

Optimal Unemployment Insurance with Sequential Search

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1 Introduction

Unemployment insurance is an asset that protects workers against the risk of failing to find a job. Its provision is limited by an important moral hazard problem: workers who are protected against unemployment risk remain unemployed for longer. A growing theoretical literature examines how optimal unemployment insurance deals with the tradeoff between insurance and moral hazard (Shavell and Weiss 1979, Atkeson and Lucas 1995, Hopenhayn and Nicolini 1997, Werning 2002). For the most part, this literature has ignored another important limitation of unemployment insurance: it does not protect workers against uncertainty regarding the type of job that they find. This paper explores the optimal provision of unemployment insurance when post-unemployment wages are uncertain and cannot be observed by the insurance provider.

We extend the McCall (1970) intertemporal job search model, a basic workhorse in decision theory, to allow for a risk-averse worker with constant absolute risk aversion (CARA) preferences. In each period, the worker receives a single job offer drawn from a known wage distribution, which she decides to accept or reject. If she accepts the job, she earns this wage forever. If she rejects it, she remains unemployed and draws another wage in the following period.

We introduce a benevolent social planner who provides insurance against unemployment risk into this environment. The planner's goal is to maximize the worker's utility while providing actuarially fair insurance, i.e. his budget is balanced in expected present value terms. We consider several different information structures, each of which offers the planner

increasingly little control over the worker’s behavior. First, we assume that he can observe whether she is employed or unemployed and he can observe her borrowing and savings; however, he cannot observe her wage offer while unemployed or actual wage while employed. At an initial date, the planner commits to a sequence of possibly stochastic transfer payments that depend on the worker’s observed employment status and reported wage draw in each subsequent period. This is analogous to Hopenhayn and Nicolini’s (1997) results in a setup with an unobservable search effort decision. The optimal unemployment insurance contract is characterized by a transfer to the worker while she is unemployed that decreases with the duration of her unemployment spell and a tax on employed workers that increases with the duration of the unemployment spell, with both the tax and the transfer independent of the wage reports.¹

We next introduce a hidden financial market. The worker may secretly borrow and lend at the same risk-free rate as the planner, subject only to the constraint that she cannot run a Ponzi scheme. The planner continues to observe the worker’s employment status but cannot observe her wage. In environments with an unobservable search effort decision, the hidden financial market reduces the effectiveness of unemployment insurance (Werning 2002). If the planner could control the worker’s savings, he would distort the timing of consumption so as to force the worker to violate her Euler equation.² This distortion is no longer feasible when borrowing and lending are unobservable, so hidden financial markets impose an additional constraint on the planner’s behavior.

To our surprise, this result does not carry over to the McCall (1970) search model. Instead, it is possible to implement the same allocation even when financial market activity is hidden. Moreover, the implementation is particularly simple: the planner commits to give a fixed unemployment benefit b to the worker in every period that she is unemployed and pays for it using a lump-sum tax τ . The tax and benefit are again independent of the history of wage reports. This last result is similar to Werning (2002).

Finally, we remove the planner from the problem entirely and instead introduce a competitive sector that provides unemployment insurance. The worker now has access to two

¹Shavell and Weiss (1979) examine an extension of the McCall (1970) search model with risk-averse workers. They introduce a search effort decision that affects the wage distribution and prove that unemployment benefits decline with unemployment duration in an economy with observable savings. Although we do not endogenize search intensity and we restrict attention to CARA utility, we extend Shavell and Weiss’s (1979) results by providing closed-form solutions, by analyzing an economy with hidden savings, and by providing a more thorough characterization of optimal unemployment insurance.

²This is Rogerson’s (1985) ‘inverse Euler equation’ result. Golosov, Kocherlakota and Tsyvinski (forthcoming) show that the planner can implement the distorted solution by a capital income tax.

financial instruments. The first is the risk-free bond, identical to the previous problem. The second is a menu of one-period actuarially fair, exclusive unemployment insurance contracts. In every period that the worker is unemployed, she chooses an unemployment benefit b and with an associated lump-sum premium T . If she accepts a job during the period, she must pay the premium T , while if she rejects her wage offer, she receives net income $b - T$. Competition ensures that unemployment insurance is actuarially fair: if in equilibrium a worker who takes an unemployment benefit of b rejects a wage offer a fraction $p(b)$ of the time, the cost of this benefit level is $T = bp(b)$. If the worker remains unemployed in the following period, she is free to choose a different unemployment benefit. Once again, this competitive provision of unemployment insurance decentralizes the solution to the original planner's problem.

After providing this characterization of optimal unemployment insurance under several different information structures, we examine whether unemployment insurance is what a layman might refer to as 'insurance', an asset with a low expected return that transfers consumption from high income to low income states. The conventional wisdom, as the name 'unemployment insurance' suggests, is yes. On the one hand, more unemployment insurance raises the unemployment rate and therefore reduces output; hence unemployment insurance offers a low expected return. On the other hand, unemployment insurance transfers income from the employed to the unemployed, allowing workers to smooth consumption across employment outcomes (Baily 1977).

Acemoglu and Shimer (1999) argue that the first part of the conventional wisdom is wrong. A moderate amount of unemployment insurance can raise output, and so unemployment insurance may have a high expected return. Although they work in a different model, their logic can be understood in terms of the McCall (1970) search model that we study. A benchmark economy consists of a risk-neutral worker without any unemployment insurance. The worker sets her reservation wage so as to maximize her expected income, which implies that the equilibrium is 'productively efficient'. In comparison, a risk-averse worker reduces her exposure to labor market uncertainty by accepting wages that are, from a productive efficiency standpoint, too low. A moderate amount of unemployment insurance allows her to raise her reservation wage. Traditionally, this is viewed as a moral hazard problem, but in this case the moral hazard offsets the adverse effects of risk-aversion and incomplete markets, raising output back to the level obtained in the benchmark economy.

This suggests that a moderate amount of unemployment insurance has a high expected return and pays off when the marginal utility of consumption is high. Acemoglu and Shimer

(1999) therefore conjecture that the optimal amount of unemployment insurance is larger than the output-maximizing amount because “at the point of maximal output, . . . a further increase in unemployment insurance leads to a second-order loss of net output” and a first-order gain in risk-sharing since it “increases the income of unemployed workers and decreases the (after-tax) income of employed workers” (pp. 907–908).

This paper shows that the other part of the conventional wisdom, and hence Acemoglu and Shimer’s (1999) conjecture, is also incorrect.³ Unemployment insurance fails to transfer income from states in which the marginal utility of consumption is low to states in which it is high. Once stated, the reason is obvious: unemployment insurance does not insure workers against ‘employment risk’, uncertainty about the type of job they take. In particular, unemployment insurance induces workers to raise their reservation wage, which may raise their expected wage. But since marginal utility decreases with consumption, very high wages may provide only moderately more utility, and hence the higher reservation wage may reduce expected utility.

We show that a risk-averse worker demands some insurance, but there is no simple relationship between her demand for insurance and the amount of insurance that maximizes output in the economy. Put differently, the reservation wage in the presence of optimal unemployment insurance is not a monotonic function of the coefficient of absolute risk-aversion. Recall that a risk-neutral worker chooses the productively efficient reservation wage. It is easy to construct examples in which an increase in risk-aversion uniformly lowers the reservation wage, examples in which the reservation wage increases with risk-aversion, and examples in which it is not monotonic. Despite this, we prove that if the wage distribution satisfies a standard restriction,⁴ optimal unemployment insurance is larger than productively efficient unemployment insurance when workers are sufficiently risk-averse. On the margin, the usual ‘equity-efficiency’ tradeoff is operative.

The crucial assumption underlying our analysis is that employment risk is not fully insurable. To understand why this is so important, suppose that the exact opposite were true, so that all wage uncertainty can be insured. Then the first best allocation is attainable by taxing all the earnings of employed workers and rebating in a constant lump sum manner

³We establish this in McCall’s (1970) intertemporal job search model, but the same result is true in Acemoglu and Shimer (1999). A counterexample to their conjecture in their model is available upon request.

⁴We require that $E(w|w \geq \bar{w})$, the expected wage conditional on the wage exceeding the reservation wage, is increasing in \bar{w} but with a slope less than one. This is weaker than log-concavity of the cumulative survivor function $1 - F(w)$ or log-concavity of the density function f . It is a common assumption in the search literature; see van den Berg (1994).

to the unemployed and employed. Workers would be indifferent about being unemployed or employed at any wage, so that any reservation wage is incentive compatible. The planner can simply recommend the reservation wage that is productively efficient, that which maximizes expected discounted output.

Although one can easily think of several reasons why such employment insurance may be impractical and attempt to incorporate these into a model,⁵ here we justify the absence of insurance in the simplest way possible, by assuming that the wages offered to the unemployed and those accepted by the employed are unobservable to the planner. This assumption allows us to treat the problem as arising endogenously from an asymmetry of information at a minimum cost, without the need to introduce several other choice variables. However, our analysis and results are likely to be relevant and shed light on the situations where the lack of complete employment insurance is motivated in other ways.

One way of reinterpreting this assumption is by assuming that the ‘wage’ variability is actually a variability in the disutility of working at a particular job that enters the worker’s utility function quasi-linearly with consumption, as a monetary cost. It is easy to imagine this idiosyncratic disutility from a particular worker-job match as being privately observed by the worker, and in particular, not observed by the planner. Under this interpretation, all jobs produce the same ‘output’ and the problem faced by unemployed workers is finding a ‘good job’ in the sense of a low disutility of work instead of a high wage. We do not propose to take this extreme assumption literally but it may be another reason why employment insurance is limited.

Our assumption that the worker has CARA preferences is important for many of our results. The critical property of CARA is that the willingness of an individual to accept a gamble is independent of her wealth level. This has a number of implications: (i) the worker chooses a constant reservation wage in response to the constant unemployment benefit and lump-sum tax, regardless of the evolution of her asset holdings; (ii) the optimal reservation wage is constant, regardless of the evolution of the utility promised to the worker; (iii) it is impossible to ask an employed worker her wage and then treat her differently according to her report; and (iv) controlling the worker’s savings does not help the planner to enforce a desired reservation wage. The first two properties are useful because they allow us to derive

⁵For example, even if the government observes total output, it may not be able to disentangle productivity, hours, and effort as in Mirrlees’s (1971) classical analysis. Another possibility is that unemployed workers may have to exert effort to find higher paying jobs, i.e. the distribution they sample from could be made to depend on an effort choice, as in ?.

closed form solutions throughout the paper. Without the closed-form solutions, an analytical comparison of optimal and productively efficient unemployment insurance would likely be impossible. The third property implies that there cannot be any ‘employment insurance’, i.e. a differential tax on workers employed at different wage levels, which simplifies the exposition of our results by reducing the class of mechanisms that the planner might contemplate using.⁶ The final property qualitatively affects our results. If workers do not have CARA preferences, we can prove that a planner who can observe the worker’s borrowing and lending will force her to violate her Euler equation. In other words, the equivalence between economies with and without financial freedom breaks down.

With these caveats, we think that CARA provides a useful starting point for an analysis of the McCall (1970) model with risk aversion. Such an analysis may be important because the characterization of optimal insurance differs significantly from the characterization in a model with an unobservable search or work effort decision (Hopenhayn and Nicolini 1997, Werning 2002, Golosov et al. forthcoming). In those models, a planner who can control workers’ savings forces any risk-averse worker to violate her Euler equation.

Section 2 describes the worker’s preferences and the income process. Section 3 sets up a very general revelation mechanism and shows that the planner does not gain from using lotteries, from asking workers to voluntarily report their wage, or from imposing time-varying taxes on a worker once she is already employed. Section 4 uses these simplifications to provide a complete characterization of the optimal transfer scheme. In Section 5 we show that if the worker has access to hidden borrowing and saving, the social optimum is easily implemented using a constant unemployment benefit and a lump-sum tax. Section 6 further decentralizes the optimum by introducing a competitive insurance sector that offers one period exclusive unemployment insurance contracts. Section 7 discusses the relationship between productively efficient and optimal unemployment insurance. We conclude in Section 8 with a brief discussion of some important avenues for future research.

⁶We conjecture that employment insurance is also infeasible with decreasing absolute risk aversion (DARA), but might be possible with (the implausible assumption of) increasing absolute risk aversion (IARA). See the discussion at the end of Section 3.

2 Environment: Preferences and Technology

There is a single risk averse worker who maximizes the expected present value of his utility from consumption,

$$U(c_0, c_1, \dots) = \mathbb{E}_{-1} \left(\sum_{t=0}^{\infty} \beta^t u(c_t) \right),$$

where $\beta < 1$ represents the discount factor and $u(c) \equiv -\frac{1}{\sigma} e^{-\sigma c}$ is the period utility function, with constant absolute risk aversion (CARA) and coefficient of absolute risk aversion σ . We allow for negative values of c . We comment throughout the paper on the extent to which our results carry over to other time-separable preferences. The consumption good is the numeraire.

The worker faces an uncertain income stream. Initially she is unemployed with income normalized to zero. As long as she remains unemployed, she draws one nonnegative wage offer w independently from a known cumulative continuous distribution function F . We assume that the support $[w_{\min}, w_{\max}]$ is compact and that the density f is bounded above and bounded away from zero.

The worker may reject any job offer and remain unemployed to continue sampling from F or she may accept the job offer w , in which case she produces before-tax income w in the current and in every future period. Consumption occurs at the end of the period, following the wage draw.

3 General Mechanisms

This section uses the revelation principle to set up the most general mechanism that a planner might contemplate given the assumed asymmetry of information. We allow the agent to make reports on the privately observed wage, we allow the planner to use lotteries, and we allow taxes to vary during an employment spell. We show that none of these capabilities are useful to the planner. Instead, the planner offers a transfer to the unemployed that depends on the duration of unemployment and sets a tax on the employed that depends on the duration of the unemployment spell but not on how long she has been employed. This induces the workers to behave optimally. The reader may skip this section and still follow the remaining analysis; its conclusions are spelled out in the problem of minimizing (8) subject to (9) and (10) at the start of the next section.

3.1 The Recursive Mechanism

For notational simplicity, we start by expressing the general mechanism in a recursive manner. This can be justified along the lines of Spear and Srivastava (1987). Consider a worker who starts the period unemployed and has been promised expected lifetime utility v . A general mechanism can be described as follows:

1. The worker receives a wage offer w from the wage distribution $F(\bar{w})$ and then makes a report \tilde{w} to the planner.
2. After receiving the report, the planner observes a (possibly infinite-dimensional) random vector z that is independent of the true wage w . This will be used to implement a lottery and may be arbitrarily rich. We denote by \mathbb{E}_z the expectation of a random variable with respect to z .
3. The planner requests that a worker who reports a wage $\tilde{w} < \bar{w}$ rejects the job and that a worker who reports a wage $\tilde{w} \geq \bar{w}$ accepts the job. Since the utility of accepting a job is increasing in w and the maximum utility attainable by rejecting a job is independent of the job rejected, the planner can only implement a reservation rule for the job acceptance decision.
4. If the worker rejects the job, the worker gets unemployment benefit $b(\tilde{w}, z)$ and continuation utility $v'(\tilde{w}, z)$.
5. If the worker accepts the job, she pays a tax $\tau(w, n, z)$, where n denotes the n^{th} period after accepting the job.

This mechanism design problem can be expressed as a constrained optimization problem in which the worker attempts to maximize her utility subject to the planner earning a particular value of discounted profits in expected value terms and subject to the two incentive compatibility constraints. The worker must find it optimal to reveal her wage truthfully, and she must find it optimal to accept a job whenever $w \geq \bar{w}$ and to reject it otherwise. It is easier to instead work with the dual problem in which the planner attempts to minimize the cost $C(v)$ of providing the worker with a given level of lifetime utility v . We later close the model with the zero profit condition $C(v) = 0$.

3.2 The Planner's Problem

The full planner's problem may be expressed recursively as follows:

$$C(v) = \min_{\{b\}, \{v'\}, \{\tau\}, \bar{w}} \int_{w_{\min}}^{\bar{w}} \mathbb{E}_z (b(w, z) + \beta C(v'(w, z))) dF(w) \\ + \int_{\bar{w}}^{w_{\max}} \mathbb{E}_z \left(\sum_{n=0}^{\infty} \beta^n \tau(w, n, z) \right) dF(w)$$

subject to the promise keeping constraint

$$v = \int_{w_{\min}}^{\bar{w}} \mathbb{E}_z (u(b(w, z)) + \beta v'(w, z)) dF(w) + \int_{\bar{w}}^{w_{\max}} \mathbb{E}_z \left(\sum_{n=0}^{\infty} \beta^n u(w - \tau(w, n, z)) \right) dF(w)$$

and a set of truth telling constraints for all w, \tilde{w} :

$$\mathbb{E}_z \sum_{n=0}^{\infty} \beta^n u(w - \tau(w, n, z)) \geq \mathbb{E}_z \sum_{n=0}^{\infty} \beta^n u(w - \tau(\tilde{w}, n, z)), \quad w, \tilde{w} \geq \bar{w} \quad (1)$$

$$\mathbb{E}_z \sum_{n=0}^{\infty} \beta^n u(w - \tau(w, n, z)) \geq \mathbb{E}_z (u(b(\tilde{w}, z)) + \beta v'(\tilde{w}, z)), \quad w \geq \bar{w} > \tilde{w} \quad (2)$$

$$\mathbb{E}_z (u(b(w, z)) + \beta v'(w, z)) \geq \mathbb{E}_z \sum_{n=0}^{\infty} \beta^n u(w - \tau(\tilde{w}, n, z)), \quad \tilde{w} \geq \bar{w} > w \quad (3)$$

$$\mathbb{E}_z (u(b(w, z)) + \beta v'(w, z)) \geq \mathbb{E}_z (u(b(\tilde{w}, z)) + \beta v'(\tilde{w}, z)), \quad \bar{w} > w, \tilde{w} \quad (4)$$

We proceed to simplify the planner's problem in steps.

Lemma 1 *At the optimum, $b(w, z)$ and $v'(w, z)$ are independent of w and z . The incentive constraints (2), (3), and (4) can be replaced with the single condition:*

$$u(b) + \beta v' = \mathbb{E}_z \sum_{n=0}^{\infty} \beta^n u(\bar{w} - \tau(\bar{w}, n, z)), \quad (5)$$

with a slight abuse of notation.

Proof. Towards a contradiction suppose $b(w, z)$ and $v'(w, z)$ are optimal but not constants. Then consider the alternative mechanism, independent of w and z :

$$\begin{aligned}\tilde{b} &= u^{-1} \left(\frac{1}{1 - F(\bar{w})} \int_{w_{\min}}^{\bar{w}} \mathbb{E}_z u(b(w, z)) dF(w) \right) \\ \tilde{v}' &= \frac{1}{1 - F(\bar{w})} \int_{w_{\min}}^{\bar{w}} \mathbb{E}_z v'(w, z) dF(w),\end{aligned}$$

with \bar{w} and $\tau(w, n, z)$ left unchanged. This change is feasible, i.e. it satisfies all the constraints. Lotteries ensure that the cost function C is convex, so this decreases the objective function, contradicting optimality.

For constant b and v' , the constraint (4) is trivially satisfied. Since the right hand side of constraint (3) is increasing in w , it may be rewritten as

$$u(b) + \beta v' \geq \mathbb{E}_z \sum_{n=0}^{\infty} \beta^n u(\bar{w} - \tau(\tilde{w}, n, z))$$

for $\tilde{w} \geq \bar{w}$. Constraint (1) implies $\tilde{w} = \bar{w}$ maximizes the right hand side of this equation, so it reduces to

$$u(b) + \beta v' \geq \mathbb{E}_z \sum_{n=0}^{\infty} \beta^n u(\bar{w} - \tau(\bar{w}, n, z)). \quad (6)$$

Next note that (2) implies that for all $w \geq \bar{w}$.

$$\mathbb{E}_z \sum_{n=0}^{\infty} \beta^n u(w - \tau(w, n, z)) \geq u(b) + \beta v'.$$

If $w > \bar{w}$, then

$$\mathbb{E}_z \sum_{n=0}^{\infty} \beta^n u(w - \tau(w, n, z)) \geq \mathbb{E}_z \sum_{n=0}^{\infty} \beta^n u(w - \tau(\bar{w}, n, z)) > \mathbb{E}_z \sum_{n=0}^{\infty} \beta^n u(\bar{w} - \tau(\bar{w}, n, z)),$$

where the first inequality uses (1) and the second uses monotonicity of the utility function. Therefore the preceding inequality is tightest when $w = \bar{w}$, giving

$$\mathbb{E}_z \sum_{n=0}^{\infty} \beta^n u(\bar{w} - \tau(\bar{w}, n, z)) \geq u(b) + \beta v'. \quad (7)$$

Inequalities (6) and (7) hold if and only if equation (5) holds, completing the proof. ■

Lemma 1 allows us to rewrite the Planner's problem as

$$\begin{aligned}
C(v) &= \min_{b, v', \{\tau\}, \bar{w}} \left\{ (b + \beta C(v')) F(\bar{w}) - \int_{\bar{w}}^{w_{\max}} \mathbb{E}_z \left(\sum_{n=0}^{\infty} \beta^n \tau(w, n, z) \right) dF(w) \right\} \\
&\text{subject to } v = F(\bar{w}) (u(b) + \beta v') + \int_{\bar{w}}^{w_{\max}} \mathbb{E}_z \left(\sum_{n=0}^{\infty} \beta^n u(w - \tau(w, n, z)) \right) dF(w) \\
&\mathbb{E}_z \sum_{n=0}^{\infty} \beta^n u(w - \tau(w, n, z)) \geq \mathbb{E}_z \sum_{n=0}^{\infty} \beta^n u(w - \tau(\tilde{w}, n, z)) \text{ for } w, \tilde{w} \geq \bar{w} \\
&\mathbb{E}_z \sum_{n=0}^{\infty} \beta^n u(\bar{w} - \tau(\bar{w}, n, z)) = u(b) + \beta v'
\end{aligned}$$

3.3 Constant Absolute Risk Aversion

So far we have not made any assumptions about the period utility function u except concavity. This section examines the implications of having constant absolute risk aversion preferences.

Lemma 2 *With CARA utility, the tax on the employed $\tau(w, n, z)$ is independent of w , n , and z at the optimum.*

Proof. With CARA utility, the planner's problem may be rewritten as

$$\begin{aligned}
C(v) &= \min_{b, v', \{\tau\}, \{v^e\}, \bar{w}} \left\{ (b + \beta C(v')) F(\bar{w}) - \int_{\bar{w}}^{w_{\max}} \mathbb{E}_z \left(\sum_{n=0}^{\infty} \beta^n \tau(w, n, z) \right) dF(w) \right\} \\
&\text{subject to } v = F(\bar{w}) (u(b) + \beta v') - \int_{\bar{w}}^{w_{\max}} \exp(-\sigma w) v^e(w) dF(w) \\
&v^e(w) \leq v^e(\tilde{w}) \text{ for } w, \tilde{w} \geq \bar{w} \\
&-\exp(-\sigma \bar{w}) v^e(\bar{w}) = u(b) + \beta v' \\
&v^e(w) = \mathbb{E}_z \sum_{n=0}^{\infty} \beta^n \exp(\sigma \tau(w, n, z)).
\end{aligned}$$

In particular, the incentive constraint for an employed worker who takes a job but considers reporting an incorrect wage, condition (1), simplifies considerably. The requirement that $v^e(w) \leq v^e(\tilde{w})$ for all $w, \tilde{w} \geq \bar{w}$ implies $v^e(w)$ is independent of w . Now fix $v^e(w)$ and

consider the minimum cost way of achieving that value:

$$\begin{aligned} \min_{\{\tau\}} \quad & -\mathbb{E}_z \sum_{n=0}^{\infty} \beta^n \tau(w, n, z) \\ \text{subject to } \quad & v^e(w) = \mathbb{E}_z \sum_{n=0}^{\infty} \beta^n \exp(\sigma \tau(w, n, z)). \end{aligned}$$

The objective function is linear in τ while the constraint is convex. Therefore the solution involves a constant tax, $\tau(w, n, z) = \tau$, with a slight abuse of notation. ■

Lemma 2 proves that private information prevents ‘employment insurance’, so the tax rate τ is independent of the wage. With CARA preferences and jobs that last forever, the wage effectively acts as a permanent multiplicative taste shock. This ensures that all employed workers have the same preferences over transfer schemes, which makes it impossible to separate workers according to their actual wages.

With non-CARA utility, workers rank different tax schemes differently depending on their wealth because their base consumption w affects their local preference for risk or intertemporal variation in the tax scheme. In some cases, it may be possible to exploit these differences in rankings to separate workers according to their wage; see Prescott and Townsend (1984) for an example. If workers have decreasing absolute risk aversion (DARA), those earning lower wages are more reluctant to take gambles or to accept intertemporal variability in wages. One can therefore induce these workers to reveal their wage by giving them a choice between an uncertain (or time-varying) employment tax with a low expected cost and a deterministic tax with a high expected cost. High wage workers would opt for the stochastic, low-cost employment tax. This does not, however, reduce the planner’s cost of providing an unemployed worker with a given level of utility, since it transfers income from low wage to high wage workers and is therefore undesirable. We conclude that with CARA or DARA preferences, lotteries or extraneous intertemporal variability in taxes are not optimal. The same logic suggests, however, that with increasing absolute risk aversion (IARA), lotteries may be part of an optimal mechanism.

4 Optimal Transfer Scheme

Lemma 2 implies that with CARA preferences, the planner's problem can be further simplified to read

$$C(v) = \min_{b, v', \tau, \bar{w}} \left((b + \beta C(v')) F(\bar{w}) - \frac{\tau}{1 - \beta} (1 - F(\bar{w})) \right) \quad (8)$$

$$\text{subject to } v = F(\bar{w}) \left(\frac{-\exp(-\sigma b)}{\sigma} + \beta v' \right) + \int_{\bar{w}}^{w_{\max}} \frac{-\exp(-\sigma(w - \tau))}{\sigma(1 - \beta)} dF(w) \quad (9)$$

$$\text{and } \frac{-\exp(-\sigma b)}{\sigma} + \beta v' = \frac{-\exp(-\sigma(\bar{w} - \tau))}{\sigma(1 - \beta)} \quad (10)$$

The planner takes the worker's promised utility v as given and chooses a payment to the worker b and a continuation utility v' if she remains unemployed, a permanent tax on the worker τ if she gets a job, and a reservation wage. Her objective (8) is to minimize the expected cost of providing the worker with this level of utility. Equation (9) represents the promise-keeping constraint and equation (10) is the incentive compatibility constraint, that the worker is willing to use the desired reservation wage \bar{w} .

This section characterizes the solution to this constrained minimization problem. We guess and verify that $C(v)$ takes the form

$$C(v) = \frac{-\log(-\sigma v(1 - \beta))}{\sigma(1 - \beta)} + K^* \quad (11)$$

for some constant K^* . Substitute this guess into the constrained minimization problem and then use the incentive constraint to substitute out the tax on employed workers τ :

$$C(v) = \min_{b, v', \bar{w}} \left(\left(b - \beta \frac{\log(-\sigma v'(1 - \beta))}{\sigma(1 - \beta)} + \beta K^* \right) F(\bar{w}) - \frac{\log(1 - \beta) + \log(\exp(-\sigma b) - \beta \sigma v') + \sigma \bar{w}}{\sigma(1 - \beta)} (1 - F(\bar{w})) \right)$$

$$\text{subject to } v = \left(\frac{-\exp(-\sigma b)}{\sigma} + \beta v' \right) \left(F(\bar{w}) + \int_{\bar{w}}^{w_{\max}} \frac{\exp(-\sigma(w - \bar{w}))}{\sigma(1 - \beta)} dF(w) \right)$$

Now take the first order necessary conditions for the choice of the payment to the unemployed b and their continuation value v' . Combining these gives

$$v' = -\frac{\exp(-\sigma b)}{\sigma(1 - \beta)}. \quad (12)$$

In other words, the continuation value of the unemployed is equal to what they would get if they earned a constant unemployment benefit forever. Use this to eliminate the continuation value v' from the optimization problem:

$$C(v) = \min_{b, \bar{w}} \left(\frac{b}{1-\beta} + \beta F(\bar{w}) K^* - \frac{\bar{w}}{1-\beta} (1 - F(\bar{w})) \right)$$

$$\text{subject to } v = \frac{-\exp(-\sigma b)}{\sigma(1-\beta)} \left(F(\bar{w}) + \int_{\bar{w}}^{w_{\max}} \frac{\exp(-\sigma(w-\bar{w}))}{\sigma(1-\beta)} dF(w) \right) \quad (13)$$

Finally, use the promise-keeping constraint to eliminate b from the objective function. This verifies the functional form assumption for $C(v)$ and implies $K^* \equiv \min_{\bar{w}} \hat{K}(\bar{w})$, where

$$\hat{K}(\bar{w}) \equiv -\frac{CE(\bar{w}, \sigma) + (1 - F(\bar{w})) \bar{w}}{(1-\beta)(1-\beta F(\bar{w}))} \quad (14)$$

and $CE(\bar{w}, \sigma)$ is the certainty equivalent for a worker with constant absolute risk aversion σ of a lottery offering the maximum of 0 and $w - \bar{w}$, where w is distributed according to the function F :

$$CE(\bar{w}, \sigma) \equiv -\frac{1}{\sigma} \log \left(F(\bar{w}) + \int_{\bar{w}}^{w_{\max}} \exp(-\sigma(w-\bar{w})) dF(w) \right). \quad (15)$$

This analysis provides the basis for a complete characterization of the optimal policy. The social planner chooses a constant reservation wage $\bar{w}^* \in \arg \max_{\bar{w}} \hat{K}(\bar{w})$, chooses the unemployment benefit b to satisfy the promise-keeping constraint (13), sets continuation utility v' to satisfy equation (9), and chooses the tax on the employed τ to satisfy the incentive constraint (10).

Proposition 1 *With CARA utility and no financial markets, the optimal policy can be characterized as follows:*

1. *The reservation wage \bar{w}^* is constant and equal to $\arg \max_{\bar{w}} \hat{K}(\bar{w})$ in equation (14).*
2. *If the expected cost of the unemployment insurance program is zero at date 0, $C(v_0) = 0$, then the unemployment benefit at date 0 is*

$$b_0 = \frac{\beta F(\bar{w}^*) CE(\bar{w}^*, \sigma) + (1 - F(\bar{w}^*)) \bar{w}^*}{1 - \beta F(\bar{w}^*)} > 0.$$

3. *The unemployment benefit falls linearly with the duration of unemployment. After t*

periods of unemployment, a worker receives benefit b_t defined recursively by $b_t = b_{t-1} - CE(\bar{w}^*, \sigma)$, where $CE(\bar{w}^*, \sigma) > 0$ is the certainty equivalent defined in equation (15).

4. The tax on the employed rises linearly with the duration of an unemployment spell. A worker who finds a job after t periods pays tax $\tau_t = \bar{w}^* - b_t$ for the remainder of her life, so that if a worker finds a job at wage w in period t , her consumption in that period is $w - \bar{w}^*$ higher than it would be if she remained unemployed.
5. The worker's consumption Euler equation holds in each period, with a gross interest rate equal to the inverse of the discount factor. In particular, while the worker is unemployed,

$$\exp(-\sigma b_{t-1}) = \exp(-\sigma b_t) F(\bar{w}^*) + \int_{\bar{w}^*}^{w_{\max}} \exp(-\sigma(w - \tau_t)) dF(w).$$

Proof. We have already proven the first part of the Proposition. To prove part 2, use equation (13) to solve for the unemployment benefit as a function of promised utility,

$$b = -\frac{1}{\sigma} \log(-\sigma v(1 - \beta)) - CE(\bar{w}, \sigma).$$

When $C(v) = 0$, equation (11) implies the first term is equal to $-(1 - \beta)K^*$. Simplifying using the definition of K^* delivers the desired result. Note that $\hat{K}(w_{\max}) = 0 > \hat{K}(\bar{w})$ for all $\bar{w} \in [0, w_{\max}]$, so $\bar{w}^* < w_{\max}$. This implies the numerator and denominator in the expression for b_0 are both positive.

To prove the third part, combine equations (12) and (13) to get a dynamic equation for the continuation value:

$$v = v' \exp(-\sigma CE(\bar{w}^*, \sigma))$$

Again use equation (12) to express this as a dynamic equation for the unemployment benefit. Part 4 is obtained by eliminating v' from equation (10) using equation (12). Part 5 follows from parts 3 and 4 with a bit of algebra. The consumption Euler equation trivially holds for an employed worker, since she gets a constant wage and pays a constant tax. ■

The last part of this Proposition is striking. The worker has no access to capital markets, and so she has no ability to smooth her consumption over time. Nevertheless, it is in the social planner's best interest to allow her to do so. This suggests that it might be possible to implement the social optimum in an economy in which workers have access to the same savings technology as the social planner and can perform the consumption smoothing

function themselves. We turn to that possibility now.

5 Implementation with Financial Freedom

We now assume that the worker can borrow and save freely at the same risk-free rate as the planner, $R = \beta^{-1}$ and demonstrate that the optimal allocation found above may be implemented through a simple actuarially fair tax-and-transfer scheme. A worker receives a constant unemployment benefit b^* in every period that she is unemployed and pays a constant tax τ^* in every period, whether she is employed or unemployed.

In addition, the worker has access to a risk-free bond. Let a denote the worker's beginning of period holdings of the bond. Since consumption may be negative, there is no natural borrowing constraint in this economy. Instead, we impose a no-Ponzi-game condition, that the value of a worker's assets must grow more slowly than the interest rate.

We start by considering how a worker behaves when faced with an arbitrary benefit and tax scheme (b, τ) . We then show that a particular scheme implements the social optimum and is actuarially fair, so it achieves a balanced budget in expected value terms.

5.1 Worker Behavior with Financial Freedom

Consider first the decision problem of an employed worker who receives a constant wage w and pays a constant tax τ . Her budget constraint implies

$$a' = \beta^{-1}(a + w - \tau - c).$$

Because her discount factor is the inverse of the interest rate, it is well-known that the worker chooses to keep her marginal utility of consumption, hence her level of consumption, constant. Given the no-Ponzi-game condition, this is only possible if she keeps assets constant, $a = a'$. It follows that she chooses to consume

$$c^e(a, w) = (1 - \beta)a + w - \tau$$

in every period, so her lifetime utility is

$$V^e(a, w) = \frac{-\exp(-\sigma((1 - \beta)a + w - \tau))}{\sigma(1 - \beta)}. \quad (16)$$

Turn next to an unemployed worker. She must choose both how much to consume and whether to accept a wage offer. If she takes a job, her utility is given by $V^e(a, w)$ in equation (16). This allows us to express her problem recursively as

$$V^u(a) = \int \max \left\langle \max_c \left(\frac{-\exp(-\sigma c)}{\sigma} + \beta V^u(\beta^{-1}(a + b - \tau - c)) \right), V^e(a, w) \right\rangle$$

We again use a guess-and-verify approach, conjecturing a value function of the form

$$V^u(a) = \frac{-\exp(-\sigma(1-\beta)(a-K))}{\sigma(1-\beta)}$$

for some constant K which remains to be determined. Given this conjecture, examine first the worker's consumption decision should she remain unemployed. The unique solution to the necessary and sufficient first order condition is

$$c^u(a) = (1-\beta)(a+b-\tau-\beta K). \quad (17)$$

Substituting this back into the value function, we can simplify to get

$$\begin{aligned} V^u(a) &= \int \max \left\langle \frac{-\exp(-\sigma(1-\beta)(a+b-\tau-\beta K))}{\sigma(1-\beta)}, \frac{-\exp(-\sigma((1-\beta)a+w-\tau))}{\sigma(1-\beta)} \right\rangle \\ &= \frac{-\exp(-\sigma(1-\beta)(a+b-\tau-\beta K))}{\sigma(1-\beta)} \left(F(\bar{w}) + \int_{\bar{w}} \exp(-\sigma(w-\bar{w})) \right), \end{aligned}$$

where

$$\bar{w} = (1-\beta)b + \beta\tau - \beta(1-\beta)K \quad (18)$$

is the reservation wage. This verifies the conjectured form of the value function and pins down the constant K :

$$K = -\frac{(1-\beta)(b-\tau) + CE(\bar{w}, \sigma)}{(1-\beta)^2}, \quad (19)$$

where $CE(\bar{w}, \sigma)$ is the certainty equivalent defined in equation (15). Plug this back into the equations for the consumption of the unemployed (17) and the reservation wage (18) to get

$$\bar{w} = b + \beta \frac{CE(\bar{w}, \sigma)}{1-\beta} \quad (20)$$

$$c^u(a) = (1-\beta)a + \bar{w} - \tau \quad (21)$$

Recall that the certainty equivalent $CE(\bar{w}, \sigma)$ is decreasing in the reservation wage \bar{w} . It follows that the reservation wage is an increasing function of the unemployment benefit, since a higher benefit raises the opportunity cost of accepting a job. In fact, a worker is indifferent between accepting her reservation wage today and rejecting it, getting her unemployment benefit today, and then earning the certainty equivalent $CE(\bar{w}, \sigma)$ thereafter. The lump-sum tax naturally does not affect the reservation wage, but instead leads to a one-for-one decrease in consumption.

It is also worth noting that the reservation wage is a decreasing function of the worker's risk aversion for a fixed unemployment benefit. To prove this, recall that the certainty equivalent of a lottery is decreasing in the coefficient of absolute risk aversion; see Mas-Colell, Whinston and Green (1995), Proposition 6.C.2, p. 191. Since $CE(\bar{w}, \sigma)$ is decreasing in both arguments, the result follows immediately from equation (20).

5.2 Implementation

To compare the economies with and without savings, it is useful to express a worker's consumption indirectly as a function of her utility $v = V^u(a)$, rather than her asset holdings a :

$$\tilde{c}^u(v) = -\frac{\log(-\sigma v(1 - \beta))}{\sigma} - CE(\bar{w}, \sigma).$$

For a given reservation wage and promised utility, this is identical to the consumption of the unemployed in the economy without savings; see part 3 of Proposition 1. Similarly, in both economies, a worker who finds a job at wage $w \geq \bar{w}$ consumes $w - \bar{w}$ more than a worker who remains unemployed, $c^e(a, w) - c^u(a) = w - \bar{w}$; see part 4 of the Proposition.

It is also useful to look at the evolution of consumption for a worker who remains unemployed. Equation (17) indicates that the change in consumption is equal to $(1 - \beta)$ times the change in the worker's asset holdings, $a' - a$. Since $a' = \beta^{-1}(a + b - \tau - c^u(a))$, one can show from equations (20) and (21) that $c^u(a') - c^u(a) = -CE(\bar{w}, \sigma)$, so consumption declines by the certainty equivalent each period. This is identical to the expression in part 3 of Proposition 1.

The preceding paragraphs imply that the unemployment benefit b implements the social optimum uniquely if it implements the socially optimal reservation wage, \bar{w}^* . All that remains is to examine the reservation wage. It follows immediately from equation (20) that

if the unemployment benefit satisfies

$$b^* = \bar{w}^* - \beta \frac{CE(\bar{w}^*, \sigma)}{1 - \beta}, \quad (22)$$

workers choose the optimal reservation wage. It is also easy to balance the government budget in expected value terms by charging a lump-sum tax

$$\tau^* = \frac{(1 - \beta)F(\bar{w}^*)}{1 - \beta F(\bar{w}^*)}. \quad (23)$$

This does not affect the reservation wage but tax receipts equal to $\tau^*/(1 - \beta)$ in present value terms. Since the expected present value of unemployment benefit payments is

$$\sum_{n=0}^{\infty} F(\bar{w}^*)^{n+1} \beta^n b^* = \frac{F(\bar{w}^*)b^*}{1 - \beta F(\bar{w}^*)},$$

taxes and benefits are equal and so the unemployment insurance is actuarially fair.

The following Proposition summarizes our findings in this section:

Proposition 2 *With CARA utility and a risk-free asset with gross return $R = \beta^{-1}$, the optimal balanced budget policy is characterized by a constant unemployment benefit b^* and a constant lump-sum tax τ^* satisfying equations (22) and (23). When workers are risk-neutral, $\sigma = 0$, the optimal unemployment benefit is equal to zero. When workers are risk-averse, the optimal unemployment benefit is strictly positive, $b^* > 0$.*

Proof. The text before the proposition establishes the main characterization results.

If $\sigma = 0$, the certainty equivalent is just equal to the expected value of the relevant lottery, and so equation (14) reduces to

$$\hat{K}(\bar{w}) = -\frac{\int_{\bar{w}}^{w_{\max}} w dF(w)}{(1 - \beta)(1 - \beta F(\bar{w}))}$$

The first order condition implies the optimal reservation wage solves

$$(1 - \beta F(\bar{w}^*)) \bar{w}^* = \beta \int_{\bar{w}^*}^{w_{\max}} w dF(w).$$

Equation (22) then indicates that the optimal benefit is equal to zero and so equation (23) indicates that the tax rate is also zero.

If $\sigma > 0$ and $\bar{w}^* = w_{\max}$, equation (14) implies $\hat{K}(w_{\max}) = 0$ and so equation (20) implies that the benefit that induces workers to hold out for the highest wage is $b = w_{\max}$.⁷

If $\sigma > 0$ and $\bar{w}^* < w_{\max}$, the optimal reservation wage satisfies $\hat{K}'(\bar{w}^*) \geq 0$ since $\bar{w}^* \in \arg \min_{\bar{w}} \hat{K}(\bar{w})$, the optimal reservation wage. Now use brute force to establish the following fact:

$$\frac{\partial CE(\bar{w}, \sigma)}{\partial \bar{w}} + 1 - F(\bar{w}) = F(\bar{w}) \left(\frac{1}{F(\bar{w}) + \int_{\bar{w}} \exp(-\sigma(w - \bar{w})) dF(w)} - 1 \right) > 0.$$

Then differentiate $\hat{K}(\bar{w})$ to get

$$\begin{aligned} \hat{K}'(\bar{w}) &= -\frac{\frac{\partial CE(\bar{w}, \sigma)}{\partial \bar{w}} + 1 - F(\bar{w}) - F'(\bar{w})\bar{w}}{(1 - \beta)(1 - \beta F(\bar{w}))} - \frac{CE(\bar{w}, \sigma) + (1 - F(\bar{w}))\bar{w}}{(1 - \beta)(1 - \beta F(\bar{w}))^2} \beta F'(\bar{w}) \\ &< \frac{((1 - \beta)\bar{w} - \beta CE(\bar{w}, \sigma)) F'(\bar{w})}{(1 - \beta)(1 - \beta F(\bar{w}))^2}. \end{aligned}$$

In particular, $(1 - \beta)\bar{w}^* > \beta CE(\bar{w}^*, \sigma)$. The result then follows from the definition of b^* in equation (22). ■

5.3 Discussion

Werning (2002) studies optimal unemployment insurance in a moral hazard setting akin to Hopenhayn and Nicolini's (1997) model, but introducing the possibility of hidden saving and borrowing for the agent. His paper obtains the optimality of constant unemployment benefits and taxes, as we do here, when considering CARA utility and a monetary cost of effort. An important difference, however, is that in his model, even with CARA utility, the introduction of hidden savings affects the optimal allocation.

As remarked in Section 4 the result that the optimal allocation obtained without hidden savings or borrowing satisfies a standard consumption Euler equation of the form:

$$u'(c_t) = \beta RE_t[u'(c_{t+1})]$$

is surprising in light of previous results in the dynamic contracting literature. Indeed, it is common in models with asymmetric information, both moral hazard and private information,

⁷As an alternative first step, it is possible to prove instead that $\bar{w}^* < w_{\max}$.

for the allocation to satisfy an ‘inverse Euler’ equation of the form:

$$\frac{1}{u'(c_t)} = E_t \left[\frac{1}{\beta R u'(c_{t+1})} \right]$$

As long as c_{t+1} is not predictable the two condition are not identical. In particular the inverse Euler implies, by the convexity of the function “ $1/x$ ” and Jensen’s inequality,

$$u'(c_t) < \beta R E_t (u'(c_{t+1})). \tag{24}$$

This inequality lead Rogerson (1985) to coin the term “savings-constrained”: at the optimal allocation the agent would prefer a marginal constant increase in c_{t+1} at the expense of c_t , if the gross return R were available for such a trade-off.

Indeed, when equation (24) holds for the optimal allocation in which the agent cannot save or borrow privately, it is easy to see why incorporating the additional assumption of hidden savings affects the optimal allocation. In such cases, hidden savings is costly since it reduces the set of incentive compatible allocations and in particular the allocation that was optimal before the introduction of the new set of incentive constraints is no longer incentive compatible. A particularly striking example of this is given by Allen (1985), recently extended by Cole and Kocherlakota (2001), who showed that with privately observed income shocks and hidden borrowing and saving, no insurance is possible. In contrast, in our version of McCall’s (1970) model, there is no cost of giving the agent complete financial freedom.

We emphasize that this result is sensitive to our assumptions. Indeed, the Euler equation no longer holds at the optimal allocation if the utility function is not CARA.

Despite this, we believe the McCall model provides a different and fresh point of departure on these issues and that our simplifying assumptions help isolate the differences. In particular, note that the unobservable search effort model implies the inverse Euler equation even with the assumptions of CARA utility. Also, even if the Euler equation no longer holds without CARA utility in a McCall model, it is highly unlikely that the inverse Euler equation will hold, as is the case in moral hazard models. Indeed, based on the CARA case it seems likely that the optimal allocation with some non-CARA utility function may have the property that the agent is “borrowing-constrained” rather than “savings-constrained”.

6 Competitive Insurance Markets

In this section, we replace the planner with a competitive sector of risk-neutral insurance agencies that offer one period unemployment insurance contracts. In each period, a worker may demand an arbitrary unemployment benefit b at cost $T(b)$. Competition drives the expected profit from selling an insurance contract to zero; equivalently, unemployment insurance contracts are actuarially fair. We assume that contracts are exclusive, so a worker cannot purchase more than one, or alternatively that the premium depends on the total quantity of insurance that the worker buys. We also continue to allow the worker to borrow and lend at the gross risk-free rate $R = \beta^{-1}$, subject only to the constraint that the present value of her debt cannot explode asymptotically.

The competitive insurance sector faces the same information restriction as the planner. An agency can observe whether a worker is employed, but it cannot observe the worker's wage or wage offer. In a generalization of the implementation of the planner's solution discussed in Section 5, here we allow the worker to purchase a different amount of unemployment insurance in each period. Our main result is that competitive insurance markets decentralize the planner's solution.

The basic structure of this problem is unchanged from the previous section and so we skip over many details. A worker who starts a period employed consumes her wage plus the annuity value of her beginning of period assets a , $c^e(a, w) = (1 - \beta)a + w$, giving her lifetime utility

$$V^e(a, w) = \frac{-\exp(-\sigma((1 - \beta)a + w))}{\sigma(1 - \beta)}.$$

If the worker is unemployed at the start of a period, she first chooses her unemployment benefit b and pays the associated premium $T(b)$. Then she draw a wage from the distribution F , decides whether to accept the job, and then consumes. Her lifetime utility may be expressed recursively as

$$V^u(a) = \max_b \int \max \left\langle \max_c (u(c) + \beta V^u(a')), V^e(a - T(b), w) \right\rangle dF(w), \quad (25)$$

where $a' = \beta^{-1}(a + b - T(b) - c)$. There are three maximization problems here. First, the worker chooses her unemployment benefit. Then she decides whether to accept the job. Finally, she chooses her consumption conditional on unemployment. If she takes a job, she consumes $c^e(a - T(b), w)$.

Our approach to solving this problem is by now standard. We conjecture and verify that

the value function of an unemployed worker has the usual functional form:

$$V^u(a) = \frac{-\exp(-\sigma(1-\beta)(a-K^*))}{\sigma(1-\beta)}.$$

Given this conjecture, we can solve first for the consumption of the unemployed $c^u(a)$ and then for the worker's reservation wage:

$$\bar{w} = (1-\beta)(b-\beta K^*). \quad (26)$$

When a worker demands an unemployment benefit b , the unemployment insurance agencies anticipate that she will fail to obtain a job a fraction $F(\bar{w})$ of the time. Actuarial fairness (or the zero profit condition) therefore pins down the lump-sum tax rate associated with a given unemployment benefit at $T(b) = F(\bar{w})b$, where \bar{w} is defined in equation (26). Putting this together, we may write the value function of an unemployed worker as

$$V^u(a) = \max_b \left(\frac{-\exp(-\sigma(1-\beta)(a+(1-F(\bar{w}))b-\beta K^*))}{\sigma(1-\beta)} \right) \\ \times \left(F(\bar{w}) + \int_{\bar{w}}^{w_{\max}} \exp(-\sigma(w-\bar{w})) dF(w) \right),$$

where \bar{w} is defined in equation (26). Alternatively, since there is a one-to-one mapping between b and \bar{w} , we may simply think of the worker choosing her reservation wage \bar{w} by demanding the unemployment benefit $b = \frac{\bar{w}}{1-\beta} + \beta K^*$. This suggests that we rewrite the value function as

$$V^u(a) = \max_{\bar{w}} \left(\frac{-\exp(-\sigma((1-\beta)a+(1-F(\bar{w}))\bar{w}-\beta(1-\beta)F(\bar{w})K^*))}{\sigma(1-\beta)} \right) \\ \times \left(F(\bar{w}) + \int_{\bar{w}}^{w_{\max}} \exp(-\sigma(w-\bar{w})) dF(w) \right),$$

Replacing $V^u(a)$ with our conjectured functional form and simplifying gives $K^* = \min_{\bar{w}} \hat{K}(w)$ defined in equation (14). This confirms the conjecture. It is then straightforward to verify that the reservation wage and consumption behavior are unchanged from the planner's problem in Section 5.

7 Efficient versus Optimal Unemployment Insurance

This section further characterizes optimal unemployment insurance. In particular, we compare the optimal unemployment benefit b^* , defined in equation (22), with the level of benefit that would induce workers to maximize the expected output produced in this economy. We refer to the latter as productively efficient, or more simply efficient, unemployment insurance. Our initial conjecture, following Acemoglu and Shimer (1999), was that optimal unemployment insurance would always exceed efficient unemployment insurance: starting from an economy with efficient unemployment insurance, a bit more insurance would induce workers to raise their reservation wage slightly, resulting in a second order loss in output; but by transferring income from employed to unemployed workers, it would improve risk-sharing and hence raise the worker's expected utility. We show by example that this conjecture is wrong. In general there is no necessary relationship between optimal and efficient unemployment insurance. If workers are sufficiently risk averse and the wage distribution satisfies a regularity condition, however, we prove that the conjecture is correct.

Rather than analyzing the optimal and efficient unemployment benefit b directly, it is analytically easier to work directly with the reservation wage \bar{w} induced by a particular unemployment benefit. Equation (20) implies that there is an increasing relationship between the two variables for a given level of risk-aversion σ , and so rather than asking whether optimal unemployment benefits are larger than efficient unemployment benefits, we ask whether the optimal reservation wage exceeds the efficient reservation wage.

7.1 Efficient Reservation Wage

The efficient reservation wage maximizes expected discounted wage income. It is straightforward to see that this is equal to the expected income of a risk-neutral worker ($\sigma = 0$) without any unemployment insurance ($b = 0$), and so an efficient reservation wage satisfies

$$\bar{w}^e \in \arg \max_{\bar{w}} \frac{\int_{\bar{w}}^{w_{\max}} w dF(w)}{1 - \beta F(\bar{w})}.$$

Alternatively, for a given value of σ define

$$\tilde{K}(\bar{w}, \sigma) \equiv -\frac{CE(\bar{w}, \sigma) + (1 - F(\bar{w})) \bar{w}}{1 - \beta F(\bar{w})},$$

analogous to the definition of \hat{K} except that we emphasize the dependence of σ .⁸ Then since the certainty equivalent is equal to the expected value of the gamble when the worker is risk-neutral,

$$\tilde{K}(\bar{w}, 0) \equiv -\frac{\int_{\bar{w}}^{w_{\max}} w dF(w)}{1 - \beta F(\bar{w})}$$

Equivalently, the efficient reservation wage solves $\bar{w}^e \in \arg \min_{\bar{w}} \tilde{K}(\bar{w}, 0)$. Note that this is independent of the worker's preferences.

7.2 Optimal Reservation Wage

Proposition 1 shows that the optimal reservation wage depends on the worker's preferences and may be expressed as $\bar{w}_{\sigma}^* \in \arg \min_{\bar{w}} \tilde{K}(\bar{w}, \sigma)$. When a worker is risk-neutral, it is clear that optimality and efficiency coincide, so $\bar{w}^e = \bar{w}_0^*$. Otherwise there is no necessary relationship between these two concepts, as a simple, if not particularly realistic, example establishes. We assume the wage distribution is uniform on $[1, 2]$ and set the discount factor to $\beta = 0.7$. We then calculate the optimal reservation wage \bar{w}_{σ}^* for different values of σ .

Figure 1 depicts the results. We find that for low values of σ , the optimal reservation wage is decreasing in σ . Then at approximately $\sigma = 1.8$, the optimal reservation wage falls discontinuously to 1, i.e. all wages are accepted. It remains there until $\sigma = 2.8$, when the reservation wage jumps up. For still higher values of σ , the reservation wage gradually increases, eventually reaching an asymptotic value of 1.41. The figure also plots the optimal unemployment benefit as a function of σ , b_{σ}^* , which is also not monotonic.

7.3 High Risk Aversion: The Usual Trade-off

It is fruitful to compare optimal and efficient unemployment insurance if workers are sufficiently risk-averse. Our approach is to define a function

$$\Delta(\bar{w}) = \tilde{K}(\bar{w}, \infty) - \tilde{K}(\bar{w}, 0),$$

and find conditions under which this is a decreasing function of \bar{w} . Standard comparative statics arguments then imply that any minimizer of $\tilde{K}(\bar{w}, \infty)$ must exceed any minimizer of $\tilde{K}(\bar{w}, 0)$, i.e. the optimal reservation wage with risk-aversion σ exceeds the efficient reservation wage.

⁸We also drop an irrelevant multiplicative constant

We first prove this is true in the limit, when $\sigma = \infty$. We later establish that these results are informative about the solution for high enough σ .

Lemma 3 *Assume*

$$\frac{\partial}{\partial \bar{w}} E(w|w \geq \bar{w}) \leq 1.$$

Then $\Delta'(\bar{w}) \leq 0$, with a strict inequality provided $\bar{w} < w_{\max}$. Assuming $\bar{w}^e > w_{\min}$, so the efficient reservation wage is interior, then $\bar{w}_\infty^* > \bar{w}^e$, so for an infinitely risk-averse worker, optimal unemployment insurance exceeds efficient unemployment insurance.

Proof. $CE(\bar{w}, \infty) = 0$, so

$$\Delta'(\bar{w}) = \frac{\int_{\bar{w}}^{w_{\max}} (w - \bar{w}) dF(w)}{1 - \beta F(\bar{w})} = g(\bar{w}) E(w - \bar{w}|w \geq \bar{w})$$

where

$$g(\bar{w}) \equiv \frac{1 - F(\bar{w})}{1 - \beta F(\bar{w})}.$$

Using this definition,

$$\Delta'(\bar{w}) = g'(\bar{w}) E(w - \bar{w}|w \geq \bar{w}) + g(\bar{w}) \frac{\partial E(w - \bar{w}|w \geq \bar{w})}{\partial \bar{w}}.$$

The first term is strictly negative when $\bar{w} < w_{\max}$ because g is decreasing. The second term is nonpositive by assumption. It follows that $\Delta(\bar{w})$ is strictly decreasing in \bar{w} .

Now we prove $\arg \min_{\bar{w}} \tilde{K}(\bar{w}, \infty) > \arg \min_{\bar{w}} \tilde{K}(\bar{w}, 0)$. Take any $\bar{w}^e \in \arg \min_{\bar{w}} \tilde{K}(\bar{w}, 0)$ and any $w < \bar{w}^e$ and note that

$$\tilde{K}(w, \infty) = \tilde{K}(w, 0) + \Delta(w) > \tilde{K}(\bar{w}^e, 0) + \Delta(\bar{w}^e) = \tilde{K}(\bar{w}^e, \infty),$$

where the strict inequality follows from Δ being a strictly decreasing function and $\bar{w}^e \in \arg \min_{\bar{w}} \tilde{K}(\bar{w}, 0)$. This proves $w \notin \arg \min_{\bar{w}} \tilde{K}(\bar{w}, \infty)$.

To rule out the possibility that any interior $\bar{w}^e \in \arg \min_{\bar{w}} \tilde{K}(\bar{w}, 0)$ can minimize $\tilde{K}(\bar{w}, \infty)$, note that

$$\frac{\partial \tilde{K}(\bar{w}^e, \infty)}{\partial \bar{w}} = \frac{\partial \tilde{K}(\bar{w}^e, 0)}{\partial \bar{w}} + \frac{\partial \Delta(\bar{w}^e)}{\partial \bar{w}}.$$

The first term is zero at an interior efficient reservation wage while the second term is always strictly negative. Finally, we can rule out the possibility that $w_{\max} \in \arg \min_{\bar{w}} \tilde{K}(\bar{w}, 0)$, since the highest possible reservation wage generates no output. ■

The assumption that

$$\frac{\partial}{\partial \bar{w}} E(w|w \geq \bar{w}) \leq 1$$

is satisfied by many distributions and has appeared elsewhere in the search literature. In particular, this assumption is implied by log-concavity of F or f . van den Berg (1994) analyzes the connection between this and several other common assumptions on wage distributions.

We next establish a technical result that is necessary to ensure that the results for $\sigma = \infty$ reflect on the behavior of workers with high but finite risk aversion.

Lemma 4 *Suppose the density f is bounded from above and bounded away from zero, so that $\inf_w f(w) > 0$ and $\sup_w f(w) < \infty$. Then the function $CE(\bar{w}, \sigma)$ is increasing in σ and $CE(\bar{w}, \sigma) \rightarrow 0$ uniformly as $\sigma \rightarrow \infty$ for all \bar{w} . Thus, $\tilde{K}(\bar{w}, \sigma) \rightarrow \tilde{K}(\bar{w}, \infty)$ uniformly as $\sigma \rightarrow \infty$.*

Proof. We first prove pointwise convergence to zero as $\sigma \rightarrow \infty$. For $\bar{w} > w_{\min}$ the result is straightforward:

$$\lim_{\sigma \rightarrow \infty} \frac{1}{\sigma} \log \left(F(\bar{w}) + \int_{\bar{w}}^{w_{\max}} e^{-\sigma(w-\bar{w})} dF(w) \right) = \lim_{\sigma \rightarrow \infty} \frac{1}{\sigma} \log (F(\bar{w})) = 0$$

while for $\bar{w} = w_{\min}$, using L'Hospital and differentiating under the integral we obtain

$$\lim_{\sigma \rightarrow \infty} \frac{1}{\sigma} \log \left(F(w_{\min}) + \int_{w_{\min}}^{w_{\max}} e^{-\sigma(w-w_{\min})} dF(w) \right) = \lim_{\sigma \rightarrow \infty} \int_{w_{\min}}^{w_{\max}} (w - w_{\min}) h(w, \sigma) dw,$$

where

$$h(w, \sigma) = \frac{e^{-\sigma(w-w_{\min})} f(w)}{\int_{w_{\min}}^{w_{\max}} e^{-\sigma(v-w_{\min})} f(v) dv}$$

can be thought of as a density. Intuitively, the probability measure associated with h is converging as $\sigma \rightarrow \infty$ to the measure with unit mass at $w = w_{\min}$, giving the result. Formally, define H as the cumulative distribution function associated with the density h .

Then $1 - H(w, \sigma) \rightarrow 0$ for all $w > w_{\min}$:

$$\begin{aligned}
0 \leq 1 - H(w, \sigma) &= \frac{\int_w^{w_{\max}} e^{-\sigma(v-w_{\min})} f(v) dv}{\int_{w_{\min}}^{w_{\max}} e^{-\sigma(v-w_{\min})} f(v) dv} \\
&= \left[\frac{\int_{w_{\min}}^w e^{-\sigma(v-w_{\min})} f(v) dv}{\int_w^{w_{\max}} e^{-\sigma(v-w_{\min})} f(v) dv} + 1 \right]^{-1} \\
&\leq \left[\frac{\int_{w_{\min}}^w e^{-\sigma(v-w_{\min})} dv \inf_{w_{\min} \leq v \leq w} f(v)}{\int_w^{w_{\max}} e^{-\sigma(v-w_{\min})} dv \sup_{w \leq v \leq w_{\max}} f(v)} + 1 \right]^{-1};
\end{aligned}$$

and note that as $\sigma \rightarrow \infty$,

$$\frac{\int_{w_{\min}}^w e^{-\sigma(v-w_{\min})} dv}{\int_w^{w_{\max}} e^{-\sigma(v-w_{\min})} dv} = \frac{1 - e^{-\sigma(w-w_{\min})}}{e^{-\sigma(w-w_{\min})} - e^{-\sigma(w_{\max}-w_{\min})}} \rightarrow \infty$$

if $w > w_{\min}$. Since in the inf and sup are positive and finite, this proves that $1 - H(w, \sigma) \rightarrow 0$ for all $w > w_{\min}$. This allows us to simplify the expression for the certainty equivalent using integration by parts:

$$\begin{aligned}
\lim_{\sigma \rightarrow \infty} \int_{w_{\min}}^{w_{\max}} (w - w_{\min}) h(w, \sigma) dw &= \lim_{\sigma \rightarrow \infty} \left[\int_{w_{\min}}^{w_{\max}} (1 - H(w, \sigma)) dw \right] \\
&= \int_{w_{\min}}^{w_{\max}} \lim_{\sigma \rightarrow \infty} (1 - H(w, \sigma)) dw = 0,
\end{aligned}$$

where we use Lebesgue's Dominated Convergence Theorem: since $1 - H(w, \sigma)$ is uniformly bounded by 1, it is integrable on a compact set. This proves pointwise convergence.

To show uniform convergence, we use Theorem 7.13 in Rudin (1976), p. 150, which establishes that pointwise convergence is uniform if it is monotonic. Recall that $CE(\bar{w}, \sigma)$ is the certainty equivalent of a particular lottery (which depends on \bar{w} but not on σ) for a consumer with CARA preference and coefficient of absolute risk aversion σ . Standard results regarding risk aversion imply that $CE(\sigma, \bar{w})$ is decreasing in σ ; see Mas-Colell et al. (1995), Proposition 6.C.2, p. 191. That $\tilde{K}(\bar{w}, \sigma) \rightarrow \tilde{K}(\bar{w}, \infty)$ uniformly as $\sigma \rightarrow \infty$ now follows directly. ■

This leads to our main result in this section.

Proposition 3 *Assume the conditions of Lemma 3 are satisfied. Then there is a $\bar{\sigma} < \infty$ such that for all $\sigma \geq \bar{\sigma}$, we have that $\bar{w}_\sigma^* > \bar{w}^e$.*

Proof. See Appendix A. ■

8 Conclusion

This paper has characterized optimal unemployment insurance in the McCall (1970) search model under a variety of informational assumptions. Our main result is that with CARA preferences, even if the planner can observe worker's savings behavior, he still chooses to offer the worker a consumption path that satisfies the consumption Euler equation. As a result, optimal unemployment insurance can be decentralized through a constant unemployment benefit funded by a constant lump-sum tax. We also compared optimal and output maximizing unemployment insurance. Although there is no general relationship between the two quantities, we found that when workers are sufficiently risk averse, it is typically optimal to provide more insurance than the amount that induces workers to set an output-maximizing reservation wage.

We conclude by briefly mentioning one important topic for future research: exogenous job separations. We have so far assumed that all jobs last forever. This assumption is not innocuous, since if jobs are temporary (e.g. end each period with probability s), the planner can separate workers earning different wages yet still induce truth-telling behavior. For example, a worker who reports a high current wage might be given a lower transfer today in return for a higher continuation value after the match ends. This may qualitatively, and perhaps quantitatively, resemble the savings behavior of employed workers in an economy with hidden savings and finite-lived jobs, although our research suggests that hidden savings will affect the nature of the social optimum in an economy with separations.

APPENDIX

A Proof of Proposition 3

We begin by establishing two preliminary results, building on Stokey, Lucas and Prescott (1989) chapter 3, pp. 63-65, as closely as possible. We extend their results in some dimensions to avoid assuming concavity of the objective functions and thus uniqueness of maximizers. We specialize to simplify the exposition because we do not have a state variable.

The first result says that, roughly speaking, if a continuous function f is close to being maximized then we must be close to the maximizing set.

Lemma 5 [Variation on Stokey et al. (1989), Lemma 3.7, p. 63] *Let X be compact*

and f a continuous function on X . Then define

$$G = \arg \max_{x \in X} f(x) \quad \text{and} \quad F = \max_{x \in X} f(x)$$

(note that F is well defined and G is nonempty by the Weierstrass Theorem and G is compact by the Theorem of the Maximum). Then for each ε there exists $\delta > 0$ such that

$$x \in X \quad \text{and} \quad |F - f(x)| < \delta \quad \text{implies} \quad |g - x| < \varepsilon \quad \text{for some } g \in G$$

Proof. For each $\varepsilon > 0$ define

$$A_\varepsilon = \{x \in X : |x - g| \geq \varepsilon \text{ for all } g \in G\} = \left\{ x \in X : \min_{g \in G} |x - g| \geq \varepsilon \right\}$$

(that is, the set that excludes values of x that are too close to the maxima of f). Note that since G is compact the second definition is valid (i.e. the minimum is well defined).

If A_ε is empty for all $\varepsilon > 0$ then it must be that f is constant over X , so that $X = G$ (i.e. $\min_{g \in G} |x - g| < \varepsilon$ for all $\varepsilon > 0$ requires $\min_{g \in G} |x - g| = 0$ so there exists some $g \in G$ such that $x = g$ or $f(x) = f(g) = F$ so that $x \in G$).

Otherwise, there exists $\hat{\varepsilon} > 0$ sufficiently small such that for all $0 < \varepsilon < \hat{\varepsilon}$, the set A_ε is nonempty and compact [this last point follows easily since X and G are compact so that the function $m(x) \equiv \min_{g \in G} |x - g|$ is continuous in x , by the Theorem of the Maximum, implying that the values for which $m(x) \geq \varepsilon$ is compact]. For any such ε , let

$$\delta = \min_{x \in A_\varepsilon} |F - f(x)|.$$

Since f is continuous and A_ε is compact, the minimum is attained. Moreover, since for any $g \in G$ then $g \notin A_\varepsilon$, it follows that $\delta > 0$. Then $x \in X$ and $|F - f(x)| < \delta$ implies that ($x \notin A_\varepsilon$ so that) $|g - x| < \varepsilon$ for some $g \in G$, as was to be shown. ■

Theorem 1 [Variation on Stokey et al. (1989), Theorem 3.8, p. 64] *Let X be compact and $\{f_n\}$ be a sequence of real continuous functions on X with $f_n \rightarrow f$ uniformly. Define:*

$$G_n = \arg \max_{x \in X} f_n(x) \quad \text{and} \quad G = \arg \max_{x \in X} f(x)$$

then for any $\varepsilon > 0$ there exists an N such that for all $n \geq N$ and $g_n \in G_n$ there exists a $g \in G$ so that $|g_n - g| < \varepsilon$.

Proof. First note that for any $g_n \in G_n$ and any $g \in G$, since g_n maximizes f_n and g maximizes f we have:

$$\begin{aligned} 0 &\leq f(g) - f(g_n) \\ &\leq f(g) - f_n(g) + f_n(g_n) - f(g_n) \\ &\leq 2\|f - f_n\| \end{aligned}$$

where $\|f\| = \max_{x \in X} |f(x)|$ for X compact and f is continuous.

Since $f_n \rightarrow f$ uniformly, it follows immediately that for any $\delta > 0$, there exists $M_\delta \geq 1$ such that $\|f - f_n\| < \delta/2$ for all $n \geq M_\delta$, so that for any $g_n \in G_n$ and any $g \in G$

$$0 \leq F - f(g_n) < \delta \text{ for all } n \geq M_\delta$$

where $F = \max_{x \in X} f(x)$.

To show that for any $\varepsilon > 0$ there exists an N such that for all $n \geq N$ and $g_n \in G_n$ there exists a $g \in G$ so that $|g_n - g| < \varepsilon$ we apply Lemma 2 above. This lemma implies that for any $\varepsilon > 0$ we can find a $\delta_\varepsilon > 0$ with the property that $0 \leq F - f(g_n) < \delta$ implies that $|g_n - g| < \varepsilon$ for some $g \in G$. But we have just shown that for any $\delta_\varepsilon > 0$ we can find a M_{δ_ε} so that for all $n \geq M_{\delta_\varepsilon}$ and any $g_n \in G_n$ we have $0 \leq F - f(g_n) < \delta$. Consequently, $N = M_{\delta_\varepsilon}$ has the required property. ■

The proof of Proposition 3 now follows directly by applying Theorem 1 to the conclusions in Lemmas 3 and 4

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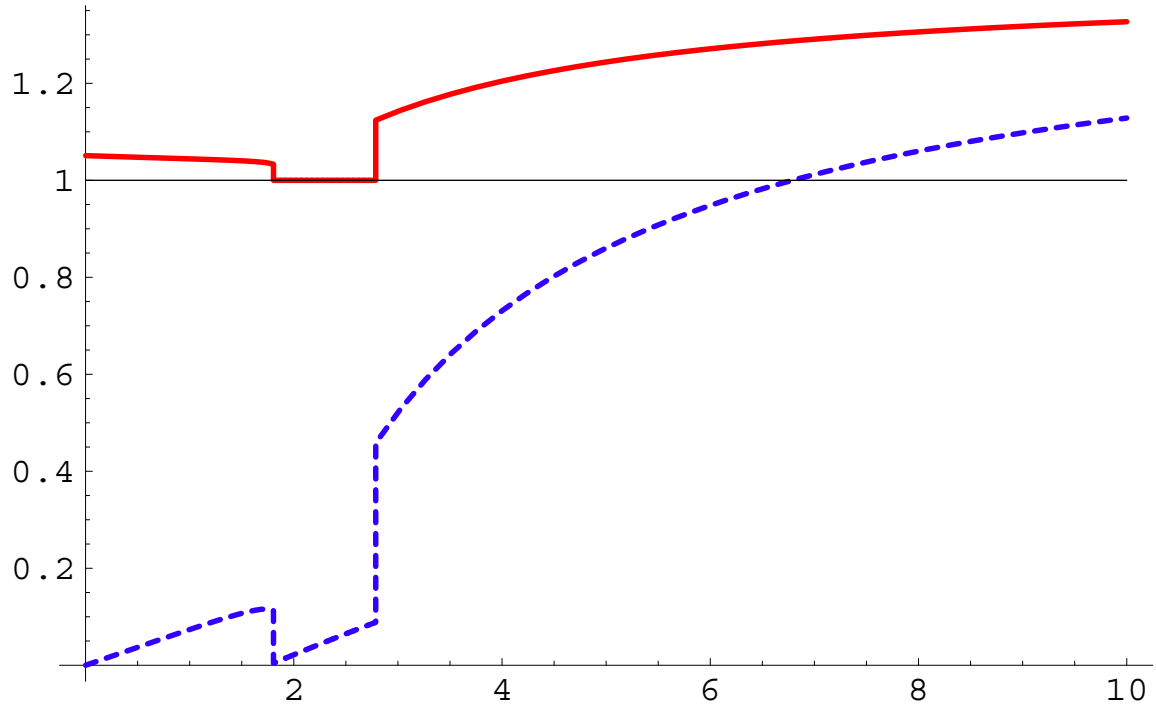


Figure 1: The optimal reservation wage \bar{w}_σ^* (solid red line) and the optimal unemployment benefit b_σ^* (dashed blue line) as functions of the coefficient of absolute risk aversion σ . The wage distribution is uniform on $[1, 2]$ and the discount factor is $\beta = 0.7$.