Endogenous Education and Job Creation

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Abstract

The paper explores the macroeconomic consequences of the complementarity between education and job creation. There is a positive feedback between education and job creation in the sense that the more workers are high-educated, the more profitable it is for firms to create high-skill vacancies, and vice versa. With this complementarity, the economy shows the properties which stand in a high contrast with those expected by the neoclassical theory of education. The gain from education is increasing in the fraction of high-educated workers. The skill-biased technological progress induces less workers to invest in the higher education if the education cost is constant, while it induces more workers to acquire the higher education if workers are heterogeneous in the education cost. The paper succeeds in modelling the stylized fact of the rise in the fraction of the high-educated associated with the rise in the education cost in the OECD countries. There are multiple equilibria if the productivity gap between jobs is relatively small. However, a switch from without to with on-the-job search induces more workers to invest in education only if workers are identical in the cost of education.

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1 Introduction

In the last decade, many authors have developed models in which firms opt to create heterogeneous jobs that are designed for specific skills. In the leading works such as Acemoglu [1999] and Albrecht and Vroman [2002], the supply of workers with different skills is assumed to be exogenous. This paper presents a model that endogenizes workers' skill choices through education and explores the interaction between education and job creation. The complementarity between education and job creation exhibits some remarkable properties that are not inherent in the neoclassical theory of education.

The increasing unemployment rate of young workers in most OECD countries has been seriously discussed in both theoretical and empirical studies of the labor markets. Workers are said to be 'over-educated' if their educational attainments surpass the skill requirements of their jobs. In the labor markets with search friction, high-educated workers may take those jobs that have been conventionally created for low-educated workers. Since the increasing mass of high-educated workers in most OECD countries has not been matched by a proportional rise in the supply of skilled vacancies, a large fraction of high-educated workers are forced to end up in mismatched low-skill jobs. As a consequence, low-educated workers have been 'crowded out' of the traditional middle-skill jobs toward very unskilled jobs. At the same time, there is a rising gap of unemployment rate between high-educated and low-educated workers.

The 'crowding-out' of low-educated workers by high-educated ones is formally analyzed by the seminar work of Albrecht and Vroman [2002]. However, their model does not produce such a large unemployment rate gap across education that fits the real unemployment data, mainly because it does not allow for workers' on-the-job search.[1] Dolado, Jansen, and Jimeno [2002]

[1] Consider an economy where there are high-educated and low-educated workers. Assume that high-educated workers are productive in both high-skill jobs and low-skill jobs, but low-educated ones are so only in low-skill jobs. This assumption results in the higher unemployment rate of low-educated workers than that of high-educated ones, no matter if on-the-job search is allowed or not. If workers are prohibited from engaging in on-the-job search, however, high-educated workers may hesitate to accept unskilled job offers, and the unemployment rate gap will be smaller than if on-the-job search is allowed. With on-the-job search, high-educated workers are more willing to accept low-skill jobs, as long as they pay a higher wage than their reservation wage. Then they 'steal' job opportunities that are more appropriate for low-educated workers, which increase the unemployment rate gap between high-educated and low-educated workers.

explicitly introduced on-the-job search and created unemployment rate gaps that match the real labor market data in Spain.

There are three main contributions of this paper to this research field. In the previous literature, the distribution of workers' education level is assumed exogenous. The first contribution of this paper is to endogenize workers' education choice. That is, the distribution of educational attainments of workers is simultaneously determined with the firm's creation of high-skill or low-skill jobs.

Our second contribution is to characterize the multiple equilibria of the model. Due to the intra-match instability (See Burdett, Imai, and Wright [2003]), there are multiple equilibria on workers' search decision while employed, as well as on the acceptance decision of the mismatched jobs. The multiplicity of equilibrium sheds a new light on the diversity of employment practices across countries that share similar technologies and institutions in common. It is widely argued that the labor market of the United States is more flexible or liquid than those of Japan and the European Union. Workers tend to change jobs more often across firms and across industries in the United States, while employees have more stable relationships with their employers in Japan or Europe. We see that this difference is, by a large part, a consequence of the multiplicity of equilibrium. If the productivity gap between the high-skill job and the low-skill one is large enough, workers always search while employed, and the labor market becomes highly liquid. On the contrary, if the productivity gap is small enough, all workers choose not to search while matched, and the labor market is not liquid. If the productivity gap is in-between, there are multiple equilibria. Even if the parameters are the same, a switch from an equilibrium without to with on-the-job search, creates larger wage gaps, both within skill and within industry, and increases the aggregate unemployment rate.

It must be emphasized that the driving force toward multiple equilibria in this paper is the intra-match instability, but not the traditional 'thick-market' externality, developed by Diamond [1982], which works as follows. The more high-educated workers decide to search while employed, the more firms will create the high-skill vacancies, which can be occupied only by high-educated workers. The increasing high-skill vacancies make on-the-job search more profitable, and produces more on-the-job searchers.

On the contrary, The intra-match instability works as follows. There is no productivity gap between high-educated workers and low-educated ones if they are employed at low-skill jobs. With on-the-job search, the joint surplus created by a low-skill job with a high-educated worker is lower than that with a low-educated worker, since the former relation faces a higher probability of destruction. This difference is reflected by a reduction of the wage for the high-educated workers, which in turn reduces the incentive for high-educated workers to stay at their current jobs, and induces them to engage in on-the-job search. Due to this instability, the same productivity gap may produce multiple equilibria. The thick market externality is existent in this paper, but is not the fundamental force to produce the multiple equilibria, and only strengthens the effects of the intra-match instability.

Third and finally, we show that the positive feedback between education and job creation produces such effects of the skill-biased technological progress on education choices that are totally opposite to what are expected from the neoclassical theory of education, under which the gain from education is *decreasing* in the fraction of high-educated workers. With endogenous creation of jobs, however, the gain from education is an *increasing* function of the fraction of high-educated workers. Due to this difference, a skill-biased productivity progress leads to different outcomes in workers' education choice and in firms' job creation.

This result sheds a new light on the discussion on the role of the skill-biased technological progress on the inequality. For example, Acemoglu (1999), in a framework with exogenous education choice, constructs a model in which the labor market is segmented with respect to workers' skill, and argues that the more workers are high-educated, the more high-skill jobs are created. However, the effects of technological progress on education are not explicitly explored in his model. In a model with endogenous education choice, however, we show that a skill-biased technological progress does not always lead to a rise in the fraction or mass of high-educated workers.

The paper is organized as follows. The next section illustrates the main results of this paper about the relationship between education, on-the-job search and technological progress. Section 3 presents the model. The model is solved and the multiple equilibria is characterized in Section 4. Some comparative statics are discussed in section 5. Section 6 concludes the paper.

2 Education and Job Creation: An Overview

In this section we contrast our model with the neoclassical theory of education, and illustrates the main contributions of this paper.

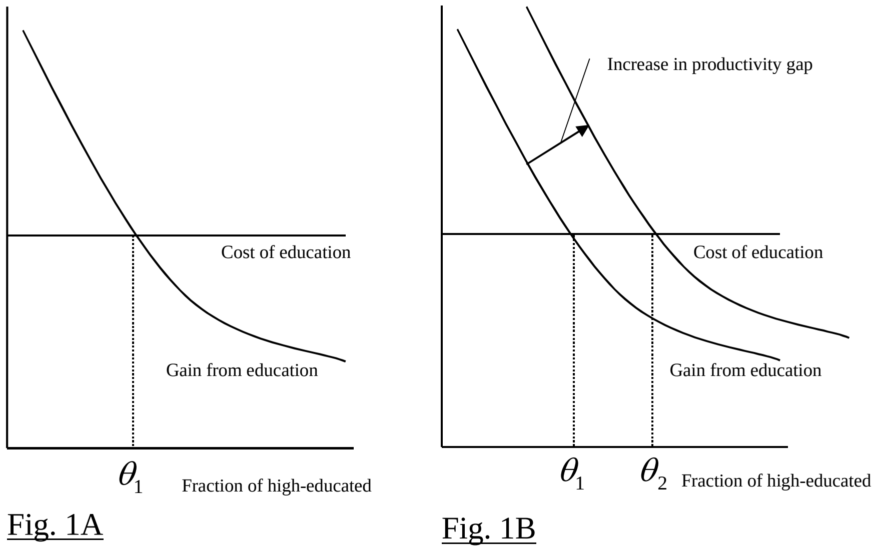

Figure 1: Education Choice with Exogenous Job Creation

Consider an economy in which job creation is exogenous. Then the gain from education is decreasing in the mass or fraction of high-educated workers, as is illustrated in Figure 1. The more workers are high-educated, the less is the return of education. That is, an increase in the mass of high-educated workers makes it more difficult for them to find their appropriate jobs, due to the reduction of their *scarcity* in the labor market. Assume that workers are homogeneous. Then the cost of education is constant and independent of the fraction of the high-educated. In the neoclassical theory of education, an agent chooses her education level as to equate the gain from education to its cost. Therefore, the fraction of the high-educated is determined at θ_1, in Fig. 1A. Then a rise in the productivity gap across education shifts up the gain from education, and encourages more workers to acquire higher education (that is, $\theta_1 \longrightarrow \theta_2$) as is illustrated in Fig. 1B.

This mechanism works only if the job creation is exogenous. In a model with *endogenous* job creation, however, the more workers are high-educated, the more profitable it is for firms to create high-skill jobs. Therefore, the gain from education is *increasing* in the fraction of high-educated workers, as is illustrated in Figure 2. In other words, there is a *positive feedback* between education and job creation. A skill-biased technological progress

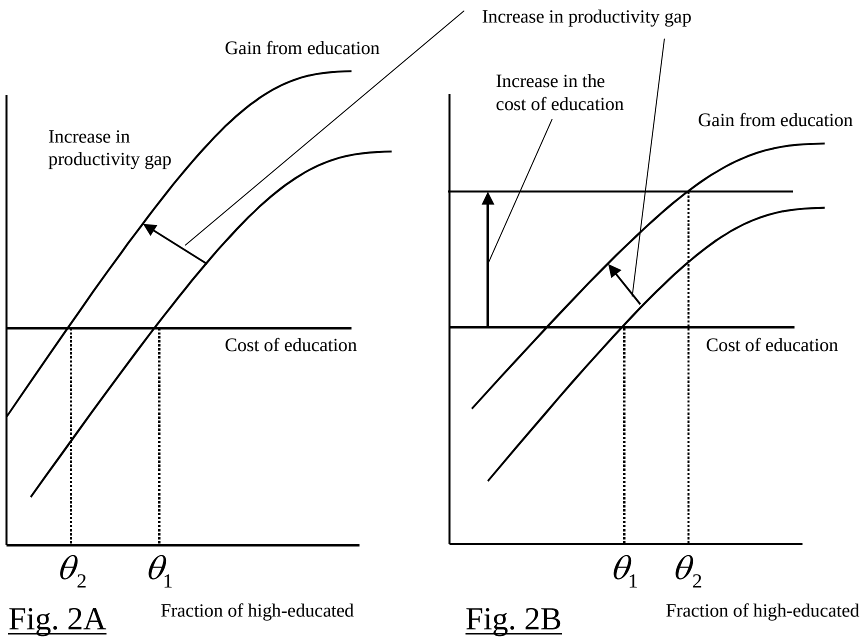

Figure 2: Education Choice with Endogenous Job Creation

increases the gain from education as is expected. However, it *reduces* the fraction of high-educated workers, if and only if the upward shift of the gain from exceeds that of the cost of education, as is illustrated in Fig. 2A. An increase in the fraction of high-educated workers occurs only if there is a more than proportional rise in the cost of education, as is illustrated in Fig. 2B. This result is consistent with the stylized fact in the OECD countries that the trend increase of high-educated workers is associated with a dramatic rise in the cost of education.

However, the equilibrium illustrated in Figure 2 is unstable, although the explicit dynamic analysis is not provided in this paper. Assume that the economy originally chooses θ_1 as an equilibrium in Fig. 2A. With a rise in the productivity gap, an agent at θ_1 finds it more profitable to invest in education. This marginal increase in the fraction of the high-educated results in a further rise in the gain from education. This positive feedback continues forever and reaches no equilibrium. In other words, the original equilibrium is unstable.

By assuming a heterogeneous population in terms of the cost of education,

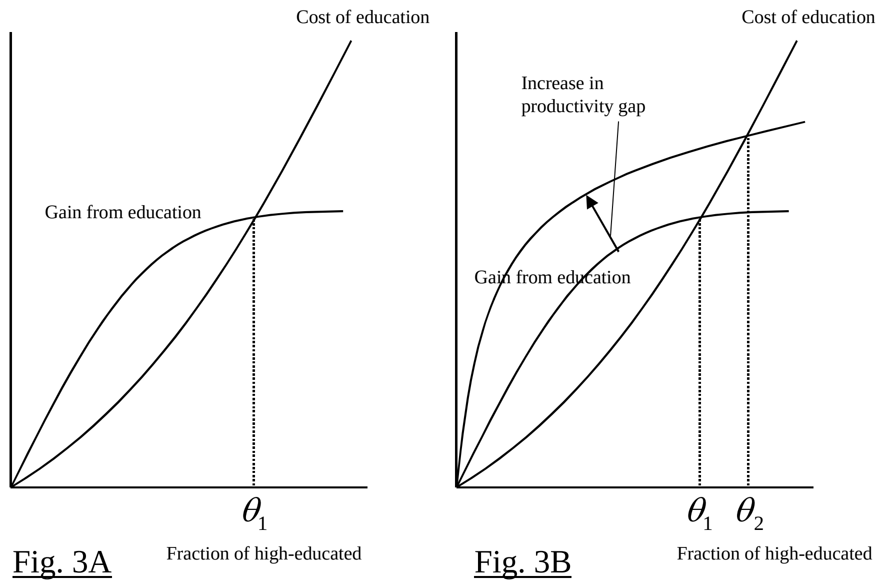

Figure 3: Heterogeneity in the Cost of Education

we can avoid this instability, as is illustrated in Figure 3. If workers are heterogeneous, the education cost of a marginal worker who is indifferent between becoming high-educated and staying low-educated, is increasing in the fraction of high-educated workers. The locus representing the gain from education crosses the cost of education from above, and the equilibrium is stable. A rise in the productivity gap always results in a larger fraction of high-educated workers, as in Fig. 3B, as long as there is no change in the type of equilibrium.

With the heterogeneity of workers in the education cost, the labor market organizes itself toward segmentation in a different way from the seemingly related models of job market signaling, originally developed by Spence (1974). In the signaling model, the asymmetric information is the source of the segmentation. In the present model, however, the segmentation is an outcome of the complementarity of education and job creation.

There is a remaining question. What will happen if the skill-biased technological progress leads to a regime switch across the multiple equilibria? As is discussed in section 5, a transition from an equilibrium without on-the-job search to another equilibrium with it *shifts down* the gain from education.

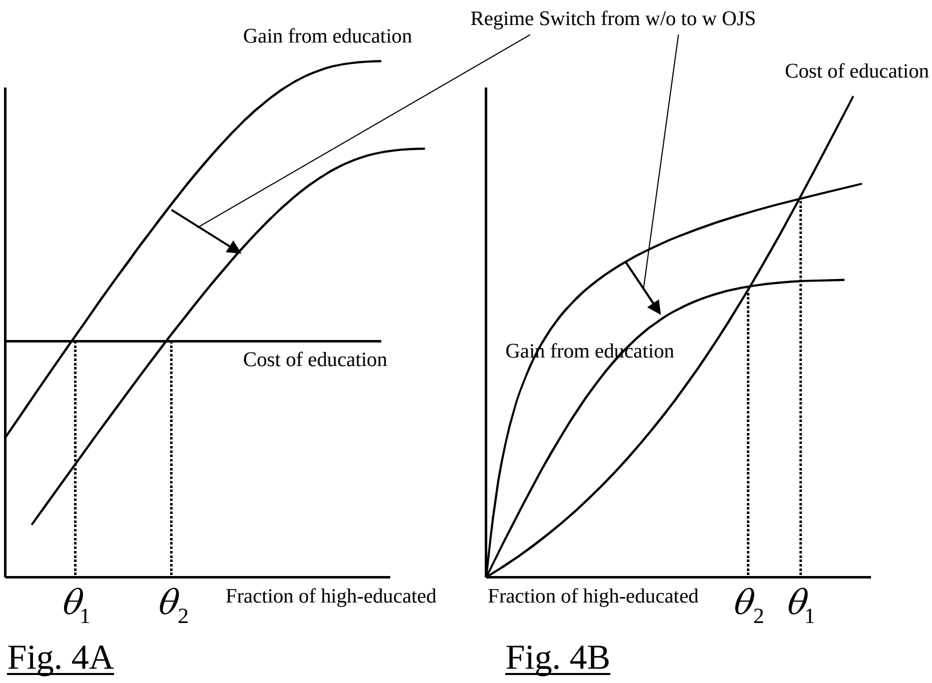

Figure 4: A Regime Switch from without to with On-the-Job Search

Then if the cost of education is constant, this regime switch is associated with a *rise* in the fraction of high-educated workers. On the contrary, if the cost of education is increasing in θ, the regime switch causes a *reduction* in the fraction of high-educated workers. These results are illustrated in Figure 4.

3 The Model

Time is continuous. There are workers and firms. Both workers and firms discount the future at rate ρ. The population of workers is normalized to one. For time interval $\triangle$, there are $\delta\triangle$ workers entering the economy, while there are $\delta\triangle$ workers dying out of it. Thus the workers' population is kept to one.

There are high-skill jobs (job A) and low-skill jobs (job B). Workers are high-educated (type H) or low-educated (type L). High-educated workers are productive in both types of jobs, while low-educated ones are so only in type B jobs. A high-skill job (A), which is filled by a high-educated worker, produces instantaneous output, y_A, while a filled low-skill job (B) produces y_B, no matter if it is filled by a high-educated worker or a low-educated one. The high-skill job is more productive than the low-skill one, that is, $y_A > y_B > 0$.

Before entering the job market, firms are identical. It takes instantaneous cost k_A to maintain a job A vacancy, while it costs k_B to hold a job B. It costs more to maintain a high-skill job vacancy than a low-skill one, that is, $k_A > k_B > 0$. Firms enter the market until their value is equal to zero. The fraction of the high-skill job vacancies to the low-skill ones is determined so as to make firms indifferent between the two choices.

The skill is only acquired through education. Workers decide whether to invest in education to obtain the skill before entering the economy.

All the employees can engage in on-the-job search. For a low-educated worker, the only job opportunity is to be hired by a type B employer. On the contrary, a high-educated worker has several options, in terms of accepting job B and searching while employed at a job B. We assume that the economy is productive enough to induce a high-educated worker to always accept a job A. Then his decision is summarized by a pair $(S, T) \in [0, 1] \times [0, 1]$, defined as follows. First, he may accept both types of jobs ($T = 1$), and search for a high-skill one on a job B ($S = 1$). Second, he may accept both types of job offers ($T = 1$), and once he is employed, he is unwilling to leave a low-skill job ($S = 0$). Third, he may accept a job A only ($T = 0$), and he would search for a job A if he were employed at a job B ($S = 1$). Fourth and finally, he may accept a job A only ($T = 0$), and he would *not* search for a job A if he were employed at a job B ($S = 0$). It seems that the difference between the third and fourth options is irrelevant. However, the strategy *off the equilibrium path* does matter with the characterization of each type of equilibrium. We consider only equilibria with pure strategies. Then we can restrict our attention to the four equilibria with the pure strategies described as above.

Firms are not allowed to fire their employees unless the match surplus is reduced to zero by an exogenous shock. Each job is destroyed by the employee's exit out of the economy, which occurs at Poisson arrival rate δ, or by the exogenous shock occurring at Poisson arrival rate σ.

The value functions of high-educated workers and low-educated ones, U_H

and U_L, are written as

$$\rho U_H = b + \alpha_A(W_{HA} - U_H) + \alpha_B t(W_{HB} - U_H) - \delta U_H. \quad (1)$$
$$\rho U_L = b + \alpha_B(W_{LB} - U_L) - \delta U_L. \quad (2)$$

Unemployed workers receive unemployment benefit b. They meet a high-skill job employer at Poisson rate α_A, and a low-skill one at rate α_B. W_{HA}, W_{HB} and W_{LB} represents the value functions of a high-educated employee hired at a high-skill job and at a low-skill one, and that of a low-educated employee hired at a low-skill job, respectively. t denotes the index function, which takes value of zero or one according to the following rule.

$$t = \begin{cases} 1, & \text{iff } W_{HB} - U_H > 0. \\ 0, & \text{otherwise.} \end{cases} \quad (3)$$

In other words, a skilled worker accepts a low-skill job offer if and only if the job produces positive surplus.

The value functions of the three types of employees, W_{SH}, W_{SL}, and W_{UL} are written respectively as

$$\rho W_{HA} = w_{HA} + \sigma(U_H - W_{HA}) - \delta W_{HA}. \quad (4)$$
$$\rho W_{HB} = w_{HB} + \sigma(U_H - W_{HB}) - \delta W_{HB} + s[\alpha_A(W_{HA} - W_{HB}) - d] \quad (5)$$
$$\rho W_{LB} = w_{LB} + \sigma(U_L - W_{LB}) - \delta W_{LB} \quad (6)$$

Workers receive the wages specific to the matches, w_{HA}, w_{HB}, and w_{LB}, which depend on the workers' skill and the types of firms. The matches are destroyed at instantaneous rates, σ and δ. A high-educated worker employed at a job B searches for a job A if and only if he obtains positive surplus net of the search cost d. In other words,

$$s = \begin{cases} 1, & \text{iff } \quad \alpha_A(W_{HA} - W_{HB}) - d > 0. \\ 0, & \text{otherwise.} \end{cases} \quad (7)$$

Let us turn to the demand side for workers. The value functions of firms with a vacancy, V_A and V_B, are written as

$$\rho V_A = -k_A + \alpha_H(J_{HA} - V_A). \quad (8)$$
$$\rho V_B = -k_B + \alpha_H t(J_{HB} - V_B) + \alpha_L(J_{LB} - V_B). \quad (9)$$

A high-skill vacancy is filled at rate α_H as the firm meets a skilled worker, while a low-skill vacancy is filled by a high-educated worker at rate $\alpha_H t$, or by a low-educated one at rate α_L, where $t = 0, 1$ represents the joint decision made by a high-educated worker and a low-skill job employer. There is no disagreement in their decision whether to form a match, since the surplus is divided through the Nash bargaining.

The Bellman equations for employers, J_{SH}, J_{SL}, and J_{UL} are written respectively as follows.

$$
\begin{aligned}
\rho J_{HA} &= y_A - w_{HA} + (\sigma + \delta)(V_A - J_{HA}), && (10)\\
\rho J_{HB} &= y_B - w_{HB} + (\sigma + \delta + s\alpha_A)(V_B - J_{HB}), && (11)\\
\rho J_{LB} &= y_B - w_{LB} + (\sigma + \delta)(V_B - J_{LB}). && (12)
\end{aligned}
$$

The interpretation of these equations are as straightforward as above. At a low-skill job, a high-educated worker is as productive as a low-educated one. The matches are destroyed by an exogenous shock at rate σ and by the employee's passing at rate δ. Furthermore, a high-educated worker leaves his low-skill employer if he engages in search and find a high-skill vacancy, which occurs at rate $s\alpha_A$.

The wages, w_{HA}, w_{HB}, and w_{LB}, are determined by the Nash Bargaining with equal bargaining power, respectively as follows.

$$
\begin{aligned}
W_{HA} - U_H &= J_{HA} - V_A,\\
W_{HB} - U_H &= J_{HB} - V_B,\\
W_{LB} - U_L &= J_{LB} - V_B. && (13)
\end{aligned}
$$

The Nash bargaining divides the surplus created by the match in a foward-looking way, and the bargaining outcome is continuously renegotiated. In other words, the bargaining participants negotiate over the surplus, taking the value of unmatched states as their threat points. Therefore, even in the bargaining between a high-skill firm and a high-educated worker currently employed at a low-skill job, the threat point for the worker is U_H, not W_{HB}.

The matching probability, α_A, α_B, α_H, and α_L, are endogenously calculated as follows. The aggregate matching function $m(\, , \,)$, which represents the mass of meetings between firms and workers per unit time, is a standard one, which exhibits constant returns to scale in the mass of vacancies, $F_A + F_B$, and job searchers, $N_H + N_L + N_{HB}$.

$$m = m(N_H + N_L + N_{HB}, \; F_A + F_B).$$

Define

$$q = \frac{F_A + F_B}{N_H + N_L + N_{HB}}, \tag{14}$$

which represents the mass of vacancies per job searchers. The higher q is, the tighter the labor market is. Then the probability that a worker meets a firm with a vacancy is given by

$$\mu(q) \equiv \frac{m(N_H + N_L + N_{HB},\ F_A + F_B)}{N_H + N_L + N_{HB}} = m\left(1, \frac{F_A + F_B}{N_H + N_L + N_{HB}}\right) = m(1, q). \tag{15}$$

Note that the probability that a firm meets a job searcher, on or off the job, is given by $\mu(q)/q$. Let

$$g = \frac{N_H + N_{HB}}{N_H + N_L + N_{HB}}, \text{ and } h = \frac{F_A}{F_A + F_B} \tag{16}$$

denote the fraction of high-educated job searchers, and that of high-skill vacancies, respectively. Then we can write

$$\alpha_A = \mu(q)h,\ \alpha_B = \mu(q)(1-h),\ \alpha_H = \frac{\mu(q)g}{q},\ \alpha_L = \frac{\mu(q)(1-g)}{q}. \tag{17}$$

Let θ denote the fraction of skilled workers entering the labor market. Then $\theta\delta$ workers enter the labor market per unit time. Using the above definitions, the steady state is characterized by the following accounting equations.

$$\begin{aligned} \theta\delta + \sigma(N_{HA} + N_{HB}) &= [\delta + \mu(q)h + \mu(q)(1-h)t]N_H \\ \mu(q)(1-h)tN_H &= [\sigma + \delta + \mu(q)hs]N_{HB} \\ \mu(q)h(N_H + sN_{HB}) &= (\sigma + \delta)N_{HA} \\ (1-\theta)\delta + \sigma N_{LB} &= [\delta + \mu(q)(1-h)]N_L \\ 1 &= N_H + N_L + N_{HA} + N_{HB} + N_{LB} \end{aligned} \tag{18}$$

Firms enter the economy until the value of holding a vacancy equals zero. That is,

$$V_A = V_B = 0. \tag{19}$$

Finally, the proportion of new workers who choose to be high-educated, θ, is determined as follows. A worker chooses to be high-educated if the gain from education exceeds its cost. If workers are identical in terms of the education cost, the fraction of high-educated is determined such that

$$U_H(\theta) - U_L(\theta) = c, \tag{20}$$

where c represents of the cost of education, which is constant. If the worker does not pay c, he stays low-educated.

Alternatively, consider that workers are heterogeneous in terms of the cost of education. Each worker is indexed by $x \in [0,1]$ with distribution function $F(x)$. The cost of education is an increasing and strictly convex function of x. It costs $c(x)$ for a worker indexed by x to become a high-educated worker, while it costs zero to stay low-educated. The mass of high-educated workers is so determined that the workers at the threshold are indifferent between being high-educated and remaining low-educated. This heterogeneity of workers does not matter once they enter the labor market.

The cost of education is an increasing and strictly convex function $c(x)$, with $c'(x) > 0,\ c"(x) > 0,\ c(0) = 0,\ \lim_{x\to 1} c(x) = \infty$. Define x^* such that

$$U_H(\theta) - U_L(\theta) \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} c(x), \quad \text{for} \left\{ \begin{array}{c} x < x^* \\ x = x^* \\ x > x^* \end{array} \right\}$$

That is, a worker indexed by x^* is indifferent between to be high-educated and to stay low-educated. Then, by definition, we have $\theta = F(x^*)$, or equivalently, $x^* = F^{-1}(\theta)$. Define also $\Gamma(\theta) \equiv c(F^{-1}(\theta))$ with $\Gamma'(\theta) > 0,\ \Gamma"(\theta) > 0,\ \Gamma(0) = 0,\ \lim_{x\to 1} \Gamma(\theta) = \infty$. That is, $\Gamma(\theta)$ is an increasing and strictly convex function of θ. Then, the fraction of the high-educated is determined such that

$$U_H(\theta) - U_L(\theta) = \Gamma(\theta). \tag{21}$$

4 Multiple Equilibria

Definition. A steady state equilibrium is a set of value functions

$$\{U_H,\ U_L,\ W_{HA},\ W_{HB},\ W_{LB},\ V_A,\ V_B,\ J_{HA},\ J_{HB},\ J_{LB}\},$$

wages

$$\{w_{HA},\ w_{HB},\ w_{LB}\},$$

steady state population distribution

$$\{N_H,\ N_L,\ N_{HA},\ N_{HB},\ N_{LB}\},$$

search and job acceptance decisions

$$\{s,\ t\},$$

the fraction of high-skill jobs h, the fraction of high-educated job searchers g, the tightness of the labor market q, and the proportion of workers who choose to be skilled when entering the labor market, θ, which satisfies the Bellman Equations and search and job acceptance conditions (1) through (12), the Nash bargaining conditions (13), the definitions of the matching probabilities (14) through (17), the steady state accounting (18), the firms' entry condition (19), and the workers' education decision (20) or (21).

There are four types of pure-strategy equilibrium, each of which is characterized by a pair $(S,\ T)$ where $S = s = 0, 1$ and $T = t = 0, 1$. S or T represents the agents' decision in equilibrium.

First, in a type (1,1) equilibrium, denoted by U (Unfaithful), a high-educated worker accepts a low-skill job vacancy, and engages in search while he is on that job. Second, in a type (0, 1) equilibrium, denoted by F (Faithful), a high-educated worker accepts a low-skill job offer and he does not search while he is on that job. In a type (1,0) equilibrium, denoted by C_{10} (Choosy), a high-educated worker does not accept a low-skill job offer, and he would search if he happened to work at a low-skill job. Finally, in a type (0, 0) equilibrium, denoted by C_{00}, a skilled worker does not accept a low-skill job, but he would not search if he happened to work at that job. One might suspect that the distinction between C_{10} and C_{00} is irrelevant. However, the assumption on the action taken by the high-educated worker *off the equilibrium path* determines the parameter range, with which each type of equilibrium exists.

Define

$$\begin{aligned} B(S,T) &\equiv \mu(q)h(W_{HA} - W_{HB}) - d \\ D(S,T) &\equiv W_{HB} - U_H \end{aligned}$$

Then, by definition, it follows.

A type (1, 1) equilibrium (U) exists if and only if $B(1,1) \geq 0$ and $D(1,1) \geq 0$.

A type (0, 1) equilibrium (F) exists if and only if $B(0,1) < 0$ and $D(0,1) \geq 0$.

A type (1, 0) equilibrium (C_{10}) exists if and only if $B(1,0) \geq 0$ and $D(1,0) < 0$.

A type (0, 0) equilibrium (C_{00}) exists if and only if $B(0,0) < 0$ and $D(0,0) < 0$.

It is hard to obtain the closed-form expressions of $B(S,\ T)$ and $D(S,\ T)$ for each equilibrium where q, g, h, and θ are endogenously determined. However, it is straightforward to characterize the parameter range in a $(y_B,\ y_A)$ plane, for which each equilibrium exists, taking q, g, h, and θ as given. The results are summarized in the following proposition.

Proposition 1 *An equilibrium U exists if and only if $y_A \geq y_1$ and $y_B \geq b+d$, where*

$$\begin{aligned} y_1 &= \left[\frac{\mu(q)+2(\rho+\sigma+\delta)}{\mu(q)(1+h)+2(\rho+\sigma+\delta)}\right] y_B + \left[\frac{\mu(q)h}{\mu(q)(1+h)+2(\rho+\sigma+\delta)}\right] b \\ &\quad + \left[\frac{\mu(q)h+2(\rho+\sigma+\delta)}{\mu(q)h} - \frac{\mu(q)+2(\rho+\sigma+\delta)}{\mu(q)(1+h)+2(\rho+\sigma+\delta)}\right] d. \end{aligned}$$

An equilibrium F exists if and only if

$$y_A < \min\{y_2,\ y_3\},$$

where

$$\begin{aligned} y_2 &= y_B + \left[\frac{2(\rho+\sigma+\delta)}{\mu(q)h}\right] d, \\ y_3 &= \left[\frac{\mu(q)h+2(\rho+\sigma+\delta)}{\mu(q)h}\right] y_B - \left[\frac{2(\rho+\sigma+\delta)}{\mu(q)h}\right] b. \end{aligned}$$

An equilibrium C_{10} exists if and only if $y_A \geq y_4$ and $y_B < b+d$.

$$\begin{aligned} y_4 &= \left(\frac{1}{2}\right)\left[\frac{\mu(q)h+2(\rho+\sigma+\delta)}{\mu(q)h+\rho+\sigma+\delta}\right] y_B \\ &\quad + \left(\frac{1}{2}\right)\left[\frac{\mu(q)h}{\mu(q)h+\rho+\sigma+\delta}\right] b + \left(\frac{1}{2}\right)\left[\frac{[\mu(q)h+2(\rho+\sigma+\delta)]^2}{\mu(q)h[\mu(q)h+\rho+\sigma+\delta]}\right] d \end{aligned}$$

An equilibrium C_{00} exists if and only if $y_3 \leq y_A < y_2$.

Proof. See APPENDIX

The regions in which each type of equilibrium exists are illustrated in Figure 5. Since we assume $y_A > y_B$, all the equilibria exist only above the 45 degree line. In the region where both the productivity of job B and the

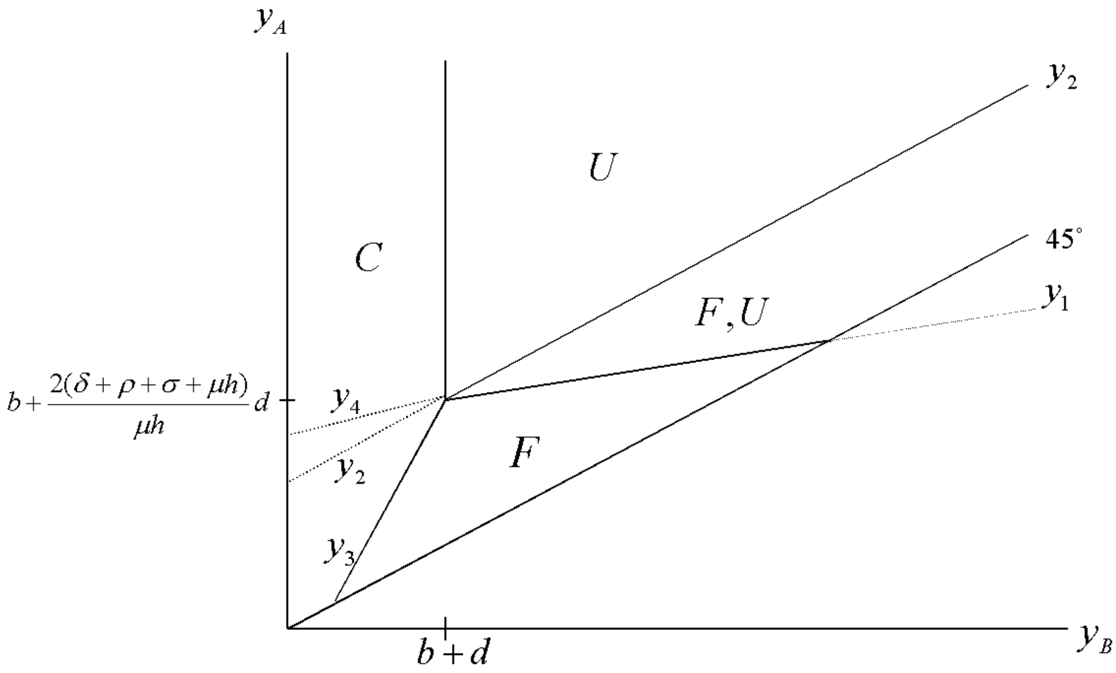

Figure 5: Multiple Equilibria

productivity gap between the two jobs are sufficiently large, there exist the Unfaithful equilibria. Below this region, there is an area where the Faithful equilibria exist, due to the relatively low productivity gap. If the productivity gap is in-between, there are both the Faithful and Unfaithful equilibria. If the productivity is too low to cover the sum of the unemployment benefit and the on-the-job search cost, high educated workers do not accept job B and the Choosy equilibria prevail. If the productivity gap is relatively large, high-educated workers would search once they were hired at low-skill jobs, while they would not if both productivity A and B are so low.

Even if the other variables, q, h, g, and θ, are endogenously determined, the equilibria have the same configuration as in Figure 4. Here is the main result of this paper.

Proposition 2 *There exists a unique steady-state equilibrium for all* s, $t = 0, 1$

Proof. See APPENDIX.

It must be emphasized that the driving force toward multiple equilibria in this paper is the intra-match instability, but not the traditional 'thick-market' externality, developed by Diamond [1982].

The intra-match instability works as follows. There is no productivity gap between high-educated workers and low-educated ones if they are employed at low-skill jobs. With on-the-job search, the joint surplus created at a low-skill job filled by a high-educated worker is lower than the one filled by a low-educated worker, since the former relation faces a higher probability of destruction. This difference is reflected by a reduction of the wage for high-educated workers, which in turn reduces the incentive for them to stay at their current jobs, and induces them to engage in on-the-job search. Due to this instability, the identical productivity gap may produce multiple equilibria.

5 Education and the Skill-Biased Technological Progress

In this section, we explore the implication of the complementarity between education choice and endogenous job creation.

It is widely argued that the increases in the gaps of both wages and unemployment rates in the past few decades have been caused by the technological progress that favors high-educated workers, who can easily acquire the high skill. Acemoglu [1999] constructs a model in which two types of jobs are endogenously created depending on the availability of high-educated workers. If the mass of high-educated workers is relatively low, both high-educated workers and low-educated ones are hired at the identical jobs. However, if the mass of high-educated workers exceeds some threshold, it becomes profitable for firms to create high-skill jobs for high-educated workers and low-skill jobs for low-educated ones, and the labor market is segmented with respect to education. In this separated equilibrium, one can observe a larger unemployment rates of both types of workers and a remarkable wage gap between high-educated and low-educated workers. In this framework, however, there is no mechanism to create the large unemployment rate *gap* across education, which is a typical evidence in the OECD economies. If the labor market is perfectly segmented with respect to education, there is no advantage or disadvantage for high-educated workers. Albrecht and Vroman [2002], and Dolado, Jansen, and Jimeno [2002] develop a plausible framework to explain the unemployment rate gap by introducing the crowding-out of low-educated workers by high-educated ones. However, in all the previous models, workers' education choice is exogenous, and there is no analysis on the channel through which the skill-biased technological progress in fact induces more workers to acquire higher education.

If jobs are endogenously created, there arises a complementarity between education and job creation. The more workers are high-educated, the more high-skill vacancies are created, which in turn creates an additional incentive for workers to acquire higher education. Therefore, the gain from education is an *increasing* function of the fraction of the high-educated. This property implies that an increase in the productivity gap between the two jobs *reduces* the fraction of high-educated workers, if workers are homogeneous in the cost of education. However, if workers are heterogeneous, a skill-biased technological progress, as is expected, results in a larger fraction of high-educated workers with larger gaps both in unemployment rates and wages across education. These outcome is illustrated in Figures 2 and 3. For these results, the complementarity, or the positive feedback between education and its gain, plays a crucial role. If the skill-biased technological progress shifts up the gain from education, all these results automatically follow. We can analytically show this result.

Proposition 3 *An increase in the productivity gap, $y_A - y_B$, taking y_B as given, increases the gain from education, $U_H(\theta) - U_L(\theta)$.*

Proof. See APPENDIX.

As is illustrated in Fig. 2B, the skill-biased technological progress reduces the fraction of high-educated workers. The possibility of the negative relationship between education choice and the skill-biased technological progress has never been pointed out before. In the previous literature, the incentive to invest in education is hindered by the holdup problem, which works as follows. The match quality depends on the level of investments which are done by the potential partners before the match is formed. Since some portion of the benefits of investment done by one partner erodes into the other partner, the amount of investment in equilibrium is lower than its social optimal level. However, the skill-biased technological progress creates more incentives to invest in the match by increasing its marginal return, even if the amount is lower than its social optimum. In other words, the larger productivity gap between high-skill jobs and low-skill ones induces firms to create more high-skill job vacancies, and encourages workers to acquire higher education. Then we obtain a positive relationship between the technological progress and education as long as all the agents take the identical actions in equilibrium. In the previous literature, all the agents choose the identical education levels, depending on the productivity gap. Then a higher productivity gap induces them to invest more in education.

However, due to the discrete nature of education, it is more plausible to consider an economy where workers choose different education levels. One must usually spend four years to complete college education. If you drop out before completion, it is difficult for you to be hired at any jobs which are designed for college graduates. You have to pay tuitions for the whole four academic years in order to obtain a bachelor's degree. Your benefit from education is close to zero even if you pay for one year. Therefore, in the neoclassical theory of education, the population of workers who complete college education is so determined as to equate the difference between the values of being with and without a bachelor's degree, to its cost.

If job creation is exogenous, the benefit of acquiring education is decreasing in the mass of high-educated agents. The more agents are high-educated, the more difficult it is for them to find high-skill jobs or get high wages. Then the skill-biased technological progress increases the fraction of high-educated workers as is illustrated in Figure 1. On the contrary, if job creation is endogenous, things are totally different. The benefit of acquiring education is increasing in the mass of high-educated agents, since the larger mass of high-educated induces firms to create more high-skill job vacancies. The skill-biased technological progress shifts up the benefit of acquiring education and reduces the fraction of high-educated workers unless there is a sufficiently rise in the cost of education, as is illustrated in Figure 2.

With the workers' heterogeneity in terms of the education cost, we can avoid this unpleasant result. The marginal gain from education is decreasing in the fraction of high-educated workers, while the marginal cost of education is increasing, and then the gain from education locus crosses the cost of education locus from above, as in Fig. 3A. Hence the skill-biased technological progress induces the marginal workers to invest in education with the higher education cost. In other words, an increase in the productivity gap creates more college-graduates and raises the average cost of education, which is the stylized fact in the OECD counties.

The result in the last paragraph is consistent with the famous 'signaling' theory of education. Being high-educated is advantageous over staying low-educated both in wages and in employment opportunities. If workers are heterogeneous in the education cost, the labor market organizes itself toward segmentation with respect to education. On the other hand, if workers are identical in the education cost, this mechanism does not work. A skill-biased technological productivity shock increases the gain from education. With the constant cost, the marginal gain from education is always greater than its marginal cost, which is equal to zero. Therefore, the equilibrium is unstable. On the contrary, with the convex cost of education, the marginal gain from education is less than its marginal cost at an equilibrium. Therefore the equilibrium is stable. Also, an increase in the productivity gap induces more workers to be high-educated.

We can parameterize the model to produce such endogenous variables that match the stylized fact. We restrict out calibration here to the Unfaithful equilibrium with workers' heterogeneity in the cost of education. In Table 1, It is shown that a skill-biased technological progress increases the fraction of high-educated workers, the aggregate unemployment rate, and the inequality in unemployment rates and wages. Here is a list of the parameters that we use to calibrate the model.

$$\begin{aligned} \rho &= .04,\ \delta = .025,\ \sigma = .01,\ b = d = 1,\ k_A = 6,\ k_B = 4, \\ \mu(q) &= q^\gamma,\ \gamma = .5,\ \Gamma(\theta) = c\theta^\lambda - 10,\ c = 70,\ \lambda = 1.5. \end{aligned}$$

We have chosen these values to produce some realistic simulation results. We examine the effects of changes in the productivity gap, $y_A - y_B$, from 20% to 40% of y_B. As shown in Table 1, an increase in the productivity gap dramatically increases the fractions of high-educated workers (θ) and of high-skill job vacancies (h), as is expected. Note that, however, its effects on the tightness of the labor market (q) and on the chance to meet a high-educated worker (g) are modest. This is a natural consequence of the zero-profits condition of the firms' entry.[2] The effects on the unemployment rates are consistent with the stylized facts. The skill-biased technological progress increases the aggregate unemployment rate (u), while it has opposite effects on between high-educated and low-educated workers. The technological progress reduces the unemployment rate of the high-educated (u_H), but increases that of the

[2] You can also confirm that, with the productivity gap as given, the effect of a rise in θ on the tightness of the labor market (q) on each locus of the gain from education, is negligibly small.

y_A	y_B	θ	h	q	g	B_{11}	u	u_H	u_L	u_L/u_H	w_{max}	w_{min}
5.6	4.0	.55	.45	.59	.58	1.20	5.84	4.34	7.68	1.65	1.49	.92
5.2	4.0	.45	.36	.56	.57	.73	5.76	4.43	6.84	1.54	1.34	.93
4.8	4.0	.34	.27	.54	.55	.24	5.56	4.51	6.10	1.35	1.19	.95

Table 1: The Effects of the Skill-Biased Technological Progress (the unemployment rates are shown in percent)

low-educated (u_L). Therefore, the unemployment rate gap (u_L/u_H) dramatically rises from 1.35 to 1.65. The measures of wage inequality are defined as follows.

$$w_{\mathsf{max}} \equiv \frac{w_{HA}}{w_{HB}}, \quad w_{\mathsf{min}} \equiv \frac{w_{HB}}{w_{LB}}.$$

In words, w_{max} represents the wage inequality *across* jobs, while w_{min} the wage inequality *within* job. Then, a rise in the productivity gap increases the wage gap across jobs considerably, but its effect on the wage gap within the low-skill job is negative and small. Note that a high-educated worker is paid lower than a low-educated one in the low-skill job.

So far, we restrict our attention to a single type of equilibrium. That is, the skill-biased technological progress either increases or decreases the fraction of high-educated workers, depending on the nature of the cost of education, as long as the economy stays at a single type of equilibrium.

However, a drastic technological progress might make the original equilibrium no longer sustainable. If the technological progress is sufficiently large, the economy will switch from an equilibrium without on-the-job search to one with on-the-job search. What is the effect of this regime switch on the gain from education? The reader might guess that the gain from education shifts up, since on-the-job search is an additional benefit of being high-educated. However, this is not true. The result is stated without proof as follows.

Proposition 4 *A switch from without to with on-the-job search reduces the gain from education $U_H(\theta) - U_L(\theta)$ for all $\theta \in [0,\ 1]$.*

At the first look, this proposition is counter-intuitive. It is interpreted as follows. With θ as given, a switch from without to with on-the-job search implies that there is a reduction in the wage for high-educated workers at low-skill jobs as long as the cost of on-the-search is sufficiently small[3], which

[3] This property is confirmed in APPENDIX A2. On-the-job search reduces the surplus

induces firms to create more low-skill job vacancies. Then high-educated workers find it harder to be employed at high-skill jobs, and the gain from education declines.

Therefore, a switch from without to with on-the-job search leads to the following consequences, depending on the nature of the cost of education. If the cost of education is constant under the homogeneous population, the regime switch *increases* the fraction of high-educated workers. On the contrary, if the population is heterogeneous in the cost of education, the switch *reduces* both the fraction of high-educated workers and the average cost of education.

Since the equilibrium with the constant cost of education is unstable, it is appropriate to choose the model with heterogeneous cost of education in describing our economy. Then the outcome of the switch from without to with on-the-job search is inconsistent with the stylized fact that both the fraction of high-educated and the cost of education have been increasing over time in the OECD countries. Therefore, the rise in the high-educated workers and the growing inequality across education in the recent decades is interpreted as a change within the Unfaithful equilibria with on-the-job search.

6 Conclusion

The paper explores the macroeconomic consequences of the complementarity between education and job creation. There is a positive feedback between education and job creation in the sense that the more workers are high-educated, the more profitable it is for firms to create high-skill vacancies, and vice versa. With this complementarity, the economy shows the properties which stand in a high contrast with those expected by the neoclassical theory of education. The gain from education is increasing in the fraction of high-educated workers. The skill-biased technological progress induces less workers to invest in the higher education if the education cost is constant, while it induces more workers to acquire the higher education if workers are heterogeneous in the education cost. The paper succeeds in modelling the stylized fact of the rise in the fraction of the high-educated associated with the rise in the education cost in the OECD countries. There are multiple

---

from a low-skill job filled by a high-educated worker, which makes it more profitable on average for frims to create low-skill vacancies.

equilibria if the productivity gap between jobs is relatively small. However, a switch from without to with on-the-job search induces more workers to invest in education only if workers are identical in the cost of education.

There are several directions into which the analysis can be extended. First, we can consider the deterioration of technology as follows. Assume that an exogenous shock hits a filled high-skill job and changes its quality from of high-skill to of low-skill. If the shock deteriorates the employee's skill only, there is an incentive for the high-skill job employer to fire him and search for a newly high-educated worker. Even if a worker acquired high education when he was young, it is very unlikely to occur that his skill is still at state-of-art after he spent many years at the job. Second, it is interesting to consider the roles of labor market institutions if there is a deterioration of the skill. If the employee's skill is no longer at state-of-art, the employer wants to fire him while the employee sticks to the job. Under some legal employment protection, the employer cannot fire his employee at his will. Then we expect that the gain from creating high-skill vacancies declines, and the positive feedback might work as to reduce the fraction of high-educated workers. All these agenda are left to our future work.

7 APPENDIX

A1. Proof of Proposition 1

It is a matter of algebra to see that

$$\begin{aligned}
B(1,1) &\geq 0, \text{ if and only if} \\
y_A &\geq \left[\frac{\mu(q)+2(\rho+\sigma+\delta)}{\mu(q)(1+h)+2(\rho+\sigma+\delta)}\right] y_B + \left[\frac{\mu(q)h}{\mu(q)(1+h)+2(\rho+\sigma+\delta)}\right] b \\
&\quad + \left[\frac{\mu(q)h+2(\rho+\sigma+\delta)}{\mu(q)h} - \frac{\mu(q)+2(\rho+\sigma+\delta)}{\mu(q)(1+h)+2(\rho+\sigma+\delta)}\right] d \\
D(1,1) &\geq 0 \quad \text{if and only if} \quad y_B \geq b+d
\end{aligned}$$

$$\begin{aligned}
B(0,1) &< 0, \text{ if and only if } y_A < y_B + \left[\frac{2(\rho+\sigma+\delta)}{\mu(q)h}\right] d \\
D(0,1) &\geq 0, \text{ if and only if } y_A < \left[\frac{\mu(q)h+2(\rho+\sigma+\delta)}{\mu(q)h}\right] y_B - \left[\frac{2(\rho+\sigma+\delta)}{\mu(q)h}\right] b
\end{aligned}$$

$$
\begin{aligned}
B(1,0) &\geq 0 \\
&\text{iff } \left(\frac{1}{2}\right)\left[\frac{\mu(q)h+2(\rho+\sigma+\delta)}{\mu(q)h+\rho+\sigma+\delta}\right] y_B \\
&+\left(\frac{1}{2}\right)\left[\frac{\mu(q)h}{\mu(q)h+\rho+\sigma+\delta}\right] b+\left(\frac{1}{2}\right)\left[\frac{[\mu(q)h+2(\rho+\sigma+\delta)]^2}{\mu(q)h[\mu(q)h+\rho+\sigma+\delta]}\right] d \\
D(1,0) &< 0 \quad \text{iff} \quad y_B < b+d
\end{aligned}
$$

$$
\begin{aligned}
B(0,0) &< 0 \quad \text{iff} \quad y_A < y_B + \left[\frac{2(\rho+\sigma+\delta)}{\mu(q)h}\right] d \\
D(0,0) &> 0 \quad \text{iff} \quad y_A \geq \left[\frac{\mu(q)h+2(\rho+\sigma+\delta)}{\mu(q)h}\right] y_B - \left[\frac{2(\rho+\sigma+\delta)}{\mu(q)h}\right] b
\end{aligned}
$$

The lemma follows straightforwardly. ¥

A2. Proof of Proposition 2

We prove the existence of an equilibrium U, which is the hardest task. For the other three equilibria, you can prove the existence in a similar way.

Set $s = t = 1$. Solving the Bellman equations and the Nash bargaining conditions, we have

$$
\begin{aligned}
w_{HA} &= \left(\frac{\rho+\sigma+\delta+\mu h}{2(\rho+\sigma+\delta)+\mu h}\right) y_A \\
&+\left(\frac{2(\rho+\sigma+\delta)(\rho+\sigma+\delta+\mu h)}{[2(\rho+\sigma+\delta)+\mu h][2(\rho+\sigma+\delta)+\mu(1+h)]}\right) b \\
&+\left(\frac{\mu(1-h)(\rho+\sigma+\delta)}{[2(\rho+\sigma+\delta)+\mu h][2(\rho+\sigma+\delta)+\mu(1+h)]}\right)(y_B-d) \\
w_{HB} &= \left(\frac{\rho+\sigma+\delta+\mu}{2(\rho+\sigma+\delta)+\mu(1+h)}\right) y_B + \left(\frac{\rho+\sigma+\delta+\mu h}{2(\rho+\sigma+\delta)+\mu(1+h)}\right)(b+d) \\
w_{LB} &= \left(\frac{\rho+\sigma+\delta+\mu(1-h)}{2(\rho+\sigma+\delta)+\mu(1-h)}\right) y_B + \left(\frac{\rho+\sigma+\delta}{2(\rho+\sigma+\delta)+\mu(1-h)}\right) b
\end{aligned}
$$

Note that

$$
w_{HB}-w_{LB} = \frac{\mu^2 h(h-1)(y_B-b)+(\rho+\sigma+\delta+\mu h)[2(\rho+\sigma+\delta)+\mu(1-h)]d}{[2(\rho+\sigma+\delta)+\mu(1+h)][2(\rho+\sigma+\delta)+\mu(1-h)]}
$$

Therefore, $w_{HB} < w_{LB}$ if $d = 0$. In other words, high-educated workers are paid lower than low-educated ones if the search cost equals zero.

Using $V_A = V_B = 0$, we can solve (10) through (12) for $J_{HA},\ J_{HB}$, and J_{LB} to obtain

$$J_{HA} = \frac{y_A - w_{HA}}{\rho + \sigma + \delta},\ J_{HB} = \frac{y_B - w_{HB}}{\rho + \sigma + \delta + \mu h},\ J_{LB} = \frac{y_B - w_{LB}}{\rho + \sigma + \delta}$$

With these expression the fact $V_A = V_B = 0$ leads to

$$\nu(q)g = \frac{[2\phi + \mu(1+h)][2\phi + \mu h]k_A}{[2\phi + \mu(1+h)]y_A - \mu(1-h)(y_B - d) - 2(\phi + \mu h)b} \tag{22}$$

$$\nu(q)(1-g) = \frac{[2\phi + \mu(1-h)]\lambda(\mu(q), h)}{(y_B - b)\{[2\phi + \mu(1+h)]y_A - \mu(1-h)(y_B - d) - 2(\phi + \mu h)b\}} \tag{23}$$

where $\phi = \sigma + \delta + \rho$ and

$$\begin{aligned}\lambda(\mu(q), h) &= [2\phi + \mu h](-y_B + b + d)k_A \\ &\quad + \{[2\phi + \mu(1-h)](y_A - b) + \mu(1-h)(-y_B + d)\}k_B\end{aligned}$$

The steady-state accounting is solved for

$$\begin{aligned}N_S &= \frac{\theta(\sigma+\delta)}{\sigma+\delta+\mu},\ N_U = \frac{(1-\theta)(\sigma+\delta)}{\sigma+\delta+\mu(1-h)} \\ N_{HA} &= \frac{\theta\mu h}{\sigma+\delta+\mu h},\ N_{HA} = \frac{\theta\mu(1-h)(\sigma+\delta)}{(\sigma+\delta+\mu)(\sigma+\delta+\mu h)},\ N_{LB} = \frac{\mu(1-h)(1-\theta)}{\sigma+\delta+\mu(1-h)}\end{aligned}$$

Then the fraction of high-educated workers to all the job searchers is computed as

$$g = \frac{\theta[\sigma + \delta + \mu(1-h)]}{\sigma + \delta + \mu h(1-\theta) + \theta\mu(1-h)} \tag{24}$$

Substitute (24) into (22) and (23), we obtain two equations that are solved for an equilibrium $(q,\ h)$ as $H(q,h) = G(q,h) = 0$ where

$$\begin{aligned}H(q,h) &\equiv H_1(q,h) - H_2(h,q) = 0, \\ G(h,q) &\equiv G_1(h,q) - G_2(h,q) = 0,\end{aligned}$$

where

$$
\begin{aligned}
H_1(h,q) &\equiv \frac{\mu(q)}{q}\left(\frac{\theta[\sigma+\delta+\mu(q)(1-h)]}{\sigma+\delta+\mu(q)[(1-2\theta)h+\theta]}\right)\\
H_2(h,q) &\equiv \frac{[2\phi+\mu(q)(1+h)][2\phi+\mu(q)h]k_A}{[(y_A+y_B-2b-d)h+y_A-y_B+d]\mu(q)+2\phi(y_A-b)}\\
G_1(h,q) &\equiv \frac{\mu(q)}{q}\left(\frac{(1-\theta)(\sigma+\delta+\mu(q)h)}{\sigma+\delta+\mu(q)[(1-2\theta)h+\theta]}\right)\\
G_2(h,q) &\equiv \left(\frac{2\phi+\mu(q)(1-h)}{y_B-b}\right)\\
&\quad \times\left(\frac{[2\phi+\mu(q)h](b+d-y_B)k_A}{[(y_A+y_B-2b-d)h+y_A-y_B+d]\mu(q)+2\phi(y_A-b)}+k_B\right).
\end{aligned}
$$

It is a matter of algebra to see

$$
\frac{\partial H_1(h,q)}{\partial h}<0,\ \frac{\partial H_2(h,q)}{\partial h}>0,\ \frac{\partial H_1(h,q)}{\partial q}<0,\ \frac{\partial H_2(h,q)}{\partial q}>0,
$$

$$
\frac{\partial G_1(h,q)}{\partial h}>0,\ \frac{\partial G_2(h,q)}{\partial h}<0,\ \frac{\partial G_1(h,q)}{\partial q}<0,\ \frac{\partial G_2(h,q)}{\partial q}>0.
$$

The exact forms of these derivatives are too complicated to be displayed here. Therefore,

$$
\begin{aligned}
H_h &\equiv \frac{\partial H(h,q)}{\partial h}=\frac{\partial H_1(h,q)}{\partial h}-\frac{\partial H_2(h,q)}{\partial h}<0,\\
H_q &\equiv \frac{\partial H(h,q)}{\partial q}=\frac{\partial H_1(h,q)}{\partial q}-\frac{\partial H_2(h,q)}{\partial q}<0. \qquad (25)
\end{aligned}
$$

Taking the total derivative of $H(h,q)=0$ and using (25), we obtain

$$
\left.\frac{dq}{dh}\right|_{H(h,q)=0}=-\left.\frac{H_h}{H_q}\right|_{H(h,q)=0}<0
$$

Similarly, we have

$$
\begin{aligned}
G_h &\equiv \frac{\partial G(h,q)}{\partial h}=\frac{\partial G_1(h,q)}{\partial h}-\frac{\partial G_2(h,q)}{\partial h}>0,\\
G_q &\equiv \frac{\partial G(h,q)}{\partial q}=\frac{\partial G_1(h,q)}{\partial q}-\frac{\partial G_2(h,q)}{\partial q}<0.
\end{aligned}
$$

Using these expressions, we obtain

$$\left.\frac{dq}{dh}\right|_{G(h,q)=0} = -\left.\frac{G_h}{G_q}\right|_{G(h,q)=0} > 0$$

Therefore, on a $(h,\ q)$ plane, $H(h,q)=0$ is a downward sloping locus, while $G(h,q)=0$ is an upward sloping one, in Figure 5.

The remaining thing we need to show is that $H(0,q) > G(0,q)$ and $H(1,q) < G(1,q)$. Define $\xi = k_A - k_B$. Then we have

$$\begin{aligned}
& H(0,q) - G(0,q) \\
= & \left(\frac{2\phi + \mu(q)}{y_B - b}\right)\left[\left(\frac{[2(y_A+b+d)-4y_B]\phi + (y_A - y_B + d)\mu(q)}{[2(y_A - b)\phi + (y_A - y_B + d)\mu(q)]}\right)k_A - \xi\right] \\
& + \frac{\mu(q)}{q}\left(\frac{\theta\mu(q) - (1-2\theta)(\sigma+\delta)}{\sigma+\delta+\theta\mu(q)}\right) \\
> & \ 0,
\end{aligned}$$

and

$$\begin{aligned}
& H(1,q) - G(1,q) \\
= & \left(\frac{(b-y_B)\mu(q)^2 + (2y_A - 4y_B + 2b + d)\phi\mu(q) + (2y_A - 4y_B + 2b + 2d)\phi^2}{[\phi+\mu(q)](y_A - b)(y_B - b)}\right)k_A \\
& -2(y_B - b)\phi\xi + \frac{\mu(q)}{q}\left(\frac{(1-\theta)\mu(q) - (1-2\theta)(\sigma+\delta)}{\sigma+\delta+(1-\theta)\mu(q)}\right) \\
< & \ 0.
\end{aligned}$$

Therefore, we have obtained the existence and uniqueness of (h,q) for any $\theta \in [0,1]$. We can write h and q as functions of θ:

$$h = h(\theta), \quad q = q(\theta)$$

Some comparative statics are obtained as follows. Taking h as given, we can write the model as

$$H(q,\ \theta) = 0, \quad G(q,\ \theta) = 0.$$

Therefore, we have

$$\left.\frac{dq}{d\theta}\right|_{H=0} = -\frac{H_\theta}{H_q} > 0, \quad \left.\frac{dq}{d\theta}\right|_{G=0} = -\frac{G_\theta}{G_q} < 0$$

since $H_q < 0$, $G_q < 0$, and

$$H_\theta = -G_\theta = \frac{\mu\theta[\sigma+\delta+\mu(1-h)](\sigma+\delta+\mu h)}{q[\sigma+\delta+\mu h(1-\theta)+\theta\mu(1-h)]^2} > 0.$$

In other words, a rise in θ shifts up the $H = 0$ locus, shifts down the $G = 0$ locus, and increases the probability that a job searchers meet a job A vacancy, while its effect on q, the tightness of the labor market, is ambiguous.

Now we are ready to prove the existence of θ. At an equilibrium, θ solves

$$D_{HL}(\theta) \equiv U_H - U_L = \text{cost of education}, \tag{26}$$

where

$$D_{HL}(\theta) = \frac{\mu\Lambda(\theta)}{(\rho+\delta)[2\phi+\mu(1-h)][2\phi+\mu(1+h)](2\phi+\mu h)},$$

and

$$\begin{aligned}\Lambda(\theta) \quad = \quad & h[2\phi+\mu(1-h)][2\phi+\mu(1+h)]gap \\ & +2\phi h(3\mu h-\mu+2\phi)(y_B-b) \\ & -2\phi(1-h)[2\phi+\mu(1-h)]d.\end{aligned}$$

Note that $h = h(\theta)$ and $\mu = \mu(q(\theta))$. It is straightforward to see that $D_{HL}(\theta)$ is continuous in $\theta \in [0, 1]$. For the existence and uniqueness, it is sufficient to show that $D_{HL}(\theta)$ is increasing in $\theta \in [0, 1]$

The comparative statics show that there is a unique and increasing relationship between θ and h. It is obvious that $h = 0$ if and only if $\theta = 0$, since no firm would create a job A if there were no high-educated worker. However, it is always not the case that $h = 1$ even if $\theta = 1$. Intuitively, there is some positive incentive for a firm to create a job B even if all the workers are high-educated, since creating a job B costs less than creating a job A. Therefore, in the steady state, some firms hold a job B vacancy. However, $h(\theta)$ approaches some number which is close to one if θ goes to one, as long as k_A is not too large. Define $h_M \equiv \lim_{\theta\to 1} h(\theta)$. It is straightforward to see that

$$\begin{aligned}D_{HL}(\theta) \quad &= \quad \frac{-\mu d}{(\rho+\delta)(2\phi+\mu)} < 0, && \text{for } \theta = 0, \\ &= \quad \frac{\mu\Lambda(1)}{(\rho+\delta)[2\phi+\mu(1-h_M)][2\phi+\mu(1+h_M)](2\phi+\mu h)} > 0, && \text{for } \theta = 1.\end{aligned}$$

The last inequality follows with an appropriate choice of parameters, since

$$D_{HL}(\theta) = \frac{\mu(y_A - b)}{(\rho+\delta)(2\phi+\mu)} > 0, \quad \text{for } h = 1,$$

and $h(\theta)$ is continuous in $\theta \in [0,1]$. Therefore we have $D_{HL}(0) < 0$, and $D_{HL}(1) > 0$. Thus there exists a θ that solves (26). The uniqueness follows as follows. Note that

$$\frac{dD_{HL}(\theta)}{d\theta} = \frac{\partial D_{HL}}{\partial h}\frac{dh(\theta)}{\partial\theta} + \frac{\partial D_{HL}}{\partial q}\frac{dq(\theta)}{\partial\theta}.$$

We can neglect the last term or assume it as positive by the following reason. First, we can neglect it since sign of $dq(\theta)/\partial\theta$ is ambiguous. Or alternatively it is positive since it is plausible to assume both $\partial D_{HL}/\partial q > 0$ and $dq(\theta)/\partial\theta > 0$. The former inequality follows since the rise in the meeting probability makes it more profitable invest in education. Also it must be the case that $dq(\theta)/\partial\theta > 0$ since an increase in the productivity gap with y_B as given implies a rise in the average productivity and induces firms to enter the economy. Then it follows that

$$\frac{dD_{HL}(\theta)}{d\theta} = \frac{\partial D_{HL}}{\partial h}\frac{dh(\theta)}{\partial\theta} + \frac{\partial D_{HL}}{\partial q}\frac{dq(\theta)}{\partial\theta} > \frac{\partial D_{HL}}{\partial h}\frac{dh(\theta)}{\partial\theta} > 0$$

where the last inequality comes from $dh(\theta)/\partial\theta > 0$, and $\partial D_{HL}/dh > 0$, since

$$\begin{aligned} \text{sign}\left\{\frac{\partial D_{HL}}{\partial h}\right\} &= \text{sign}\{[(2\phi+\mu)^2 - 3(\mu h)^2]gap \\ &\quad +2\phi(2\phi - \mu + 6\mu h)(yl - b) + 4\phi(\phi + \mu - \mu h)d\}. \end{aligned}$$

Then $dD_{HL}(\theta)/d\theta > 0$ for $\theta \in [0,1]$. In summary, $D_{HL}(\theta)$ is uniformly increasing in θ with $D_{HL}(\theta) = 0$ and $D_{HL}(1)$ is bounded. Note that $\Gamma(\theta)$ is increasing and strictly convex in θ with $\Gamma(\theta) = 0$ and $\lim_{x\to 1}\Gamma(\theta) = \infty$. With the cost of education appropriately defined, as is illustrated in Figure 2 and 3, the uniqueness follows. ¥

Proof of Proposition 3

Again we prove the proposition for the Unfaithful equilibrium. For the Faithful equilibrium, the proof is left to the reader.

Define $gap = y_A - y_B$. Our goal is to show that $U_H - U_L$ is increasing in gap. The derivatives of H and G with respect to gap is given as

$$\begin{aligned} H_{gap} &\equiv \frac{\partial H(h,q)}{\partial gap} = \frac{\partial H_1(h,q)}{\partial gap} - \frac{\partial H_2(h,q)}{\partial gap} > 0, \\ G_{gap} &\equiv \frac{\partial G(h,q)}{\partial gap} = \frac{\partial G_1(h,q)}{\partial gap} - \frac{\partial G_2(h,q)}{\partial gap} < 0, \end{aligned}$$

since

$$\frac{\partial H_1(h,q)}{\partial gap} = \frac{\partial G_1(h,q)}{\partial gap} = 0,$$

and

$$\begin{aligned} \frac{\partial H_2(h,q)}{\partial gap} &= \frac{-[(1+h)\mu + 2\phi]^2(2\phi + \mu h)k_H}{\{[2\phi + \mu(1+h)]gap + 2(\phi + \mu h)(y_B - b) + (1-h)\mu d\}^2} < 0, \\ \frac{\partial G_2(h,q)}{\partial gap} &= \frac{[2\phi + \mu(1-h)][2\phi + \mu(1+h)](2\phi + \mu h)(y_B - b - d)k_H}{\{[2\phi + \mu(1+h)]gap + 2(\phi + \mu h)(y_B - b) + (1-h)\mu d\}^2 (y_B - b)} > 0. \end{aligned}$$

Therefore,

$$\left.\frac{dq}{dgap}\right|_{H=0} = -\frac{H_{gap}}{H_q} > 0, \quad \left.\frac{dq}{dgap}\right|_{G=0} = -\frac{G_{gap}}{G_q} < 0$$

In other words, an increase in the productivity gap, $y_A - y_B$, shifts the $H = 0$ locus up, and the $G = 0$ locus down, which results in a higher fraction of high skill vacancies, h. Its effect on the tightness of the labor market, q, is negligible. ¥

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