Asymmetric Cycles

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Abstract

I estimate a model in which new technology entails random adjustment costs. Rapid adjustments may cause productivity slowdowns. These slowdowns last longer when retooling is costly. The model explains why

1. growth-rate disasters are more likely than miracles,
2. volatility of growth relates negatively to growth over time.

1 Introduction

RBC research takes technology shocks as given and studies how a model economy responds to them. But one can decompose the shocks themselves. The shocks probably depend on the technologies we adopt. I study technology adoption in an "Ak" growth model with endogenous shocks and thereby help explain a few business-cycle facts. I assume that a technology has specific skills it requires. The exact nature of the needed skills is not known before a technology is adopted. Having committed to a technology, firms may face unexpectedly large training costs.

The model generates left-skewed distributions for the growth rates of output, consumption, investment, stock prices, and interest rates. Such skewness is seen in U.S. data. The model also generates growth-rates that are more volatile in recessions than in booms. This explains the time-series findings of Ramey and Ramey (1991) which I have updated. The model contains similar economics as Ramey and Ramey's, but the details differ and I make more progress analytically. The growth process obeys a simple difference equation and I provide estimates of the model's parameters

Plan of paper.—Section 2 starts with a sketch of the quantitative puzzle and the intuition. Section 3 presents the model and compares it to some evidence. Section 4 discusses the literature and concludes.

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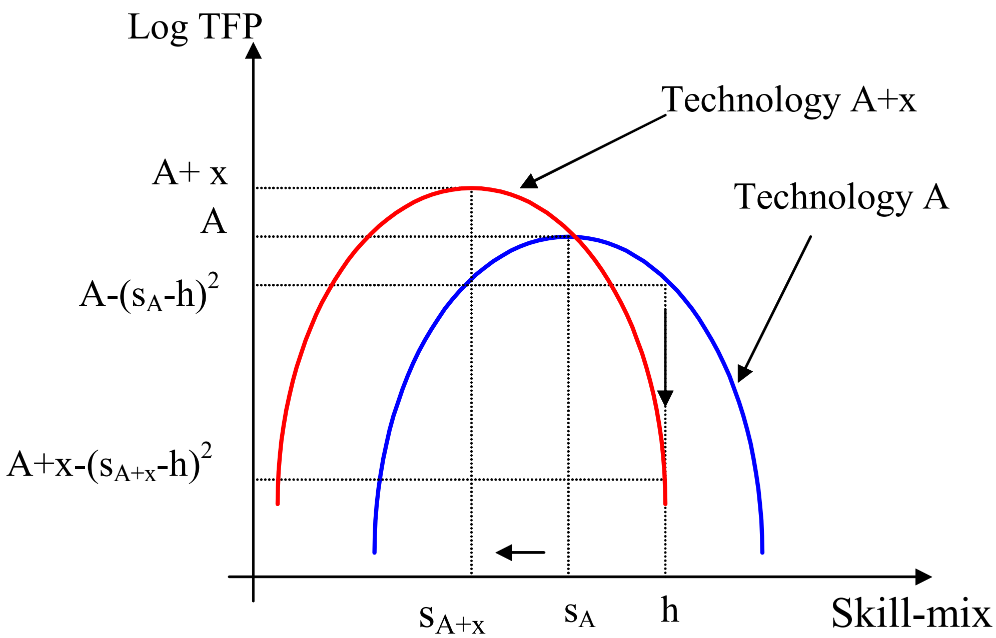


Figure 1: Benefits and costs of technology adoption

2 Intuitive explanation

The model assumes technological commitment and random adoption costs. Figure 1 explains why the asymmetry arises. Technology is indexed by the log of potential TFP, denoted by A. Actual TFP, however, is $A - \frac{1}{2}\lambda\left(s_A - h\right)^2$, where h is the skill mix. Committing to a potential TFP-growth rate of x exposes the adopter to uncertainty about s_{A+x}. The law of motion for s is $s_{A+x} = s_A + x\varepsilon$, and ε is unknown until after the commitment to $A + x$ is made. Thus the variance of the increment of s is of the order x^2. If ε is, say, a normal variate, the quadratic loss transforms the loss into a Chi-squared distributed variate that is highly skewed. Thus disasters are possible, and miracles aren't. The size of the disaster depends on λ, σ_ε^2, and on the costs of adjusting h.

Figure 1 shows what could happen if h cannot be adjusted at all. It shows an unlucky outcome in which the new technology $A + x$ has an ideal-skill mix, s_{A+x}, that is so far away from h that actual output falls. The TFP miracle cannot exceed $x + \frac{1}{2}\lambda\left[s_A - h\right]^2$, whereas disasters are unbounded. More generally, even when we allow costly adjustment of h, TFP declines will remain a possibility as long as the dispersion in ε is large enough.

3 Model

The model has two types of capital. The first, k, is the quantity of capital. The second, h, is a non-hierarchical index of expertise and physical-capital type, which I think of as the skill mix.

Production function.—With k units of capital, firm has a potential output of

$$y^p = zk.$$

The productivity parameter, z, is endogenous and given by

$$z = \exp\left\{A - \frac{\lambda}{2}\left(s_A - h'\right)^2\right\}. \tag{1}$$

Here A is the firm's technology, h' is the firm's skill mix, and s_A is the type of skill mix that is ideal for technology A. The cost of technological imbalance is indexed by $\lambda > 0$.

Adoption of technology.—Adoption of a better technology is free. A firm can choose a technology level by any amount, x, so that starting today at A, tomorrow's technology is

$$A' = A - \delta\left(A\right) + x. \tag{2}$$

where $\delta\left(A\right)$ is the rate of obsolescence, an exogenously given increasing function of A. The firm commits to using technology A for at least one period. But A' makes unpredictable demands on the skill mix. Assume that

$$s_{A'} = s_A + x\varepsilon, \tag{3}$$

where ε is a zero-mean random variable with variance σ_ε^2. The parameter ε is time specific.[1] The firm chooses x before seeing ε. Assume $x \geq 0$. I.e., once abandoned, a technology cannot be recalled.

Adjustment of h.—The firm starts with skill mix h. Before producing, it can adjust its skill mix from h to h' at a cost of

$$C\left(y^p, h, h'\right) \equiv \left[1 - \exp\left\{-\frac{\theta}{2}\left(h - h'\right)^2\right\}\right] y^p$$

The cost of redressing technological imbalance is indexed by $\theta > 0$. I refer to this loosely as a retooling cost.

Outline of firm's decision problem.—Firms will choose their x and h' so as to maximize the productivity of the capital that they raised in the previous period. The decision rules taken in this decentralized market economy will be similar to the rules that a planner would choose. All firms will choose the same (x, h') pair. A firm produces for one period and then liquidates. In the pre-pre-production period it

[1] If ε were technology-specific, it could create incentives to free ride on the information generated by adoption decisions of other agents. A symmetric, representative firm equilibrium may then fail to exist. I discuss this complication in Appendix 1.

1. raises capital k from shareholders,

2. chooses x which commits it to using technology A' as given by (2),

3. freely inherits the prevailing skill mix h.

In the production period the firm does the following in sequence: It

1. observes $s_{A'}$ as given by (3),

2. chooses h',

3. produces and pays a dividend

$$y = y^p - C\left(y^p, h, h'\right)$$

4. liquidates; the salvage value of its k and h' is zero.

Choice of h'.—Suppose that at the start of the production period the firm has observed that $s_{A'} = s'$. The firm then chooses h' to solve

$$\max_{h'} \left\{y^p - C\left(y, h, h'\right)\right\} = k \max_{h'} \exp\left\{A' - \frac{\lambda}{2}\left(s' - h'\right)^2 - \frac{\theta}{2}\left(h' - h\right)^2\right\} \qquad (4)$$

The first-order condition is $\lambda\left(s' - h'\right) - \theta\left(h' - h\right) = 0$ and at its solution, the second-order derivative w.r.t. h' is negative. The optimal h' is a convex combination of starting skill mix h, and ideal skill mix s':

$$h' = \alpha h + (1 - \alpha)\, s' \qquad (5)$$

where

$$\alpha = \frac{\theta}{\lambda + \theta}. \qquad (6)$$

Substituting into (4), its maximized value is the firm's output:

$$y = Z\left(A', s' - h\right) k$$

where

$$Z\left(A', s' - h\right) \equiv \exp\left(A' - \frac{\alpha\lambda}{2}\left(s' - h\right)^2\right) \qquad (7)$$

is the average product of capital. Maximized TFP depends only on $s' - h$, the "skill-mix gap" that exists at the start of the production period, after s' has been drawn, but before the firm has adjusted h.

The choice of x.—The firm chooses its technology in the pre-production period, before knowing s'. The state-of-the-art technology is summarized by the pair (A, s), and the skill mix is h. All firms face the same shock ε and so tomorrow's aggregate output and consumption will depend on ε. This means that the firm's dividend will be correlated with tomorrow's aggregate consumption. Let $p(A, \varepsilon)$ be today's price of a unit of consumption tomorrow if the aggregate shock is ε. In (16) we shall see that if all other firms choose the value x^*,

$$p(A, \varepsilon) = \frac{1}{Z(A + x^*, s + x^*\varepsilon - h)} \tag{8}$$

The optimal x maximizes the pre-production value of the firm per unit of k raised. This value, v, depends on the firm's pre-production state $(A, s - h)$ as follows:

$$\begin{aligned} v(A, s - h) &\equiv \max_x \int p(A, \varepsilon) Z(A + x, s + x\varepsilon - h) \, dF(\varepsilon) \\ &= 1. \end{aligned} \tag{9}$$

The amount the market is willing to pay for owning the rights to receive the firm's dividend in the next period is $v(A, s - h)$. It must equal unity because cost of capital is 1. At this price and value, a firm breaks even on each unit of k that it raises. We now differentiate the RHS of (9) w.r.t. x in and substituting from (8) into the resulting expression. We then evaluate the FOC at the symmetric equilibrium $x = x^*$, and obtain

$$\int [1 - \varepsilon\lambda\alpha(s + x\varepsilon - h)] \, dF(\varepsilon) = 0.$$

Since ε has mean zero and variance σ_ε^2, and since (x, s, h) are predetermined,

$$x = \frac{1}{\sigma_\varepsilon^2 \lambda\alpha} = \frac{1}{\sigma_\varepsilon^2}\left(\frac{1}{\lambda} + \frac{1}{\theta}\right). \tag{10}$$

Now we see clearly what the barriers to technological improvement are. If λ or θ or σ_ε^2 were zero, x would be infinite.

Preferences.—Households are infinitely lived with preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t \ln c_t. \tag{11}$$

Asset markets.—The number of households and the number of firms are both normalized to unity. This double normalization is fine because firm-size is indeterminate. Then y and k are output and capital per consumer. A household owns one-period shares of firms, and dividends are its only income. Because the representative firm grows over time, let us define shares in terms of pieces of capital rather than firms. That is, let n be the number of units of capital that the household owns. From (9), the price of a share is unity.

The behavior of the aggregate state.—The pricing of assets will not depend on the capital stock so that for the consumer's savings problem, at least, the aggregate state will be (A, s, h_{-1}). Let $u = s - h_{-1}$. From (7) it follows that (s, h_{-1}) matters for aggregate output only through u. We shall show that u follows the Markov process

$$u' = \alpha u + x\varepsilon,$$

so that its transition function is

$$\Phi(u', u) = F\left(\frac{u' - \alpha u}{x}\right),$$

where F is the C.D.F. of ε.

The savings decision.—If it owns n shares, a household's wealth is $Z(A, u)\, n$. Its budget constraint therefore is

$$n' + c = Z(A, u)\, n. \tag{12}$$

The consumer's state is (n, u), and the Bellman equation is

$$w(A, n, u) = \max_{n'} \left\{ \ln\left(Z(A, u)\, n - n'\right) + \beta \int w\left(A - \delta(A) + x, n', u'\right) d\Phi(u', u) \right\}. \tag{13}$$

The first appendix shows that optimal consumption is

$$c = (1 - \beta)\, Zn. \tag{14}$$

and saving is

$$n' = \beta Z n$$

At equilibrium,

$$n = k. \tag{15}$$

so that $k' = \beta Z(A, u)\, k$ and

$$\begin{aligned} \frac{U'(c')}{U'(c)} &= \frac{c}{c'} = \frac{(1-\beta)\, Zk}{(1-\beta)\, Z'(\beta Z k)} = \frac{1}{\beta Z(A', u')} \\ &= \frac{1}{\beta Z\left(A - \delta(A) + x^*, s + x^*\varepsilon - h\right)}, \end{aligned} \tag{16}$$

which proves (8).

3.1 The growth process

In interpreting the data I will treat the adjustment costs of h as an unmeasured investment. Measured output is[2]

$$y = Z(A, u)\, k.$$

[2] When firms train workers they produce less output. On the other hand, schooling is also an investment in human capital and it comes out of measured output. It is not clear what corresponds to measured GNP more closely.

The process for A has a deterministic steady state A^* that uniquely solves

$$x = \delta(A). \tag{17}$$

I will assume that A is already at A^*, and this simplifies the expressions. Then (14) implies

$$\begin{aligned} \ln y &= \ln k + A^* - \frac{\lambda}{2}(s-h')^2 - \frac{\theta}{2}(h-h')^2 \\ &= \ln\beta + A^* + \ln y_{-1} - \frac{\lambda}{2}(\alpha u)^2 - \frac{\theta}{2}([1-\alpha]u)^2 \\ &= \psi_0 + \ln y_{-1} - \psi u^2 \end{aligned} \tag{18}$$

where

$$\psi_0 = \ln\beta + A^* \quad \text{and} \quad \psi = \frac{\lambda}{2}\alpha^2 + \frac{\theta}{2}(1-\alpha)^2. \tag{19}$$

Thus letting $\Delta \ln y_t \equiv \ln y_{t+1} - \ln y_t$, we have the following representation for the growth rate of output

$$\Delta \ln y_t = \psi_0 - \psi u_{t+1}^2. \tag{20}$$

Long-run growth.—The long-run-average growth rate of output is

$$\begin{aligned} \psi_0 - \psi E\left(u_{t+1}^2\right) &= \ln\beta + A^* - \left[\frac{\lambda}{2}\alpha^2 + \frac{\theta}{2}(1-\alpha)^2\right]\frac{x^2\sigma_\varepsilon^2}{1-\alpha^2} \\ &= \ln\beta + \delta^{-1}\left(\frac{1}{\sigma_\varepsilon^2\lambda\alpha}\right) - \left[\frac{\lambda\alpha^2}{1-\alpha^2} + \theta\right]\frac{1}{2\sigma_\varepsilon^2\lambda^2} \end{aligned}$$

using (17), (28), (10) and (22).

The process for u.—From (5), $h' = \alpha h + (1-\alpha)s'$, so that

$$u' = s' - h = s + x\varepsilon - \alpha h_{-1} - (1-\alpha)s = \alpha(s - h_{-1}) + x\varepsilon.$$

Since ε is independent of u we adopt the convention of dating it at $t+1$ and we therefore have the time-series process

$$u_{t+1} = \alpha u_t + x\varepsilon_{t+1}. \tag{21}$$

The case where ε is normally distributed.—If ε_t is normally distributed, the stationary distribution of u_t is also normal with mean zero an variance:

$$\sigma_u^2 = \frac{x^2\sigma_\varepsilon^2}{1-\alpha^2} \tag{22}$$

Now, the stationary distribution of the square of a standard normal variate, is $\chi^2_{(1)}$. Denote by v the square of such a variable, i.e.,

$$v = \left(\frac{\sqrt{1-\alpha^2}}{x\sigma_\varepsilon}u\right)^2 \tag{23}$$

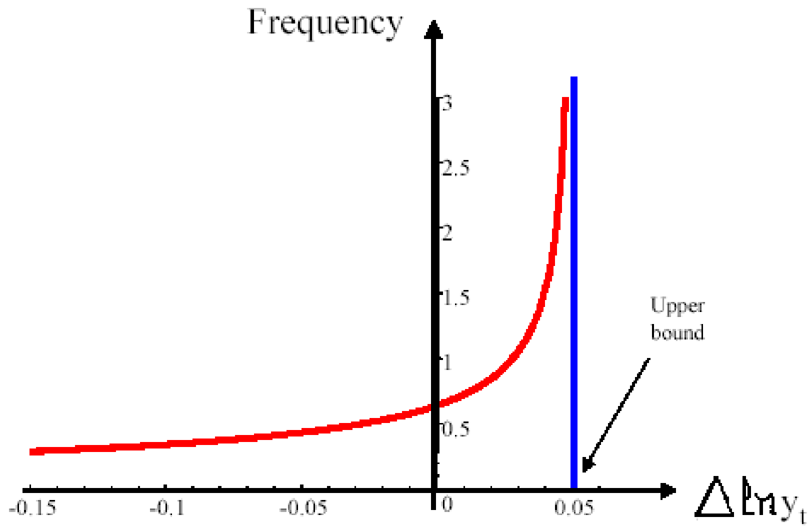


Figure 2: Predicted distribution of the growth rate

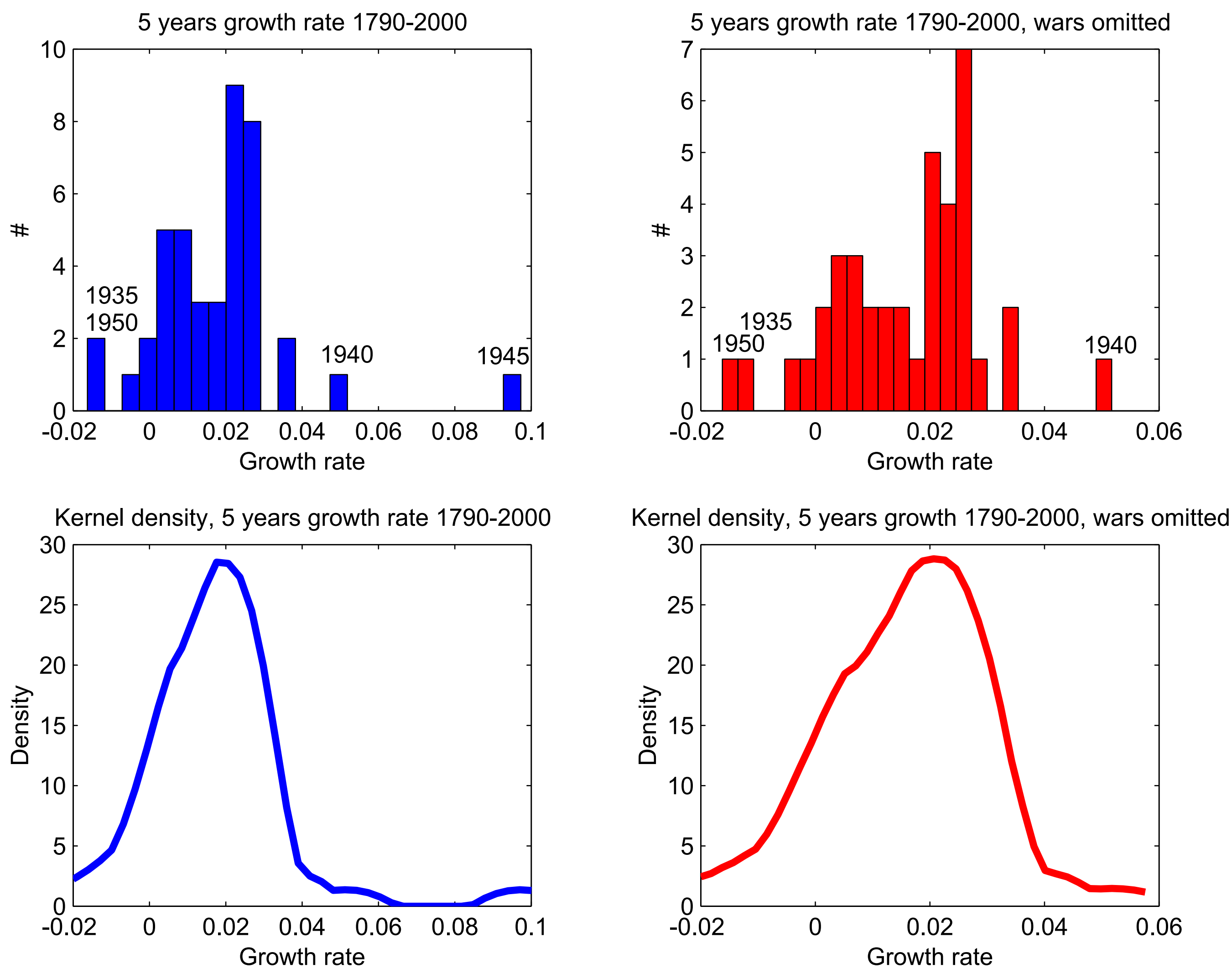
Then v has a Chi-squared distribution with 1 degree of freedom:

$$v^{-\frac{1}{2}} \frac{1}{2\pi} \exp\left(-\frac{1}{2} v^{\frac{1}{2}}\right) \equiv g(v), \tag{24}$$

for $v \geq 0$.

Figure 2 shows the long-run distribution of output growth (given in [20]). It is also distributed χ_1^2, except that the tail is on the left. Output growth is negative if $u_t^2 > \psi_0/\psi$. The reader may wonder, as I did, why u_t is missing in (20). It is because savings exactly offset the influence of u_t: Savings are proportional to y_t so that fluctuations in y_t do not affect the growth rate – a drop in y_t simply translates into an equal percentage drop in k_{t+1}. On the other hand, fluctuations in y_{t+1} do get into the growth rate between t and $t+1$, and the distribution of the level y_{t+1} is skewed to the left. Hence the asymmetry in the growth rate of y. This asymmetry should also show up in consumption and investment growth.

The top panels of the next figure show the frequency distribution of growth rates of per-capita output at a five-year frequency. The labeling refers to the last year of a five-year interval so, for example, the growth rate for 1940 means $\ln y_{1940} - \ln y_{1935}$. With the three observations the three wars (Civil 1860-65, WW1 1915-20, WW2 1940-45) taken out, the numbers are decidedly skewed to the left. Omitted were those 5-year intervals that most naturally contain the most intense war-time years). The two histograms look a little different because the number of bins in both histograms is the same – 25 bins. As a result, bin size is slightly different and, hence, the 2



Distribution of 5 years growth rates

left-most observations are paired in the right histograms and not paired in the left one. The kernel density estimates are also reported in the bottom panels.

Other distributions for ε.—Normality of ε is not necessary for the distribution of u^2 – and, hence of $\Delta \ln y$ – to be skewed to the left. The latter will be true whenever the stationary distribution of u is symmetric and uni-modal. The latter is likely to be approximately true even if ε is neither symmetric nor uni-modal, as long as α is fairly close to unity.

TFP growth is symmetric.—Since TFP does not depend on the rate of saving, the above argument does not apply to TFP growth. From (18) and (28), $\ln TFP \equiv \ln Z = A - \psi u^2$. Then

$$\Delta \ln TFP_t = \ln Z_{t+1} - \ln Z_t = x + \frac{\lambda}{2}\left(u_t^2 - u_{t+1}^2\right),$$

and the stationary distribution of $\left(u_t^2 - u_{t+1}^2\right)$ is symmetric and so, therefore, is that of TFP growth.

The perverse effect of σ_ε^2 on volatility.—A rise in σ_ε^2 lowers growth and, paradoxically, lowers the volatility of growth. Substituting for x from (10), the stationary distribution of u has variance

$$Var\,(u) = \frac{x^2\sigma_\varepsilon^2}{1-\alpha^2} = \frac{1}{\sigma_\varepsilon^2\lambda^2\left(1-\alpha^2\right)},$$

so that a rise in σ^2 reduces $Var\,(\Delta y)$.

3.2 Growth and retooling

Recessions are retooling episodes here. Let r denote the retooling cost relative to potential output:

$$r \equiv \frac{1}{y^p} C\left(y^p, h, h'\right).$$

Then we have

Proposition 1

$$r = 1 - \exp\left\{(1-\alpha)\left(\Delta \ln y_t - \psi_0\right)\right\} \tag{25}$$

Proof. From the definition of $C\,()$,

$$r = 1 - \exp\left\{-\frac{\theta}{2}\left(h-h'\right)^2\right\} = 1 - \exp\left\{-\frac{\theta}{2}\left(1-\alpha\right)^2 u^2\right\}$$

Then (20) implies

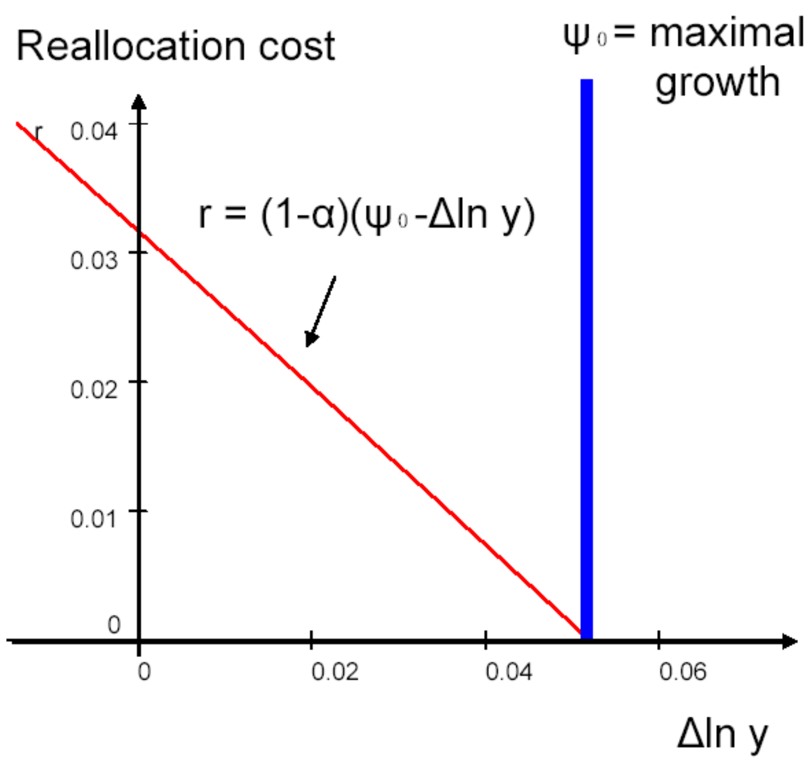
$$r = 1 - \exp\left\{\frac{\theta}{2}\left(1-\alpha\right)^2\left[\frac{\Delta \ln y_t - \psi_0}{\psi}\right]\right\}.$$

But

$$\frac{\theta}{2\psi}(1-\alpha)^2 = \frac{\theta(1-\alpha)^2}{\lambda\alpha^2 + \theta(1-\alpha)^2} = \frac{\theta\lambda^2}{\lambda\theta^2 + \theta\lambda^2} = \frac{\lambda}{\lambda+\theta} = 1-\alpha.$$

■

The trade-off between growth Figure plots the relation evaluated at the estimated values parameter values ("no wars") in column 2: $\alpha = \sqrt{0.14} = 0.38$ and $\psi_0 = 0.052$.



The plot shows, we essentially have a linear relation between growth and retooling:

$$r = (1-\alpha)(\psi_0 - \Delta \ln y).$$

The ex-post cost of restoring the economy to its maximal growth rate of ψ_0 – which would require that $u_{t+1} = 0$ – is just is $\alpha(\psi_0 - \Delta \ln y)$.

3.3 Growth vs. volatility of Δy_t over time

When $\theta > 0$, and hence when $\alpha > 0$, the model predicts a negative correlation between output growth and output-growth variability over time. This is seen intuitively in Figure 2. The conditional variance is higher if we know that $\Delta \ln y$ is likely to be low. The latter, in turn, follows because u us autocorrelated — when u strays far from the origin, it will probably remain far from the origin in the next period as well. Conditional on u, this implies lower expected growth but, because u^2 is an increasing and convex function of $|u|$, it also implies a higher variance of growth. Note that this is meant to explain the time-series relation between the two moments of growth, and not any cross section relation. Formally

Proposition 2 *The time-series relation between growth and its variability is negative.*

Proof. In (20) we condition the mean and variance of $\Delta \ln y_t$ on the lagged value of u, i.e., on u_t. As u_t varies over time, the conditional mean and variance of $\Delta \ln y_t$ shift. Showing that the two move in opposite ways when u_t shifts is equivalent to showing that the conditional mean and conditional variance of u_{t+1}^2 move in the same direction as u_t changes. Note that

$$\begin{aligned} E\left(u_{t+1}^2 \mid u\right) &= \left[E\left(u_{t+1} \mid u\right)\right]^2 + Var\left(u_{t+1} \mid u\right) \\ &= \alpha^2 u^2 + \sigma_\varepsilon^2 \end{aligned} \tag{26}$$

On the other hand

$$Var\left(u_{t+1}^2 \mid u\right) = E\left(u_{t+1}^4 \mid u\right) - \left[E\left(u_{t+1}^2 \mid u\right)\right]^2$$

But

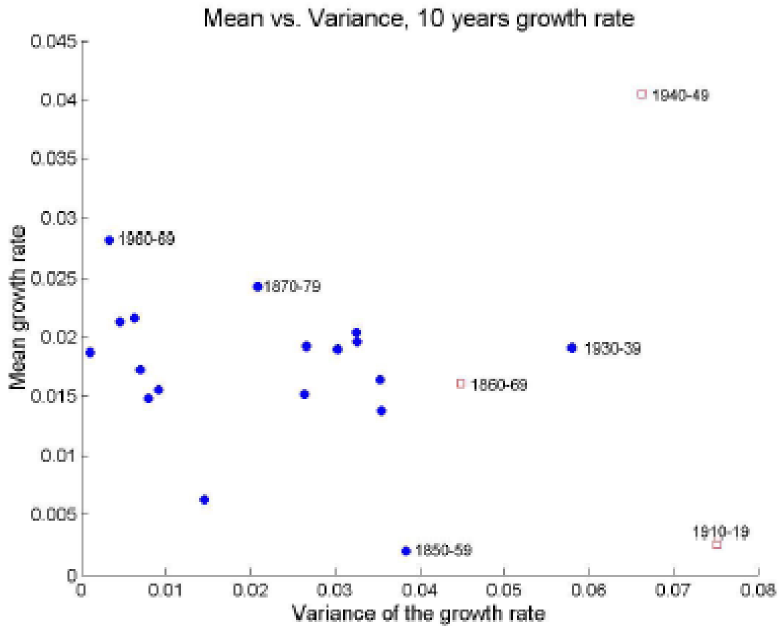
$$\begin{aligned} E\left(u_{t+1}^4 \mid u\right) &= \left[E\left(u_{t+1} \mid u\right)\right]^4 + 3\left[Var\left(u_{t+1} \mid u\right)\right]^2 + 6\left[E\left(u_{t+1} \mid u\right)\right]^2 Var\left(u_{t+1} \mid u\right) \\ &= \alpha^4 u^4 + 6\alpha^2\sigma_\varepsilon^2 u^2 + 3\sigma_\varepsilon^4. \end{aligned}$$

Thus, as long as $\alpha > 0$, the mean and variance of u^2 are both increasing in lagged u^2. ■

The negative time-series relation is confirmed in Figures 7 and 8 of Ramey and Ramey (1991) for annual data. My model seems ill-suited to annual data, however, and I look at the relation between mean and variance of growth at 5-year and 10-year frequencies. A negative relation emerges for 5-year intervals whether we include wars or not. For decades, the relation is negative only if we exclude wars. Generally, decades do not support the model well as 5-year periods and, in any case, we have too few observations at that low a frequency.[3] Figure A1 of the appendix reports the entire growth-rate series in decade and 5-year form, along with the standard deviations.[4]

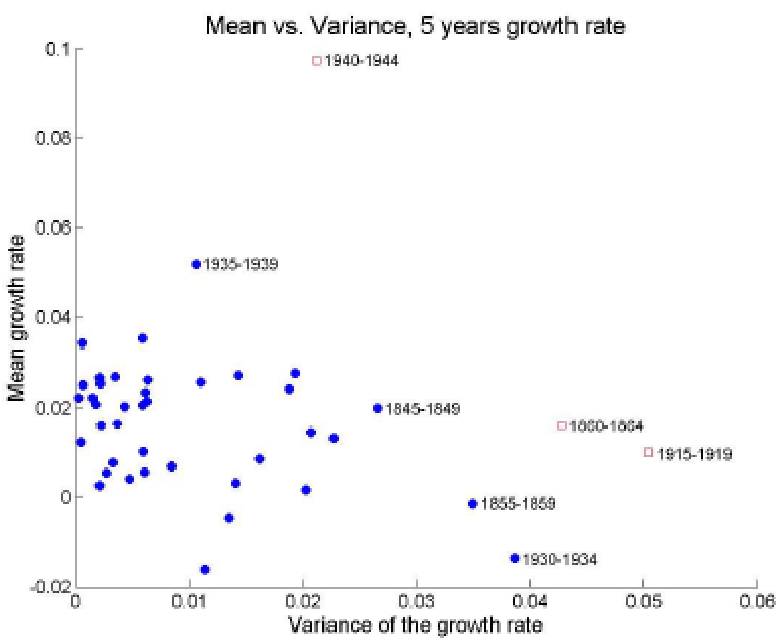
[3] If we are willing to assign country-specific parameters, then we can get a positive cross-country co-variation between growth and volatility. For example, suppose that some country has a very inflexible labor market so that its θ. is high. Such a country would have low growth and low volatility.

[4] The statistical program used required the shading of the wars to be shifted to the left by 2.5 years.



Mean growth rate vs.Variance, 10 years periods

Figure 3:



Mean growth rate vs.Variance, 5 years periods

3.4 Estimates of the parameters

I shall use per-capital GDP data from 1790 until the present. This model is better for low frequencies, so we need a long time series, at least while we deal with one country only. We do feel like we have a ballpark estimate of the rate of obsolescence of technology: at the steady state, we assume

$$x = \delta(A) = 0.05$$

This is based roughly on the rate at which capital gets cheaper over time (6% per year since WW2, more slowly earlier) and the patent law that protects inventions for 18 years. The 4 parameters are ψ_0, λ, θ, and σ_ε^2, but not all 4 are identified:

Claim 3 *The model's likelihood function depends on* $(\lambda, \theta, \sigma_\varepsilon^2)$ *only through the two parameters*

$$\left(\lambda\sigma_\varepsilon^2, \theta\sigma_\varepsilon^2\right).$$

Proof. The expressions in (6) and (10) do not change. From (22) the variance of u is proportional to σ_ε^2, so that the distribution of u/σ_ε is invariant to changes in σ_ε. Therefore the variance of ψu^2 is of order $\psi^2\sigma_\varepsilon^4 = (\psi\sigma_\varepsilon^2)^2$. But from (28), ψ is homogeneous of degree 1 in (λ, θ), and this implies that the distribution of ψu^2 depends only on $(\lambda\sigma_\varepsilon^2, \theta\sigma_\varepsilon^2)$. ■

In other words, if we double the penalties λ and θ but halve the variance σ_ε^2 of the innovations, the equilibrium remains the same. We shall measure λ and θ in units of σ_ε^{-2} by constraining the last parameter as follows:

$$\sigma_\varepsilon^2 = 1.$$

I fit the model to both 5-year and 10-year frequencies but the procedure took us to a corner ($\alpha = 1$) in the case of decades. So we are left with the estimates for 5-year-long periods. This is the only set of estimates that Table 1 reports. With its assumption of 100% depreciation of k, the model seems inappropriate for frequencies higher than that. The estimates come from data on per-capita GDP since 1790, and no other series were used. The estimates are reported in Table 1

Note that λ is far more precisely estimated than θ. The order-statistic estimator for ψ_0 apparently becomes very precise when a χ_1^2 distribution is involved, because there is so much mass in the right tail – see Figure (2). The estimate of α ($\sqrt{0.43} = 0.66$ and $\sqrt{0.14} = 0.37$, respectively) when we drop the 3 wartime observations.

Table 1: Parameter estimates, 5-year periods: 1790-2000

Standard errors in parentheses

	All years	no wars
α^2	0.43	0.14
	(0.18)	(0.18)
ψ_0	0.097	0.052
	(0)	(0)
λ	304.3	530.1
	(6.44)	(33.8)
θ	583.4	321.2
	(348.0)	(308.6)
$\#Obsrv.$	41	38

3.5 Micro evidence

The model casts recessions as technological mistakes and booms as choices of technology that are well suited to existing skill mix. Technological choices are irreversible and adoption costs are hard to predict. What micro evidence can we find for such assumptions?

3.5.1 Irreversible technological choices

In the standard convex adjustment cost model faster growth can raise adjustment costs even without uncertainty. Here we have decomposed the adjustment cost into an unpredictable skill content of new technology together with an adjustment cost for the skill mix. Here, then, are some examples of ex-post mistakes in adoption. Some affected to small a part of the economy to affect per-capita output; one such is the Concorde which BA and AirFrance are retiring, 27 years after its first commercial flight (Cowell 2003). Of larger import are mistaken adoptions of standards that can cause a group of otherwise independent firms to lock into an inferior technology – e.g. Choi (1994) – and this would reduce per capita output significantly if the technology is used widely enough. Cowan (1990) argues that this was so with nuclear power in the U.S., and Henderson (1977) argues similarly for the U.K. Nuclear power was used widely in both countries. The conglomerate merger wave of the 60s was probably a mistake caused by an overestimate of managerial reach, and the resulting organizational structure of the early 1970's probably contributed to the post-1974 productivity slowdown.

3.5.2 Costs of adjusting h

Bartel and Sicherman (1998) and Brynjolfsson and Yang (2000) find that new technology is accompanied by investments in human and organization capital. Formal company training is the most elastic of the various sources of skill improvement, and yet it does not usually count as investment. Bartel and Sicherman (1998) find that company training relates more to technological change than other training. Table 1 describes changes in the incidence of different kinds of training used by workers to improve their skills in existing jobs between 1983 and 1991. The categories are not mutually exclusive since re-training can come from more than a single source. Half of the increase came from formal company programs.

Table 2: Sources of Skill Improvement

Source: Barton (1993). The numbers are percentages

Year	All	School	OJT	Formal Company Training	Other
1983	35	12	14	11	4
1991	41	13	15	16	7

This increase in training (which is not properly measured as investment in the data that we tend to use) comes too late to explain the early part of the productivity slowdown in the 1970's.

Workers may need to change sectors in response to unfavorable shocks some of which may be like an unfavorable draw of ε that raises industry costs. Neal (1995) finds that a change of industries reduces lifetime wages by about 15%. DeJong and Ingram (2001) report a correlation between schooling hours and output of -0.36. I have been assuming that investments in h appear as subtractions from measured output, and schooling is a component of measured output. Therefore this evidence is only suggestive, in that one expects that the measured parts of the costs of adjusting h should be correlated with the unmeasured parts. Finally, this is not evidence *per se*, but it has been held that Europe has less flexible labor markets – which we can think of as a higher θ – and that this reduces Europe's willingness to adopt new technology – (10) would imply. Topel and Kim (1995), e.g., found that the shift from farming to manufacturing in Korea was accomplished by new cohorts entering manufacturing and not by older people switching, suggesting that θ is high.

3.5.3 Sectoral growth and volatility

The model could be about sectors as well, except that the rate of saving would depend on the aggregate economy and not on individual sectors. Imbs (2002) finds that the cross-sector correlation between growth and volatility is positive. This could happen in the model if technological opportunity, as expressed, e.g., in the parameter λ, were to vary over sectors. For instance, (**??**) implies that a fall in λ raises x and it raises volatility of output so that growth and volatility both rise Imbs also finds that the correlation remains substantial even after controlling for investment, suggesting that the explanation is technological, such as the one advanced here. d

4 Related work

The model relates to work on appropriate technology. In the model skill is "appropriate" to a technology in a sense similar to Basu and Weil (1998). In a partial equilibrium setting, Jovanovic and Rob (1990) and Jovanovic and Nyarko (1996) endogenize the shocks by making their variance proportional to the size of the technological leap. Jovanovic and Nyarko (1996) and Jovanovic and Rob (1990) take a closely related information-theoretic approach to technology choice. Comin (2000) argued that the productivity slowdown of the 70s and 80s was the result of a rise in technological uncertainty in the 1970's which raised the demand for less productive but more flexible capital. I get a similar effect from a rise in σ^2 that reduces x and, hence, TFP.

The model also relates to work on delay in order to observe what happens to other agents. If the ε's were technology specific and not vintage effects, we would have the possibility that some firms would free ride on the adoption efforts of others. Models in which agents learn from the decisions taken by others because these decisions are informative, e.g. Chamley and Gale (1993) and Caplin and Leahy (1994). My model is in the same spirit, but I have shut off the imitation incentive by making the ε's time-specific variates. A similar assumption is in Greenwood, Hercowitz and Huffman (1988) who assume that there are shocks to the marginal efficiency of investment so that

$$k_{t+1} = (1 - \delta_K)\, k_t + \varepsilon_t i_t, \tag{27}$$

A similar assumption holds for the evolution of productivity in Aghion and Howitt's (1992) model of research-driven growth. Greenwood *et al* assume that ε_t is known before i_t is chosen, so that the issue of waiting for the information of others does not arise. Aghion and Howitt assume that their analog of ε_t is not known ahead of time, but they assume, as I do, that it is time-specific, so that delay is of no informational value.

The model certainly relates to lots of other work on technology and the business cycle. The model also relates to Barlevy (2002), where research is countercyclical because a negative shock to productivity in production reduces the foregone-output cost of research. So it is in my model, in which I_H is higher when the gap is higher. Jones, Manuelli and Stacchetti (1999) study how a change in the variance of the shocks to productivity influences growth, Scott and Uhlig (1999) study the growth effects of a change in the volatility of investment. Chalkley and Lee (1998) and Veldcamp (2002) argue cycles are asymmetric because we are more likely to detect negative shocks than positive ones.

Finally, this paper relates to the topic of costs of business cycles. If we could get rid of cycles but keep the trend of consumption unchanged, Lucas (1987) argued the benefits would be small. But it is probable that getting rid of cycles would reduce growth. That is the implication of my model, but also of several others. Benhabib and Nishimura (1984), e.g., get the result if the consumption sector is capital intensive. Matsuyama (1999) models an economy that alternates between periods of high investment and the periods of higher innovation. Growth is also higher in a cyclical equilibrium in the paper by Ellis and Francois (2001). Aghion and Howitt (1992) get a positive relation between growth and cycles because research produces growth at random dates. Shleifer (1986) obtains it because product introductions are bunched. Caballero and Hammour (1994) argue that recessions are reallocative, for reasons similar to those I have modelled. On the other hand, Barlevy (2001) and Krebs (2002) argue that cycles are bad for growth, and Rampini and Eisfeldt (2003) find that reallocation is pro-cyclical, and so the debate goes on.

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5 Appendix

Several arguments are listed in separate Appendixes.

5.1 Estimation procedure

I estimate the model using the following sequence of steps:

A. Use the consistent estimate of ψ_0

$$\hat{\psi}_0 = \max_t \Delta \ln y_t \tag{28}$$

B. We now show that (20) implies

$$\Delta \ln y_t = a_0 + \alpha^2 \Delta \ln y_{t-1} + \omega_t, \tag{29}$$

where

$$\omega_t = \psi u_{t+1}^2.$$

Now,

$$u_{t+1} = \alpha u_t + x\varepsilon_t$$

so that

$$u_{t+1}^2 = \alpha^2 u_t^2 + \zeta_t, \tag{30}$$

where

$$\zeta_t = x^2 \varepsilon_{t+1}^2 + 2\alpha x u_t \varepsilon_{t+1}.$$

where $u_t \sim N\left(0, \sigma_\varepsilon^2 / \left[1 - \alpha^2\right]\right)$. Lagging (20) by a period and solving for u_t^2,

$$u_t^2 = \frac{\psi_0 - \Delta \ln y_{t-1}}{\psi}. \tag{31}$$

Substituting from (31) into (30),

$$\frac{\psi_0 - \Delta \ln y_t}{\psi} = \alpha^2 \frac{\psi_0 - \Delta \ln y_{t-1}}{\psi} + \zeta_t$$

i.e.,

$$-\psi_0 + \Delta \ln y_t = -\alpha^2 \left(\psi_0 - \Delta \ln y_{t-1}\right) - \psi \zeta_t$$

i.e.,

$$\Delta \ln y_t = \left(1 - \alpha^2\right) \psi_0 + \alpha^2 \Delta \ln y_{t-1} - \psi \zeta_t, \tag{32}$$

which is the same as (29) if $a_0 = \left(1 - \alpha^2\right) \psi_0$. Estimate a_0 and α using OLS. The estimates $\hat{a}_0$ and $\hat{\alpha}$ are unbiased because ε_{t+1} is independent of y_{t-1} and therefore $\mathrm{Cov}(\Delta \ln y_{t-1}, \omega_t) = 0$. This will be constrained least squares. Calculate the expected value of a_0 as follows: Since $\psi = \frac{\lambda}{2}\alpha^2 + \frac{\theta}{2}(1 - \alpha)^2$,

$$E\left(a_0\right) = \left(1 - \alpha^2\right) \hat{\psi}_0 - \psi E\left(u_{t+1}^2\right).$$

This will be the constraint we shall impose on the OLS estimation of (29). We need to figure out the explicit version of the right-hand side. Since $\sigma_{\varepsilon}^2 = 1$,

$$\begin{aligned}
\psi E\left(u_{t+1}^2\right) &= \frac{1}{2}\left[\lambda\alpha^2 + \theta\left(1-\alpha\right)^2\right] E\left(u^2\right) \\
&= \frac{1}{2}\left[\lambda\alpha^2 + \theta\left(1-\alpha\right)^2\right] x^2 \frac{1}{1-\alpha^2} \\
&= \frac{1}{2}\left[\lambda\alpha^2 + \theta\left(1-\alpha\right)^2\right] \frac{1}{\lambda^2\alpha^2}\frac{1}{1-\alpha^2} \\
&= \frac{1}{2}\left[\frac{\lambda\theta^2 + \theta\lambda^2}{\left(\lambda+\theta\right)^2}\right] \frac{1}{\lambda^2\alpha^2}\frac{1}{1-\alpha^2} \\
&= \frac{1}{2}\alpha\left(\frac{\lambda\theta + \lambda^2}{\lambda+\theta}\right) \frac{1}{\lambda^2\alpha^2}\frac{1}{1-\alpha^2} \\
&= \frac{1}{2}\left(\frac{\theta + \lambda}{\lambda+\theta}\right) \frac{1}{\lambda\alpha}\frac{1}{1-\alpha^2} \\
&= \frac{1}{2}\frac{1}{\lambda\alpha}\frac{1}{1-\alpha^2}
\end{aligned}$$

Therefore we run OLS version of $\Delta \ln y_t = a_0 + b\Delta \ln y_{t-1} + \xi_t$ subject to the constraint

$$a_0 = \left(1-b\right)\hat{\psi}_0 - \frac{1}{2}\frac{1}{\lambda\alpha}\frac{1}{1-b} \tag{33}$$

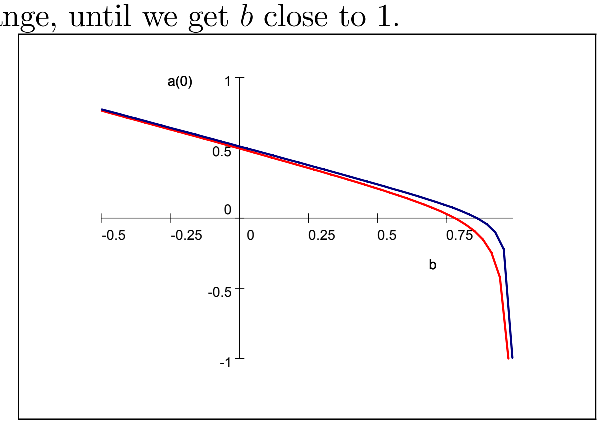
Now we use the constraint that

$$x = \frac{1}{\lambda\alpha} = \delta\left(A\right) = 0.05.$$

Then our constraint for the OLS estimates is

$$a_0 = \left(1-b\right)\hat{\psi}_0 - \frac{1}{2}\delta\left(A\right)\frac{1}{1-b}$$

Just to see if this constraint is well defined, I plot the RHS of this constraint assuming that (as in the decade data with no wars) $\hat{\psi}_0 = 0.052$. The bottom line assumes that $\delta\left(A\right) = 0.025$, and the top line assumes that $\delta\left(A\right) = 0.01$. Things are not too sensitive to this change, until we get b close to 1.



a_0 as a function of b

This gives us the constrained estimate of α.

C. We then estimate λ as

$$\hat{\lambda} = \frac{1}{\delta(A)\,\hat{\alpha}}$$

D. The standard errors were computed by linearization: Generally, suppose that the parameter vector β is distributed under the null as

$$\beta \sim N(\beta_0, \Omega).$$

For any function $g(\beta)$, its Taylor expansion is

$$g(\beta) \simeq g(\beta_0) + J(\beta_0)(\beta - \beta_0)$$

where $J(\beta_0)$ is the matrix of first derivatives (the Jacobian). Combining

$$g(\beta) \sim N\left(g(\beta_0), J(\beta_0)\Omega J(\beta_0)'\right).$$

In our case $\beta = (a_0,\, \alpha^2, \psi_0)$ and $g(\beta) = (\theta, \lambda) = (\frac{\lambda\alpha}{1-\alpha}, \frac{1}{0.005\lambda})$.

5.2 Derivation of optimal savings

Let us now analyze the savings problem defined in (13) and derive the optimal consumption rule expressed in (14). To save space I do it only under the assumption that $A = A^*$ so that we can drop A from the vector of states.

Lemma 4 *(13) has a solution of the form*

$$w(n,u) = W(u) + \frac{1}{1-\beta}\ln n,$$

where

$$W(u) = \max_{\xi}\left\{\ln(Z(u) - \xi) + \frac{\beta}{1-\beta}\ln\xi + \beta\int W(u)\,d\Phi(u', u)\right\}. \qquad (34)$$

Proof. We can change variables and let $\xi = n'/n$ so that substituting into the RHS an equation of the form $w(n,u) = W(u) + B\ln n$, (13) becomes

$$\begin{aligned} w(n,u) &= \max_{\xi}\left\{\ln(nZ[u] - n\xi) + \beta[B\ln n + B\ln\xi] + \beta\int W(u')\,d\Phi(u', u)\right\} \\ &= \ln n + \beta[B\ln n] + \max_{\xi}\left\{\ln(Z - \xi) + \beta B\ln\xi + \beta\int W(u')\,d\Phi\right\} \end{aligned}$$

which works if

$$B = 1 + \beta B,$$

i.e., if $B = 1/(1-\beta)$. Since the right hand side is a contraction operator on a complete metric space, there exists exactly one function $W(u)$ such that (34 holds. ■

Then the FOC for ξ in (34) is

$$-\frac{1}{Z-\xi} + \frac{\beta}{1-\beta}\frac{1}{\xi} = 0, \tag{35}$$

Proposition 5 *Optimal consumption is*

$$c = (1-\beta)\, Zn.$$

Proof. (12) implies

$$\frac{c}{n} = Z - \xi. \tag{36}$$

Since shares are one-period, consumer wealth is the same as aggregate output. We posit consumption to be a constant fraction of wealth

$$c = \omega Z n,$$

Together with (36) this implies

$$\xi = Z - \frac{c}{n} = Z(1-\omega).$$

Substituting for ξ into (35), we find that it holds if and only if

$$\omega = 1 - \beta.$$

■

5.3 Technology-specific ε

If we wanted ε to be technology-specific rather than time-specific, then Eq. (2) could be interpreted as

$$s_{A+x} = s_A + \sigma \int_A^{A+x^2} dW(a). \tag{37}$$

where $W(a)$ is Brownian Motion. Jovanovic and Rob (1990) and Jovanovic and Nyarko (1996) make the more natural assumption: When upgrading from technology A to technology $A+x$, the technological parameter s evolves as follows:

$$s_{A+x} = s_A + \sigma \int_A^{A+x} dW(a) \tag{38}$$

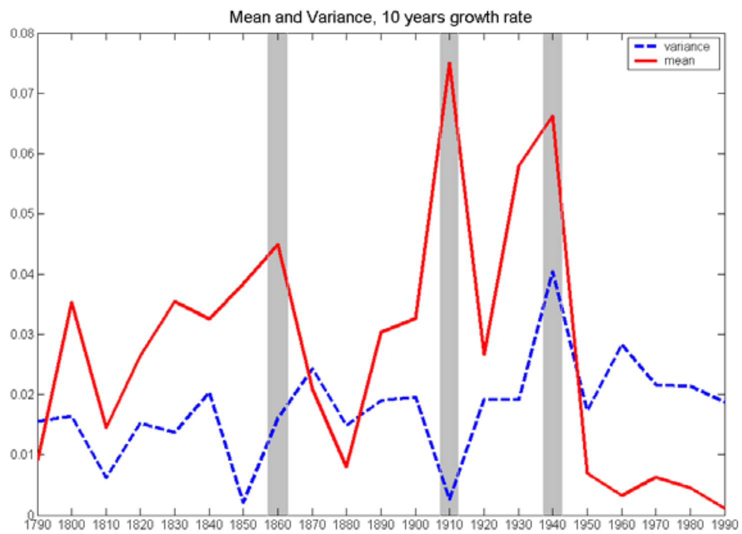
This leads to the exact discrete representation

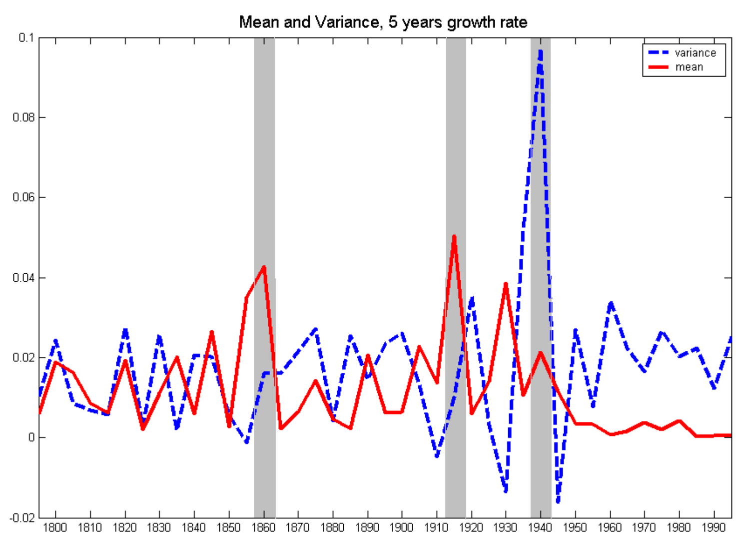
$$s_{A+x} = s_A + x^{1/2}\varepsilon$$

where $\varepsilon \sim N(0, \sigma)$. Even without the free riding complications, however, the process in (38) would not work for our present purposes: since

$$Var(s_{A+x} \mid s_A) = x\sigma_\varepsilon^2$$

the costs of adoption rise linearly with x, and since the same is true of benefits, firms would want x to be either infinite or zero.


Mean and Variance, 10 years growth rate
variance
mean
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
1790 1800 1810 1820 1830 1840 1850 1860 1870 1880 1890 1900 1910 1920 1930 1940 1950 1960 1970 1980 1990


Mean and Variance, 5 years growth rate
variance
mean
0.1
0.08
0.06
0.04
0.02
0
-0.02
1800 1810 1820 1830 1840 1850 1860 1870 1880 1890 1900 1910 1920 1930 1940 1950 1960 1970 1980 1990