A Parsimonious Macroeconomic Model for Asset Pricing: Habit Formation or Cross-sectional Heterogeneity?*

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June 23, 2003

Abstract

In this paper we study the asset pricing implications of a parsimonious two-agent macroeconomic model with two key features: limited participation in the stock market and heterogeneity in the elasticity of intertemporal substitution. The parameter values for the model are taken from the business cycle literature and are not calibrated to match any financial statistic. Yet, with a risk aversion of two, the model is able to explain a large number of asset pricing phenomena including all the facts matched by the external habit model of Campbell and Cochrane (1999). Examples in this list include a high equity premium and a low risk-free rate; a counter-cyclical risk premium, volatility and Sharpe ratio; predictable stock returns with coefficients and R^2 values of long-horizon regressions matching their empirical counterparts, among others. In addition the model generates a risk-free rate with low volatility (5.7 percent annually) and with high persistence. We also show that the similarity of our results to those from an external habit model is not a coincidence: the model has a reduced form representation which is remarkably similar to Campbell and Cochrane's framework for *asset pricing*. However, the *macroeconomic implications* of the two models are quite different, favoring the limited participation model. Moreover, we show that policy analysis yields dramatically different conclusions in each framework.

Keywords: Limited stock market participation, asset pricing, the equity premium puzzle, incomplete markets, habit formation, elasticity of intertemporal substitution.

JEL classification: E32, E44, G12

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[*] I would like to thank Daron Acemoglu, Narayana Kocherlakota, Per Krusell, Tony Smith and seminar participants at the Universities of Montréal, Pittsburgh, Ohio State, Texas at Austin, and Simon School of Business at Rochester for helpful comments. The usual disclaimer applies.

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1 Introduction

No other puzzle in the last two decades has probably generated as much interest and subsequent research as the equity premium puzzle of Mehra and Prescott (1985). These authors essentially showed that the excess return of stocks over bonds—which averages about 6 percent annually in the U.S. historical data—was puzzling in the context of a canonical general equilibrium portfolio-choice model. The long quest for a resolution has uncovered further puzzling facts such as the risk-free rate puzzle of Weil (1989), adding to the list of challenges. After many extensions and enrichments of the basic framework, a satisfactory explanation still remained elusive.[1]

Building on the earlier insight of Abel (1990) and Constantinides (1990) a number of recent papers have introduced models featuring preferences with endogenous habit formation (Jermann 1998; Boldrin, Christiano and Fisher 1999) or "catching-up with Joneses" (Campbell and Cochrane 1999) which can explain both the equity premium and the risk-free rate puzzles. In fact, Campbell and Cochrane (1999) go even further and are able to match a wide variety of financial statistics including the time-series and cyclical patterns of asset prices as well as the predictability of excess returns.[2]

Although habit persistence could be an appealing description of behavior at an introspective level, it is fair to say that it is probably not the best understood aspect of individual preferences. At a more concrete level, the empirical evidence seems mixed. Studies using aggregate data usually find evidence in favor of habit[3] with quite large persistence (Ferson and Constantinides 1991; Heaton 1995) whereas individual-level data does not seem to reveal even the slightest hint of such behavior (Naik and Moore 1996; Dynan 2000; McKenzie 2002). Furthermore, Otrok, Ravikumar and Whiteman (2002) show that both the equity premium and the risk-free rate implied by these models are extremely sensitive to the specification of the shock process. For example, even small changes in the persistence of consumption growth—much more modest than those observed in the U.S. historical data—imply movements in *average* returns in the order of 15 to 25 percentage points. Thus it is still useful to study asset prices in a model with standard preferences (for example, CRRA) and analyze the role of other elements, such as sources of heterogeneity which are observable or empirically well-established.

In this paper we follow this second route and study asset prices in a parsimonious two-agent macroeconomic model with two key features: limited participation in the stock market and heterogeneity in the elasticity of intertemporal substitution. In other respects the framework is a standard real business cycle model. More specifically, we consider an economy with a neoclassical production technology and competitive markets. There are two types of agents. The majority of households (first type) do not participate in the stock market where claims to the firm's future dividends are traded. However, a risk-free bond is available to all households, so non-stockholders can also accumulate wealth and smooth consumption intertemporally. We

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[1] For excellent surveys of the literature on these puzzles, see Kocherlakota (1996) and Campbell (1999).

[2] See also Chan and Kogan (2002). Section 8 reviews the existing literature in more detail.

[3] Some authors have argued that time aggregation might strongly bias results in favor of habit persistence even when there is none. See for example Porter and Wheatley (1999) and the references therein.

also model the capital adjustment costs faced by firms (as in Lucas and Prescott 1971, and Jermann 1998). Finally, consistent with the empirical evidence documented in Section 3, we assume that stockholders have a higher elasticity of substitution than non-stockholders. With CRRA utility this also implies heterogeneity in risk aversion, but as we show in the robustness analysis using Epstein-Zin (1989) preferences the results of the paper remain essentially the same when both agents' risk aversions are set equal to two, while keeping the elasticities fixed at their baseline values.

We find that the model is surprisingly successful in explaining a large number of asset pricing phenomena, many of them considered to be puzzles. To provide a benchmark, the model can match all the facts studied in Campbell and Cochrane (1999) *as well as* a number of others on which that model is silent. This is true even though the model has no free parameters, and all the structural ones are calibrated to standard values from the business cycle literature. This approach is in contrast to the one taken in some recent asset pricing papers where a number of parameters are chosen specifically to match certain moments of financial variables.

Here is an overview of the results. With a risk aversion of two for both agents, the Sharpe ratio is 23 percent (and 29 percent with a risk aversion of three) compared to 25 to 30 percent in the U.S. data. The model also matches the *levels* of the stock return and the risk-free rate—and hence the equity premium—as well as their *volatilities.* Especially the low variability of the interest rate has proved to be a particularly tough challenge for habit persistence models—a risk-free rate volatility above 20 percent is common (c.f., Boldrin et. al. 1999). In contrast, the annual volatility of the interest rate in our model is 5.7 percent compared to 5.4 percent in the U.S. data. Moreover, the model generates a countercyclical equity premium, conditional volatility, and Sharpe ratio, all of which are documented features of asset prices (Schwert 1989; Nelson 1991; and Chou, Engle and Kane 1992). Furthermore, stock returns are predictable and both the coefficients and the R^2 values of the long-horizon regressions of future stock returns on the price-dividend ratio match those obtained in the finance literature (Campbell and Shiller 1988; Fama and French 1989). Finally, the model explains a number of other features of asset prices, such as their autocorrelation patterns, their cross-correlations with each other, and so on.

These results do not rely on high risk aversion, idiosyncratic shocks or binding borrowing constraints. Limited participation is the critical element behind these results. At the same time, preference heterogeneity has a big impact on the first two moments of returns, but is not as critical for the rest. The driving force behind the time-series results (e.g., predictability, etc.) is the evolution of the wealth distribution over the business cycle. More specifically, a positive persistent technology shock increases the value of equity significantly benefiting stockholders immediately. On the other hand, non-stockholders gain only gradually as their wages grow with increasing capital and they respond by accumulating more wealth. Thus, initially the share of wealth held by stockholders go up coming back down slowly. As a result, each group's influence over the determination of asset prices change endogenously through the business cycle giving rise to interesting time-series behavior.

As mentioned earlier, Campbell and Cochrane (1999, hereafter C-C) are also able to explain many of the phenomena mentioned above. They study a representative-agent exchange economy with a slow-moving external habit term in the utility function.[4] After reverse-engineering the parameters of the habit process to match certain financial moments, they find that the model performs impressively in other dimensions as well. The striking similarities between the results of the two models suggest an interesting possibility: Even though the two frameworks look very different on the surface, could there be a deeper connection between them? We find that this is indeed the case. *The limited participation model has a reduced form which is remarkably similar to Campbell and Cochrane's framework in terms of asset pricing implications.* In particular, the elaborate process they assume for the exogenous habit stock turns out to be just the consumption process of non-stockholders in our model.

A simple way to see this point is by considering the Euler equation for stockholders in our model. Let X be non-stockholders' consumption, and C^A be aggregate consumption. So, stockholders' consumption is: $C^h = C^A - X$, and we have:

$$E_t\left[\beta\left(\frac{C^A_{t+1} - X_{t+1}}{C^A_t - X_t}\right)^{-\alpha}\left(R^S_{t+1} - R^f_t\right)\right] = 0,$$

where α and β are the risk aversion and time discount parameters respectively, and R^S_{t+1} and R^f_t are the risky and risk-free rates. Note that a similar Euler equation does *not* hold for aggregate consumption due to the presence of non-stockholders. Now consider a representative-agent who has external habit preferences ($U\left(C^A_t, X_t\right) = \frac{\left(C^A_t - X_t\right)^{1-\alpha}}{1-\alpha}$ where we now let X_t to denote the exogenous habit stock). It is immediately clear that the same Euler equation above holds for this representative-agent, but now X_t is reinterpreted as an externality and C^A_t as his actual consumption (rather than $C^A - X$). In other words, ignoring limited participation would make it look as if the representative agent was displaying habit persistence. Of course, for this argument to have any validity, one has to establish that the properties of X_t are very much the same in the two models. In Section 5 we show that the exogenous habit process assumed in C-C has virtually the same statistical properties as non-stockholders' consumption in our model. In a way this finding verifies their conjecture that the representative agent preferences they recover "could result from aggregation of heterogenous consumers with quite different preferences" (p. 241).

What do we learn from these results for studying macroeconomic problems? This question is critical because macroeconomists' interest in financial anomalies is partly motivated by the fact that these puzzles challenge the foundations of the very framework used for policy analysis: it would be hard to place a lot of confidence in the *quantity* implications of a framework if it has notoriously poor *pricing* implications. Unfortunately, the extension of the external habit model to study macroeconomic questions has not been straightforward. For example, Lettau and Uhlig (2000) show that incorporating external habit results in a number of anomalies in

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[4] In this paper we often refer to the C-C model alternatively as "the external habit model," although it should be noted that other authors have implemented different versions of these preferences with somewhat different properties.

an otherwise standard business cycle model. One has to introduce a number of frictions in production and more habits in leisure and so on, in order to carefully balance and counteract its effect. Fortunately, this is where the similarities between the current model and the habit framework ends. The macroeconomic implications of the limited participation model without adjustment costs are as good as a standard business cycle model and are still in the right ball park with the frictions imposed. A major difference between the two models leading to these different results is the extremely high risk aversion (around 80) assumed in C-C compared to a risk aversion of only two used in our framework.[5]

In section 6.1, we conduct a simple policy experiment (a capital income taxation problem) and find that the two models yield drastically different policy conclusions. This point suggests extreme caution as the success of habit persistence models encouraged many researchers to address policy questions in that framework (among others, McCallum and Nelson 1998; Fuhrer 2000; Ljungqvist and Uhlig 2000; Christiano, Eichenbaum and Evans 2001; and Abel 2001).

The paper is organized as follows. Section 2 introduces the model and the parametrization is discussed in Section 3. We then analyze asset prices in Section 4 and establish the connection between the two models in Section 5. Starting with Section 6 we look at macroeconomic behavior where the similarities end. Section 6 analyzes the positive implications and Section 6.1 presents a normative perspective in the context of a capital income taxation problem. Section 8 contains sensitivity analysis and relates the paper to the existing literature, and Section 9 concludes.

2 The Model

The model is an extension of the framework studied in Guvenen (2000). For transparency of results, our modeling goal is to stay as close to the standard real business cycle model as possible and only introduce two key features: limited participation in the stock market and heterogeneity in the elasticity of intertemporal substitution. We consider an economy populated by two types of agents who live forever. The population is constant and is normalized to unity. Let λ ($0 < \lambda < 1$) denote the measure of the first type of agents (who will be called "stockholders" later) in total population.

PREFERENCES

Both agents have time separable expected utility functions defined over future consumption streams: $E_t\left[\sum_{j=1}^{\infty}\beta^{t+j}u^i\left(c_{t+j}\right)\right]$, for $i = h,\ n,$ where the superscripts h and n denote stock*h*olders and *n*on-stockholders respectively, and β is the subjective discount rate. As for the parameterization of the momentary utility function, we have two considerations in mind. On the one hand, we want to keep preferences standard to highlight the effect of limited participation and other endogenous features of the model. This suggests a standard CRRA utility function: $u\left(C\right) = \frac{C^{1-\alpha}}{1-\alpha}$ and we adopt it throughout much of the paper. On the other hand,

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[5] Although endogenous habit models imply a low risk aversion they still result in a very low EIS which is typically the key parameter in macroeconomic models. In the current model, capital is controlled by stockholders who have a high EIS (at least 0.5) with our calibration.

it is well-known that with this specification the parameter α controls both the relative risk aversion (RRA) and the elasticity of intertemporal substitution (EIS) which are different aspects of individuals' tastes. To clarify the intuition of the results in Section 8 we will generalize the preferences to the recursive utility function of Epstein and Zin (1989). This extension will also allow us to illustrate the distinct roles played by the risk aversion and the elasticity of intertemporal substitution.

THE FIRM

There is a single aggregate firm producing the consumption good using capital (K_t) and labor (L_t) inputs according to a Cobb-Douglas technology:

$$Y_t = Z_t K_t^{\theta} L_t^{1-\theta},$$

where $\theta \in (0,1)$ is the factor share parameter. The logarithm of the stochastic technology level evolves as an AR(1) process:

$$\begin{aligned} \log(Z_{t+1}) &= \rho \log(Z_t) + \varepsilon_{t+1}, \\ & \varepsilon \sim N\left(0, \sigma_{\varepsilon}^2\right). \end{aligned}$$

The goal of the firm's managers is to maximize the value of the firm to owners:[6]

$$P_t^S = \underset{\{I_{t+j}, L_{t+j}\}}{Max} E_t \left[\sum_{j=1}^{\infty} \beta^j \frac{\Lambda_{t+j}}{\Lambda_t} \left(Z_{t+j} K_{t+j}^{\theta} L_{t+j}^{1-\theta} - W_{t+j} L_{t+j} - I_{t+j} \right) \right]$$

subject to the technology constraint which features adjustment costs in investment

$$K_{t+1} = (1-\delta) K_t + \Phi\left(\frac{I_t}{K_t}\right) K_t,$$

where P_t^S is the ex-dividend value of the firm in a given period, and $\beta^j \frac{\Lambda_{t+j}}{\Lambda_t}$ is the discount rate (i.e., the marginal rate of substitution between periods t and $t+j$). The adjustment cost function $\Phi(\cdot)$ is concave in investment which captures the difficulty of quickly changing the level of capital installed in the firm. Consequently, the prices of capital and consumption goods are not equal and Tobin's q is not necessarily equal to unity. This specification is the same as the one used in Jermann (1998) and Boldrin, et. al. (1999) which makes comparison easier.

The firm is 100 percent equity financed as commonly assumed in the real business cycle literature. It is relatively straightforward to introduce leverage into this framework (say, as a fixed proportion of the firm's capital) although we will not pursue that approach here. A share

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[6] To save on notation (but at the expense of slightly abusing notation) we use K_t to denote the firm's capital choice and we will also use the same variable to denote aggregate capital which the firm takes as given when making its choice. In equilibrium, of course, the two are the same.

in this firm entitles its owner to the entire stream of future dividends given by

$$D_t = Z_t K_t^{\theta} L_t^{1-\theta} - W_t L_t - I_{t.}$$

The firm does not issue new shares and finances investment through retained earnings. For convenience we normalize the number of shares outstanding to unity so that P_t^S is also the stock price. In this environment, the basic asset valuation condition holds:

$$P_t^S = E_t \left[\beta \frac{\Lambda_{t+1}}{\Lambda_t} \left(D_{t+1} + P_{t+1}^S \right) \right]. \tag{1}$$

Finally, the firm's first order conditions together with competitive labor markets imply that workers are paid their marginal products: $W_t = (1-\theta)\, Z_t \left(K_t / L_t \right)^{\theta}$.

STOCKHOLDERS AND NON-STOCKHOLDERS

Both agents have one unit of time endowment in each period, which they supply inelastically to the firm. Besides the productive capital asset there is also a one-period riskless household bond (in zero net supply) traded in this economy. The crucial difference between the two groups is in their investment opportunity sets: "Non-stockholders" can freely trade the risk-free asset, but as their name suggests, they are restricted from participating in the capital market. "Stockholders," on the other hand, have access to both markets and hence are the sole capital owners in the economy. Following the incomplete markets literature we impose portfolio constraints as a convenient way to prevent Ponzi schemes. As we discuss in the quantitative analysis these constraints can be quite loose (and in fact they almost never bind in our simulations).

The timing of events is as follows: each period starts with production; agents are paid their wages and asset returns are realized after production takes place. Then consumption and portfolio choice decisions are made and asset trading is carried out. Finally, consumption takes place and the period ends. Before we move to agents' problem a final remark is in order.

Remark: It is possible to think of the participation structure assumed here as an endogenous outcome of a model where there is a one-time fixed cost of participating. With a cost of appropriate magnitude, type 1 agents (with low risk aversion) will enter the stock market whereas the other group will stay out. The resulting equilibrium is identical to the one studied here; see Guvenen (2002) for a further discussion about endogenizing participation. We feel that for the purposes of the current paper this is a reasonable assumption and our main conclusion is not likely to be overturned by this extension.

INDIVIDUALS' DYNAMIC PROBLEM AND THE EQUILIBRIUM

In order to state the individual's problem recursively, we need to specify the aggregate state-space for this economy. The Markov characteristic of the exogenous driving force naturally suggests concentrating on equilibria which are dynamically simple. Thus, the portfolio holdings of each group together with the exogenous technology shock constitute a sufficient state space which summarizes all the relevant information for the equilibrium functions.

In a given period, the portfolios of each group can be expressed as functions of the *beginning-*

of-period capital stock, K, the aggregate bond holdings of non-stockholders *after* production, B, and the technology level, Z. Let us denote the financial wealth of an agent in the current period by ω where we suppress superscripts for clarity of notation. The dynamic programming problem of a stockholder can be expressed as follows:

$$\begin{aligned}
V^h(\omega; K, B, Z) &= \max_{b',\ s'} \left\{ U(c) + \beta E\left[V^h(\omega'; K', B', Z') \mid \Omega \right] \right\} \\
& \quad s.t \\
c + P^B(K, B, Z) * b' + P^S(K, B, Z) * s' &\leq \omega + W(K, Z) \\
\omega' &= b' + s' * \left(P^S(K', B', Z') + D(K', B', Z') \right) \\
K' &= \Gamma_K(K, B, Z) \\
B' &= \Gamma_B(K, B, Z) \\
b' &\geq \underline{B}^h,
\end{aligned}$$

where the expectation is conditional on the set Ω containing all the information at the time of decision, and b' and s' denote bond and stock holdings respectively. The endogenous functions Γ_K and Γ_B denote the laws of motion for aggregate wealth distribution which are determined in equilibrium; P^B denotes the equilibrium bond pricing function. Note that each agent is facing a constraint on bond holdings with possibly different (and negative) lower bounds. The problem of the non-stockholder can be written as above with $s' \equiv 0$.

A stationary recursive competitive equilibrium for this economy is given by a pair of value functions $V^i(\omega^i; K, B, Z)$, $(i = h, n)$, bond holding decision rules for each agent $b^i(\omega^i; K, B, Z)$, stockholding decision for the stockholder, $s(\omega^h; K, B, Z)$, stock and bond pricing functions, $P^S(K, B, Z)$ and $P^B(K, B, Z)$, a competitive wage function, $W(K, Z)$, an investment function for the firm, $I(K, B, Z)$, and laws of motion for aggregate capital and aggregate bond holdings of non-stockholders, $\Gamma_K(K, B, Z)$, $\Gamma_B(K, B, Z)$, such that:

1) Given the pricing functions and the laws of motion, the value functions and decision rules of each agent solve that agent's dynamic problem

2) Given the equilibrium discount rate process $\frac{\Lambda_{t+j}}{\Lambda_t}$ and $W(K, Z)$, the investment function $I(K, B, Z)$ and labor choice are optimal.

3) The bond market clears: $\lambda b^h(\varpi^h; K, B, Z) + (1-\lambda) b^n(\varpi^n; K, B, Z) = 0$, where ϖ^i denote the aggregate wealth of a given group; and the labor market clears: $L = \lambda * 1 + (1-\lambda) * 1 = 1$.

4) Aggregates result from individual behavior:

$$\begin{aligned}
K_{t+1} &= (1-\delta) K_t + \Phi\left(\frac{I_t}{K_t}\right) K_t, && (2) \\
B_{t+1} &= (1-\lambda) b^n(\varpi^n, K_t, B_t, Z_t). && (3)
\end{aligned}$$

5) There exists an invariant probability measure $\mathbf{P}$ defined over the ergodic set of equilibrium distributions.

3 Quantitative Analysis

Since an analytical solution is not possible, we use numerical methods to solve for the equilibrium. The task is quite challenging requiring solutions to three dynamic programs (one for each agent plus one for the firm) where each program depends on the solution of other problems in a quite nonlinear way. For example, the firm takes the discount factor (stockholders' MRS) as given where in fact that MRS is obtained from the solution to the stockholder's problem who in turn takes the investment decision of the firm and the resulting stock prices as given. The crucial step is the first one: getting good initial guesses for equilibrium laws of motion and pricing functions. We relegate the discussion of these and other computational issues (as well as the accuracy of the solution) to the Appendix.

BASELINE PARAMETERIZATION

A recently common method for calibrating general equilibrium asset pricing models is to choose a number of parameters to match certain financial statistics, such as the risk-free rate and the equity premium, the persistence of the price-dividend ratio, and so on. Then additional moments of the data serve as overidentifying restrictions to be examined. In contrast, we follow the real business cycle tradition and calibrate the parameters to replicate the long-run macroeconomic facts of the U.S. economy such as the average capital-output ratio, the persistence of the Solow residuals and so on. In particular, no parameter is chosen to replicate a financial statistic.

The time period in the model corresponds to 3 months of calendar time. The capital share of output, θ, is set equal to 0.4 following Cooley and Prescott (1995). As for the technology shock, we match the persistence of the quarterly Solow residual, $\rho = 0.95$, and set the standard deviation of the innovation equal to 2 percent. Although this latter number is larger than the one reported in Cooley and Prescott (1995), it is consistent with the estimates obtained by Christiano and Eichenbaum (1992), and the values used in Danthine and Donaldson (2001), and Storesletten, Telmer and Yaron (1999) among others. Moreover, we will compare the model to data extending back to 1890, and given that output and consumption were significantly more volatile prior to World War II, this seems like a sensible choice. We use a 12-state Markov chain to discretize the AR(1) process for Z_t following Tauchen and Hussey's (1991) method. As Table 12 in the Appendix shows, the autocorrelation structure (from lag 1 to 5) of the approximation tracks the AR(1) process quite closely, and the standard deviation is equal to the true value.

To complete the description of technology, we choose the adjustment cost parameter, $\xi = 0.23$, which is the value used in a number of recent studies (Jermann 1998; Boldrin et. al 1999; Francis and Ramey 2002).[7] This value is near the low end of the empirical estimates for this variable (the elasticity of investment with respect to Tobin's q), so we will also conduct sensitivity analysis with respect to the value of this parameter.

Participation rates. The stock market participation rate has gradually increased from

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[7] The functional form for Φ is $\frac{a_1}{1-1/\xi}\left(\frac{I_t}{K_t}\right)^{1-1/\xi} + a_2$, where a_1 and a_2 are chosen such that the steady state is not affected by the value of ξ.

around 5 percent in the 1950s to approximately 19 percent in 1982 (Survey of Consumer Finances). During the 1990s, due to a variety of factors—ranging from the emergence of mutual funds and reduced costs of (on-line) trading to the retirement saving by baby-boomers—this trend has accelerated in the U.S. as in the rest of the world. As a result, in 2002 the stockholding rate has reached 52 percent.[8] Since in this paper we study a stationary economy, we want to focus on the pre-1990s U.S. economy. Moreover, a significant fraction of households are holding very small amounts of stocks. For example, in the 1984 PSID data, 27.6 percent of households declare themselves as stock owners whereas the fraction holding more than $10,000 worth of stocks is less than 12 percent (Mankiw and Zeldes 1991). Even after the participation boom of the last decade, 95 percent of all equity is still held by the top 10 percent of stock owners (*SCF* 1998). With these considerations in mind we set the fraction of stockholders, λ, equal to 20 percent. We also report results for $\lambda = 30\%$, which is close to the average participation rates in the 1980s. We conclude that this change does not affect the main message of the paper.

Borrowing constraints are harder to measure and calibrate. We want to choose these bounds to reflect the fact that stockholders can potentially accumulate capital which can then be used as collateral in the bond market, whereas non-stockholders have to pay all their debt through future wages. For the baseline case, we allow stockholders to borrow up to 12 quarters of expected labor income ($\underline{B}^h = 12 * E(W)$). As for non-stockholders, we calibrate their borrowing limit to two quarters of expected income, which is close to twice the credit limit most short-term creditors, such as credit card companies, impose. These constraints rarely bind in our simulations and can be relaxed without affecting any of the main results.

PREFERENCE PARAMETERS

The subjective discount factor, β, is set equal to 0.99 in order to match the U.S. capital-output ratio of 3.3 reported by Cooley and Prescott (1995). We calibrate the curvature parameter α based on the implied elasticity of intertemporal substitution. There is a large body of empirical work documenting significant heterogeneity in the EIS across population (Blundell, Browning and Meghir 1994; Attanasio and Browning 1995; Atkeson and Ogaki 1996; Barsky, Juster, Kimball and Shapiro 1997; Attanasio, Banks and Tanner 2002; Vissing-Jorgensen 2002, etc.).[9] Furthermore, using individual level data, these studies find that stockholders (or the wealthy in general) have significantly higher elasticity of substitution than the poor. To capture these differences in a parsimonious way we set $\alpha^h = 2$ and $\alpha^n = 10$ which is close to values obtained by Attanasio et. al (2002). Although with CRRA utility these different curvature parameters also give rise to heterogeneity in the risk aversion, interestingly this heterogeneity plays *no* essential role in our results. In Section 8, we disentangle the two parameters using Epstein-Zin preferences and show that with the same EIS values used here, if we set both agents' risk aversion parameters to two, the results of the paper remain intact. Table 1 summarizes our baseline parameterization.

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[8] In terms of wealth-weighted participation rates, participation boom is less pronounced because old stockholders still own most of the equity outstanding (see Investment Company Institute 2002).

[9] See Browning, Hansen and Heckman (1999) and Guvenen (2002) for detailed descriptions of this evidence and discussions of the importance of heterogeneity in the EIS in various contexts.

Table 1: BASELINE PARAMETRIZATION

Quarterly Model		
Parameter		Value
β	Time discount rate	0.99
α^h	Risk aversion of stockholders	2
α^n	Risk aversion of non-stockholders	10
λ	Participation rate	0.2
ρ	Persistence of aggregate shock	0.95
σ_ε	Standard deviation of shock	0.02
θ	Capital share	0.4
ξ	Adjustment cost coefficient	0.23
δ	Depreciation rate	0.02
$\underline{B}^h$	Borrowing limit of stockholders	$12\overline{W}$
$\underline{B}^n$	Borrowing limit non-stockholders	$2\overline{W}$

Note: The baseline model assumes CRRA utility function for both agents implying that the respective elasticities of intertemporal substitution are 0.5 and 0.1 for stockholders and non-stockholders respectively. The borrowing limits are indexed to the average wage rate, $\overline{W}$.

4 Results: Asset Prices

In this section we study the asset pricing implications of our model. We will look at a large number of pricing phenomena that have received attention in the literature.

We start with the first two moments of the stock and bond returns. Table 2 displays the statistics from the simulated model along with their empirical counterparts from the U.S. historical return data taken from Campbell (1999). The "long sample" corresponds to the period $1890 - 1991$, and the "post-war period" covers $1947 - 1991$. When calculating the stock return in the post-war data there is one point worth noting. In a recent paper McGrattan and Prescott (2001) argue that approximately 1.8 percent of yearly stock returns from 1960 onward can be attributed to favorable changes in the tax code (and specifically to the reduction of dividend and corporate income taxes). Since the model here abstracts from all taxes we subtract 1.8 percent from stock returns each year after 1960. With this adjustment the equity premium is roughly 5 percent per year in both sample periods.

Comparing the model to the data, we first note that the average risk-free rate closely matches the low empirical values with a plausible time discount rate of 0.99, hence resolving the risk-free rate puzzle. Second, the stock return is 5.3 percent compared to 7 percent in the data and consequently the equity premium in the model is around 3.3 percent, which is approximately 2/3 of the historical value. Although, by slightly increasing the risk aversion of stockholders (from two to three), the equity premium rises to 4.9 percent and the Sharpe ratio to 0.29 without any pronounced side effects (column 4), we will not pursue this strategy here. The reason is that the risky return in the model corresponds to the stock of a firm which is entirely

Table 2: MOMENTS OF ASSET PRICES AND RETURNS

	US Data		Model		RBC
	Long Sample	Post-War	$\alpha^h = 2$	$\alpha^h = 3$	
	Panel A: The Risky Rate and Risk-free Rate				
$E\left(R^S\right)$	6.89	7.04	5.30	5.83	4.16
$\sigma\left(R^S\right)$	18.2	16.7	14.1	16.7	0.37
$E(R^f)$	1.91	1.68	1.98	0.93	4.16
$\sigma(R^f)$	5.44	2.23	5.73	6.51	0.18
$E(R^S - R^f)$	**4.82**	**5.36**	**3.32**	**4.90**	**.004**
$\sigma(R^S - R^f)$	19.1	16.8	14.7	17.1	0.27
$\frac{E(R^S - R^f)}{\sigma(R^S - R^f)}$	**0.25**	**0.32**	**0.23**	**0.29**	**.014**
$\rho(R^S, R^f)$	−0.04	0.02	−0.01	0.02	0.97
	Panel B: The Stock Price and Dividend				
$\sigma\left(d\right)$	13.3	13.6	11.2	10.6	–
$p - d$	21.1	24.7	23.2	23.8	–
$\sigma(p - d)$	27.0	26.2	15.6	19.6	–

Notes: The mean and standard deviation of each variable are annualized and reported in percentages. The data is from Campbell (1999) and covers 1890-1991. Long sample covers the entire period and post-war data is from 1947 to 1991. Following McGrattan and Prescott (2001) we subtract 1.8 percent per year from stock returns after 1960 to adjust for tax changes as described in the text.

equity financed (hence unlevered) unlike the empirical counterpart which measures the return to U.S. firms which have significant debt in their capital structures.[10] The effects of leverage in production economies are well understood: it raises both the level and the volatility of the stock return (see Benninga and Protopapadakis 1991, and Cecchetti, Lam and Mark 1993). Thus, it does not seem reasonable to try hard to match the risky rate in the current framework otherwise introducing debt into the capital structure would cause us to overshoot the risk premium.[11]

Second, the volatility of the risky rate is around 14 percent which is slightly lower than in the data ($17 - 18$ percent). This small discrepancy is again consistent with the absence of leverage from the model. Also in the data the risky rate and the equity premium have very similar variabilities (18.2 versus 19.1) which is captured well by the model (14.1 versus 14.7). As can be anticipated from these figures, stock and bond returns are almost uncorrelated: $\rho(R_t^S, R_t^f) \approx 0.0$ in historical data as well as in our simulations. For comparison the two returns have a correlation of 0.97 in the standard RBC model. We also report the corresponding statistics for the standard RBC model in the last column, but since the shortcomings of that framework are quite well-known we do not comment on them here (see Rouwenhurst, 1995).

Now we turn to the volatility of the interest rate that has proved quite a challenge even for

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[10] Jermann (1998) reports from Masulis (1988) that the debt-to-value ratio varies between 0.13 to 0.44 for market values and between 0.53 to 0.75 for book values.

[11] In principle it is easy to incorporate leverage into the current model. However, the computational demand of the current version is already very high. So we postpone that exercise for now.

Table 3: VOLATILITY OF THE RISK-FREE RATE IN VARIOUS MODELS

		US Data Long Sample	This Paper	Boldrin et. al.	Jermann	Campbell-Cochrane
β		–	0.99	0.99999	0.99	0.97
$E(R^f)$	%	1.91	1.98	1.20*	0.82*	0.94*
$\sigma(R^f)$	%	5.44	5.73	24.6	11.46	0.0*

Notes: A "*" indicates that some parameters in that particular model were chosen to match the statistic. The volatility statistic from Boldrin et. al is the result in their baseline model (preferred two-sector). In Jermann (1998) the figure is from the baseline case with stationary shocks; the volatility is 11.98 percent with random walk shocks.

recent models which have been quite successful otherwise (Table 3). For example, in Boldrin et. al (1999) the variability ranges from 17.4 percent all the way up to 25.4 percent and in fact the risk-free asset is more volatile than the risky asset in all specifications they consider (except when they introduce leverage in which case the volatility of the risky rate goes up to 25 percent). Cognizant of this fact C-C chose the functional form and the free parameters for the law of motion of the habit process judiciously to generate a constant risk-free rate.

In contrast, and to our pleasant surprise, the volatility of the interest rate closely matches that in the U.S. data without any degrees of freedom to choose. This outcome will appear even more surprising later when we further document many similarities between the asset pricing implications of this model and the habit persistence framework. So, what explains the difference?

To clearly see the difference consider the bond market diagram in Figure 1. The left panel depicts the case of a representative agent with habit formation in consumption. Even though external and endogenous habit differ in the risk aversion they imply, both specifications result in very low EIS and hence in a very inelastic bond demand curve. The interaction of this steep demand curve with the bond supply which is also *perfectly inelastic* at zero (due to the representative agent assumption) means that even small shocks to demand will generate large movements in the bond price and hence substantial volatility in the risk-free rate.[12] On the other hand, in the current framework the explanation is quite simple (right panel). First, notice that the majority of the population (non-stockholders) have a very inelastic demand function in our model too. The crucial difference here is that the bond supply is not inelastic at all. In fact, stockholders' supply schedule is quite flat both because of their high intertemporal elasticity ($EIS \geq 0.5$) and also because they have another asset to substitute for bond. Hence, a movement in the demand curve of similar magnitude now results only in small changes in the interest rate and the rest will be reflected in the variability of trade volume.[13] This explana-

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[12] C-C specify the habit process such that the effect of intertemporal substitution and precautionary saving on bond demand cancel each other exactly leaving the demand curve constant in response to shocks.

[13] The terms demand and supply are rather arbitrary here since both groups do demand bonds depending

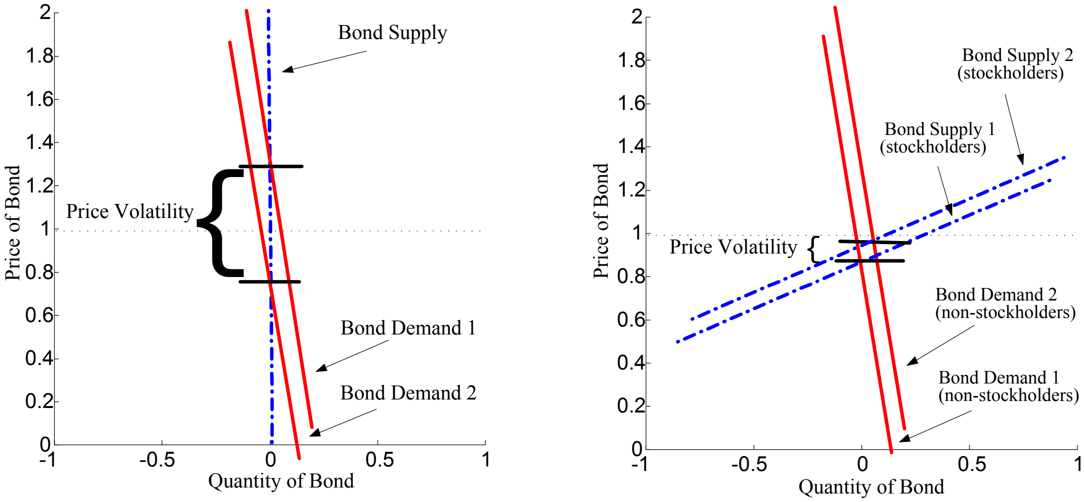

Figure 1: Determination of Bond Price Volatility in A Representative Agent Model (Left) versus in the Limited Participation Model (Right)

tion also captures the fact that individuals do trade in bond markets (in contrast to the zero bond holding implicit in representative agent models), and it seems that heterogeneity among participants is an important factor determining asset prices.

Finally, we move from returns to prices. The lower panel in Table 2 displays the moments of the stock price and the dividend process. First, the volatility of the dividend process is around 11 percent in the model which is close to what is in the data. Second, the average price-dividend ratio in the model is around 23, in line with the empirical evidence. The P/D ratio however seems a little too smooth (but higher when $\alpha^h = 3$) which could increase once leverage is introduced into the capital structure.

HOW DOES THE MODEL GENERATE A LARGE RISK PREMIUM?

How does limited participation and preference heterogeneity combine to generate a large equity premium with standard preferences and low risk aversion? It is useful to relate the mechanism here to the existing literature and highlight the differences. In an insightful paper, Saito (1995) studied an economy with limited participation where both agents consume out of wealth. He showed that the equity premium is linearly increasing in the share of wealth owned by non-stockholders. A large premium is then possible only if stockholders borrow a significant fraction of the capital invested from non-stockholders. This is basically a leverage story: high borrowing makes stockholders' portfolio risky and they demand a high premium to hold stocks. However, because there is no labor income and hence no precautionary saving in that model, non-stockholders' share of wealth goes to zero in the long-run and so does the premium.[14] But

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on the interest rate. We denote $-b^h$ as the bond supply of stockholders for convenience and because they are actually net borrowers in equilibrium.

[14] Furthermore, in reality even though U.S. firms have significant leverage ratios, a large fraction of this debt is

non-stockholders hold very little wealth—in fact, less than 10 percent of aggregate wealth—in our economy too (Table 10), which means there must be another mechanism generating the sizeable risk premium.

The large premium arises in our model from the interaction of (i) limited participation; (ii) preference heterogeneity; and (iii) precautionary saving resulting from persistent labor income shocks. (The last two components are absent from Saito's model as well as from Basak and Cuoco (1998) and create more demand for precautionary wealth in our model.) All three elements are essential for a high price of risk. Basically, both agents want to accumulate a buffer wealth in order to self-insure against persistent income shocks. Non-stockholders can only hold the risk-free asset, and can only accumulate a buffer wealth if stockholders are willing to borrow. On the other hand, stockholders can also smooth shocks by savings and dissaving in physical capital—an option not available to non-stockholders. Further exacerbating this asymmetric situation is the heterogeneity in the EIS: those who have less room for self-insurance (non-stockholders) are exactly those who want it the most, whereas those who tolerate fluctuations better also have better insurance opportunities. As a result, non-stockholders need the bond market much more than stockholders, who demand a hefty premium (a low borrowing rate) to participate in the bond market. The high premium in turn implies a low return on the savings of non-stockholders (low risk-free rate) who end up with very little wealth in equilibrium.

The Effect of Adjustment Costs

The mechanism just described for the equity premium more precisely determines the price of risk, or in other words, the premium for each unit of the risk. The *level* of the premium is also determined by the volatility of the return. In a frictionless production economy with small aggregate shocks this volatility is naturally low. Introducing adjustment costs is one way to reduce the elasticity of capital supply and increase the volatility of returns and as a result the level of the equity premium. However, adjustment costs have little effect on the price of risk which is displayed in Table 4. The Sharpe ratio goes from 24 percent down to 22 percent when we move essentially from an exchange economy ($\xi = 0.01$) to the frictionless limit ($\xi = \infty$). Furthermore, other results studied below are not sensitive to the adjustment cost: even setting $\xi = 2$, which is higher than empirical estimates has no appreciable effect.

Finally, we should also stress that we use adjustment costs as a simple way to introduce a friction into the technology and also to make our results comparable to recent papers which employed the same specification. However, this particular type of friction is not an essential part of the model. Alternative ways to increase the volatility of return such as stochastic depreciation (as in Storesletten et. al 1999) could also generate a high *level* of risk premium.

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in fact held by other stockholders (either as corporate bonds or other non-equity financial assets). An arguably better measure of stockholder's borrowing from the rest of the population is to look at the fraction of productive capital owned by stockholders (capital in publicly traded companies plus private businesses) as a fraction of the total. By this measure stockholders own more than 90 percent of all financial assets and private capital which means that total borrowing cannot be more than 10 percent of aggregate capital (Poterba and Samwick 1995). This amount is too small to yield any significant risk premium.

Table 4: Sharpe Ratio for Different Values of the Adjustment Cost Parameter

	Adjustment Cost Parameter (ξ)				
	0.01	0.10	0.23	0.5	∞
Sharpe Ratio	0.241	0.234	0.229	0.223	0.216

The reported Sharpe ratios are from a version of the baseline model with a two point approximation to the AR(1) process (due to computation time) where the standard deviation is adjusted to keep the same unconditional standard deviation of Z.

4.1 Cyclical Properties of Returns

Even a few years ago matching the first two moments of returns would probably be enough to declare success. However, several recent papers raised the bar higher with increasingly more successful results. We will thus examine the implications of our model in a number of further dimensions that have received attention both in the finance and macroeconomic literatures.

A large literature in finance documents the predictability of stock returns, and in particular, the fact that many of the variables predicting the returns also predict the business cycle (Campbell and Shiller, 1988; Poterba and Summers, 1988; Fama and French, 1989, among others). These studies focus on the predictive power of such variables as the dividend-price (D/P) ratio, the default spread or the term premium which also closely track business cycle fluctuations.[15] Furthermore, Fama and French (1989) document the *long term* co-movement between the D/P ratio and business conditions that extend well beyond the duration of a typical business cycle. They state:

> The major movements in these variables [D/P ratio and default premium] and in the expected return components they track, seem to be related to longer term business episodes that span several measured business cycles. The dividend yield and the default spread forecast high returns when conditions are persistently weak and low returns when conditions are persistently strong.

Figure 2 plots the D/P ratio together with the NBER business cycle dates. In addition to the short-run co-movement with the business cycle, there is also a striking long-run relation with business conditions. The D/P ratio is high during the 1930s and World War II, and is persistently low from 1955 to 1973, a period of stronger economic conditions that still includes four recessions, and increases after the oil shock of 1973 and remains high until mid-80s.

To further substantiate their claim Fama and French (1989) regress future stock returns on the D/P ratio *and* the term premium where each variable captures the long- and short-term

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[15] The statistical evidence on return predictability has been the focus of intense debate in the finance literature mainly because even a century-long data is quite short considering the persistence of relevant time-series. There are also a number of other issues such as the bias introduced by time averaging, the distribution of the R^2 statistics and so on. We do not discuss these problems here. The point of our exercise is to show that by applying the same methods to both the data and the model one obtains similar results.

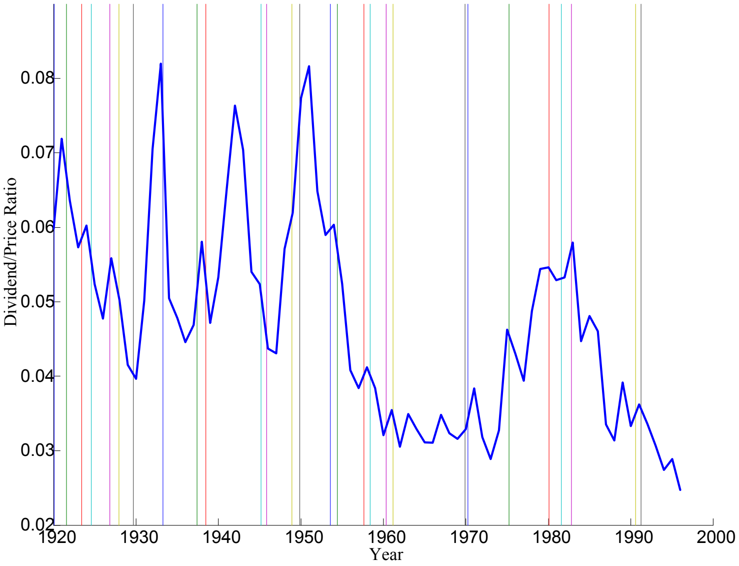

Figure 2: Co-movement Between the Dividend/Price Ratio and Business Conditions

co-movement with output respectively and find *both* of them to be significant. Moreover each variable increases the R^2 of the regression when added separately.

In order to test whether the model can capture the twists and turns of the relationship between financial variables and business conditions we conduct the following exercises. We first investigate if the model can account for the cyclical movement of asset prices including the countercyclicality of the expected equity premium, the conditional volatility and the Sharpe ratio documented in the literature. Moreover, in the next subsection we also replicate the long-horizon regressions of expected returns on the D/P ratio of Campbell and Shiller.

One advantage of having a fully specified model is that we can compute $E_t\left(R^S_{t+1}\right)$ and $\sigma_t\left(R^S_{t+1}\right)$ from the equilibrium functions exactly (rather than having to regress the return data on variables predicting business conditions) and examine their co-movements with aggregate output directly. First, in order to separate high frequency fluctuations typically associated with business cycles from longer term movements we follow the real business cycle tradition and filter the output series using a Hodrick-Prescott filter. We then document the correlation both in the short-run and in the long-run as stressed by Fama and French. Table 5 presents the relevant statistics.

Column 1 shows that besides the long-term positive co-movement with output depicted in Figure 2, the price-dividend ratio is also pro-cyclical at business cycle frequencies. Both of these correlations are captured by the model. Second, a curious observation is that the realized equity premium is procyclical even though ex ante excess return is strongly countercyclical. This feature is also true in the model: the correlation of realized excess returns from the model is equal to its empirical counterpart (0.30), even though the expected return is countercyclical (-0.31).

Table 5: CROSS-CORRELATION OF FINANCIAL VARIABLES WITH OUTPUT

	US Data		Model	
Variable	Short	Long	Short	Long
$p_t - d_t$	0.13	p-cycl*	0.53	0.94
$R^S_{t+1} - R^f_t$	0.31	p-cycl	0.30	0.33
$E_t\left(R^S_{t+1}\right)$	c-cycl*	c-cycl	-0.64	-0.92
$E_t(R^S_{t+1} - R^f_t)$	c-cycl	c-cycl	-0.31	-0.51
$\sigma_t(R^S_{t+1} - R^f_t)$	c-cycl	c-cycl	-0.28	-0.47
$\frac{E_t(R^S_{t+1}-R^f_t)}{\sigma_t(R^S_{t+1}-R^f_t)}$	c-cycl	c-cycl	-0.34	-0.51

Notes: All statistics are annual from the long sample covering 1890-1991. The columns with headings Short and Long report the correlation with output at business cycle frequencies and in the long-run respectively. The notation p-cycl and c-cycl denote pro- and counter-cyclical.

As for the conditional volatility of returns, there are two well-documented facts. One, conditional volatility is countercyclical in the U.S., which is also true in our model both in the long-run (-0.47) and at business cycle frequencies (-0.28). Two, conditional volatility is highly persistent and tends to cluster over time. This feature is clearly seen in the model as well: the quarterly first order autocorrelation of $\sigma_t\left(R^S_{t+1} - R^f_t\right)$ is 0.87. Moreover, peeking ahead to Table 8 (bottom of Panel A) we see that absolute stock returns are positively autocorrelated and die down very slowly as in the data, which again is another indication of persistence in variability.

The countercyclical variation in volatility could potentially provide a justification for the countercyclical movement in the risk premium. After all, the premium can be expressed as the product of the price of risk $(\frac{E_t(R^S-R^f)}{\sigma_t(R^S-R^f)})$ and the quantity of risk $\left(\sigma_t(R^S - R^f)\right)$, and the movement in the latter will give rise to a change in the risk premium even if the former is constant. However, empirical evidence does not seem to back up this argument. Expected returns move a lot more than conditional volatility does, resulting in a Sharpe ratio which is also countercyclical (Chou, Engle and Kane 1992). The same pattern emerges in the limited participation model both in the long-run (-0.51) and in the short-run (-0.34). This result is due to limited participation and is not sensitive to preference parameters: even when both agents are assumed to have identical preferences, $\alpha^h = \alpha^n = 2$, the price of risk continues to be countercyclical (-0.74 in the long-run and -0.45 in the short-run).

This result is generated by the significant countercyclical redistribution of wealth away from stockholders which makes them effectively more risk averse in recessions because of the concavity of the utility function (as a result of precautionary motives). As a result, they demand a higher return to hold the risky asset. At the same time, the fluctuations in non-stockholders' wealth is much more modest over the business cycle, so the bond price jointly determined by both groups is not affected as much creating countercyclical movement in the price of risk.

Table 6: PREDICTABILITY OF STOCK RETURNS: LONG-RUN REGRESSION ON PRICE/DIVIDEND RATIO

	US. Data					Model		
	Long Sample			Post-War				
Δd included?	No	No	Yes	No	No	No	No	Yes
Horizon	Coeff.	R^2	R^2	Coeff.	R^2	Coeff.	R^2	R^2
1	−0.08	.06	.06	−0.10	.11	−0.12	.12	.12
3	−0.16	.12	.13	−0.18	.31	−0.20	.33	.33
5	−0.25	.21	.22	−0.33	.51	−0.28	.41	.41
7	−0.34	.27	.28	−0.44	.57	−0.36	.48	.48
10	−0.36	.39	.39	−0.64	.73	−0.41	.53	.53

Notes: The data is from Campbell (1999) and covers $1890 - 1991$. The coefficients for the regression when Δd are very similar to those with dividends left out and hence are not reported.

4.2 Predictability of Returns: Long Horizon Regressions

The previous section has documented the fact that expected returns are correlated with the business conditions and hence are predictable in our model. Although these patterns of time-variation are interesting they do not tell us directly the *size* of the predictable component in stocks. The long-horizon regressions of Campbell-Shiller, and Fama-French provide a more direct answer. Furthermore, unlike in the theoretical model, the ex ante premium cannot be calculated directly from data, so these regressions form the basis of most empirical evidence on predictability. Thus we replicate these regressions in our framework which also serve as a more direct comparison of the model to existing evidence.

We first regress stock returns on the price/dividend ratio for the long sample and post-war U.S. data (Table 6). The classic pattern documented in the literature can seen here: the coefficients are negative indicating that a high P/D ratio forecasts lower returns in the future. Moreover both the coefficients and the R^2 values are increasing with horizon and reach impressive levels.

The model counterpart is reported in the last three columns. Both the coefficient estimates and the R^2 values are strikingly similar to empirical results: predictability is modest at one year horizon but increases steadily and reaches 50 percent at 10 year horizon. The coefficients also increase quickly first and then grow more slowly. Another finding by Campbell and Shiller is also presented in column 3: lagged dividend growth has almost no predictive power once the P/D ratio is included in the regression. The R^2 values remain virtually unchanged and the coefficients (not reported) are not significant. The same result is replicated in simulated data where the R^2 values do not change up to the third digit and the coefficients are mostly insignificant even with a simulated sample size of 15,000 years.

An alternative manifestation of return predictability is the excess volatility of stock prices. A simple way to see this is by first decomposing the variance of the log P/D ratio following

Table 7: VARIANCE OF PRICE-DIVIDEND RATIO EXPLAINED BY FUTURE COVARIANCES

Variance explained by	U.S. Data		Model
	Long Sample	Post-War	
r^S_{t+j}	101	137	124
Δd_{t+j}	−10	−31	−27

Notes: Each cell reports the percentage of variance explained by the corresponding variable (that is, $\left(\sum_{j=1}^{\infty} \gamma^j cov\left(p_t - d_t, x_{t+j}\right)\right) / var(p_t - d_t)$ where x is Δd and $-r^S$ in each case). The formula is calculated using 15 lags (years) both in the data and in the model.

Cochrane (1992). Defining $\gamma = \left(\overline{P}^S/\overline{D}\right) / \left(1 + \left(\overline{P}^S/\overline{D}\right)\right)$ at the steady state, we have:

$$var(p_t - d_t) \approx \sum_{j=1}^{\infty} \gamma^j cov\left(p_t - d_t, \Delta d_{t+j}\right) - \sum_{j=1}^{\infty} \gamma^j cov\left(p_t - d_t, r^S_{t+j}\right)$$

In the U.S. data (Table 7) a substantial fraction of total volatility is accounted for by the covariance of the log P/D ratio with future returns and only a small component is explained by varying expectations of future dividend growth. Moreover, both autocovariances are negative, consistent with the idea that a high P/D ratio signals low dividend growth which in turn means low returns in the future. The model captures both aspects of the data, and the covariances fall between the values observed in the post-war data and in the long-sample.

4.3 Time-series Properties: Autocorrelation and Cross-correlation

The autocorrelation structure of financial variables display some interesting patterns. For example, the risk-free rate is highly persistent whereas the risky rate (as well as the equity premium) has no significant persistence and displays mild mean reversion. At the same time, the absolute value of the risky rate is positively autocorrelated both at short- and long-horizons indicating clustering of volatility. Finally, as mentioned earlier the dividend yield (D/P) is extremely persistent and its fluctuations extend well beyond the duration of typical business cycle episodes.

In the standard RBC model, on the other hand, the time-series behavior of many variables closely mimic that of the technology shock. This is not totally surprising because the commonly assumed functional forms (with little curvature) together with a single source of uncertainty ties most variables tightly to the exogenous shock. In this sense the wide variety of autocorrelation patterns observed in financial data might seem too rich to be explained with our model which is based on the same framework.

The model turns out to be surprisingly successful in explaining the time-series of financial variables. For example, as reported in Table 8, the autocorrelation structure of the P/D ratio matches that in the data all the way from 1-year to 10-year horizon. C-C view this persistence to be key for their results and calibrate one of the free parameters to match this autocorrelation

Table 8: AUTOCORRELATION STRUCTURE OF KEY FINANCIAL VARIABLES

		LAG (years)					
		1	2	3	5	7	10
		AUTOCORRELATION					
$p-d$							
US Data		.79	.59	.52	.35	.32	.23
Model		.80	.64	.52	.35	.23	.18
r^s-r^f							
US Data		.03	$-.22$	.08	$-.14$	.10	.12
Model		$-.02$	$-.04$	.01	$-.01$	$-.01$	$-.02$
r^s							
US Data		$-.02$	$-.17$	.10	$-.11$	.08	.09
Model		$-.12$	$-.07$	$-.05$	$-.03$	$-.02$	$-.03$
r^f							
US Data	(R^*)	.53	.36	.23	.14	.15	.10
US Data	(N^*)	.83	.73	.69	.60	.57	.43
Model		.84	.66	.52	.34	.22	.11
$\|r^s\|$							
US Data		.13	.09	.06	.14	.15	.07
Model		.08	.06	.04	.04	.04	.03
		CROSS-CORRELATION					
$p_t-d_t, \|r^s_{t+j}\|$							
US Data		$-.12$	.02	$-.06$	$-.10$	$-.05$	$-.04$
Model		$-.16$	$-.13$	$-.10$	$-.07$	$-.04$	$-.03$

Notes: The empirical statistics are calculated for the period 1890-1991. The rows denoted by R* and N* report the correlation for real and nominal interest rates respectively.

structure. In contrast, the limited participation framework delivers this result as a natural outcome.

Second, the model generates weak mean reversion in the equity premium similar to that in the data. The autocorrelation is very small and tends to revert to its mean. For comparison, in the RBC model the first annual autocorrelations of the risky rate and the excess return are 0.63 and 0.55 respectively and are still above 0.30 at 10 year horizon

Third, since the risk-free rate is time-varying in our model we can examine its time-series behavior. The interest rate is highly persistent in the model. Measuring the empirical counterpart is somewhat tricky because in reality bonds are only nominally riskless due to inflation risk.[16] Using the ex-post real rate is one possible approach (the row marked by R^*) but the

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[16] One could potentially look at index bonds to get a measure of the real interest rate. Index bonds however are very new in the US, and they have been traded in the UK only since 1982. Also these bonds have an indexation lag of 8 months which means they are essentially nominal at shorter horizons. Thus with few data points at annual frequency it is not clear how useful that data will be for our purposes and we do not pursue it here.

autocorrelation structure calculated this way is downward biased because of unanticipated inflation. An alternative option is to use the ex-ante nominal interest rate which might be a better indicator of the risk-free rate investors anticipate (the row marked by N^*). This series is significantly more persistent. The truth probably lies somewhere in between and so does the interested rate implied by the model. The persistence is the same as that of the nominal rate for short horizons and falls to levels closer to the autocorrelation of real interest rates at longer horizons (for 5 to 10 years).

Finally, the model also generates the correct pattern of volatility clustering and predictability: absolute returns are positively autocorrelated implying that high volatility is usually followed by more volatility. This evidence complements the high persistence revealed in the conditional standard deviation ($\rho(\sigma_t, \sigma_{t+1}) = 0.87$) reported earlier. Furthermore, as shown in the lower panel, a low P/D ratio predicts higher volatility in the future and both the sign and size of the autocorrelation structure are similar to empirical values. This is the leverage effect documented by Schwert (1989) and Nelson (1991) among others.

Correlation of Consumption Growth with Asset Returns

One curious observation about the linkage between asset prices and quantities is that the market return usually does a better job in explaining individual stock returns than does consumption growth. An alternative statement of this fact is that the static CAPM outperforms consumption-based asset pricing models. C-C show that when consumption is aggregated over time before calculating the growth rate, the same relation holds in their model as well which is quite surprising since the true data generating process for asset prices is the consumption-based model.

At this point, may be not too surprisingly, the same result holds in our model too. The correlation of the true discount factor (M) with the growth rate of time-aggregated consumption falls very quickly (Table 9). On the other hand the risky rate has a substantially higher correlation of 0.96 with M even when aggregated over 12 periods. This suggests, for example, that if our baseline economy was simulated at a monthly frequency and then annual averages were constructed along the lines of empirical applications, it would seem as if the stock market return, R^S, was a much better proxy for the true discount factor compared to annual consumption growth. Moreover, although C-C attribute this outcome to the existence of habit, here the same result obtains even when $\left(\frac{C^h_{t+1}}{C^h_t}\right)^{-\alpha}$ is calculated from stockholders' (time-aggregated) consumption and by construction there is no habit, which suggests that this outcome is a result of time aggregation.

Table 9: CORRELATION OF THE TRUE DISCOUNT FACTOR WITH DIFFERENT VARIABLES

	Correlation of M with:	
Aggregated over (periods)	$\left(\frac{C^h_{t+1}}{C^h_t}\right)^{-\alpha}$	R^S
1	1.000	0.992
4	0.713	0.975
8	0.572	0.963
12	0.534	0.959

Notes: The ratio $\left(\frac{C^h_{t+1}}{C^h_t}\right)^{-\alpha}$ is calculated from simulated data by *first* aggregating consumption over N periods (horizon given on each row) as is typically done with actual data. M is the true discount factor at each horizon.

5 Habit versus Limited Participation: What is the Connection?

In the previous section we showed that a macroeconomic model with limited participation and heterogeneity in the EIS is able to explain a large number of asset pricing phenomena including all the results presented in C-C.[17] The similarities between the two models' results are quite remarkable. Although the current framework arguably has further advantages (such as the time-varying interest rate with plausible properties) the similarities are still many.

A natural question that comes to mind is whether the similarities are just a coincidence or whether there might be a deeper connection between the two frameworks. On the surface though the two models look quite different: representative-agent versus heterogeneous-agents; external habit versus standard CRRA utility; exchange versus production economy; random walk versus autoregressive shocks, to name a few. Nevertheless, we will show that our model has a reduced form representation which looks strikingly similar to the external habit formulation of C-C including their parameterization of the free parameters!

To develop the argument, first recall that the Euler equations for stockholders' portfolio choice decision (in the stock and bond markets) imply:

$$E_t\left[\beta\left(\frac{C^h_{t+1}}{C^h_t}\right)^{-\alpha^h}\left(R^S_{t+1}-R^f_t\right)\right]=0.$$

Notice that this condition only holds for stockholders, and not for aggregate consumption. Making the substitution, $C^h = C^A - X$, where C^A denotes aggregate consumption, and letting $X \equiv C^n$ for reasons that will become clear in a moment, we get:

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[17] There are a few other results discussed in C-C (such as the correlation of consumption growth with the risky rate, the actual time series of stock price data generated by the model, etc.) that we have not reported here to save space. They are all as similar to C-C's findings as the ones reported. Available upon request.

$$E_t\left[\beta\left(\frac{C^A_{t+1}-X_{t+1}}{C^A_t-X_t}\right)^{-\alpha^h}\left(R^S_{t+1}-R^f_t\right)\right]=0. \tag{4}$$

But this is the same Euler equation that one would obtain (as in C-C) for a representative-agent with external habit preferences: $u=\frac{(C-X)^{1-\alpha}}{1-\alpha}$ where X now denotes the exogenous habit stock. In other words, if asset prices are in fact determined by condition (4) because some households do not participate in the stock market, a representative-agent model (which necessarily abstracts away from this fact) would still have to subtract these households' consumption (X_{t+1}) from the aggregate (C^A_{t+1}) to mimic the true Euler equation and thus to successfully explain asset prices. Thus ignoring limited participation would make it look as if the representative agent was displaying habit persistence.

Of course for this claim to have any validity, one has to show that the properties of the habit term X_t in C-C's model are very much the same as the neglected term, that is, non-stockholder's consumption in our model. To that end, we follow C-C and define $S_t\equiv\frac{C^A_t-X_t}{C^A_t}$, which is the key figure in their model and the properties of this ratio drives most of the action. In that framework S_t is the consumption above habit as a percentage of total, and is called "the surplus consumption ratio." In the current model, $S_t\equiv\frac{C^h_t}{C^A_t}$, is simply the share of stockholder's consumption in the aggregate. Now manipulating (4) we obtain:

$$E_t\left[\beta\left(\frac{C^A_{t+1}-X_{t+1}}{C^A_{t+1}}\right)^{-\alpha^h}\left(\frac{C^A_t}{C^A_t-X_t}\right)^{-\alpha^h}\left(\frac{C^A_{t+1}}{C^A_t}\right)^{-\alpha^h}\left(R^S_{t+1}-R^f_t\right)\right]=0$$

$$\Longrightarrow E_t\left[\beta\left(\frac{S_{t+1}}{S_t}\right)^{-\alpha^h}\left(\frac{C^A_{t+1}}{C^A_t}\right)^{-\alpha^h}\left(R^S_{t+1}-R^f_t\right)\right]=0.$$

This alternative expression also holds in both frameworks, and can be viewed as adding an extra state variable, S_t, to the standard Euler equation of Mehra and Prescott (1985). For either model to have any hope of success S_t must be doing something magical. Stressing this critical role C-C introduce a fairly elaborate exogenous process for $s_t\equiv\log(S_t)$:[18]

$$s_{t+1}=(1-\phi)\,\overline{s}+\phi s_t+\lambda\left(s_t\right)\left(c^A_{t+1}-c^A_t\right) \tag{5}$$

where $c^A_t\equiv log\left(C^A_t\right)$ and calibrate it to match a number of financial moments. Apart from the persistence term ϕ, the nonlinear sensitivity function, $\lambda\left(s_t\right)$, is chosen to satisfy three conditions viewed desirable for matching asset prices (more on this later). As C-C acknowledge, this is

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[18] We set the growth rate of consumption (g) in their formulation equal to zero since we study a stationary model.

a reverse engineering exercise as it is difficult to justify the specific functional form and the parameter values chosen for $\lambda(s_t)$ on economic grounds.

So how similar are the two models really? To answer this question, we examine the statistical properties of S_t as well as the features of $\lambda(s_t)$ assumed in C-C and the one implied by our model. The first obvious step is to check if S_t in our model is procyclical which is, a priori, not at all clear. In the habit model the surplus ratio increases in good times because habit evolves very slowly and lags behind actual consumption, so this ratio has a strong positive correlation with output. Similarly, in our model S_t (stockholders' share of consumption) has a correlation of 0.92 with output. This correlation is driven by two separate effects. First, stockholders have a higher EIS so their consumption rise more in response to a positive shock. And second, a technology shock increases the stock price and thus stockholders' wealth significantly on impact, whereas the immediate effect on non-stockholders wealth is much weaker: their savings increase only gradually as their wage grow slowly with capital.

After passing this first test of sensibility we can now proceed to take a closer look. First, we compare the densities of S_t which are displayed in Figure 3. The one in the right panel is obtained by simulating the AR(1) process above with the parameter choices in C-C. As can be seen, its mode is at its upper bound and the distribution has significant negative skewness. In the left panel we plot the unconditional density of $\left(C^h/C^A\right)$ in our model. To us, the similarity seems quite remarkable, especially considering that $\left(C^h/C^A\right)$ is an endogenous outcome and does not have any of the above specifications or free parameters.[19] The two distributions almost have the same shape including the location of the mode, the sharp upper bound and the fat left tail.[20] Moreover, the left skew in the ratio is despite the fact that neither C^h nor C^A have significant skewness[21] and the two are highly correlated at business cycle frequencies.

Second, turning to the conditional information one can get an even clearer picture. C-C calibrate the persistence parameter ϕ above to match the autocorrelation structure of the price/dividend ratio. Their parameter choice implies a quarterly persistence of 0.96 for S_t which is virtually the same as the autocorrelation of $\left(C^h/C^A\right)$ in our model: 0.954! This could be expected at some level since our model was able to match the autocorrelation of the P/D ratio without calibrating any extra parameters (Table 8).

Third, as C-C emphasize the properties of the sensitivity function, $\lambda(s_t)$, is at the heart of the external habit model. Especially the fact that the sensitivity is decreasing in S_t so that habit is more sensitive to consumption shocks in recessions is credited for generating many of the results. By substituting ϕ (0.96) and the model generated sequences of s_t and c_t^A into the AR(1) process (5) we can back out the implied sensitivity function from our model which we

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[19] The small dip at the right end of the distribution of $\left(C^h/C^A\right)$ is due to the Markov approximation. Even though the 12-state process is able to match the autocorrelation and standard deviation of the AR(1), it is still truncated at both ends. When we reduce the persistence of the AR(1) process we get very similar distributions with slightly less dispersion but without the dip in $\left(C^h/C^A\right)$.

[20] Note that in C-C's benchmark the interest rate is assumed to be constant. If this restriction is relaxed (as in equation 6 in the main text) and the interest rate volatility is calibrated to 6 percent per year the density for λ becomes slightly less skewed and looks even more similar to that in the limited participation model.

[21] The skewness coefficients are -0.07 and -0.01 for C^h and C^A respectively, but it is -0.43 for C^h/C^A.

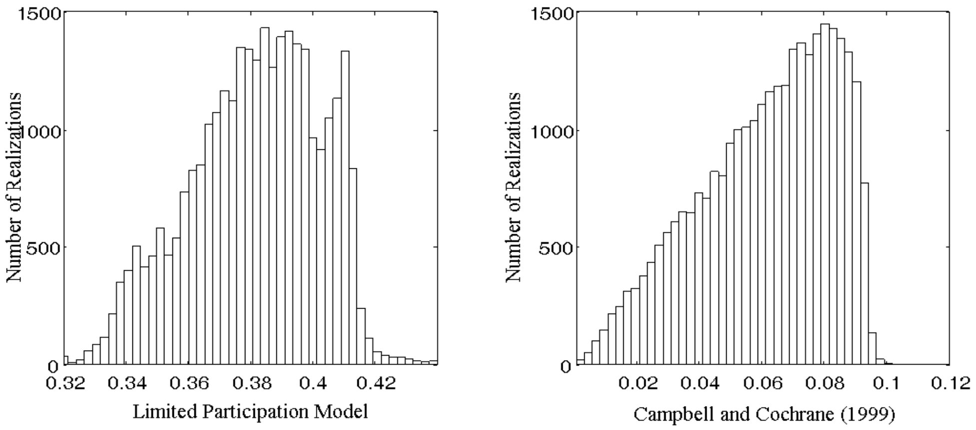

Figure 3: Comparing Empirical Distributions: Stockholders' share in Aggregate Consumption (Left) Versus the Surplus Consumption Ratio in Campbell-Cochrane (Right)

call $\widehat{\lambda}\left(s_t\right)$.

Figure 4 plots both λ and $\widehat{\lambda}$ on the same graph. The cloud of points representing $\widehat{\lambda}$ are downward sloping and almost parallel to λ as if it was shifted down. However, observe that the two functions could not possibly be identical since one of the goals C-C had in mind when choosing λ was to get a constant risk-free rate which is not the case in our model. One can relax this assumption in the external habit model and allow for a variable risk-free rate:

$$r_t^f = r_0^f - B\left(s_t - \overline{s}\right), \tag{6}$$

where B is another free parameter.[22] This extension modifies some of the equations in C-C and in particular implies a higher steady state surplus ratio which at the baseline case is quite low compared to the average value in our model. Furthermore, this modification scales λ down by $\sqrt{\frac{1-\phi}{1-\phi-B/2}}$. By choosing B such that the risk-free rate has roughly 1.5 percent quarterly standard deviation as in our model, the implied $B = 0.039$ and the scaling factor is 1.78. The adjusted λ function now falls almost on top of $\widehat{\lambda}$!

Finally, the two models can be compared in yet another dimension by looking at the implied habit process X_t directly. When choosing the λ function, C-C discuss three desirable properties that X_t should satisfy. First, it is a smoothly evolving process and in particular $\sigma_C^2 \gg \sigma_X^2$. Second, $C > X$ everywhere so that the power utility function is well-defined. Third, around the steady state X_t is predetermined and moves positively with consumption elsewhere. This last assumption essentially implies that locally C and X are uncorrelated and globally they have a positive correlation.

All three of these conditions are naturally satisfied by non-stockholders' consumption. First,

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[22] See Campbell and Cochrane (1995) for details.

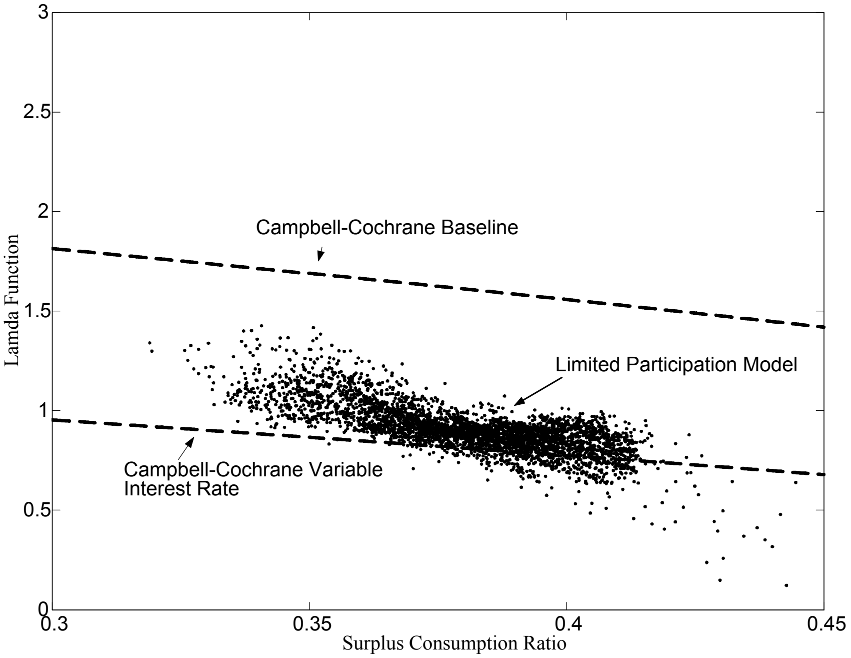

Figure 4: Comparison of the sensitivity function $\lambda(s_t)$ assumed in Campbell-Cochrane to the one implied by the Limited Participation model

aggregate consumption is much more variable than that of non-stockholders' ($\frac{\sigma_C^2}{\sigma_X^2} \approx 4-5$) even though non-stockholders comprise eighty percent of the population (more on this in the next section). Second, $C^A - X > 0$ holds naturally since stockholders' consumption is always positive. And third, the two group's consumption has a relatively low correlation (0.72) so that C^A and X are positively—but less than perfectly—correlated.

Given that asset prices in both models are determined by the Euler equation above, the close correspondence between S_t (and X_t) in the two models largely explains how the two models' implications are virtually identical confirming our conjecture that the external habit specification can be viewed as a reduced form representation of the limited participation model. The endogenous re-distribution of wealth between the two groups with different utility curvatures is reflected in a time-varying risk aversion in the representative-agent framework with external habit.

Before concluding this section we want to show how our reinterpretation of external habit preferences eliminates an unattractive feature of that model: an incredibly high risk aversion. To see this recall that the utility function used by C-C is $u = \frac{(C^A - X)^{1-\alpha}}{1-\alpha}$ and with the representative agent assumption C^A is the actual consumption of the agent and X is an externality. When calculating risk aversion one would then differentiate all expressions with respect to C_t^A and get

$$\begin{aligned} RRA_t &= -\omega \frac{V_{\omega\omega}}{V_\omega} = \psi_t \frac{\partial \ln C_t^A}{\partial \ln \omega_t}, \\ \psi_t &\equiv -C^A \frac{U_{cc}}{U_c} = \alpha C^A \frac{\left(C^A - X\right)^{-\alpha-1}}{\left(C^A - X\right)^{-\alpha}} = \frac{\alpha C_t^A}{C_t^A - X_t} = \frac{\alpha}{S_t} \end{aligned}$$

where the second equality is obtained by using the envelope condition, $u_c = V_\omega$, and then substituting for u_c. With C-C's parameterization $\frac{\alpha}{S_t}$ is approximately 40 at steady state and grows without bound as C_t^A falls near X_t during a recession. Moreover, $\frac{\partial \ln C_t^A}{\partial \ln W_t}$ is greater than 1 resulting in an average risk aversion of 80, and in hundreds during recessions.

In the context of our model, however, if one correctly interprets X_t as non-stockholders' consumption, then the agent who prices stocks consumes *not* the aggregate consumption, C_t^A, but only $C_t^h = C_t^A - X_t$. Consequently, differentiating u correctly with respect to $(C^A - X)$ we obtain

$$\psi_t = -C^h \frac{U_{cc}}{U_c} = \alpha \left(C^A - X\right) \frac{\left(C^A - X\right)^{-\alpha-1}}{\left(C^A - X\right)^{-\alpha}} = \alpha!$$

Moreover, $\frac{\partial \ln C_t^h}{\partial \ln \omega_t}$ is just 1 with CRRA utility in the absence of borrowing constraints (and is only slightly above 1 in our model since stockholders are far from their constraints) which implies $RRA_t \approx \alpha = 2$ independent of X_t!

6 Results: Macroeconomics

The interest of macroeconomists in the equity premium puzzle—despite being primarily an asset pricing phenomenon—is not all that surprising. Mehra and Prescott (1985) essentially showed that the historical excess return was puzzling in the context of a standard general equilibrium (consumption-portfolio choice) model which is also the foundation of most neoclassical macroeconomic analysis.[23] Naturally, one would wonder, how much confidence can be placed in a framework which has grossly counterfactual implications for most pricing problems? Thus it is compelling, from a macroeconomist's perspective, to examine if a model proposed to resolve these asset pricing puzzles also displays plausible macroeconomic behavior.

Despite its success in explaining asset prices, the extension of the external habit model to study macroeconomic problems has not been as impressive. Lettau and Uhlig (2000) embed the habit specification employed by C-C in a standard real business cycle model and found that it creates a number of macroeconomic anomalies. For example, consumption turns out to be extremely smooth: the quarterly standard deviation is a mere 0.02 percent which is 70 times smaller than the volatility observed in the U.S. data. Introducing another habit formation,

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[23] Although Mehra and Prescott used an exchange economy version of this canonical model, further extensions to include production, more general preferences, as well as many other elements to enrich the model did not alter the conclusion appreciably until the introduction of habit persistence models discussed here.

Table 10: MACROECONOMIC STATISTICS – TIME SERIES AND CROSS-SECTION

	US Data	Model		
	Post-War	$\xi = \infty$	$\xi = 0.23$	$\xi = 0.5$
	Panel A: time-series			
$\sigma(Y)$	1.9	2.3	2.4	2.5
$\sigma(C^A)$	1.4	1.3	2.2	1.6
$\sigma(C^h)$	see text	1.8	4.4	3.6
$\sigma(C^n)$	see text	1.0	1.1	1.0
$\sigma(I)$	8.2	8.6	3.0	5.3
	Panel B: Cross-Sectional			
ω^h/ω^A	88%	85%	89%	86%
C^h/C^A	32%	34%	37%	35%

Notes: The statistics reported in panel A are from *quarterly* U.S. data covering 1947.1-1991.4. Standard deviations are calculated from data after the trend is removed with a Hodrick-Prescott filter from the log of raw data with a bandwidth parameter of 1600. The investment figure includes changes in inventories; if excluded the volatility is 5.8 percent. The source for the distribution of financial wealth is Wolff (2000), and for consumption data, is Guvenen (2002).

one in leisure, does not help much but instead gives rise to other puzzles of its own: labor hours become too smooth and countercyclical. Overall, they conclude that "introducing habit formation in consumption and leisure yields counterfactual cyclical behavior in an otherwise standard real business cycle model."[24]

It is then natural to ask if the successful asset pricing results of the limited participation framework also come at the cost of implausible macroeconomic behavior. In Guvenen (2002) we analyze the quantity implications of this model (without adjustment costs) in detail and find that it not only delivers plausible statistics, but it also helps us understand some facts which appear puzzling in a standard RBC model. In this section we present a brief overview of the macroeconomic performance of the model both with and without adjustment costs imposed. Table 10 displays a summary of both aggregate business cycle and cross-sectional statistics.

We first start with the case when $\xi = \infty$ (no adjustment costs) which is more directly comparable to Lettau and Uhlig's analysis. In contrast to their findings, here the volatilities of output, aggregate consumption and investment are all quite close to their empirical values. Note that this is true even though most households have a very low elasticity of substitution. If these households were holding a significant fraction of wealth (which is not the case; see panel B), their preference for very smooth consumption would induce them to save and dissave vigorously in response to shocks causing aggregate investment and thus output to be overly

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[24] Boldrin et. al (1998) tackle this problem by imposing restrictions on the production side of the economy such as immobility of labor and/or capital between sectors; another assumption which forces agents to make savings decisions before they observe the aggregate shocks and so on. Moreover, the EIS is not as low in their framework as in C-C.

volatile. Instead, virtually all the wealth is held by households with a high EIS, so both output and investment display more plausible fluctuations.

Second, when adjustment costs are imposed, with the baseline calibration ($\xi = 0.23$) output remains fairly unchanged, but the volatility of aggregate consumption goes up. When we decompose consumption across the two groups, it becomes clear that the increase is mainly due to stockholders. One possible way to reconcile this finding with empirical evidence is to observe that stockholding is very unequal even among stockholders. For example, the top 1 percent of households own about 48 percent of all equity (Survey of Consumer Finances, 1998). Since consumption is much more evenly distributed than wealth in the U.S. (panel B), even if these wealthy stockholders have very volatile consumption paths, their contribution to aggregate consumption volatility is likely to be quite modest.[25] In other words, by capturing heterogeneity in the population parsimoniously with two agents, the model necessarily attributes a lot of variability to the consumption of *all* stockholders whereas in reality most of the aggregate risk is borne by a much smaller fraction within this group.

Nevertheless, the model still predicts the *average* consumption volatility of stockholders to be higher than that of non-stockholders. There is considerable evidence that this is indeed the case. For example, Mankiw and Zeldes (1991) report that the variance of stockholders' consumption growth is more than twice that of non-stockholders. This is true even though their consumption measure consists of only food expenditures from PSID. Furthermore, Attanasio, et. al. (2002) use non-durables and services as their consumption measure from the Family Expenditure Survey on British households and calculate stockholders' consumption growth variance to be *four times* larger than non-stockholders'. In a recent paper, Aït-Sahalia, Parker and Yogo (2001) focus on the sales of luxury goods as a measure of the wealthy's consumption.[26] They find for example that the annual standard deviation of the sales of luxury retailers is between 17 to 23 percent, and the volatility of charitable contributions is 21 percent. Although there is certainly some durability in these expenditures, these numbers are even much larger than the standard deviation of durable consumption expenditures which is around 8 percent annually in the post-war NIPA data. Again, comparing between durable goods, the sales of Porsche cars have an annual volatility of 34.3 percent compared to 8.5 percent for overall luxury cars (defined as having a price higher than \$24,000). Hence, although indirect, this evidence also seems to suggest that the wealthy (and similarly the stockholders) have more volatile consumption paths than non-stockholders consistent with the model.

Finally, the baseline parameter choice for adjustment costs ($\xi = 0.23$) which we took from previous studies is on the low side of empirical estimates. If one sets $\xi = 0.5$ then, as shown in column 4, the volatilities of both consumption and investment move closer to empirical values. The only noticeable change in financial statistics is a small reduction in the *level* of the premium without affecting the Sharpe ratio (Table 4) or any other asset pricing questions investigated before. As we discussed before the level of the premium can be increased by leverage or a

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[25] See Poterba (2000) for an elaboration of this point.

[26] Of course the link from the sales of luxury goods to the consumption of a certain group is clearly not a direct one, but the evidence should still be informative.

slightly higher risk aversion. Alternatively, adjustment costs may not be the most appealing and realistic friction to consider. Irreversibilities in the investment decision, stochastic depreciation of capital or frictions in inter-sectoral movements of capital are other realistic features of the production process which can also generate high return volatility.

Turning to the cross-section (Panel B), the model reveals an interesting pattern of heterogeneity: in the baseline case: stockholders account for only 37 percent of aggregate consumption despite holding 89 percent of aggregate wealth, which is exactly the same kind of distribution observed in the U.S. data. This result is intuitive since consumption is proportional to income in the long-run which is more evenly distributed than wealth. Moreover, these cross-sectional distributions are largely unaffected by adjustment costs (and remain unchanged even in the limit when capital is perfectly elastic).

Putting all this evidence together it is clear that the close correspondence between the two models do not extend to macroeconomic analysis. Both models have implications not captured by stockholders' Euler equation (4). In particular, in the limited participation model, though non-stockholders hold little wealth they account for a substantial fraction of aggregate consumption and affect its properties.

6.1 How Different Are The Two Models?: A Policy Experiment

The success of habit formation models (both endogenous and external variety) has encouraged researchers to perform policy analysis in these frameworks. For example, Fuhrer (2000) documents this success and proceeds to conduct monetary policy analysis in an endogenous habit model. In a similar framework, Christiano, Eichenbaum and Evans (2001) analyze optimal monetary and fiscal policy. On the fiscal policy side, Ljungqvist and Uhlig (2000) and Abel (2001) study optimal taxation in environments with external habit (For other examples, see also McCallum and Nelson 1998, Levin and Williams 2002). In this section we conduct a simple policy experiment to demonstrate that one can reach dramatically different conclusions in the limited participation model compared to the habit persistence model.

Consider a simple tax reform problem similar to the one studied by Lucas (1990). We imagine that initially the government imposes a flat-rate tax on capital income and returns the proceeds to households in a lump-sum fashion. Suppose that, at a certain date, capital income tax is completely eliminated and agents have not previously anticipated it. We set the initial tax rate $\tau^k = 36$ percent which roughly corresponds to the average rate in the U.S. All aspects of the baseline model remain intact. Also, in order to make our results comparable to the previous literature, we first consider the welfare gain from this reform in a representative agent framework. If the agent has $\rho = 0.5$, the welfare benefit of this policy is 0.75 percent of consumption per period taking the transition path into account. As Lucas argues, this number is an order of magnitude larger than the gain from eliminating the business cycle fluctuations, and almost twice the gain from eliminating a 10 percent inflation. If, on the other hand, we assume $\rho = 0.02$, (which is even higher than the EIS implied by C-C's model) the welfare gain is reduced to 0.2 percent of consumption instead, mainly because now the transition takes approximately 250 years compared to 20 years in the former case!

Table 11: FIRST TWO MOMENTS OF RETURNS FOR DIFFERENT PARAMETERIZATIONS

Difference from the Baseline	Baseline	rra^h=rra^n=2 eis^h=eis^n=.5 (CRRA)	rra^h=rra^n=2 eis^h= 0.5 eis^n=0.1	$\lambda = 30\%$	Exchange economy ($\xi = 0.01$)
$E\left(R^S\right)$	5.30	4.73	5.25	5.15	5.84
$\sigma\left(R^S\right)$	14.1	8.71	13.7	12.9	20.8
$E(R^f)$	1.98	3.92	2.01	2.26	0.51
$\sigma(R^f)$	5.73	3.64	5.85	5.23	7.46
$E(R^S - R^f)$	**3.32**	**0.81**	**3.24**	**2.89**	**5.33**
$\sigma(R^S - R^f)$	14.7	8.92	14.2	13.5	22.1
$\frac{E(R^S-R^f)}{\sigma(R^S-R^f)}$	**0.23**	**0.09**	**0.23**	**0.21**	**0.24**
$\rho(R^S, R^f)$	−0.01	0.03	0.02	0.04	−0.02

Now we subject the limited participation economy to the same tax experiment. The welfare gain is 0.71 percent of total consumption.[27] In effect, this economy behaves as if it was populated only by agents with unit elasticity and non-stockholders' preferences virtually vanished from the problem.

In sum, both kinds of habit persistence imply a low EIS for the representative agent, when in fact almost all productive capital is owned by households with a high elasticity. Thus, any policy question which depend on intertemporal substitution is likely to give radically different answers in each framework.

7 Sensitivity Analysis and Discussion of Results

This section investigates the robustness of the results from our baseline economy to changes in the parameterization. In most cases considered below, variations in the values of these parameters mainly affect the first two moments of returns with only minor consequences for other statistics. Hence, to save space, we only report the counterpart to Table 2 here. The counterparts of Tables 5 to 9 for the parameterizations discussed in this section are available in an appendix which can be obtained from the author's website.[28]

ELIMINATING PREFERENCE HETEROGENEITY: When both agents are assumed to have *identical* CRRA utility functions with a relative risk aversion coefficient of 2 (and thus an elasticity of intertemporal substitution of 0.5) the equity premium falls to 0.8 percent and the volatility falls to 8.9 percent reducing the Sharpe ratio to 9 percent (column 2 in Table 11). Most of the other results are not affected: for example, the conditional equity premium, volatility

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[27] This calculation is based on the assumption that there is a utilitarian government which tries to attain the same social welfare index as without taxes and makes transfers to agents in such a way to minimize the total amount of transfers.

[28] The URL is http://www.econ.rochester.edu/guvenen/research.htm. To economize on computational time these simulations (including the results in Table 11 in the text) have been performed using an 8 point Markov process instead of the 12 point used to solve the baseline model.

and Sharpe ratio continue to be countercyclical; stock returns are still predictable (although the R^2s fall between 10 to 35 percent); and the autocorrelation patterns in Table 8 remain largely the same. So, the low EIS of non-stockholders seems critical for the first two moments of returns, with the rest of the results driven mainly by limited participation. Nevertheless, in this case non-stockholders' consumption becomes more volatile than stockholders' (inconsistent with macro data), and the share of stockholder's consumption (S_t) ceases to be procyclical, breaking the close link between this model and the external habit model. In this sense preference heterogeneity is essential for the results overall.

Disentangling RRA from EIS: The only difference between the case considered above and the baseline is the curvature assumed in the utility function of non-stockholders. But because RRA and EIS are entangled with CRRA preferences, it is not clear whether the significant fall in the equity premium is due to a lower risk aversion or a higher elasticity. To address this question we solve the model assuming that agents have Epstein-Zin (1989) recursive preferences which allows us to disentangle the two parameters. In particular, we leave the EIS of both agents at their baseline values ($EIS^h = 0.5$, $EIS^n = 0.1$) but eliminate the heterogeneity in risk aversion: $RRA^h = RRA^n = 2$. As reported in column 3, the level and volatility of the equity premium as well as of other returns are remarkably close to their baseline values. Reducing the risk aversion of non-stockholders—who constitute 80 percent of the population—from 10 to 2 has virtually no impact on asset prices! It is clear then that the fall in the equity premium in column 2 is due to the higher EIS of non-stockholders and not lower risk aversion. Furthermore, as noted earlier (column 4 in Table 2) the equity premium increases almost linearly with the risk aversion of stockholders suggesting that what matters most for our results is the RRA of stockholders and the EIS of non-stockholders.

Participation Rates: If the participation rate is assumed to be 30 percent, the equity premium falls to 2.96 percent from 3.3 percent in the baseline case while the volatility falls to 13.8 percent, bringing down the Sharpe ratio—rather modestly—from 23 percent to 21.5 percent. All other results remain essentially the same. It should be emphasized though that one cannot infer much from this exercise—of comparing economies with low and high participation rates—for the consequences of the participation boom of the last decade, a question that would require solving for the entire transition path of the economy to the new stochastic steady state.

Exchange Economy versus Production Economy: Following Mehra and Prescott (1985) a large number of studies used exchange economy models to address asset pricing puzzles. To make our results more directly comparable to these studies, it is useful to investigate the behavior of our model as the cost of adjustment goes to infinity ($\xi \to 0$). With such prohibitive costs, the capital stock remains fixed and the model becomes equivalent to an exchange economy (column 5).[29] The level of the equity premium rises to 5.3 percent accompanied by a higher volatility of 22.1 percent bringing the Sharpe ratio up slightly to 24 percent. As we argued before this exercise illustrates that adjustment costs raise both the level and volatility of the premium without significantly affecting the price of risk.

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[29] We solve the model for $\xi = 0.01$ to approximates this limiting case. The annual standard deviation of the capital stock is a mere 0.2 percent suggesting this is a reasonable approximation.

Borrowing Constraints: In the simulations of the baseline economy, borrowing constraints are binding about 0.1 to 0.5 percent of the time. Clearly then relaxing these constraints is not likely to change the results much and indeed they do not. When stockholders and non-stockholders are able to borrow up to 4 years and 1 year of income respectively ($B^h = 16 * \overline{W}, B^n = 4 * \overline{W}$), the first two moments of returns remain pretty much unchanged (c.f., additional appendix).

7.1 Relation to the Literature

There is a vast and still expanding literature on the asset pricing puzzles addressed in this paper. For recent surveys of this literature see Campbell (1999) and Cochrane (2001). This paper is more closely related to the strand of literature which emphasizes the role of limited participation starting with Mankiw and Zeldes (1991). Saito (1995), and Basak and Cuoco (1998) studied general equilibrium models with limited participation in the stock market. Saito was in fact the first one to draw attention to a possible link between limited participation and habit persistence based on equation (4), though he did not pursue that correspondence further in his paper. Also, that link is not nearly as tight in his model as it is in the current framework due to the absence of preference heterogeneity and labor income risk which play essential roles in generating the results in this paper.[30] In a recent paper, Guo (2002) studies an exchange economy model with limited participation and is able to match some of the financial statistics discussed in this paper. However, as he also emphasizes his results also rely on large income shocks to each group (36 percent per year) in addition to aggregate shocks, as well as frequently binding borrowing constraints (about 40 percent of the time). Dai (2001) analyzes a related model with a capitalist and a worker where the worker consumes his income and hence does not participate in any market. He shows that for a particular (exogenous) wage process for the worker the economy is isomorphic to C-C's model. In contrast, the wage process is endogenous in our model and very different than the consumption of non-stockholders, yet the latter corresponds to the habit term in an external habit model. Finally, Heaton and Lucas (1999) investigate if the booming stock prices in the 1990s could be justified by booming participation in the stock market. Our paper complements this literature by showing that limited participation together with preference heterogeneity of plausible form can provide a compelling explanation for a large number of asset pricing phenomena.

8 Conclusions

In this paper we introduced a macroeconomic model with limited stock market participation and heterogeneity in the EIS parameter. This particular two-agent representation is parsimonious compared to the traditional (fully) heterogenous-agent frameworks of Aiyagari (1994), Krusell

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[30] Although Basak and Cuoco derive some theoretical results allowing for preference heterogeneity, they restrict non-stockholders to logarithmic utility which is hard to reconcile with the empirical evidence of Section 3. Moreover, in their numerical analysis they assume identical preferences.

and Smith (1998), and Storesletten et. al (1999), yet it captures a lot of interesting heterogeneity absent from representative agent models, which turns out to be crucial for understanding asset prices. We calibrate the parameters of the model to standard values from the business cycle literature and in particular we do not choose them to match any financial statistic.

The model is able to explain the behavior of a large number of financial variables surprisingly well. These include the level and the volatility of the stock return and the interest rate as well as the cyclical movement of the price-dividend ratio, the equity premium, its volatility and the Sharpe ratio. Moreover, stock prices have a large predictable component in the model as documented in the finance literature.

Many of the results explained here are also features of the external habit model of Campbell and Cochrane (1999). We find that this is not a coincidence. The limited participation model has a reduced form which is intimately related to the external habit model. In particular, the part of aggregate consumption accounted for by non-stockholders—which is necessarily ignored in representative-agent models, since there are no non-stockholders—resurfaces as an elaborate habit process in the external habit model. One important difference between the two models is the value of the risk aversion: it is just two in our model compared to an average value of 80 in the external habit model.

The model also generates plausible macro statistics even when there are no frictions in production whereas the external habit model is hard to be reconciled with aggregate data (Lettau and Uhlig 2000) mainly due to the extremely low EIS implied by that model. Furthermore, policy analysis in the two frameworks also yield quite different conclusions. This point suggests extreme caution as the success of habit persistence models encouraged many researchers to address policy questions in that framework.

Another lesson that we draw from these results is about the potential dangers of reverse-engineering. Although reverse-engineering can do an impressive job in matching a certain dimension of data (such as asset prices) this success is in no way a guarantee that the model can be straightforwardly used for other questions of interest (such as for policy analysis).

We believe that these results would encourage further research on the reasons behind limited participation which is not addressed in this paper. Furthermore, given the central role played by non-participation, another important research avenue should investigate the consequences of the recent trends in participation observed in most countries for asset prices as well as for wealth inequality and welfare.

A Appendix

The discretization method understates the true persistence of the AR(1) process. In order to get a first order autocorrelation of 0.95 we simulate an AR(1) process with $\rho = 0.96$ for 30,000 periods. Applying Tauchen and Hussey's (1991) method with 12 state points we obtain the desired transition matrix. Table 11 shows that the generated Markov process behaves almost exactly like an AR(1) process with $\rho = 0.95$.

Table 12: Autocorrelation Structure of the Markov Chain Approximation

	$\sigma\left(\varepsilon\right)$	Autocorrelation at Lag				
		1	2	3	4	5
AR(1)	2.000%	.950	.903	.857	.815	.774
Markov Approximation	2.009%	.950	.902	.856	.814	.772

A.1 Computational Algorithm

[To be added...]

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