Rule-of-Thumb Consumers and the Design of Interest Rate Rules*

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Abstract

We introduce rule-of-thumb consumers in an otherwise standard dynamic sticky price model, and show how their presence can change dramatically the properties of widely used interest rate rules. In particular, the existence of a unique equilibrium is no longer guaranteed by an interest rate rule that satisfies the Taylor principle. Thus, when the weight of rule-of-thumb consumers is large, a strong anti-inflationary bias may be needed in order to avoid indeterminacy whenever the central bank follows a rule that responds to contemporaneous inflation and output. On the other hand, when that rule is either forward or backward-looking, the opposite result often holds in a similar environment: only specifications of the rule that violate the Taylor principle (i.e., passive rules) can guarantee a unique equilibrium.

Keywords: Taylor principle, interest rate rules, sticky prices, rule-of-thumb consumers.

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1 Introduction

The study of the properties of alternative monetary policy rules, and the assessment of their relative merits, has been one of the central themes of the recent literature on monetary policy. Many useful insights have emerged from that research, with implications for the practical conduct of monetary policy, and for our understanding of its role in different macroeconomic episodes.

Among some of the recurrent themes, much attention has been drawn to the potential benefits and dangers associated with simple interest rate rules. Thus, while it has been argued that simple interest rate rules can approximate well the performance of complex optimal rules in a variety of environments,[1] those rules have also been shown to contain the seeds of unnecessary instability when improperly designed.[2]

A sufficiently strong feedback from endogenous target variables to the short-term nominal interest rate is often argued to be one of the requirements for the existence a locally unique rational expectations equilibrium and, hence, for the avoidance of indeterminacy and fluctuations driven by self-fulfilling expectations. For a large number of models used in applications that determinacy condition can be stated in a way that is both precise and general: the policy rule must imply an eventual increase in the real interest rate in response to a sustained increase in the rate of inflation. In other words, the monetary authority must adjust (possibly gradually) the short-term nominal rate more than one-for-one with changes in inflation. That condition, which following Woodford (2001) is often referred to as *the Taylor principle*, has also been taken as a benchmark for the purposes of evaluating the stabilizing role of central banks' policies in specific historical periods. Thus, some authors have hypothesized that the large and persistent fluctuations in inflation and output in the late 60s and 70s in the U.S. may have been a consequence of the Federal Reserve's failure to meet the Taylor principle in that period; by contrast, the era of low and steady inflation that has characterized most of Volcker and Greenspan's tenure seems to have been associated with interest rate policies that satisfied the Taylor principle.[3]

In the present paper we show how the presence of rule-of-thumb consumers may alter dramatically the properties of simple interest rate rules, and overturn some of the conventional results found in the literature. In particular, we analyze a standard new Keynesian model modified to allow for a fraction of consumers who do not borrow or save in order to smooth consumption, but instead follow a simple rule of thumb: they consume their current labor income.

To anticipate our main result: we show that if the weight of such rule-of-thumb consumers is large enough, a more than one-for-one change in the nominal interest

[1] This is possibly the main conclusion from the contributions to the Taylor (1999a) volume.

[2] See, e.g., Kerr and King (1996), Bernanke and Woodford (1997), Taylor (1999b), Clarida, Galí and Gertler (2000), and Benhabib, Schmitt-Grohé, and Uribe (2001a,b), among others.

[3] See, e.g., Taylor (1999b), and Clarida, Galí, and Gertler (2000). Orphanides (2001) argues that the Fed's failure to satisfy the Taylor principle was not intentional; instead it was a consequence of a persistent bias in their real-time measures of potential output.

rate in response to a change in inflation is often needed in order to guarantee the uniqueness of equilibrium. In particular, when the central bank follows a rule that implies an adjustment of the nominal interest rate in response to variations in current inflation and output, the size of the inflation coefficient that is required in order to rule out multiple equilibria is an increasing function of the weight of rule-of thumb consumers in the economy (for any given output coefficient). Hence, the Taylor principle becomes too weak a criterion for stability when the share of rule-of-thumb consumers is large.

We also find that, independently of their weight in the economy, the presence of rule-of-thumb consumers cannot *in itself* overturn the conventional result on the sufficiency of the Taylor principle. Instead, we argue that it is the interaction of those consumers with countercyclical markups (resulting from sticky prices in our model) that lies behind our main result.

In addition, we show that the presence of rule-of-thumb consumers affects the conditions for a unique equilibrium when the central bank follows some alternative interest rate rules, namely, forward looking or backward looking ones. The specific form in which those conditions are affected depends on the share of rule-of-thumb consumers as well as features of the rule. In particular, we show that when that share is sufficiently large, the existence of a unique equilibrium requires that interest rates respond less than one-for-one to changes in expected (or lagged) inflation.

As discussed in more detail below, there is an intuitive economic mechanism underlying the above results: the presence of rule-of-thumb consumers makes a component of aggregate demand insensitive to changes in real interest rates; instead that component is highly responsive to changes in real wages which, in turn, are affected by the evolution of markups. That feature of our economy influences the effectiveness of any given interest rate rule as a stabilizing tool.

Our framework shares most of the features of recent dynamic optimizing sticky price models.[4] The only difference lies in the presence of rule-of-thumb consumers, in coexistence with conventional, intertemporally optimizing (Ricardian) ones. While the behavior that we assume for rule-of-thumb consumers is admittedly simplistic (and justified only on tractability grounds) we believe that their presence captures an important aspect of actual economies which is missing in conventional models. Empirical support of non-Ricardian behavior among a substantial fraction of households in the U.S. and other industrialized countries can be found in Campbell and Mankiw (1989). It is also consistent, at least prima facie, with the findings of a myriad of papers rejecting the permanent income hypothesis on the basis of aggregate data. While many authors have stressed the consequences of the presence of rule-of-thumb consumers for fiscal policy,[5] the study of its implications for the design of monetary

[4]See, e.g., Rotemberg and Woodford (1999), Clarida, Gali and Gertler (1999), or Woodford (2001).

[5]See, e.g., Mankiw (2000) and Galí, López-Salido and Vallés (2002).

policy is largely non-existent.[6]

The rest of the paper is organized as follows. Section 2 lays out the basic model, and derives the optimality conditions for consumers and firms, as well as their log-linear counterparts. Section 3 contains an analysis of the equilibrium dynamics and its properties under our baseline interest rate rule, with a special emphasis on the conditions that the latter must satisfy in order to guarantee uniqueness. Section 4 examines the robustness of those results and the required modifications when alternative interest rate rules are assumed. Section 5 concludes.

2 A New Keynesian Model with Rule-of-Thumb Consumers

The economy consists of two types households, a continuum of firms producing differentiated intermediate goods, a perfectly competitive final goods firm, and a central bank in charge of monetary policy. Next we describe the objectives and constraints of the different agents. Except for the presence of rule-of-thumb consumers, our framework corresponds to a conventional New Keynesian model with staggered price setting à la Calvo used in numerous recent applications.[7]

2.1 Households

We assume a continuum of infinitely-lived households, indexed by $i \in [0, 1]$. A fraction $1 - \lambda$ of households have access to capital markets where they can trade a full set of contingent securities, and buy and sell physical capital (which they accumulate and rent out to firms). We use the term *optimizing* or *Ricardian* to refer to that subset of households. The remaining fraction λ of households do not own any assets or have any liabilities, and just consume their current labor income. We refer to them as *rule of thumb* (or *non-Ricardian*) consumers. Different interpretations for the latter include myopia, lack of access to capital markets, fear of saving, ignorance of intertemporal trading opportunities, etc. Campbell and Mankiw (1989) provide some evidence, based on estimates of a modified Euler equation, of the quantitative importance of such rule-of-thumb consumers in the U.S. and other industrialized economies.

[6] A recent paper by Amato and Laubach (2002) constitutes an exception. In that paper the authors derive the appropriate loss function that a benevolent central banker should seek to minimize in the presence of habit formation and rule-of-thumb consumers.

[7] Many recent monetary models with nominal rigidities abstract from capital accumulation. A list of exceptions includes King and Watson (1996), Yun (1996), Dotsey (1999), Kim (2000) and Dupor (2002). In our framework, the existence of a mechanism to smooth consumption over time is critical for the distinction between Ricardian and rule-of-thumb consumers to be meaningful, thus justifying the need for introducing capital accumulation explicitly.

2.1.1 Optimizing Households

Let C_t^o, and L_t^o represent consumption and leisure for optimizing households (henceforth we use a "o" superscript to refer to optimizing households' variables). Preferences are defined by the discount factor $\beta \in (0,1)$ and the period utility $U(C_t^o, L_t^o)$. Optimizing households seek to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \; U(C_t^o, L_t^o) \tag{1}$$

subject to the sequence of budget constraints

$$L_t^o + N_t^o = 1 \tag{2}$$

$$P_t \; (C_t^o + I_t^o) + R_t^{-1} B_{t+1} = W_t N_t^o + R_t^k K_t^o + B_t + D_t \tag{3}$$

and the capital accumulation equation

$$K_{t+1}^o = (1-\delta) \; K_t^o + \phi \left(\frac{I_t^o}{K_t^o} \right) \; K_t^o \tag{4}$$

Hence, at the beginning of the period the consumer receives labor income $W_t N_t^o$ (where W_t denotes the nominal wage), and income from renting his capital holdings K_t^o to firms at the (nominal) rental cost R_t^k. B_t is the quantity of nominally riskless one-period bonds carried over from period $t-1$, and paying one unit of the numéraire in period t . R_t denotes the gross nominal return on bonds purchased in period t. D_t are dividends from ownership of firms. $P_t C_t^o$ and $P_t I_t^o$ denote, respectively, nominal expenditures on consumption and capital goods. Capital adjustment costs are introduced through the term $\phi\left(\frac{I_t^o}{K_t^o}\right) \; K_t^o$, which determines the change in the capital stock (gross of depreciation) induced by investment spending I_t^o. We assume $\phi' > 0$, and $\phi'' \leq 0$, with $\phi'(\delta) = 1$, and $\phi(\delta) = \delta$.

In what follows we specialize the period utility to take the form:

$$U(C, L) \equiv \frac{1}{1-\sigma} \left(C \; L^{\nu} \right)^{1-\sigma}$$

where $\sigma \geq 0$ and $\nu > 0$.

The first order conditions for the optimizing consumer's problem can be written as:

$$\frac{C_t^o}{L_t^o} = \frac{1}{\nu} \frac{W_t}{P_t} \tag{5}$$

$$1 = R_t \; E_t \left\{ \Lambda_{t,t+1} \right\} \tag{6}$$

$$P_t Q_t = E_t \left\{ \Lambda_{t,t+1} \left[R^k_{t+1} + P_{t+1} Q_{t+1} \left((1-\delta) + \phi_{t+1} - \left(\frac{I^o_{t+1}}{K^o_{t+1}} \right) \phi'_{t+1} \right) \right] \right\} \tag{7}$$

$$Q_t = \frac{1}{\phi'\left(\frac{I^o_t}{K^o_t}\right)} \tag{8}$$

where $\Lambda_{t,t+k}$ is the stochastic discount factor for nominal payoffs given by:

$$\Lambda_{t,t+k} \equiv \beta^k \left(\frac{C^o_{t+k}}{C^o_t} \right)^{-\sigma} \left(\frac{L^o_{t+k}}{L^o_t} \right)^{\nu(1-\sigma)} \left(\frac{P_t}{P_{t+k}} \right) \tag{9}$$

and where Q_t is the (real) shadow value of capital in place, i.e., Tobin's Q. Notice that, under our assumption on ϕ, the elasticity of the investment-capital ratio with respect to Q is given by $-\frac{1}{\phi''(\delta)\delta} \equiv \eta$.

2.1.2 Rule-of-Thumb Households

Rule-of-thumb households do not attempt (or are just unable) to smooth their consumption path in the face of fluctuations in labor income. Each period they solve the static problem

$$\max \; U(C^r_t, L^r_t) \tag{10}$$

subject to the constraint that all their labor income is consumed, that is:

$$P_t C^r_t = W_t N^r_t \tag{11}$$

and where an "r"superscript is used to denote variables specific to rule-of-thumb households.

The associated first order condition is given by:

$$\frac{C^r_t}{L^r_t} = \frac{1}{\nu} \frac{W_t}{P_t} \tag{12}$$

which combined with (11) yields

$$N^r_t = \frac{1}{1+\nu} \equiv N^r \tag{13}$$

hence implying a constant employment for rule-of-thumb households, as well as a consumption level proportional to the real wage:[8]

$$C^r_t = \frac{1}{1+\nu} \frac{W_t}{P_t} \tag{14}$$

[8] Alternatively we could have directly assumed a constant labor supply.

2.1.3 Aggregation

Aggregate consumption and leisure are a weighted average of the corresponding variables for each consumer type. Formally:

$$C_t \equiv \lambda \; C_t^r + (1-\lambda) \; C_t^o \tag{15}$$

and

$$N_t \equiv \lambda \; N_t^r + (1-\lambda) \; N_t^o \tag{16}$$

Similarly, aggregate investment and capital stock are given by

$$I_t \equiv (1-\lambda) \; I_t^o$$

and

$$K_t \equiv (1-\lambda) \; K_t^o$$

We can combine (15) and (16) with the optimality conditions (5), (12), and (13) to obtain,

$$N_t = \frac{\lambda}{1+\nu} + (1-\lambda) \; N_t^o$$

and

$$C_t = \frac{1}{\nu} \left(\frac{W_t}{P_t} \right) \; (1 - N_t) \tag{17}$$

which will be used below.

2.2 Firms

We assume the existence of a continuum of monopolistically competitive firms producing differentiated intermediate goods. The latter are used as inputs by a (perfectly competitive) firm producing a single final good.

2.2.1 Final Goods Firm

The final good is produced by a representative, perfectly competitive firm with a constant returns technology:

$$Y_t = \left(\int_0^1 X_t(j)^{\frac{\varepsilon-1}{\varepsilon}} \; dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

where $X_t(j)$ is the quantity of intermediate good j used as an input. Profit maximization, taking as given the final goods price P_t and the prices for the intermediate goods $P_t(j)$, all $j \in [0,1]$, yields the set of demand schedules

$$X_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} Y_t$$

as well as the zero profit condition $P_t = \left(\int_0^1 P_t(j)^{1-\varepsilon}\, dj\right)^{\frac{1}{1-\varepsilon}}$.

2.2.2 Intermediate Goods Firm

The production function for a typical intermediate goods firm (say, the one producing good j) is given by:

$$Y_t(j) = K_t(j)^{\alpha}\ N_t(j)^{1-\alpha} \tag{18}$$

where $K_t(j)$ and $N_t(j)$ represents the capital and labor services hired by firm j.[9] Cost minimization, taking the wage and the rental cost of capital as given, implies the optimality condition:

$$\frac{K_t(j)}{N_t(j)} = \left(\frac{\alpha}{1-\alpha}\right) \left(\frac{W_t}{R_t^k}\right)$$

Real marginal cost is common to all firms and given by:

$$MC_t = \frac{1}{\Phi} \left(\frac{R_t^k}{P_t}\right)^{\alpha} \left(\frac{W_t}{P_t}\right)^{1-\alpha}$$

where $\Phi \equiv \alpha^{\alpha}(1-\alpha)^{1-\alpha}$.

Price Setting Intermediate firms are assumed to set nominal prices in a staggered fashion, according to the stochastic time dependent rule proposed by Calvo (1983). Each firm resets its price with probability $1-\theta$ each period, independently of the time elapsed since the last adjustment. Thus, each period a measure $1-\theta$ of producers reset their prices, while a fraction θ keep their prices unchanged

A firm resetting its price in period t will seek to maximize

$$\max_{P_t^*} E_t \sum_{k=0}^{\infty} \theta^k\ E_t \left\{\Lambda_{t,t+k}\ Y_{t+k}(j)\ \left(P_t^* - P_{t+k}\ MC_{t+k}\right)\right\}$$

subject to the sequence of demand constraints $Y_{t+k}(j) = X_{t+k}(j) = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\varepsilon} Y_{t+k}$ and where P_t^* represents the price chosen by firms resetting prices at time t.

The first order conditions for the above problem is:

$$\sum_{k=0}^{\infty} \theta^k\ E_t \left\{\Lambda_{t,t+k}\ Y_{t+k}(j)\ \left(P_t^* - \frac{\varepsilon}{\varepsilon-1}\ P_{t+k}\ MC_{t+k}\right)\right\} = 0 \tag{19}$$

[9] Without loss of generality we have normalized the level of total factor productivity to unity.

Finally, the equation describing the dynamics for the aggregate price level is given by:

$$P_t = \left[\theta \ P_{t-1}^{1-\varepsilon} + (1-\theta) \ (P_t^*)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{20}$$

2.3 Monetary Policy

In our baseline specification the central bank is assumed to set the nominal interest rate $r_t \equiv R_t - 1$ every period according to a simple linear interest rate rule:

$$r_t = r + \phi_\pi \ \pi_t \tag{21}$$

where $\phi_\pi \geq 0$ and r is the steady state nominal interest rate. Notice that the rule above implicitly assumes a zero inflation target.

An interest rate rule of the form (21) is the simplest specification in which the conditions for indeterminacy and their connection to the Taylor principle can be analyzed. Notice that it is a particular case of the celebrated Taylor rule (Taylor (1993)), one with a zero coefficient on the output gap and a zero inflation target. Rule (21) is said to satisfy the Taylor principle whenever $\phi_\pi > 1$. As discussed below, and in the absence of rule-of-thumb consumers, the Taylor principle provides a useful criterion to determine whether a rule like (21) guarantees a unique equilibrium or whether local indeterminacy and the possibility of sunspot fluctuations arises instead.[10]

In addition to the above baseline specification of monetary policy, we also examine the robustness of our findings to alternative interest rate rules. Among those we consider rules in which the interest rate responds (i) to current output, (ii) to expected inflation and output, and (iii) to lagged inflation, respectively. We refer the reader to the discussion below for details.

2.4 Market Clearing

The clearing of factor and good markets requires that the following conditions are satisfied for all t:

$$N_t = \int_0^1 N_t(j) \ dj$$

$$K_t = \int_0^1 K_t(j) \ dj$$

$$Y_t(j) = X_t(j) \qquad \text{for all } j \in [0,1]$$

and

[10] See, e.g., Bullard and Mitra (2002) for the case without capital accumulation, and Dupor (2002) for the case with capital accumulation and adjustment costs like ours.

$$Y_t = C_t + I_t \tag{22}$$

2.5 Linearized Equilibrium Conditions

Next we derive the log-linear versions of the key optimality and market clearing conditions that will be used in our analysis of the model's equilibrium dynamics. Some of these conditions hold exactly, while others represent first-order approximations around a zero inflation steady state. In general, we use lower case letters to denote the logs of the corresponding original variables, (or their log deviations from steady state), and ignore constant terms throughout.

2.5.1 Households

The log-linearized versions of the households' optimality conditions, expressed in terms of aggregate variables, are presented next, while leaving the derivations and proofs to the appendix. Some of these optimality conditions turn out to be independent of λ, the weight of rule-of-thumb consumers in the economy. Among the latter we have the aggregate labor supply schedule

$$c_t + \varphi\ n_t = w_t - p_t \tag{23}$$

where $\varphi \equiv \frac{N}{1-N}$. The latter coefficient, which can be interpreted as the inverse of the Frisch aggregate labor supply elasticity, can be shown to be independent of λ.

The log-linearized equations describing the dynamics of Tobin's Q and its relationship with investment are also independent of λ, and given respectively by

$$q_t = \beta\ E_t\{q_{t+1}\} + [1 - \beta(1-\delta)]\ E_t\{(r_{t+1}^k - p_{t+1})\} - (r_t - E_t\{\pi_{t+1}\}) \tag{24}$$

and

$$i_t - k_t = \eta\ q_t \tag{25}$$

The same invariance to λ holds for the log-linearized capital accumulation equation:

$$k_{t+1} = \delta\ i_t + (1-\delta)\ k_t \tag{26}$$

The only aggregate equilibrium condition that is affected by the weight of rule-of-thumb consumers turns out to be the log-linearized aggregate Euler equation, which takes the form

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}\ (r_t - E_t\{\pi_{t+1}\} - \rho) - \Theta\ E_t\{\Delta n_{t+1}\} \tag{27}$$

where

$$\Theta \equiv \frac{\varphi\lambda_c}{1-\lambda_c} + \frac{(1-\frac{1}{\sigma})\nu\varphi_o}{(1-\lambda_n)}$$

with $\varphi_o \equiv \frac{N^o}{1-N^o}$ being the (inverse) Frisch labor supply elasticity for optimizing consumers. Coefficients $\lambda_n \equiv \frac{\lambda N^r}{N}$ and $\lambda_c \equiv \frac{\lambda C^r}{C}$ are the steady state shares of rule-of-thumb households in aggregate employment and aggregate consumption. As shown in Appendix 1, both λ_n and λ_c lie in the $[0,1]$ interval, are strictly increasing in λ, and satisfy the limiting conditions $\lim_{\lambda \to 0} \lambda_n = \lim_{\lambda \to 0} \lambda_c = 0$. The previous results, combined with the fact that $\lim_{\lambda \to 0} \varphi_o = \varphi$, imply that $\lim_{\lambda \to 0} \Theta = (1 - \frac{1}{\sigma})\nu\varphi = (1 - \frac{1}{\sigma})\frac{(\rho+\delta)(1-\alpha)}{\rho+\delta(1-\alpha)+\mu(\rho+\delta)} \in (-\infty, 1)$, where the second equality makes use of a result derived in Appendix 2.

Notice that the possibility of a non-separable utility ($\sigma \neq 1$) justifies in itself the presence of the term involving expected employment growth in the aggregate Euler equation. Nevertheless, the presence and weight of rule-of-thumb consumers will, through its impact on λ_n and λ_c, alter the size of Θ, i.e. the coefficient associated with that variable. As discussed below, that effect can potentially alter the local stability properties of the dynamical system describing the equilibrium.

2.5.2 Firms

Log-linearization of (19) and (20) around the zero inflation steady state yields the familiar equation describing the dynamics of inflation as a function of the deviations of the average (log) markup from its steady state level

$$\pi_t = \beta \; E_t\{\pi_{t+1}\} - \lambda_p \; \mu_t \tag{28}$$

where $\lambda_p = \frac{(1-\beta\theta)(1-\theta)}{\theta}$ and (ignoring constant terms)

$$\mu_t = (y_t - n_t) - (w_t - p_t) \tag{29}$$

or, equivalently,

$$\mu_t = (y_t - k_t) - (r_t^k - p_t) \tag{30}$$

Furthermore, it can be shown that the following aggregate production function holds, up to a first order approximation:

$$y_t = \alpha \; k_t + (1-\alpha) \; n_t \tag{31}$$

2.5.3 Market clearing

Log-linearization of the market clearing condition of the final good around the steady state yields:

$$y_t = \gamma_c \; c_t + (1-\gamma_c) \; i_t \tag{32}$$

where $\gamma_c \equiv \frac{C}{Y}$ represents aggregate consumption share in the steady state. As shown in Appendix 1 the latter variable is independent of the weight of rule-of-thumb consumers.

3 Analysis of Equilibrium Dynamics

We can now combine equilibrium conditions (23)-(32) to obtain a system of difference equations describing the log-linearized equilibrium dynamics of our model economy. After several straightforward though tedious substitutions described in Appendix 3, we can reduce that system to one involving four variables:

$$\mathbf{A}\ E_t\{\mathbf{x}_{t+1}\} = \mathbf{B}\ \mathbf{x}_t \tag{33}$$

where $\mathbf{x}_t \equiv (n_t,\, c_t,\, \pi_t,\, k_t)'$. Notice that i_t, n_t, k_t are expressed in terms of log deviations from their values in the zero inflation steady state. The elements of matrices $\mathbf{A}$ and $\mathbf{B}$ are all functions of the underlying structural parameters, as shown in Appendix 3.

Notice that $\mathbf{x}_t = 0$ for all t , which corresponds to the perfect foresight zero inflation steady state, always constitutes a solution to the above system. This should not be surprising, given that for simplicity we have not introduced any fundamental shocks in our model. In the remainder of the paper we study the conditions under which the solution to (33) is unique and converges to the steady state, for any given initial capital stock. In doing so we restrict our analysis to solutions of (33) (i.e., equilibrium paths) which remain within a small neighborhood of the steady state.[11] Before we turn to that task, we discuss briefly the calibration that we use as a baseline for that analysis.

3.1 Baseline Calibration

The model is calibrated to a quarterly frequency. The rate of depreciation δ is set to 0.025 (implying a 10 percent annual rate). The elasticity of output with respect to capital, α, is assumed to be $\frac{1}{3}$, a value roughly consistent with the labor income share given any reasonable steady state markup. With regard to preference parameters, we set the discount factor β equal to 0.99 (implying a steady state real annual return of 4 percent). The elasticity of substitution across intermediate goods, ε, is set to 6, a value consistent with a steady state markup μ of 0.2. The previous parameters are kept at their baseline values throughout the present section.

Next we turn to the parameters for which some sensitivity analysis is conducted, by examining a range of values in addition to their baseline settings. We set the baseline value for parameter ν in a way consistent with a *unit* Frisch elasticity of labor supply (i.e., $\varphi = 1$) in our baseline calibration. That choice is associated with a fraction of time allocated to work in the steady state given by $N = \frac{1}{2}$. We choose a baseline value of one for σ, which corresponds to a separable (log-log) utility specification. The fraction of firms that keep their prices unchanged, θ, is given a baseline value of 0.75, which corresponds to an average price duration of one year.

[11] See, e.g., Benhabib, Schmitt-Grohé and Uribe (2001a) for a discussion of the caveats associated with that approach.

This is consistent with the findings reported in Taylor (1999c). Following King and Watson (1996), we set η, the elasticity of investment with respect to Tobin's Q, equal to 1.0 under our baseline calibration.[12] Much of the sensitivity analysis below focuses on the weight of rule-of-thumb households (λ) and its interaction with θ, σ, φ, η, and ϕ_π.

3.2 Determinacy Analysis

Vector $\mathbf{x}_t$ contains three non-predetermined variables (hours, consumption and inflation) and a predetermined one (capital stock). Hence, the solution to (33) is unique if and only if three eigenvalues of matrix $A^{-1}B$ lie outside the unit circle, and one lies inside.[13] Alternatively, if there is more than one eigenvalue of $A^{-1}B$ inside the unit circle the equilibrium is locally indeterminate: for any initial capital stock there exists a continuum of deterministic equilibrium paths converging to the steady state, and the possibility of stationary sunspot fluctuations arises. On the other hand, if all the eigenvalues $A^{-1}B$ lie outside the unit circle, there is no solution to (33) that converges to the steady state, unless the initial capital stock happens to be at its steady state level (in which case $\mathbf{x}_t = 0$ for all t is the only non-explosive solution). Below our focus is on how the the presence of rule-of-thumb consumers may influence the configuration of eigenvalues of the dynamical system, and hence the properties of the equilibrium.

3.3 Rule-of-Thumb Consumers, the Taylor Principle, and Indeterminacy

We start by exploring the conditions for the existence of a unique equilibrium as a function of the degree of price stickiness (indexed by parameter θ) and the weight of rule-of-thumb households (indexed by parameter λ). We focus on a version of our economy in which the interest rate rule satisfies, albeit marginally, the Taylor principle. A key finding of our paper is illustrated by Figure 1. That figure represents the equilibrium properties of our model economy for all configurations of λ and θ, *under the assumption of* $\phi_\pi = 1.001$.

In particular, the figure displays the regions in the parameter space (λ, θ) that are associated with the presence of uniqueness, multiplicity, and non-existence of a rational expectations equilibrium in a neighborhood of the steady state. Notice that each graph corresponds to an alternative pair of settings for the risk aversion coefficient σ and the inverse labor supply elasticity φ.

[12] Other authors who have worked with an identical specification of capital adjustment costs have considered alternative calibrations of that elasticity. Thus, e.g., Dotsey (1999) assumes an elasticity of 0.25; Dupor (2002) assumes a baseline elasticity of 5; Baxter and Crucini (1993) set a baseline value of 15; Abel (1980) estimates that elasticity to be between 0.3 and 0.5 .

[13] See, e.g., Blanchard and Kahn (1980)

A key finding emerges clearly: the combination of a high degree of price stickiness with a large share of rule-of-thumb consumers rules out the existence of a unique equilibrium converging to the steady state. Instead, the economy is characterized in that case by indeterminacy of equilibrium (dark region) or–for a smaller range of parameter configurations–non-existence (white region). Conversely, if (a) prices are sufficiently flexible (low θ) and/or (b) the share of rule-of-thumb consumers is sufficiently small (low λ), the existence of a unique equilibrium is guaranteed. That finding holds irrespective of the assumed values for σ and φ, even though the relative size of the different regions can be seen to depend on those parameters. In particular, the size of the uniqueness region appears to shrink as σ and φ increase. In sum, as made clear by Figure 1, *the Taylor principle may no longer be a useful criterion for the design of interest rate rules in economies with strong nominal rigidities and a substantial presence of rule-of-thumb consumers.*

Importantly, while the previous result has been illustrated under the assumption of an inflation coefficient ϕ_π very close to unity, similar patterns arise for higher values of that parameter. However, the size of the indeterminacy and non-existence regions can be shown to shrink gradually, as the interest rate response to inflation increases (while keeping other parameters constant). Thus, for any configuration of parameter settings, there exists a minimum threshold value for the inflation coefficient that guarantees the existence and uniqueness of a stationary equilibrium in a neighborhood of the steady state. In other words, a strengthened condition on the size of the response of interest rates to changes in inflation is required in that case. Next, we provide an explicit analysis of the variation in the threshold value for ϕ_π, as a function of different parameter values and, most importantly, as a function of the share of rule-of-thumb households.

3.4 Interest Rate Rules and Rule-of-Thumb Consumers: Requirements for Stability

Figure 2 displays the threshold value for ϕ_π that guarantees the existence and uniqueness of a rational expectations equilibrium (in a neighborhood of the steady state), as a function of the share of rule-of-thumb consumers λ. The different line types correspond to alternative settings for the price stickiness parameter θ. For convenience, Figure 2 (like many of the subsequent figures) plot the *inverse* of such a threshold value.[14]

Notice first that, in every case considered, the threshold value for ϕ_π is equal to *one*, so long as the weight of rule-of-thumb consumers is sufficiently low. That finding is consistent with the results discussed above, and corresponds to the conventional wisdom regarding the Taylor principle as a criterion for stability. Secondly, and most interestingly, once the share of rule-of-thumb consumers attains a certain level (which

[14] The inverse of the threshold value is bounded within the interval [0,1], which facilitates graphical display.

depends on other parameter values) the lower bound for ϕ_π that guarantees existence and uniqueness starts to increase sharply. Thus, for instance, under our baseline calibration ($\theta = 0.75$), the central bank needs to vary the nominal rate in response to changes in inflation on a more than one-for-one basis whenever the share of rule-of-thumb consumers is above 0.57. In particular, when $\lambda = \frac{2}{3}$, the inflation coefficient ϕ_π must lie above 6 (approximately) in order to guarantee a unique equilibrium. Notice also how the threshold value for the inflation coefficient goes down as the degree of price stickiness is reduced, conditional on any given λ.

Figures 3, 4, and 5 display similar information for alternative calibrations of φ, η, and σ, respectively, with all other parameters set at their baseline values in each case. The picture that emerges is, qualitatively, similar to that of Figure 2: the lower bound for ϕ_π guaranteeing a unique equilibrium is shown to be increasing in the share of rule-of-thumb consumers for any given value of those parameters. Regarding the influence of those parameters, the main qualitative findings can be summarized as follows: the deviation from the Taylor principle criterion seems to become more likely and/or quantitatively larger the lower is the labor supply elasticity (i.e., the higher is φ), the more convex capital adjustment costs are (i.e., the lower is η), and the higher is the risk aversion parameter σ.

3.5 Impulse Responses and Economic Mechanisms

As discussed above, in the standard new Keynesian framework with a representative consumer, the Taylor principle generally constitutes the appropriate criterion for determining whether an interest rate rule of the sort considered in the literature will guarantee or not a unique equilibrium, and thus rule out the possibility of sunspot-driven fluctuations. The basic intuition goes as follows. Suppose that, in the absence of any shock to fundamentals that could justify it, there was an increase in the level of economic activity, with agents anticipating the latter to return only gradually to its original (steady state) level. That increase in economic activity would be associated with increases in hours, lower markups (because of sticky prices), and persistently high inflation (resulting from the attempts by firms adjusting prices to re-establish their desired markups). But an interest rate rule that satisfied the Taylor principle would generate high real interest rates along the adjustment path, and hence, would call for a low level of consumption and investment relative to the steady state. The implied impact on aggregate demand would make it impossible to sustain the initial boom, thus rendering it inconsistent with a rational expectations equilibrium.

Consider instead the dynamic response of the economy to such an exogenous revision in expectations when the weight of rule-of-thumb consumers is sufficiently high to allow for multiple equilibria even though the interest rate rule satisfies the Taylor principle. That response is illustrated graphically in Figure 6, which displays the simulated responses to an expansionary sunspot shock for a calibrated version of our model economy meeting the above criteria. The presence of rule-of-thumb consumers, combined with the countercyclical markups, makes it possible to break the logic that served us to rule out a sunspot-driven economic boom. Two features are critical here. First, the decline in markups resulting from sluggish price adjustment allows real wages to go up (this effect is stronger in economies with a low labor supply elasticity) in spite of the decline in labor productivity associated with higher employment. Secondly, and most importantly, the increase in real wages generates a boom in consumption among rule-of-thumb consumers; if the weight of the latter in the economy is sufficiently important, the rise in their consumption will more than offset the decline in that of Ricardian consumers, as well as the drop in aggregate investment (both generated by the rise in interest rates). As a result, aggregate demand will rise, thus making it possible to sustain the persistent boom in output that was originally anticipated by agents. That possibility is facilitated by the presence of highly convex adjustment costs (low η), which will mute the investment response, together with a low elasticity of intertemporal substitution (a high σ), which will dampen the response of the consumption of Ricardian households.

4 Alternative Specifications of the Interest Rate Rule

In the present section we explore how changes in the specification of the interest rate rule affect some of the conclusions of the determinacy analysis described above. In particular, we modify our baseline rule in three directions often considered in the literature, by allowing the central bank to respond (i) to current output, (ii) to expected inflation and output, and (iii) to lagged inflation, respectively. We examine each of the those cases in turn.

4.1 The Baseline Rule Augmented with Output

In this section we consider the properties of our economy when the central bank follows the interest rate rule

$$r_t = r + \phi_\pi \ \pi_t + \phi_y \ y_t$$

where $\phi_\pi \geq 0$ and $\phi_y \geq 0$. As shown by Bullard and Mitra (2002) and Woodford (2001), in the context of a version of the new Keynesian model *without* capital it is possible for the equilibrium to be unique for values of ϕ_π less than one, as long as as the central bank raises the interest rate sufficiently in response to an increase in output (i.e., if ϕ_y lies above a certain threshold, which is decreasing in ϕ_π). In other words, in the canonical model there is some substitutability between the size of the response to output and that of the response to inflation.

The previous result carries over, at least in a qualitative sense, to our model with rule-of-thumb consumers (and capital accumulation). This is illustrated by Figure 7 which plots the threshold value of ϕ_π that is required for a unique equilibrium as a function of the share of rule-of-thumb consumers, for three alternative values of ϕ_y: 0 (our baseline case), 0.5 (as in Taylor (1993)) and 1.0 (as in the modified Taylor rule considered in Taylor (1999c)). We notice that as ϕ_y increases, the threshold value for ϕ_π falls, for any given share of rule-of-thumb consumers. Yet, as the Figure makes clear, the fact that the central bank is responding to output does not relieve it from the need to respond to inflation on a more than one-for-one basis, once a certain share of rule-of-thumb consumers is attained. Furthermore, as in our baseline case, the size of the minimum required response is increasing in that share.

4.2 A Forward Looking Rule

We have also analyzed the properties of our model when the central bank follows a forward-looking interest rate rule of the form

$$r_t = r + \phi_\pi \; E_t\{\pi_{t+1}\} + \phi_y \; E_t\{y_{t+1}\} \tag{34}$$

The rule above corresponds to a particular case of the specification originally proposed by Bernanke and Woodford (1997), and estimated by Clarida, Galí and Gertler (1998, 2000). Dupor (2002) analyzes the equilibrium properties of a rule identical to (34) in the context of a new Keynesian model with capital accumulation similar to the one used in the present paper, though without rule-of-thumb consumers. His analysis suggests that the Taylor principle remains a useful criterion for this kind of economies, but with an important qualification: the existence of a unique rational expectations equilibrium now requires that ϕ_π lies within an interval $(1, \phi_\pi^u)$, for some upper limit $\phi_\pi^u > 1$. In other words, in addition to the usual Taylor principle-related condition $\phi_\pi > 1$, there is an upper bound to the size of the response to expected inflation that must be satisfied in order for a unique equilibrium to exist; if that upper bound is overshot the equilibrium becomes indeterminate. A similar result has been shown analytically in the context of a similar model without capital. See, e.g., Bernanke and Woodford (1997), and Bullard and Mitra (2002).[15]

How does the presence of rule-of-thumb consumers affect the previous result? Figure 8 represents graphically the interval of ϕ_π values for which a unique equilibrium exists, as a function of the weight of rule-of-thumb consumers λ, and given $\phi_y = 0$ (the latter assumption is relaxed below). Notice that the three graphs correspond to three alternative values of η, our parameter measuring the importance of capital adjustment costs. The main results here can be summarized as follows.[16]

[15] More recently, Levin, Wieland and Williams (2002) have shown that the existence of such an upper threshold is inherent to a variety of forward-looking rules, with the uniqueness region generally shrinking as the forecast horizon is raised.

[16] Similar qualitative findings emerge when we replace expected output with current output in the interest rate rule.

First, for low values of λ (roughly below 0.6) the qualitative result found in the literature carries over to our economy: the uniqueness requires that ϕ_π lies within some interval bounded below by 1. Interestingly, for this region, the size of that interval shrinks gradually as λ increases. Also, and as long as we are in this region, the size of that interval is increasing with the convexity of capital adjustment costs (i.e., decreasing with elasticity η), for any given λ. The latter result is consistent with the findings of Dupor (2002) for the particular case of $\lambda = 0$ (no rule-of-thumb consumers).

Most interestingly (and surprisingly), when the weight of rule-of-thumb consumers λ lies above a certain threshold, the properties of the forward-looking rule change dramatically. In particular, a value for ϕ_π below unity is needed in order to guarantee the existence of a unique rational expectations equilibrium. In other words, the central bank would be ill advised if it were to follow a forward-looking rule satisfying the Taylor principle, since that policy would necessarily generate an indeterminate equilibrium.

How can a large presence of rule-of-thumb consumers make it possible for a rule that responds less than one-for-one to (expected) inflation to be consistent with a unique equilibrium? In order to gain some intuition about that result, in Figure 9 we present the dynamic responses of several variables to a unit cost-push shock (i.e., an additive shock to (28)), when $\phi_\pi = 0.2$, $\lambda = 0.85$ and $\eta = 1$. The shock is assumed to follow an AR(1) process with autoregressive coefficient equal to 0.9. As shown in the figure, output declines persistently in response to the shock, thus dampening (though not offsetting) its impact on inflation. Most importantly, and as could be anticipated from the calibration of the rule, the (ex-ante) real interest rate remains below its steady state value throughout the adjustment in response to the higher inflation. In the model with no rule-of-thumb consumers the response of the real rate would lead to an output expansion and, as a result, an explosive path for inflation. The presence of rule of thumb consumers, however, allows for an equilibrium in which output decline in spite of the lower real rate, as a result of the lower real wages and hence lower consumption by those households.

In summary, the presence of rule-of-thumb consumers either shrinks the interval of ϕ_π values for which the equilibrium is unique (in the case of low λ), or makes a passive policy necessary to guarantee that uniqueness (for high values of λ).

How do the previous results change when we the central bank respond to output, as well as inflation? The answer to that question is summarized by means of Figure 10, which displays four graphs corresponding to alternative values of ϕ_y, given baseline settings for the remaining parameters. A systematic response of the interest rate to changes in output, even if small in size, has a significant impact on the stability properties of our model economy. Thus, for low values of λ, a positive setting for ϕ_y tends to raise the upper threshold for ϕ_π consistent with a unique equilibrium. As seen in the four consecutive graphs of Figure 10, the effect of ϕ_y on the size of the uniqueness region appears to be non-monotonic, increasing very quickly for low values of ϕ_y and shrinking back gradually for higher values. On the other hand, for higher values of λ, the opposite effect takes place: the interval of ϕ_π values for which there is a unique equilibrium becomes smaller as we increase the size of the output coefficient relative to the $\phi_y = 0$ case. In fact, under our baseline calibration, when $\phi_y = 0.5$ and for λ sufficiently high, an indeterminate equilibrium arises regardless of the value of the inflation coefficient.

The high sensitivity of the model's stability properties to the size of the output coefficient in a forward-looking interest rate rule in a model with capital accumulation (but no rule-of-thumb consumers) had already been noticed by Dupor (2002). Our previous analysis raises an important qualification (and warning) on such earlier results: in the presence of rule-of-thumb consumers an aggressive response to output does not seem warranted, for it can only reduce the region of inflation coefficients consistent with a unique equilibrium. On the other hand, a small response to output has the opposite effect: it tends to enlarge the size of the uniqueness region.

4.3 A Backward Looking Rule

Finally we analyze the stability properties of our model economy when the central bank follows a rule of the form:

$$r_t = r + \phi_\pi \; \pi_{t-1} \tag{35}$$

The previous rule has been proposed by McCallum (1999) as a "realistic" alternative to the more commonly assumed (21), given that data on inflation and other variables is only released with a certain time lag, thus making it impossible in practice for central bank to respond to contemporaneous inflation (or output).

We are not aware of any systematic analysis of the stability properties of a backward-looking rule like (35) in the context of a sticky price with capital accumulation. The analysis of Bullard and Mitra (2002), though restricted to an economy without capital (and without rule-of-thumb consumers), points to the need of setting a value for ϕ_π within an interval $(1, \phi_\pi^u)$ with an upper limit identical to the one found in the case of a forward-looking rule.

Figure 11 summarizes the properties of the backward -looking rule for an economy with capital accumulation and rule-of-thumb consumers. It shows, for each possible weight λ of rule-of-thumb consumers, the interval of ϕ_π values for which an equilibrium exists and is locally unique under (35), and given baseline settings for the remaining parameters. Several results stand out. First, for values of λ below a certain threshold (about 0.6 under our baseline calibration) there exists an interval $(1, \phi_\pi^u)$ for the coefficient on lagged inflation such that the equilibrium exists and is locally unique. That result is similar to the one obtained above for the forward-looking rule. As in the latter case, and for the low λ region considered here, the upper limit ϕ_π^u of the range of ϕ_π values for which the equilibrium is unique becomes smaller as the weight of rule-of-thumb consumers increases.

As in the forward-looking rule case, the Taylor principle ceases to be a desirable property for rule like (35) once a certain threshold value for λ is exceeded. In particular, and for an intermediate range of λ values, no rational expectations equilibrium converging to the steady state will exist if $\phi_\pi > 1$ (this corresponds to the white region in the figure). In that case the central bank should follow a passive rule ($\phi_\pi < 1$) in order to guarantee a locally unique equilibrium.

Finally, for a higher range of λ values, we notice that the equilibrium is locally indeterminate whenever $\phi_\pi > 1$, a result similar to the one uncovered for the forward-looking rule. The difference here is that indeterminacy also arises for a range of values for ϕ_π below unity; in fact, only if that inflation coefficient takes very low value can a locally unique equilibrium be restored in that case. That region can be appreciated in the zoomed version of the graph displayed in Figure 11.

Hence, under a backward-looking interest rate rule, the presence of rule-of-thumb consumers also complicates substantially the central bank's task, by narrowing considerably the range of acceptable responses to changed in lagged inflation consistent with a unique equilibrium.

5 Concluding Remarks

The Taylor principle, i.e., the notion that central banks should raise (lower) nominal interest rates more than one-for-one in response to a rise (decline) in inflation, is generally viewed as a *prima facie* criterion in the assessment of a monetary policy. Thus, an interest rate rule that satisfies the Taylor principle is viewed as policy with stabilizing properties, whereas the failure to meet the Taylor criterion is often pointed to as a possible explanation for periods characterized by large fluctuations in inflation and widespread macroeconomic instability.

In the present paper we have provided a simple but potentially important qualification to that view. We have shown how the presence of rule-of-thumb (non-Ricardian) consumers in an otherwise standard dynamic sticky price model, can alter the properties of simple interest rate rules dramatically. The intuition behind the important role played by rule-of-thumb consumers is easy to grasp: the behavior of those households is, by definition, insulated from the otherwise stabilizing force associated with changes in real interest rates. We summarize our main results as follows.

1. Under a contemporaneous interest rate rule, the existence of a unique equilibrium is no longer guaranteed by the Taylor principle when the weight of rule-of-thumb consumers attains a certain threshold. Instead the central bank may be required to pursue a more anti-inflationary policy than it would otherwise be needed.

2. Under a backward-looking of forward-looking interest rate rules, the presence of rule-of-thumb consumers also complicates substantially the central bank's

task, by shrinking the range of responses to inflation consistent with a unique equilibrium (when the share of rule-of-thumb consumers is relatively low), or by requiring that a passive interest rate rule is followed (when the share of rule-of-thumb consumers is large).

The previous results call for caution on the part of central banks when designing their anti-inflation strategies. Overall, they suggest that in the presence of a significant, but not too large a share of rule-of-thumb consumers (or more broadly speaking, procyclical components of aggregate demand that are insensitive to interest rates), the strength of the response to contemporaneous inflation may need to be increased. When that share reaches larger values the response to inflation required to guarantee a unique equilibrium may be too large to be credible or to be consistent with a non-negative nominal rate. In that case, our findings suggest that the central bank should consider adopting a passive rule that responds to expected inflation only (as an alternative to a rule that responds to current inflation with a very high coefficient). It is clear, however, that such an alternative would have practical difficulties, especially from the viewpoint of communication to the public.

The above discussion notwithstanding, it is not the objective of the present paper to come up with specific recommendations for central banks of economies populated by households who may not respond to interest rate changes; our model is clearly too simplistic to be taken at face value. On the other hand we believe our analysis is useful in at least one regard: it points to some important limitations of the Taylor principle as a simple criterion for the assessment of monetary policy when rule-of-thumb consumers (or the like) are present in the economy. In doing so it also raises some warning flags about the need to carefully think out what the proper (and robust) design of monetary policy should be in those cases.

More generally, we believe that the introduction of rule-of-thumb consumers in dynamic general equilibrium models not only enhances significantly the realism of those models, but it can also allow us to uncover interesting insights that may be relevant for the design of policies and helpful in our efforts to understand many macroeconomic phenomena. An illustration of that potential usefulness can be found in a companion paper, where we have argued that the presence of rule-of-thumb consumers may help account for the observed effects of fiscal policy shocks, some of which are otherwise hard to explain with conventional new Keynesian or neoclassical models.[17]

[17] See Galí, López-Salido and Vallés (2002).

Appendix 1: Steady State Analysis

In the zero inflation steady state the real marginal cost is constant and given by $MC = 1 - \frac{1}{\varepsilon}$. Labor market clearing requires

$$\frac{W}{P} = \left(1 - \frac{1}{\varepsilon}\right)(1-\alpha)\,\frac{Y}{N} = \nu\,\frac{C}{1-N}$$

Rearranging terms we obtain

$$\frac{N}{1-N} = \frac{1}{\nu}\frac{(1-\frac{1}{\varepsilon})(1-\alpha)}{\gamma_c}$$

In turn, the market clearing condition for final goods implies:

$$\begin{aligned}
\gamma_c &= 1 - \frac{I}{Y} \\
&= 1 - \frac{\delta\alpha}{\alpha\left(\frac{Y}{K}\right)} \\
&= 1 - \frac{\delta\alpha\left(1-\frac{1}{\varepsilon}\right)}{(\rho+\delta)}
\end{aligned}$$

where the last equality follows from the fact that in the steady state $\frac{R^k}{P} = \left(1 - \frac{1}{\varepsilon}\right)\alpha\frac{Y}{K}$ (implied by the constant marginal cost) and $\frac{R^k}{P} = (\rho+\delta)$ (implied by a constant Q).

Thus, combining the above expressions we obtain an equation relating steady state aggregate hours, N, to structural parameters

$$\varphi \equiv \frac{N}{1-N} = \frac{1}{\nu}\;\frac{(\rho+\delta)(1-\alpha)}{\rho+\delta(1-\alpha)+\mu(\rho+\delta)}$$

where $\mu \equiv \frac{1}{\varepsilon-1}$, is the net markup in the steady state. Solving for N, one gets:

$$N = \frac{(\rho+\delta)(1-\alpha)}{\nu[\rho+\delta(1-\alpha)+\mu(\rho+\delta)]+(\rho+\delta)(1-\alpha)}$$

Notice that aggregate hours do not depend on λ (and, hence, neither does the inverse Frisch labor supply elasticity φ). Notice also that $\varphi\nu \in (0,1)$.

In addition, using expression (13) we can define

$$\lambda_n \equiv \frac{\lambda N^r}{N} = \frac{\lambda}{(1+\nu)N}$$

Using the steady state relationships associated with equations (14) and (17) yields

$$\lambda_c \equiv \frac{\lambda C^r}{C} = \left(\frac{\lambda\nu}{1+\nu}\right)\left(\frac{1}{1-N}\right)$$

Notice that both λ_n and λ_c are increasing in λ. Finally, from equation (16) and the definition of λ_n we obtain an expression for N^o

$$N^o = \frac{1-\lambda_n}{1-\lambda} N$$

which can be used to compute the parameter $\varphi_o \equiv \frac{N^o}{1-N^o}$ (see below).

Appendix 2: Rewriting the Euler equation in terms of aggregate variables

A first order Taylor expansion of (6) and (9) yields the following log-linear Euler equation for optimizing consumers

$$c_t^o = E_t\{c_{t+1}^o\} - \frac{1}{\sigma}\,(r_t - E_t\{\pi_{t+1}\} - \rho) + \left(1 - \frac{1}{\sigma}\right)\nu\ E_t\{\Delta l_{t+1}^o\} \tag{36}$$

Next we rewrite the above conditions in terms of aggregate consumption and hours. Notice that the log-linearized time endowment constraint for optimizing consumers (2) can be written as

$$l_t^o = -\frac{N^o}{1 - N^o}\ n_t^o \equiv -\varphi_o\ n_t^o$$

Combining the above expression with the log-linearized definition of aggregate hours (16) and using the fact that N_t^r is constant (13) yields

$$l_t^o = -\frac{\varphi_o}{1 - \lambda_n}\ n_t \tag{37}$$

The previous expression establishes the desired relationship between optimizing consumers' leisure and aggregate hours.

In addition, the log-linearized definition of aggregate consumption (15) takes the form

$$c_t = \lambda_c\ c_t^r\ +\ (1 - \lambda_c)\ c_t^o$$

Notice that log-linearization of expressions (14) and (17) yields

$$\begin{aligned} c_t^r &= w_t - p_t \\ &= c_t + \varphi\ n_t \end{aligned}$$

Combining both results we obtain,

$$c_t^o = c_t - \frac{\varphi\lambda_c}{1 - \lambda_c}\ n_t \tag{38}$$

which establishes a relationship between optimizing consumers' consumption and aggregate consumption and hours.

Substituting expressions (37) and (38) into expression (36)

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}\ (r_t - E_t\{\pi_{t+1}\}) - \Theta\ E_t\{\Delta n_{t+1}\}$$

where $\Theta \equiv \frac{\varphi\lambda_c}{1-\lambda_c} + \frac{(1-\frac{1}{\sigma})\nu\varphi_o}{(1-\lambda_n)}$, which is the Euler equation in terms of aggregate variables shown in the text.

Appendix 3: Derivation of the Reduced Dynamical System

The equilibrium conditions describing the model dynamics are given by expressions (23)-(32). Now we reduce those conditions to the four variable system (33) in terms of hours, consumption, inflation and capital.

The first equation in the (33) corresponds to the linearized capital accumulation equation (26), with i_t substituted out using market clearing condition (32) and replacing y_t subsequently using the production function (31):

$$k_{t+1} = \left(1 - \delta + \frac{\delta\alpha}{1-\gamma_c}\right) \; k_t + \frac{\delta(1-\alpha)}{1-\gamma_c} \, n_t - \frac{\delta \; \gamma_c}{1-\gamma_c} \; c_t \tag{39}$$

In order to derive the second equation in (33) we start by rewriting the inflation equation (28) in terms of variables contained in $\mathbf{x}_t$. Using (29) and (23) we obtain an expression for the marginal cost as a function of the consumption output ratio and aggregate hours

$$\mu_t = y_t - c_t - (1+\varphi) \; n_t \tag{40}$$

Substituting the previous expression (40) into (28), and making use of (31) yields the second equation in (33)

$$\begin{aligned} \pi_t &= \beta \; E_t\{\pi_{t+1}\} + \lambda_p \; \left[c_t - y_t + (1+\varphi) \; n_t\right] \\ &= \beta \; E_t\{\pi_{t+1}\} + \lambda_p \; c_t - \alpha\lambda_p \; k_t + (\alpha+\varphi)\lambda_p \; n_t \end{aligned} \tag{41}$$

Plugging the interest rate rule into the aggregate Euler equation yields the third equation in (33):

$$c_t - \Theta \; n_t + \frac{\phi_\pi}{\sigma} \; \pi_t = E_t\{c_{t+1}\} + \frac{1}{\sigma} \; E_t\{\pi_{t+1}\} - \Theta \; E_t\{n_{t+1}\} \tag{42}$$

In order to derive the fourth equation we first combine (40) and (30) to obtain $r_t^k - p_t = c_t - k_t + (1+\varphi)n_t$. The latter expression and the interest rate rule (21), allows us to rewrite the equations describing the dynamics of Tobin's q and investment as follows:

$$\begin{aligned} i_t - k_t \; = \; & \beta \; E_t\{(i_{t+1} - k_{t+1})\} \\ & + \eta[1-\beta(1-\delta)] \; \left[E_t\{c_{t+1}\} - k_{t+1} + (1+\varphi) \; E_t\{n_{t+1}\}\right] \\ & - \eta\phi_\pi \; \pi_t + \eta \; E_t\{\pi_{t+1}\} \end{aligned}$$

Finally, substituting the relationship $i_t - k_t = \left(\frac{1}{1-\gamma_c}\right) [(1-\alpha)n_t - \gamma_c c_t - (1-\gamma_c-\alpha)k_t]$ (which can be derived by combining the goods market clearing condition with the production function) into the previous equation and rearranging terms we obtain the fourth equation of our dynamical system

$$\begin{aligned}(1-\alpha)\ n_t - \gamma_c\ c_t - (1-\gamma_c-\alpha)\ k_t + (1-\gamma_c)\eta\phi_\pi\ \pi_t \quad = \quad & [\omega(1+\varphi)+\beta(1-\alpha)]\ E_t\{n_{t+1}\} \\ & +(\omega-\beta\gamma_c)\ E_t\{c_{t+1}\} \\ & -[\omega+\beta(1-\gamma_c-\alpha)]\ k_{t+1} \\ & +(1-\gamma_c)\eta\ E_t\{\pi_{t+1}\} \qquad (43)\end{aligned}$$

where $\omega \equiv \eta[1-\beta(1-\delta)](1-\gamma_c) > 0$.

Hence the system of equations (39), (41), (42), and (43) can be written in a matrix form as follows

$$\mathbf{A}\ E_t\{\mathbf{x}_{t+1}\} = \mathbf{B}\ \mathbf{x}_t$$

where $\mathbf{x}_t \equiv [n_t,\ c_t,\ \pi_t,\ k_t]'$, and

$$\mathbf{A} \equiv \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & \beta & 0 \\ -\Theta & 1 & \frac{1}{\sigma} & 0 \\ \omega(1+\varphi)+\beta(1-\alpha) & \omega-\beta\gamma_c & (1-\gamma_c)\eta & -[\omega+\beta(1-\gamma_c-\alpha)] \end{bmatrix}$$

$$\mathbf{B} \equiv \begin{bmatrix} \frac{\delta(1-\alpha)}{1-\gamma_c} & -\frac{\delta\ \gamma_c}{1-\gamma_c} & 0 & 1-\delta+\frac{\delta\alpha}{1-\gamma_c} \\ -(\alpha+\varphi)\lambda_p & -\lambda_p & 1 & \alpha\lambda_p \\ -\Theta & 1 & \frac{\phi_\pi}{\sigma} & 0 \\ 1-\alpha & -\gamma_c & (1-\gamma_c)\eta\phi_\pi & \gamma_c+\alpha-1 \end{bmatrix}$$

References

Abel, Andrew. (1980): "Empirical Investment Equations. An Integrated Framework," *Carnegie Rochester Conference Series on Public Policy* , 12, 39-91.

Amato, Jeffery, and Thomas Laubach (2002): "Rule-of-Thumb Behavior and Monetary Policy," *European Economic Review*, forthcoming.

Baxter, Marianne. and Mario Crucini, (1993): "Explaining Saving Investment Correlations," *American Economic Review,* vol. 83, no. 1, 416-436.

Benhabib, Jess, Stephanie Schmitt-Grohe, and Martin Uribe (2001a): "The Perils of Taylor Rules," *Journal of Economic Theory* 96, 40-69.

Benhabib, Jess, Stephanie Schmitt-Grohe, and Martin Uribe (2001b): "Monetary Policy and Multiple Equilibria," *American Economic Review* vol. 91, no. 1, 167-186.

Bernanke, Ben S., and Michael Woodford (1997): "Inflation Forecasts and Monetary Policy," *Journal of Money, Credit and Banking*, vol. 24, 653-684:

Blanchard, Olivier and Charles Kahn (1980), "The Solution of Linear Difference Models under Rational Expectations", *Econometrica*, 48, 1305-1311.

Bullard, James, and Kaushik Mitra (2002): "Learning About Monetary Policy Rules," *Journal of Monetary Economics*, vol. 49, no. 6, 1105-1130.

Campbell, John Y. and N. Gregory Mankiw (1989): "Consumption, Income, and Interest Rates: Reinterpreting the Time Series Evidence," in O.J. Blanchard and S. Fischer (eds.), *NBER Macroeconomics Annual 1989*, 185-216, MIT Press

Calvo, Guillermo, 1983, "Staggered Prices in a Utility Maximizing Framework," *Journal of Monetary Economics*, 12, 383-398.

Clarida, Richard, Jordi Galí, and Mark Gertler (1998): "Monetary Policy Rules in Practice: Some International Evidence", *European Economic Review*, 42 (6), June 1998, pp. 1033-1067.

Clarida, Richard, Jordi Galí, and Mark Gertler (1999): "The Science of Monetary Policy: A New Keynesian Perspective," *Journal of Economic Literature*, vol. 37, 1661-1707.

Clarida, Richard, Jordi Galí, and Mark Gertler (2000): "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory," *Quarterly Journal of Economics*, vol. 105, issue 1, 147-180.

Dotsey, Michael (1999): "Structure from Shocks," Federal Reserve Bank of Richmond Working Paper 99-6.

Dupor, Bill (2002): "Interest Rate Policy and Investment with Adjustment Costs," manuscript.

Galí, Jordi, J. David López-Salido, and Javier Vallés (2003): "Technology Shocks and Monetary Policy: Assessing the Fed's Performance," *Journal of Monetary Economics*, vol. 50, no. 4.

Galí, Jordi, J. David López-Salido, and Javier Vallés (2002): "Understanding the Effects of Government Spending Shocks," mimeo, *Bank of Spain*.

Kerr, William, and Robert G. King: "Limits on interest rate rules in the IS model," *Economic Quarterly*, spring, 47-75.

Kim, Jinill. (2000), "Constructing and Estimating a Realistic Optimizing model of Monetary Policy," *Journal of Monetary Economics*, vol. 45, Issue 2 , April 2000, 329-359

King, Robert, and Mark Watson (1996), "Money, Prices, Interest Rates and the Business Cycle" *Review of Economics and Statistics*, 78, 35-53

Levin, Andrew, Volker Wieland, John C. Williams (2002): "The Performance of Forecast-Based Monetary Policy Rules under Model Uncertainty," *American Economic Review*, forthcoming.

Mankiw, N. Gregory (2000): "The Savers-Spenders Theory of Fiscal Policy," *American Economic Review*, vol. 90, no. 2, 120-125.

McCallum, B. (1999): "Issues in the Design of Monetary Policy Rules", in J.B. Taylor and M. Woodford eds., *Handbook of Macroeconomics*, vol. 1c, 1341-1397, Elsevier, New York.

Orphanides, A. (2001), "Monetary Policy Rules based on Real Time Data", *American Economic Review, vol.* 91, no. 4, *964-985*

Rotemberg, J. and M. Woodford (1999): "Interest Rate Rules in an Estimated Sticky Price Model," in J.B. Taylor ed., *Monetary Policy Rules*, University of Chicago Press, 57-119.

Taylor, John B. (1993): "Discretion versus Policy Rules in Practice," *Carnegie Rochester Conference Series on Public Policy* , December 1993, 39, 195-214.

Taylor, John B. (1999a): *Monetary Policy Rules*, University of Chicago Press and NBER.

Taylor, John B. (1999b): "An Historical Analysis of Monetary Policy Rules," in J.B. Taylor ed., *Monetary Policy Rules*, University of Chicago Press.

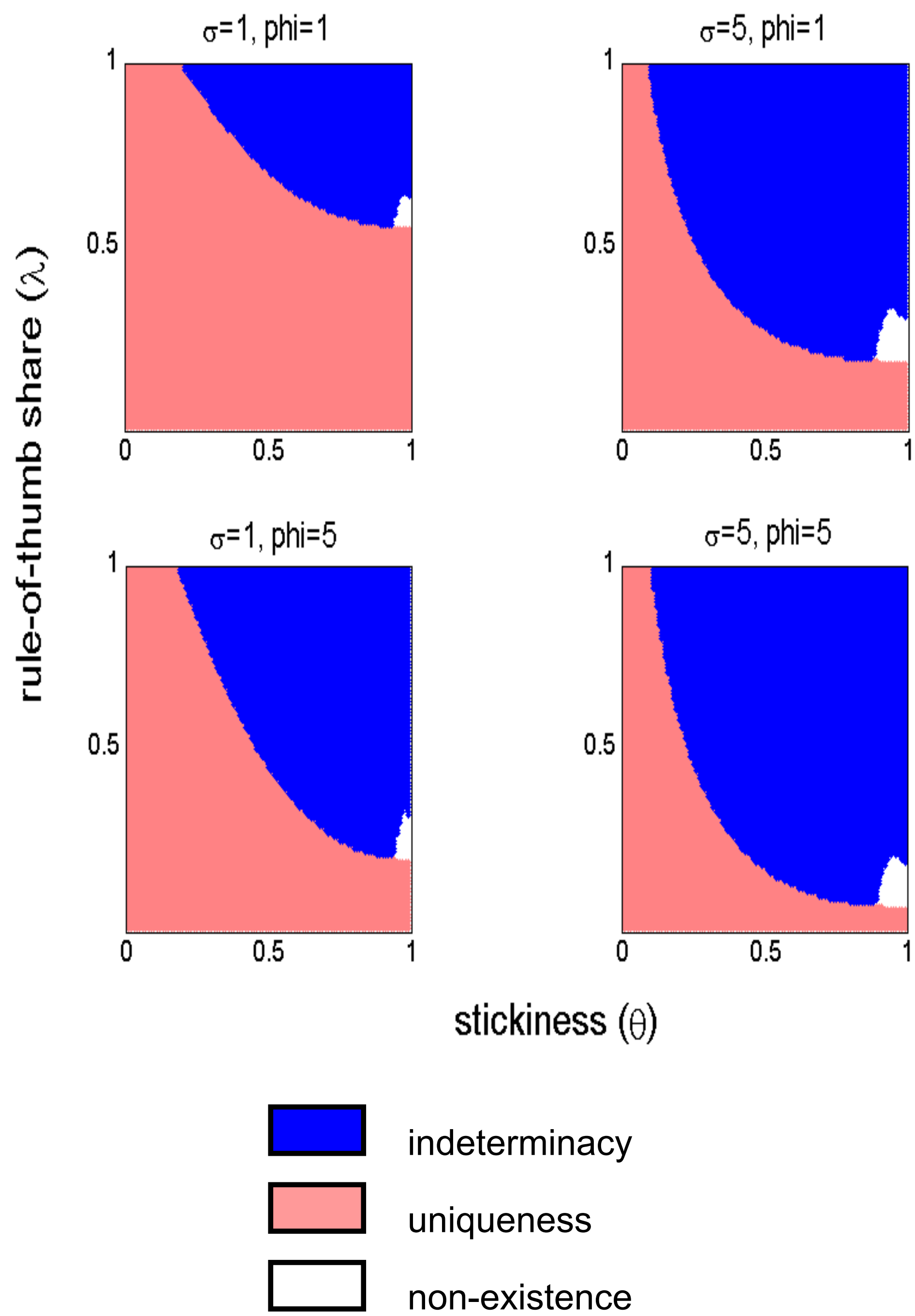
Taylor, John B. (1999c): "Staggered Price and Wage Setting in Macroeconomics," in J.B. Taylor and M. Woodford eds., *Handbook of Macroeconomics*, chapter 15, 1341-1397, Elsevier, New York.

Woodford, Michael (2001): "The Taylor Rule and Optimal Monetary Policy," *American Economic Review* vol. 91, no. 2, 232-237.

Yun, Tack (1996): "Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles," *Journal of Monetary Economics* 37, 345-370.

Figure 1

Indeterminacy and the Taylor Principle



Note: Simulations based on Φ_{π}=1.001.

Figure 2

Rule-of-Thumb Consumers and the Threshold Inflation Coefficient

The Role of Price stickiness

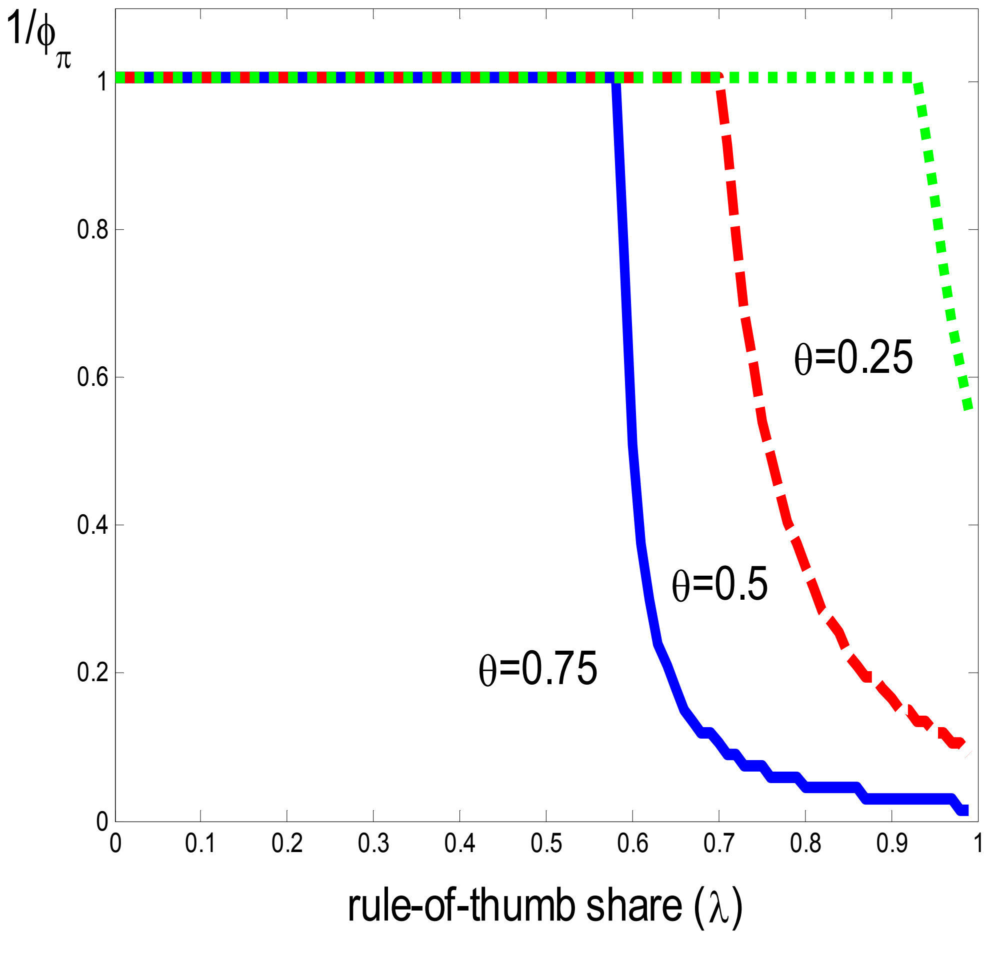


Figure 3

Rule-of-Thumb Consumers and the Threshold Inflation Coefficient

The Role of Labor Supply Elasticity

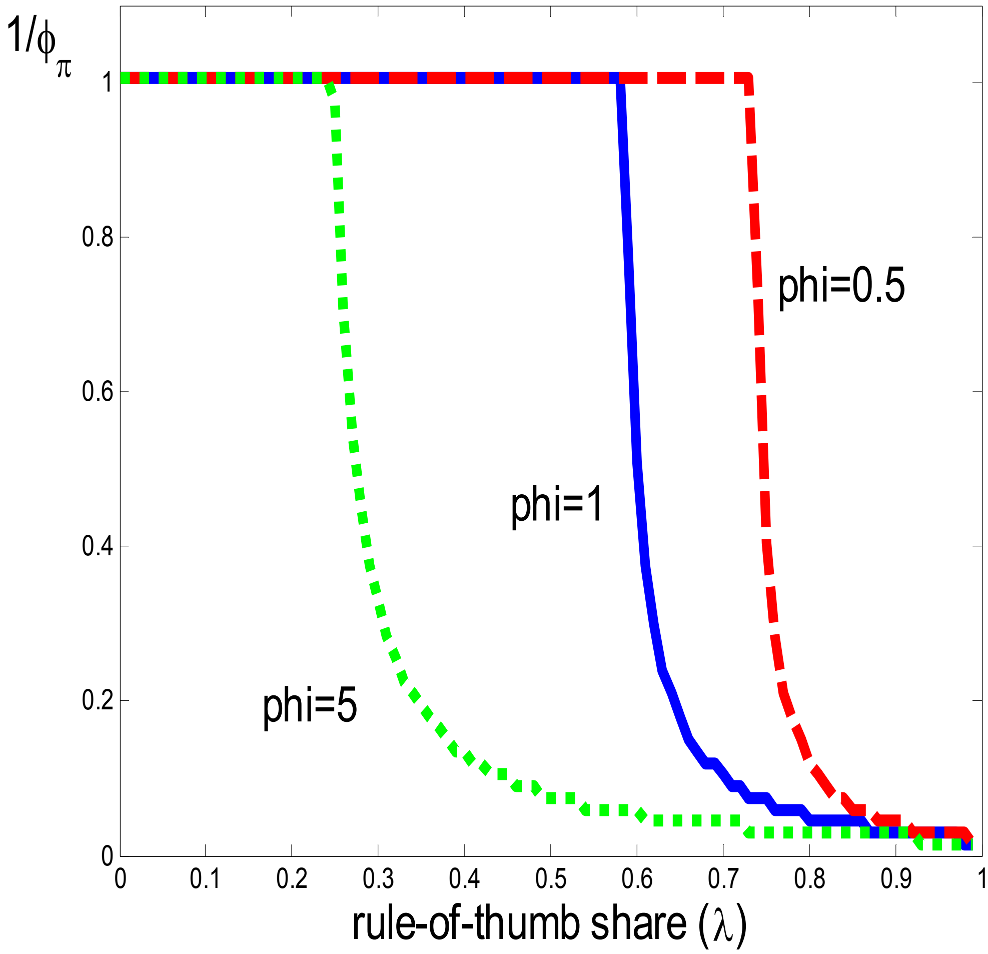


Figure 4

Rule-of-Thumb Consumers and the Threshold Inflation Coefficient

The Role of Capital Adjustment Costs

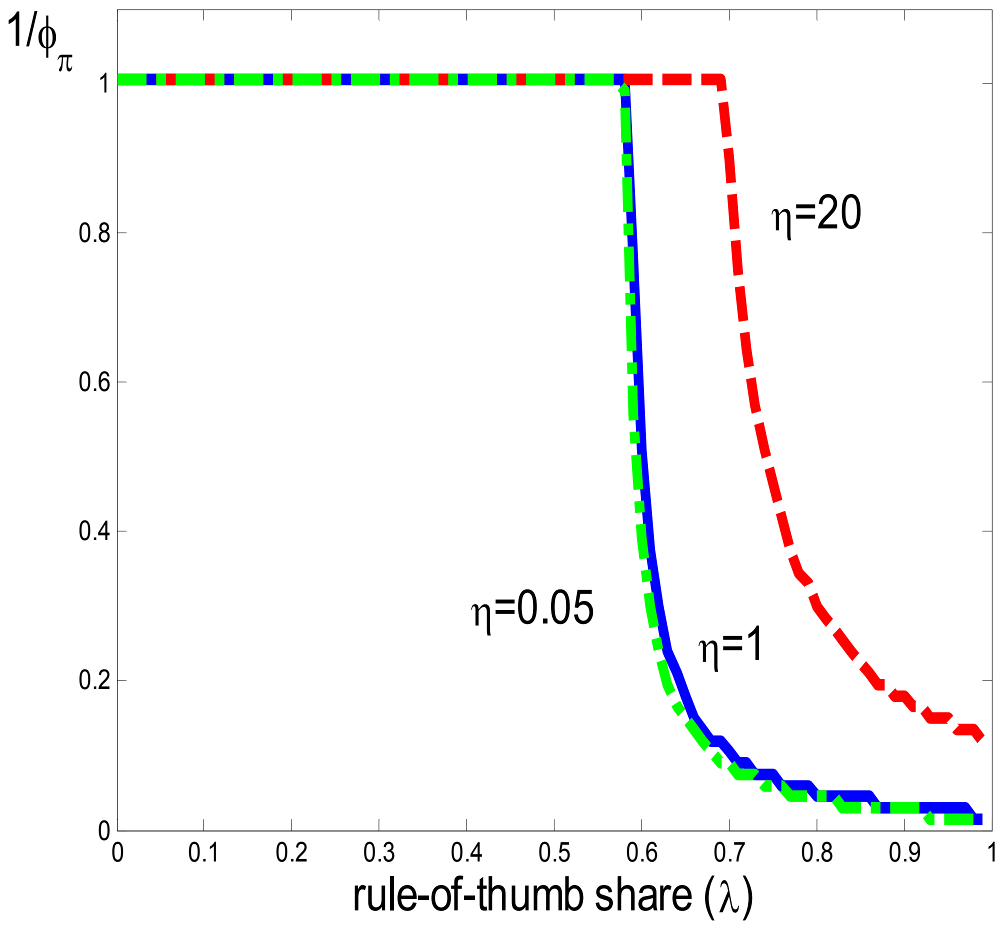


Figure 5

Rule-of-Thumb Consumers and the Threshold Inflation Coefficient

The Role of Risk Aversion

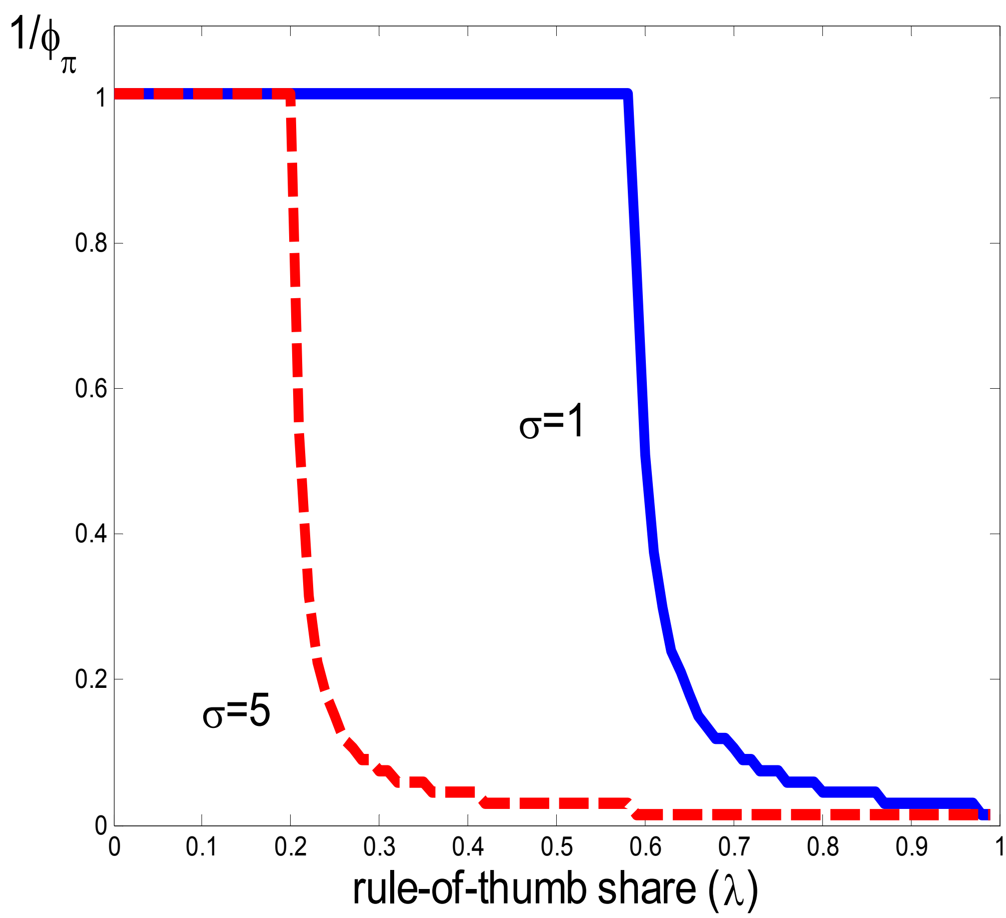


Figure 6
Dynamic Responses to a Sunspot Shock
Baseline Rule (Φ_{π}=1.1, λ=0.85)

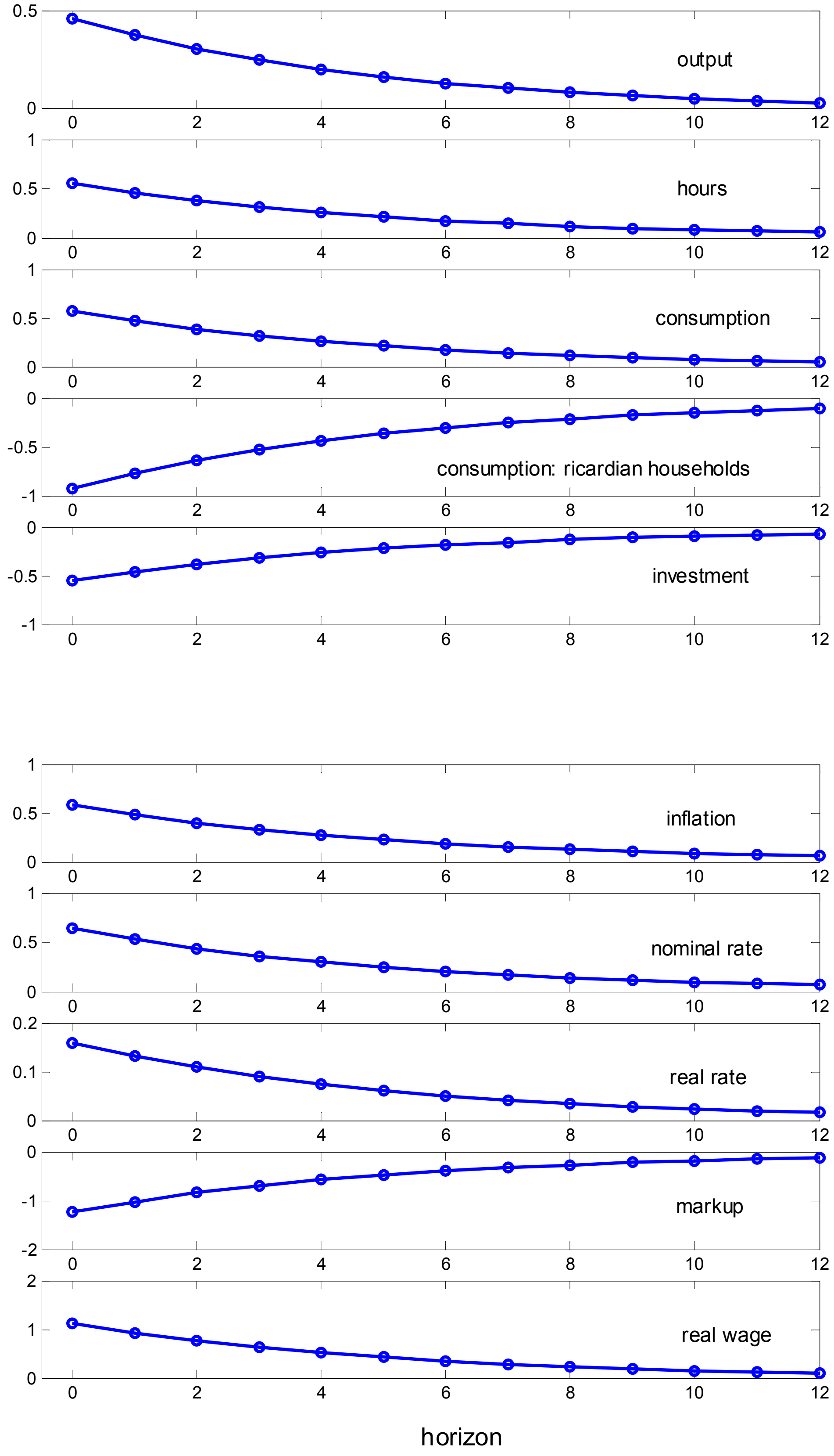


Figure 7

Rule-of-Thumb Consumers and the Threshold Inflation Coefficient

A Rule with both Inflation and Output

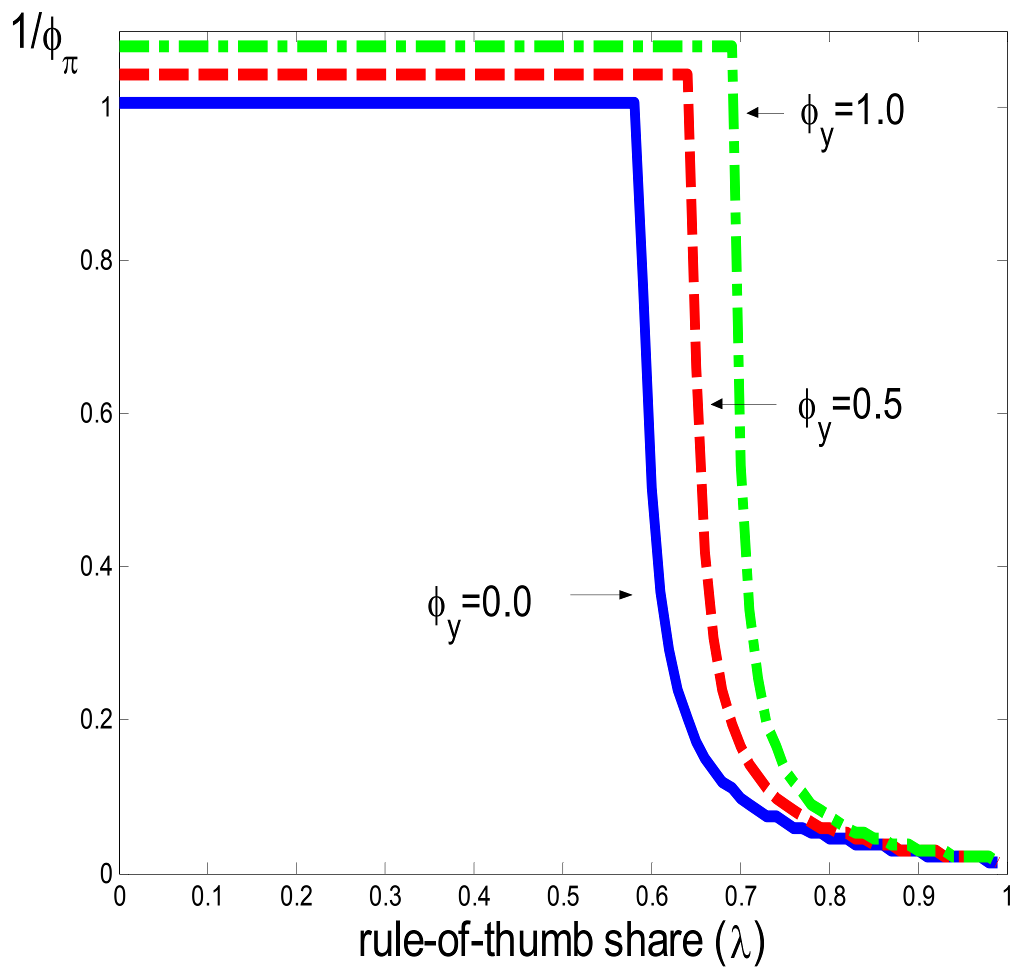


Figure 8
Rule-of-Thumb Consumers and Indeterminacy
Forward Looking Rule

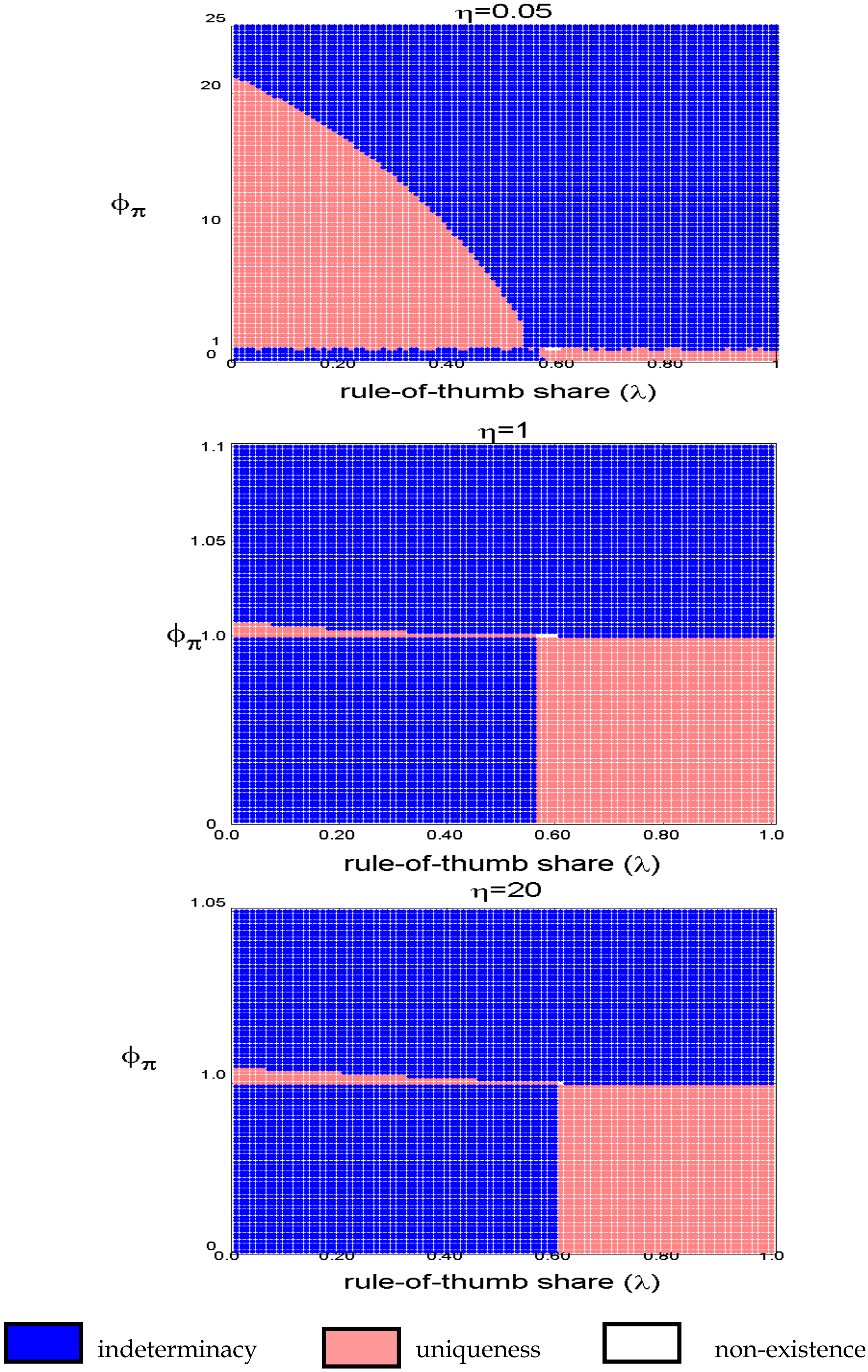


Figure 9
Dynamic Responses to a Cost Push Shock when the Taylor Principle is not Met
Forward Looking Rule (Φ=0.2 and λ=0.85)

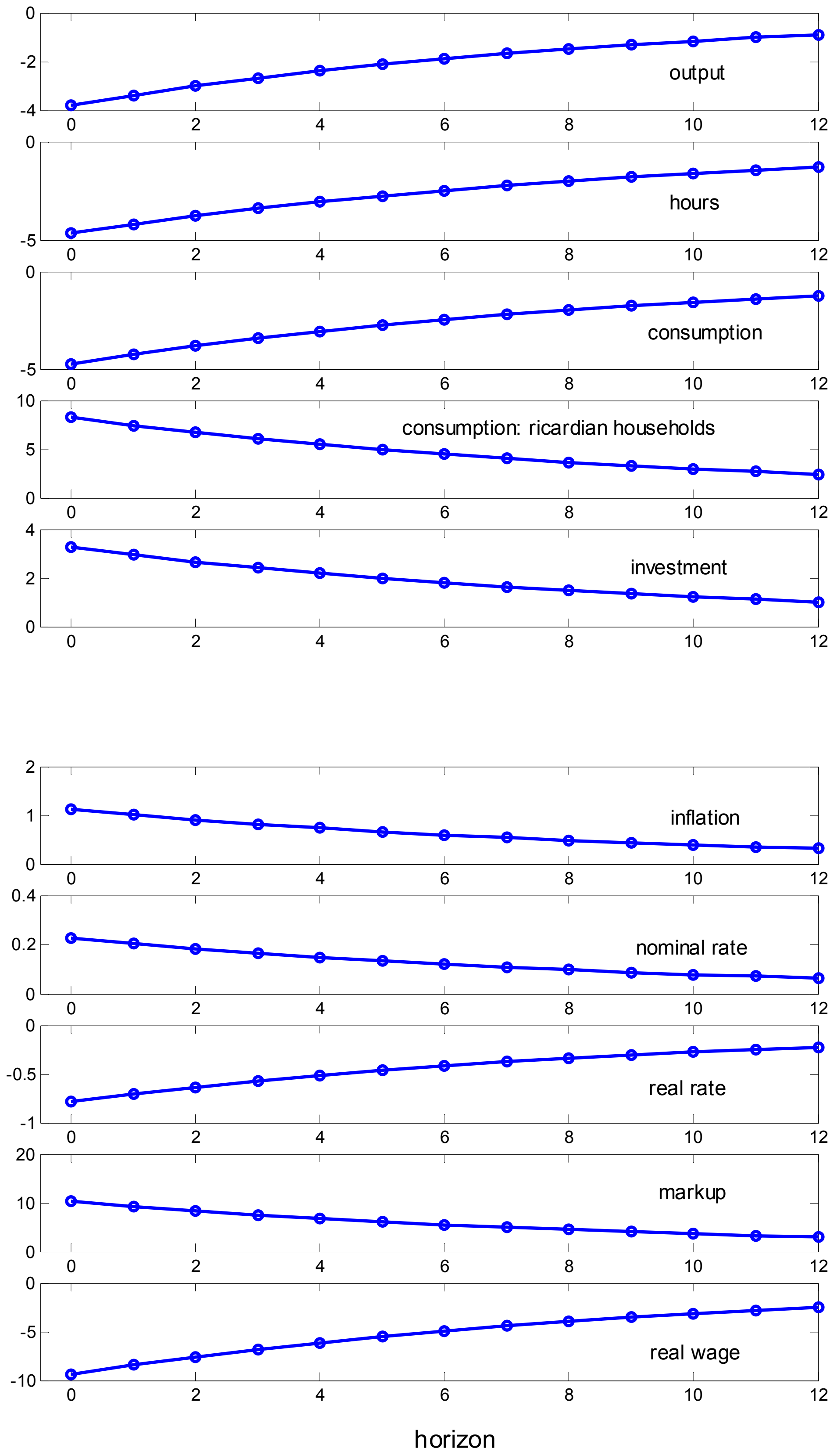


Figure 10
Rule-of-Thumb Consumers and Indeterminacy
The Forward Looking Rule augmented with Expected Output

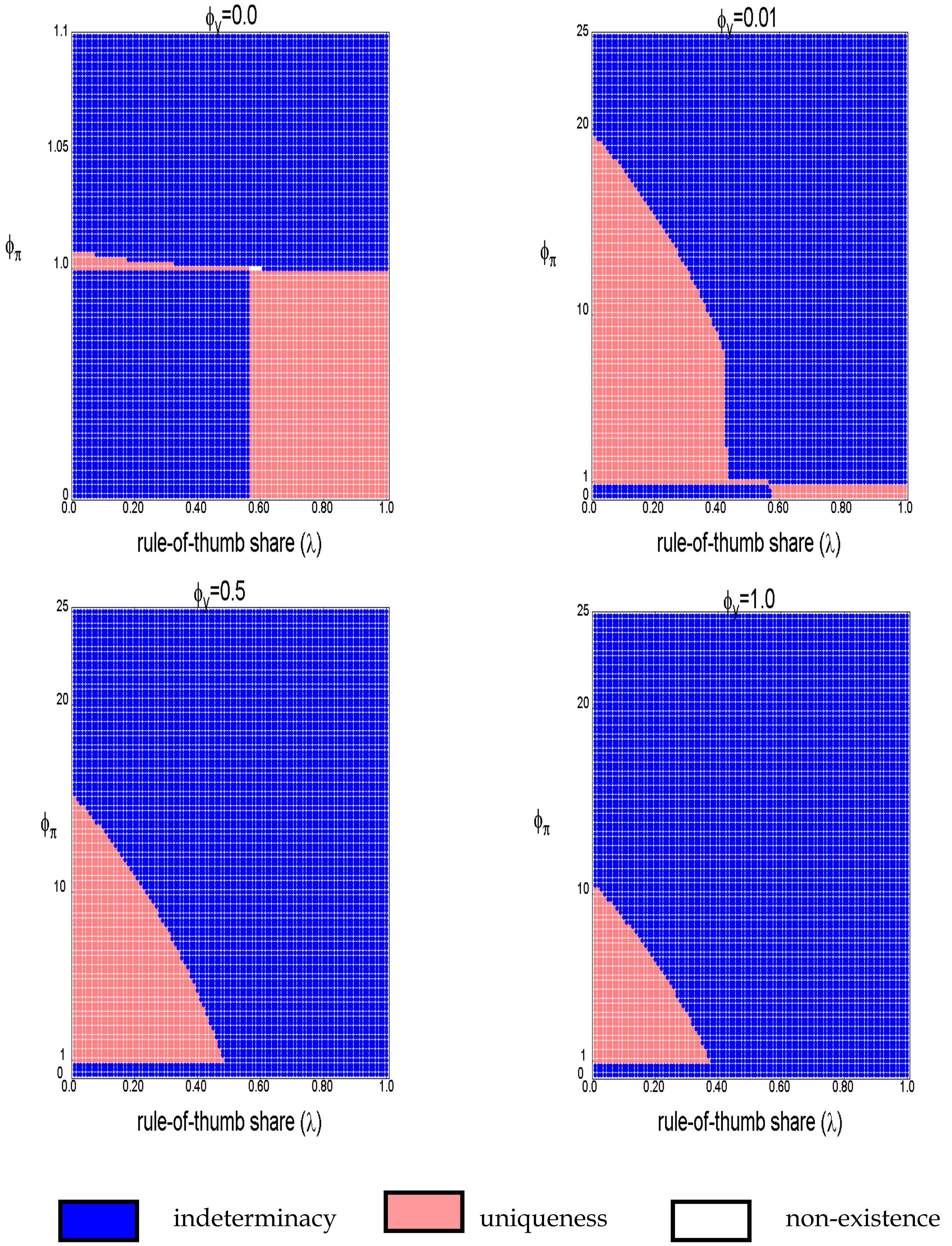


Figure 11

Rule-of-Thumb Consumers and Indeterminacy

Backward Looking Rule