{\delta}_1 \hat{e}_t + \hat{\delta}_2 \hat{u}_t$ and the elements of $\Omega^4_{t,\mathbb{US}}$, and $\hat{C}_{t,\mathbb{US}} - E^4_{t,,\mathbb{UK}} \hat{C}_{t,\mathbb{US}}$ as a function of $\delta_1 e_t + \delta_2 u_t$ and the elements of $\Omega^4_{t,\mathbb{UK}}$. Inverting these functions, gives

$$
\hat{\delta}_1 \hat{e}_t + \hat{\delta}_2 \hat{u}_t = \ln\left(\frac{2\left(C_{t,\mathbb{UK}} - E^4_{t,\mathbb{US}} C_{t,\mathbb{UK}}\right)}{\mu\left(2K_t - W^4_{t,\mathbb{US}}\right)} + E^4_{t,\mathbb{US}} \exp\left(\delta_{t,\mathbb{UK}}\right)\right) - \hat{\delta} \tag{A21}
$$

$$
\delta_1 e_t + \delta_2 u_t = \ln\left(\frac{2S^3_t\left(\hat{C}_{t,\mathbb{US}} - E^4_{t,,\mathbb{UK}} \hat{C}_{t,\mathbb{US}}\right)}{\mu\left(2S^3_t \hat{K}_t - W^4_{t,\mathbb{UK}}\right)} + E^4_{t,\mathbb{UK}} \exp\left(\delta_{t,\mathbb{US}}\right)\right) - \delta \tag{A22}
$$

Since $\hat{e}_t + \hat{u}_t \in \Omega^4_{t,\mathbb{US}}$ and $e_t + u_t \in \Omega^4_{t,\mathbb{UK}}$ (from period-2 trading), US consumers can infer the values of $\hat{e}_t$ and $\hat{u}_t$ from (A21) and $\Omega^4_{t,\mathbb{US}}$, while UK can infer the values of $e_t$ and $u_t$ from (A22) and $\Omega^4_{t,\mathbb{UK}}$. Hence, $\{\hat{e}_t, \hat{u}_t\} \in \Omega^1_{t+1,\mathbb{US}}$ and $\{e_t, u_t\} \in \Omega^1_{t+1,\mathbb{UK}}$ consistent with our conjectured information structure. We can also use the equations above to compute components of $\phi_{t,z}$, the wedge between the log marginal utility of consumption and wealth. Using the definitions $\zeta_t \equiv (C_{t,\mathbb{UK}} - E^4_{t,\mathbb{US}} C_{t,\mathbb{UK}})/W^4_{t,\mathbb{US}}$ and $\hat{\zeta}_t \equiv \left(\hat{C}_{t,\mathbb{US}} - E^4_{t,,\mathbb{UK}} \hat{C}_{t,\mathbb{US}}\right)/W^4_{t,\mathbb{UK}}$ we have

$$
\begin{aligned}
\mathbb{CV}_{t,US}\left(r^k_{t+1}, \zeta_t\right) &= \frac{\mu}{2}\left(\frac{2K_t}{W^4_{t,\mathbb{US}}} - 1\right)\exp\left(\hat{\delta}\right)\mathbb{CV}_{t,US}\left(r^k_{t+1}, \hat{\delta}_1 \hat{e}_t + \hat{\delta}_2 \hat{u}_t\right), \\
\mathbb{CV}_{t,US}\left(\hat{r}^k_{t+1}, \hat{\zeta}_t\right) &= \frac{\mu}{2S^3_t}\left(\frac{2S^3_t \hat{K}_t}{W^4_{t,\mathbb{UK}}} - 1\right)\exp\left(\delta\right)\mathbb{CV}_{t,US}\left(\hat{r}^k_{t+1}, \delta_1 e_t + \delta_2 u_t\right).
\end{aligned}
$$

Given the capital returns processes, the covariance terms on the right hand side of each equation are equal to zero, so $\mathbb{CV}_{t,US}\left(r^k_{t+1}, \zeta_t\right) = \mathbb{CV}_{t,US}\left(\hat{r}^k_{t+1}, \hat{\zeta}_t\right) = 0$.

To identify the information conveyed by period-4 currency trading, we start with the market clearing condition for UK bonds held overnight: $\int \hat{B}^1_{t+1,z} dz = 0$. Combining this condition with the definitions of $\alpha_{t,z}$, and $\gamma_{t,z}$ gives

$$
\varsigma_t = \int \alpha_{t,z} \frac{W^4_{t,z}}{W^4_t} dz - \frac{S^3_t \hat{K}_{t+1}}{\hat{R}^k_{t+1} W^4_t}
$$

where $W^4_t = \int W^4_{t,z} dz$ is world wealth which is equal to $S^3_t \hat{K}_t + K_t$. Using (31) to substitute for $\hat{K}_{t+1}$, we can rewrite this expression in two ways

$$
\varsigma_t = \alpha_{t,\mathbb{UK}} + \left(\alpha_{t,\mathbb{US}} - \alpha_{t,\mathbb{UK}}\right)\frac{W^4_{t,\mathbb{US}}}{2K_t} - \frac{1-\mu}{2}\exp\left(\frac{-\mu}{2(1-\mu)}\left(\delta_{t,\mathbb{US}} + \delta_{t,\mathbb{UK}}\right)\right), \tag{A23}
$$

$$
\varsigma_t = \alpha_{t,\mathbb{US}} + \left(\alpha_{t,\mathbb{UK}} - \alpha_{t,\mathbb{US}}\right)\frac{W^4_{t,\mathbb{UK}}}{2S^3_t \hat{K}_t} - \frac{1-\mu}{2}\exp\left(\frac{-\mu}{2(1-\mu)}\left(\delta_{t,\mathbb{US}} + \delta_{t,\mathbb{UK}}\right)\right). \tag{A24}
$$

Substituting our solutions for $\delta_{t,z}$ and $\alpha_{t,z}$ into (A23) shows that $\varsigma_t$ is a function of $\hat{e}_t, \hat{u}_t$ and elements of US consumers period-4 information, $\Omega^4_{t,\mathbb{US}}$. Similarly, (A24) implies that $\varsigma_t$ can also be written as a function of $e_t, u_t$ and the elements of $\Omega^4_{t,\mathbb{UK}}$. These observations further support the conjectured information structure.

We can also use (A19), (A20), (A23) and (A24) to study the covariance between order flow and spot rates. In particular, taking a log approximation to $\xi_t$ and $\varsigma_t$ around the steady state gives

$$\begin{aligned}
\xi_t &\cong \frac{1}{2}\hat{\lambda}_1 \left(\hat{e}_t + \hat{u}_t\right) + \frac{1}{2}\lambda_1 \left(e_t + u_t\right) - \frac{1}{4}\left(s^3_t - s^1_t\right), \\
\varsigma_t &\cong \frac{1}{2}\left(\hat{\alpha}_1 \hat{e}_t + \hat{\alpha}_2 \hat{u}_t\right) + \frac{1}{2}\left(\alpha_1 e_t + \alpha_2 u_t\right) + \frac{1}{2}\left(\alpha - \hat{\alpha}\right)\left(w^4_{t,us} - k_t\right) \\
&\quad + \frac{\mu}{4}\left(\delta_1 e_t + \delta_2 u_t + \hat{\delta}_1 \hat{e}_t + \hat{\delta}_2 \hat{u}_t\right)
\end{aligned}$$

Using these approximations, the equilibrium process for spot rates, and the conjectured information structure, we obtain:

$$\begin{aligned}
CV^4_{t,\mathbb{US}}\left(s^1_{t+1}, \varsigma_t\right) &\cong \left(\frac{\mu}{4}\hat{\delta}_1 + \frac{1}{2}\hat{\alpha}_1\right)(1-\kappa)\sigma^2_e, \\
CV^4_{t,\mathbb{UK}}\left(s^1_{t+1}, \varsigma_t\right) &\cong \left(\frac{\mu}{4}\delta_1 + \frac{1}{2}\alpha_1\right)(1-\kappa)\sigma^2_e, \\
CV^2_{t,\mathbb{US}}\left(s^3_{t,}\xi_t\right) &\cong \frac{1}{2} - \frac{1}{4}\left(\sigma^2_e + \sigma^2_u\right), \\
CV^2_{t,\mathbb{UK}}\left(s^3_{t,}\xi_t\right) &\cong \frac{1}{2} - \frac{1}{4}\left(\sigma^2_e + \sigma^2_u\right).
\end{aligned}$$

Since we have established that $\mathbb{CV}_{t,US}\left(r^k_{t+1}, \zeta_t\right) = \mathbb{CV}_{t,US}\left(\hat{r}^k_{t+1}, \hat{\zeta}_t\right) = 0$, these approximations imply that $\phi_{t,z} = \phi_z$, as conjectured above.