# That's Where the Money Was. Home Bias and English Investments Abroad, 1866-1885

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Abstract

Why did Victorian Britain send so much of its capital abroad? We make use of a recently collected data set to estimate the return to investments in British and foreign assets between 1866 and 1885. We introduce a formal test of the hypothesis that British investors did not benefit from foreign investments. We reject this hypothesis for most foreign assets. In fact, we estimate that as a result of foreign investment, British investors enjoyed an increase in utility equivalent to a 4-8% increase in permanent consumption.

## 1 Introduction

"Never before or since has one nation committed so much of its national income and savings to capital formation abroad." – Michael Edelstein<sup>1</sup>

It is estimated that between 1865 and 1914 Great Britain's capital exports amounted to roughly £4 billion nominal pounds<sup>2</sup>. For Great Britain, a nation that until 1850 had invested less than two percent of its gross domestic product abroad, this was a prodigious sum that represented 5.2 percent of GDP. At the same time that British capital was leaving the island at unprecedented levels, British industry suffered a relative decline that signaled the beginning of the transformation of the British economy from the "workshop of the world" into the "world's banker". While it was no surprise that the United States would eventually surpass Britain in industrial might, the speed of the reversal caused much consternation among the British elite. C.K. Hobson, writing in 1914 noted

"A few years ago the British public was startled by a new cry-the cry that capital was being driven abroad...Foreign investment was regarded as a new and portentous phenomenon, without precedent in the history of the country, as a running sore, sapping the life blood of British

<sup>&</sup>lt;sup>1</sup>Edelstein (1981) "Foreign Investment and Empire 1860-1914" in Floud and McCloskey, *The Economic History of Britain Since 1700 Vol 2.* 

 $<sup>^{2}</sup>$ The estimate is from Cottrell (1975) p. 27. To put this number in perspective, the United Kingdom's nominal GDP was £.97B in 1865 and £2.44B in 1914.

industry...The matter was discussed in Parliament. A well-known statesman made the discovery that all the great ships going westward across the Atlantic were carrying bonds and stocks in ballast...Other speakers lamented the increase in unemployment and the stagnation of trade, which they attributed to the unparalleled outflow of capital" – C.K. Hobson (1914), The Export of Capital, p.i

While Hobson is no doubt engaging in some hyperbole, the city of London, with its perceived propensity to funnel capital overseas rather than into domestic industry, was suspected of hastening the decline of British industry. According to this view, London's capital markets systematically discriminated against domestic industry by ignoring potentially profitable investments in Britain, preferring instead to invest in inferior projects overseas<sup>3</sup>. The feeling that "the city of London and its financial institutions were the single greatest threat to the prosperity of England"<sup>4</sup> was widespread by 1931 when Keynes and his colleagues on the Macmillian Committee accused the London capital markets of a long history of foreign bias. Since 1931, the charge of capital market failure due to foreign bias has resonated throughout the literature with prominent voices on both sides.

The proponents of market failure have argued that British investors sent capital abroad due to a bias or ignorance which prevented them from investing in domestic assets that presumably would have yielded at least as much as foreign investments if only given the chance.

"There is strong evidence that it [the London capital market] was not perfect, that there was virtually total ignorance among financial institutions and advisors about investment opportunities in home industry, and that banks and other institutional lenders operated with traditional and irrational prejudices as to which type of investments they should support and which they should not." — Pollard (1987) p.460

According to this view, Britain would have enjoyed a far higher standard of living had London investors only channeled more investment towards domestic industry<sup>5</sup>. Defenders of rational markets responded with an appeal to revealed preferences<sup>6</sup>. The London investors who sent capital abroad must have believed that this was the optimal investment. To proponents of rational agents, this was strong evidence that the returns on the foregone British investments must have somehow been inferior to their international counterparts.

It is important to note that both sides of the debate framed their arguments in the context of which investment (domestic or foreign) had a higher expected return. If a Victorian investor expected to earn a

<sup>&</sup>lt;sup>3</sup>See O'Rourke and Williamson (1999) Chapter 12 for an excellent review and critique of the capital market failure view of British overseas investment.

<sup>&</sup>lt;sup>4</sup>Rosenstein-Rodan (1967) Capital Movements and Economic Development p.68

<sup>&</sup>lt;sup>5</sup>For instance Crafts (1979), Pollard (1985) and Kennedy (1974, 1987).

<sup>&</sup>lt;sup>6</sup>For example, McCloskey (1970,1979), Temin (1987, 1989) and Michie (1988).

higher return investing overseas, this could explain capital flows abroad without having to resort to claims of bias. Therefore, past tests of capital market bias were often based on arguments about the relative returns to foreign versus domestic investments. In their 1999 text, O'Rourke and Williamson reviewed the literature

"The claim is that the City of London systematically discriminated against domestic borrowers, preferring instead to channel funds into overseas ventures. The result was that domestic British industry, starved of capital, grew more slowly than it would otherwise have done. An obvious implication of the hypothesis is that domestic (British) rates of return must have exceeded those available on foreign investments"

-O'Rourke and Williamson, p.226 in (1999) Globalization and History : The Evolution of a Nineteenth-Century Atlantic Economy

There is one obvious shortcoming with any test that attempts to identify investor bias based on the means of asset returns. The riskiness of foreign and domestic assets may not be the same. The implication that domestic rates of return must have exceeded those available on foreign investments is a valid test of capital market bias if, and only if, the returns have equal variances and investors must chose between investing all of their savings either at home or abroad. When investors are given the choice of investing a portion of their money both at home *and* abroad, however, the expected rate of return on domestic assets can exceed the expected rate of return on foreign assets and an unbiased (rational) investor may still choose to invest a large portion of her wealth overseas, provided the correlation between foreign and domestic assets is low enough to provide diversification benefits.

Diversification seems like a likely explanation for British overseas investment. After all, foreign investments probably had a lower correlation with the consumption of British investors than their domestic counterparts. Nineteenth Century investors were certainly sophisticated enough to realize the benefits of diversification. During the 1860s, a new investment vehicle, the closed-end investment trust, appeared for the first time in the London market. As the 1868 prospectus of the Foreign and Colonial Government Trust demonstrated, investment trusts successfully marketed themselves as instruments of portfolio diversification:

"The object of this trust is to give the investor of moderate means the same advantages as the large capitalist in diminishing the risk of investing in foreign and colonial government stocks, by spreading the investment over a number of different stocks"<sup>7</sup>

The Foreign and Colonial Government Trust was not an isolated case. From the very beginning, British investment trusts had a decidedly international flavor. In his study of U.K. capital markets, William Kennedy noted that diversifying institutions in the form of financial and investment trusts appeared in the 1860s

<sup>&</sup>lt;sup>7</sup>Quoted in Robinson (1930)

"holding portfolios that were almost completely composed of foreign assets"<sup>8</sup>. These trusts proved wildly popular with British investors. The par value of all investment company securities listed on the London stock exchange grew from roughly £7.3 million in 1873 to £33.5 million by 1883<sup>9</sup>. This number increased almost fourfold to £ 124.6 million by 1893. There was a brief slowdown in international investment after the Barings crises and the U.S. railroad defaults of the mid 1890s. Nonetheless, by 1903 the institutional market had grown to £176.5 million and it would reach £248.7 million by the eve of W.W.L<sup>10</sup>. If the success of the investment trust is any indication, C.K. Hobson was correct when he noted that Victorian investors had a "desire to spread risks by investing in various countries or in diverse industries"<sup>11</sup>.

In retrospect, it's surprising that one of the dominate explanations of the high level of British foreign investment ignores diversification and relies upon a market failure (an irrational bias) to explain international investment. After all, economists, who have long understood the diversification benefits of international investment, lament the refusal of modern investors to hold foreign assets. Ironically, market failures due to bias, transaction costs or an information asymmetry are often cited as explanations for the *low* level of international diversification in modern portfolios. The logical extension of this argument is that we should see high levels of international investment when agents are rational and markets are informationally and operationally efficient.

Why haven't we viewed the high level of international investment present in 19th century capital markets as a sign of rationality and efficiency? In a word, "covariance". When economists assert that modern investors should invest a higher percentage of their wealth in foreign assets, they are basing this conclusion on the diversification benefits of holding foreign assets that have a low covariance with investors' consumption. Economists studying the 19th century have largely ignored covariance (and hence diversification) when evaluating the optimality of portfolio choices.

There are two reasons for this omission. First, the debate was framed in the years before mean-variance analysis of portfolio decisions became standard. At the time that Rostow (1949) and Cairncross (1953) literally wrote the books on Victorian overseas investment, an investment was evaluated by its return, not its effect on the return *and* risk of one's portfolio. Despite advances in our understanding of portfolio choice under uncertainty, the optimality of Victorian foreign investment continued to be framed as an "either or" decision. That is, investors had a choice of investing either at home or abroad (but not both). When faced with such a choice, the rational investor will choose the "best" return, where "best" denotes the highest return or the highest return for a given unit of variance. In such a setting, any evidence that domestic investments

 $<sup>^{8}</sup>$ Kennedy (1987) p.130

<sup>&</sup>lt;sup>9</sup>Morgan derived these estimates by aggregating securities listed in the Investors Monthly Manual.

 $<sup>^{10}</sup>$ In real 2001 pounds the size of the trust investments were £309 million in 1873, £1.7 billion in 1883, £8.3 billion in 1893, £10.75 billion in 1903 and £14.9 billion in 1913.

 $<sup>^{11}</sup>$ Hobson (1914) .xiii.

had a higher return than foreign investments (which received capital) was viewed as evidence of bias. Tests of capital market failure were therefore tests of the hypothesis that domestic assets had a higher return than their foreign counterparts. This paradigm survived even as economists stopped evaluating investments by their risk and return and instead began to rank investments according to their effect on both the risk and return of one's *portfolio*.

The second, and probably more important, reason that diversification has been ignored as a potential cause of foreign investment is the more practical problem of a lack of data. Before we can hope to evaluate the diversification benefits of foreign investment we must be able to measure the covariance between home and foreign assets.

One would have thought that the debate about the cause of overseas investment would therefore have ended in 1982 when Michael Edelstein published Overseas Investment in the Age of High Imperialism. In this impressive work, Edelstein computed the realized annual return of 566 foreign and domestic assets listed on the London stock exchange between 1870 and 1913. He concluded that the return on foreign assets was, on average, slightly higher than domestic assets and this difference was insignificant, even after controlling for risk. Even Edelstein, who brought more data to bear on this problem than anyone to date, did not perform mean-variance analysis or highlight the diversification benefits of foreign investment, however<sup>12</sup>.

Rather than end the debate, Edelstein's work simply revitalized the antagonists. The proponents of efficient markets cited the lack of high returns on domestic assets as evidence of market efficiency. The proponents of market failure responded by noting that the relative spread between foreign and domestic returns fluctuated with long periods of higher domestic returns<sup>13</sup>. In short, Edelstein's average returns were too similar and the returns too noisy to convince either side that their position was wrong.

In our opinion, diversification seems like a likely explanation for British investors' overseas investments. Before we can evaluate the hypothesis that diversification drove British investment abroad, we must first have access to a sample of foreign and domestic asset returns that are both broad enough to control for idiosyncratic risks and sampled at a high enough frequency that one can estimate a covariance matrix with confidence<sup>14</sup>. To this end, we have collected a huge cross section of asset returns from stocks and bonds trading in London and the United States. We will employ these heretofore unknown data to investigate

 $<sup>^{12}</sup>$ Due to the low frequency (annual) of his observations, mean-variance analysis would have been inappropriate with Edelstein's data. Edelstein did the next best thing and appealed to theory. He controlled for risk by comparing the return of each security to one covariance – that with the market. This eliminated the need to estimate a multi-asset covariance matrix and, if the assumptions of the CAPM hold, is the perfect analysis of risk adjusted return.

 $<sup>^{13}</sup>$ See Pollard (1985)

<sup>&</sup>lt;sup>14</sup>We eliminate much of the idiosyncratic risk by forming portfolios, hence the need for a broad sample of securities. Many of our tests, require the estimation of a mean vector and covariance matrix for 10 or more portfolios. With data sampled at an annual frequency, one would need over 165 years of data to estimate a covariance matrix with a saturation ratio of 3! Obviously, we need a sample at a higher frequency than Edelstein's data.

the effects of international diversification on 19th century investors' optimal portfolios. This is the first study to use 19th century data sampled at a high enough frequency to apply the mean-variance spanning tests common in the modern home-bias literature. This is also the first study to include assets trading on the exchanges of the United States as well as London. There is considerable evidence that in addition to purchasing foreign assets trading in London, British investors held a large number of assets trading on the exchanges of the United States<sup>15</sup>. By adding U.S. assets to the choice set, we hope to better reflect the true set of investment opportunities available to British investors.

# 2 19th Century Stock Exchanges

"The London Stock Exchange is the only really international market of the world. Its interests branch over all parts of our globe" – R.M. Bauer 1911<sup>16</sup>.

British investors did not have to go far to find international investments. During the latter half of the 19th century, foreign governments and industrialists increasingly looked to London for financing. The nominal value of all securities listed on the London Stock Exchange was £1.68 billion in 1863 and foreign government and railroad securities accounted for £278 million or 16.5% of this total. Between 1863 and 1873, the nominal value of listed securities grew to £2.27 billion while foreign government and railroad securities grew to £2.27 billion while foreign government and railroad securities grew to £2.68 billion in 28.8% of the total. By 1883, the same foreign securities accounted for 41.7% of the £3.63 billion London market<sup>17</sup>.

If the offerings of the London market were not satisfactory, a British investor could easily invest in America by purchasing securities listed on the stock exchanges of the United States. Besides the well known New York Stock Exchange, the post-bellum United States had a number of regional exchanges providing markets for the ever expanding capital demands of the railroad, canal, financial, and mining industries. The telegraph, undersea cable (first completed in 1866), and stock ticker (introduced in 1867) provided fast, reliable communications between British investors and the U.S. exchanges. Brokerage houses like Heseltine, Powell & Co. and E. Satterthwaite specialized in overseas's investment and were more than willing to offer British investors the opportunity to invest in securities trading on American exchanges<sup>18</sup>. With so many options at their fingertips, British investors could easily purchase foreign assets if they so desired.

Overseas investments offered British capitalists the opportunity to invest in the promising opportunities of the new world while potentially diversifying their holdings. To quantify the gains to overseas investment, we have collected the prices and dividends of 2,757 individual securities that traded in London, New York,

<sup>&</sup>lt;sup>15</sup>See Mira Wilkins 1989 Chapters 4-5

 $<sup>^{16}</sup>$ Quoted in Michie (1999) p.11

<sup>&</sup>lt;sup>17</sup>Michie (1999) Tables 32.-3.3 P.88-89.

<sup>&</sup>lt;sup>18</sup>Michie (1999) p.127-129

Baltimore, Boston, Cincinnati, Charleston, Louisville, Philadelphia, St. Louis, or San Francisco. The prices were sampled every 28-days between January 1866 and December 1885. In addition to the closing prices, we have also collected a history of dividend payments and shares outstanding for each security. The combination of prices, dividends and shares allows us to compute market values and 28-day holding period returns that accurately reflect dividend payments and stock splits. In total, the data set contains 226,497 individual stock and bond returns.

The data was hand entered from 19th century financial publications. To eliminate errors, all data was double entered and compared.

### 2.1 Test Portfolios

Before we can evaluate the effect of foreign investments on the risk and return of 19th century British investors' portfolios, we must separate assets into foreign and domestic portfolios. Identifying which assets are foreign and which assets are domestic is not a trivial task, however. In some cases, such as British railway securities, foreign government bonds, or securities trading in the United States, the nationality of the security is obvious. In other cases, it is not so clear whether a security belongs in a domestic or foreign portfolio. A number of financial corporations listed and headquartered in London existed to funnel capital abroad. For example, the Egyptian Land Development Company, was a real estate investment trust. This British company was headquartered in London and listed on the London exchange. The capital raised by the IPO and latter offerings, however, was for the most part invested in real estate in Egypt. If a British investor invested in this company was this an investment in a domestic or a foreign asset? We treat multinationals who list in London but do their primary business abroad as foreign investments. If a company lists a foreign country or city as its place of business we also treat this company as foreign. If the location of a company's business could not be determined, we treat it as domestic security. This has the potential drawback of mis-labeling some foreign assets as domestic and underestimating the diversification benefits of holding an international portfolio.

We sort assets according to their type and location to form 11 value-weighted test portfolios. Details of the portfolio compositions, and the average 28-day gross returns, standard deviations and correlation coefficients can be found in Table I. Graphs of the ex-post returns an investor would enjoy had she held a various value-weighted combination of combinations of these portfolios can be found in Figures 1-2.

Foreign assets had a higher return than their British counterparts. This extra return came with added risk (a higher standard deviation), however. If one were to use the "either or" criteria to compare these investments, it is not clear which would be preferable to a risk-averse British investor.

Note the low correlations between foreign government bonds and other securities. This is *prima facie* evidence of diversification benefits from international investing. Also, note that unlike foreign government

bonds, foreign corporate debt and equity was highly correlated with London corporate stocks and bonds. Both Temin (1987) and Kennedy (1987) have suggested that British investors had a "fear of equities" and preferred to invest in foreign government bonds. Ex-post, this seemed to be a wise decision as foreign government bonds simultaneously delivered high returns and diversification benefits.

The ex-post returns and correlations found in Table I suggest a diversification motive for foreign investment. The foreign government bond portfolio had a high return and a low correlation with other assets. Likewise, foreign corporate stocks and bonds had higher returns than their domestic counterparts with roughly the same risk. An investor who combined all foreign and domestic assets into a well-diversified portfolio enjoyed higher ex-post returns and lower variance than an investor who held a well diversified domestic portfolio.

The diversification benefits apparent in Table I cry out for a formal test. Are the increased returns and decreased variances observed in the diversified portfolios real or merely the result of sampling error? To answer this question we require a method to evaluate the mean-variance trade-offs available to 19th century investors.

## 3 Evaluating the Benefits of International Investing

We wish to utilize a method that encompasses both risk and return to measure the affect of the addition of foreign assets into the portfolios of British investors. To this end, we present two measures of the benefits of international diversification. The first, is a straightforward statistical evaluation of the null hypothesis that the addition of foreign assets provided no diversification benefits. The second measure, is an estimation of the permanent consumption gain a British investor would demand before willingly refraining from international investment. Specifically, we wish to ask if the addition of foreign assets expanded the mean-variance frontier of asset returns available to 19th century British investors and if so, how valuable was this expansion in terms of permanent consumption?

### 3.1 The Mean-Variance Frontier

The mean-variance frontier is the set of all possible portfolios with the minimum variance for a given expected return. Let **R** denote the gross return vector of N assets available to an investor. Given expected returns  $\boldsymbol{\mu} = E[\mathbf{R}]$ , and a non-singular covariance matrix  $\boldsymbol{\Sigma} = Cov(\mathbf{R})$ , the investor's portfolio choice can be represented by a N-vector of weights **w** such that the weights sum to one and the n-th weight is the fraction of money invested in the n-th asset. How would an investor choose between potential portfolios comprised of these assets? If the investor has mean-variance preferences, she will choose a portfolio on the mean-variance efficient frontier. That is, she will choose a portfolio with the minimum variance of all portfolios with a

given expected return. We can, therefore, define a mean-variance efficient portfolio as the set of weights that minimize portfolio variance for a given expected return  $\alpha$ .

$$\min_{w} \mathbf{w}' \Sigma \mathbf{w}$$
(1)  
s.t.  $\mathbf{w}' \boldsymbol{\mu} = \alpha$  and  $\mathbf{w}' \mathbf{1} = 1$ 

We can trace the entire mean-variance frontier by solving this minimization problem for different values of  $\alpha$ . Graphs of the mean-standard deviation frontier formed by different combinations of British and foreign benchmark portfolios can be found in Figures 1-3. The figures appear to confirm the diversification benefits of international investing. For most expected returns, the mean-standard deviation frontier formed from domestic and foreign securities lies well to the left of the mean-standard deviation frontier formed from British securities alone. Once British investors add foreign government bonds to their portfolios, however, the diversification benefits of adding foreign corporate securities is rather small. Likewise, once an investor has diversified between British securities, foreign government bonds and either U.S. or other foreign assets, there is virtually no benefit to further diversification.

A word of caution is in order. Did the addition of foreign securities actually expand the mean-variance frontier of British investors or are the observed differences the result of sampling error? Ex-post estimates of the mean-variance frontier were formed by replacing  $\mu$  and  $\Sigma$  with their sample estimates  $\hat{\mu}$  and  $\hat{\Sigma}$ . In any finite sample, our consistent estimates of the assets' expected returns and covariance matrix will be equal to the true population parameters plus some sampling  $\operatorname{error}^{19}$ ,  $\hat{\mu} = \mu + \xi_{\mu} \operatorname{and} \hat{\Sigma} = \Sigma + \xi_{\Sigma}$ . When we replace  $\mu$  and  $\Sigma$  with the sample estimates  $\hat{\mu}$  and  $\hat{\Sigma}$  and minimize (1), the resulting minimum variance will be smaller than the minimum variance one would find with the population values  $\mu$  and  $\Sigma$ . This sampling bias assures that ex-post efficient frontier estimates always span the actual ex-ante mean-variance frontier. Thus, even if a portfolio of domestic assets was ex-ante mean-variance efficient, the domestic portfolio will lie well within the ex-post frontier in any finite sample. In short, with the benefit of hindsight it is easy to say "I could have increased my return and decreased my risk by buying X and shorting Y". What we require is a method to take sampling error into account when we ask if the addition of international assets expanded the mean-variance frontier of British investors.

### 3.2 Spanning Tests

Under what conditions would the inclusion of foreign assets *fail* to expand the mean-variance set of potential investments? The inclusion of foreign assets would certainly have made British investors better off if the foreign assets had a higher expected return and lower variance than their domestic counterparts. Even, in the

<sup>&</sup>lt;sup>19</sup>Throughout this paper covariance matrixes are estimated via Newey and West's (1987) frequency zero spectral density estimator with a fixed bandwidth lag criteria.

absence of high returns, foreign investments would have made British investors better off if the correlation between British and foreign investments was sufficiently low. In fact, as long as the foreign assets offered different risk and return characteristics than their domestic counterparts, the inclusion of foreign assets would have made British investor's better off. Foreign assets offered different risk and return characteristics if the potential portfolios formed from foreign investments were not spanned by domestic assets.

Let  $\mathbf{R}_t^d$  denote the time t vector of gross returns on  $N_d$  domestic assets and  $\mathbf{R}_t^f$  denote the time t vector of gross returns on  $N_f$  foreign assets. We say  $\mathbf{R}_t^d$  spans  $\mathbf{R}_t^f$  if it is possible to replicate the expected return of each asset in  $\mathbf{R}^f$  with a linear combination of assets in  $\mathbf{R}^d$  such that

$$\mathbf{R}_{t}^{f} = \mathbf{a} + \boldsymbol{\delta} \mathbf{R}_{t}^{d} + \boldsymbol{\varepsilon}_{t}$$

$$E[\mathbf{a}] = E[\boldsymbol{\varepsilon}_{t}] = 0$$
(2)

If (2) holds, we can replicate the expected return of each foreign asset with a portfolio of domestic assets. If this is the case, the foreign assets are redundant and the inclusion of foreign assets had no effect on the ex-ante mean-variance frontier available to British investors. If  $\mathbf{R}^d$  did not span  $\mathbf{R}^f$ , however, then the inclusion of foreign assets expanded the mean-variance frontier available to British investors.

A test of the null hypothesis that the inclusion of foreign assets had no effect on the mean-variance efficient frontier available to British investors is equivalent to the test of the null hypothesis that domestic British assets spanned foreign assets.

Often, it is convenient to express the spanning restrictions in (2) as restrictions on investors' stochastic discount factors. A random variable  $m_{t+1}$  is a valid stochastic discount factor if  $m_{t+1}$  discounts future payoffs such that the time t price of a time t + 1 payoff,  $X_{t+1}$ , is

$$P_t = E_t[m_{t+1}X_{t+1}]$$
(3)

(2) implies that a stochastic discount factor formed from domestic assets alone  $m_t = \alpha + R_t^d \beta$  is a valid discount factor for domestic as well as foreign securities. The insight that mean variance spanning implies a valid discount factor linear in  $\mathbf{R}^d$  is the basis of the mean-variance spanning tests of De Santis (1993), Bekaert and Urias (1996), Cochrane (2001), DeRoon and Nijman (2001), and Nijman and Werker (2001).

#### 3.2.1 A Generalized Method of Moments Spanning Test

Recall that a valid discount factor implies  $P_t = E_t[m_{t+1}X_{t+1}]$ . If we divide both sides by  $P_t$ , we are left with the moment condition  $E_t[m_{t+1}R_{t+1}] = 1$ . A valid  $m_{t+1}$  is guaranteed to exist. Hansen and Jagannathan (1991) show us how to derive the unique valid discount factor in the space of domestic returns. Imagine a regression of the valid discount factor  $m_{t+1}$  on  $\mathbf{R}_{t+1}^d$ 

$$m_{t+1} = \alpha + \{ \mathbf{R}_{t+1}^d - E[\mathbf{R}_{t+1}^d] \} \boldsymbol{\beta} + \varepsilon_{t+1}$$

$$\tag{4}$$

Where  $\alpha$  is equal to the mean of the discount factor and  $\varepsilon_{t+1}$  is mean zero and un-correlated with  $\mathbf{R}_{t+1}^d$ by construction. The left hand side variable,  $m_{t+1}$  is unobservable (a considerable burden when estimating a regression!). Nonetheless, for a given  $\alpha$ , we can estimate  $\boldsymbol{\beta}$  from the moment conditions  $E_t[m_{t+1}\mathbf{R}_{t+1}^d] = \mathbf{1}$ 

$$\boldsymbol{\beta}^* = [\mathbf{cov}(\mathbf{R}_{t+1}^d \mathbf{R}_{t+1}^{d\prime})]^{-1} (1 - \alpha E[\mathbf{R}_{t+1}^d])$$
(5)

Thus, for any  $\alpha$ , there exists a valid discount factor  $m(\alpha)_{t+1} = \alpha + \mathbf{R}_{t+1}^d \boldsymbol{\beta}^*$ . The discount factor  $m(\alpha)_{t+1}$ prices the domestic assets (and all linear combinations of the domestic assets) by construction. If the domestic assets span the foreign assets, recall that we can write  $\mathbf{R}_{t+1}^f$  as a linear combination of  $\mathbf{R}_{t+1}^d$ . Therefore, if  $\mathbf{R}_{t+1}^d$  spans  $\mathbf{R}_{t+1}^f$ ,  $m(\alpha)_{t+1}$  must be a valid discount factor for the foreign assets as well.

De Santis (1993), and Bekaert and Urias (1996) exploit the restrictions implied by spanning and the two-fund theorem to derive a GMM spanning test. The two-fund theorem states that every portfolio on the mean-variance frontier can be replicated by a liner combination of any two mean-variance efficient portfolios. But every portfolio on the mean-variance frontier of domestic assets has a corresponding valid stochastic discount factor of the form  $m(\alpha)_{t+1} = \alpha + \mathbf{R}_{t+1}^d \boldsymbol{\beta}^*$ . Therefore, if domestic assets span foreign assets we should be able to find two different mean-variance efficient portfolios of domestic assets that imply two discount factors that are valid for both the domestic and foreign assets. If, on the other hand, the domestic assets do not span the foreign assets, the discount factors formed from domestic assets alone will be unable to assign the correct prices to the foreign assets.

#### The Test Statistic

To evaluate the null hypothesis that  $\mathbf{R}_{t+1}^d$  spans  $\mathbf{R}_{t+1}^f$ , combine the  $T \, xN_d$  matrix of domestic returns and the  $TxN_f$  matrix of foreign returns into a single TxN matrix<sup>20</sup>

$$\mathbf{R} = [R_1^d, ..., R_{N_d}^d, R_1^f, ..., R_{N_f}^f]^T$$

A test of the null hypothesis that the set of domestic assets,  $\mathbf{R}^d$ , spans the set of foreign assets,  $\mathbf{R}^f$ , amounts to a test of the null hypothesis that there exists two constants  $\alpha_1 \neq \alpha_2$  and two valid discount factors such that

$$\mathbf{m}_{1} = \alpha_{1} + \mathbf{R}^{d} \boldsymbol{\beta}_{1}$$

$$\mathbf{m}_{2} = \alpha_{2} + \mathbf{R}^{d} \boldsymbol{\beta}_{2},$$
(6)

$$E_t[m_1 \mathbf{R}] = \mathbf{1}$$

$$E_t[m_2 \mathbf{R}] = \mathbf{1},$$
(7)

 $<sup>^{20}</sup>$ We have suppressed the time subscript for ease of notation.

Let  $\boldsymbol{\theta} = [\alpha_1, \alpha_2, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2]$  and  $\alpha_1 \neq \alpha_2$ . The model is over-identified. That is, we have  $2N_d$  free parameters but  $2(N_d + N_f)$  assets to price. Let  $\mathbf{G}_T(\boldsymbol{\theta}) = [\mathbf{g}_T(\boldsymbol{\theta})'_1 \mathbf{g}_T(\boldsymbol{\theta})'_2]'$  denote the average mispricings that result when one uses the estimated discount factors to price assets

$$\mathbf{g}_{T}(\boldsymbol{\theta})_{1} = E_{T}[(\alpha_{1} + \mathbf{R}^{d}\boldsymbol{\beta}_{1})\mathbf{R} - \mathbf{1}] = \frac{1}{T}\sum_{t=1}^{T}[(\alpha_{1} + \mathbf{R}_{t}^{d}\boldsymbol{\beta}_{1})\mathbf{R}_{t} - \mathbf{1}]$$

$$\mathbf{g}_{T}(\boldsymbol{\theta})_{2} = E_{T}[(\alpha_{2} + \mathbf{R}^{d}\boldsymbol{\beta}_{2})\mathbf{R} - \mathbf{1}] = \frac{1}{T}\sum_{t=1}^{T}[(\alpha_{2} + \mathbf{R}_{t}^{d}\boldsymbol{\beta}_{2})\mathbf{R}_{t} - \mathbf{1}].$$
(8)

Given two values of  $\alpha_1, \alpha_2$ , the goal is to pick the free parameters  $\hat{\boldsymbol{\theta}} = [\hat{\boldsymbol{\beta}}_1, \hat{\boldsymbol{\beta}}_2]$  to minimize  $J(\hat{\boldsymbol{\theta}}) = \mathbf{G}_T(\hat{\boldsymbol{\theta}})' \mathbf{W} \mathbf{G}_T(\hat{\boldsymbol{\theta}})$  for a positive definite weighting matrix,  $\mathbf{W}$ .

The choice of  $\mathbf{W}$  has obvious ramifications for the parameter estimates. For example, if we set  $\mathbf{W} = \mathbf{I}$ , we will select  $\boldsymbol{\theta}$  to price assets equally well. That is, we will try to minimize an equally weighted average of each asset's pricing error. Unfortunately, we only observe an estimate of each asset's pricing error. Recognizing that we may be more confident in the measurement of some pricing errors than others, we should adjust the weighting matrix to pay more attention to the moments that are measured with confidence. Therefore, we use the following weighting matrix

$$W = \begin{bmatrix} S_1^{-1} 0\\ 0 & S_2^{-1} \end{bmatrix}$$

Where  $\mathbf{S}_n^{-1}$  is the inverse of the sample covariance matrix of the moment conditions  $g_T(\boldsymbol{\theta})_n$ .

The spanning test is based on the insight that if the foreign assets can be written as a linear combination of domestic assets, the discount factors that price the domestic assets will also assign the correctly price to the foreign assets. Under the null that  $m(\hat{\theta})_1$  and  $m(\hat{\theta})_2$  are valid discount factors, Hansen (1982) demonstrates that<sup>21</sup>

$$J_T = \mathbf{G}(\widehat{\boldsymbol{\theta}})' \{ cov[\mathbf{G}(\widehat{\boldsymbol{\theta}})] \}^{-1} \mathbf{G}(\widehat{\boldsymbol{\theta}}) \sim \chi^2_{(2N-k)}$$
(9)

Where 2N - k is the total number of moment conditions minus the number of free parameters in  $\theta$ .

#### 3.2.2 The Spanning Test with Short Restrictions

The mean-variance spanning tests above, although widely utilized for testing diversification benefits, may not be restrictive enough to capture the constraints faced by real-world investors. If we reject the null hypothesis that  $\mathbf{R}^d$  spans  $\mathbf{R}^f$ , this implies that British investors could have been made better-off by holding foreign assets. Specifically, by buying the assets with abnormally high returns and shorting the assets with

 $<sup>{}^{21}</sup>cov[\mathbf{G}(\widehat{\theta})]^{-1}$  is the pseudo-inverse of the 2N by 2N covariance matrix of moment conditions. See Cochrane (2001) p.210-12 for a discussion of the use of a pre-specified weighting matrix when estimating this model.

abnormally low returns, British investors could have expanded their mean-variance frontiers. Shorting assets is often costly, however, and in the aggregate we can not all be short.

Therefore, whenever we reject the null hypothesis of spanning, we should always ask if the apparent diversification benefits of international investing rely upon the ability to short assets and form highly leveraged portfolios. If the results rely upon short sales, we should question whether the apparent gains from diversification would still be available to investors who face transaction costs when forming leveraged portfolios.

We can measure the affect of market frictions by constraining investors to long positions. Although it was often easier to short stocks in the 19th century than it is today, for many investors short restrictions were a realistic constraint when choosing their optimal portfolios.

DeRoon, Nijman, and Werkers (2001) show us how to manipulate the set of domestic assets and moment conditions to derive tests for spanning in the face of short sale constraints. The moment condition  $P_t = E_t[m_{t+1}X_{t+1}]$  assumes that investors can buy and sell, (go long and short) to exploit any mispricings that might arise. If investors are short sale constrained, however,  $P_t$  may be greater than  $E_t[m_{t+1}X_{t+1}]$ , or in terms of returns,  $1 \ge E_t[m_{t+1}R_{t+1}]$ .

Imagine a set of investors who choose the optimal portfolio of domestic assets that maximizes their utility by solving the following problem subject to the constraint that they do not short any asset.

$$\max_{\mathbf{w}>\mathbf{0}} \mathbf{w}' E[\mathbf{R}_{t+1}^d] - \frac{1}{2} \gamma \mathbf{w}' \cos [\mathbf{R}_{t+1}^d] \mathbf{w} \text{ subject to } \mathbf{w}' \boldsymbol{\iota} = 1$$
(10)

 $\gamma$  is the coefficient of risk aversion.

By altering  $\gamma$ , we can trace the short sale constrained mean-variance efficient frontier and derive a test of the null hypothesis that  $\mathbf{R}_{t+1}^d$  spans  $\mathbf{R}_{t+1}^f$  even in the presence of short sale constraints.

Order the domestic assets from highest expected return to lowest expected return. Start with a risk neutral investor ( $\gamma = 0$ ) and solve (10) for her optimal portfolio. The risk neutral investor will choose the trivial solution where all her money is invested in the asset with the highest expected return. Let  $\mathbf{R}^{d1} = [R_1^d]$ denote the single asset that the risk neutral investor places her money in.

Next increase  $\gamma$  by a very small amount and solve (10) for a slightly higher coefficient of risk aversion. If the change in  $\gamma$  is sufficiently small, the investor will still choose to place all her money in  $\mathbf{R}^{d1}$ . If we keep increasing  $\gamma$ , however, eventually the investor will become sufficiently risk-averse that the benefits of diversification will outweigh the cost of lower returns and she will take some of her money out of the asset with the highest expected return and divide her money between this asset and another asset. Let  $\mathbf{R}^{d2} = [R_1^d]$  denotes the set of two domestic assets that the investor divides her money between and let  $\gamma_1$  denote the largest coefficient of risk aversion for which the investor only invests in  $\mathbf{R}^{d1}$ .

Continue to increase  $\gamma$  until the investor chooses to invest in assets other than  $\mathbf{R}^{d_2}$ . Let  $\mathbf{R}^{d_3} = [R_i^d \dots R_{i+j}^d]$ denote the new set of domestic assets the investor divides her money between and  $\gamma_2$  denote the largest coefficient of risk aversion for which the investor only invests in  $\mathbf{R}^{d2}$ . Continue to increase  $\gamma$  and keep track of each new set of assets until  $\gamma$  becomes sufficiently large that the investor chooses to place all of her money in the assets that make up the minimum variance portfolio. Let  $\mathbf{R}^{dJ} = [R_i^d \dots R_{i+j}^d]$  denote the set of assets that make up the minimum variance portfolio and  $\gamma_{\max}$  denote the largest coefficient of risk aversion for which the investor chooses to invest in  $\mathbf{R}^{dJ-1}$ . Once the investor has reached a level of risk aversion sufficiently large that she chooses the minimum variance portfolio, she will continue to hold this portfolio for all  $\gamma > \gamma_{\max}$ .

We now have J-subsets of domestic assets that correspond to the set of assets investors choose to invest in when they are short constrained. Arbitrarily pick the j-th subset  $\mathbf{R}^{dj}$  and note that the following maximization problems have the same solution for every  $\gamma \in (\gamma_{j-1}, \gamma_j)$ .

$$\max_{\mathbf{w}} E[\mathbf{R}^{dj}]\mathbf{w} - \frac{1}{2}\gamma \mathbf{w}' cov[\mathbf{R}^{dj}]\mathbf{w} \text{ subject to } \mathbf{w}'\boldsymbol{\iota} = 1$$
(11)

$$\max_{\mathbf{w}>\mathbf{0}} \mathbf{w}' E[\mathbf{R}_{t+1}^d] - \frac{1}{2} \gamma \mathbf{w}' cov[\mathbf{R}_{t+1}^d] \mathbf{w} \text{ subject to } \mathbf{w}' \boldsymbol{\iota} = 1,$$

Therefore, the short-sale constrained mean-variance frontier of  $\mathbf{R}_{t+1}^d$  is equal to the unconstrained meanvariance frontier of  $\mathbf{R}^{dj}$  for every optimal portfolio corresponding to a coefficient of risk aversion between  $\gamma \in (\gamma_{j-1}, \gamma_j)$ .

If we solve (10) for a given coefficient of risk aversion, the Kuhn-Tucker multiplier for the restriction that  $w'\iota = 1$  is equal to  $\eta = \frac{1}{v}$ , were v is to the mean of the stochastic discount factor that prices  $\mathbf{R}_{t+1}^{dj}$  by construction

$$m_{R}(v)_{t+1} = v + \lambda^{(\nu)\prime} (\mathbf{R}_{t+1}^{dj} - E[\mathbf{R}_{t+1}^{dj}])$$

$$\lambda^{(\nu)} = cov[\mathbf{R}_{t+1}^{dj}]^{-1} (\iota - \nu E[\mathbf{R}_{t+1}^{dj}])$$
(12)

Again, define  $\mathbf{R}^{f}$  and  $\mathbf{R}^{d}$  as our foreign and domestic assets respectively.  $\mathbf{R}^{d}$  spans  $\mathbf{R}^{f}$  subject to short sale constraints if, for all values of v, there exists an  $m_{R}(v)_{t+1}$  that correctly prices  $\mathbf{R}^{f}$ 

$$E[m_R(v)_{t+1}\mathbf{R}_{t+1}^f] \quad \boldsymbol{\iota}_N, \tag{13}$$

How do we check "all values of v"? As we alter  $\gamma$  between 0 and  $\gamma^{\max}$ , we identify the J subsets of  $\mathbf{R}^d$ and 2J values of v that correspond to each subset of assets  $\mathbf{R}^{d1}...\mathbf{R}^{dJ}$ .

Recall that in the spanning test without short sales restrictions we used two values of  $\alpha$  to test for two intersections on the mean variance frontier formed by the domestic assets. Given the subset  $\mathbf{R}^{dj}$ , a test of the null hypothesis that  $\mathbf{R}^{dj}$  spans the foreign assets  $\mathbf{R}^{f}$  again amounts to a test of the null hypothesis that there exist two constants  $v_1 \neq v_2$  and two valid discount factors such that

$$m_{R1}(v_1)_{t+1} = v_1 + (\mathbf{R}_{t+1}^{dj} - E[\mathbf{R}_{t+1}^{dj}])\boldsymbol{\beta}_1$$

$$m_{R2}(v_2)_{t+1} = v_2 + (\mathbf{R}_{t+1}^{dj} - E[\mathbf{R}_{t+1}^{dj}])\boldsymbol{\beta}_2$$
(14)

$$E[m_R(v_1)_{t+1}\mathbf{R}_{t+1}^f] \qquad \boldsymbol{\iota}_N \tag{15}$$
$$E[m_R(v_2)_{t+1}\mathbf{R}_{t+1}^f] \qquad \boldsymbol{\iota}_N$$

Once again let  $\mathbf{R} = [\mathbf{R}^{dj}, R_1^f, \dots R_{N_f}^f]'$  denote the set of domestic and foreign assets we wish to price. The average mispricings that results when one uses the restricted discount factors to price assets are

$$\mathbf{g}(\boldsymbol{\theta})_j = \begin{bmatrix} E_T[m_R(v_1)\mathbf{R}] - \mathbf{1} \\ E_T[m_R(v_2)\mathbf{R}] - \mathbf{1} \end{bmatrix}$$

We estimate and evaluate each candidate  $m_R(v)_{t+1}$  by choosing  $\boldsymbol{\theta} = [\boldsymbol{\beta}_1 \boldsymbol{\beta}_2]$  to minimize

$$\xi(\boldsymbol{\theta})_{j} = \min[\mathbf{g}(\boldsymbol{\theta})_{j}]' W[\mathbf{g}(\boldsymbol{\theta})_{j}]$$
(16)

To evaluate the null hypothesis that  $\mathbf{R}^d$  spans  $\mathbf{R}^f$ , estimate two candidate discount factors for each subset of assets  $\mathbf{R}^{d_1}...\mathbf{R}^{d_J}$ . The result is 2*J* candidate discount factors and *J* vectors of pricing errors. Stack the vectors of pricing errors  $\mathbf{G}(\boldsymbol{\theta})_T = [\mathbf{g}(\boldsymbol{\theta})_1...\mathbf{g}(\boldsymbol{\theta})_J]'$ . If the investor is able to short foreign assets, then under the null hypothesis that the short-sale constrained domestic assets span foreign assets

$$\Upsilon_T = [\mathbf{G}(\boldsymbol{\theta})_T]' \{ \cos[\mathbf{G}(\boldsymbol{\theta})_T] \}^{-1} [\mathbf{G}(\boldsymbol{\theta})_T] \sim \chi^2_{(\#assets - \#free \ parameters)}$$

If the investor is unable to short foreign assets, set the negative elements of  $\mathbf{G}(\boldsymbol{\theta})_T$  to zero and under the null hypothesis that  $R^d$  spans  $R^f$ ,  $\Upsilon_T$  is distributed as a weighted chi-squared statistic

$$\Pr(\Upsilon_T > c) = \sum_{i=1}^{N} \Pr\{\chi_i^2 \ge c\} \mathbf{w}(N, i, Var[\mathbf{g}_T(v)])$$

where  $\mathbf{w}(N, i, Var[u_t(v)])$  is a probability weight equal to the probability that N-i of the N elements of a vector distributed  $N(\mathbf{0}, cov[\mathbf{G}(\boldsymbol{\theta})_T])$  are strictly negative<sup>22</sup>.

 $<sup>^{22}</sup>$ See Kodde and Palm (1986) and DeRoon, et al, (2001).

### **3.3** Measuring the Utility Gains from International Diversification

The spanning tests above suffer from the well known problem of statistical versus economic significance. The spanning tests asks a simple question: If domestic assets span foreign assets, what is the probability of observing the given mispricing  $\mathbf{g}_T(\hat{\theta})$ ? If we fail to reject the null of spanning, this implies that British investors were made better-off by holding foreign assets By how much did the introduction of foreign assets increase the welfare of British investors? The spanning test offers no guidance. To give the shift in ex-post frontiers an economic interpretation we require a measure of the welfare gains of diversification. To this end, we calculate the gains in lifetime utility associated with expanding the portfolio of assets available to British investors.

We follow Cole and Obstfeld (1991), Lewis (1996), and Rowland and Tesar  $(1998)^{23}$  and measure the utility gain associated with international diversification as the percentage reduction in permanent consumption that makes an individual indifferent between the optimal portfolio when the investor can hold foreign and domestic assets and the optimal portfolio when the investor is restricted to holding domestic assets only.

Let  $C_t$  denote permanent consumption at time t of an investor who holds the optimal portfolio of domestic assets. Let  $C_t^*$  denote permanent consumption at time t of an investor who holds the optimal portfolio of domestic and foreign assets. Define  $\Phi$  as the utility gain given by the relationship

$$U_0(C_0) = U_0\{C_0^*(1-\Phi)\}$$

Following Lewis (1996), we evaluate the utility gain for an investor with an Epstein-Zin-Weil expected utility function<sup>24</sup>

$$U_t = \left[C_t^{(1-\theta)} + \beta \left[E_t(U_{t+1}^{1-\gamma})\right]^{\frac{(1-\theta)}{(1-\gamma)}}\right]^{\frac{1}{(1-\theta)}}$$
for  $\gamma, \theta > 0; \gamma, \theta \neq 1$ 

$$(17)$$

Where  $\gamma$  and  $\theta$  are the coefficients of risk aversion and inverse of the elasticity of intertemporal substitution respectively.  $\beta$  is the discount rate which we set equal to .99. Both foreign and domestic asset returns are assumed to be jointly log normally distributed. The expected utility of consumption for an investor who holds the optimal domestic portfolio may be written as

$$E_t U(C_t) = W_t \{ 1 - \beta \exp[(1 - \theta)(\mu_D - \frac{1}{2}\gamma\sigma_D^2)] \}^{\frac{-1}{(1 - \theta)}}$$
(18)

Where  $W_t$  is wealth at time t and assumed to be exogenous.  $\mu_D$  and  $\sigma_D^2$  are the expected return and variance of the domestic portfolio. Likewise, the expected utility of the investor who is holding the optimal

 $<sup>^{23}</sup>$ Much of what following discription can be found in Section 3 of Rowland and Tesar (1998).

<sup>&</sup>lt;sup>24</sup>It is customary to use the Epstein-Zin-Weil utility function because this specification allow the risk-aversion parameter,  $\gamma$ , to differ from the inverse of the elasticity of intertemporal substitution parameter,  $\theta$ .

portfolio of foreign and domestic assets may be written as

$$E_t U(C_t) = W_t \{ 1 - \beta \exp[(1 - \theta)(\mu_{DF} - \frac{1}{2}\gamma\sigma_{DF}^2)] \}^{\frac{-1}{(1 - \theta)}}$$
(19)

 $\mu_{DF}$  and  $\sigma_{DF}^2$  are the expected return and variance of the optimal portfolio formed with domestic and foreign stocks.

Given a set of domestic and foreign assets,  $\mathbf{R} = [R_1^d, ..., R_{N_d}^d, R_1^f, ..., R_{N_f}^f]$ , with expected return vector  $\boldsymbol{\mu}$ and covariance matrix  $\Sigma$ , the utility gain from diversification can be computed by choosing a portfolio to maximizing (17) and comparing the resulting utility to the maximum utility possible when the investor is constrained to hold domestic assets alone<sup>25</sup>.

The utility gain from expanding the choice set to include foreign stocks is

$$\Phi = 1 - \left\{ \frac{1 - \beta \exp[(1 - \theta)(\mu_D^* - \frac{1}{2}\gamma\sigma_D^{*2})]}{1 - \beta \exp[(1 - \theta)(\mu_{DF}^* - \frac{1}{2}\gamma\sigma_{DF}^{*2})]} \right\}^{\frac{1}{(1 - \theta)}}$$
(20)

The diversification benefits in (20) depend upon the investor's risk aversion and elasticity of intertemporal substitution. The is no consensus about the true magnitude of risk aversion and intertemporal substitution, however. As a result we report the value of  $\Phi$  for a range of risk aversion and intertemporal substitution parameters.

## 4 Results

### 4.1 Spanning Tests

We group assets into four nested sets, where each set corresponds to a differing levels of international diversification. The first set, which we call benchmark 1, contains only British domestic assets. These include the British Government bond portfolio, the British corporate bond portfolio, and the British corporate stock portfolio. We evaluate the ability of these three portfolios to span the foreign portfolios. This is equivalent to asking if a British investor who held domestic assets could expand her mean-variance frontier by adding the foreign portfolios.

The second set of assets, which we call benchmark 2, consists of all the domestic portfolios contained in the first benchmark plus the foreign government bond portfolio. When we evaluate the hypothesis that the assets in benchmark 2 span the remaining foreign assets, we are asking if an investor who has already

$$\mu_p^* = \{(cb - a^2)/\gamma c\} - (a/c)$$

<sup>&</sup>lt;sup>25</sup>The optimal weights are  $w^* = g + h\mu_p^*$  with  $h = (1/d) * [c(\Sigma^{-1}\mu) - a(\Sigma^{-1}\iota)]$  and  $g = (1/d) * [b(\Sigma^{-1}\iota) - a(\Sigma^{-1}\mu)]$ 

 $a=\iota\Sigma^{-1}\mu$ 

 $b=\mu\Sigma^{-1}\mu$ 

 $c=\iota\Sigma^{-1}\iota$ 

 $d=bc-a^2$ 

diversified her portfolio between domestic assets and foreign government bonds was able to further expand the mean-variance frontier by adding the remaining foreign assets.

Benchmark 3 contains all the portfolios in benchmark 2 plus U.S. stocks and bonds. Again, a test of the hypothesis that the assets in benchmark 3 span the remaining assets is equivalent to a test of the hypothesis that once an investor had diversified her portfolio by holding domestic assets, foreign government bonds and U.S. assets the addition of further assets had no effect on the mean and variance of her optimal portfolio.

Finally, benchmark 4 consists of all the portfolios in benchmark 2 plus the non-U.S. foreign stocks. A test of the hypothesis that the assets in benchmark 4 span the U.S. assets is equivalent to asking if a British investor who held all assets except U.S. assets could expand her mean-variance frontier by adding U.S. assets to her portfolio.

#### Benchmark 1: Did British Securities Span International Assets?

The results of the spanning tests can be found in Table II. We reject the null hypothesis that British assets spanned the foreign government bond portfolio. It appears that the addition of foreign bonds significantly expanded the mean-variance frontier of British investors. Foreign government bonds were by far the most popular foreign investment among Victorian-era British investors<sup>26</sup>. When one considers the high expected returns and low correlation between foreign government bonds and the British domestic assets, foreign government bonds appear to be a particularly wise investment. Indeed, given the diversification benefits apparent in Figure I and Table II, the British appetite for foreign government bonds is easy to understand.

We reject the hypothesis that the portfolios in benchmark 1 span the non-U.S. foreign stocks. However, we fail to reject the hypothesis that the portfolios in benchmark 1 span the portfolio of all foreign stocks (U.S. plus non-U.S.) or the portfolio of U.S. stocks alone. The gains from adding U.S. stocks to the assets in benchmark 1 are either too small, or the data too noisy, to reject the null hypothesis of spanning.

As long as British investors were able to short assets, the spanning tests soundly reject the hypothesis that British domestic portfolios spanned portfolios comprised of foreign bonds (both U.S. and non-U.S. foreign bonds). When we restrict investors to portfolios with positive weights, however, we are unable to reject the null hypothesis that the portfolios in benchmark 1 span the U.S. and foreign bonds. This is a suspicious result. If investors could not replicate the expected return on U.S. bonds with no constraints on their domestic portfolio weights, then obviously they could not replicate U.S. bond returns with a portfolio constrained to have positive weights. That said, there is nothing to assure that the p-values of the short-restricted spanning tests will be smaller than the p-value of the same spanning test without short restrictions. These are different tests with different moment conditions and weighting matrices. One can not compare p-values in table II and conclude that one candidate discount factor does a "better" job of pricing assets than another. In fact,

<sup>&</sup>lt;sup>26</sup>Temin (1987) provides an alternative explanation of demand for foreign government bonds.

if we price assets with the candidate discount factors estimated with short constraints (the discount factor we do not reject), the resulting pricing errors are larger than the mispricings that result when we use the candidate discount factor estimated without short constraints (the discount factor we reject). However, due to the high volatility of the constrained candidate discount factor, we can not reject the null hypothesis that the pricing errors are in fact mean zero. This is a problem of low power that is common when the number of over-identifying conditions is small relative to the total number of free parameters. Luckily, the problem of low power is less severe when we evaluate the ability of a candidate discount factor to simultaneously price multiple foreign assets.

In Table II, test assets a through h are single portfolios. When we evaluate the null hypothesis that the assets in benchmark 1 span these test assets, we are evaluating the effect of the addition of one portfolio to the set of assets available to British investors. In practice, however, when a British investors decided to invest in foreign assets, she had the option of simultaneously investing in a number of different assets and regions.

Tests assets i through l are sets of assets corresponding to different geographic locations or asset types  $(\text{debt or equity})^{27}$ . When we evaluate the null hypothesis that the assets in benchmark 1 span these test assets, we are evaluating the affect of adding multiple portfolios to the set of assets available to British investors.

With two exceptions, we always reject the null hypotheses that the assets in benchmark 1 simultaneously spanned the set of foreign assets, i through 1. Our inability to reject the hypothesis that benchmark 1 spanned k with short restrictions combined with our ability to reject the same hypothesis without short restrictions appears to be another case of too little power.

The second, and more notable exception, is our inability to reject the hypothesis that, when short sales were possible, the assets in benchmark 1 spanned the set of assets in j. This is notable because test assets j include all foreign stocks but no foreign bonds. One explanation for British foreign investment was that British investors preferred debt to equity and therefore sough out foreign government and U.S. railroad bonds<sup>28</sup>. The fact that we can not reject the hypothesis that domestic assets spanned foreign stocks lends some credence to the theory, as British investors had little incentive to seek out equity investments once they had a well diversified domestic portfolio. When we rule out short sales, we do, however, reject the hypothesis that benchmark 1 spanned the set of assets in j.

The fact that we reject the null of spanning in the vast majority of cases involving test assets i through l is strong evidence that British domestic assets did not do a good job of spanning foreign assets. In most cases, the addition of foreign assets appears to have significantly expanded the mean-variance frontier of

 $<sup>^{27}</sup>$ Some of the portfolios in a thru h are nested. For example, the assets in b are a subset of the assets in e.

 $<sup>^{28}</sup>$ See Kennedy (1987)

British investors.

#### Benchmark 2: Was There a Need To Add More Than Foreign Government Bonds?

The second benchmark contains the first benchmark portfolios plus the foreign government bond portfolio. A test of the null hypothesis that the assets in the set of second benchmark portfolios spanned the remaining assets is equivalent to asking if a British investor who held domestic assets and foreign government bonds could have expanded her mean-variance frontier by adding other portfolios.

When we rerun the spanning tests with Benchmark 2 instead of Benchmark 1, we again fail to reject the null of spanning for the individual foreign portfolios that do not include U.S. bonds. Once British investors had added foreign government bonds to their domestic assets, the addition of equity or non-U.S. foreign debt added little diversification value.

When we evaluate the ability of the assets in benchmark 2 to span j and k, we reject spanning for the case of no short sales. We do not reject the null hypothesis that foreign equity was redundant when British investors were able to short assets, however. The addition of U.S. corporate debt did expand the mean-variance frontier of British investors, even after they had added foreign government bonds to their portfolios.

#### Benchmark 3: Investing in Foreign Government Bonds and the United States

British investors could still expand their mean-variance frontiers, even after they had added Foreign Government Bonds and the U.S. securities. the spanning tests soundly reject the hypothesis that the assets in benchmark 3 span the non-U.S. foreign equity.

#### Benchmark 4: Investing in Foreign Nations Other Than United States

An investor who invested in all the assets in benchmark 4 had divided her money between every region and class of asset except the United States. Once the investor had diversified so thoroughly, would the addition of U.S. assets have added any diversification benefits? The answer appears to be yes. Once again, the addition of U.S. bonds appears to significantly expand the mean-variance frontier. The return to U.S. equity, on the other hand, is too similar to the other assets in Benchmark 4 to reject the hypothesis of spanning in all but the special case where the investors can not short domestic assets but can short U.S. assets.

#### Benchmarks 5-7: Debt Versus Equity

One explanation for British investment abroad is that British investors preferred debt to equity and were biased towards foreign government and U.S. railroad debt to the detriment of domestic debt and equity. We evaluate this hypothesis by forming 3 nested sets of debt assets, benchmarks 5-7, and evaluating the ability of these assets to span equity and debt portfolios. Again, except for the special case of being able to short foreign but not domestic assets, we can not reject the null hypothesis that once a British investor had purchased domestic government and corporate bonds, the addition of equity added little diversification benefit while the addition of U.S. bonds added significant diversification benefits.

#### Table III: Debt versus Equity

As a whole, the results from the tests in Table II suggest that the diversification benefits from investing in foreign bonds far outweighed the benefits from investing in foreign stocks. Table III reports the results from tests of the hypotheses that a value-weighted portfolio of all equities span bond portfolios and the hypotheses that the bond portfolios span equities.

In the case where investors were unable to short, we reject the hypothesis that British investors could have exhausted all diversification opportunities by holding all of their wealth in bonds or all of their wealth in stocks. In every case, we reject the hypothesis that the equity portfolio spans bonds or that the bond portfolios span equities.

When investors can short, however, we find that the addition of equities does not significantly expand the mean-variance frontier once investors have purchased British domestic bonds and either foreign government bonds or U.S. corporate bonds.

#### Table IV: Did the Empire Matter?

So far, we have treated all non-U.S. foreign investments the same. However, Victorian British investors had the opportunity to invest abroad without having to risk their capital in a land beyond British rule. Great Britain's vast 19th Century empire provided the London investor with ample opportunity to diversify her holdings and invest in the high return infrastructure projects of the developing world. Was the British empire so vast that it provided British investors with the ability to diversify their holdings under the relative safety of British legal protections? Or, was there something unique about investment in the United States, Latin America and elsewhere that could not be replicated with empire investments? To answer these questions, we sort all assets into value-weighted British, empire and non-empire stock and bond portfolios.

Table IV reports the results of the test that investing in the empire expanded the mean-variance frontier of British investors who held domestic assets, as well as, the test that even after these investors had diversified their portfolios with empire investments, the non-empire securities still added diversification value. We reject the hypothesis of spanning in every case. Investing in the empire did provide diversification benefits, but the empire assets we not enough to exhaust the diversification value of U.S. and other non-empire investments.

### 4.2 Utility Gains

What about the welfare benefits of international diversification? Figure 4 contains 10 graphs of the permanent consumption gain required to make an investor indifferent between investing in the optimal combination of benchmark portfolios and the optimal combination of benchmark plus other portfolios. Recall that our estimate of the consumption gain relies upon knowledge of investor's risk aversion and intertemporal marginal rate of substitution. We do not want to take a stand on the risk aversion and intertemporal marginal rate of substitutions of 19th century investors. We therefore estimate  $\Phi$  for a range of I.E.S between 1.5 and 3 and risk aversion coefficients between 1 and 30.

In general, the utility measures correspond closely with the results from the spanning tests. The spanning tests soundly rejected the hypothesis that British domestic assets spanned foreign government bonds. The utility measures imply that, conditional on the level of risk aversion and intertemporal substitution, the addition of foreign government bonds to a British investor's portfolio was equivalent to a 4-8% increase in lifetime consumption! Furthermore, this gain was not dependent on British investor's ability to short assets. To put this in perspective, Lucus's (1987) famous estimate of the cost of business cycle fluctuations is 5-10 times smaller than our estimate of the gain British investor's enjoyed from adding foreign government bonds were so popular with British investors?

The increase in utility associated with the addition of foreign corporate stocks and bonds to a British investor's choice set ranged from 4-8% or .75-5% of lifetime consumption, depending on the investor's ability to short assets. Likewise, the consumption gain associated with adding government securities to a corporate stock and bond portfolio ranged from 10-100% with short sales and 0-100% without.

The utility measures also highlight some of the differences between economic and statistical significance. Despite our ability to soundly reject the null hypothesis that domestic plus empire securities spanned non-empire securities, the consumption gains from diversifying beyond the empire were small. Once British investors diversified between domestic and empire securities, the gains from adding non-empire securities were a measly 0-.25% of lifetime consumption. If the investor was able to short, however, then the gains to investing both within the empire and outside the empire ranged from 5-60% of permanent consumption depending on risk aversion<sup>29</sup>.

<sup>&</sup>lt;sup>29</sup>This incredibly high level of consumption gain is the result of a near arbitrage between U.S. and British corporate bonds. U.S. railroad bonds had a 28-day average return of 44 basis points, while British corporate bonds had an average return of 28 basis points. British and U.S. corporate bonds had a correlation of .87, however. By shorting the low yielding British corporate bonds and buying the highly correlated but higher yielding U.S. bonds, a British investor could form a low risk, high return portfolio.

## 5 Conclusion

By insisting on an "either or" evaluation of foreign investment, the proponents of 19th century market failure have argued that foreign investment, combined with any evidence of domestic returns commensurate with the return to overseas investment, must be the result of bias. When one considers the low correlation between domestic and foreign investments, however, it becomes obvious that the test of market failure is far more stringent. Before we can deem Victorian investors irrational, we must not only show that the domestic assets had a higher return than their foreign counterparts, but we must also show that it was possible to form a domestic portfolio with the low variance of an internationally diversified portfolio.

What about the claim that British investors and the British economy could have done better by investing at home? These counter-factual investments never occurred, so we can not evaluate their benefits directly. Nonetheless, given the assets that did exist, we can reject the claim that British Victorian-era investors were acting irrationally when they purchased foreign assets. To the contrary, the consumption gains associated with the purchase of foreign assets was between 4-8% of permanent consumption.

Before one can argue that foreign investment is evidence of irrational bias on the part of investors, one must explain exactly how domestic investors would be better off remaining at home. We can ask ourselves whether it was likely that the forgone investment in British industry that did not occur could have yielded the same returns while *simultaneously* providing the diversification benefits that international investment did. Claims that forgone domestic investments would have yielded comparable (or even excess) returns are not sufficient. Before one can argue that a rational Victorian should have invested exclusively at home, one must explain why a rational investor would willingly forgo the diversification benefits of international assets. This is quite a challenge. Think about the assumptions one must make regarding domestic and foreign returns. It is not enough that the foregone investments in British industry return a higher profit than their international counterparts. It is not even enough that these investments return a higher profit with less variance. Before one can argue that British investors would have been better off with domestic assets alone, one must argue that the observed domestic investments and the foregone domestic investments together spanned the actual foreign investments. Only then, would a rational investor not seek to diversify overseas.

Why did British investors send so much of their capital overseas? Because that's where the returns were. The benefits of overseas investment were not limited to competitive returns, however. The real benefit of international investing was the diversification benefit of holding foreign assets with a low correlation to their domestic counterparts. Foreign government bonds, with there low correlations and relatively high returns provided just such a benefit. By sending a portion of their capital abroad, Victorian investors were able to increase their returns while simultaneously decreasing the riskiness of their investments.

Given the diversification benefits of international investments (especially foreign government bonds), it

is no surprise that London investors' sent capital overseas. The preponderance of evidence suggests that Victorians did not invest overseas due to bias or ignorance. Instead, Victorians sent capital overseas in search of both the high returns and the diversification that rational investors crave.

## 6 References

Bekaert, Geert and Michael S. Urias (1996) "Diversification, Integration and Emerging Market Closed-End Funds." NBER Working Paper 4990.

Cairncross, Sir Alec (1953) Home and foreign investment, 1870-1913; studies in capital accumulation. Cambridge: Cambridge University Press.

**Chancellor, Edward** (2000) Devil Take the Hindmost: A History of Financial Speculation. New York: Penguin Books.

Cochrane, John H. (2001) Asset Pricing. Princeton: Princeton University Press.

Cole, H. L., and M. Obstfeld (1991): "Commodity Trade and International Risk Sharing: How Much Do Financial Markets Matter?," *Journal of Monetary Economics*, 28, 3–24

Crafts, N.F.R (1979) "Victorian Britain Did Fail." Economic-History-Review 32(4) 533-37.

**De Santis, Giorgio** (1993) "Volatility Bounds for Stochastic Discount Factors: Tests and Implications from International Financial Markets." Dissertation, University of Chicago.

**DeRoon, Frans A. and Theo Nijman** (2001) "Testing for mean-variance spanning: a survey." Journal of Empirical Finance 8, 111-155.

**DeRoon, Frans A., Theo Nijman and B. Werke**r (2001) "Testing for MV-Spanning with Short Sales Constraints and Transaction Costs: The Case of Emerging Markets." *Journal of Finance* 56, 723-744

Edelstein, Micheal (1981) "Foreign Investment and Empire 1860-1914" in Floud and McCloskey, *The Economic History of Britain Since 1700*, Vol 2: 1860 to the 1970s, Cambridge: Cambridge University Press.

Edelstein, Micheal (1982) Overseas investment in the age of high imperialism : the United Kingdom, 1850-1914. New York : Columbia University Press.

Hansen, Lars P. (1982) "Large Sample Properties of Generalized Methods of Moments Estimators," *Econometrica* 50(4), 1029-1054.

Hansen, Lars P. and Ravi Jagannathan (1991) "Implications of security market data for models of dynamic economies." *Journal of Political Economy* 99, 225-262.

Hansen, Lars P. and Ravi Jagannathan (1995) "Econometric Evaluation of asset pricing models." *Review of Financial Studies* 8, 237-274.

Kennedy, William P. (1974) "Foreign Investment, trade, and growth in the United Kingdom, 1870-1913." *Explorations in Economic History* 11, 415-443. \_\_\_\_\_(1987) Industrial Structure, Capital Markets and the Origins of British Economic Decline, Cambridge University Press

Keynes, John M (1924) "Foreign Investment and National Advantage." Nation and Athenaeum, 584-7.
Lewis, Karen K. (1996) "Consumption, Stock Returns, and the Gains from International Risk-Sharing,"

NBER Working Papers 5410, National Bureau of Economic

Lucas, Robert E., Jr. (1987) Models of Business Cycles Basil-Blackwell Ltd., Oxford

McCloskey, Donald N. (1970) "Did Victorian Britain Fail?" *Economic-History-Review*, Second-Series; 23(3), 446-59.

(1979) "No It Did Not: A Reply to Crafts (in Comment)" The Economic History Review, New Series, Vol. 32, No. 4. (Nov., 1979), pp. 538-541

Michie, Ronald (1988) The London and New York Stock Exchanges, 1850-1914, Unwin Hyman Press. Morgan, Victor E. and W.A. Thomas (1962) *The Stock Exchange*. London: Elek Books.

O'Rourke and Williamson (1999) Globalization and History : The Evolution of a Nineteenth-Century Atlantic Economy Cambridge: MIT Press

**Ogaki, Masao** (1993) "Generalized Method of Moments: Econometric Applications." in *Handbook of Statistics Vol 11: Econometrics* Amsterdam: Vinod, 455-488.

Pollard, Sidney (1985) "Capital Exports, 1870-1914: Harmful or Beneficial?", *Economic History Review*, Vol. 38, No. 4. (Nov., 1985), pp. 489-514

\_\_\_\_\_ (1987) "Comment on Peter Temin's Comment (in Comments)", *Economic History Review*, Vol. 40, No. 3. (Aug., 1987), pp. 459-460.

Rowland, Patrick and Linda L. Tesar, "Multinationals and the Gains from International Diversification," NBER Working Paper.

**Rostow, W.W.** (1948) The British Economy of the Nineteenth Century. Oxford: Oxford University Press.

Temin, Peter (1987) "Capital Exports, 1870-1914: An Alternative Model" *Economic History Review*, Vol. 40, No. 3. (Aug., 1987), pp. 453-458.

\_\_\_\_\_(1989) "Capital Exports, 1870-1914: A Reply" *Economic History Review*, Vol. 42, No. 2. (May, 1989), pp. 265-266

Wilkins, M. (1989) The History of Foreign Investment in the United States to 1914. Cambridge: Harvard University Press.

# TABLE I

# 28-day Gross Returns, in pounds, Standard Deviations and Correlation Coefficients

# Value-weighted portfolios

Portfolio	All Assets	British Gov. Bonds	British Corp. Bonds	British Corp. Stocks	Foreign Gov. Bonds	Foreign Corp. Stocks Trading in London	Foreign Corp. Bonds Trading in London	All Foreign Corp Stocks	All Foreign Corp. Bonds	US Corp. Stocks	US Corp. Bonds	Non-US Foreign Corp. Stocks
Average 28-day Return	1.0034	1.003	1.0025	1.0028	1.0059	1.0033	1.0027	1.0031	1.0039	1.0033	1.0044	1.002
Standard Deviation	.0162	0.0106	0.0354	0.0364	0.0244	0.0449	0.0303	0.0413	0.0264	0.0483	0.0227	0.0339
				C	Correlation	Coefficients						
	All Assets	British Gov. Bonds	British Corp. Bonds	British Corp. Stocks	Foreign Gov. Bonds	Foreign Corp. Stocks Trading in London	cks Trading Bonds Trading Corp		All Foreign Corp. Bonds	US Corp. Stocks	US Corp. Bonds	Non-US Foreign Corp. Stocks
All Assets	1	0.4408	0.7586	0.7932	0.1745	0.7472	0.7852	0.8042	0.836	0.7287	0.8011	0.8045
British Gov. Bonds		1	0.1211	0.1559	0.1262	0.2185	0.1493	0.2119	0.1743	0.1821	0.1966	0.239
British Corp. Bonds			1	0.8882	-0.1545	0.5857	0.8026	0.5873	0.8657	0.4791	0.8725	0.83
British Corp. Stocks				1	-0.0526	0.6094	0.7753	0.5691	0.8048	0.4615	0.8032	0.8338
Foreign Gov. Bonds					1	0.0184	-0.0143	-0.0392	-0.1058	-0.0467	-0.1523	0.0478
Foreign Corp. Stocks (in Lon)						1	0.7019	0.8598	0.7111	0.8102	0.7064	0.7338
Foreign Corp. Bonds (in Lon)							1	0.6537	0.9539	0.5646	0.8705	0.8084
All Foreign Corp. Stocks								1	0.7234	0.9844	0.7307	0.6391
All Foreign Corp. Bonds									1	0.64	0.9457	0.8043
US Corp. Stocks										1	0.655	0.5083
US Corp. Bonds											1	0.7676
Non-US Foreign Corp. Stocks												1

#### TABLE II

#### **Spanning Tests**

#### Table Reports the P-value of the null hypothesis that Benchmark Spans Test Assets

Benchmark 1: British Gov. Bonds, British Corp. bonds, and British Corp. Stocks

Benchmark 2: Benchmark 1 plus Foreign Government Bonds

Benchmark 3: Benchmark 2 plus US Stocks and Bonds

Benchmark 4: Benchmark 2 plus Non-US For. Stocks

Benchmark 5: British Government Bonds and British Corporate Bonds

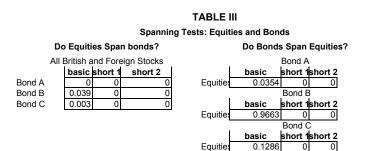
Benchmark 6: Benchmark 5 plus Foreign Government Bonds

Benchmark 7: Benchmark 6 plus Foreign Corporate Bonds

Basic Denotes a normal spanning test without short sales constraints short 1 denotes the spanning test when investors can not short domestic assets short 2 denotes the spanning test when investors can not short any asset

	Benchmark	1	E	Benchmark	2	E	Benchmark	3		Benchmark	4		Benchma	ırk 5		Benchma	ark 6		Benchma	ark 7	
<u>Test Assets</u> a. Foreign Government Bonds	basic 0	short 1 0	short 2 0	basic	short 1	short 2	basic	short 1	short 2	basic	short 1	short 2	basic 0	<b>short 1</b> 0	<b>short 2</b> 0	basic	short 1	short 2	basic	short 1	short 2
b. Foreign Corporate Bonds In London	0.7801	0.122	1E-04	0.8208	0	0.995							0.8498	0.0285	0.5998	0.8964	0	0.9952			
c. Foreign Stocks in London	0.7738	0.998	0.976	0.6907	0	0.965							0.7632	0.9656	0.7776	0.6136	0	0.9694	0.2072	0	0
d. All Foreign Bonds (b plus US bonds)	0.0002	0.853	0.834	0	0	0							0.0006	0	0	0	0	0			
e. All Foreign Stocks, including trading	ii 0.8851	0.999	0.952	0.8637	0	0.347							0.8702	0.9427	0.7756	0.8294	0	0.3469	0.3021	0	0
f. US Stocks	0.9124	0.998	0.906	0.9101	0	0.142				0.9	0	0.142	0.9044	0.9451	0.764	0.8976	0	0.1416	0.4149	0	0
g. US Bonds*	0.0013	0.619	0.68	0	0	0				0	0	0	0.0001	0	0	0	0	0			
h. Foreign Stocks not Including US	0.5511	0.002	0	0.3301	0	0	0.0003	0	0				0.575	0.2884	0.8032	0.3091	0	0	0.1634	0	0
i. Portfolio comprised of a, b, f, g*, and h	0	0	0										0	0	0						
j. Portfolio comprised of a, b, d, and e	0.0035	0.004	0										0.0633	0	0						
k. Portfolio comprised of f and h	0.8521	5E-04	0	0.6711	0	0							0	0	0	0.3091	0	0			
I. Portfolio comprised of f and g	0.00001	0.236	0.215	0	0	0							0	0	0	0.0005	0	0			
m. Portfolio comprised of a, d, and e	0	0	0										0	0	0						

\* g is the value-weighted portfolio of US bonds trading only in US



Bond A: British Corp. and Govt. Bonds, Foreign Bonds, and US bonds Bond B: British Corp. and Govt. Bonds, and US bonds Bond C: British Govt. Bonds and Foreign Govt. Bonds

#### TABLE IV

Spanning Test: Empire v. Non-Empire Do British Securities Span Empire?

basic short 1 short 2

0.0002 0 0

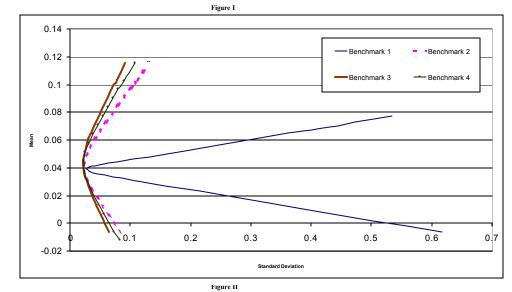
Do British and Empire Securities Span Rest of World?

basic	short 1	short 2				
0	0	0				

### Figures 1-3

#### Mean-Variance Frontiers

- Benchmark 1: British Gov. Bonds, British Corp. bonds, and British Corp. Stocks
- Benchmark 2: Benchmark 1 plus Foreign Government Bonds
- Benchmark 3:
- Benchmark 2 plus US Stocks and Bonds Benchmark 4: Benchmark 2 plus Non-US For. Stocks



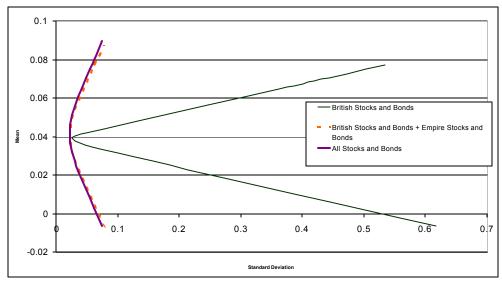


Figure III

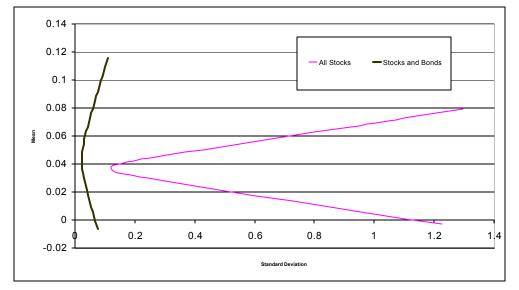


Figure 4 **Plots of Consumption Gains** 

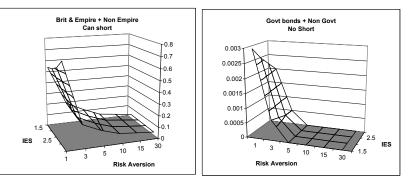
Benchmark1 + a Benchmark1+a Benchmark1+d+e Benchmark1+d+e Can not short Can short Can not short Can short 0.09--0.1 -0.06 0.09 0.08 0.08 0.08 0.07 0.05 .06 0.07 0.06 0.04 0.06 0.05 0.05 0.04 n 02 0.03 0.04 0.03-0.02 0.03 30 15 0.02-1.5 0.02 0.01 0.01 10 0.01 1.5 Λ Risk 25 2.5 5 IES IES **Risk Aversion** 2.5 1 Aversion 3 IES 1.5 2 2.5 3 3 3 5 10 15 10 5 - 3 15 30 30 **Risk Aversion** IES 1 **Risk Aversion** Private Securites + a+b Benchmark2+d+e Benchmark2 +d+e Private Securities + a+b Can short can short Can not short Can not short 0.16 0.03-0.9 0.14 0.025-0.8 0.12 07 0.02 0.1 0.6 0.5 0.08 0.015 0.5 0.4 0.06 n 4 0.01 0.3 0.3 0.04 02 n 2 0.005 0.02 IES <sup>2.5</sup> 25 2.5 2.5 IES IES IES 10 15 30 30 1 15 3 3 10 15 30 10 5 1.5 3 1.5 5 10 15 5 1.5 5 3

30

**Risk Aversion** 

Risk Aversion





1

**Risk Aversion** 

**Risk Aversion**