Housing Collateral, Consumption Insurance and Risk Premia

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Abstract

In a model with housing collateral, the ratio of housing wealth to human wealth shifts the conditional distribution of asset prices and consumption growth. A decrease in house prices reduces the collateral value of housing, increases household exposure to idiosyncratic risk, and increases the conditional market price of risk. Using aggregate data for the US, we find that a decrease in the ratio of housing wealth to human wealth predicts higher returns on stocks. *Conditional* on this ratio, the covariance of returns with aggregate risk factors explains up to eighty percent of the cross-sectional variation in annual size and book-to-market portfolio returns.

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1 Introduction

We introduce housing into Lucas' (1978) endowment economy. The households in this economy trade contingent claims to insure against labor income risk. These claims have to be fully backed by the value of their housing wealth. An increase in the value of housing wealth relative to human wealth, *the housing collateral ratio*, increases the scope for risk sharing and decreases the conditional dispersion of consumption growth across households. This endogenously lowers the conditional market price of aggregate risk. We show how this mechanism helps to explain some of the variation in US stock returns over time and across assets.

When the collateral constraints do not bind, our model collapses to the standard consumption-based capital-asset-pricing model of Lucas (1978) and Breeden (1979). That model prices only aggregate consumption growth risk. It has been rejected by the data (e.g. Hansen & Singleton (1983)). Our paper addresses two shortcomings of the consumption-based capital-asset-pricing model (CCAPM).

First, because US aggregate consumption growth is approximately i.i.d., the CCAPM implies a market price of risk that is approximately constant. However, in the data, stock market returns are predictable. This suggests that the market price of aggregate risk varies over time (e.g. Fama & French (1988), Campbell & Shiller (1988), Ferson, Kandel, & Stambaugh (1987), Whitelaw (1997), Lamont (1998), Lettau & Ludvigson (2003) and Campbell (2000) for an overview). Our model delivers time variation in the market price endogenously through the housing market.

Second, the covariance of asset returns with consumption growth explains only a small fraction of the variation in the cross-section of stock returns of firms sorted in portfolios according to size (market capitalization) and value (book-value to market-value ratio) characteristics (Fama & French (1992)). In response to this failure, Fama & French (1993) drop the connection between the stochastic discount factor and consumption growth and directly specify the stochastic discount factor as a linear function of the market return, the return on a small minus big firm portfolio, and a high minus low book-to-market firm portfolio. The empirical success of this three-factor model has motivated quite some more recent research on the underlying macroeconomic sources of risk for which their factors proxy (e.g. Bansal, Dittmar, & Lundblad (2002), Lettau & Ludvigson (2001b), Santos & Veronesi (2001) and Cochrane (2001) for an overview). Conditional on the housing collateral ratio, consumption growth betas can explain an important part of the variation in stock returns.

Our paper addresses these issues in the context of an endowment economy. We follow Alvarez & Jermann (2000) in relaxing the assumption that contracts are perfectly enforceable. As in Lustig (2001), we allow households to forget their debts. The new feature of our model is that each household owns part of the housing stock. Housing provides utility services and collateral services. When a household chooses to forget its debts, it loses all its housing wealth but its labor income is protected from creditors. The household is not excluded from trading. The lack of commitment gives rise to collateral constraints. Their tightness depends on the abundance of housing collateral. We measure this by the *housing collateral ratio*: the ratio of collateralizable housing wealth to non-collateralizable human wealth.

The collateral constraints are motivated by the empirical importance of housing as a collateral asset. In the US, two-thirds of households own their house. For the median-wealth homeowner, home equity represents seventy percent of household net worth (Survey of Consumer Finance, 1998). Residential real estate wealth accounts for twenty-eight percent of total household net worth and sixty-eight percent of non-financial assets, while home mortgages make up sixty-four percent of household liabilities (Flow of Funds, Federal Reserve, averages for 1952-2002). Currently, the value of residential wealth exceeds the total household stock market wealth ($13.6 trillion) and the mortgage market is the largest credit market in the US ($6.1 trillion).

Relative to the benchmark model with fully enforceable contracts, our theory adds a new component to the stochastic discount factor. The household's consumption share of the total endowment is governed by its Pareto-Negishi weight. This Pareto-Negishi weight increases whenever the household switches to a state with a binding constraint. The new component of the stochastic discount factor is the growth rate of one cross-sectional moment of the Pareto-Negishi weight distribution. We label it *the aggregate weight shock*. When a large fraction of households is constrained this growth rate is high. We call this a liquidity shock.

The housing collateral ratio changes the conditional moments of the aggregate weight shock. When the housing collateral ratio is low, households run into binding collateral constraints more frequently. The conditional standard deviation of the aggregate weight shock, in turn increasing the market price of risk. Thus, endogenous movements in the housing collateral ratio induce heteroskedasticity and counter-cyclicality in the Sharpe ratio. This collateral mechanism is a novel feature of the model.

The empirical strategy is to directly specify a stochastic process for the aggregate weight shock. The aim is to link the aggregate weight shock to the data on housing collateral. We achieve this in two steps. First, we fully calibrate and solve the model for an economy with a continuum of agents. The equilibrium Pareto-Negishi weight processes are functions of the primitives of the model: the preferences, the household endowment process, the aggregate endowment process and the aggregate rental price process. We obtain a quasi-recursive formulation and numerically solve for the equilibrium Pareto-Negishi weight processes. Agents forecast the aggregate wight shock to price aggregate risk. Second, we impose that this forecasting function be linear in the housing collateral ratio and Markov in the aggregate factors. In the model, this linear forecasting function performs very well. This linear factor structure for the weight process connects our model to the linear factor models in the empirical finance literature.

Our model delivers a conditional version of the CCAPM with the housing collateral ratio as the conditioning variable. The housing collateral ratio summarizes the investor's time-varying information set. The risk of binding collateral constraints is captured by the housing collateral ratio and the interaction terms of the housing collateral ratio and the aggregate sources of risk. With non-separable preferences over housing services and consumption, the aggregate risk factors are consumption growth and rental price growth.

Our theory has two main asset pricing predictions. First, households demand a larger compensation for a given amount of aggregate consumption risk in times when the housing collateral ratio is low. This implies that the housing collateral ratio predicts aggregate stock returns over time. Second, a particular asset earns a larger risk premium if its returns are more correlated with consumption growth when the housing collateral ratio is low.

We test these predictions using the following data. First, for the time-series predictability of returns, we use annual return data for the aggregate US stock market index. We measure the aggregate stock of housing collateral in three different ways: by the value of outstanding mortgages, by the value of residential real estate (structures and land) and by the value of residential fixed assets (structures). The housing collateral ratio is measured as the deviation from the cointegration relationship between the value of the aggregate housing stock and aggregate labor income. Second, for the cross-sectional exercise, we use twenty-five size and book-to-market portfolios, and the value-weighted market return.

Table 1 summarizes the predictions of our model and contrasts them with the predictions of the Breeden-Lucas model. The last column shows the data we use to test them. The ratio of collateralizable housing wealth to non-collateralizable human wealth is labelled my.

	Perfect risk-sharing Consumption-CAPM	Limited risk-sharing Collateral-CAPM	Data Period
Time-series Predictability	no return predictability constant price of risk	my predicts returns my-varying price of risk	Excess market return and my (1889-2001)
Pricing Portfolios	Covariance of returns with risk factors	Covariance conditional on my	Aggregate factors (1926-01)

Table 1: Predictions and Data for Empirical Exercises.

We find strong empirical support for each of the predictions. First, in the time series, the housing collateral ratio *does* predict stock returns, mainly at lower frequencies. Second, in the cross-section, our model explains between seventy and eighty percent of the variability in annual returns of the Fama-French portfolios. For annual returns, this matches the empirical success of the Fama & French (1993) three-factor model and recent conditional consumption-based asset pricing models (e.g. Lettau & Ludvigson (2001b)).

The failure of the CCAPM reflects its imposing of perfect consumption insurance. In the data, there is strong empirical evidence against full consumption insurance at different levels of aggregation: at the household level (e.g. Attanasio & Davis (1996) and Cochrane (1991b)), the regional level (e.g. Hess & Shin (1998)) and the international level (e.g. Backus, Kehoe, & Kydland (1992)). Blundell, Pistaferri, & Preston (2002) find evidence for a degree of consumption insurance that varies over time. Lustig & VanNieuwerburgh (2002) provide direct empirical support for the underlying time-variation in risk-sharing. Using a data set for US metropolitan areas, we reject full consumption insurance. The degree of partial insurance between regions decreases when the housing collateral ratio is low. It varies substantially over time. This time variation in risk-sharing is direct evidence for the mechanism that drives our model and leads to the asset pricing predictions discussed here.

The empirical evidence for the collateral effect is strong whether preferences over nondurable and housing consumption are separable or non-separable. In recent work, Piazzesi, Schneider, & Tuzel (2002) argue that non-separability is important for pricing assets. They consider a representative agent who consumes nondurables and housing services. If housing services and consumption are complements then households command a larger risk premium if returns and rental prices are positively correlated. They show that this *composition effect* increases the explanatory power of the standard consumption capital asset pricing model for stock and bond returns. Yogo (2003) makes a similar point.

We organize the paper as follows. In section 2, we briefly discuss other related literature. Section 3 describes the environment and characterizes efficient and equilibrium allocations. The fourth section discusses the results from a fully calibrated version of the model. Section 5 contains a discussion of our empirical strategy which bridges the gap between theory and data. Section 6 describes the data and section 7 shows how we measure the housing collateral ratio. Our empirical findings are summarized in section 8. Section 9 concludes. Appendix A contains details of the model, the computational method and the data. The most important figures and tables appear in the main text, all others in Appendix B.

2 Related Literature

Our paper is close in spirit to the work of Lettau & Ludvigson (2001b). As in their paper, we develop a scaled version of the CCAPM. Our state variable *my* summarizes information about future returns on housing *relative* to human capital. It does not contain any direct information on the future returns on stocks. In contrast, the scaling variable in Lettau & Ludvigson (2001b) is the consumption-wealth ratio, which summarizes household expectations about future returns on the entire market portfolio, including financial wealth.

Cochrane (1996) explores the explanatory power of residential and non-residential investment for equity returns in the context of his production-based asset pricing framework (Cochrane (1991a)). Li, Vassalou, & Xing (2002) find that investment growth, including household sector investment which is largely residential, can help account for a large fraction of the cross-sectional variation in equity returns. Similarly, Kullmann (2002) uses returns on residential and commercial real estate to improve the performance of the capital asset pricing model. Our model studies the role of housing in a consumption-based asset pricing model. As such it is silent on what distinguishes value firms from growth firms. However, we report empirical evidence that the dividend process of value firms is more sensitive to the housing collateral ratio than the dividend process for growth firms. When feeding in dividend processes with different sensitivity to the housing collateral ratio, the model is able to generate endogenously a value premium of the size observed in the data.

Our model contains two further important features. First, we model the outside option as bankruptcy with loss of all collateral assets. In Kehoe & Levine (1993), Krueger (2000), krueger &

Perri (2002), and Kehoe & Perri (2002) limited commitment is also the source of incomplete risk-sharing across US households and across countries respectively. In contrast, the outside option upon default is exclusion from future participation in financial markets. In our model, all promises are backed by all collateral assets. Geanakoplos & Zame (2000) think of individual assets collateralizing individual promises in an incomplete markets economy.

Second, our paper features a general equilibrium economy with aggregate uncertainty and endogenous house price fluctuations. In contrast, life-cycle and portfolio choice models such as Fernandez-Villaverde & Krueger (2001), Cocco (2000), Yao & Zhang (2002), Flavin & Yamashita (2002) posit an exogenous price process for housing.

3 Model

This section starts with a complete description of the environment in section 3.1. The next section, section 3.2, sets up the household problem in time zero trading environment. We provide a complete characterization of these allocations using stochastic Pareto-Negishi weight processes. We show that the growth rate of an aggregated Pareto-Negishi weight process drives the consumption growth of the off-corner households and these households price the random payoffs. Section 3.3 introduces sequential trading and discusses conditions under which these equilibria coincide with time zero trading equilibria.

3.1 Environment

We consider an endowment economy with a continuum of households on the unit interval. These households are infinitely-lived.

Uncertainty $s = (y, z)$ is an event that consists of a household specific component $y \in Y$ and an aggregate component $z \in Z$. These events take on values on a discrete grid $S = Y \times Z$. We use $s^t = (y^t, z^t)$ to denote the history of events. S^t denotes the set of possible histories up until time t. s follows a Markov process with transition probabilities π that obey:

$$\pi(z'|z) = \sum_{y' \in Y} \pi(y', z'|y, z) \; \forall z \in Z, y \in Y.$$

Because of the law of large numbers, $\pi_z(y)$ denotes both the fraction of households drawing y when the aggregate event is z and the probability that a given household is in state y when the aggregate state is z.

Preferences We use $\{x\}$ to denote an infinite stream $\left\{x_t(s^t)\right\}_{t=0}^{\infty}$. There are two types of commodities in this economy: a consumption good and housing services. The consumption good cannot be stored. We let $\{c(\theta_0, s_0)\}$ denote the stream of consumption and we let $\{h(\theta_0, s_0)\}$ denote the stream of housing services of a household of type (θ_0, s_0). The households rank consumption streams according to the criterion:

$$U\left(\{c\},\{h\}\right) = \sum_{s^t|s_0} \sum_{t=0}^{\infty} \delta^t \pi(s^t|s_0) u\left(c_t(\theta_0, s^t), h_t(\theta_0, s^t)\right), \tag{1}$$

where δ is the time discount factor. The households have power utility over a CES-composite consumption good:

$$u(c_t, h_t) = \frac{\left[c_t^{\sigma} + \psi h_t^{\sigma}\right]^{\frac{1-\gamma}{\sigma}}}{1-\gamma}.$$

$\psi > 0$ converts the housing stock into a service flow. The elasticity of substitution between c and h is $(1-\sigma)^{-1}$. Housing and non-durable consumption are complements if $1-\gamma-\sigma > 0$. Otherwise the two goods are substitutes.[1] We define $\phi = \left(\frac{1-\gamma-\sigma}{1-\sigma}\right)$.

Endowments The aggregate endowment of the non-durable consumption good is denoted $\{e\}$. The growth rate of the aggregate endowment depends only on the current aggregate state: $e_{t+1}(z^{t+1}) = \lambda(z_{t+1})e_t(z^t)$. Each of the households is endowed with a claim to a labor income stream $\{\eta\}$. The labor income share $\hat{\eta}(y_t, z_t)$, given by $\eta(y_t, z^t) = \hat{\eta}(y_t, z_t)e(z^t)$, only depends on the current state of nature. The aggregate endowment is the sum of the individual endowments:

$$\sum_{y' \in Y} \pi_z(y')\hat{\eta}_t(y', z) = 1, \ \forall z, t \geq 0.$$

The aggregate endowment of housing services is denoted $\{h^a\}$. We use r to denote the ratio of the aggregate housing stock to the non-durable endowment by r:

$$r_t(z^t) = \frac{h_t^a(z^t)}{e_t(z^t)}.$$

Trading We use $p_t(s^t|s_0)$ to denote the price of a unit non-durable consumption to be delivered in state s^t, in units of time zero consumption. $\rho_t(s^t)$ denotes the relative price of a unit of housing services. Finally, we let $\Pi_{s^t}\left[\{d\}\right]$ denote the price of claim to $\{d\}$ in units of s^t consumption, $\Pi_{s^t}\left[\{d\}\right] = \sum_{s^\tau|s^t} \sum_{\tau=0}^{\infty} \left[p_{t+\tau}\left(s^\tau|s^t\right) d_{t+\tau}\left(s^\tau|s^t\right)\right]$.

Markets open only at time zero. Households purchase a complete, state-contingent consumption plan $\{c(\theta_0, s_0), h(\theta_0, s_0)\}$ subject to a single, time zero budget constraint:

$$\Pi_{s_0}\left[\{c(\theta_0, s_0) + \rho h(\theta_0, s_0)\}\right] \leqslant \theta_0 + \Pi_{s_0}\left[\{\eta\}\right], \tag{2}$$

where θ_0 is the initial non-labor wealth. We use Θ_0 to denote the initial distribution of non-labor wealth holdings.

[1] The preferences belong to the class of homothetic power utility functions of Eichenbaum & Hansen (1990). Special cases are separability ($1-\gamma-\sigma = 0$) and Cobb-Douglas preferences ($\gamma, \sigma = 0$).

Solvency Constraints Households can forget their debts. When the household defaults, it keeps its labor income in all future periods. The household is not excluded from trading, even in the same period. To keep households from defaulting, they face a sequence of solvency constraints, one for each node s^t:

$$\Pi_{s^t}\left[\{c(\theta_0, s_0) + \rho h(\theta_0, s_0)\}\right] \geq \Pi_{s^t}\left[\{\eta\}\right]. \tag{3}$$

The solvency constraints keep the households from defaulting, but they are not too tight, in the sense of Alvarez & Jermann (2000). Lustig (2001) provides a formal derivation of this result. If households were to be excluded for a number of periods, the solvency constraints would loosen.

3.2 Equilibrium Prices and Allocations

We define an equilibrium in the spirit of Kehoe and Levine (1993).

Definition 1. *For given initial state z_0 and for given distribution Θ_0, an equilibrium consists of prices $\left\{p_t(s^t|s_0), \rho(z^t|z_0)\right\}$ and allocations $\left\{c_t(\theta_0, s^t), h_t(\theta_0, s^t)\right\}$ such that*

- *For given prices $\left\{p_t(s^t|s_0)\right\}$, the allocations solve the household's problem of maximizing (1) subject to (2) and (3) (except possibly on a set of measure zero).*
- *Markets clear for all t, z^t:*

$$\sum_{y^t} \int c_t(\theta_0, y^t, z^t) d\Theta_0 \frac{\pi(y^t, z^t|y_0, z_0)}{\pi(z^t|z_0)} = e_t(z_t). \tag{4}$$

$$\sum_{y^t} \int h_t(\theta_0, y^t, z^t) d\Theta_0 \frac{\pi(y^t, z^t|y_0, z_0)}{\pi(z^t|z_0)} = h_t^a(z_t) \tag{5}$$

To determine the equilibrium consumption of households, it is helpful to examine the dual of this household maximization problem. Let $U_0(\{c\}, \{h\})$ denote the total utility from consuming $\{c\}$ and $\{h\}$. For given prices $\{p, \rho\}$ a household with label (w_0, s_0) minimizes the cost $C(\cdot)$ of delivering initial utility w_0 to itself:

$$\begin{aligned} C(w_0, s_0) &= \min_{\{c,h\}} \left(c_0(w_0, s_0) + h_0(w_0, s_0)\rho_0(s_0)\right) \\ &\quad + \sum_{s^t} p(s^t|s_0) \left(c_t(w_0, s^t|s_0) + h_t(w_0, s^t|s_0)\rho_t(s^t|s_0)\right) \end{aligned}$$

subject to the initial promised utility constraint: $U_0(\{c\}, \{h\}) \geq w_0$, and the collateral constraints (3), one for each node s^t. The initial promised value w_0 is determined such that the household spends its entire initial wealth: $C(w_0, s_0) = \theta_0 + \Pi_{s_0}\left[\{\eta\}\right]$. There is a monotone relationship between θ_0 and w_0.

Pareto-Negishi Weights Let $\{\gamma(\theta_0, s_0)\}$ denote the sequence of multipliers on the solvency constraints imposed on household (θ_0, s_0). We define $\xi_t(\theta_0, s^t)$ to be household (θ_0, s_0)'s cumulative

Lagrange multiplier:

$$\xi_t(\theta_0, s^t) = \ell(\theta_0, s_0) + \sum_{\tau=0}^{t} \sum_{s^\tau \preceq s^t} \gamma_\tau(\theta_0, s^\tau).$$

We refer to $\xi_t(\theta_0, s^t)$ as the *Pareto-Negishi weight* in state s^t for household (θ_0, s_0). The initial weight $\ell(\theta_0, s_0)$ is the inverse of the Lagrange multiplier on the initial promised utility constraint, $\xi_0(s_0) = \ell$. $\{\xi(\theta_0, s_0)\}$ is a non-decreasing stochastic process. When a household solvency constraint binds, its weight increases to a cutoff level that depends only on s, the current event. If the constraint does not bind, its weight remains unchanged. This imputes limited memory to the allocations: a household's individual history is erased whenever it switches to a state with binding constraints.

Risk-sharing Rule There is a mapping from the multipliers at s^t to the equilibrium allocations of both commodities. We refer to this mapping as the risk-sharing rule. This rule flows from the optimality conditions of the dual household problem and the market clearing conditions. First, the solution to the dual household problem requires that for any pair of households (θ_0', θ_0''), at the same node s^t, their consumption satisfy:

$$\left(\frac{c_t(\theta_0', s^t)}{c_t(\theta_0'', s^t)}\right)^{-\gamma} = \frac{\xi_t(\theta_0'', s^t)}{\xi_t(\theta_0', s^t)}. \tag{6}$$

Second, all households equate the ratio of marginal utilities for these commodities:

$$\rho_t(z^t) = \psi \left(\frac{h_t(\theta_0, s^t)}{c_t(\theta_0, s^t)}\right)^{\sigma-1} = \psi \left(\frac{h_t^a(z^t)}{e_t(z^t)}\right)^{\sigma-1}.$$

The shadow state price of rental services is a function of the aggregate history z^t only.

Henceforth, we express individual-specific variables as functions of (ℓ, s^t) rather than (θ_0, s^t). We conjecture a linear risk sharing rule: the consumption share and the housing services share is a function of the household's own Pareto-Negishi weight and an aggregate sum of these weights:

$$c_t(\ell, s^t) = \frac{\xi_t(\ell, s^t)^{\frac{1}{\gamma}}}{\xi_t^a(z^t)} c_t^a(z^t) \text{ and } h_t(\ell, s^t) = \frac{\xi_t(\ell, s^t)^{\frac{1}{\gamma}}}{\xi_t^a(z^t)} h_t^a(z^t), \tag{7}$$

where $\xi_t^a(z^t)$ denotes the aggregate weight process $\int \xi_t(y^t, z^t)^{\frac{1}{\gamma}} d\Phi_t(z^t)$. Φ_0 is the initial distribution over $\ell(\theta_0, s_0)$, implied by the initial wealth distribution Θ_0. $\Phi_t(z^t)$ is the distribution over weights after aggregate history z^t. It is easy to verify that this rule satisfies (6) and the market clearing conditions.

When a household switches to a state with a binding constraint, its consumption share increases. Everywhere else, its consumption share is drifting downwards. Shocks to $\xi_t^a(z^t)$ reflect aggregate shocks to the wealth distribution, which can be interpreted as liquidity shocks.

The perfect commitment environment is an ideal laboratory for understanding this risk sharing rule. Because households are never constrained, the individual weight stays constant and is equal to the initial Pareto-Negishi weight: $\xi_t(s^t) = \xi_0(s_0) = \ell$. The aggregate weight process reflects the initial wealth distribution and is constant: $\xi^a(z_0) = \int \ell(y_0, z_0)^{\frac{1}{\gamma}} d\Phi_0(z_0)$.

Stochastic Discount Factor A household's intertemporal marginal rate of substitution (IMRS) can be determined directly off the risk sharing rule we have just derived. In each state, the payoffs are priced by the household with the highest IMRS. If not, there would be an arbitrage opportunity. The implied stochastic discount factor is

$$m_{t+1} = m^a_{t+1}(z^{t+1}) g^{\gamma}_{t+1}, \tag{8}$$

where m^a_{t+1} denotes the IMRS of an agent who consumes the aggregate non-durable and housing endowment. The stochastic discount factor consists of two parts: (1) the intertemporal marginal rate of substitution m^a_{t+1} of the representative agent[2] and (2) the growth rate g_{t+1} of the aggregate Pareto-Negishi weight process $\xi^a_{t+1}(z^{t+1})$. When many households are severely constrained in state z^{t+1}, that state's price increases, because the unconstrained households experience high marginal utility growth. When nobody is constrained, the Breeden-Lucas stochastic discount factor reemerges.

Optimal Weight Policy The optimal policy for the individual weights $\xi(\ell, s_0)$ is a to apply a cutoff rule $\ell^c(y_t, z^t)$. Consider a household starting period t with weight ξ_{t-1}. If its weight exceeds $\ell^c(y_t, z^t)$, its weight stays constant; if not, its increased to the cutoff level:

$$\begin{aligned} \xi_t &= \xi_{t-1} \text{ if } \xi_{t-1} > \ell^c(y_t, z^t), \\ \xi_t &= \ell^c(y_t, z^t) \text{ otherwise.} \end{aligned}$$

More housing collateral lowers these cutoff weights, allowing for more consumption smoothing. Conversely, a decrease in the supply of collateral brings the cutoff rules closer to their upper bound: the labor income shares. In the limit, as the collateral disappears altogether, the households revert to autarky. The following proposition makes this point more formally.

Proposition 1. *Assume utility is separable. Consider 2 economies with $r^1_\tau(z^\tau) < r^2_\tau(z^\tau)$ for all $z^\tau \geq z^t$. Then the cutoff rules satisfy $\ell^{1,c}(y_t, z^t) \geq \ell^{2,c}(y_t, z^t)$. If $\sigma < 0$, as $r_\tau(z^\tau)^\sigma \rightarrow 0$ for all $z^\tau \geq z^t$, $\ell^c(y_t, z^t) \rightarrow \hat{\eta}(y_t, z^t)$. Conversely, as $r_\tau(z^\tau) \rightarrow 0$ for all $z^\tau \geq z^t$, $\ell^c(y_t, z^t) \rightarrow 0$*

Perturbations of the r process also change the equilibrium aggregate weight process. An economy with a uniformly higher r process and less collateral as a result has higher liquidity shocks

[2] The intertemporal marginal rate of substitution of the representative agent is a function of the aggregate endowment growth rate and the growth rate of the housing-to-non-durable endowment ratio $r_t = h^a_t / e_t$,:

$$m^a_{t+1}(z^{t+1}) = \delta \left(\frac{e_{t+1}(z^{t+1})}{e_t(z^t)} \right)^{-\gamma} \left(\frac{1 + \psi r^\sigma_{t+1}(z^{t+1})}{1 + \psi r^\sigma_t(z^t)} \right)^{\frac{1-\gamma-\sigma}{\sigma}}.$$

and lower interest rates on average.

Corollary 2. *Consider 2 economies with $r_t^1(z^t) < r_t^2(z^t)$ for all z^t. Fix the distribution of initial multipliers across economies: $\Phi_0^1(z_0) = \Phi_0^2(z_0)$. If $\sigma < 0$, then $\left\{\xi_t^{a,1}(z^t)\right\} \geq \left\{\xi_t^{a,2}(z^t)\right\}$.*

This corollary is a good starting point for understanding the mechanism that underlies the time-variation in the equilibrium market price of aggregate risk in this economy. However, instead of comparing two different economies, we really are interested in the equilibrium changes in the conditional moments of the aggregate weight process in a single economy. The simulations of the model in section 4 will help us understand this mechanism.

Collateral Supply The housing-endowment ratio $\{r\}$ indexes how much risk-sharing the economy can achieve. To see this, we add up the solvency constraints across households. The tightness of the constraints depends on the ratio of aggregate housing wealth to aggregate human wealth. We define the housing collateral ratio $my\left(z^t\right)$ as:

$$my\left(z^t\right) = \frac{\Pi_{z^t}\left[\{\rho h^a\}\right]}{\Pi_{z^t}\left[\{e\}\right]} = \psi \frac{\Pi_{z^t}\left[\{r^\sigma e\}\right]}{\Pi_{z^t}\left[\{e\}\right]} \tag{9}$$

If r is sufficiently persistent, then r and my are tightly correlated. For $\sigma < 0$, they are inversely correlated.

3.3 Sequential Trading

This section describes an equivalent sequential trading arrangement. It illustrates the nature of the collateral constraints in a more intuitive way. We then argue that the equilibrium with sequential trading can be mapped into a time zero trading, Kehoe-Levine equilibrium.

The financial markets are complete. Households trade a complete set of contingent claims a in forward markets. $a_t(\ell, s^t, s')$ is a promise made by agent (ℓ, s_0) to deliver one of unit the consumption good if event s' is realized in the next period. These claims trade at a price $q_t(s^t, s')$. All prices are quoted in units of the non-durable consumption good. ρ_t denotes the rental price; $p_t^h(z^t)$ denotes the price of the housing stock.

Household Problem The household problem is to maximize utility over non-durable consumption and rental services (1) subject to the following collateral constraints and wealth constraints. At the start of the period, the household purchases goods in the spot market $c_t(\ell, s^t)$, rental services in the rental market $h_t^r(\ell, s^t)$, contingent claims in the financial market and shares in the housing stock $h_{t+1}^o(\ell, s^t)$ subject to a wealth constraint:

$$c_t(\ell, s^t) + \rho_t(z^t) h_t^r(\ell, s^t) + \sum_{s'} q_t(s^t, s') a_t(\ell, s^t, s') + p_t^h(s^t) h_{t+1}^o(\ell, s^t) \leq W_t(\ell, s^t).$$

Next period wealth is:

$$W_{t+1}(\ell, s^t, s') = \eta_{t+1}(s^t, s') + a_t(\ell, s^t, s') + h^o_{t+1}(\ell, s^t)\left[p^h_{t+1}(s^t, s') + \rho_{t+1}(s^t, s')\right].$$

All of a household's state-contingent promises are backed by the cum-dividend value of its housing h^o_{t+1}, owned at the end of period t. In each node s^t, households face a separate *collateral constraint* for each event s':

$$-a_t(\ell, s^t, s') \leq h^o_{t+1}(\ell, s^t)\left[p^h_{t+1}(s^{t+1}) + \rho_{t+1}(s^{t+1})\right], \text{ for all } s^t, s'. \tag{10}$$

Competitive Equilibrium

Definition. *Given an initial wealth distribution Θ_0 or equivalently Φ_0, a competitive equilibrium is a feasible allocation $\{c(\ell, s_0), h^r(\ell, s_0), a_{t-1}(\ell, s_0), h^o(\ell, s_0)\}$ and a price vector $\{q_{t-1}, p^h, \rho\}$ such that (1) for given prices and initial wealth, the allocation solves each household's maximization problem and (2) the markets for the consumption good, the housing services, the contingent claims and housing shares clear.*

The equilibria in the economy with sequential trading are equivalent to Kehoe & Levine (1993) equilibria, if the equilibrium interest rates are high enough (Alvarez & Jermann (2000)). These Kehoe-Levine equilibria are essentially Arrow-Debreu equilibria and hence the underlying allocations are (constrained) efficient. Appendix A.1 defines the Kehoe-Levine equilibrium and gives details on a recursive formulation.

To show the equivalence, we define the market state price $p_t(z^t)$ as the product of the Arrow prices for the events along a path z^t:

$$p_t(z^t) = q_{t-1}(z^{t-1}, z')q_{t-2}(z^{t-1}) \dots q_0(z^1),$$

where $p_t(z^t)$ is the price at time 0 of a unit of consumption to be delivered at node z^t.

By iterating forward on the collateral constraints in (10), substituting for the time 0 budget constraint, and imposing a no-arbitrage condition on $\{p^h\}$, the sequence of collateral constraints can be restated as a non-negativity constraint on net wealth in every history (see Appendix A.2):

$$\Pi_{s^t}\left[\{c(\ell, s_0) + \rho h(\ell, s_0)\}\right] \geq \Pi_{s^t}\left[\{\eta\}\right], \ \forall s^t, t \geq 0. \tag{11}$$

Proposition 2. *If the interest rates are high enough, the sequential equilibrium allocations can be supported as a Kehoe-Levine equilibrium (Alvarez & Jermann (2000)).*

4 Computation

To solve the model numerically, we rely on an approximation of g, the growth rate of the aggregate weight process using a truncated history of aggregate shocks. This is discussed in subsection 4.1.

To bring the model to the data, we take a similar approach. The econometrician uses only a finite history of the aggregate factors to predict aggregate weight growth. In 4.2, we fully calibrate the model. We simulate the model and discuss the results in section 4.3.

4.1 Approximating Stationary Equilibria

In general, the aggregate weight process depends on the entire history of shocks z^∞. To avoid the curse of dimensionality, we truncate aggregate histories (Lustig (2001)). Households do not keep track of the entire aggregate history, only the last k lags: $z_t^k = (z_t, z_{t-1}, \cdots, z_{t-k})$ and the current housing-endowment ratio $r_t(z^t)$. The current housing-endowment ratio r_t contains information not present in the truncated history z^k, namely r_{t-k}.

Let R be the domain of the housing endowment ratio r. For a household starting the period with weight ξ, $l(y', z'; \xi, r, z^k) : L \times R \times Z^k \to \Re$ produces the new individual weight in state (y', z'). There is one policy function $l(\cdot)$ for each pair $(y', z') \in Y \times Z$. $g^*(z'; r, z^k) : R \times Z^k \to \Re$ forecasts the aggregate weight shock when moving to state z' after history z^k.

Definition. *A stationary stochastic equilibrium is a joint distribution over individual weights, individual endowments, current housing-endowment ratio and truncated aggregate histories,* $\Phi^*_{(r,z^k)}(\xi, y)$, *which is time invariant, and updating rules* $l(\cdot)$ *and* $g^*(\cdot)$*. For each* $\left(z^{k\prime}, z^k\right)$ *with* $z^{k\prime} = \left(z', z^k\right)$

$$\Phi^*_{(r,z^{k\prime})} = \sum_{z^k} \pi(z^{k\prime}|z^k) \int Q\left(\xi, y, r, z^k\right) \Phi^*_{(r,z^k)}(d\xi \times dy)$$

where $Q\left(\xi, y, r, z^k\right)$ *is the transition function induced by the policy functions.*

The forecast of the aggregate weight shock satisfies

$$g^*(z'; r, z^k) = \sum_{y' \in Y} \int l(y', z'; \xi, r, z^k)^{\frac{1}{\gamma}} \Phi^*_{r,z^k}(d\xi \times dy) \frac{\pi(y', z'|y, z)}{\pi(z'|z)}, \tag{12}$$

for each z'. Prices are determined using the stochastic discount factor in equation (8), and using $g^*(\cdot)$ as an approximation to the actual $g(\cdot)$.

For any given realization $\{z\}$, the actual aggregate weight shock $g(\cdot)$ differs from the forecast $g^*(\cdot)$ because the distribution over individual weights and endowments $\Phi^*(\cdot)$ differs from the actual distribution $\Phi(\cdot)$, which depends on z^∞. The definition of stationary equilibrium implies that,*on average*, $\Phi^*(\cdot) = \Phi(\cdot)$, and markets clear. That is, for every aggregate state z', the allocation error

$$c^a(z'; r, z^k) - e(z') = \frac{g^*(z'; r, z^k) - g(z'; r, z^\infty)}{g^*(z'; r, z^k)} \tag{13}$$

is *on average* zero.[3] As k increases, the approximation error decreases because market clearing holds on average in long histories.

[3]There is an *exact* aggregation result if aggregate uncertainty is i.i.d., with k=0. See Lustig (2001) for a proof in a model without housing.

Algorithm We compute the approximating equilibrium as follows. We use the full insurance values as initial guesses for the aggregate weight shock process and compute the corresponding stochastic discount factor. We compute the cutoff rule for the individual weight shocks and simulate the economy by drawing $\{z_t\}_{t=1}^T$ for $T = 10,000$ and $\{y_t\}_{t=1}^T$ for a cross-section of $5,000$ households. For each truncated history, we compute the sample mean of the aggregate weight shock $\{g_t^*(z', r, z^k)\}_{t=1}^T$and the resulting stochastic discount factor $\{m_t^*(z', r, z^k)\}_{t=1}^T$. A new cut-off rule is computed with these new forecasts. These two steps are iterated on until convergence. Throughout we use $k = 5$ and report percentage allocation errors as a measure of closeness to the actual equilibrium.

4.2 Calibration

In this section we fully calibrate the model and report the approximation errors.

Benchmark Parametrization The only driving force in the model is the Markov process for the non-durable endowment. It contains an aggregate and an idiosyncratic component.

The aggregate endowment growth process is taken from Mehra & Prescott (1985) and replicates the corresponding moments in aggregate consumption growth data (1871-1975). The growth rate of the aggregate endowment, λ, follows an AR(1) process:

$$\lambda_t(z_t) = \rho\lambda_{t-1}(z_{t-1}) + \varepsilon_t,$$

with $\rho = -.14, E(\lambda) = .0183$ and $\sigma(\lambda) = .0357$. We discretize the AR(1) process with two aggregate growth states $z = (z^{exp}, z^{rec}) = \{1.0402, .9602\}$ and an aggregate state transition matrix [.17 .83; .31 .68]. The implied ratio of the probability of an expansion to the probability of a recession is 2.65. The unconditional probability of a recession is 27.4 percent.

The calibration of a heteroskedastic labor income process is taken from Storesletten, Telmer, & Yaron (2001). Log labor income shares follow an AR(1) with autocorrelation of .92 and a conditional variance of .181 in recessions and .0467 in booms.[4] Again the AR(1) process is discretized into a two-state Markov chain. The individual income states are $\left(\eta^{hi}, \eta^{lo}\right)$.

Assumption 1: *idiosyncratic uncertainty depends on aggregate endowment:*
$\hat{\eta}(lo, rec) \leq \hat{\eta}(lo, \exp)$ and $\hat{\eta}(hi, rec) \leq \hat{\eta}(hi, \exp)$.

This results in the following 4 states for the income share [.2048 .3422 .7952 .6578]. The 4×4

[4]The only difference with the Storesletten et al. (2001) calibration is that recessions are shorter in our calibration. In their paper the economy is in a recession 50 percent of the time. That implies that the unconditional variance of our labor income process is lower.

transition matrix π is given by:

$$\begin{bmatrix} .1710 & .8186 & .0040 & .0063 \\ .3020 & .5757 & .0172 & .1051 \\ .0040 & .0063 & .1710 & .8186 \\ .0172 & .1051 & .3020 & .5757 \end{bmatrix}$$

The rental price process (the relative price of housing services in terms of non-durable consumption) is calibrated as an AR(1) process, with endowment growth, measured as innovations in real per household GDP, as an exogenous variable.

Assumption 2: *aggregate rental price growth depends on aggregate endowment growth:*

$$\rho_{t+1} = \bar{\rho} + 0.87\rho_t + 0.88\lambda_{t+1} + \eta_{t+1},$$

where η is an i.i.d. process with mean zero. The coefficients are estimated for the post-war period (standard errors are .05 and .15), and the data are described in detail in section 6. In the data, the standard deviation of η_{t+1} equals .043. In order to attain sufficient movement in the state variable r in a simulation of the model, we set the standard deviation equal to .25 in our benchmark calibration.

$\bar{\rho}$ is chosen such that the average amount of housing wealth tot total wealth is 0.05 in a deterministic economy. $\bar{\rho}$ is .62 in the benchmark calibration. We fix $\psi = 1$ throughout.

In the benchmark calibration we take $\delta = .95$, $\gamma = 8$, $\sigma = -3$, for the preference parameters. The effective relative risk aversion, defined as $\frac{-cu_{cc}}{u_c}$, is a linear combination of γ and σ: $(1-A_t)\gamma + A_t\sigma$. The housing expenditure share A_t is the fraction of housing services consumption in total consumption $\frac{\rho_t h_t}{\rho_t h_t + c_t}$. We focus on the region $\gamma > 1-\sigma$, so that the effective degree of risk aversion is strictly smaller than γ. For the benchmark model, the 95 percent confidence interval for the effective degree of relative risk aversion over non-durables is [7.57,7.96].

Approximation Errors The approximation errors for $k = 5$ are very small. The average error in equation 13 in a simulation of 20,000 periods is 0.0011 with standard deviation .0035. The largest error in absolute value is 0.0282.

4.3 Simulation Results

The heteroskedasticity of the stochastic discount factor comes from two mechanisms. The first is the counter-cyclical labor income dispersion in assumption 1 present in Constantinides & Duffie (1996) and Lustig (2001). The second one is the collateral mechanism, which is new in this paper. Assumption 2 and endogenous movements in the stochastic discount factor drive movements in the housing collateral ratio. For comparison, we compute and simulate three models: a representative agent economy with non-separable preferences, the collateral model, and the collateral model where the Constantinides & Duffie (1996) mechanism is switched off. The discussion pertains to the full collateral model with the benchmark parameters unless otherwise mentioned.

Housing Collateral and Housing Endowment Ratios Figure 2 shows the housing collateral ratio my, computed according to equation (9), plotted together with the housing endowment ratio r. It is a one hundred period window of a long simulation of the model. The housing collateral ratio is the mirror image to the housing-endowment process. It is also a very persistent process. Using simulated data, the estimate of the autocorrelation coefficient in an AR(1) specification for my_t is .86.

Risk Sharing Figure 3 plots the income and consumption profile for one agent and one hundred simulation periods. The income is one on average. The household's consumption share increases when it runs into a binding constraint. This happens when its income share switches from the low to the high idiosyncratic state, as in figure 3 or when there is a decrease in collateral in the economy, as illustrated in figure 4. To illustrate the latter case further, figure 5 plots the aggregate weight shocks g^* and the housing collateral ratio my for a simulation of the model. The largest aggregate weight shocks occur when the housing collateral ratio is low. More households are constrained. There is less risk-sharing in such times.

Conditional Asset Pricing Moments Figure 6 shows that the market price of risk is higher in times with a lower housing collateral ratio. Correspondingly, the excess return on stocks is higher in such periods (figure 7). Variation in the housing collateral induces conditional heteroskedasticity in the market price of risk. This is the central feature of the collateral model. We find the same conditional increase in expected excess returns occurs for the model without the Constantinides & Duffie (1996) mechanism (figure 8).

The model generates the countercyclicality of the Sharpe ratio found in the data (e.g. Whitelaw (1997)). It generates a highly volatile Sharpe ratio. The standard deviation of the Sharpe ratio is .26 compared to .09 for the Campbell & Cochrane (1999) model and the consumption volatility model of Lettau & Ludvigson (2003). However, it shares with other equilibrium models the feature that the conditional mean and volatility of the excess market return are positively correlated (.9). Recent empirical work indicates a negative correlation.[5]

Unconditional Asset Pricing Moments As a byproduct of the model simulations, we obtain the *unconditional* first and second moments for excess returns, stock returns and the risk-free rate. Table 2 summarizes the statistics for the full collateral model and the data. For comparison, all parametrizations of the model are such that the ratio of housing wealth to total wealth is around 5.8 percent. For $\gamma > 6$, the model is able to generate a high and volatile equity risk premium. The

[5] For quarterly excess returns between 1952:4 and 2000:4, Lettau & Ludvigson (2003) find that the dividend yield, the default spread, the term spread, the relative risk-free rate and the consumption wealth ratio plus two lags of volatility jointly explain roughly 30 percent of the time series variation in the conditional second moment and 9 percent of the time series variation in the conditional first moment of the excess stock market return. They using this set of variables to form conditional moments and to compute the Sharpe ratio. The correlation between the conditional first and second moments is -.60 and the implied unconditional standard deviation of the Sharpe ratio is .45.

benchmark model ($\gamma = 8, \sigma = -3$) comes close to matching the historical mean and volatility of the excess return (see data section 6).

We contrast the results of the collateral model with a representative agent economy with non-separable preferences and identical parameters as the collateral model. Table 3 shows a mean stock return and risk-free rate are very high (22 percent) and not volatile enough. The excess returns are on the order of 2-3 percent. Increasing risk aversion γ increases the excess return, but increases the risk-free rate as well through the intertemporal substitution effect. We recall that the effective degree of risk-aversion $(1 - A_t)\gamma + A_t(1 - \sigma)$ moves over time. However, the movements are too small to generate an equity risk premium. As we vary the intratemporal elasticity of substitution $(1 - \sigma)^{-1}$, the excess return and risk-free rate move very little. For a large range of the parameter space, the non-separability generates little action.

The collateral model is able to bridge most of the gap between the representative agent model and the data. Our model can generate a low average level of the risk-free rate (3 percent) for $\gamma \approx 13$. However, it shares with most other equilibrium asset pricing models that the risk-free rate is too volatile relative to the data.

No Labor Income Heteroskedasticity Table 4 shows the asset pricing moments computed for an economy in which the counter-cyclical labor income dispersion mechanism is switched off. That is, the labor income share in the good and bad idiosyncratic states is the same in a recession as in an expansion.

It generates equity premia that are 80 percent as high as is the full collateral model. The risk-free rate is 2 to 3 percent higher than in the full model. We conclude that the endogenous variation in the housing collateral ratio not only generates time-varying market prices of risk, it is also an alternative to the Constantinides & Duffie (1996) mechanism to generate realistic unconditional asset pricing moments .

Value Premium Value firms, with a high ratio of book equity to market equity, historically pay higher returns than growth firms, with a low book-to-market ratio. The annual excess return on a zero-cost investment strategy that goes long in the highest book-to-market decile and short in the lowest decile is 5.5 percent for 1927-2002. The value premium is 5.7 percent for quintile portfolios. Similar value premia are found for monthly and quarterly returns.

The model is able to generate a value premium. To illustrate this, we specify dividend processes, normalized by the aggregate non-durable endowment, that vary with the housing collateral ratio:

$$log(d_t^s) - log(e_t) = A^s + B^s my_t + \epsilon_t, \; s \in \{1, ..., 8\} \tag{14}$$

where d_t^s is the dividend on the s^{th} value portfolio. We let the sensitivity to the housing collateral ratio, B^s, vary from -1 to +7. We price the different dividend streams *inside the model* and compute the returns. The value premium in the model is defined as the excess return on the "value" portfolios (B^8) over the "growth" portfolios (B^1). The baseline model ($\gamma = 8, \; \sigma = -3$)

generates a value premium of 8.3 percent (see table 5). The value premium is 4.3 percent for a calibration with $\gamma = 6$ and 20 percent for $\gamma = 10$.

5 From Model to Data

In the previous section we solved numerically for the equilibrium Pareto-Negishi weight processes. In this section we show the conditions on the aggregate weight process $G(\cdot)$ under which the model gives rise to a linear asset pricing model of the form

$$m_{t+1} = -\theta F_{t+1},$$

where θ is a vector of constants and F_{t+1} is a vector of asset pricing factors. This connects our model to the linear factor model tradition in the empirical asset pricing literature. In particular, we propose a linear, Markov structure for the weight process $G(\cdot)$ in section 5.2). The numerical results from section 4 lend validity to this specification. The linear factor model gives rise to the β-representation described in 5.3, which is later estimated in section 8.3.

5.1 Empirical Strategy

F^a denotes the vector of macro-economic variables that summarizes new information revealed to household at t. The household's information at t includes the entire history of realizations $(F^a)_t^\infty$. The aggregate factors we use are consumption growth and rental price growth, scaled by the housing expenditure share $A_t = \frac{\rho_t h_t}{c_t^a + \rho_t h_t^a}$:

$$F_{t+1}^a = (\Delta \log c_{t+1}^a, A_t \Delta \log \rho_{t+1})$$

On this basis of this entire history, households can exactly forecast g_{t+1} for each F_{t+1}^a tomorrow. We let households forecast g using the current housing collateral ratio my_t instead of r_t and a truncated history of aggregate factors $(F^a)_t^k$. Households predict the aggregate weight shock g

$$G_{t+1}(z'; my_t, (F^a)_t^k) \approx \log g_{t+1}(z'; (F^a)_t^\infty),$$

We use $(\Upsilon_c, \Upsilon_\rho)$ to denote the unconditional mean of the aggregate factors.

5.2 Linear Pricing Model

Definition. *A complete description of the linear pricing model $m_{t+1}(my_t, F_{t+1}^a)$ consists of (1) a specification for the aggregate weight shocks $G_{t+1}\left(my_{t+1}, (F^a)_{t+1}^k\right)$ and (2) a process for the housing collateral ratio $my_{t+1}(my_t, F_{t+1}^a)$.*

The Housing Collateral Ratio $\{my\}$ is specified as an autoregressive process whose innovations are a linear combination C of the innovations to F^a_{t+1}:

$$my_{t+1} = \rho my_t + C_1 \left(\Delta \log c^a_{t+1} - \Upsilon_c \right) + C_2 \left(A_t \Delta \log \rho_{t+1} - \Upsilon_\rho \right). \tag{15}$$

The innovations to the aggregate factors are the structural innovations in our model. In the model, the housing-endowment ratio r maps monotonically into the housing collateral ratio my. The persistence in my is inherited from the persistence in r.

Aggregate Weight Shocks As discussed in section 4, we approximate the actual weight shock g_{t+1} by keeping track of a truncated aggregate history and the current housing-endowment ratio: $G_{t+1}\left(my_{t+1}, (F^a)^k_{t+1}\right)$. We propose a *linear* expression for $G_{t+1}(\cdot)$:

$$\begin{aligned}\log G_{t+1} = {} & (my^{max} - my_{t+1}) \sum_{j=0}^{k} B_{1,j} \left(\Delta \log(c^a)_{t+1-j} - \Upsilon_c \right) + \\ & (my^{max} - my_{t+1}) \sum_{j=0}^{k} B_{2,j} \left(A_{t-j} \Delta \log \rho_{t+1-j} - \Upsilon_\rho \right) + \varepsilon_{t+1},\end{aligned} \tag{16}$$

where B_1 and B_2 are $k \times 1$ vectors of constants.

The ratio my governs how much consumption can be transferred from good states to bad states by the planner. If this ratio is high enough, the planner can sustain perfect risk sharing. This occurs at $my_t = my^{max}$. On the other hand, if this ratio is low enough, the planner cannot improve upon the autarkic outcome. The housing collateral ratio shifts the conditional distribution of tomorrow's aggregate Pareto-Negishi weights. We assume that the shifting occurs in a linear fashion.

A negative consumption growth shocks has two effects. First, a recession increases r (decreases my) which makes the risk-sharing bounds narrower. Second, a recession coincides with an increase in the income dispersion, which makes the bounds narrower as well. In either case, the extent to which a recession narrows the bounds depends on the level of r or, equivalently, the housing collateral ratio. When the risk-sharing bounds are narrower, agents run more frequently into them and the aggregate weight growth is high. When housing collateral is scarce, $my^{max} - my_{t+1}$ is large. A negative consumption growth shock increases G_{t+1} for $B_1 < 0$. When $my_{t+1} = my^{max}$, there is no effect of innovations to aggregate consumption and rental price growth on the expression for the aggregate weights: G_{t+1} is one.

The Markov Assumption Unless the aggregate shocks are i.i.d., the actual aggregate weight shock g_{t+1} depends on the entire aggregate history z^{t+1} and the initial housing-endowment ratio r_0. In the empirical analysis below, we choose to impose a *Markov* structure on the specification $\{G(\cdot)\}$. This is, we set $k = 0$. The benefit is that we obtain a parsimonious factor model.

$$\log G_{t+1} = (my^{max} - my_{t+1}) \left(B_1 \left(\Delta \log(c^a)_{t+1} - \Upsilon_c \right) + B_2 \left(A_t \Delta \log \rho_{t+1} - \Upsilon_\rho \right) \right) + \varepsilon_{t+1}, \tag{17}$$

where B_1 and B_2 are now 1×1 vectors of constants.

Accuracy of the linear Markov specification We undertake two tests.

First, we simulate the benchmark model for 20,000 periods and compute $\{g_t^*\}$ as described in section 4, using $k = 5$. Then, we run the regression

$$g_{t+1}^* = c_0 + c_1 my_{t+1} + c_2 \left(my_{t+1}\right)^2 + \eta_{t+1} \tag{18}$$

on the simulated data. The aggregate shocks z only take on 2 different values because of the Markov nature. In a first regression, we group all periods with the same current and previous shock (z_t, z_{t-1}), find the corresponding $\{g_t^*, my_t\}$ and estimate equation (18). The R^2 is 70.11 percent without and 70.35 percent with the quadratic term in my. In a second and third regression we group all periods with identical (z_t, z_{t-1}, z_{t-2}) and $(z_t, z_{t-1}, z_{t-2}, z_{t-3})$. The R^2 are 92.86 (93.08) and 95.88 (96.09), where the numbers in parentheses refer to the regressions with squared term. We conclude that keeping track of just the current and the previous shock approximates the weight shock very well. Furthermore, the linearity in my is not restrictive.

Second, we test for additional history dependence in the estimation exercise by including up to four lags of the factors, F_{t-k}^a for $k = 1, 2, 3, 4$, on the right hand side of equation (16). We discuss the results in section 8.3.

Linear Factor Model The factor model for the weight shocks and the autoregressive process for my provide a complete description of the pricing model. By combining $G_{t+1}\left(my_{t+1}, F_{t+1}^a\right)$ and $my_{t+1}(my_t, F_{t+1}^a)$, the stochastic discount factor in (8) can be stated in terms of aggregate factors F_{t+1}^a and the state variable my_t: $m_{t+1}(my_t, F_{t+1}^a)$. A first-order Taylor approximation of this expression delivers our linear factor model:

$$m_{t+1} \approx \tilde{\delta}(const - \theta^a F_{t+1}^a - \theta^c F_{t+1}^c + \gamma \varepsilon_{t+1}), \tag{19}$$

where the representative agent factors F_{t+1}^a and constraint factors F_{t+1}^c are:

$$\begin{aligned}
F_{t+1}^a &= \left(\Delta \log(c_{t+1}^a), A_t \Delta \log(\rho_{t+1})\right)' \\
F_{t+1}^c &= \left(my^{max} - my_t, \left(my^{max} - my_t\right) \Delta \log c_{t+1}^a, \left(my^{max} - my_t\right) A_t \Delta \log \rho_{t+1}\right)',
\end{aligned}$$

with associated factor loadings

$$\begin{aligned}
\theta^a &= \left(\gamma \left(1 + \Upsilon_c\right)^{-1} - \gamma(1-\rho) B_1 my^{max}, \phi \left(1 + \frac{\sigma}{\sigma - 1} \Upsilon_\rho\right)^{-1} - \gamma(1-\rho) B_2 my^{max}\right) \\
\theta^c &= \left(-\gamma \rho B_1 \Upsilon_c - \gamma \rho B_2 \Upsilon_\rho, -\gamma \rho B_1, -\gamma \rho B_2\right)
\end{aligned}$$

The constraint factors interact the aggregate factors F^a_{t+1} with the state variable my_t.[6]

Case 1: Separable Preferences When utility is separable, the equity risk premium is determined by the conditional covariance of its returns with consumption growth and a state-varying market price of risk:

$$E_t\left[R^{e,j}_{t+1}\right] \approx \tilde{\delta} R^f_t \gamma \left[(1+\Upsilon_c)^{-1} - B_1\gamma(1-\rho)my^{\max} - B_1\rho\left(my^{\max} - my_t\right)\right] Cov_t\left(\Delta \log c^a_{t+1}, R^{e,j}_{t+1}\right) \tag{20}$$

where R^f_t is the risk-free rate at time t. If B_1 is zero, the expression collapses to the standard CCAPM of Lucas (1978) and Breeden (1979). The market price of consumption risk is determined by the coefficient of relative risk aversion γ. In contrast, our theory predicts an increase in the size of the aggregate weight shock when aggregate consumption growth is low, driven by an increase in idiosyncratic risk. Consumption growth has an effect on the liquidity shock: $B_1 < 0$. When housing collateral is scarce ($my^{\max} - my_t$ is large), the market price of consumption risk is high.

Case 2: Non-Separable Preferences Non-separability introduces a second covariance in the risk premium equation: the covariance with rental price changes. Under complementarity of nondurable consumption and housing ($\phi > 0$), households want to hedge by investing in assets that deliver high returns when consumption is scarce, that is when the rental price of housing services increases. This hedging risk is the focus of recent work by Piazzesi et al. (2002).

If B_2 is zero, the market price of rental price risk is constant. The market price of rental price risk is determined by the degree of complementarity between consumption and housing services in the utility function ϕ. In contrast, if $B_2 < 0$, the market price of rental price risk is high when housing collateral is scarce ($my^{\max} - my_t$ is large).

[6] The Markov assumption on the aggregate weight process implies

$$\begin{aligned} E_t\left[(my^{max} - my_{t+1})\left(\Delta \log c^a_{t+1} - \Upsilon_c\right)\varepsilon_{t+1}\right] &= 0, \\ E_t\left[(my^{max} - my_{t+1})\left(A_t \Delta \log \rho_{t+1} - \Upsilon_\rho\right)\varepsilon_{t+1}\right] &= 0. \end{aligned}$$

We assume that the specification error is orthogonal to the financial returns:

$$E_t\left[\varepsilon_{t+1} R^{j,e}_{t+1}\right] = 0 = E\left[\varepsilon_{t+1} R^{j,e}_{t+1}\right],$$

where the second equality follows from the law of iterated expectations. The constant and $\tilde{\delta}$ in the factor representation are given by

$$\begin{aligned} const &= 1 + \left[\gamma\left(1+\Upsilon_c\right)^{-1} - B_1\gamma(1-\rho)my^{\max}\right]\Upsilon_c + \left[\phi\left(1 + \frac{\sigma}{\sigma-1}\Upsilon_\rho\right)^{-1} - B_2\gamma(1-\rho)my^{\max}\right]\Upsilon_\rho, \\ \tilde{\delta} &= \delta\left(1+\Upsilon_c\right)^{-\gamma}\left(1 + \frac{\sigma}{\sigma-1}\Upsilon_\rho\right)^{\frac{1-\sigma-\gamma}{\sigma}}. \end{aligned}$$

5.3 Unconditional β-Representation

To summarize, the discount factor is decomposed into a representative agent and a constraint component:

$$m_{t+1} = -\theta F_{t+1}, \tag{21}$$

where θ is a vector of constants, $\theta = \left(const, \tilde{\theta}\right)$ and $\tilde{\theta} = (\theta^a, \theta^c)$ and $F_{t+1} = \left(1, \tilde{F}_{t+1}\right)$. $\tilde{F}_{t+1} = \left(F^{a\prime}_{t+1}, F^{c\prime}_{t+1}\right)'$ is a vector of representative agent and constraint risk pricing factors.

If θ were time-varying, the conditional orthogonality conditions in $E_t\left[m_{t+1}R^j_{t+1}\right]$ would not imply unconditional orthogonality conditions. Here, the vector of constraint factors contains the original factors scaled by the housing collateral ratio my_t. my_t is the conditioning variable that summarizes the investor's information set. The stochastic discount factor contains the conditioning information through the scaled constraint factors. The model (21) can be tested using the unconditional orthogonality conditions of the discount factor and excess asset returns j:

$$E\left[m_{t,t+1}R^{e,j}_{t+1}\right] = 0. \tag{22}$$

Using the definition of the risk-free rate and the covariance, the unconditional factor model in (21) implies an unconditional β-representation:

$$\begin{aligned} E\left[R^{e,j}_{t+1}\right] &= \tilde{\delta}\bar{R}^f\tilde{\theta}Cov\left(\tilde{F}_{t+1}, R^{e,j}_{t+1}\right) \\ &= \tilde{\lambda}\tilde{\beta}^j, \end{aligned} \tag{23}$$

where $\bar{R}^f$ is the average risk-free rate, $\tilde{\beta}^j$ is asset j's risk exposure and $\tilde{\lambda}$ is a transformation of the parameter vector $\tilde{\theta}$[7]:

$$\tilde{\beta}^j = Cov\left(\tilde{F}, \tilde{F}'\right)^{-1} Cov\left(\tilde{F}, R^{e,j}\right)$$

$$\tilde{\lambda} = \tilde{\delta}\bar{R}^f\tilde{\theta}Cov\left(\tilde{F}, \tilde{F}'\right)$$

The unconditional β-representation in (23) is the equation we estimate in section 8.3.

6 Data

In the empirical section (section 8) we use two sets of variables: financial variables and aggregate macroeconomic variables. All variables are annual and for the United States.

[7]Lettau & Ludvigson (2001b) point out that $\tilde{\lambda}$ does not have a straightforward interpretation as the vector of market prices of risk. The market prices of risk λ depend on the conditional covariance matrix of factors which is unobserved.

6.1 Financial Data

In a first time-series exercise we just use the return on the aggregate stock market. In a second exercise we use a cross-section of stock portfolios, sorted by size and value characteristics.

Market Return The market return is the cum-dividend return on the Standard and Poor's composite stock price index. The market return is expressed in excess of a risk-free rate, the annual return on six-month prime commercial paper. The returns are available for the period 1889-2001 from Robert Shiller's web site.

Size and Book-to-Market Portfolios We use twenty-five portfolios of NYSE, NASDAQ and AMEX stocks, grouped each year into five size bins and five value (book-to-market ratio) bins. Size is market capitalization at the end of June. Book-to-market is book equity at the end of the prior fiscal year divided by the market value of equity in December of the prior year. Portfolio returns are value-weighted. We also include the market return R^{vw}, the value-weighted return on all NYSE, AMEX and NASDAQ stocks. We refer to this set of 26 test assets as $T1$. All returns are expressed in excess of an annual return on a one-month Treasury bill rate (from Ibbotson Associates). The returns are available for the period 1926-2001 from Kenneth French's web site and are described in more detail in Fama & French (1992). The first column of table 6 shows mean and standard deviation for the 26 excess returns in $T1$.

Dividends on Value Portfolios We use annual dividend data on each of the 10 book-to-market decile portfolios. Book-to-market is book equity at the end of the prior fiscal year divided by the market value of equity in December of the prior year. We follow Bansal et al. (2002) by constructing dividends from value-weighted total returns and price appreciation rates on the decile value portfolios (both from Kenneth French). We construct nominal annual dividends by summing up monthly nominal dividends. The data are for 1952-1999.

6.2 Aggregate Macroeconomic Data

Price Indices Aggregate rental prices ρ_t are constructed as the ratio of the CPI rent component p_t^h and the CPI food component p_t^c. Data are for urban consumers from the Bureau of Labor Statistics for 1926-2001. The price of rent is a proxy for the price of shelter and the price of food is a proxy for the price of non-durables. We use the rent and food components because the shelter and non-durables components are only available from 1967 onwards. Two-thirds of consumer expenditures on shelter consists of owner-occupied housing. The BLS uses a rental equivalence approach to impute the price of owner-occupied housing. Because ρ_t is a relative *rental* price, our theory is conceptually consistent with the BLS approach. We also use the all items CPI, p_t^a, which goes back to 1889. All indices are normalized to 100 for the period 1982-84.

Housing Collateral We use three distinct measures of the housing collateral stock HV: the value of outstanding home mortgages (mo), the market value of residential real estate wealth (rw) and the net stock current cost value of owner-occupied and tenant occupied residential fixed assets (fa). The first two time series are from the Historical Statistics for the US (Bureau of the Census) for the period 1889-1945 and from the Flow of Funds data (Federal Board of Governors) for 1945-2001. The last series is from the Fixed Asset Tables (Bureau of Economic Analysis) for 1925-2001.

We use both the value of mortgages HV^{mo} and the total value of residential fixed assets HV^{rw} to be robust to changes in the extent to which housing can be used as a collateral asset. We use both HV^{rw}, which is a measure of the value of housing owned by households, and HV^{fa} which is a measure of the value of housing households live in, to be robust to changes in the home-ownership rate over time. Appendix A.5 provides detailed sources. Real per household variables are denoted by lower case letters. The real, per household housing collateral series $hv^{mo}, hv^{rw}, hv^{fa}$ are constructed using the all items CPI from the BLS, p^a, and the total number of households, N, from the Bureau of the Census.

Consumption and Income Consumption is non-durable consumption C, measured by total expenditures minus apparel and minus rent and imputed rent. The housing expenditure share, A, is the ratio of rent expenditures to non-durable consumption.

The income endowment in the model corresponds to an after-government income concept; it includes net transfer income. Aggregate income Y is labor income plus net transfer income. Nominal data are from the Historical Statistics of the US for 1926-1930 and from the National Income and Product Accounts for 1930-2001. Consumption and income are deflated by p^c and p^a and divided by the number of households N.

7 Measuring the Housing Collateral Ratio

This section measures the new state variable, the housing collateral ratio my. my is defined as the ratio of collateralizable housing wealth to non-collateralizable human wealth. Human wealth is unobserved. Following Lettau & Ludvigson (2001a), we assume that the non-stationary component of human wealth H is well approximated by the non-stationary component of labor income Y. In particular, $\log(H_t) = \log(Y_t) + \epsilon_t$, where ϵ_t is a stationary random process. The assumption is valid in a model in which the expected return on human capital is stationary (see Jagannathan & Wang (1996) and Campbell (1996)).

Cointegration Log, real, per household real estate wealth ($\log hv$) and labor income plus transfers ($\log y$) are non-stationary. According to an augmented Dickey-Fuller test, the null hypothesis of a unit root cannot be rejected at the 1 percent level. This is true for all three measures of housing wealth ($hv = mo, rw, fa$).

If a linear combination of $\log hv$ and $\log y$, $\log(hv_t) + \varpi \log(y_t) + \chi$, is trend stationary, the components $\log hv$ and $\log y$ are said to be stochastically cointegrated with cointegrating vector

$[1, \varpi, \chi]$. We additionally impose the restriction that the cointegrating vector eliminates the deterministic trends, so that $\log(hv_t) + \varpi \log(y_t) + \vartheta t + \chi$ is stationary. A likelihood-ratio test (Johansen & Juselius (1990)) shows that the null hypothesis of no cointegration relationship can be rejected, whereas the null hypothesis of one cointegration relationship cannot. This is evidence for one cointegration relationship between housing collateral and labor income plus transfers. Table 7 reports the results of this test and of the vector error correction estimation of the cointegration coefficients:

$$\begin{bmatrix} \Delta \log(hv_t) \\ \Delta \log(y_t) \end{bmatrix} = \alpha \left[\log(hv_t) + \varpi \log(y_t) + \vartheta t + \chi\right] + \sum_{k=1}^{K} D_k \begin{bmatrix} \Delta \log(hv_{t-k}) \\ \Delta \log(y_{t-k}) \end{bmatrix} + \varepsilon_t. \tag{24}$$

The K error correction terms are included to eliminate the effect of regressor endogeneity on the distribution of the least squares estimators of $[1, \varpi, \vartheta, \chi]$. The housing collateral ratio my is measured as the deviation from the cointegration relationship:

$$my_t = \log(hv_t) + \hat{\varpi} \log(y_t) + \hat{\vartheta} t + \hat{\chi}.$$

The OLS estimators of the cointegration parameters are superconsistent: They converge to their true value at rate $1/T$ (rather than $1/\sqrt{T}$). The superconsistency allows us to use the housing collateral ratio my as a regressor without need for an errors-in-variables standard error correction (see section 8).

We also estimate the constant and trend in the cointegrating relationship while imposing the restriction $\varpi = -1$. This is the second block of each panel in table 7. For mo and fa, we find strong evidence for one cointegrating relationship. The coefficient on $\log ylt$ is precisely estimated (significant at the 1 percent level, not reported), varies little between subperiods, and the 95 percent confidence interval contains -1. The resulting time-series are stationary. The null hypothesis of a unit root is rejected for $mymo$ and $myfa$. For each subperiod, the correlation between the residual estimated assuming $\varpi = -1$ and the one with ϖ freely estimated is higher than 0.95. For rw, the evidence for a cointegrating relationship is weaker, except for the 1925-2002 period. Furthermore, the slope coefficient in the cointegration relationship varies considerably between subperiods and does not contain -1 in its 95 percent confidence interval. The correlation between the residual estimated assuming $\varpi = -1$ and the one with ϖ freely estimated is 0.81 for the entire sample, 0.88 for 1925-2002 and 0.89 for the post-war period.

For consistency we work with the three series that impose $\varpi = -1$. The housing collateral ratios are labelled $mymo$, $myrw$ and $myfa$. For the common sample period 1925-2001, the correlation between $mymo$ and $myrw$ is 0.89, 0.76 between $mymo$ and $myfa$ and 0.86 between $myrw$ and $myfa$. Figure 9 displays my between 1889 and 2002. All three series exhibit large persistent swings. They reach a maximum deviation in 1932-33. Residential wealth and residential fixed assets are 30 and 34 percent above their respective joint trends with human wealth. Mortgage debt is 53 percent above its trend. The series reach a minimum in 1944-45, when $mymo$ is $-.92$, $myrw$ is $-.57$ and $myfa$ is $-.38$. $mymo$ and $myrw$ have increased considerably since the year 2000: from

.24 to .36 and from 0.19 to 0.30 respectively. Figure 10 shows the cointegration residuals my for that post-war period. Housing collateral wealth fluctuates within 30 percent below and above the long-run trend with human wealth.

When housing wealth deviates from its long-run ratio with labor income, the equilibrium relationship is restored by transitory movements in both housing wealth and labor income. Table 8 (in appendix B) shows the estimation results of a bivariate vector autoregression of changes in housing wealth and labor income. The lagged housing collateral ratio, my_{t-1}, is an exogenous regressor. The coefficients on my_{t-1} in both equations have about the same size (and opposite signs). This suggests that the transitory return to the common trend is done by both variables.

8 Empirical Evidence of the Collateral Effect

We address two empirical failures of the CCAPM. In section 8.1 we provide evidence that the housing collateral ratio predicts stock returns. This suggests that the market price of risk is not a constant but a function of my. Second, we show that dividend processes for growth and value firms respond differently to innovations in the housing collateral ratio (8.2). Third, in contrast to the CCAPM, our model can help account for a large fraction of the cross-sectional variation in size and book-to-market portfolio returns (section 8.3).

8.1 Time-Series Predictability

The model generates predictable variation in risk premia on stocks. The reward for risk is higher when housing collateral is scarce. We find empirical support for this negative relationship.

VAR A bivariate vector autoregression of one-year excess returns on the aggregate stock market and the housing collateral ratio provides a first look at the predictability question. We study the response of excess returns to an innovation to my. Figure 12 shows the negative response of the excess return to an orthogonal innovation in $myfa$, the my measure for fixed assets. The initial drop in the equity risk premium is followed by a further decrease which persists for multiple years.[8] The effect is large: A 4 percentage point innovation to $myfa$ causes a 2.4 percentage point decrease in the equity risk premium. Between 1941 and 1942, $myfa$ declined by 20 percentage points in one year. The impulse response estimates suggest a 12 percentage point increase in the risk premium. Figures 13 and 14 (at the end of the text) show a similar pattern for the other two measures of the housing collateral ratio.

Long Horizon Predictability To illustrate the economic effect of return forecastibility over a longer period, we study long-horizon excess returns. We define the K-year continuously compounded excess return as $r^{vw,K}_{t+K} = (r^{vw,1}_{t+1} + ... + r^{vw,1}_{t+K})$ where $r^{vw,1}_t$ equals $\log(1 + R^{vw,e}_t)$. Figure 15

[8] The optimal lag length for the VAR is two years according to the Aikake Information criterion. The covariance matrix of innovations has small off-diagonal elements, i.e. their innovations have a small common component. Therefore, changing the ordering of the variables $R^{vw,e}$ and my in the VAR does not affect the impulse-responses.

shows the housing collateral ratio ($mymo$) and the annualized ten-year excess return. The series exhibit a negative correlation of -0.52. Regressions of the one- to ten-year cumulative stock returns on the housing collateral ratio ($mymo$) provide further evidence of predictability.

Row 1 of table 9 shows the least squares coefficient estimate on the housing collateral ratio for the period 1889-2001. Row 5 contains the estimates for the postwar period. All coefficients on the rescaled housing collateral ratio are positive: A low housing collateral ratio predicts a high future risk premium. The R^2 of the least-squares regression increases with the horizon, to 43 percent in the postwar period (row 5, $k = 10$).

There are two econometric problems with the ordinary least squares regression:

$$r^{vw,K}_{t+K} = b_0 + b_{my} my_t + e_{t+1}. \tag{25}$$

First, because the forecasting variable my is a slow-moving process, the least squares estimator of the coefficient on my, b^{LS}_{my}, suffers from persistent regressor bias in small samples (Stambaugh (1999)). Second, because $r^{vw,K}_{t+K}$ contains overlapping observations, the standard errors on b^{LS}_{my} need to be corrected for serial correlation in the residuals e. Asymptotic corrections as advocated by Hansen & Hodrick (1980) have poor small sample properties. Ang & Bekaert (2001) find that use of those standard errors leads to over-rejection of the no-predictability null. To address the persistent regressor bias and the serial correlation issues we conduct a bootstrap exercise, detailed in appendix A.3.

Rows 3 and 7 reports the small-sample coefficient estimates, generated by bootstrap. At every horizon, the small sample coefficient estimates are positive. They are slightly lower than the least squares estimates in the entire sample, but slightly higher in the post-war sample. Rows 4 and 8 show the p-value of a two-sided test of no predictability, generated by bootstrap. It measures one minus the likelihood of observing the least squares coefficient estimate when returns are in fact unpredictable. For $K \geq 5$, there is evidence against the null-hypothesis at the 15 percent level.

Comparison Across Models Many financial and macroeconomic variables have forecasting power for the market return. The dividend-price and dividend-earnings ratio, the treasury bill rate, the term spread between long-term government bonds and treasury bills and the default spread between low- and high-grade corporate bonds are financial variables with forecasting power for excess returns (see Cochrane (2001), Ch. 20 for an overview). A subset of those variables, such as the investment-capital ratio (Cochrane (1991a)), the consumption-wealth ratio, cay, (Lettau & Ludvigson (2001a)) and the labor income - consumption ratio, lc, (Santos & Veronesi (2001)), are macroeconomic variables. Most of these macroeconomic variables are correlated with or forecast the business cycle. In contrast, the housing collateral ratio is a low-frequency variable. A spectral decomposition reveals that at least three-quarters of the variation in the housing collateral ratio is situated at horizons longer than 20 years. The power spectrum in figure 11 reaches its peak at frequencies below $2\pi/20$. As for the cyclical properties of my, the spectrum displays a smaller hump at $2\pi/8$, a frequency associated with a long recession (8 years).

We compare the return-forecasting ability of the housing collateral ratio with that of the dividend yield ($\log dp$), the consumption-wealth ratio (cay), and the labor income-consumption ratio (lc). The two samples are the longest available (1926-2002) and the postwar sample (1945-2002). Table 10 displays the results for the long-horizon regressions. The dividend yield is a strong predictor of excess returns in both samples (lines 2 and 9). For $K \geq 5$, the predicting power of the housing collateral ratio is almost as strong (lines 1 and 8). Lines 5 and 12 show that the housing collateral ratio contains information that is relevant for predicting returns beyond what is included in the dividend yield. Both coefficients remain jointly significant for $K > 5$. The R^2 goes up to 57 percent in the postwar period for 10-year returns, 13 percent more than in each of the individual regressions.

Lettau & Ludvigson (2001b) explore a conditional version of the CCAPM with the consumption to wealth ratio as conditioning variable. The ratio is measured as the deviation from the common trend in consumption, labor income and financial wealth (cay).[9] Periods with high cay indicate high expected future returns, thereby rationalizing a high propensity to consume out of wealth. In the representative agent economy of Santos & Veronesi (2001), the ratio of labor income to consumption lc predicts stock returns.[10] Times in which investors finance a large fraction of consumption out of labor income rather than out of stock dividend income (lc is high), are less risky. For the entire sample, cay and lc have no forecasting ability (lines 3 and 4). Moreover, the coefficients have the wrong sign. This problem goes away in the post-war period (lines 10 and 11), but both variables still have very little forecasting power for annual returns when used in a univariate regression. When both the housing collateral ratio and the labor income to consumption ratio are included, lc has the right sign for $K \geq 5$ (line 6). The coefficient on the housing collateral ratio remains significant and the R^2 of the regression increases to 47 percent. In the postwar period, my and lc enter jointly significantly and explain up to 54 percent of the time-series variation in excess returns (line 13). The highest predictive power is obtained when all three variables my, lc, and dp are included (lines 7 and 14). In the postwar period the R^2 goes up to 64 percent for $K = 10$, and all three coefficients are measured precisely.

8.2 Dividends on Value Portfolios

We find in table 11 that dividends of the high book-to-market portfolios (normalized by the aggregate labor income plus transfers) are strongly positively correlated with the housing collateral ratio (e.g. the tenth decile portfolio B10). The normalized dividend process for the low book-to-market

[9]We construct the cay variable for the period 1926-2001 using log real per household total consumption expenditures (c), log real per household labor income plus transfers (ylt) and log real per household financial wealth (fw). We find evidence for one cointegration relationship between the three variables. The estimated relationship we find with annual data is $cay = c - 0.233fw - 0.799ylt + 0.385$. We follow Lettau and Ludvigson and rescale the scaling variable, making sure it remains positive cay: $\tilde{cay}_t = 2.5 + \frac{cay_t - E(cay)}{std(cay)}$. For 1926-2001, the correlation between cay and the housing collateral ratio is .69 for $mymo$, .78 for $myrw$, and .69 for $myfa$. In contrast to the consumption-wealth ratio (cay), our conditioning variable does not contain direct information on future returns.

[10]We construct their scaling variable as the ratio of annual labor income to total consumption expenditures, for the period 1926-2001. We rescale the scaling variable lc: $\widetilde{lc}_t = 1 + \frac{lc_t - E(lc)}{std(lc)}$.

ratio portfolios is strongly negatively correlated with my (e.g. the first decile portfolio B1). We showed earlier that the collateral model endogenously generates a value premium when the dividend process for value firms is highly positively correlated and the dividend process for growth firms is either negatively or not very strongly correlated with the housing collateral ratio.

VAR A vector autoregression further demonstrates the different dynamics of growth and value dividend processes. We study the response of the normalized dividend process on the growth (B1) and value (B10) portfolios to an impulse in the housing collateral ratio. The responses are long-lived because of the persistence in the housing collateral ratio. The coefficients on the lagged housing collateral ratio in the dividend share equation have the opposite sign and are significantly different for B1 and B10. As a result, the responses of the normalized dividends to an innovation to myfa also go in opposite directions. Figures 16 and 17 show the impulse response graphs.

8.3 Cross-Sectional Results

Size and book-to-market value are asset characteristics that challenge the standard CCAPM. Historically, small firm stocks and high book-to-market firm stocks have higher returns. In the post-war period, the size premium has largely disappeared, but the value premium is still prominent. The CCAPM yields large pricing errors on book-to-market stocks: This is the value premium puzzle. The new asset pricing factors in our model substantially improve the fit of the cross-section of returns. The average pricing errors are cut in half (section 8.3.2). In section 8.3.3, we compare the fit of our model to other asset pricing models. First we briefly discuss the computational procedure.

8.3.1 Computational Procedure

The coefficient vector θ in equation (22) can be estimated using the Fama & MacBeth (1973) two-stage regression procedure or the Hansen & Singleton (1982) generalized method of moments procedure. Jagannathan & Wang (2001) compare both methods and argue that both have similar properties in terms of estimation efficiency and finite sample performance. We opt for the two-stage Fama-MacBeth procedure and estimate the unconditional β-representation $E\left[R^{e,j}_{t+1}\right] = \tilde{\lambda}\tilde{\beta}^j$. In a first time-series stage, for each asset j separately, excess returns are regressed on factors to uncover the $\tilde{\beta}$'s. In a second cross-sectional stage, average excess returns are regressed on the $\tilde{\beta}$'s from the first stage to obtain the market prices of risk $\tilde{\lambda}$. Appendix A.4 describes the procedure in more detail.

8.3.2 Results: Aggregate Asset Pricing Factors

We use aggregate macroeconomic data and the Fama-MacBeth procedure to investigate the explanatory power of the aggregate asset pricing factors in (19) for the cross-section of excess returns on size and book-to-market portfolios $T1$.

Table 12 reports the estimates for the market price of risk $\tilde{\lambda}$ obtained from the second-stage of the Fama-MacBeth procedure. Below the estimates for $\tilde{\lambda}$, we report conventional standard errors and Shanken (1992) standard errors, which correct for the fact that the $\tilde{\beta}$'s are generated regressors from the first time-series step. Since all returns are in excess of a risk free rate, according to the theory, the intercept in the cross-sectional regressions should be zero.

Row 1 shows the standard CCAPM. It explains 9 percent of the cross-sectional variation in excess returns of the size and book-to-market portfolios between 1926 and 2002. Unsurprisingly, the coefficient of relative risk aversion γ implied by the market price of consumption risk $\tilde{\lambda}_c$ is very high (22, not reported). With non-separable preferences but perfect commitment, the change in relative rental prices scaled by the housing expenditure share is an additional asset pricing factor. This is the HCAPM of Piazzesi et al. (2002). The non-separability effect increases the R^2 to 50 percent (row 2). Rows 3 through 8 investigate the collateral effect. With separable preferences, the new asset pricing factors are the housing collateral ratio my and consumption growth scaled by my. The fit improves to 73 - 88 percent for the respective measures of the housing collateral ratio (rows 3-5). The coefficients on the interaction terms are positive and significant. With non-separable preferences, the interaction term of my with rental price growth is an additional asset pricing factor (rows 6-8). The new interaction term has a positive factor loading, but does not enter statistically significantly. Except for the conditioning variable $myfa$, non-separability does not add much to the explanatory power of the collateral CAPM.

The coefficient estimates for $\tilde{\lambda}$ can be related back to the structural parameters of the model. The time-invariant market price of consumption risk, predicted by the standard CCAPM, is overly restrictive. A decrease in the housing collateral ratio my_t increases $(my^{max} - my_t)$ and increases the market price of consumption risk: We estimate $\tilde{\lambda}_{my.c} > 0$. We recall that $\tilde{\lambda} = \tilde{\theta}\left[\tilde{\delta}\bar{R}^f Cov\left(\tilde{F}, \tilde{F}'\right)\right]$. The time-varying reward for consumption risk is a crucial feature of our model, and we find it in the estimation. The estimates for $\tilde{\theta}$ implied by $\tilde{\lambda}$ give a positive coefficient on the factor $(my^{max} - my_t)\,\Delta\log c^a_{t+1}$ in equation (20). For $\rho > 0, \gamma > 0$, $-\gamma\rho B_1 > 0$ implies that $B_1 < 0$. B_1 is the coefficient on aggregate consumption growth in equation (16). A negative consumption growth shock (a recession) increases the extent to which households run into binding collateral constraints (the aggregate weight shock) and hence the equity risk premium. This is the effect predicted by the theory.

We find that the asset pricing data are inconclusive with respect to the sign of ϕ, which captures the degree of substitutability between non-durable and housing services consumption in the utility function. The implied estimate for $\tilde{\theta}^a_2$, which is approximately equal to ϕ in the representative agent framework (line 2), is -.60. However, once the collateral effect is taken into account (e.g. line 7) the implied parameter estimate for $\tilde{\theta}^a_2$ is positive (1.03).

The intercept in the cross-sectional regression, $\tilde{\lambda}_0$ is a measure of the difference between the borrowers' and the lenders' risk-free rate. It should be close to zero. Its estimate is positive and significant in rows 1 and 2, but becomes insignificant for the collateral CCAPM.

Figure 18 compares the CCAPM and the collateral-CAPM under separability. The left panel plots the sample average excess return on each of the 26 portfolios in $T1$ against the return predicted by the standard CCAPM. It also shows the 45 degree line. This panel illustrates that the CCAPM fails to account for the variation in excess return across portfolios: The predicted returns spread along a horizontal line. The right panel, which corresponds to the estimates in row 4 of table 12, shows the returns predicted by the collateral-CAPM. The size and value portfolios line up along the 45 degree line.

Table 13 at the end of the text reports the sample average pricing errors on each of the 26 portfolios. Relative to the CCAPM, the collateral-CAPM largely eliminates the overpricing of growth stocks and the underpricing of value stocks. The average pricing error across portfolios is 3.27 percentage points for the CCAPM (first column, second to last row) but less than half as large for the collateral-CAPM (1.21 percent, last column). The errors are comparable in size and sign to the Fama & French (1993) three-factor model (second column of table 13, see also section 8.3.3). Especially the pricing errors on the small growth firms (S1B1 and S1B2) and large growth and value firms (S5B1, S5B4, S5B5) are lower than for the three-factor model. The last row of the table shows a χ^2-distributed test statistic for the null hypothesis that all pricing errors are zero. The collateral-CAPM is the only model for which the hypothesis of zero pricing errors cannot be rejected at the 5 percent level.[11]

As a robustness check, we relax the Markov assumption that we imposed on the aggregate weight shock $\{G\}$ in section (5.2). We include additional lags of the aggregate factors in the empirical specification of the aggregate weight process: $G_{t+1}\left(my_{t+1}, (F^a)_{t+1}^k\right)$. This introduces additional asset pricing factors in the unconditional β-representation. Figure 1 reports the estimation results for $k \in 1, 2, 3, 4$ for the sample 1929-2002. They show that the fit of the cross-sectional estimation does not improve significantly and the extra factors enter insignificantly. Only for $k = 4$ is there some additional explanatory power. We conclude that the Markov assumption on $\{G\}$ fits the data well.

As second robustness exercise, we estimate the cross-sectional regression for the postwar period (1945-2002). The results are in table 14. The benchmark consumption CAPM performs much better in the postwar sample (49 percent R^2). The collateral-CAPM with separable preferences explains between 70 and 83 percent and the collateral-CAPM with non-separable preferences between 76 and 84 percent of the cross-sectional variation in the 26 portfolios. The implied coefficient estimates for B_1 are still negative, so that the post-war results confirm the presence of the collateral effect.

Time-Varying Betas Why does the collateral-CAPM help explain the value premium? In the model, a stock's riskiness is determined by the covariance of its returns with aggregate risk factors *conditional* on the state variable my. The conditional covariance reflects time-variation in risk

[11] Because of the sampling error in the regressors the Shanken correction for the χ^2 test statistics is large. This is because the macro-economic factors have a low sample variance and the size of the standard-error correction is inversely related to this variability. While increasing the standard errors, this correction reduces the χ^2 test statistic (see A.4). The result that the collateral-CAPM fails to reject the null hypothesis of zero pricing errors should be interpreted in this light.

premia. If time variation in risk premia is important for explaining the value premium, then stocks with high book-to-market ratios should have a larger covariance with aggregate risk factors in risky times, when my is low ($my^{max} - my_t$ is high), than in less risky times, when my is high ($my^{max} - my_t$ is low). This is the pattern we find in the data.

We estimate the risk exposure (the β's) for each of the twenty-five size and book-to-market portfolios and the value weighted market return. This is the first step of the Fama-MacBeth two-step procedure. To make the point more forcefully we impose separability on the preferences over housing and non-durable consumption:

$$R^{e,j}_{t+1} = \tilde{\beta}^j_0 + \tilde{\beta}^j_c \Delta \log c_{t+1} + \tilde{\beta}^j_{my}(my^{max} - my_t) + \tilde{\beta}^j_{my.c}(my^{max} - my_t)\Delta \log c_{t+1}. \tag{26}$$

Equation (26) allows the covariance of returns with consumption growth to vary with my. For each asset j, we define the conditional consumption beta as $\beta^j_t = \tilde{\beta}^j_c + (my^{max} - my_t)\,\tilde{\beta}^j_{my.c}$. We estimate equation (26) and compute the average consumption beta in good states, defined as times in which my is one standard deviation above zero, and in bad states (risky times) when my is one standard deviation below zero. Table 15 shows that the high book-to-market portfolios (B4 and B5) have a consumption β that is large when housing collateral is scarce and small in times of collateral abundance. The opposite is true for growth portfolios (B1 and B2). Moreover, the value stocks have higher consumption betas than the growth stocks in bad states, and vice versa for the good states. This is the sense in which value portfolios are riskier than growth portfolios.

The left panel of figure 19 shows that the value portfolios (B4, B5) have a high return and the growth portfolios (B1, B2) have a low return. The right panel plots realized excess returns against $\tilde{\beta}^j_{my.c}$, the exposure to the interaction term of the housing collateral ratio with aggregate consumption growth. Growth stocks in the lower left corner have a low exposure to collateral constraint risk whereas value stocks have a large exposure. So, value stocks, are riskier than growth stocks because their returns are more highly correlated with the aggregate factors when risk is high ($my^{max} - my_t$ is high) than when risk is low ($my^{max} - my_t$ is low). Because both the estimates of $\tilde{\lambda}_{my.c}$ and of $\tilde{\beta}^j_{my.c}$ are positive, value stocks are predicted to have a higher risk premium. The value premium is the compensation for the fact that high book-to-market firms pay low returns when housing collateral is scarce and constraints bind more frequently.

When preferences are non-separable, the change in rental prices and its interaction term with the housing collateral ratio enter as additional regressors in equation (26). Table 16 shows the $\tilde{\beta}$-estimates for the value weighted market return. Not only does the covariance of the market return with consumption growth increase when collateral is scarce ($my^{max} - my_t$ is high), the covariance with rental price growth does as well ($\tilde{\beta}^{vw}_{my.\rho} > 0$). The fit of the time-series regression improves from five to twenty percent once the scaled factors (the interaction terms) are included. This result shows that the covariance of the aggregate US stock market return with the aggregate risk factors is not constant as predicted by the static CCAPM, but varies with the housing collateral ratio.

8.3.3 Comparison Across Models

The cross-sectional explanatory power of the collateral-CAPM proposed in this paper compares favorably to other asset pricing models. Table 17 compares return-based asset pricing models in rows 1-3 with consumption-based models in rows 4-6.

The capital asset pricing model relates the returns on stocks to their correlation with the return on the wealth portfolio. In the standard CAPM of Lintner (1965), the return on the wealth portfolio is proxied by the market return R^{vw} (row 1). It explains 28 percent of annual returns.

Because stock market wealth is an incomplete total wealth measure, Jagannathan & Wang (1996) include the return on human wealth in the return on the wealth portfolio. That return is measured by the growth rate in labor income (plus transfers). The R^2 in row 2 increases slightly to 37 percent. In contrast to Jagannathan & Wang (1996), this paper assumes that human wealth cannot be traded. Human wealth affects financial returns through the housing collateral ratio, and not directly. In addition, our model points towards another often ignored source of wealth: housing. Kullmann (2002) investigates the improvements to the CAPM when residential housing wealth is incorporated into the definition of wealth. Here, housing wealth affects returns only through the collateral ratio.

In the economy of Santos & Veronesi (2001), times in which investors finance a large fraction of consumption out of labor income (lc is low), are less risky. Their conditional CAPM explains 53 percent of the annual returns (row 3).[12]

The Fama & French (1993) three-factor model adds a size and a book-to-market factor to the standard CAPM. The size factor is the return on a hedge portfolio that goes long in small firms and short in big firms (smb). The value factor is the return on a hedge portfolio that goes long in high book-to-market firms and short in low book-to-market firms (hml). This model accounts for 78 percent of the cross-sectional variation in annual returns (row 7). There is a 2.7 percent per annum size premium and a 6.3 percent value premium in our sample. Given its good fit, this model serves as the empirical benchmark.

In contrast to the previous models, the consumption-based asset pricing models measure the riskiness of an asset by its covariance with marginal utility growth. One of the objectives of this literature has been to identify macroeconomic sources of risk that can explain the empirical success of the Fama & French (1993) size and book-to-market factors. The fourth row reports the standard CCAPM of Breeden (1979). Lettau & Ludvigson (2001b) explore a conditional version of the CCAPM with the consumption-wealth ratio as scaling variable. The market price of consumption risk increases in times with low cay (recessions). The Lettau-Ludvigson model explains 89 percent of the annual cross-sectional variation.

Model 6 is our collateral-CCAPM under separability and scaling variable $myrw^{max} - myrw_t$. The model goes a long way in accounting for the cross-sectional differences in returns on the 25 Fama-French portfolios and the market return. The R^2 of 88 percent improves upon the fit

[12] The authors also investigate a scaled version of the CCAPM, as we do, but their results for the scaled CCAPM are not as strong as for the scaled CAPM.

of the Fama-French model. Table 18 investigate residual explanatory power of the idiosyncratic portfolio characteristics, size and value. The top panel includes the log market capitalization into the regression, the bottom panel the log value weighted book-to-market ratio of the portfolio. In contrast with the return-based models and the static CAPM, there are no residual size nor value effects in the collateral-CCAPM. The same conclusion is found for the post-war sample.

9 Conclusion

This paper develops a general equilibrium asset pricing model with housing collateral. Agents have to back up their state-contingent promises with the value of their house. Time variation in the price of housing induces time variation in the economy's ability to share labor income risk. In recessions, the collateral value is low and there is an endogenous increase in idiosyncratic risk. Agents demand a higher risk premium to hold equity. The housing collateral mechanism endogenously generates heteroskedasticity and counter-cyclicality in the market price of risk. This is a novel feature of the model.

A fully calibrated version of the model, in which the housing collateral ratio moves endogenously, delivers a market price of risk that is high when housing collateral is scarce. The collateral effect works in a similar way as the Constantinides & Duffie (1996) mechanism, where the labor income dispersion goes up in recessions. In contrast, the movements in the housing collateral ratio are endogenous. We also find that, for calibrated dividend processes on growth and value stocks, the model generates endogenously a value premium of the magnitude observed in the data. Finally, the model is capable of generating a realistic unconditional excess stock market return.

We specify conditions under which the stochastic discount factor in the model is a linear function of current, aggregate variables only. The calibrated model supports this specification. This support solidifies an empirical test of the model using a linear asset pricing kernel in only a few aggregate factors.

Asset pricing evidence supports the collateral effect. First, the housing collateral ratio predicts aggregate returns, mainly at lower frequencies. Second, conditional on the housing collateral ratio, the consumption-CAPM explains the cross-sectional variation in size and book-to-market portfolio returns at least as well as the Fama & French (1993) model.

Support for the collateral effect is also found in a model with housing market frictions, discussed in Lustig & VanNieuwerburgh (2002). That model generates similar cross-sectional asset pricing results, but the asset pricing factors are constructed with regional data for U.S. metropolitan areas. Using the same regional data set, we find direct evidence for the mechanism underlying the pricing results: time variation in the extent of risk-sharing. Our theory only predicts strong consumption growth correlations when my is high. The data seem to support this qualification; conditioning on my weakens the consumption growth puzzle for US regions.

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A Appendix

A.1 Equilibrium Prices and Allocations

This appendix spells out the household problem in an economy where all trade takes place at time zero. It defines a competitive equilibrium and shows the equivalence between this problem and the sequential trade problem in section 3.3 of the main text. Finally, we outline a procedure to compute time-zero equilibria.

Household Problem A household of type (θ_0, s_0) purchases a complete contingent consumption plan $\{c(\theta_0, s_0), h(\theta_0, s_0)\}$ at time-zero market state prices $\{p, p\rho\}$. The household solves:

$$\sup_{\{c,h\}} U(c(\theta_0, s_0), h(\theta_0, s_0))$$

subject to the time-zero budget constraint

$$\Pi_{s_0}\left[\{c(\theta_0, s_0) + \rho h(\theta_0, s_0)\}\right] \leqslant \theta_0 + \Pi_{s_0}\left[\{\eta\}\right],$$

and an infinite sequence of collateral constraints for each t

$$\Pi_{s^t}\left[\{c(\theta_0, s_0) + \rho h(\theta_0, s_0)\}\right] \geq \Pi_{s^t}\left[\{\eta\}\right], \forall s^t.$$

Computation It is more convenient to work with the dual problem for the household. Given Arrow-Debreu prices $\{p, \rho\}$ the household with label (w_0, s_0) minimizes the cost $C(\cdot)$ of delivering initial utility w_0 to itself:

$$\begin{aligned} C(w_0, s_0) \quad &= \quad \min_{\{c,h\}} \left(c_0(w_0 s_0) + h_0(w_0, s_0)\rho_0(s_0)\right) \\ &\quad + \sum_{s^t} p(s^t|s_0)\left(c_t(w_0, s^t|s_0) + h_t(w_0, s^t|s_0)\rho_t(s^t|s_0)\right) \end{aligned}$$

subject to the promise-keeping constraint

$$U_0(\{c\}, \{h\}; w_0, s_0) \geq w_0$$

and the collateral constraints

$$\Pi_{s^t}\left[\{c(w_0, s_0) + \rho h(w_0, s_0)\}\right] \geq \Pi_{s^t}\left[\{\eta\}\right], \forall s^t.$$

The initial promised value w_0 is determined such that the household spends its entire initial wealth:

$$C(w_0, s_0) = W_0 = \theta_0 + \Pi\left[\{\eta\}\right].$$

There is a monotone relationship between θ_0 and w_0.

The above problem is a convex programming problem. We set up the saddle point problem and then make it recursive by defining cumulative multipliers (Marcet & Marimon (1999)). Let ℓ be the Lagrange multiplier on the promise keeping constraint and $\gamma_t(w_0, s^t)$ be the Lagrange multiplier on the collateral constraint in history s^t. Define a cumulative multiplier at each node: $\zeta_t(w_0, s^t) = 1 - \sum_{s^t} \gamma_t(w_0, s^t)$. Finally, we rescale the market state price $\hat{p}_t(s^t) = p_t(z^t)/\delta^t \pi_t(s^t|s_0)$. By using Abel's partial summation formula and the law of iterated expectations to the Lagrangian, we obtain an objective function that is a function of the cumulative multiplier process ζ^i :

$$D(c, h, \zeta; w_0, s_0) = \sum_{t \geq 0} \sum_{s^t} \left\{ \delta^t \pi(s^t|s_0) \left[\begin{array}{c} \zeta_t(w_0, s^t|s_0)\hat{p}_t(s^t)\left(c_t(w_0, s^t) + \rho_t(s^t)h_t(w_0, s^t)\right) \\ +\gamma_t(w_0, s^t)\Pi_{s^t}\left[\{\eta\}\right] \end{array} \right] \right\}$$

such that

$$\zeta_t(w_0, s^t) = \zeta_{t-1}(w_0, s^{t-1}) - \gamma_t(w_0, s^t), \ \zeta_0(w_0, s_0) = 1$$

Then the **recursive dual** saddle point problem is given by:

$$\inf_{\{c,h\}} \sup_{\{\zeta\}} D(c,h,\zeta;w_0,s_0)$$

such that

$$\sum_{t\geq 0}\sum_{s^t}\delta^t \pi(s^t|s_0)u(c_t(w_0,s^t),h_t(w_0,s^t)) \geq w_0$$

To keep the mechanics of the model in line with standard practice, we re-scale the multipliers. Let

$$\xi_t(\ell,s^t) = \frac{\ell}{\zeta_t(w_0,s^t)},$$

where ℓ is the Lagrange multiplier on the promise keeping constraint. The cumulative multiplier $\xi(\ell,s^t)$ is a non-decreasing stochastic sequence (sub-martingale). If the constraint for household (ℓ,s_0) binds, it goes up, else it stays put.

First Order Necessary Conditions The f.o.c. for $c(\ell,s^t)$ is :

$$\hat{p}(s^t) = \xi_t(\ell,s^t)u_c(c_t(\ell,s^t),h_t(\ell,s^t)).$$

Upon division of the first order conditions for any two households ℓ' and ℓ'', the following restriction on the joint evolution of marginal utilities over time and across states must hold:

$$\frac{u_c(c_t(\ell',s^t),h_t(\ell',s^t))}{u_c(c_t(\ell'',s^t),h_t(\ell'',s^t))} = \frac{\xi_t(\ell'',s^t)}{\xi_t(\ell',s^t)}.$$

Growth rates of marginal utility of non-durable consumption, weighted by the multipliers, are equalized across agents:

$$\frac{\xi_{t+1}(\ell',s^{t+1})}{\xi_t(\ell',s^t)}\frac{u_c(c_{t+1}(\ell',s^{t+1}),h_{t+1}(\ell',s^{t+1}))}{u_c(c_t(\ell',s^t),h_t(\ell',s^t))} = \frac{\hat{p}_{t+1}(s^{t+1})}{\hat{p}_t(s^t)} = \frac{\xi_{t+1}(\ell'',s^{t+1})}{\xi_t(\ell'',s^t)}\frac{u_c(c_{t+1}(\ell'',s^{t+1}),h_{t+1}(\ell'',s^{t+1}))}{u_c(c_t(\ell'',s^t),h_t(\ell'',s^t))}.$$

The time zero ratio of marginal utilities is pinned down by the ratio of multipliers on the promise-keeping constraints. For $t>0$, it tracks the stochastic weights ξ. From the first order condition w.r.t. $\xi_t(\ell,s^t)$ we obtain a reservation weight policy:

$$\xi_t(\ell,s^{t+1}) = \begin{cases} \underline{\xi}_t(\ell,s^{t+1}) \text{ if } \xi_t(\ell,s^t) < \underline{\xi}_t(\ell,s^{t+1}) \\ \xi_t(\ell,s^t) \text{ elsewhere} \end{cases} \tag{28}$$

and the collateral constraints hold with equality at the bounds:

$$\Pi_{s^t}\left[\left\{c_t(\ell,s^t;\underline{\xi}_t(\ell,s^t)) + \rho h^i(\ell,s^t;\underline{\xi}(\ell,s^t))\right\}\right] = \Pi_{s^t}\left[\{\eta\}\right].$$

Optimal Weight Policy Proof of Proposition 1: Let $C(\ell,y_t,z^t)$ denote the cost of claim to consumption in state (y_t,z^t) for a household who enters the period with weight ξ . The cutoff rule $\ell^c(y_t,z^t)$ is determined such that the solvency constrain binds exactly: $\Pi_{y,z^t}\left[\{\eta\}\right] = C(\xi,y_t,z^t)$, where $C(\xi,y_t,z^t)$ is defined recursively as:

$$C(\xi,y_t,z^t) = \frac{\ell^c(y_t,z^t)}{\xi^a_t(z^t)}\left(1+\psi r^\sigma_t\right) + \beta\sum_{z_{t+1}}\pi(z_{t+1}|z_t)\sum_{y'}\frac{\pi(y_{t+1},z_{t+1}|y_t,z_t)}{\pi(z_{t+1}|z_t)}m_{t+1}(z^{t+1})C(\xi',y_{t+1},z^{t+1}),$$

where ξ' is determined by the cutoff rule. Note that $m_{t+1}(z^{t+1})$ does not depend on $r_t(z^t)$ because we assumed that utility is separable. This also implies that the cost of a claim to labor income $\Pi_{y,z^t}\left[\{\eta\}\right]$ does not depend on r.

We first proof the result for a finite horizon version of this economy. In the last period T, the cutoff rule is determined such that:

$$\eta(y_{T-1},z^{T-1}) = \frac{\ell^c(y_{T-1},z^{T-1})}{\xi^a_T(z^{T-1})}\left(1+\psi r^\sigma_{T-1}\right) + \beta\sum_{z_{t+1}}\pi(z_T|z_{T-1})\sum_{y'}\frac{\pi(y_T,z_T|y_{T-1},z_{T-1})}{\pi(z_T|z_{T-1})}m_T(z^T|z_{T-1})\frac{\xi'^{1/\gamma}}{\xi^a_T(z^T)},$$

where $\frac{\xi'^{1/\gamma}}{\xi^a_T(z^T)} \geq \eta\left(y_T, z^T\right)$. Given $r^1_{T-1} < r^2_{T-1}$, this implies that $\ell^{1,c}(y_{T-1}, z^{T-1}) < \ell^{2,c}(y_{T-1}, z^{T-1})$ for all (y_{T-1}, z^{T-1}). By backward induction we get that, for a given sequence of $\left\{\xi^a_t(z^t)\right\}$, $\ell^{1,c}(y_t, z^t) < \ell^{2,c}(y_t, z^t)$ for all nodes (y_t, z^t) in the finite horizon economy. This in turn implies that $\left\{\xi^{a,1}_t(z^t)\right\} \leq \left\{\xi^{a,2}_t(z^t)\right\}$ for all z^t. This follows directly from the definition of

$$\xi^a_t(z^t) = \sum_{y^t} \int \xi_t(\ell, y^t, z^t) \frac{\pi(z^t, y^t|z_0, y_0)}{\pi(z^t|z_0)} d\Phi_0 \tag{29}$$

$$= \sum_{y^t} \int_{\ell^c(y_t,z^t)} \xi_{t-1}(\ell, y^t, z^t) \frac{\pi(z^t, y^t|z_0, y_0)}{\pi(z^t|z_0)} d\Phi_0 \tag{30}$$

$$+ \sum_{y^t} \int^{\ell^c(y_t,z^t)} \ell^c(y_t, z^t) \frac{\pi(z^t, y^t|z_0, y_0)}{\pi(z^t|z_0)} d\Phi_0 \tag{31}$$

$\xi^a_t(z^t)$ is non-decreasing in $\ell^c(y_t, z^t)$.

The proof extends to the infinite horizon economy if the transition matrix has no absorbing states. The reason is that $lim_{T\to\infty} E_t \left[\beta^{T-t} m_T(z^T|z_t)\pi_{z^T,y_T}\right]$ does not depend on the current state (y_t, z_t). q.e.d.

Proof of Corollary 2: Follows from the definition of the cutoff level in the previous proof. For a given sequence of $\left\{\xi^a_t(z^t)\right\}$, it is obvious that $\ell^{1,c}(y_t, z^t) > \ell^{2,c}(y_t, z^t)$ for all nodes (y_t, z^t). This in turn implies that $\left\{\xi^{a,1}_t(z^t)\right\} \geq \left\{\xi^{a,2}_t(z^t)\right\}$. This follows directly from the definition of the aggregate weight shock (31). As a result, $\xi^a_t(z^t)$ is non-decreasing in $\ell^c(y_t, z^t)$. q.e.d.

A.2 Sequential versus Time-Zero Constraints

We show under which conditions the sequence of budget constraints and collateral constraints in the sequential market setup can be rewritten as one time-zero budget constraint and the collection of collateral constraints shown in equation (11). The proof strategy follows Sargent (1984) (Ch. 8).

Budget Constraint First, we show how the Arrow-Debreu budget constraint obtains from aggregating successive sequential budget constraints. The sequential budget constraint is:

$$c_t(\ell, s^t) + \rho_t(z^t)h^r_t(\ell, s^t) + \sum_{s'} q_t(s^t, s')a_t(\ell, s^t, s') + p^h_t(s^t)h^o_{t+1}(\ell, s^t) \leq W_t(\ell, s^t).$$

Next period wealth is:

$$W_{t+1}(\ell, s^t, s') = \eta_{t+1}(s^t, s') + a_t(\ell, s^t, s') + h^o_{t+1}(\ell, s^t)\left[p^h_{t+1}(s^t, s') + \rho_{t+1}(s^t, s')\right].$$

Multiply the second equation by $q_{t+1}(s')$ and sum over states. Then substitute the expression for $\sum q_{t+1}(s')a_{t+1}(s')$ into the first equation.

$$c_t + \rho_t h^r_t + \sum_{s'} q_{t+1}(s')W_{t+1}(s')$$
$$\leqslant W_t + \sum_{s'} q_{t+1}(s')\eta_{t+1}(s') +$$
$$h^o_{t+1}\left(\sum_{s'} q_{t+1}(s')\left[p^h_{t+1}(s') + \rho_{t+1}(s')\right] - p^h_t\right).$$

Similarly, for period $t+1$:

$$c_{t+1} + \rho_{t+1} h^r_{t+1} + \sum_{s''} q_{t+2}(s'') W_{t+2}(s'') \quad \leqslant \quad W_{t+1} + \sum_{s''} q_{t+2}(s'') \eta_{t+2}(s'') + h^o_{t+2} \left(\sum_{s''} q_{t+2}(s'') \left[p^h_{t+2}(s'') + \rho_{t+2}(s'') \right] - p^h_{t+1} \right).$$

Substituting the expression for $t+1$ into the expression for t by substituting out W_{t+1}, we get:

$$c_t + \rho_t h^r_t + \sum_{s'} q_{t+1}(s') \left[c_{t+1} + \rho_{t+1} h^r_{t+1} \right] + \sum_{s'} \sum_{s''} q_{t+1}(s') q_{t+2}(s'') W_{t+2}(s'') \leqslant$$

$$W_t + \sum_{s'} q_{t+1}(s') \eta_{t+1}(s') + \sum_{s'} \sum_{s''} q_{t+1}(s') q_{t+2}(s'') \eta_{t+2}(s'') + h^o_{t+1} \left(\sum_{s'} q_{t+1}(s') \left[p^h_{t+1}(s') + \rho_{t+1}(s') \right] - p^h_t \right) +$$

$$\sum_{s'} q_{t+1}(s') h^o_{t+2}(s') \left(\sum_{s''} q_{t+2}(s'') \left[p^h_{t+2}(s'') + \rho_{t+2}(s'') \right] - p^h_{t+1} \right).$$

Let Π_{s^t} be the value of a dividend stream $\{d\}$ starting in history s^t priced using the market state prices $\{p\}$:

$$\Pi_{s^t}\left[\{d\}\right] = \sum_{j \geq 0} \sum_{s^{t+j}|s^t} p_{t+j}(s^{t+j}) d_{t+j}(s^{t+j}),$$

where for a given path s^{t+j} following history s^t, p is defined as

$$p_{t+j}(s^{t+j}|s^t) = q_{t+j}\left(s^{t+j}|s^{t+j-1}\right) q_{t+2}(s^{t+2}|s^{t+1}) \ldots q_{t+1}(s^{t+1}|s^t).$$

Repeating the successive substitutions, the budget set is given by

$$\Pi_{s^t}\left[\{c + \rho h^r\}\right] \leqslant W_t - \eta_t + \Pi_{s^t}\left[\{\eta\}\right] \tag{32}$$

under 2 assumptions: (1) the transversality condition

$$\lim_{j \to \infty} \sum_{s^{t+j}} p_{t+j}(s^{t+j}) W_{t+j}(s^{t+j}) = 0, \tag{33}$$

is satisfied and (2) there are no arbitrage opportunities:

$$p^h_{t+j-1}(s^{t+j-1}) = \sum_{s^{t+j}|s^{t+j-1}} q_{t+j}(s^{t+j}) \left[p^h_{t+j}(s^{t+j}) + \rho_{t+j}(s^{t+j}) \right], \qquad \forall j \geq 0, \forall s^{t+j} \tag{34}$$

If the latter condition were not satisfied, a household could achieve unbounded consumption by investing sufficiently high amounts in housing shares h^o and financing this by borrowing. This is a feasible strategy because ownership shares in the housing tree are collateralizable.

Because $W_0 = \eta_0 + \theta_0$, and relabelling $h^r_t = h_t$, we obtain the Arrow-Debreu budget constraint

$$\Pi_{s^0}\left[\{c + \rho h\}\right] \leqslant \theta_0 + \Pi_{s^0}\left[\{\eta\}\right]$$

Collateral Constraints Second, we show the equivalence between the collateral constraints of the sequential markets setup and the solvency constraint in the static economy. The sequential collateral constraints are:

$$\left[p^h_t(z^t) + \rho_t(z^t) \right] h^o_{t-1}(s^{t-1}) + a_{t-1}(s^{t-1}, s_t) \geq 0,$$

and the collateral constraints in a history s^t:

$$\Pi_{s^t}\left[\{c+\rho h\}\right] \geq \Pi_{s^t}\left[\{\eta\}\right]. \tag{35}$$

The equivalence follows if and only if

$$a_{t-1}(s^{t-1}, s_t) + h^o_{t-1}(s^{t-1})\left[p^h_t(z^t) + \rho_t(z^t)\right] = \Pi_{s^t}\left[\{c+\rho h - \eta\}\right].$$

But this follows immediately from the budget constraint (32) holding with equality and the definition of W:

$$W_t(s^t) - \eta_t(s) = a_{t-1}(s^{t-1}, s_t) + h^o_{t-1}(s^{t-1})\left[p^h_t(z^t) + \rho_t(z^t)\right].$$

Under conditions (33) and (34) an allocation that is feasible and immune to the threat of default in sequential markets is feasible and immune to the threat of default in time-zero markets.

The equivalence implies that the allocation of home-ownership h^o is indeterminate in the sequential economy.

A.3 Bootstrap Procedure

The bootstrap procedure addresses the persistent-regressor bias of the OLS coefficient estimators and serial correlation in the OLS residuals in the estimation of equation (25). We compute small-sample coefficient estimates and small-sample p-values for the null hypothesis of no predictability.

A univariate specification test shows that my is best described by an AR(2) process.

$$\left(\frac{my^{max} - my_{t+1}}{my^{max} - my^{min}}\right) = c_0 + c_1\left(\frac{my^{max} - my_t}{my^{max} - my^{min}}\right) + c_2\left(\frac{my^{max} - my_{t-1}}{my^{max} - my^{min}}\right) + v_{t+1}, \tag{36}$$

where v is i.i.d. mean zero.

If annual returns are truly unforecastable, the data generating process for $\left(r^{vw,e}_{t+1}\right)$ is

$$\begin{aligned} r^{vw,1}_{t+1} &= b^{np}_{0,1} + e^1_{t+1} \\ r^{vw,2}_{t+2} &= b^{np}_{0,2} + e^1_{t+1} + e^1_{t+2} \\ r^{vw,K}_{t+K} &= b^{np}_{0,K} + e^1_{t+1} + e^1_{t+2} + ... + e^1_{t+K} \end{aligned} \tag{37}$$

where e^1 is an i.i.d. mean zero process of innovations to the 1-year log excess return. Under the no-predictability null, K-period returns have a MA(K) error structure because of overlapping observations.

Similarly, with predictability

$$\begin{aligned} r^{vw,1}_{t+1} &= b^{wp}_{0,1} + b^{wp}_{1,1}\left(\frac{my^{max} - my_t}{my^{max} - my^{min}}\right) + u^1_{t+1}. \\ r^{vw,2}_{t+2} &= b^{wp}_{0,2} + b^{wp}_{1,2}\left(\frac{my^{max} - my_t}{my^{max} - my^{min}}\right) + u^1_{t+1} + u^1_{t+2}. \\ r^{vw,K}_{t+K} &= b^{wp}_{0,K} + b^{wp}_{1,K}\left(\frac{my^{max} - my_t}{my^{max} - my^{min}}\right) + u^1_{t+1} + u^1_{t+2} + ... + u^1_{t+K} \end{aligned} \tag{38}$$

where u^1 is an i.i.d. mean zero process of innovations to the 1-year log excess return.

In a preliminary step, we estimate the no-predictability coefficients $b^{np}_{0,1}$ through $b^{np}_{0,K}$ in K OLS regressions of the form of equation (37). We do the same for the with-predictability coefficients $\left(b^{wp}_{0,1}, b^{wp}_{1,1}\right)$ through $\left(b^{wp}_{0,K}, b^{wp}_{1,K}\right)$ in K OLS regressions of the form of equation 38. We store the residuals $\left\{e^1, u^1\right\}$.

The bootstrap exercise for the small sample bias consists of the following steps.

step 1 Draw a sample of length T with replacement from the residuals $\left\{u^1, v\right\}$ obtained in preliminary step.

step 2 For given my_0, my_1 (which we set equal to the sample values) and parameter estimates from preliminary step, build up time series for $r^{e,K}$ and my_{t+1} recursively from equations (38) and (36), for $K \in \{1, ..., 10\}$.

This takes into account the MA(K) structure of the innovations.

step 3 Estimate by OLS the intercept and the coefficient on the rescaled housing collateral ratio in the return equation. Let the coefficient on my be $b_{1,K}^{n,*}$, for $K \in \{1, ..., 10\}$.

step 4 Repeat steps 1 through 4 $N = 20,000$ times.

The small sample coefficient estimate is $\frac{1}{N}\sum b_{1,K}^{n,*}$. The bias equals $\frac{1}{N}\sum b_{1,K}^{n,*} - b_{1,K}^{LS}$.

The second bootstrap exercise (for the no-predictability p-value) proceeds as the first, except it imposes the null hypothesis of no predictability in step 1. It consists of the following steps.

step 1 Draw a sample of length T with replacement from the residuals $\{e^1, v\}$ obtained in preliminary step.

step 2 For given my_0, my_1 (which we set equal to the sample values) and parameter estimates from preliminary step, build up time series for $r^{e,K}$ and my_{t+1} recursively from equations (37) and (36), for $K \in \{1, ..., 10\}$. This takes into account the MA(K) structure of the innovations.

step 3 Estimate by OLS the intercept and the coefficient on the rescaled housing collateral ratio in the return equation. Let the coefficient on my be $b_{1,K}^{n,**}$, for $K \in \{1, ..., 10\}$.

step 4 Repeat steps 1 through 4 $N = 20,000$ times.

The p-value is the frequency of observing estimates $b_{1,K}^{n,**}$ smaller than the least-squares estimate $b_{1,K}^{LS}$:

$$p = 1 - \frac{1}{N}\sum_{n=1}^{N} \mathbb{I}_{b_{1,K}^{n,**} > b_{1,K}^{LS}}.$$

It's the p-value of a two-sided test of no predictability.

A.4 Fama-MacBeth Procedure

The estimation problem is

$$E\left[R_{t+1}^{e,j}\right] = \tilde{\lambda}\tilde{\beta}^{j\prime},$$

with

$$\begin{aligned} \tilde{\lambda} &= \theta Cov\left(F_{t+1}, F_{t+1}'\right), \\ \tilde{\beta}^j &= Cov\left(F_{t+1}, F_{t+1}'\right)^{-1} Cov\left(F_{t+1}, R_{t+1}^{e,j}\right). \end{aligned}$$

First, for each j, the vector $\tilde{\beta}^j$ is obtained from the time-series regression of returns on the factors. Given the limited length of the time series, the β are estimated using 1 regression over the entire sample instead of a rolling regression.

$$R_t^{e,j} = \beta_0^j + \tilde{\beta}_a^{j\prime} F_t^a + \tilde{\beta}_c^{j\prime} F_t^c + \epsilon_t^j \qquad t = 1, 2, ..., T. \tag{39}$$

Let $\Sigma = E\left[\epsilon_t \epsilon_t'\right]$ be the $N \times N$ covariance matrix of the errors $\epsilon_t = \left[\epsilon_t^1, ..., \epsilon_t^N\right]$. Second, for each t, a cross-sectional regression of returns on the estimated $\tilde{\beta}^i$ uncovers estimates for $\left(\tilde{\lambda}_a, \tilde{\lambda}_c\right)$ and the zero-β return $\tilde{\lambda}_0$:

$$R_t^{e,j} = \tilde{\lambda}_0 + \tilde{\beta}_{a,t}^{j\prime} \tilde{\lambda}_{a,t} + \tilde{\beta}_{c,t}^{j} \tilde{\lambda}_{c,t} + \alpha_t^j \qquad j = 1, 2, ..., J.$$

The estimator for the price of risk is the time series average of the estimated second stage coefficients: $\tilde{\lambda} = \frac{1}{T}\sum_{t=1}^{T} \tilde{\lambda}_t$. The α_t^j are the pricing errors, $\hat{\alpha}^j = \frac{1}{T}\sum_{t=1}^{T} \hat{\alpha}_t^j$. The sampling error for $\hat{\lambda}$ and the covariance matrix for $\hat{\alpha}$ are given by:

$$\begin{aligned} var\left(\hat{\lambda}_{FMB}\right) &= \frac{1}{T^2}\left[\sum_{t=1}^{T}\left(\hat{\lambda}_t - \hat{\lambda}\right)^2\right] \\ cov\left(\hat{\alpha}_{FMB}\right) &= \frac{1}{T^2}\left[\sum_{t=1}^{T}\left(\hat{\alpha}_t - \hat{\alpha}\right)\left(\hat{\alpha}_t - \hat{\alpha}\right)'\right] \end{aligned}$$

The pricing errors are the basis for a goodness of fit statistic $\hat{\alpha} cov\left(\hat{\alpha}\right)^{-1}\hat{\alpha}$. In a regression with K factors the statistic has a χ^2 distribution with $J-K$ degrees of freedom. A second measure of fit is the R^2 constructed from the cross-sectional variance of the time-averages (denoted by a bar) of errors and returns for each of the portfolios:

$$R^2_{FMB} = 1 - \frac{var\left(\overline{\hat{\alpha}^i_t}\right)}{var(\overline{R^i_{t+1} - R^f_t})}$$

The $\beta' s$ are generated regressors from a first stage time-series analysis. Generally, this error in variables problem gives rise to an underestimation of the asymptotic variance-covariance matrix. The standard errors for λ and α are corrected following Shanken (1992). Let the matrix Σ_f be the covariance matrix of the factors F. The Shanken correction to the variance of the estimator $\hat{\lambda}$ and the covariance matrix of pricing errors α is:

$$\begin{aligned} Var\left(\hat{\lambda}_{corr}\right) &= Var\left(\hat{\lambda}_{uncorr}\right)\left(1+\hat{\lambda}'\Sigma_F^{-1}\hat{\lambda}\right) + \frac{1}{T}\Sigma_F \\ cov\left(\hat{\alpha}_{corr}\right) &= \left(1+\hat{\lambda}'\Sigma_F^{-1}\hat{\lambda}\right) cov\left(\hat{\alpha}_{uncorr}\right). \end{aligned}$$

Cochrane (2001) (pp.241-242) describes a GMM procedure that carries out the time series and cross-sectional estimation jointly. It corrects for serial correlation and conditional heteroskedasticity in the residuals ϵ^i_t and for correlation of α^i_t across assets.

A.5 Data Appendix

See www.stanford.edu/˜ svnieuwe/data_aggr.pdf.

B Tables and Figures

γ	σ	$E[R^e]$	$\sigma[R^e]$	$E[R^s]$	$\sigma[R^s]$	$E[R^f]$	$\sigma[R^f]$	$E[my]$
6	-3	3.90	10.64	16.13	12.78	12.23	7.80	5.76
7	-3	5.01	12.14	16.08	14.86	11.06	9.42	5.75
8	-3	6.15	13.25	16.37	16.27	10.22	10.60	5.73
9	-3	7.60	14.99	16.17	18.79	8.57	12.77	5.73
10	-3	9.13	16.17	16.67	20.19	7.54	13.67	5.72
8	-2	6.34	13.80	15.85	17.00	9.51	11.24	5.70
8	-3	6.15	13.25	16.37	16.27	10.22	10.60	5.73
8	-4	6.34	13.46	16.20	16.65	9.86	10.99	5.68
8	-5	6.36	13.16	16.01	16.36	9.65	10.72	5.79
8	-6	6.33	13.49	16.09	16.60	9.76	11.14	5.85
Data	FF	7.95	20.92	11.79	20.62	3.84	3.17	
Data	MP	5.94	16.84	6.80	16.54	1.04	5.56	

Table 2: Unconditional Asset Pricing Moments Collateral Model Averages from a simulation of the model for 5,000 agents and 20,000 periods. All parameters are as in the benchmark model, except for the ones in the first column. $\bar{\rho}$ is chosen differently in each line so that the average amount of housing collateral is the same in each row: 5.8. The first block of computations holds the intratemporal elasticity of substitution constant (σ) and varies the utility curvature parameter γ. The second block holds γ constant and varies σ. The last block gives historical averages for 1927-2002 (data from Kenneth French) and for 1889-1979 (data from Mehra and Prescott).

γ	σ	$E[R^e]$	$\sigma[R^e]$	$E[R^s]$	$\sigma[R^s]$	$E[R^f]$	$\sigma[R^f]$	$E[my]$
6	-3	1.58	6.56	21.83	7.68	20.25	4.19	5.86
7	-3	1.81	7.17	23.17	8.70	21.36	5.09	5.76
8	-3	2.34	7.71	24.55	9.57	22.21	5.92	5.85
9	-3	2.95	8.21	25.79	10.52	22.84	6.78	5.85
10	-3	3.60	8.74	26.89	11.48	23.30	7.64	5.86
8	-2	2.44	7.67	24.60	9.51	22.15	5.89	5.76
8	-3	2.34	7.71	24.55	9.57	22.21	5.92	5.85
8	-4	2.47	7.68	24.58	9.53	22.10	5.90	5.86
8	-5	2.56	7.65	24.60	9.48	22.04	5.88	5.93
8	-6	2.41	7.73	24.54	9.62	22.13	5.94	5.95
Data	FF	7.95	20.92	11.79	20.62	3.84	3.17	
Data	MP	5.94	16.84	6.80	16.54	1.04	5.56	

Table 3: Unconditional Asset Pricing Moments Representative Agent Model Averages from a simulation of the representative agent model for 20,000 periods. All parameters are as in the benchmark model, except for the ones in the first column. $\bar{\rho}$ is chosen differently in each line so that the average amount of housing collateral is the same in each row: 5.8. The first block of computations holds the intratemporal elasticity of substitution constant (σ) and varies the utility curvature parameter γ. The second block holds γ constant and varies σ. The last block gives historical averages for 1927-2002 (data from Kenneth French) and for 1889-1979 (data from Mehra and Prescott).

Model	$E[R^e]$	$\sigma[R^e]$	$E[R^s]$	$\sigma[R^s]$	$E[R^f]$	$\sigma[R^f]$	$E[my]$
$\gamma = 7, \sigma = -3$	4.06	11.33	16.87	14.18	12.81	9.08	5.75
$\gamma = 8, \sigma = -3$	4.96	12.56	17.38	15.93	12.42	10.44	5.74
$\gamma = 9, \sigma = -3$	6.13	13.85	17.65	17.77	11.52	11.88	5.65
$\gamma = 10, \sigma = -3$	7.02	15.06	17.77	19.61	10.75	13.38	5.72

Table 4: No Heteroskedastic Labor Income Averages from a simulation of the model for 5,000 agents and 20,000 periods. The Storesletten, Telmer,Yaron mechanism is switched off, and only the collateral effect causes heteroskedasticity in the stochastic discount factor.

	B1	B2	B3	B4	B5	B6	B7	B8
B^s	-1	0	2	3	4	5	6	7
$\gamma = 6, \sigma = -3$	15.26	15.39	15.66	15.79	15.91	16.08	16.59	19.51
$\gamma = 8, \sigma = -3$	15.50	15.62	15.86	15.97	16.08	16.28	17.03	23.84
$\gamma = 10, \sigma = -3$	15.79	15.89	16.11	16.23	16.35	16.61	17.76	35.12

Table 5: The Value Premium in the Model. The results are for the benchmark calibration of the model. The first row gives the slope coefficient on the housing collateral ratio, B^s in the dividend share process: $\log\left(\frac{d}{e}\right) = B^s my$. The second through fourth rows report the model-implied average stock returns $E\left[R^{vw}\right]$ on the portfolios for the corresponding dividend process.

VW	Mean	St. Dev	Mean	St. Dev.	B/M
RVW	7.9	20.9			
S1B1	3.8	38.0	7.3	40.4	0.35
S1B2	9.7	37.4	15.6	45.2	0.70
S1B3	13.8	35.9	17.6	40.1	1.03
S1B4	17.8	44.6	22.1	53.4	1.55
S1B5	18.2	37.6	26.2	48.6	5.52
S2B1	6.9	32.3	7.1	35.5	0.38
S2B2	11.8	30.3	12.6	32.6	0.70
S2B3	13.7	30.5	15.0	33.6	1.03
S2B4	14.7	32.8	15.3	35.1	1.52
S2B5	15.1	33.0	16.5	36.2	3.76
S3B1	8.5	30.5	8.0	30.2	0.38
S3B2	11.4	28.0	11.7	29.8	0.69
S3B3	12.3	27.2	12.8	28.2	1.02
S3B4	13.1	27.8	13.7	28.1	1.51
S3B5	13.9	32.6	14.9	32.8	3.40
S4B1	8.4	24.0	8.4	24.5	0.37
S4B2	9.2	25.6	9.4	26.2	0.69
S4B3	11.1	25.9	11.4	26.9	1.01
S4B4	12.1	27.0	12.4	27.8	1.49
S4B5	13.6	34.5	14.3	36.6	3.35
S5B1	7.6	21.6	6.9	21.1	0.33
S5B2	7.2	19.5	8.4	20.3	0.68
S5B3	8.8	22.1	9.5	23.7	1.00
S5B4	9.5	25.4	10.6	27.3	1.50
S5B5	11.0	33.7	11.5	34.4	1.59

Table 6: Annual Portfolio Returns 1927-2002. Time-series mean and standard deviation of gross portfolio returns. All returns are in excess of a 1 month T-bill return. The first two columns are value-weighted portfolios, the next two for equally-weighted portfolios and the last column denotes the value weighted portfolio average book-market ratio. All data are from Kenneth French.

$mymo$	ϖ	ϑ	χ	LHR	ADF
1889-2002	-1.5164	-.0066	1.8010	21.07*	-3.46**
1925-2002	-1.2064	-.0164	2.3546	35.04***	-5.38***
1945-2002	-1.2987	-.0176	2.6511	30.77***	-3.06 **
1889-2002	-1	-.0102	1.6974		-3.08**
1925-2002	-1	-.0148	2.0624		-4.16***
1945-2002	-1	-.0233	2.8302		-2.89*
$myrw$	ϖ	ϑ	χ	LHR	ADF
1889-2002	-1.8255	.0084	-.3659	15.16	-3.46**
1925-2002	-.5480	-.0120	.1895	34.00***	-4.01***
1945-2002	-.4108	-.0147	.3311	25.00*	-3.32**
1889-2002	-1	.0011	-.4434		-2.29
1925-2002	-1	-.0023	-.1720		-3.42**
1945-2002	-1	-.0083	.3784		-3.51**
$myfa$	ϖ	ϑ	χ	LHR	ADF
1925-2001	-1.0137	-.0004	-.2257	52.01***	-4.70***
1945-2001	-1.0055	-.0011	-.1624	28.45**	-3.41**
1925-2001	-1	-.0005	-.2254		-4.65***
1945-2001	-1	-.0026	-.0365		-2.88*

Table 7: Cointegration Relationship. The second through fourth columns show coefficient estimates for the cointegration relationship. The cointegration relationship is estimated for 1889-2002, 1925-2002, and 1945-2002 for $hv = mo$ and $hv = rw$, and for 1925-2001 and 1945-2001 for $hv = fa$. Coefficient estimates for D_k are not reported. We set $K = 8$ in the VECM. The fifth column shows the likelihood ratio statistic of the Johansen cointegration test. It assumes a constant and a trend in the cointegration relationship. The last column shows the value of the ADF test statistic (with K=8 lags) of the null hypothesis of a unit root in the resulting my series. For both test, significance at the 10% level is denoted by a *, significance at the 5% level by **, and at the 1% level by ***. The second part of each block gives the parameter estimates of an OLS regression of $\log hv - \log ylt$ on a constant and a trend.

Dependent var:	$\Delta\log(hv_t^{mo})$	$\Delta\log(y_t)$	$\Delta\log(hv_t^{rw})$	$\Delta\log(y_t)$	$\Delta\log(hv_t^{fa})$	$\Delta\log(y_t)$
$\Delta\log(hv_{t-1})$	.75	-.15	.14	-.39	.33	.21
(s.e.)	(.09)	(.13)	(.10)	(.13)	(.12)	(.19)
$\Delta\log(hv_{t-2})$	-.20	.07	-.17	-.11	-.29	-.50
(s.e.)	(.09)	(.13)	(.11)	(.13)	(.12)	(.19)
$\Delta\log(y_{t-1})$	-.09	.24	-.06	.34	.04	.73
(s.e.)	(.07)	(.10)	(.09)	(.11)	(.06)	(.11)
$\Delta\log(y_{t-2})$	-.11	-.05	-.02	-.01	-.05	-.18
(s.e.)	(.07)	(.10)	(.09)	(.11)	(.07)	(.12)
my_{t-1}	**-.038**	**.023**	**-.037**	**.042**	**-.072**	**.110**
(s.e.)	(.015)	(.021)	(.027)	(.033)	(.032)	(.052)
R^2	49.3	7.6	5.8	15.3	22.8	48.4

Table 8: Estimates from Bivariate VAR. For each of the three measures of the housing collateral ratio, a bivariate VAR is estimated. The dependent variables are the current growth rate in housing wealth and in labor income plus transfers. The dependent variables are the first two lags of these variables and the one-period lagged housing collateral ratio (mymo, myrw, and myfa respectively). A constant is included in the system, but its coefficients are not reported. The sample period is 1889-2002 for the first two VAR's and 1925-2001 for the last VAR.

89-02	1	2	3	4	5	6	7	8	9	10
b_{my}^{LS}	.09	.18	.26	.33	.42	.54*	.70**	.87**	1.04***	1.14***
R^2	.01	.02	.03	.03	.04	.07	.10	.14	.18	.20
b_{my}^{ss}	.08	.16	.23	.29	.38	.49	.64	.80	.97	1.07
p	[.14]	[.14]	[.15]	[.16]	[.15]	[.13]	[.11]	[.08]	[.07]	[.06]
45-02	1	2	3	4	5	6	7	8	9	10
b_{my}^{LS}	-.01	.08	.23	.41**	.62***	.81***	1.00***	1.22***	1.49***	1.71***
R^2	.00	.00	.03	.08	.13	.18	.23	.29	.37	.43
b_{my}^{ss}	.00	.11	.28	.46	.68	.88	1.08	1.32	1.59	1.83
p	[.58]	[.43]	[.32]	[.25]	[.19]	[.16]	[.14]	[.11]	[.09]	[.07]

Table 9: Long-Horizon Predictability Regressions. The results are for the regression $R^{vw,e}_{t+K} = b_0 + b_{my}\left(\frac{my^{max} - my_t}{my^{max} - my^{min}}\right) + \epsilon_{t+K}$, where $R^{vw,e}_{t+K}$ are cumulative excess returns on the S&P Composite Index over a K-year horizon. The housing collateral ratio my is $mymo$, estimated on the entire sample in the first panel and on the postwar sample in the second panel. my^{max} and my^{min} are the maximum and minimum observation on $mymo$ in the respective samples. The first row reports least squares estimates for b_{my}. Newey-West HAC standard errors are used to denote significance at the 1 percent (***), 5 percent (**) and 10 percent level (*). The second row reports the R^2 for this OLS regression. The third row reports small sample coefficient estimates (see A.3). The fourth row gives the p-value of the null hypothesis of no predictability. For 1889-2002, the sample size decreases from 113 observations for K=1 to 104 years for K=10. For 1945-2002, it decreases from 57 observations for K=1 to 48 years for K=10.

	1926-2002	1	2	3	4	5	6	7	8	9	10
1	*mymo*	.16	.28	.39	.48	.60	.73*	.89**	1.05***	1.20***	1.31***
	R^2	.02	.03	.04	.05	.07	.10	.16	.20	.23	.24
2	$\log d - \log p$	.14***	.25***	.33**	.36*	.42	.51*	.66**	.83***	1.03***	1.20***
	R^2	.07	.09	.09	.08	.08	.10	.17	.23	.31	.35
3	*cay*	-.13	-.27	-.51	-.72	-.81	-.75	-.67	-.64	-.55	-.55
	R^2	.00	.01	.02	.03	.04	.03	.02	.02	.01	.01
4	*lc*	.22	.41	.57	.66	.73	.72	.73	.74	.74	.81
	R^2	.01	.02	.03	.04	.04	.04	.04	.04	.03	.03
5	*mymo*	.09	.17	.25	.35	.46	.57*	.69**	.80***	.90***	.97***
	$\log d - \log p$	.13**	.22**	.30*	.30	.34	.40	.53*	.67**	.85***	1.00***
	R^2	.07	.10	.11	.10	.12	.16	.25	.34	.43	.47
6	*mymo*	.14	.23	.30	.44	.65	1.00	1.42**	1.83***	2.22***	2.43***
	lc	.04	.10	.19	.09	-.12	-.59	-1.12	-1.63	-2.14*	-2.35*
	R^2	.02	.03	.04	.05	.07	.11	.19	.27	.34	.35
7	*mymo*	-.08	-.14	-.21	-.08	.11	.45	.80	1.09**	1.33***	1.36***
	$\log d - \log p$	.15**	.27**	.38**	.39*	.42	.43	.50*	.59**	.73***	.89***
	lc	.32	.61	.89	.82	.65	.21	-.21	-.54	-.81	-.74
	R^2	.08	.11	.14	.12	.13	.17	.26	.35	.44	.48
	1945-2002	1	2	3	4	5	6	7	8	9	10
8	*mymo*	-.01	.08	.23	.41**	.62***	.81***	1.00***	1.22***	1.49***	1.71***
	R^2	.00	.00	.03	.08	.13	.18	.23	.29	.37	.43
9	$\log d - \log p$	.14**	.24**	.32**	.41**	.54**	.64**	.79***	.93***	1.12***	1.27***
	R^2	.11	.14	.17	.20	.22	.23	.28	.33	.39	.44
10	*cay*	.15	.25	.50	.91	1.10*	1.24*	1.38*	1.60**	1.35	1.07
	R^2	.00	.01	.03	.09	.09	.09	.09	.10	.06	.03
11	*lc*	-.42	-.75	-1.18	-1.59	-1.76	-1.91	-2.06	-2.46	-1.93	-1.61
	R^2	.01	.02	.04	.06	.06	.05	.05	.06	.03	.02
12	*mymo*	-.08	-.07	-.01	.11	.24	.39*	.52**	.67**	.86***	1.01***
	$\log d - \log p$	.15**	.24**	.32**	.39	.47*	.51*	.59**	.66**	.76***	.83***
	R^2	.12	.14	.17	.21	.24	.26	.33	.41	.50	.57
13	*mymo*	.00	.07	.21	.42**	.63***	.85***	1.07***	1.33***	1.59***	1.80***
	lc	-.42	-.81	-1.36	-1.97*	-2.39*	-2.83**	-3.33**	-4.17***	-4.11***	-4.16***
	R^2	.01	.03	.07	.15	.20	.26	.34	.44	.50	.54
14	*mymo*	-.06	-.04	.04	.19	.34	.52*	.70**	.92***	1.12***	1.28***
	$\log d - \log p$	.19***	.31***	.40***	.47***	.53**	.54**	.58***	.62***	.67***	.72***
	lc	-1.01**	-1.60**	-2.12**	-2.57**	-2.81**	-3.00**	-3.28**	-3.93***	-3.61**	-3.43***
	R^2	.18	.23	.29	.36	.37	.38	.45	.55	.60	.64

Table 10: Long-Horizon Predictability Regressions.

The regressor in the first row is the rescaled housing collateral ratio *mymo*, estimated for the period 1925-2002. In row 2, it is the log dividend yield. In row 3, $.1 * \left(2.5 + \frac{cay - E(cay)}{\sigma(cay)}\right)$ is the rescaled consumption-wealth ratio $cay = \log c - .23 \log fw - .80 \log y + 0.38$, estimated for the period 1926-2002. In row 4, the regressor is the labor income plus transfers to total consumption expenditures ratio. Row 5 includes both the housing collateral ratio and the log dividend yield as regressors. Row 6 includes both the housing collateral ratio and the labor income to consumption ratio as regressors. Row 7 includes the housing collateral ratio, the log dividend yield, and the labor income to consumption ratio as regressors. The regressor in the row 8 is the housing collateral ratio *mymo*, estimated for the post-war period. In row 10, $x = .1 * \left(3.5 + \frac{cay - E(cay)}{\sigma(cay)}\right)$ is the rescaled consumption-wealth ratio $cay = \log c + .23 \log fw - 1.35 \log y - 0.10$, estimated for the period 1945-2002. The first row of each panel reports least squares estimates for b_1. Newey-West HAC standard errors are used to denote significance at the 1 percent (***), 5 percent (**) and 10 percent level (*). The second row reports the R^2 for this regression.

		B1	B2	B3	B4	B5	B6	B7	B8	B9	B10
$mymo$	b_0	-4.60	-4.47	-4.26	-4.46	-3.75	-3.80	-4.05	-3.27	-3.50	-3.44
	b_1	-1.93	-.81	-1.35	-1.62	-.76	-.53	-1.12	-.10	1.01	1.79
	R^2	.27	.07	.22	.21	.10	.18	.49	.01	.30	.34
	$s.e.$	.47	.41	.36	.46	.33	.16	.17	.18	.22	.36
$myrw$	b_0	-4.69	-4.51	-4.32	-4.53	-3.78	-3.83	-4.10	-3.27	-3.45	-3.36
	b_1	-3.08	-1.72	-2.30	-2.52	-2.05	-1.09	-1.08	.21	1.69	2.57
	R^2	.57	.29	.56	.43	.62	.65	.39	.03	.72	.60
	$s.e.$	.36	.36	.27	.39	.21	.11	.18	.18	.14	.28
$myfa$	b_0	-4.69	-4.51	-4.32	-4.53	-3.78	-3.83	-4.10	-3.28	-3.46	-3.36
	b_1	-3.53	-3.15	-3.4	-3.52	-3.03	-1.38	-.91	1.59	2.28	2.39
	R^2	.13	.17	.22	.15	.24	.19	.05	.24	.23	.09
	$s.e.$	.51	.39	.37	.48	.30	.16	.23	.16	.23	.42

Table 11: Divided Share and the Housing Collateral Ratio. The results are for the regression $log(d_t) - log(y_t) = b_0 + b_1 my_t + \epsilon_t$, where d_t is the nominal dividend on each of 10 decile value portfolios and y_t is the nominal aggregate labor income plus transfers. The regressor in the first (second and third) row is the annual housing collateral ratio $mymo$ ($myrw$ and $myfa$), estimated for the period 1889-2002. The first row of each panel reports least squares estimates for the intercept b_0, the second row for the slope coefficient b_1. The third row reports the OLS R^2 for this regression. The last row gives the standard error of the regression (i.e. of ϵ_t). The dividend data are annual for 1952 to 1999, constructed from the monthly dividend yield provided by Bansal, Dittmar and Lundblad (2002).

	$\tilde{\lambda}_0$	$\tilde{\lambda}_c$	$\tilde{\lambda}_\rho$	$\tilde{\lambda}_{my}$	$\tilde{\lambda}_{my.c}$	$\tilde{\lambda}_{my.\rho}$	R^2
1	8.87	1.61					**9.4**
CCAPM	(2.55)	(1.01)					5.6
	[2.77]	[1.18]					
2	5.02	.84	-.76				**50.1**
HCAPM	(2.27)	(.93)	(.27)				45.8
	[2.83]	[1.23]	[.36]				
3	4.22	1.94		-.03	2.23		**86.5**
Separable Prefs.	(2.29)	(1.05)		(.06)	(.79)		84.6
mymo	[3.31]	[1.58]		[.09]	[1.17]		
4	3.52	2.12		-.03	1.36		**87.8**
Separable Prefs.	(2.25)	(1.02)		(.03)	(.47)		86.1
myrw	[3.33]	[1.58]		[.05]	[.72]		
5	2.81	.97		-.00	.66		**73.3**
Separable Prefs.	(2.27)	(.94)		(.02)	(.35)		69.7
myfa	[2.93]	[1.28]		[.03]	[.47]		
6	4.71	1.91	.00	-.04	2.18	.12	**86.6**
Non-Sep. Prefs.	(2.60)	(1.06)	(.40)	(.06)	(.81)	(.43)	83.3
mymo	[3.74]	[1.58]	[.59]	[.09]	[1.20]	[.63]	
7	4.28	2.12	-.07	-.03	1.33	.03	**88.3**
Non-Sep. Prefs.	(2.58)	(.95)	(.46)	(.03)	(.45)	(.28)	85.3
myrw	[3.79]	[1.47]	[.69]	[.05]	[.69]	[.41]	
8	4.03	1.30	.15	-.01	.91	.05	**82.3**
Non-Sep. Prefs.	(2.40)	(.91)	(.42)	(.02)	(.35)	(.19)	77.8
myfa	[3.94]	[1.54]	[.70]	[.04]	[.59]	[.32]	

Table 12: Cross-Sectional Results with Aggregate Pricing Factors. The sample period is 1926-2002. The asset pricing factors are $\Delta\log(c_{t+1})$ in row 1, $\Delta\log(c_{t+1})$ and $A_t\Delta\log(\rho_{t+1})$ in row 2, $\Delta\log(c_{t+1})$, $my^{max} - my_t$, $(my^{max} - my_t)\,\Delta\log(c_{t+1})$ in rows 3-5 and $\Delta\log(c_{t+1})$, $A_t\Delta\log(\rho_{t+1})$, $my^{max} - my_t$,$(my^{max} - my_t)\,\Delta\log(c_{t+1})$ and $(my^{max} - my_t)\,A_t\Delta\log(\rho_{t+1})$ in rows 6-8. The housing collateral variable is $mymo$ in rows 3 and 6, $myrw$ in row 4 and 7 and $myfa$ in row 5 and 8. my is estimated with data from 1925-2002. The variables used are Δc_1, $\Delta\rho_1$, and A_1 described in the data appendix. The estimation is done using the Fama-MacBeth procedure. The set of test assets is $T1$. OLS standard errors are in parenthesis, Shanken (1992) corrected standard errors are in brackets. The last column reports the R^2 and the adjusted R^2 just below it.

Portfolio	CCAPM	Fama-French	Collateral-CCAPM
RVW	2.97	-.20	.07
S1B1	7.97	3.96	1.79
S1B2	1.89	2.51	1.23
S1B3	-1.01	-1.08	-.34
S1B4	-7.10	-2.07	1.53
S1B5	-5.29	-2.78	-.62
S2B1	4.64	1.94	2.03
S2B2	-.30	-1.15	-1.27
S2B3	-2.65	-1.23	-.96
S2B4	-3.31	-.79	-1.14
S2B5	-3.00	-.08	-1.72
S3B1	1.99	-1.41	2.00
S3B2	-.24	-1.12	.28
S3B3	-.50	-.62	-.55
S3B4	-1.28	.03	-1.40
S3B5	-1.92	1.58	.38
S4B1	1.65	-2.54	-1.25
S4B2	.90	.70	.85
S4B3	-.22	.24	-2.08
S4B4	-1.18	.64	.62
S4B5	-3.90	-.11	-2.38
S5B1	3.41	-2.52	-.07
S5B2	3.31	.73	-.51
S5B3	2.13	.46	-.59
S5B4	1.19	1.96	-.10
S5B5	-3.95	-.86	.10
Average	3.27	1.61	1.21
χ^2	72.1^{***}	61.1^{***}	35.1^{*}

Table 13: Average Pricing Errors. Pricing errors from the Fama-MacBeth Regressions with aggregate pricing factors. The set of returns is the value weighted market return and the 25 size and book-to-market portfolio returns. The sample is 1926-2002. The second column reports errors from the consumption CAPM, the third from the three-factor Fama-French model and the last column reports average errors from the collateral CAPM with scaling variable *myrw* and separability in preferences (line 4 in table 12). The last two rows report the square root of the average squared pricing errors and the χ^2 statistic for the null hypothesis that all pricing errors are zero. The degrees of freedom are 25, 23 and 23 respectively. Three stars denote rejection of the null hypothesis at the 1 percent level, 2 stars at the 5 percent level and 1 star at the 10 percent level.

Figure 1: Testing the Markov Assumption

	$\tilde{\lambda}_0$	$\tilde{\lambda}_c$	$\tilde{\lambda}_\rho$	$\tilde{\lambda}_{my}$	$\tilde{\lambda}_{my.c}$	$\tilde{\lambda}_{my.\rho}$	$\tilde{\lambda}_{my.c_{-1}}$	$\tilde{\lambda}_{my.\rho_{-1}}$	$\tilde{\lambda}_{my.c_{-2}}$	$\tilde{\lambda}_{my.\rho_{-2}}$	$\tilde{\lambda}_{my.c_{-3}}$	$\tilde{\lambda}_{my.\rho_{-3}}$	R^2
1	3.28	2.51		-.03	1.52								**90.3**
Separable Prefs.	(2.31)	(1.00)		(.03)	(.47)								89.0
myrw	[3.55]	[1.60]		[.06]	[.75]								
2	3.47	2.39		-.02	1.48		.13						**90.7**
Separable Prefs.	(2.20)	(1.12)		(.03)	(.51)		(.81)						88.9
myrw	[3.37]	[1.77]		[.05]	[.80]		[1.25]						
3	3.48	2.38		-.02	1.48		.13		-.65				**90.7**
Separable Prefs.	(2.26)	(.98)		(.04)	(.46)		(.81)		(.98)				88.3
myrw	[3.45]	[1.55]		[.06]	[.72]		[1.25]		[1.51]				
4	3.42	1.44		-.04	.79		-.17		-.62		-1.42		**94.9**
Separable Prefs.	(2.26)	(1.10)		(.04)	(.52)		(.88)		(.97)		(.60)		93.2
myrw	[3.45]	[1.72]		[.06]	[.81]		[1.36]		[1.49]		[.94]		
5	3.13	2.51	.18	-.03	1.52	.20							**90.3**
Non-Sep. Prefs.	(2.65)	(1.00)	(.46)	(.03)	(.48)	(.28)							87.9
myrw	[4.09]	[1.61]	[.71]	[.06]	[.77]	[.44]							
6	2.93	2.45	.23	-.03	1.50	.22	.07	.02					**90.8**
Non-Sep. Prefs.	(2.66)	(.85)	(.37)	(.03)	(.42)	(.25)	(.59)	(.22)					87.2
myrw	[4.12]	[1.39]	[.58]	[.05]	[.67]	[.39]	[.93]	[.35]					
7	2.91	2.38	.33	-.03	1.45	.32	.40	-.18	-.76	-.10			**92.0**
Non-Sep. Prefs.	(2.58)	(.85)	(.32)	(.03)	(.43)	(.18)	(.51)	(.19)	(.49)	(.18)			87.4
myrw	[4.36]	[1.50]	[.55]	[.05]	[.74]	[.30]	[.88]	[.33]	[.85]	[.31]			
8	3.61	.19	.16	-.03	.22	.16	-.72	-.14	-.60	.11	-.92	-.23	**97.1**
Non-Sep. Prefs.	(2.59)	(.83)	(.32)	(.03)	(.41)	(.18)	(.61)	(.19)	(.46)	(.19)	(.42)	(.16)	94.8
myrw	[4.41]	[1.47]	[.56]	[.06]	[.73]	[.31]	[1.05]	[.32]	[.81]	[.33]	[.74]	[.28]	

Table 1: Cross-Sectional Results with Lagged Aggregate Pricing Factors. The asset pricing factors are $my^{max} - my_t$, $(my^{max} - my_t)\,\Delta\log(c_{t+1})$ in row 1, row 2-4 contain in addition the one through four period lagged interaction terms $(my^{max} - my_t)\,\Delta\log(c_{t+1-k})$, $k = 1-4$. Row 5 contains the factors $\Delta\log(c_{t+1})$, $A_t\Delta\log(\rho_{t+1})$, $my^{max} - my_t$, $(my^{max} - my_t)\,\Delta\log(c_{t+1})$ and $(my^{max} - my_t)\,A_t\Delta\log(\rho_{t+1})$. Rows 6-8 contain in addition the one through four period lagged interaction terms $(my^{max} - my_t)\,\Delta\log(c_{t+1-k})$, $k = 1-4$ and $(my^{max} - my_t)\,A_{t-k}\Delta\log(\rho_{t+1-k})$, $k = 1-4$. The housing collateral variable is *myrw* in all rows. The estimation is done using the Fama-MacBeth procedure. The same returns are used in each row: 1930-2002, i.e. in row 4, consumption growth up to 1926 is used. OLS standard errors are in parenthesis, Shanken (1992) corrected standard errors are in brackets. The last column reports the R^2 and the adjusted R^2 just below it.

	$\tilde{\lambda}_0$	$\tilde{\lambda}_c$	$\tilde{\lambda}_\rho$	$\tilde{\lambda}_{my}$	$\tilde{\lambda}_{my.c}$	$\tilde{\lambda}_{my.\rho}$	R^2
1	9.30	2.43					**49.1**
CCAPM	(2.69)	(.84)					47.0
	[4.64]	[1.47]					
2	3.85	1.99	.45				**64.8**
HCAPM	(3.05)	(.77)	(.12)				61.8
	[5.67]	[1.45]	[.22]				
3	7.12	1.66		-.11	-.03		**70.0**
Separable Prefs.	(2.19)	(.66)		(.09)	(.31)		65.4
mymo	[4.43]	[1.36]		[.18]	[.54]		
4	9.61	1.15		.02	-.04		**69.9**
Separable Prefs.	(3.23)	(.67)		(.06)	(.13)		65.8
myrw	[6.50]	[1.37]		[.12]	[.27]		
5	4.30	1.28		-.06	-.15		**83.8**
Separable Prefs.	(2.62)	(.68)		(.05)	(.13)		81.6
myfa	[5.37]	[1.42]		[.10]	[.27]		
6	5.44	1.56	.37	-.11	.05	.22	**80.4**
Non-Sep. Prefs.	(2.66)	(.64)	(.10)	(.09)	(.32)	(.06)	75.5
mymo	[5.59]	[1.37]	[.22]	[.18]	[.68]	[.14]	
7	5.02	1.58	.34	.01	.09	.06	**75.6**
Non-Sep. Prefs.	(2.79)	(.65)	(.11)	(.03)	(.14)	(.02)	69.5
myrw	[5.31]	[1.27]	[.22]	[.06]	[.28]	[.04]	
8	5.32	1.10	.22	-.06	-.17	.05	**84.3**
Non-Sep. Prefs.	(3.12)	(.63)	(.12)	(.04)	(.16)	(.03)	80.4
myfa	[6.20]	[1.27]	[.24]	[.09]	[.33]	[.06]	

Table 14: Post-war Cross-Sectional Results. The sample period is 1945-2002. The asset pricing factors are $\Delta\log(c_{t+1})$ in row 1, $\Delta\log(c_{t+1})$ and $A_t\Delta\log(\rho_{t+1})$ in row 2, $\Delta\log(c_{t+1})$, $my^{max} - my_t$, $(my^{max} - my_t)\,\Delta\log(c_{t+1})$ in rows 3-5 and $\Delta\log(c_{t+1})$, $A_t\Delta\log(\rho_{t+1})$, $my^{max} - my_t$,$(my^{max} - my_t)\,\Delta\log(c_{t+1})$ and $(my^{max} - my_t)\,A_t\Delta\log(\rho_{t+1})$ in rows 6-8. The housing collateral variable is $mymo$ in rows 3 and 6, $myrw$ in row 4 and 7 and $myfa$ in row 5 and 8. my is estimated with data from 1945-2002. The variables used are Δc_2, $\Delta\rho_2$, and A_2 described in the data appendix. The estimation is done using the Fama-MacBeth procedure. The set of test assets is $T1$. OLS standard errors are in parenthesis, Shanken (1992) corrected standard errors are in brackets. The last column reports the R^2 and the adjusted R^2 just below it.

Portfolio	All States	Good States	Bad States
RVW	1.56	0.89	2.34
S1B1	1.48	1.27	1.76
S1B2	2.28	.47	4.34
S1B3	3.20	1.70	4.95
S1B4	3.39	-.70	8.00
S1B5	3.96	.85	7.50
S2B1	1.94	.98	3.04
S2B2	2.23	1.25	3.37
S2B3	2.42	.33	4.80
S2B4	2.68	.49	5.17
S2B5	2.89	.58	5.51
S3B1	1.80	.33	3.46
S3B2	2.27	.61	4.17
S3B3	2.52	1.16	4.08
S3B4	2.53	.91	4.39
S3B5	2.99	.53	5.79
S4B1	1.05	.37	1.84
S4B2	1.58	.04	3.33
S4B3	1.71	.65	2.93
S4B4	2.34	.04	4.93
S4B5	1.62	-1.10	4.67
S5B1	1.54	1.20	1.95
S5B2	1.21	.76	1.74
S5B3	1.57	.77	2.50
S5B4	1.69	.32	3.25
S5B5	1.57	-1.66	5.18

Table 15: Consumption Betas. Consumption betas are computed as $\beta_t = \beta_c + \beta_{c.my}(my^{max} - my_t)$. Good states are states where $(my^{max} - myrw_t)$ is below below zero and bad states are times where $myrw$ is one standard deviation above zero (11 observations each). The third and fourth column report the average consumption betas in good states and bad states respectively. The sample is 1926-2002. Lettau and Ludvigson (2001b) do the same exercise for their scaling variable, the consumption-wealth ratio.

	$\tilde{\beta}_0$	$\tilde{\beta}_c$	$\tilde{\beta}_\rho$	$\tilde{\beta}_{my}$	$\tilde{\beta}_{my.c}$	$\tilde{\beta}_{my.\rho}$	R^2
1	6.46	1.28					5.16
	(2.63)	(.79)					
2	5.84	1.37	-3.44				8.44
	(2.45)	(.73)	(3.58)				
3	-6.89	.93	-11.33	17.61	.26	13.30	21.22
	(5.68)	(.87)	(2.47)	(7.40)	(1.39)	(2.74)	
4	-4.67	.71	-11.23	25.12	1.08	23.44	21.60
	(5.82)	(.80)	(2.41)	(13.40)	(2.34)	(5.00)	
5	-.83	-.34	-11.45	17.78	5.63	27.70	19.35
	(10.10)	(1.45)	(2.98)	(32.00)	(5.83)	(6.27)	

Table 16: Time-Series Analysis for Aggregate Stock Market Return. The regression is: $R^{e,j}_{t+1} = \tilde{\beta}^j_0 + \tilde{\beta}^j_c \Delta \log c_{t+1} + \tilde{\beta}^j_\rho A_t \Delta \log \rho_{t+1} + \tilde{\beta}^j_{my}(my^{max} - my_t) + \tilde{\beta}^j_{my.c}(my^{max} - my_t)\Delta \log c_{t+1} + \tilde{\beta}^j_{my.\rho}(my^{max} - my_t)A_t \Delta \log \rho_{t+1}$. The first row just includes $\Delta \log(c_{t+1})$. The second row adds $A_t \Delta \log(\rho_{t+1})$. The last three rows add the interaction terms with my_t. The scaling variable is $mymo$ in regression 3, $myrw$ in regression 4, and $myfa$ in regression 5. Data are for 1926-2002. Newey-West heteroskedasticity and autocorrelation corrected standard errors are in parentheses.

Model	$\tilde{\lambda}_0$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_3$	R^2
1	$-.30$	9.35			28.3
Static CAPM	(4.02)	(4.68)			25.3
Sharpe-Lintner	[4.40]	[5.65]			
2	2.24	7.13	3.91		37.2
Human Capital-CAPM	(3.93)	(4.58)	(1.28)		31.7
Jagannathan-Wang	[4.87]	[6.16]	[1.71]		
3	1.47	7.48	$-.00$	5.57	49.3
lc-conditional CAPM	(4.17)	(4.82)	(0.02)	(4.99)	42.4
Santos-Veronesi	[5.49]	[6.42]	[0.03]	[6.62]	
4	8.88	1.61			9.4
Static CCAPM	(2.55)	(1.01)			5.6
Breeden-Lucas	[2.77]	[1.18]			
5	3.22	2.86	.23	4.87	89.2
cay-conditional CCAPM	(2.33)	(1.09)	(.17)	(3.19)	87.7
Lettau-Ludvigson	[3.87]	[1.86]	[.31]	[5.53]	
6	3.52	2.12	$-.03$	1.36	87.8
Collateral-CAPM	(2.25)	(1.02)	(.03)	(.47)	86.1
this paper	[3.33]	[1.58]	[.05]	[.71]	
7	10.21	-2.46	2.71	6.30	78.1
Three-factor model	(4.63)	(5.17)	(1.68)	(1.74)	75.1
Fama-French	[5.24]	[6.32]	[2.52]	[2.56]	

Table 17: Model Comparison: 7 models, 1926-2002. Row 1: factor is $R^{vw,e}_{t+1}$. Row 2: factors are $R^{vw,e}_{t+1}$ and $R^{hc,e}_{t+1}$. Row 3 factors: $R^{vw,e}_{t+1}$, lc_t and $lc_t R^{vw,e}_{t+1}$. Row 4: $\Delta \log(c_{t+1})$. Row 5: $\Delta \log(c_{t+1})$, cay_t, $cay_t \Delta \log(c_{t+1})$. Row 6 is the collateral model under separability: $\Delta \log(c_{t+1})$, $myrw^{max} - myfa_t$, and $(myrw^{max} - myfa_t)\Delta \log(c_{t+1})$. Row 7: $R^{vw,e}_{t+1}$, $R^{smb,e}_{t+1}$, and $R^{hml,e}_{t+1}$. The second column gives the zero-β return $\tilde{\lambda}_0$. OLS standard errors are in parenthesis, Shanken corrected standard errors are in brackets.

Model	$\tilde{\lambda}_0$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_3$	Size	R^2
1	2.64	6.31			-1.28	44.2
Static CAPM	(4.11)	(4.72)			(.35)	39.2
Sharpe-Lintner	[5.27]	[6.49]			[.48]	
2	5.53	3.80	4.12		-1.32	54.5
Human Capital-CAPM	(4.08)	(4.67)	(1.29)		(.35)	48.0
Jagannathan-Wang	[5.93]	[7.18]	[1.99]		[.55]	
3	12.93	-4.00	.05	-5.26	-1.46	64.7
lc-conditional CAPM	(5.67)	(5.96)	(0.03)	(6.09)	(.43)	57.6
Santos-Veronesi	[10.09]	[10.87]	[0.05]	[11.11]	[.78]	
4	6.20	.86			-1.40	44.2
Static CCAPM	(2.33)	(.93)			(.36)	39.2
Breeden-Lucas	[3.14]	[1.33]			[.52]	
5	4.57	.64	$-.62$		-1.03	57.5
HCAPM	(2.26)	(.91)	(.27)		(.32)	51.4
Non-Separabililty	[2.91]	[1.25]	[.36]		[.45]	
6	3.60	2.34	$-.04$	1.43	.07	87.8
Collateral-CAPM	(2.21)	(.90)	(.04)	(.43)	(.45)	85.4
this paper	[3.38]	[1.45]	[.07]	[.68]	[.71]	
Model	$\tilde{\lambda}_0$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_3$	Value	R^2
1	4.30	4.20			.24	34.8
Static CAPM	(4.28)	(4.87)			(.09)	28.9
Sharpe-Lintner	[4.62]	[5.76]			[.12]	
2	7.80	.99	3.43		.23	43.4
Human Capital-CAPM	(4.37)	(4.93)	(1.25)		(.08)	35.3
Jagannathan-Wang	[5.30]	[6.44]	[1.65]		[.12]	
3	14.91	-6.92	.05	-8.04	.22	68.8
lc-conditional CAPM	(6.08)	(6.45)	(0.02)	(6.62)	(.08)	62.5
Santos-Veronesi	[9.38]	[10.23]	[0.04]	[10.49]	[.14]	
4	8.05	2.10			-.06	17.7
Static CCAPM	(2.46)	(.83)			(.05)	10.2
Breeden-Lucas	[2.82]	[1.05]			[.09]	
5	7.20	1.37	$-.70$		-.10	43.1
HCAPM	(2.42)	(.79)	(.28)		(.05)	35.0
Non-Separabililty	[3.13]	[1.10]	[.39]		[.10]	
6	3.53	2.71	$-.01$	1.76	-.03	78.0
Collateral-CAPM	(2.33)	(.77)	(.04)	(.42)	(.06)	73.6
this paper	[3.95]	[1.37]	[.06]	[.73]	[.13]	

Table 18: Residual Size and Value Effects, 1926-2002. Row 1: factor is $R_{t+1}^{vw,e}$. Row 2: factors are $R_{t+1}^{vw,e}$ and $R_{t+1}^{hc,e}$. Row 3 factors: $R_{t+1}^{vw,e}$, lc_t and $lc_t R_{t+1}^{vw,e}$. Row 4: $\Delta \log(c_{t+1})$. Row 5: $\Delta \log(c_{t+1})$, $A_t \Delta \log(\rho_{t+1})$. Row 6 is the collateral model: $\Delta \log(c_{t+1})$, , $myrw^{max} - myrw_t$, and $(myrw^{max} - myrw_t)\Delta \log(c_{t+1})$. Size is log of portfolio's market capitalization. Value is log of portfolio's book-to-market ratio.

Figure 2: Housing Collateral Ratio Against the Housing-Endowment Ratio in Benchmark Calibration.

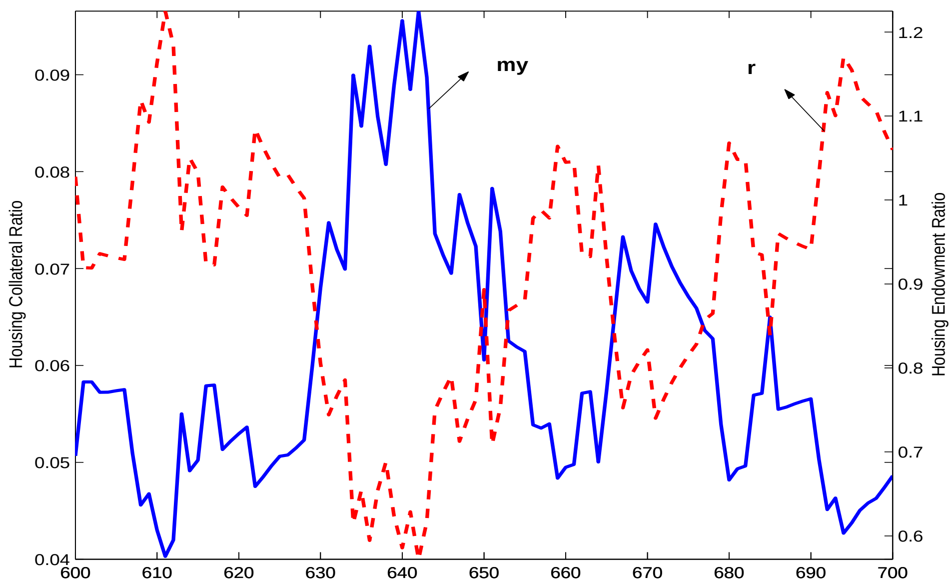

Figure 3: The Consumption and Income Share of One Household.

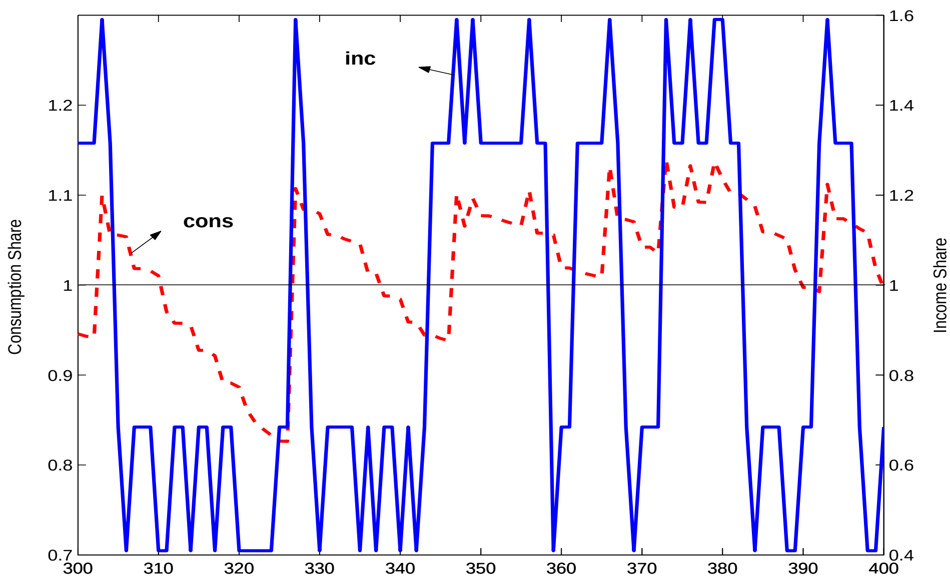

Figure 4: The Housing Collateral Ratio and Consumption Share of One Household.

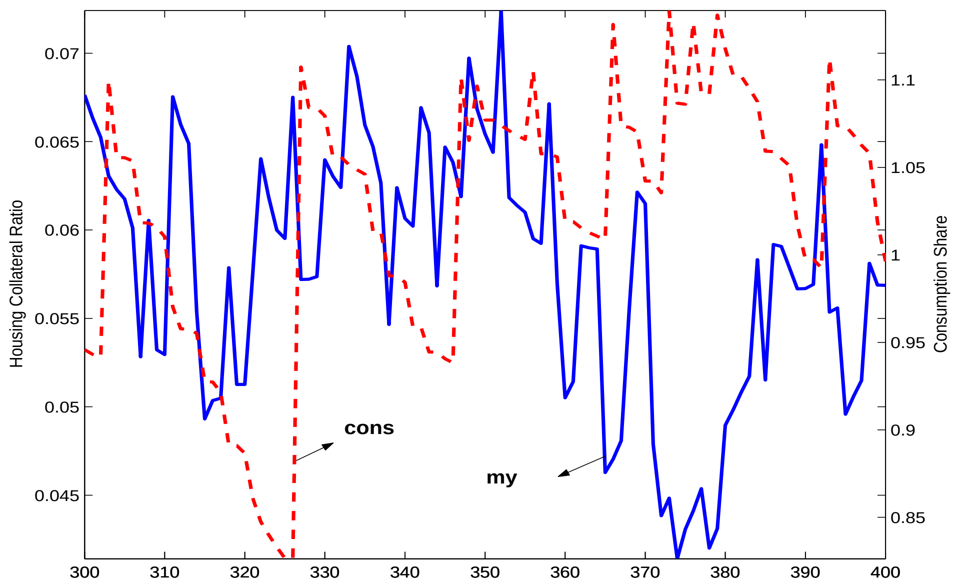

Figure 5: Housing Collateral Ratio Against the Aggregate Weight Shock.

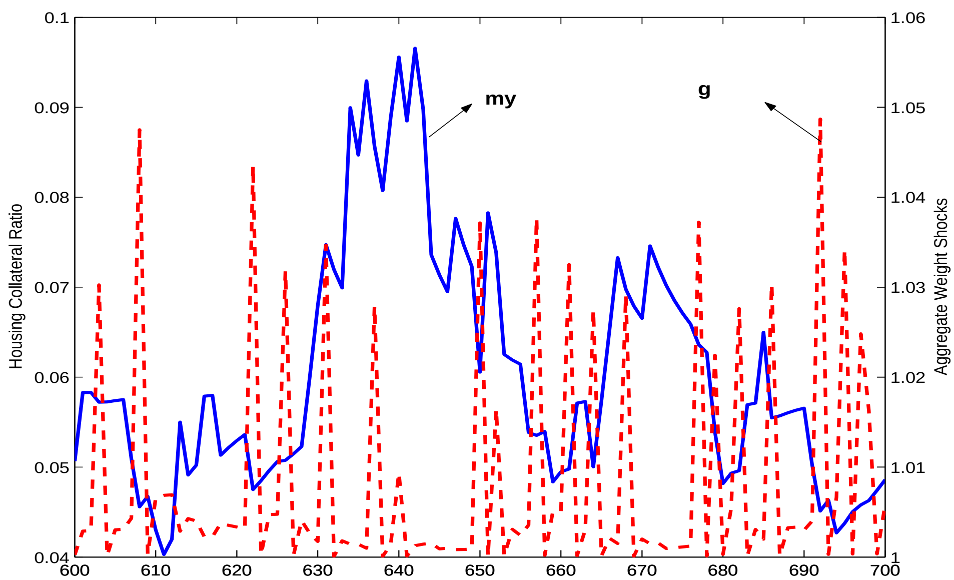

Figure 6: Housing Collateral Ratio Against the Market price of Risk $\frac{\sigma_t[m_{t+1}]}{E_t[m_{t+1}]}$.

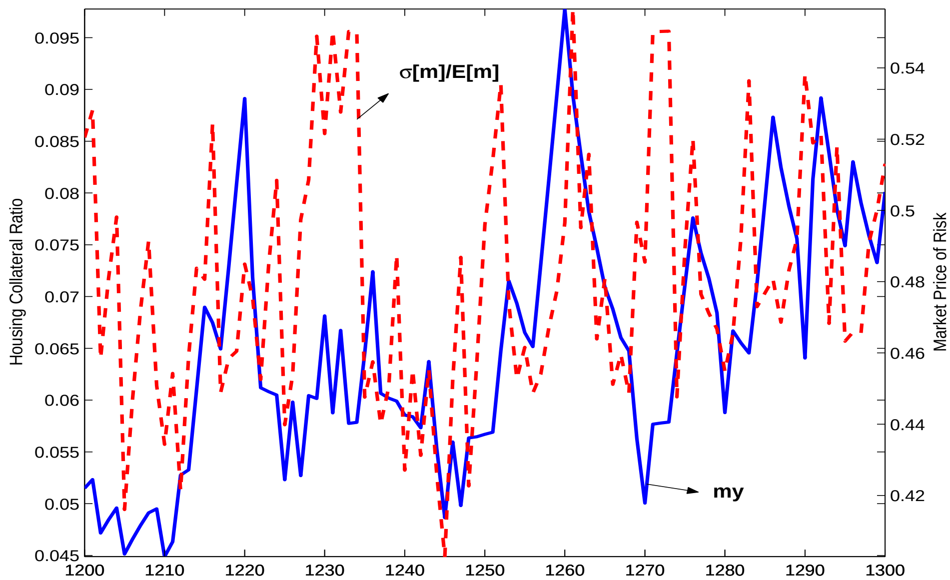

Figure 7: Housing Collateral Ratio Against the expected Excess Return on Stocks $E_t\left[R^e_{t+1}\right]$.

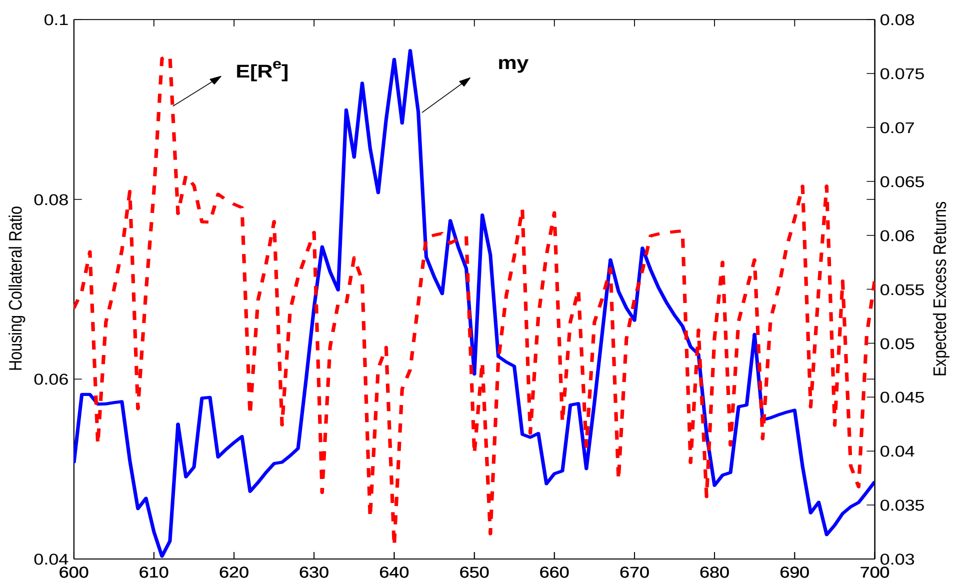

Figure 8: Housing Collateral Ratio Against the expected Excess Return on Stocks $E_t\left[R^e_{t+1}\right]$ in the Model without Heteroskedasticity in Labor Income.

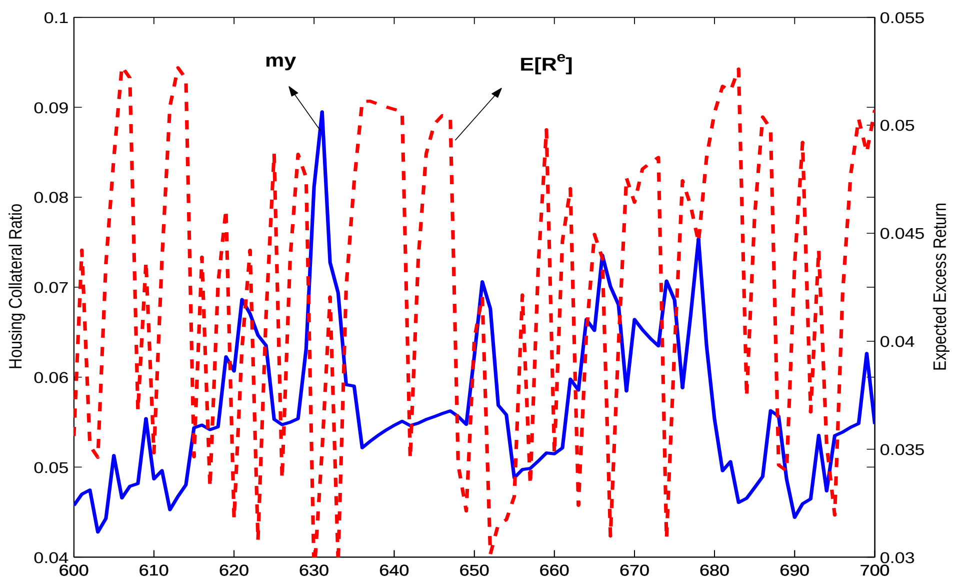

Figure 9: Housing Collateral Ratio 1889-2002. Measured by outstanding home mortgages (mo), non-farm residential wealth (rw) and residential fixed asset wealth (fa) relative to human wealth (y)

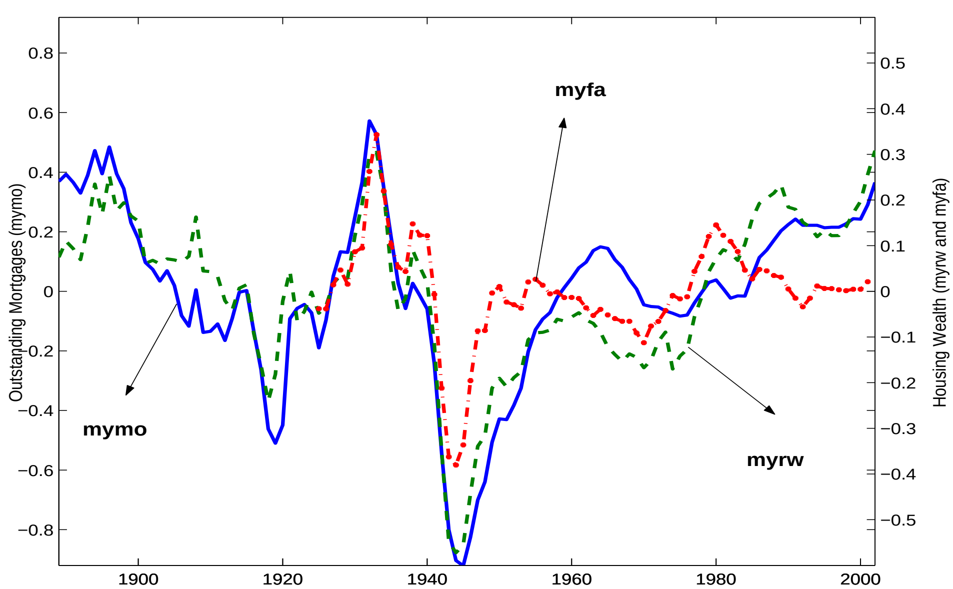

Figure 10: Housing Collateral Ratio 1945-2002.

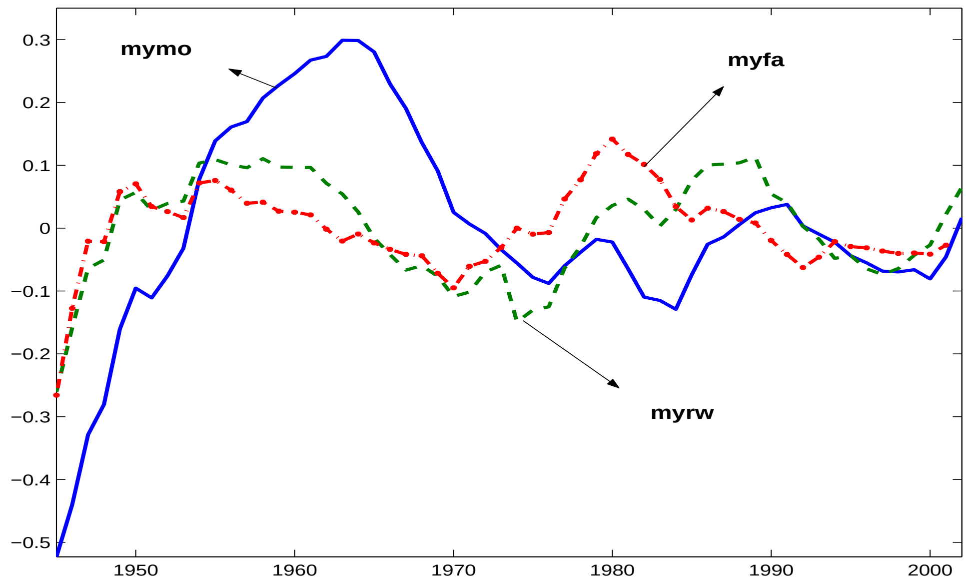

Figure 11: Power Spectral Density of Housing Collateral Ratio. The three lines are the cointegration deviations between labor income and one of our three different measures of housing collateral. For mymo, 90 percent of the variance occurs at frequencies below $2\pi/20$. For myrw that is 82 percent and for myfa 77 percent.

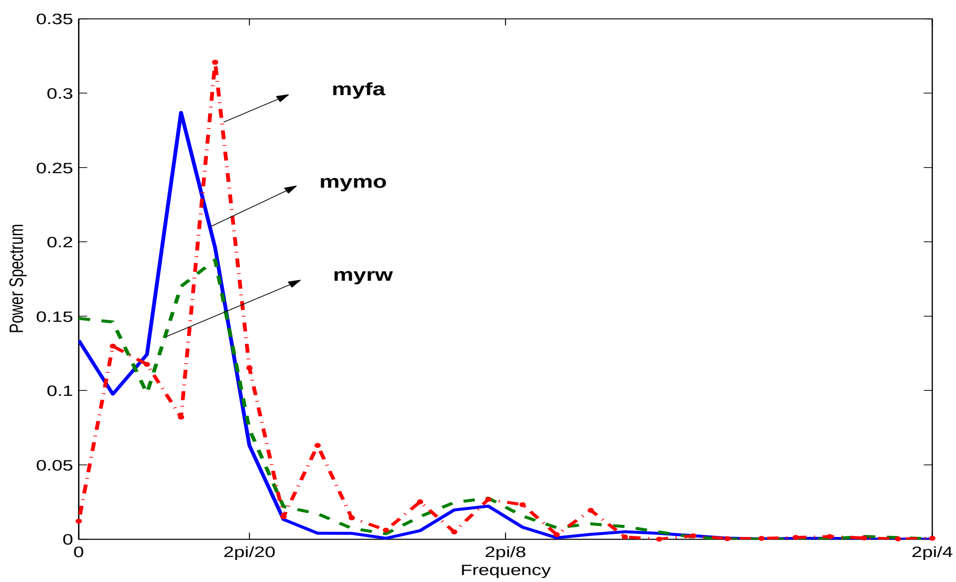

Figure 12: Response of the One-Year Excess Return to Impulse in Collateral Ratio $myfa$. Sample is 1925-2002, standard errors are compute from 50,000 Monte carlo repetitions.

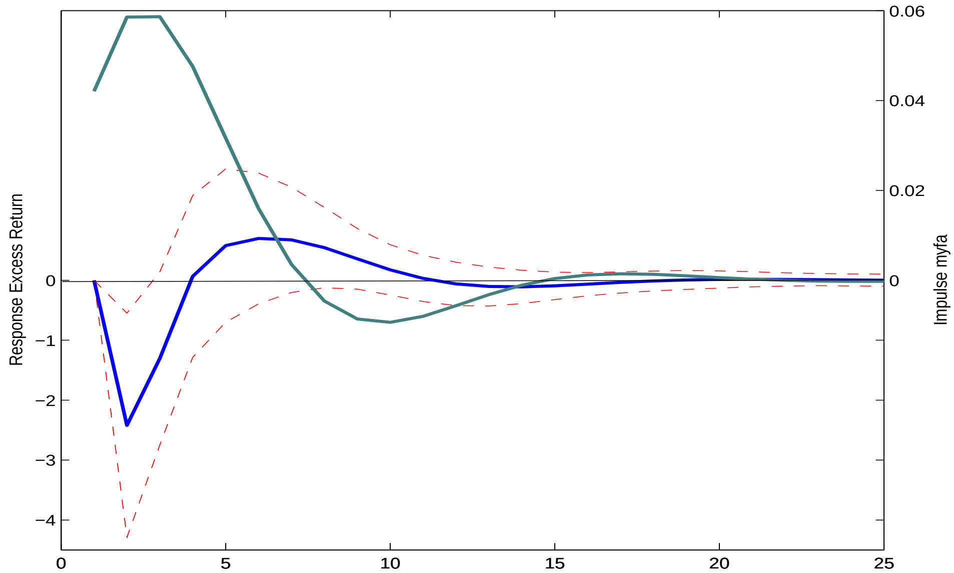

Figure 13: Response of the One-Year Excess Return to Impulse in Collateral Ratio $mymo$, 1925-2002.Sample is 1925-2002, standard errors are compute from 50,000 Monte carlo repetitions.

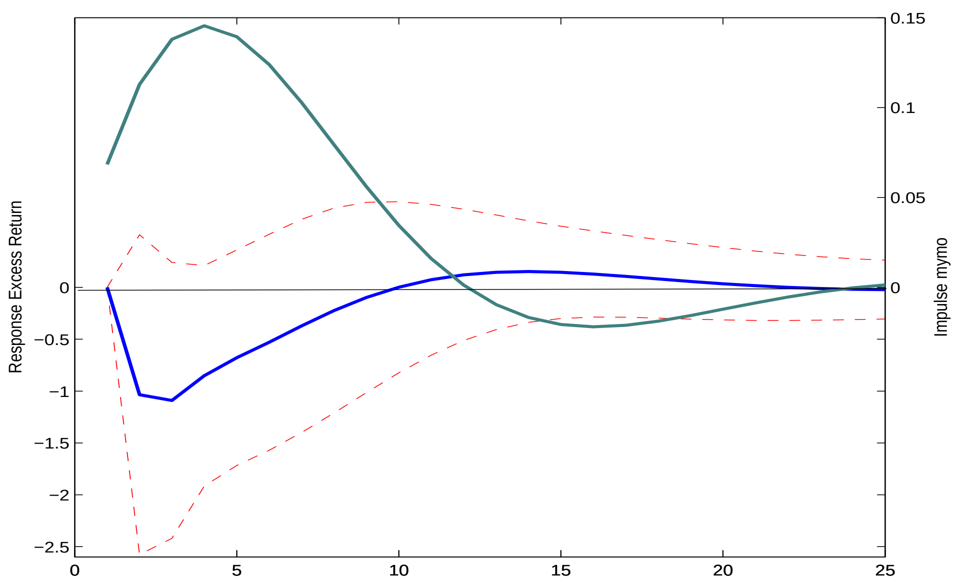

Figure 14: Response of the One-Year Excess Return to Impulse in Collateral Ratio *myrw*. Sample is 1925-2002, standard errors are compute from 50,000 Monte carlo repetitions.

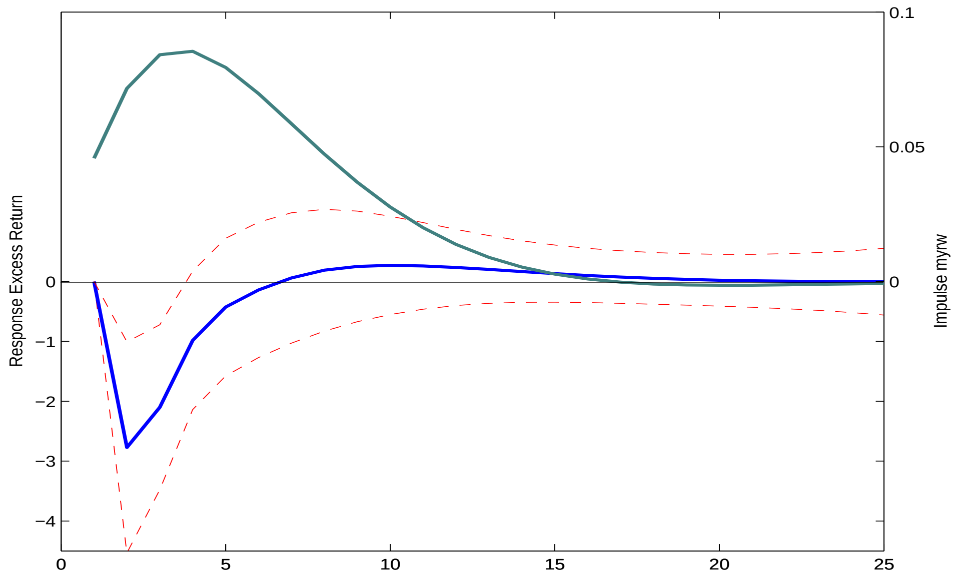

Figure 15: 10-year Excess Market Return on the Collateral ratio *mymo*.

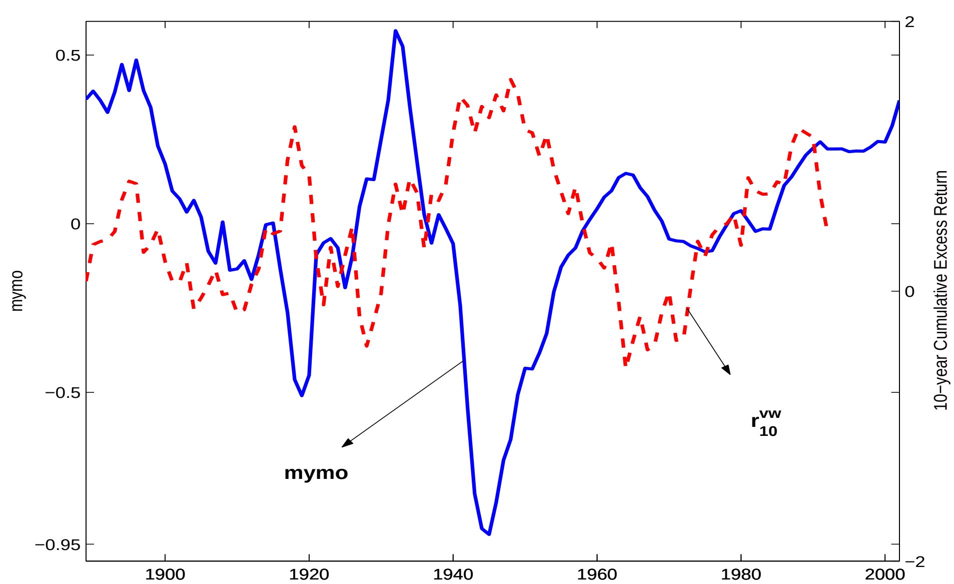

Figure 16: Response of the the log Dividend Share on Portfolio B1 to Impulse in Collateral Ratio $myfa$. Sample is 1952-2002, standard errors are compute from 5,000 Monte carlo repetitions.

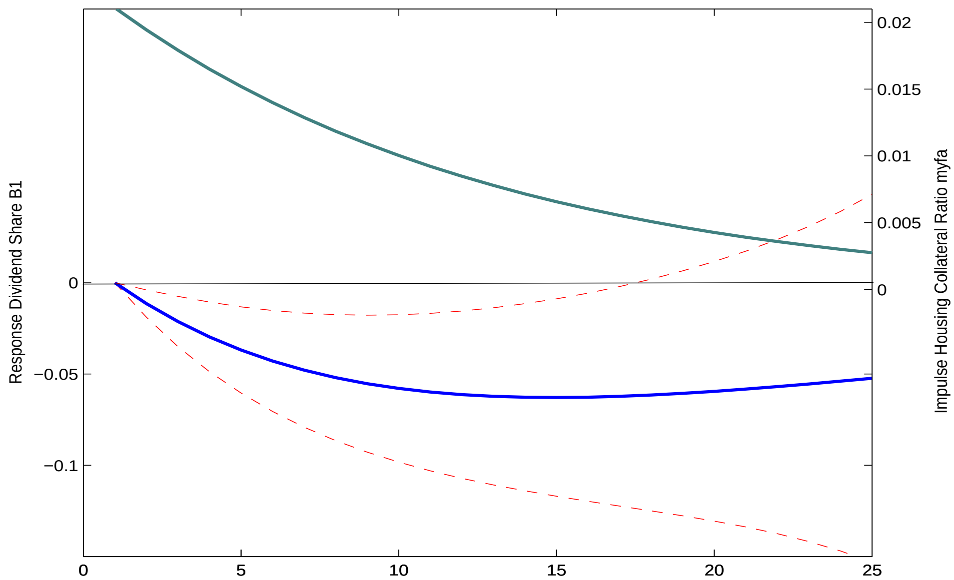

Figure 17: Response of the log Dividend Share on Portfolio B10 to Impulse in Collateral Ratio $myfa$. Sample is 1952-2002, standard errors are compute from 5,000 Monte carlo repetitions.

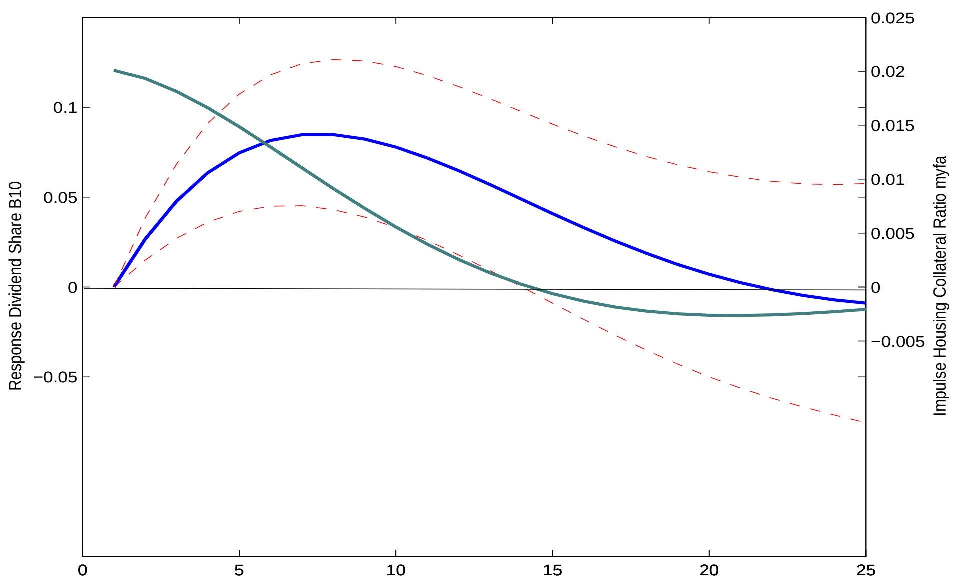

Figure 18: CCAPM and Collateral-CAPM - Aggregate Pricing Factors. Left Panel: Realized average excess returns on 25 Fama-French portfolios and the value weighted market return against predicted excess returns by standard Consumption-CAPM. Right Panel: against predicted returns by Collateral-CAPM (under separability).

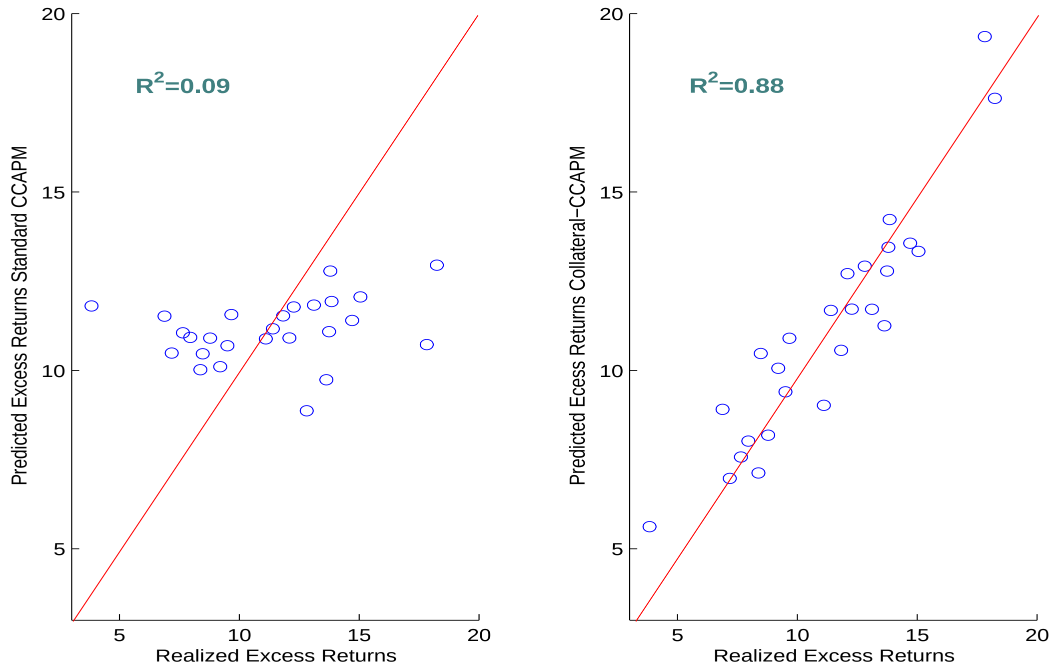

Figure 19: Collateral CAPM: The Value Premium. Left Panel: Realized average excess returns on 25 Fama-French portfolios and the value weighted market return against excess returns predicted by the collateral-CAPM with $myrw$. Right Panel: Realized average excess returns against $\tilde{\beta}_{my.c}$ exposure to interaction term of my_t and $\Delta \log c_{t+1}$.