Intraday Returns and Heterogeneous Liquidity Sources

Wenjin Kang *

Anderson Graduate School of Management
University of California, Los Angeles
Telephone: (310) 825-8207
Email: wkang@anderson.ucla.edu.

---

* Anderson Graduate School of Management at UCLA, Telephone (310) 825-8207, Email address: wkang@anderson.ucla.edu. Mailing address: Suite C513, Anderson Graduate School of Management at UCLA, 110 Westwood Plaza, Los Angeles, CA, 90095.

** I am extremely grateful to the members of my committee, Avanidhar Subrahmanyam (chair), Jack Hughes, Rossen Volkanov, and Bryan Ellickson. I have also benefited a lot from Antonio Bernardo, Michael Brennan, Carla Hayn, Marc Junkunc, Feifei Li, Jun Liu, Bruce Miller, and Eduardo Schwartz.

Intraday Returns and Heterogeneous Liquidity Sources

Abstract

This paper studies the relation between stock market liquidity and intraday stock returns. My model characterizes a momentum-reversal intraday return pattern within which, during the reversal, a negative return can be associated with a positive order imbalance. The implied momentum-reversal return pattern is observed by comparing the post returns of past winners and past losers generated in an intraday context. The empirical study also shows that in the reversal, past winners underperform past losers even though investors still prefer to buy past winners and sell past losers. The explanation for these phenomena lies in the recognition of the market maker as a specialist and limit orders as two heterogeneous liquidity sources. In the momentum phase there are relatively few limit orders and the market maker charges a high premium to compensate for his inventory risk. Later when sufficient limit orders arise in the reversal phase, the market maker can switch to this inexpensive liquidity source and thus reduce his premium. Although the momentum-reversal pattern could be a Nash equilibrium for limit-order traders, the market-order traders' welfare is not optimized in the equilibrium.

JEL Classification: G19.

Keywords: intraday return, liquidity, limit order.

1. Introduction

How liquidity is associated with stock market returns has been extensively explored in a growing literature [Amihud and Mendelson (1986), Eleswarapu and Reinganum (1993), Brennan and Subrahmanyam (1996), Datar, Naik, and Radcliffe (1998), Chordia, Roll, and Subrahmanyam (2002a), Pastor and Stambaugh (2002)]. Most of these studies measure returns over daily, monthly, or yearly horizons. For example, Chordia, Roll, and Subrahmanyam (2002a) find that liquidity and order imbalances are strongly related to past daily market returns. Brennan and Subrahmanyam (1996) investigate the relation between monthly stock returns and measures of illiquidity obtained from the intraday data and find a significant return premium associated with both the fixed and variable elements of the transacting cost. Pastor and Stambaugh (2002) find that on average the annual return of stocks with high sensitivities to innovations in aggregate liquidity exceeds that of stocks with low sensitivities by 7.5%. However, there are relatively few papers addressing how liquidity can affect intraday returns.

In order to gain more insight into the relation between liquidity and intraday returns, I link liquidity to intraday stock price movements measured in minutes instead of days, months, or years. Dividing the liquidity supply into two heterogeneous sources; the market maker as a specialist and limit orders, I model how their interaction could affect intraday returns. I then apply the portfolio sorting method to intraday data to empirically examine the return difference between past winners and past losers. This research complements recent work by Chordia, Roll, and Subrahmanyam (2002b) who find that in an intraday context sophisticated traders react to order imbalances by undertaking countervailing trades and thereby remove serial dependence over the daily horizon by studying twenty large-cap and twenty mid-cap stocks. My study differentiates itself from their work by explicitly considering the role of limit orders and examining the return of past winner (loser) portfolio instead of individual stock returns.

As the motivation of my empirical study, I begin with a model of how prices react to order imbalances when the market maker has an inventory concern and needs to accommodate the competition from limit-order traders. My model follows the inventory paradigm represented by Stoll (1978), Ho and Stoll (1983), O'Hara and Oldfield (1986), Grossman and Miller (1988), Spiegel and Subrahmanyam (1995), Chordia and Subrahmanyam (2002). In the spirit of Grossman and Miller (1988), I model market liquidity as being determined by the demand and supply of immediacy. There are three types of players; the market maker, market-order traders, and limit-order traders. Market-order traders perceive a gap between their current holdings and their desired holdings of a particular asset. To fill this gap, they choose to trade immediately rather than to wait. If, in aggregate, market-order buyers cannot be matched with market-order sellers perfectly, a demand of immediacy is created. The market maker then has the responsibility to meet this demand in order to clear the market. The other liquidity source is limit-order traders. They do not ask for an immediate execution of their orders and compete against the market maker for the liquidity provision service [Seppi (1997)].

The two liquidity sources differ in the following respect. Limit-order traders cannot determine the bid and ask quotes posted on the market. However, on a NYSE-type exchange, they are protected by the price priority and public priority. Price priority means that if the market maker as a specialist wants to sell (buy) at a specific price, he must first fully fill all limit sell (buy) orders at the lower (higher) position. Public priority means that the limit order at the same price level also must be filled first. These regulatory policies restrain the market maker's monopoly power. A distinguishing feature of my theoretical framework is that it explicitly addresses the relation between the limit-order depth and market returns.

In my model there is one risky asset traded in two periods. Prior to the first trading period, an information signal arrives on the market. I assume that, if the signal conveys good news, it increases the market maker's expectation of the risky asset's value and will cause the aggregate market orders placed in both trading periods to be buy orders. In the first trading period, confronted with a buy pressure, the market maker will adjust his quote higher than his expectation of the risky asset's value. Hence, the stock price movement in the first trading period will be in the same direction as the price jump caused by the information signal itself and a momentum effect will be observed.

During the second trading period, it is interesting to examine whether reversal can occur even though the order imbalance in this period has the same sign as that of the previous period. After clearing the market in the first period, the market maker holds a short position as he enters the second period. The positive order imbalance continues to create a buy pressure, causing the market maker to increase his risk premium and adjust his quotes upward. If he is the only liquidity supplier, the momentum effect will continue in the second trading period. To make the reversal occur, it is necessary for the market maker's risk premium to be reduced. This can only occur if an alternative liquidity source, limit orders, is present. If there are enough limit sell orders, the market maker can match the market buy orders, cover at least a portion of his previous short position, and thereby reduce his risk premium. As the model analysis indicates, for the reversal to occur, a sufficient and necessary condition is that the ratio of the limit-order density to the order imbalance size in the second trading period be larger than that in the first trading period. Limit orders thus play a crucial role in determining the direction of return in the second period. If there are enough limit orders, a negative return accompanied by a positive order imbalance can be observed and overall the market will witness a "momentum-reversal" return pattern.

In order to test the "momentum-reversal" pattern indicated by my model analysis, I examine the performance of past winners and past losers, which are generated and compared in an intraday context. Over time horizons from six to twelve months, Jegadeesh and Titman (1993) show that for U.S. stocks past winners will outperform past losers. When the time horizon is expanded to 3-5 years, the autocorrelation in stock returns becomes negative, as documented by Fama and French (1988) and Poterba and Summers (1988). In general, over long term horizons such as months and years the return reversal occurs after the momentum effect expires. It is an open question whether any similar return pattern can be expected in an intraday context.

The empirical results show that the momentum effect is significant after the portfolio formation and lasts for a few hours for most stocks. After that, the return difference between the past winners and the past losers turns from positive to negative, implying that reversal replaces the momentum effect. The exception is that extreme losers do not show any momentum. Instead, the reversal occurs immediately after the portfolio formation and lasts until the end of the day.

Competing explanations for these patterns include various versions of "underreaction-overreaction" story, such as the investor's sentiment model of Barberis, Shleifer, and Vishny (1998), and the information diffusion model of Hong and Stein (1999). However, order imbalances accompanying the return pattern provide support for my model. No matter whether the market is in the momentum phase or in the reversal phase, the order imbalance difference between the past winners and past losers is always positive. The implication is that in the intraday context investors always have more incentive to buy past winners and sell past losers in both the momentum phase and the reversal phase. Then a paradox is why in the reversal phase past winners underperform past losers even though there are more investors who still want to buy past winners rather than to buy past losers.

This question is answered by my model. Let us construct a security of buying past winners and short-selling past losers and think of it as the risky asset in my model. The return and order imbalance of this security is the return difference and order imbalance difference between past winners and past losers. Thus, the momentum phase can be interpreted as the first trading period in my model with positive return and positive order imbalance and the reversal phase can be interpreted as the second trading period in my model with negative return and positive order imbalance.

As shown in the later part of my model analysis, under certain condition the number of available limit orders during the reversal phase should be larger than that during the momentum phase so that the market maker's risk premium can be reduced in the reversal. This proposition is supported indirectly by my empirical observation of spread width. Regarding the security constructed in the previous paragraph, its spread width is the sum of the spread widths of the past winners and that of the past losers. Empirically, I find a decreasing trend in this spread width after the portfolio formation. Chung et al (1999) show that the market maker's own account is an expensive liquidity source and will cause large spread width and the limit-order book is relatively inexpensive and will lead to small spread width. Thus, the decrease of spread width implies that after the informational event the market is turning gradually from the expensive liquidity source (market maker's own specialist account) to the inexpensive liquidity source (limit-order book). This conjecture is also consistent with Kavajecz (1999) who finds that the depth of the limit order book will decrease around an informational event and then recover afterwards.

The organization of the paper is as follows. Section 2 presents a theoretical model that derives empirical implication for the intraday return. Section 3 describes the data and sample selection method and documents the empirical finding about the patterns of intraday return, order imbalance, and spread. Section 4 concludes the paper.

2. Theoretical Framework

A. Model Setting

In the model there is a simple world with four periods: 0, 1, 2, and 3. An informational event arrives during period 0. Periods 1 and 2 are the trading periods. Liquidation occurs during period 3. I assume two assets: a risky asset the liquidation value of which is a random variable Y with normal distribution $N(Y_0, \sigma_y^2)$ and a riskless asset (cash) with a rate of return equal to zero.

A monopolistic risk-averse market maker has constant absolute risk aversion (CARA) utility $U(W_t) = -\exp(-RW_t)$. W_t, his total wealth at time t, is equal to the sum of his cash balance W_{Ct} and inventory balance W_{It} at that time, i.e., $W_t = W_{Ct} + W_{It}$. I assume that the ask-bid spread covers only the market maker's order-processing cost. His inventory risk is compensated by shifting the quotes away from the rational expectation of the asset's liquidation value. The market maker does not suffer from adverse selection.

There are two types of traders. Limit-order traders compete with the market maker to provide liquidity service. During both trading periods, each limit-order trader submits a unit-size limit order. The number of limit-order traders is exogenously generated. Market-order traders cannot wait for a better price and need to ensure the immediate execution of their trading orders. The asynchronization of market-order trading creates a temporary order imbalance. A positive (negative) order imbalance indicates that in aggregate the market-order traders have excess demand (supply) of the risky asset.

An informational event occurs during period 0 and changes the market maker's rational expectation of the risky asset's value from Y_0^- to Y_0. Let P_0^- and P_0^+ be the market maker's midquote right before and right after the informational event. I assume period 0 is so short that in this period there is no order flow. Thus, the market maker's midquotes will be $P_0^- = Y_0^-$ and $P_0^+ = Y_0$ respectively. If the informational event is good (bad) news, the midquotes will jump up (down). I assume the market maker's initial position is $W_{C0} = W_{I0} = 0$. Hence, if the market maker needs to use his own specialist account he must either borrow the cash or borrow the stock. The cost of borrowing is non-negative. In other words, it implies that the market maker's own account is an expensive resource of liquidity compared with the limit order book.

During period 1, the informational event arrived during period 0 causes a market order imbalance, denoted by j_1. I assume that the good (bad) news will lead to a positive (negative) order imbalance, i.e., $j_1 > 0$ ($j_1 < 0$). Correspondingly, the market maker will accumulate a negative (positive) inventory, $J_1 < 0$ ($J_1 > 0$). Since the market maker is risk averse, he will post a midquote P_1, which is higher (lower) than his rational expectation Y_0. Let i_1 and I_1 be the absolute value of j_1 and J_1, and $p_1 = |P_1 - Y_0|$ be the inventory risk premium. Using the absolute values i, I, and p helps to simplify the analysis. The market maker's cash balance at time 1 is $W_{C1} = -P_1 J_1$ and his inventory balance is $W_{I1} = YJ_1$. Let M_1 be the number of limit-order traders who submit a unit-size limit order per capita during period 1.

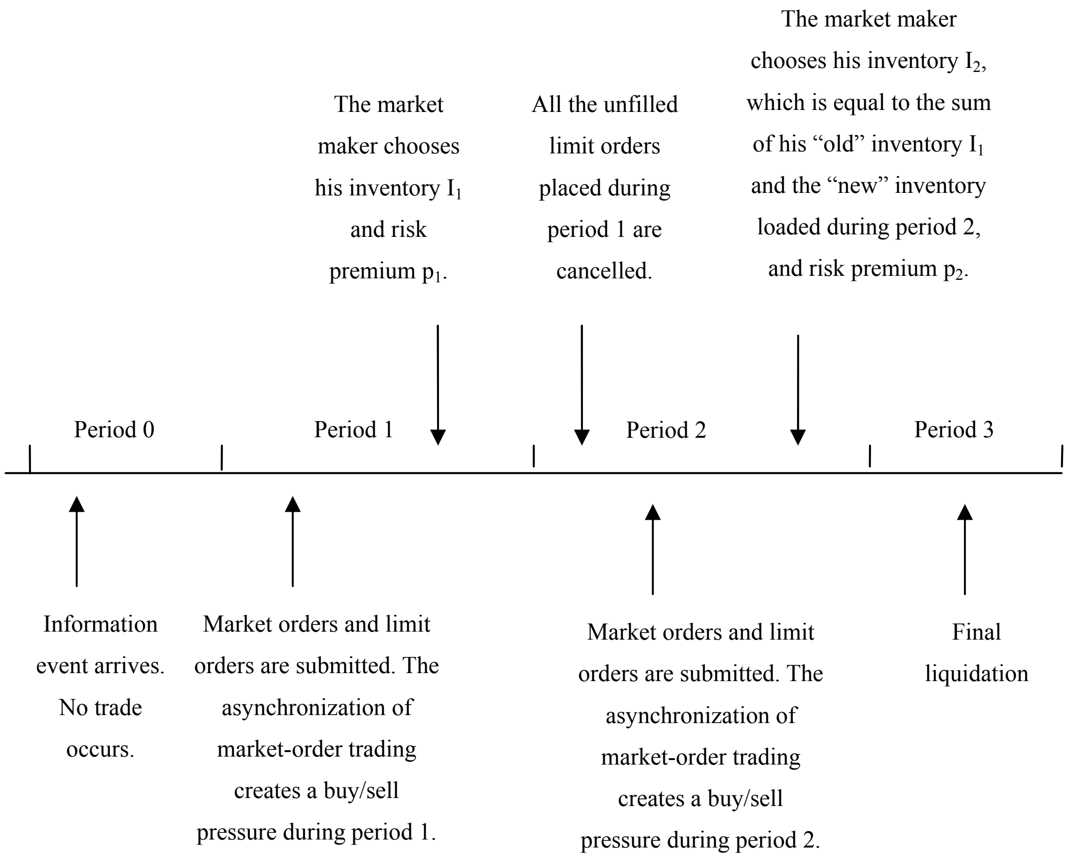
Each unit-size limit order specifies a price that, in turn, implies a premium that varies across limit-order traders.

During period 2 I suppose that the order imbalance j_2 has the same sign as j_1, that is, $j_1 j_2 > 0$. The market maker's inventory and midquote during period 2 is J_2 and P_2 respectively. Similarly, let i_2 and I_2 be the absolute value of j_2 and J_2, and $p_2 = |P_2 - Y_0|$ be the inventory risk premium at time 2. In this period, the market maker's cash balance is $W_{C2} = W_{C1} - P_2(J_2 - J_1) = (P_2 - P_1)J_1 - P_2 J_2$ and his inventory balance is $W_{I2} = YJ_2$. The number of unit size limit orders that arise during period 2 is M_2. I assume that before the beginning of period 2 all the unexecuted limit orders placed during period 1 get cancelled.

The market maker's terminal wealth at time 3 before liquidation is $W_3 = W_{C2} + YJ_2$ $= (P_2 - P_1)J_1 + (Y - P_2)J_2$. If the information event is good news and there is a buy pressure in the market, the market maker should build a negative inventory to alleviate the buy pressure according to the stabilization requirement. If the market maker accumulates a positive inventory, it implies that he actually ignores the market buy pressure and destabilizes the price. This is inconsistent with the exchange policy and is prohibited in this model. As discussed above, under the buy pressure the market maker's inventory will be negative ($J_1 < 0$ and $J_2 < 0$) and his midquote will be higher than the expected risky asset's value ($P_1 > Y_0$ and $P_2 > Y_0$). Hence, his terminal wealth W_3 can be rewritten as $W_3 = [(P_1 - Y_0) - (P_2 - Y_0)](-J_1) + (P_2 - Y)(-J_2)$. If the information event is bad news and it is a sell pressure that appears on the market, the market maker's inventory will be positive

($J_1 > 0$ and $J_2 > 0$) and his midquote will be lower than the expected risky asset's value ($P_1 < Y_0$ and $P_2 < Y_0$). It implies $W_3 = [(Y_0 - P_1) - (Y_0 - P_2)]J_1 + (Y - P_2)J_2$. Hence, it is straightforward to show $E(W_3) = (p_1 - p_2)I_1 + p_2 I_2$ and $Var(W_3) = \sigma_y^2 I_2^2$ no matter whether the informational event is good news or bad news. The first item in the expected terminal wealth is the profit the market maker earns from the premium difference between p_1 and p_2. The second item in the expected terminal wealth is the reward for him to carry the risky asset until liquidation. In Appendix 1 I also show that optimizing on the absolute-value variable I_t is equivalent to the optimization on the signed-value variable J_t ($t \in \{1,2\}$). The benefit of optimizing on absolute value variables is that it unifies the optimization procedures under buy pressure and sell pressure and bring considerable analytical convenience. Before ending the model introduction, I provide Diagram 1 as a visual illustration of the model setting. A list of the notations used in this paper can be found in Appendix 1.

Diagram 1



B. Market Maker's Optimization

In this part I solve the model by backward induction. As the first step I solve the market maker's optimization during period 2. In period 2 the market maker maximizes the expected utility of his terminal wealth $E_2(U(W_3))$. Also, the market clearing condition implies that the market maker also needs to abide by a limit order constraint. According to the limit-order priority rules, all the limit orders that imply a premium less than or equal to p_2 must be fulfilled first. This constraint implies $I_2 = (I_1 + i_2) - F(M_2, p_2)$, where M_2 is the total number of unit-size limit orders in period 2 and $F(M_2, p_2)$ is the number of limit orders that will be fulfilled conditional on p_2. Particularly, I assume $F(M_2, p_2) = \lambda_2 p_2 M_2$ when $p_2 \in (0,1/\lambda_2]$ and $F(M_2, p_2) = M_2$ when $p_2 > 1/\lambda_2$. Furthermore, I assume that the exchange does not allow the market maker to ask a risk premium higher than $1/\lambda_2$. It implies that within the interval $(0,1/\lambda_2]$ the number of fulfilled limit orders is a linear function of p_2 and the position of the unit-size limit order is uniformly distributed. Based on the assumption above, the limit-order constraint can be written as $I_2 = (I_1 + i_2) - \lambda_2 M_2 p_2$ $= (I_1 + i_2) - m_2 p_2$, where $m_2 = \lambda_2 M_2$ can be viewed as the "density" of limit orders placed in period 2. Using this definition of m_2 implies that the number of limit orders that will be fulfilled is $m_2 p_2$ if the market maker decides to charge a premium of p_2.

Providing a numerical explanation of the limit-order density, I assume that there are two stocks A and B. The expected value of stock A is Y_0^A=\$30.00 and there are 50,000 shares of limit orders that uniformly distribute in the price interval from \$30.01 to \$30.50. Stock B's expected value is also Y_0^B=\$30.00 and 60,000 shares of its limit orders uniformly distribute in the price interval from \$30.01 to \$30.80. It implies that $p^A \in (0,50¢]$, $\lambda^A = 0.02(1/¢)$, $p^B \in (0,80¢]$, and $\lambda^B = 0.0125(1/¢)$. Thus, the limit-order density of stock A is $m^A = 1{,}000$ (share/¢), higher than Stock B's limit-order density that is $m^B = 750$(share/¢), although the number of Stock B's limit orders, M^B, is larger than that of Stock A's limit orders, M^A. If the market maker of stock A increases his quote by 1¢, he needs to fulfill 1,000 shares of limit orders, while the market maker of stock B only needs to fill 750 shares of limit orders when increasing his quote by 1¢. In the following analysis I show that it is the density of limit orders that determines the return pattern. Also, I assume that there are always enough limit orders lying on the limit-order book to match the imbalanced market orders and the question is how many of them the market maker wants to fill. Specifically, I assume that $M_1 > i_1$ and $M_2 > i_1 + i_2$.[1]

The market maker maximizes his expected utility in period 2 as follows.

$$\max_{I_2} \; E_2(U(W_3)) = E_2(-\exp(-RW_3)) = -\exp\{-R((p_1 - p_2)I_1 + p_2 I_2) + \frac{1}{2}R^2\sigma_y^2 I_2^2\} \tag{1}$$

$$\text{with } I_2 = (I_1 + i_2) - m_2 p_2 \tag{2}$$

Given that, having chosen his inventory, the market maker has implicitly chosen his required premium, the optimization above is equivalent to

$$\max_{I_2} \; (p_1 - p_2)I_1 + p_2 I_2 - \frac{1}{2}R\sigma_y^2 I_2^2, \text{ with } p_2 = \frac{I_1 + i_2 - I_2}{m_2}.$$

Standard optimization procedure yields that

$$I_2^* = \frac{2I_1 + i_2}{2 + m_2 R\sigma_y^2} \tag{3}$$

$$p_2^* = \frac{m_2 R\sigma_y^2 I_1 + (1 + m_2 R\sigma_y^2)i_2}{m_2(2 + m_2 R\sigma_y^2)} \tag{4}$$

*(*See appendix for the detailed optimization procedure.)*

---

1. Kavajecz (1999) shows that the average volume of existing limit orders is 54,211 shares on the buy side and 37,457 shares on the sell side according to the TORQ data in 1991. A market order with a comparable size arrives rarely, and even if it does, it would be more likely to be settled upstairs rather than entering the specialist's floor trading.

Next, I solve the market maker's optimization problem during period 1. In order to solve the model I assume the market maker has the foresight knowledge of i_2 and M_2 when optimizing during period 1. Admittedly this is a strong assumption. However, it adds considerable tractability and enables the closed-form solutions of the model. The total market order imbalance the market maker and the limit-order traders together need to absorb is i_1. Hence, the market maker's inventory amount I_1 is $I_1 = i_1 - L(M_1, p_1) = i_1 - \lambda_1 M_1 p_1$, where M_1 is the total number of unit-size limit orders in period 1 and $L(M_1, p_1)$ is the number of limit orders that will be fulfilled conditional on p_1. I also assume that $L(M_1, p_1) = \lambda_1 p_1 M_1$ when $p_1 \in [0, 1/\lambda_1]$ and $L(M_1, p_1) = M_1$ when $p_1 > 1/\lambda_1$ and any premium higher than $1/\lambda_1$ is not allowed by the exchange. Similarly, I introduce $m_1 = \lambda_1 M_1$ as the density of limit orders during period 1. Now I can rewrite the limit-order constraint in period 1 as

$$p_1 = (i_1 - I_1)/m_1 \tag{5}$$

Thus, the market maker's optimization during period 1 is

$$\max_{I_1} \; E_2(U(W_3)) = E_2(-\exp(-RW_3)) = -\exp\{-R[(p_1 - p_2^*)I_1 + p_2^* I_2^*) - 1/2R\sigma_y^2 {I_2^*}^2]\}$$

with $I_2^* = \dfrac{2I_1 + i_2}{2 + m_2 R\sigma_y^2}$; $p_2^* = \dfrac{m_2 R\sigma_y^2 I_1 + (1 + m_2 R\sigma_y^2) i_2}{m_2(2 + m_2 R\sigma_y^2)}$; $p_1 = (i_1 - I_1)/m_1$

and s.t. $I_1 > 0$.

The optimal inventory amount and premium in period 1 implied by the Kuhn-Tucker conditions are as follows.

If $i_1 / m_1 > R\sigma_y^2 i_2 /(2 + m_2 R\sigma_y^2)$,

$$I_1^* = (\frac{1}{m_1} i_1 - \frac{R\sigma_y^2}{2 + m_2 R\sigma_y^2} i_2) / [2(\frac{1}{m_1} + \frac{R\sigma_y^2}{2 + m_2 R\sigma_y^2})] \quad (6a)$$

$$p_1^* = [(\frac{1}{m_1} + \frac{2R\sigma_y^2}{2 + m_2 R\sigma_y^2}) i_1 + \frac{R\sigma_y^2}{2 + m_2 R\sigma_y^2} i_2] / [2(\frac{1}{m_1} + \frac{R\sigma_y^2}{2 + m_2 R\sigma_y^2})] \quad (7a)$$

If $i_1 / m_1 < R\sigma_y^2 i_2 /(2 + m_2 R\sigma_y^2)$,

$$I_1^* = 0 \quad (6b)$$

$$p_1^* = i_1 / m_1 \quad (7b)$$

*(*See appendix for the detailed optimization procedure.)*

C. The Return Pattern

To examine the pattern of returns, I assume that during period 0 the informational event is good news. Hence, the market maker's quote will jump up from P_0^- to P_0^+. This can be viewed as the prior return in the following empirical part. It is easy to show that the post return $(P_1 - P_0^+)/P_0^+$ during period 1 is positive, since the market maker will ask a price higher than the rational expectation $Y_0 = P_0^+$ to compensate the risk of his short inventory position due to the positive order imbalance during period 1. Therefore, period 1 witnesses the momentum effect. The interesting question is whether during period 2 any reversal can be expected after the momentum effect, even if the order imbalance in this period has the same sign as that of period 1. In my model, the sign of the return in period 2 depends on whether the market maker's risk premium p_2 during period 2 is lower than p_1. As the optimization above implies, the density of limit orders at both periods (m_1, m_2) will play an important role of determining the market maker's risk premium. The proposition below states the sufficient and necessary condition for the reversal to occur in period 2.

Proposition 1: During period 2, the reversal will occur if and only if $m_2 / i_2 > m_1 / i_1$ and the momentum will occur if and only if $m_2 / i_2 < m_1 / i_1$. If $m_2 / i_2 = m_1 / i_1$, the midquote will not change in period 2.

(Proof: See Appendix)

After going through some tedious algebra, I get a surprisingly elegant result. Proposition 1 implies that, in order for the reversal to occur in period 2, the ratio of the limit order density to the order imbalance size in period 2 should be greater than the same ratio in period 1. This is a necessary and sufficient condition. The occurrence of reversal only depends on the relation between the limit order density and the order imbalance.

Corollary 1: (Assume $i_1 = i_2$) The reversal (momentum) will occur during period 2 if and only if $m_2 > m_1$ ($m_2 < m_1$).

Corollary 2: (Assume $\lambda_1 = \lambda_2$) The reversal (momentum) will occur during period 2 if and only if $M_2 / i_2 > M_1 / i_1$ ($M_2 / i_2 < M_1 / i_1$).

Corollary 3: (Assume $i_1 = i_2$, and $\lambda_1 = \lambda_2$) The reversal (momentum) will occur during period 2 if and only if $M_2 > M_1$ ($M_2 < M_1$).

(Proof: See Appendix)

Corollary 1 implies that, if the buy (sell) pressure in period 2 is the same as that in period 1, whether the reversal can occur depends on whether limit orders are more dense in period 2 than that in period 1. Corollary 2 indicates that, if the unit-size limit order position distributions in both periods are the same, the occurrence of reversal relies on the ratio of the limit order number to the order imbalance size. Corollary 3 shows that, after combining the assumptions in Corollaries 1 and 2, the occurrence of reversal relies on whether there are more limit orders in period 2. As a simplified version of Proposition 1, Corollary 3 lays out the setting in which I address the limit-order traders' choice and the welfare of market-order traders in the next part.

D. The Behavior of Limit Order Traders

The model analysis in the previous part focuses on the strategic choice of the market maker and the number and density of limit orders are taken as exogenous variables. In this part, this restrictive setting is modestly relaxed. Now limit-order traders have discretion over whether they trade in period 1 or in period 2. For analytical convenience, I assume that the size of the order imbalance and the distribution of limit order positions in both periods are the same ($i_1 = i_2$ and $\lambda_1 = \lambda_2$). Corollary 3 implies that the reversal occurrence in trading period 2 requires $M_2 > M_1$, i.e., more limit orders in the second period. How the aggregate trading cost of market-order traders is affected by the choice of limit-order traders is also addressed in this part.

I assume that each limit-order trader is atomistic. If he chooses to trade in the first (second) trading period, the limit-order trader earns a profit of p_1 (p_2). Also, I suppose that once his limit order is fulfilled, the limit-order trader needs to bear a non-negative cost c_1 (c_2). This cost could possibly stem from his monitoring cost and opportunity cost. The limit-order trader in period 1 needs to monitor the market more frequently so that he can identify the information event in time and respond earlier than the limit-order trader in period 2. Also, the limit-order trader in period 1 is engaged in his trading strategy for a longer time than the limit-order trader in period 2 and correspondingly endures a higher, or at least the same, opportunity cost. Hence, I assume that c_1 is larger than or equal to c_2. For tractability, I assume that if the limit order is not fulfilled, the trader will bear a zero cost. Since he is atomistic, the limit-order trader does not need to consider how other limit-order traders and the market maker will respond to his action. But he has the knowledge of the market maker's optimization procedure.

Proposition 2: (Assume $i_1 = i_2$ and $\lambda_1 = \lambda_2$) If all the limit order traders bear the same cost, i.e., $c_1 = c_2$, a Nash equilibrium is established when $M_2 = M_1$, that is, there are equal number of limit orders in both trading periods and the midquote return in the second trading period is zero.

(Proof: See Appendix)

Proposition 3: (Assume $i_1 = i_2$ and $\lambda_1 = \lambda_2$) If the cost of the limit-order trader in period 1 is higher than that in period 2, i.e., $c_1 > c_2$, in the Nash equilibrium there are more limit orders in period 2 ($M_2 > M_1$) and a "momentum-reversal" return pattern exists.

(Proof: See Appendix)

As Corollary 3 shows above, if there are more limit orders in the second trading period, the model implies a "momentum-reversal" pattern, which is observed in the following empirical test. Hence, a natural question is whether this "momentum-reversal" pattern can be a stable equilibrium. Proposition 3 indicates that, as a Nash equilibrium, it is stable as long as the cost of the limit-order trader in the first trading period is higher than that in the second trading period. Given the heterogeneous cost structure, a small group of limit-order traders chooses to pay the high cost c_1 and trade in period 1, and a large group of limit-order traders decides to trade in period 2 with the low cost c_2. In the Nash equilibrium, their profit margin is the same ($p_1 - c_1 = p_2 - c_2$). Thus, no one has the incentive to deviate. At the first sight, Proposition 2 seems to be nothing more than a degenerate version of Proposition 3. However, I am going to show that in terms of the market-order traders' welfare, the Nash equilibrium described in Proposition 2 is optimal compared to the Nash equilibrium in Proposition 3.

The net welfare of the market-order traders is determined by the benefit they receive minus the cost they pay. In my model, given that i_1, i_2 are exogenously generated, their benefit is the immediate execution of their market orders in period 1 and period 2, which does not vary according to the decision of the market maker and limit-order traders. Their cost is the total premium they pay to the market maker and limit-order traders as the fee for their liquidity service. I denote C_{Mkt} to be the aggregate trading cost of market-order traders. Clearly, $C_{Mkt} = p_1 i_1 + p_2 i_2$. I assume that the total supply of limit orders is fixed, that is, $M_1 + M_2$ is a constant. Proposition 4 states how C_{Mkt} is affected by the limit-order trader's cost structure.

Proposition 4: (Assume $i_1 = i_2$, $\lambda_1 = \lambda_2$, and $M_1 + M_2$ is a constant) The aggregate trading cost of market-order traders, C_{Mkt}, is minimized when $M_2 = M_1$.

If $c_1 = c_2$, in the limit-order traders' Nash equilibrium the market-order traders' aggregate cost is minimized. If $c_1 > c_2$, this cost is not minimized in the limit-order traders' Nash equilibrium. Given that everything else is the same, a larger $\Delta c = c_1 - c_2$ leads to a higher C_{Mkt}.

(Proof: See Appendix)

Ideally, the market-order traders prefer the same amount of limit orders in both trading periods and a flat midquote schedule posted by the market maker. This requires the cost of the limit-order trader in the period 1 to be equal to that in period 2. However, this may be unrealistic from a practical standpoint, as suggested by the empirical observation of the "momentum-reversal" intraday return pattern instead of a flat midquote level. Proposition 4 implies that the market-order trader's welfare can be improved by narrowing the cost difference between limit-order traders who trade in different periods.

The model analysis suggests that a higher ratio of the limit order density to the order imbalance size in period 2 relative to period 1 leads to a “momentum-reversal” intraday return pattern. To empirically test the model’s implication, in the next section I generate past winner and past loser portfolios and compare their post performance in an intraday context.

3. Empirical Study

A. Data and Methodology

The transaction data source is the NYSE Trades and Automated Quotations (TAQ) database. The sample space includes all the S&P 500 non-NASDAQ components. The virtue of these stocks is that they are actively traded and are unlikely to have a long interval without trading and quote adjustments. Since NASDAQ market has multiple market makers for the same security who may compete with each other, while in the model I assume the market maker is a monopolistic specialist, and the trade rules in NASDAQ are different with NYSE, I exclude securities listed on NASDAQ. To avoid the ask/bid bounce, I use quote midpoints instead of price. The sample period is from Jan 07th to May 17th, 2002.

The sign of the trade is decided by Lee and Ready (1991) logarithm: if a transaction occurs above (below) the prevailing quote mid-point, it is regarded as buyer-initiated (seller-initiated). If a transaction occurs exactly at the quote mid-point, it is signed by the tick test based on the previous transaction price. Any quote less than five seconds prior to the trade is ignored and the first one at least five seconds prior to the trade is retained. Order imbalance (OIB) for any stock over any time interval can be calculated as the number of buyer-initiated trades less the number of seller-initiated trades. Order imbalance in dollar amount (OIB$) is correspondingly the dollars paid by buyer-initiators less the dollars paid by seller-initiators. The inclusion rules are as follows. If the stock price at any day-end during the sample period was greater than $999, the stock is deleted from the sample. Any trade and quote observation is excluded if it is before the opening of the market, or after the close, or has special settlement condition. All the negative bid-ask spreads are discarded.

My trading strategy is the intraday version of the portfolio sorting method applied in the momentum literature such as Jegadeesh and Titman (1990). It selects stocks based on their returns over the past 30, 60, and 90 minutes. I also consider holding periods that vary from 30 to 180 minutes. Let us take the example of prior-30-minute/post-30-minute strategy. At the beginning of each 30-minute interval, stocks in the sample are ranked in ascending order on the basis of their returns in the past 30 minutes. Ten decile portfolios are formed based on these rankings. Each portfolio equally weights the stocks contained. Since it is possible that trades do not occur exactly at the end of a certain interval, if the closest quote revision is more than 150 seconds away from the end of the interval, the return for that interval will not be used. I should mention that in order to ensure the observation period for post returns is long enough the winner (loser) portfolios are formed no later than 1:00pm.

The rationale for applying the portfolio sorting method to test the model is as follows: if I assume an information shock arrives on the market being good news for some stocks and bad news for some others during the portfolio formation period, then the past winners could be interpreted as the beneficiary and the past losers could be interpreted as the victim due to the information shock. If I construct the security of longing past winners and shorting past losers, it can be thought of as the risky asset in my model. Hence, the price jump caused by the information event can be regarded as the prior return and the quote drift after the jump can be regarded as the post return. According to the model, the post quote drift should follow the "momentum-reversal" pattern. Of course, what I am doing here is approximation. However, given the difficulty of identifying numerous individual information shocks with their exact occurrence time and whom they affect, the portfolio sorting method should be a reasonable approach.

B. Portfolio Sorting Result

Applying the portfolio sorting method in the intraday data leads to the results summarized as follows:

(a) For most portfolios, the momentum effect is significant after the portfolio formation and lasts for a few hours. After that, the reversal effect replaces the momentum effect.

(b) As the exception, the extreme loser portfolio does not show any momentum. Instead, the reversal occurs immediately after the portfolio formation and lasts until the end of the day. The extreme winner portfolio may also expect a similar reversal without momentum, but it is far less significant than that of the extreme loser portfolio.

Figures 2a and 2b show the post 60-minute and 90-minute return patterns of the portfolios constructed according to their prior 60-minute and 90-minute returns. If I am allowed to put aside the exception of the extreme portfolios, a clear momentum trend in Figures 1a and 1b can be observed, which can be summarized as the positive return difference between portfolio #9 (past winners) and portfolio #2 (past losers). From now on, I will construct a security buying portfolio #9 as the past winners and shorting portfolio #2 as the past losers and study the return of this constructed security in the following work.

The next question is whether there is any reversal after the momentum effect. In order to answer this question, I examine the return difference between portfolio 9 (past winners) and portfolio 2 (past losers) of various time lengths of portfolio construction and post return observation. The results are provided in Tables 2a and Table 2b. Table 2a is the cumulative return difference between the past winners and past losers and Table 2b is the corresponding incremental return difference. Table 2b can be interpreted as the result of operating first-order auto-difference calculations on Table 2a. If the market is in the momentum phase, the cumulative return difference should increase and the incremental return difference should be positive. If the market is in the reversal phase, the cumulative return difference should decrease and the incremental return difference should be negative. The "momentum-reversal" pattern is clear in Tables 2a and 2b. For example, Let us choose the prior-60-minute strategy. The return differences between the past winners and past losers are significantly positive at 99% confidence level from the first 30-minute interval to the third 30-minute interval after the portfolio formation, with a t-statistics no less than 2.84. During the fourth 30-minute interval, it is still positive, but less significant, with a t-statistics 1.19 only. The cumulative return difference between the past winners and past losers reaches its peak at the end of the forth 30-minute interval, that is, 120 minutes after the portfolio formation. The peak value of the cumulative return difference is 6.47 basis points with a t-statistics as large as 4.88. After that, past losers begin to outperform past winners. The return differences between the past winners and past losers in the fifth 30-minute interval and in the sixth 30-minute interval are -0.45 basis points and -1.21 basis points respectively. The result implies that the first two hours after the portfolio formation is the momentum phase. After that, the reversal phase takes place.

Given the existence of the “momentum-reversal” quote drift pattern, how can the readers are convinced that my heterogeneous liquidity source model is the underlying reason for this phenomenon? Ideally, if I had the data including all the information about the limit orders for our sample, I could directly examine the change of depth of the limit-order book and how much liquidity is provided by it to test our model. However, the NYSE TORQ (Trades, Orders, Reports, and Quotes) database that includes necessary limit order information is not publicly available. In addition, even if I had the same access to the part of the TORQ database that was used in previous empirical literature such as Chung et al (1999) and Kavajecz (1999), the database covers only 144 NYSE securities from November 1990 to January 1991. Therefore, I examine the related microstructure variables, that is, order imbalance and spread, to obtain the indirect supportive evidence for the model.

The order imbalance (OIB) study reveals the dollar-amount order imbalance difference between the past winners and past losers is always positive and fluctuates within a fair range in both the momentum phase and the reversal phase, as shown in Tables 3a and 3b. Order imbalance is the number of buyer-initiated transactions less the number of seller-initiated transactions and the dollar-amount order imbalance is the dollar amount of buyer-initiated transactions less the dollar amount of seller-initiated transactions. A positive dollar-amount order imbalance means that there is more money buying the asset than selling it. Hence I know that the investor's incentive, on net, to buy the past winners never decays during the same trading day, even when the past winners actually underperform past losers. Based on this fact, I think there exists a positively auto-correlated excess demand for the past winners in the intraday scenario. In particular, it should be reasonable to post the restricting assumption for the net pooled market order to be of the same sign during two trading periods in my model. Also, the positive dollar-amount order imbalance difference between the past winners and past losers in the reversal phase indicates that it might not be the optimal choice to explain the reversal as the price correction from the previous over-reaction. Otherwise, investors should on average sell the past winners rather than buy them in the reversal phase to correct the price.

Another piece of the indirect evidence is the spread width. Given the risky asset in our model is the combination of buying the past winners and selling the past losers, its spread is naturally the sum of spreads of past winners and losers (portfolio #9 and portfolio #2). However, I should mention that there is an inherent intraday spread pattern for NYSE stocks that spread widths are largest right after the market opens and then decrease, as shown by McInish and Wood (1992), and Lee, Mucklow, and Ready (1993). This inherent pattern could obscure the empirical study of spread in this paper. Therefore I use the portfolios in the midway (portfolio #5 and portfolio #6) as the benchmark and calculate the difference between the spread of past winners and past losers (portfolio #9 and portfolio #2) and midway portfolios (portfolio #5 and portfolio #6) to remove the inherent intraday spread pattern mentioned above. The results are provided in Tables 4a(b) and 5a(b). Clearly, the spread width shrinks as time goes on. Assuming that the market maker's own account is the expensive liquidity source and causes large spread width, and the limit-order book is relatively inexpensive, and therefore leads to small spread width, the decrease of spread width implies that after the informational shock the market is turning gradually from the expensive liquidity source (market maker's own account) to the inexpensive liquidity source (limit-order book).

C. Subperiod Test

The message from the empirical study up to now is that in the intraday context there are a "momentum-reversal" return pattern, an always positive order imbalance difference between the past winners and the past losers, and a decreasing trend of spread width. In order to test whether these results are robust to different market situations, I apply the same empirical method to a bull market subperiod and a bear market subperiod separately. During the whole sample period I identify a bull trend, which is from February 02nd to March 19th when the S&P 500 index went up by 6.92%, and a bear trend, which is from March 20th to May 10th when S&P 500 went down by 11.10%. Tables 6a and 6b reports the cumulative and increment return differences between the past winners and past losers in the bull market and the bear market correspondingly.

Like the overall sample period, the subperiods witness a "momentum-reversal" intraday return pattern. Let us still take the example of the prior-60-minute strategy. In both markets, the return differences between the past winners and past losers keep increasing until the 120 minutes after the portfolio formation. In the bull market, the peak value of the cumulative return difference is 5.56 basis points with a t-statistics of 2.39. In the bear market, the peak value of the cumulative return difference is 8.83 basis points with a t-statistics of 4.49. After that, past winners underperform past losers. The return differences between the past winners and past losers in the fifth 30-minute interval and in the sixth 30-minute interval are -0.12 basis points and -1.80 basis points in the bull market and -0.60 basis points and -1.23 basis points in the bear market respectively. Hence in both markets the momentum phase lasts for the first two hours after the portfolio formation and after that reversal occurs. It seems that the significance of the result in the bear market is stronger than that in the bull market. But I should mention that the bear subperiod includes 38 trading days, while the bull subperiod contains only 31 trading days.

Similarly with the return pattern, the patterns of order imbalance and spread width are also qualitatively replicated in the bull and the bear subperiods, as shown in Tables 7a, 7b, 8a, 8b, 9a, and 9b. They imply that the previous empirical results about return, order imbalance, and spread are robust in both the bull and the bear markets.

Before ending this section, I should mention that it still remains an open question regarding why for the extreme loser (winner) portfolio the reversal occurs immediately after the portfolio formation without any momentum period. One possible explanation is that this occurs because of arbitrageurs. Suppose there exist a group of arbitrageurs on the exchange floor who has a threshold of cost C. If the market maker's quote has been shifted away from the rational expectation more than their cost threshold C, the floor arbitrageurs will trade to earn a profit from the difference. Otherwise, they will choose to do some other profitable business. It might be possible that for the extreme loser (winner) portfolio the difference between the market maker's quote and the rational expectation is larger than C and the arbitrageurs counter-trade by buying the past losers at the discounted price or selling past winners at the overpriced quote. Once this arbitrageur-effect is strong enough, it may override other factors and lead to the immediate reversal for the extreme losers (winners).

4. Conclusion

In this paper I study the association among liquidity, order imbalance, and intraday stock market returns. I derive implications about the relation between market liquidity and intraday returns by developing a two-trading-period model in which the market liquidity source is composed of two heterogeneous parts; the market maker as a specialist and limit-order traders. The risk-averse market maker needs to adjust his inventory risk and accommodate the competition from limit-order traders in providing liquidity. The limit-order traders cannot determine the current quotes on the market but are protected by the price priority and public priority. The interaction between these two liquidity sources affects the return pattern.

I assume that an information event causes the order imbalance in the same direction during two successive trading periods within an intraday context. In the first trading period, the order imbalance forces the market maker to ask for a risk premium by shifting his midquote away from his rational expectation of the asset value. Thus, a momentum effect can be observed. The interesting part in my model is whether reversal can occur during the second trading period. I should mention that in the second period the order imbalance is assumed to be in the same direction as that in the first period. Hence, in order to make the market maker reduce his risk premium, the amount of limit orders should be sufficiently large to help the market maker offload part of his previous inventory.

My empirical study provides results consistent with the theory. A "momentum-reversal" intraday return pattern is observed by applying the portfolio sorting method to the intraday data except for the extreme losers. Further, the order imbalance difference between the past winners and past losers is always positive in both the momentum phase and the reversal phase. Hence, if I construct a security of buying past winners and selling past losers, in the reversal phase there is a negative return with a positive order imbalance. This provides supportive evidence for my model.

From a practical standpoint, my result implies that intraday traders could consider timing their trade to provide liquidity when the market needs it most. For example, the empirical "momentum-reversal" pattern implies after the arrival of good news a profitable trading strategy is to sell short around the end of momentum phase and then cover the short position after the reversal. This profit is not generated from superior information. It is because smart traders can help to alleviate the order imbalance pressure when the market needs liquidity suppliers most. They will be compensated for their correct liquidity timing.

In the future study, it would be interesting to consider how the "direct-reversal" pattern of the extreme losers and winners and the "momentum-reversal" pattern of the rest of the portfolios can be unified. How to incorporate the adverse selection concern is another topic worth more thought.

Reference

Barberis N., Shleifer A., and Vishny R., 1998, A Model of Investor Sentiment. Journal of Financial Economics 49, 307-343.

Chordia, T., and Subrahmanyam, A., 2002a, Order Imbalance and Individual Stock Returns. UCLA Working Paper.

Chordia, T., Roll, R., and Subrahmanyam, A., 2002b, Evidence on the Speed of Convergence to Market Efficiency. UCLA Working Paper.

Chung, K., Van Ness, B., and Van Ness, R. 1999. Limit Orders and the Bid-Ask Spread. Journal of Financial Economics 53, 255-287.

Daniel, K., Hirshleifer, D., and Subrahmanyam, A., 1998, Investor Psycology and Security Market Under- and Overreactions. Journal of Finance 53, 1839-1885.

Datar V., Naik N., and Radcliffe R., 1998, Liquidity and stock returns: An alternative test. Journal of Financial Markets 1, 203-219

Eleswarapu V. and Reinganum M., 1993, The seasonal behavior of the liquidity premium in asset pricing, Journal of Financial Economics 34, 373-386.

Fama, E. and French, K., 1988, Permanent and Temporary Components of Stock Prices, Journal of Political Economy 96, 246-273.

Fama, E. and French, K., 1996, Multifactor Explanations of Asset Pricing Anomalies, Journal of Finance 51, 55-84.

Hasbrouck, J. 1991. Measuring the Information Content of Stock Trades. Journal of Finance 46, 179-207.

Hong, H. and Stein, J., 1999. A Unified Theory of Underreaction, Momentum Trading, and Overreaction in Asset Markets. Journal of Finance Volume LIV, 2143-2184.

Ho, T. and Stoll, H., 1983, The Dynamics of Dealer Markets Under Competition, Journal of Finance 38, 1053-1074.

Huang, R. and Stoll, H., 1994. Market Microstructures and Stock Return Predictions. Review of Financial Studies 7, 179-213.

Jegadeesh, N. and Titman, S., 1993, Returns to Buying Winners and Selling Losers: Implication for Stock Market Efficiency. Journal of Finance 48, 65-91

Kavajecz, K., 1999, A Specialist's Quoted Depth and the Limit Order Book. Journal of Finance, Volume LIV, 747-771.

Lee, C. and Ready, M., 1993. Spreads, Depths, and the Impact of Earnings Information: An Intraday Analysis. Review of Financial Studies 6, 345-374.

Madhavan, A., Richardson M., and Roomans, M. 1997 Why Do Security Prices Change? A Transaction-Level Analysis of NYSE Stocks. Review of Financial Studies 10, 1035-1064.

O'Hara, M. and Oldfield, G., 1986, The Microeconomics of Market Making, Journal of Financial and Quantitative Analysis 21, 361-376.

Poterba, J. and Summers, L., 1988, Mean Reversion in Stock Returns: Evidence and Implications. Journal of Financial Economics 22, 27-59.

Rouwenhorst, G., 1998, International Momentum Strategies, Journal of Finance 53, 267-284.

Seppi, D., 1997, Liquidity Provision with Limit Orders and Strategic Specialist. Review of Financial Studies 10, 103-150.

Spiegel M. and Subrahmanyam, A., 1995, On Intraday Risk Premia, Journal of Finance 50, 319-339.

Stoll, H., 1978, The Supply of Dealer Services in Securities Markets, Journal of Finance 33, 1133-1151.

Appendix 1

Some notations used in the paper:

Y_0, σ_y^2 : The first and second moments of the risky asset's value Y;

j_t: The order imbalance during period t;

i_t: The absolute value of the order imbalance during period t;

J_t: The market maker's inventory during period t;

I_t: The absolute value of the market maker's inventory during period t;

P_t: The midquote during period t;

p_t: The absolute value of the difference between the midquote and the expectation of the risky asset's value during period t ($p_1 = |P_1 - Y_0|$, $p_2 = |P_2 - Y_0|$);

M_t: The number of unit-size limit orders during period t;

$L(p_1)$: The cumulative probability for a unit-size limit order in period 1 to be executed conditional on p_1. I assume that $F(p_2)$ is a linear function of p_2, i.e., $L(p_1) = \lambda_1 p_1$

$F(p_2)$: The cumulative probability for a unit-size limit order in period 2 to be executed conditional on p_2. I assume that $F(p_2)$ is a linear function of p_2, i.e., $F(p_2) = \lambda_2 p_2$

m_t: The density of limit orders during period t ($m_t = \lambda_t M_t$). The interpretation of the density concept is that if the market maker decides to charge a premium of p_t, the number of limit orders that will be executed is $m_t p_t$.

* $t \in \{1,2\}$

Market maker's optimization during period 2:

At period 2 the market maker maximizes his expected utility as

$$\max_{I_2} E_2(U(W_3)) = E_2(-\exp(-RW_3)) \ = E_2[-\exp(-R((P_2 - P_1)J_1 + (Y - P_2)J_2))]$$

$$= -\exp\{-R((p_1 - p_2)I_1 + p_2 I_2) + 1/2R^2\sigma_y^2 I_2^2\}$$

s.t. $I_2 = (I_1 + i_2) - M_2 F(p_2) = (I_1 + i_2) - \lambda_2 M_2 p_2$

This is equal to $\max_{I_2} \ (p_1 - p_2)I_1 + p_2 I_2 - 1/2R\sigma_y^2 I_2^2$, with $p_2 = \dfrac{I_1 + i_2 - I_2}{\lambda_2 M_2}$.

The objective function can be written as

$$(p_1 - p_2)I_1 + p_2 I_2 - 1/2R\sigma_y^2 I_2^2$$

$$= p_1 I_1 + \frac{I_1 + i_2 - I_2}{\lambda_2 M_2}(I_2 - I_1) - 1/2R\sigma_y^2 I_2^2$$

$$= p_1 I_1 + \frac{I_1 + i_2}{\lambda_2 M_2} I_2 - \frac{1}{\lambda_2 M_2} I_2^2 - \frac{I_1 + i_2}{\lambda_2 M_2} I_1 + \frac{I_1}{\lambda_2 M_2} I_2 - \frac{1}{2} R\sigma_y^2 I_2^2$$

$$= -(\frac{1}{2} R\sigma_y^2 + \frac{1}{\lambda_2 M_2}) I_2^2 + \frac{2I_1 + i_2}{\lambda_2 M_2} I_2 + p_1 I_1 - \frac{I_1 + i_2}{\lambda_2 M_2} I_1$$

The first order condition implies

$$I_2^* = \frac{\dfrac{2I_1 + i_2}{\lambda_2 M_2}}{2(\dfrac{1}{2} R\sigma_y^2 + \dfrac{1}{\lambda_2 M_2})} = \frac{I_1 + \dfrac{1}{2} i_2}{1 + \dfrac{1}{2}\lambda_2 M_2 R\sigma_y^2} = \frac{2I_1 + i_2}{2 + m_2 R\sigma_y^2}, \text{ where } m_2 = \lambda_2 M_2 .$$

The equilibrium price will be

$$p_2^* = \frac{I_1 + i_2 - I_2}{\lambda_2 M_2} = \frac{(I_1 + i_2)(1 + \dfrac{1}{2}\lambda_2 M_2 R\sigma_y^2) - (I_1 + \dfrac{1}{2} i_2)}{\lambda_2 M_2 (1 + \dfrac{1}{2}\lambda_2 M_2 R\sigma_y^2)}$$

$$= \frac{\dfrac{1}{2}\lambda_2 M_2 R\sigma_y^2 I_1 + \dfrac{1}{2}(1 + \lambda M_2 R\sigma_y^2) i_2}{\lambda_2 M_2 (1 + \dfrac{1}{2}\lambda M_2 R\sigma_y^2)} = \frac{m_2 R\sigma_y^2 I_1 + (1 + m_2 R\sigma_y^2) i_2}{m_2 (2 + m_2 R\sigma_y^2)}$$

Since the second order condition is $-(\frac{1}{2}R\sigma_y^2 + \frac{1}{\lambda_2 M_2}) < 0$, the objective function is maximized because of its concavity. Also, it is straightforward to show that $I_2^* = \frac{2I_1 + i_2}{2 + m_2 R\sigma_y^2} \geq 0$ and $p_2 = \frac{I_1 + i_2 - I_2}{\lambda_2 M_2} < \frac{i_1 + i_2}{\lambda_2 M_2} < \frac{1}{\lambda_2}$, given the assumption $M_2 > i_1 + i_2$.

Market maker's optimization during period 1:

According to the limit-order constraint, the market maker's inventory amount I_1 is

$I_1 = i_1 - M_1 L(p_1) = i_1 - \lambda_1 M_1 p_1 \Rightarrow p_1 = \frac{i_1 - I_1}{\lambda_1 M_1} = \frac{i_1 - I_1}{m_1}$, where $m_1 = \lambda_1 M_1$.

Thus, the market maker's optimization during period 1 is

$$\max_{I_1} \; -\exp\{-R[(p_1 - p_2^*)I_1 + p_2^* I_2^*) - 1/2 R\sigma_y^2 I_2^{*2}]\}$$

with $I_2^* = \frac{2I_1 + i_2}{2 + m_2 R\sigma_y^2}$; $p_2^* = \frac{m_2 R\sigma_y^2 I_1 + (1 + m_2 R\sigma_y^2) i_2}{m_2(2 + m_2 R\sigma_y^2)}$; $p_1 = (i_1 - I_1)/m_1$

Subject to I_1>0.

The objective function can be written as

$$[(p_1 - p_2^*)I_1 + p_2^* I_2^*] - 1/2 R\sigma_y^2 I_2^{*2}$$

$$= [\frac{1}{m_1}(i_1 - I_1) - \frac{m_2 R\sigma_y^2 I_1 + (1 + m_2 R\sigma_y^2) i_2}{m_2(2 + m_2 R\sigma_y^2)}]I_1 + \frac{m_2 R\sigma_y^2 I_1 + (1 + m_2 R\sigma_y^2) i_2}{m_2(2 + m_2 R\sigma_y^2)} \frac{2I_1 + i_2}{2 + m_2 R\sigma_y^2}$$

$$- \frac{1}{2} R\sigma_y^2 (\frac{2I_1 + i_2}{2 + m_2 R\sigma_y^2})^2$$

$$=[\frac{i_1}{m_1}I_1-\frac{1}{m_1}I_1^2-\frac{R\sigma_y^2}{2+m_2R\sigma_y^2}I_1^2-\frac{(1+m_2R\sigma_y^2)i_2}{m_2(2+m_2R\sigma_y^2)}I_1]$$

$$+[\frac{2R\sigma_y^2}{(2+m_2R\sigma_y^2)^2}I_1^2+\frac{(2+3m_2R\sigma_y^2)i_2}{m_2(2+m_2R\sigma_y^2)^2}I_1+\frac{(1+m_2R\sigma_y^2)i_2^2}{m_2(2+m_2R\sigma_y^2)^2}]$$

$$-[\frac{2R\sigma_y^2}{(2+m_2R\sigma_y^2)^2}I_1^2+\frac{2R\sigma_y^2 i_2}{(2+m_2R\sigma_y^2)^2}I_1+\frac{R\sigma_y^2 i_2^2}{2(2+m_2R\sigma_y^2)^2}]$$

$$=-[\frac{1}{m_1}+\frac{R\sigma_y^2}{2+m_2R\sigma_y^2}]I_1^2+[\frac{i_1}{m_1}-\frac{(1+m_2R\sigma_y^2)i_2}{m_2(2+m_2R\sigma_y^2)}+\frac{(2+3m_2R\sigma_y^2)i_2}{m_2(2+m_2R\sigma_y^2)^2}$$

$$-\frac{2R\sigma_y^2 i_2}{(2+m_2R\sigma_y^2)^2}]I_1+[\frac{(1+m_2R\sigma_y^2)i_2^2}{m_2(2+m_2R\sigma_y^2)^2}-\frac{R\sigma_y^2 i_2^2}{2(2+m_2R\sigma_y^2)^2}]$$

$$=-[\frac{1}{m_1}+\frac{R\sigma_y^2}{2+m_2R\sigma_y^2}]I_1^2+[\frac{i_1}{m_1}-\frac{R\sigma_y^2 i_2}{2+m_2R\sigma_y^2}]I_1+[\frac{(1+m_2R\sigma_y^2)i_2^2}{m_2(2+m_2R\sigma_y^2)^2}-\frac{R\sigma_y^2 i_2^2}{2(2+m_2R\sigma_y^2)^2}]$$

Hence, the optimization can be rewritten as

$$\max_{I_1}\ -[\frac{1}{m_1}+\frac{R\sigma_y^2}{2+m_2R\sigma_y^2}]I_1^2+[\frac{i_1}{m_1}-\frac{R\sigma_y^2 i_2}{2+m_2R\sigma_y^2}]I_1+[\frac{(1+m_2R\sigma_y^2)i_2^2}{m_2(2+m_2R\sigma_y^2)^2}-\frac{R\sigma_y^2 i_2^2}{2(2+m_2R\sigma_y^2)^2}]$$

s.t $I_1>0$.

The Lagrange function is

$$L=-[\frac{1}{m_1}+\frac{R\sigma_y^2}{2+m_2R\sigma_y^2}]I_1^2+[\frac{i_1}{m_1}-\frac{R\sigma_y^2 i_2}{2+m_2R\sigma_y^2}]I_1$$

$$+[\frac{(1+m_2R\sigma_y^2)i_2^2}{m_2(2+m_2R\sigma_y^2)^2}-\frac{R\sigma_y^2 i_2^2}{2(2+m_2R\sigma_y^2)^2}]+k_1(I_1-0)$$

The Kuhn-Tucker conditions for this optimization with inequality constraint are as follows.

$$-2[\frac{1}{m_1}+\frac{R\sigma_y^2}{2+m_2R\sigma_y^2}]I_1+[\frac{i_1}{m_1}-\frac{R\sigma_y^2 i_2}{2+m_2R\sigma_y^2}]+k_1=0$$

$k_1I_1=0$ and $I_1\geq 0$

If $i_1/m_1 > R\sigma_y^2 i_2/(2+m_2R\sigma_y^2)$, I have

$$I_1^* = (\frac{1}{m_1}i_1 - \frac{R\sigma_y^2}{2+m_2R\sigma_y^2}i_2)/[2(\frac{1}{m_1}+\frac{R\sigma_y^2}{2+m_2R\sigma_y^2})] > 0,\ k_1 = 0,$$

and $p_1^* = [(\frac{1}{m_1}+\frac{2R\sigma_y^2}{2+m_2R\sigma_y^2})i_1 + \frac{R\sigma_y^2}{2+m_2R\sigma_y^2}i_2]/[2m_1(\frac{1}{m_1}+\frac{R\sigma_y^2}{2+m_2R\sigma_y^2})]$ and

If $i_1/m_1 \le R\sigma_y^2 i_2/(2+m_2R\sigma_y^2)$, I have

$I_1^* = 0$, $k_1 = \frac{R\sigma_y^2 i_2}{2+m_2R\sigma_y^2} - \frac{i_1}{m_1} \ge 0$, and $p_1^* = i_1/m_1$.

Since $\partial^2 L/\partial I_1^2 = -2[\frac{1}{m_1}+\frac{R\sigma_y^2}{2+m_2R\sigma_y^2}] < 0$, the objective function is maximized because of its concavity. Also, from $p_1 = (i_1 - I_1)/m_1$ and $I_1 \ge 0$ I have $p_1 \le i_1/m_1 = i_1/(\lambda_1 M_1) < 1/\lambda_1$, given the assumption that $M_1 > i_1$.

Justifying the use of absolute-value variables.

(A) The information event is good news and there is a buy pressure in the market. Then the market maker's inventories are negative ($J_1 < 0$ and $J_2 < 0$) and his midquotes are higher than the expected risky asset's value ($P_1 > Y_0$ and $P_2 > Y_0$). His terminal wealth is $W_3 = (P_2 - P_1)J_1 + (Y - P_2)J_2 = [(P_1 - Y_0) - (P_2 - Y_0)](-J_1) + (P_2 - Y)(-J_2)$.

Thus, his second period optimization is

$$\max_{J_2} E_2(U(W_3)) = E_2(-\exp(-RW_3)) = E_2[-\exp(-R((P_2 - P_1)J_1 + (Y - P_2)J_2))]$$

$$= -\exp\{-R\{[(P_1 - Y_0) - (P_2 - Y_0)](-J_1) + (P_2 - Y_0)(-J_2)\} + (1/2)R^2\sigma_y^2 J_2^2\}.\ (Y_0 = E(Y))$$

Given $p_2 = P_2 - Y_0$ and $I_2 = -J_2$, it is straightforward to show that the maximization above is equal to $\max_{I_2} E_2(U(W_3)) = -\exp\{-R((p_1 - p_2)I_1 + p_2 I_2) + 1/2R^2\sigma_y^2 I_2^2\}$, which is the maximization used in this paper.

Similarly, the market maker's optimization in period 1 is

$$\max_{J_1} E_1(U(W_3)) = E_1(-\exp(-RW_3)) = E_1[-\exp(-R((P_2 - P_1)J_1 + (Y - P_2)J_2))]$$

$$= -\exp\{-R[(P_1 - Y_0) - (P_2 - Y_0)](-J_1) + (P_2 - Y_0)(-J_2)] + (1/2)R^2\sigma_y^2 J_2^2\}.$$

Given $p_1 = P_1 - Y_0$ and $I_1 = -J_1$, the optimization above is also the equivalent of

$\max_{I_1} \; -\exp\{-R[(p_1 - p_2^*)I_1 + p_2^* I_2^*) - 1/2R\sigma_y^2 I_2^{*2}]\}$, which is the maximization used in the paper.

In order to let the reversal occur in the second period, P_2 should be lower than P_1, since the prior return is positive and the return in the first period is also positive. This is equal to $p_2 < p_1$.

Hence I show that under the buy pressure the optimization and analysis of reversal occurrence based on absolute-value variables are the same as that based on signed-value variables.

(B) The information event is bad news and there is a sell pressure in the market. The market maker's inventories are positive ($J_1 > 0$ and $J_2 > 0$) and his midquotes are lower than the expected risky asset's value ($P_1 < Y_0$ and $P_2 < Y_0$). His terminal wealth is $W_3 = (P_2 - P_1)J_1 + (Y - P_2)J_2 = [(Y_0 - P_1) - (Y_0 - P)]J_1 + (Y - P_2)J_2$.

Thus, his second period optimization is

$$\max_{J_2} E_2(U(W_3)) = E_2(-\exp(-RW_3)) = E_2[-\exp(-R((P_2 - P_1)J_1 + (Y - P_2)J_2))]$$

$$= -\exp\{-R\{[(Y_0 - P_1) - (Y_0 - P)]J_1 + (Y - P_2)J_2\} + (1/2)R^2\sigma_y^2 J_2^2\}.$$

Given $p_2 = Y_0 - P_2$ and $I_2 = J_2$, it is straightforward to show that the maximization above is equal to $\max_{I_2} E_2(U(W_3)) = -\exp\{-R((p_1 - p_2)I_1 + p_2 I_2) + 1/2R^2\sigma_y^2 I_2^2\}$, the maximization used in this paper.

Similarly, the market maker's optimization in period 1 is

$$\max_{J_1} E_1(U(W_3)) = E_1(-\exp(-RW_3)) = E_1[-\exp(-R((P_2 - P_1)J_1 + (Y - P_2)J_2))]$$

$$= -\exp\{-R\{[(Y_0 - P_1) - (Y_0 - P)]J_1 + (Y - P_2)J_2\} + (1/2)R^2\sigma_y^2 J_2^2\}.$$

Given $p_1 = Y_0 - P_1$ and $I_1 = J_1$, the optimization above is the equivalent of $\max_{I_1} -\exp\{-R[(p_1 - p_2^*)I_1 + p_2^* I_2^*) - 1/2R\sigma_y^2 I_2^{*2}]\}$, the maximization used in this paper.

In order to let the reversal occur in the second period, P_2 should be higher than P_1, since the prior return is negative and the return in the first period is also negative. This is equal to $p_2 < p_1$.

Hence, I show that under the sell pressure the optimization and analysis of reversal occurrence based on absolute-value variables are also the same as that based on signed-value variables.

In general, the analysis above shows that using absolute-value variables I_t, p_t is equal to using signed-value variables J_t, P_t.

Proof of Proposition 1.

The sufficient and necessary condition for the occurrence of reversal is that the market maker's risk premium p_2 in period 2 should be lower than p_1 in period 1. Hence, I need

$$p_1 = \frac{i_1 - I_1}{m_1} > p_2^* = \frac{m_2 R\sigma_y^2 I_1 + (1 + m_2 R\sigma_y^2) i_2}{m_2(2 + m_2 R\sigma_y^2)} \Leftrightarrow I_1 < \frac{\frac{1}{m_1} i_1 - \frac{1 + R\sigma_y^2 m_2}{m_2(2 + R\sigma_y^2 m_2)} i_2}{\frac{1}{m_1} + \frac{R\sigma_y^2}{2 + R\sigma_y^2 m_2)}}$$

So the reversal occurs if and only if

$$I_1^* = \frac{\frac{1}{m_1}\frac{i_1}{i_2} - \frac{R\sigma_y^2}{2 + R\sigma_y^2 m_2}}{2[\frac{1}{m_1} + \frac{R\sigma_y^2}{2 + R\sigma_y^2 m_2}]} i_2 < \frac{\frac{1}{m_1}\frac{i_1}{i_2} - \frac{1 + R\sigma_y^2 m_2}{m_2(2 + R\sigma_y^2 m_2)}}{\frac{1}{m_1} + \frac{R\sigma_y^2}{2 + R\sigma_y^2 m_2}} i_2$$

$$\Leftrightarrow \frac{1}{m_1}\frac{i_1}{i_2} - \frac{R\sigma_y^2}{2 + R\sigma_y^2 m_2} < 2[\frac{1}{m_1}\frac{i_1}{i_2} - \frac{1 + R\sigma_y^2 m_2}{m_2(2 + R\sigma_y^2 m_2)}]$$

$$\Leftrightarrow -R\sigma_y^2 m_2 < \frac{1}{m_1}\frac{i_1}{i_2} m_2(2 + R\sigma_y^2 m_2) - 2(1 + R\sigma_y^2 m_2)$$

$$\Leftrightarrow R\sigma_y^2 \frac{1}{m_1}\frac{i_1}{i_2} m_2^2 + (2\frac{1}{m_1}\frac{i_1}{i_2} - R\sigma_y^2) m_2 - 2 > 0$$

$$\Leftrightarrow (\frac{1}{m_1}\frac{i_1}{i_2} m_2 - 1)(R\sigma_y^2 m_2 + 2) > 0$$

Given $R\sigma_y^2 > 0$ and $m_2 > 0$, I can conclude that reversal happens if and only if $\frac{m_2}{i_2} > \frac{m_1}{i_1}$.

Correspondingly, momentum occurs if and only if $\frac{m_2}{i_2} < \frac{m_1}{i_1}$.

The analysis above applies to $I_1^* > 0$. If $I_1^* = 0$, then $p_1^* = i_1 / m_1$. This happens only

when $\frac{i_1}{m_1} < \frac{R\sigma_y^2 i_2}{2 + m_2 R\sigma_y^2}$, which is equal to $\frac{m_1}{i_1} > \frac{m_2}{i_2} + \frac{2}{R\sigma_y^2 i_2} > \frac{m_2}{i_2}$. It is straightforward to show that, given $I_1^* = 0$ and $\frac{i_1}{m_1} < \frac{R\sigma_y^2 i_2}{2 + m_2 R\sigma_y^2}$, $p_1^* = \frac{i_1}{m_1} < p_2^* = \frac{(1 + m_2 R\sigma_y^2) i_2}{(2 + m_2 R\sigma_y^2) m_2}$ and momentum happens in period 2. Hence the conclusion above also holds true.

Proof of Corollaries 1,2, and 3.

If $i_1 = i_2$, then I have $\frac{m_2}{i_2} > \frac{m_1}{i_1} \Leftrightarrow m_2 > m_1$ as the necessary and sufficient condition for the reversal to occur in period 2. If $\lambda_1 = \lambda_2$, then I have $\frac{m_2}{i_2} > \frac{m_1}{i_1} \Leftrightarrow \frac{m_2 \lambda_2}{i_2} > \frac{m_1 \lambda_1}{i_1}$

$\Leftrightarrow \frac{M_2}{i_2} > \frac{M_1}{i_1}$ as the necessary and sufficient condition for the reversal to occur in period 2.

It is straightforward to show that if $i_1 = i_2$ and $\lambda_1 = \lambda_2$, the necessary and sufficient condition for the reversal to occur in period 2 is $M_2 > M_1$

Proof of Proposition 2.

Assuming $i_1 = i_2 = i$ and $\lambda_1 = \lambda_2 = \lambda$, the optimal midquote and inventory choice of the market maker can be written as

$$I_1^* = \frac{(2 + R\sigma_y^2 m_2) - R\sigma_y^2 m_1}{2[(2 + R\sigma_y^2 m_2) + R\sigma_y^2 m_1]} i = \frac{1 + R\sigma_y^2 (m_2 - m_1)/2}{2 + R\sigma_y^2 (m_1 + m_2)} i$$

$$p_1^* = \frac{2 + R\sigma_y^2 m_2 + 3R\sigma_y^2 m_1}{2m_1[2 + R\sigma_y^2 (m_1 + m_2)]} i$$

$$I_2^* = \frac{\frac{2(2 + R\sigma_y^2 m_2)}{[(2 + R\sigma_y^2 m_2) + R\sigma_y^2 m_1]}}{2 + R\sigma_y^2 m_2} i = \frac{2}{[2 + R\sigma_y^2 (m_1 + m_2)]} i$$

$$\Delta I_2^* = \frac{2}{[(2+R\sigma_y^2 m_2)+R\sigma_y^2 m_1]}i - \frac{1+R\sigma_y^2(m_2-m_1)/2}{[(2+R\sigma_y^2 m_2)+R\sigma_y^2 m_1]}i = \frac{1-R\sigma_y^2(m_2-m_1)/2}{[2+R\sigma_y^2(m_1+m_2)]}i$$

$$p_2^* = \frac{2+3R\sigma_y^2 m_2 + R\sigma_y^2 m_1}{2m_2[2+R\sigma_y^2(m_1+m_2)]}i$$

Given $c_1 = c_2$, the ex ante profit of a limit order trader who trades in period 1 (period 2) is $p_1 - c_1$ ($p_2 - c_2$). Nash equilibrium requires that no one has the motivation to deviate. Hence if $M_1 < M_2$, I have $p_1 - c_1 > p_2 - c_2$, and the limit order trader in period 2 has the incentive to deviate and choose to trade in period 1. If $M_1 > M_2$, then $p_1 - c_1 < p_2 - c_2$, and the limit order trader in period 1 has the incentive to deviate and choose to trade in period 2. Only when $M_1 = M_2$ and $p_1 = p_2$, the limit order traders in both trading period have no incentive to deviate and, thus, a Nash equilibrium is established. If $p_1 - c_1$ or $p_2 - c_2$ is negative, some limit-order traders will quit the money-losing position until the premium is larger than or equal to the limit-order traders' cost.

Proof of Proposition 3.

If the cost c_1 of the limit order trader who trades in period 1 is higher than the cost c_2 of the limit order trader who trades in period 2, the no-deviation condition $p_1 - c_1 = p_2 - c_2$ implies that $p_1 > p_2$ and correspondingly $m_2 > m_1$, which is equivalent to $M_2 > M_1$ since $\lambda_1 = \lambda_2$.

Proof of Proposition 4.

Let us assume that the total number of limit orders $M_1 + M_2$ is fixed as a constant M . Then the sum of limit order densities in both trading periods is also a constant, denoted as $m \equiv m_1 + m_2 = M / \lambda$. Then we can further rewrite the optimal price and inventory amount as

$$I_1^* = \frac{1 + R\sigma_y^2(m_2 - m_1)/2}{2 + R\sigma_y^2 m} i, \quad p_1^* = \frac{2 + R\sigma_y^2 m + 2R\sigma_y^2 m_1}{2m_1(2 + R\sigma_y^2 m)} i = (\frac{1}{2m_1} + \frac{R\sigma_y^2}{2 + R\sigma_y^2 m})i ,$$

$$I_2^* = \frac{2}{2 + R\sigma_y^2 m} i, \quad \Delta I_2^* = \frac{1 + R\sigma_y^2(m_1 - m_2)/2}{2 + R\sigma_y^2 m} i ,$$

and $p_2^* = \frac{2 + R\sigma_y^2 m + 2R\sigma_y^2 m_2}{2m_2(2 + R\sigma_y^2 m)} i = (\frac{1}{2m_2} + \frac{R\sigma_y^2}{2 + R\sigma_y^2 m})i$.

The market-order trader's total cost C_{Mkt} is $C_{Mkt} = p_1 i_1 + p_2 i_2$. I show that C_{Mkt} is minimized when $M_1 = M_2$ given $M_1 + M_2$ is the constant M . Remember now $M_1 = M_2$ is equal to $m_1 = m_2$.

Suppose the limit-order density in period 1 is changed from m_1 to $m_1' = m_1 + dm$ and the limit-order density in period 2 is changed from m_2 to $m_2' = m_2 - dm$, since $m_1 + m_2 = m = M / \lambda$ is also a constant. Then the market maker's optimal midquote and inventory level will change accordingly. It is straightforward to show the follows.

$$dI_1 = I_1' - I_1^* = [\frac{1}{2 + R\sigma_y^2 m} + \frac{R\sigma_y^2(m_2' - m_1')}{2(2 + R\sigma_y^2 m)}]i - [\frac{1}{2 + R\sigma_y^2 m} + \frac{R\sigma_y^2(m_2 - m_1)}{2(2 + R\sigma_y^2 m)}]i$$

$$= [\frac{1}{2 + R\sigma_y^2 m} + \frac{R\sigma_y^2(m_2 - m_1)}{2(2 + R\sigma_y^2 m)} - \frac{R\sigma_y^2}{2 + R\sigma_y^2 m} dm]i - [\frac{1}{2 + R\sigma_y^2 m} + \frac{R\sigma_y^2(m_2 - m_1)}{2(2 + R\sigma_y^2 m)}]i$$

$$= -\frac{R\sigma_y^2 i}{2 + R\sigma_y^2 m} dm$$

$$dp_1 = p_1' - p_1^* = [\frac{1}{2m_1'} + \frac{R\sigma_y^2}{2 + R\sigma_y^2 m}]i - [\frac{1}{2m_1} + \frac{R\sigma_y^2}{2 + R\sigma_y^2 m}]i$$

$$= (\frac{1}{2(m_1 + dm)} + \frac{R\sigma_y^2}{2 + R\sigma_y^2 m})i - (\frac{1}{2m_1} + \frac{R\sigma_y^2}{2 + R\sigma_y^2 m})i$$

$$= -\frac{i}{2m_1^2} dm$$

$$d(\Delta I_2) = \Delta I_2' - \Delta I_2^* = [\frac{1}{2 + R\sigma_y^2 m} + \frac{R\sigma_y^2 (m_1' - m_2')}{2(2 + R\sigma_y^2 m)}]i - [\frac{1}{2 + R\sigma_y^2 m} + \frac{R\sigma_y^2 (m_1 - m_2)}{2(2 + R\sigma_y^2 m)}]i$$

$$= [\frac{1}{2 + R\sigma_y^2 m} + \frac{R\sigma_y^2 (m_1 - m_2)}{2(2 + R\sigma_y^2 m)} + \frac{R\sigma_y^2}{2 + R\sigma_y^2 m} dm]i - [\frac{1}{2 + R\sigma_y^2 m} + \frac{R\sigma_y^2 (m_1 - m_2)}{2(2 + R\sigma_y^2 m)}]i$$

$$= \frac{R\sigma_y^2 i}{2 + R\sigma_y^2 m} dm$$

$$dp_2 = p_2' - p_2^* = [\frac{1}{2m_2'} + \frac{R\sigma_y^2}{2 + R\sigma_y^2 m}]i - [\frac{1}{2m_2} + \frac{R\sigma_y^2}{2 + R\sigma_y^2 m}]i$$

$$= (\frac{1}{2(m_2 - dm)} + \frac{R\sigma_y^2}{2 + R\sigma_y^2 m})i - (\frac{1}{2m_2} + \frac{R\sigma_y^2}{2 + R\sigma_y^2 m})i$$

$$= \frac{i}{2m_2^2} dm$$

In terms of the market-order trader's total cost $C_{Mkt} = p_1 i_1 + p_2 i_2$, its derivative is $dC_{Mkt} = (dp_1 + dp_2)i = i(\frac{i}{2m_2^2} - \frac{i}{2m_1^2})dm$.

Obviously, when $m_1 < m_2$, I have $dC_{Mkt} / dm < 0$. When $m_1 > m_2$, I have $dC_{Mkt} / dm > 0$. $dC_{Mkt} / dm = 0$ if and only if $m_1 = m_2$, i.e. $M_1 = M_2$.

(If the optimal inventory I_1 is the boundary solution $I_1 = 0$, then I have $I_2 = i/(2 + m_2 R\sigma_y^2)$, $p_2 = (1 + m_2 R\sigma_y^2)i/[(2 + m_2 R\sigma_y^2)m_2]$, and $p_1 = i/m_1$. Thus, I have

$$dp_1/dm = -\frac{i}{m_1^2},\quad dp_2/dm = \frac{(m_2 R\sigma_y^2)^2 + 2m_2 R\sigma_y^2 + 2}{[(R\sigma_y^2 m_2^2 + 2m_2) - (2m_2 R\sigma_y^2 + 2)dm](R\sigma_y^2 m_2^2 + 2m_2)} i,$$

Since $I_1 = 0$ only happens when $m_1 \geq m_2 + 2/R\sigma_y^2 > m_2$, I have

$$dp_1/dm \geq -\frac{i}{(m_2 + 2/R\sigma_y^2)^2} = -\frac{(R\sigma_y^2)^2}{(R\sigma_y^2 m_2 + 2)^2} i \text{ and}$$

$$dp_2/dm \geq [\frac{(R\sigma_y^2)^2}{(R\sigma_y^2 m_2 + 2)^2} + \frac{2m_2 R\sigma_y^2 + 2}{(R\sigma_y^2 m_2^2 + 2m_2)^2}]i$$

It implies that $dC_{Mkt}/dm = (dp_1/dm + dp_2/dm)i \geq \frac{2m_2 R\sigma_y^2 + 2}{(R\sigma_y^2 m_2^2 + 2m_2)^2} i > 0$, and then the results above keep intact.)

Hence if $c_1 = c_2$, in the Nash equilibrium I have $m_1 = m_2$ and the aggregate trading cost of the market-order traders is minimized. If $c_1 > c_2$, I have $m_1 > m_2$ and C_{Mkt} is not minimized. What's more, given that in the Nash equilibrium $\Delta c = c_1 - c_2 = p_1 - p_2 = \frac{i}{2}(\frac{1}{m_1} - \frac{1}{m_2})$, it is straightforward that a larger Δc leads to a larger $m_2 - m_1$ and then a higher C_{Mkt} .

Appendix 2

Figure 1a: The post returns of the portfolios based on *prior 60-minute* return

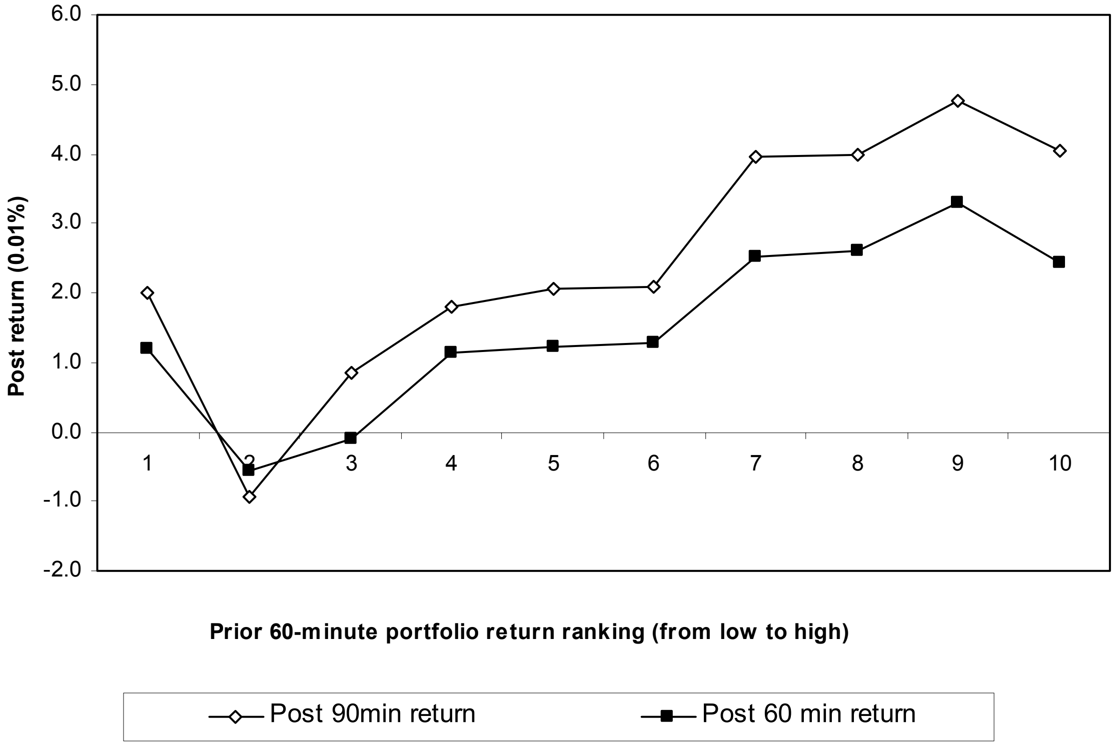


Figure 1b: The post returns of the portfolios based on *prior 90-minute* return

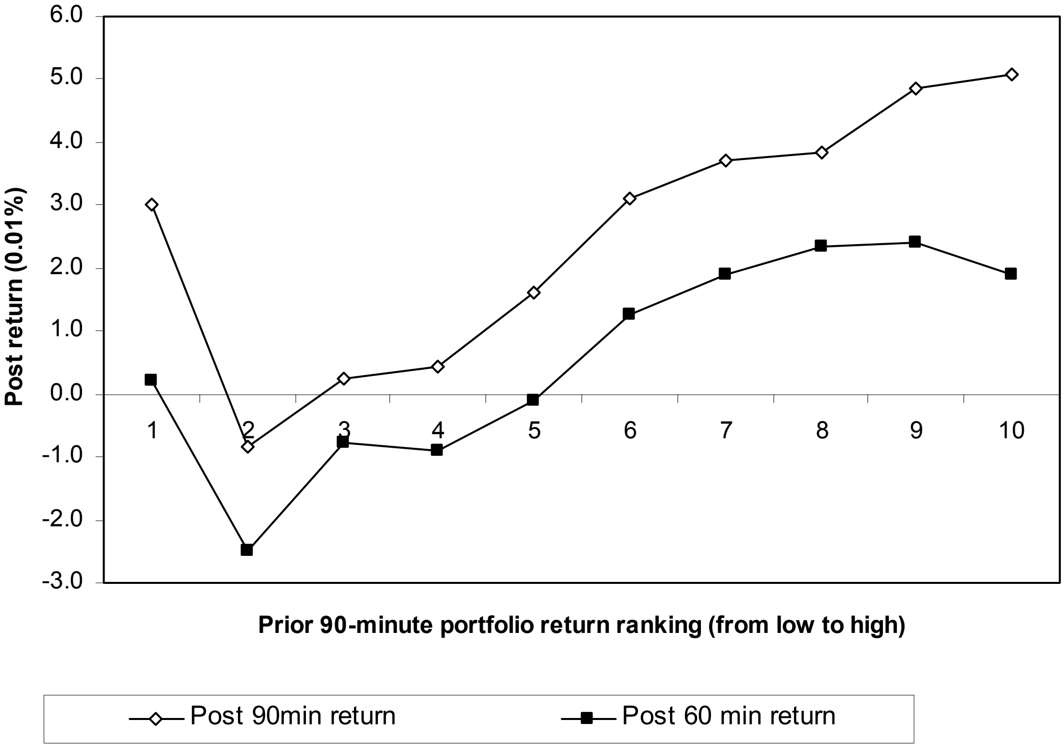


Table 1: Sample Descriptive Statistics

* Descriptive statistics are the average price level, market capitalization, and daily trading volume of the sample firms during the sample period.

	Mean	Median	Min	Max
Price ($)	36.74	34.05	5.88	121.74
Market Capitalization ($MM)	18,236	7,672	705	309,499
Daily Volume (1,000 shares)	2,665	1,422	191	28,139

Table 2a: The Cumulative Return Differences from Post 30-Minute to Post 180-Minute

* The sample firms are sorted in ascending order on the basis of their returns in the past 30, 60, and 90 minutes. Ten decile portfolios are formed based on these rankings. The extreme portfolios (portfolio #1 and portfolio #10) are excluded. The return difference is the return of past winners (portfolio #9) minus the return of past losers (portfolio #2). The portfolio return is the equally-weighted average of the returns of the stocks contained. Cumulative return difference is the return difference from the portfolio formation to the end of each post-return observation period. The sample period is from Jan 07th to May 17th, 2002. The corresponding t-statistics are included in the parenthesis below the value of return difference

	Return Difference = Return of portfolio #9 (past winners) - Return of portfolio #2 (past losers) (unit:0.01%)					
Portfolio Formation Time	Post 30 minutes	Post 60 minutes	Post 90 minutes	Post 120 minutes	Post 150 minutes	Post 180 minutes
30 minutes	1.77 (3.46)	3.09 (4.56)	3.90 (4.63)	4.70 (5.09)	3.85 (3.72)	3.06 (2.87)
60 minutes	2.04 (3.07)	3.85 (3.95)	5.70 (4.95)	6.47 (4.88)	6.01 (4.01)	4.80 (3.07)
90 minutes	1.21 (1.60)	4.88 (4.65)	5.69 (4.16)	4.49 (2.90)	4.12 (2.39)	0.80 (0.41)

Table 2b: The Incremental Return Differences from Post 30-Minute to Post 180-Minute

* The sample firms are sorted in ascending order on the basis of their returns in the past 30, 60, and 90 minutes. Ten decile portfolios are formed based on these rankings. The extreme portfolios (portfolio #1 and portfolio #10) are excluded. The return difference is the return of past winners (portfolio #9) minus the return of past losers (portfolio #2). The portfolio return is the equally-weighted average of the returns of the stocks contained. Incremental return difference is the return difference during each 30-minute interval after the portfolio formation. The sample period is from Jan 07th to May 17th, 2002. The corresponding t-statistics are included in the parenthesis below the value of return difference

	Return Difference = Return of portfolio #9 (past winners) - Return of portfolio #2 (past losers) (unit:0.01%)					
Portfolio Formation Time	Post 0~30 minutes	Post 30~60 minutes	Post 60~90 minutes	Post 90~120 minutes	Post 120~150 minutes	Post 150~180 minutes
30 minutes	1.77 (3.46)	1.31 (2.95)	0.81 (1.81)	0.80 (1.92)	-0.85 (-2.05)	-0.79 (-1.90)
60 minutes	2.04 (3.07)	1.81 (2.84)	1.85 (2.96)	0.77 (1.19)	-0.45 (-0.71)	-1.21 (-1.86)
90 minutes	1.21 (1.17)	3.67 (4.44)	0.81 (1.06)	-1.21 (-1.54)	-0.37 (-0.45)	-3.32 (-4.21)

Table 3a: The Cumulative \$OIB Difference from Post 30-Minute to Post 180-Minute

* \$OIB difference is the difference between the equally-weighted dollar-amount order imbalance of the past winners (portfolio #9) and that of the past losers (portfolio #2). Trades are signed using the Lee and Ready (1991) algorithm. Cumulative \$OIB difference is the \$OIB difference from the portfolio formation to the end of each post-return observation period. The sample period is from Jan 07th to May 17th, 2002. The corresponding t-statistics are included in the parenthesis below the value of \$OIB difference.

	\$OIB Difference = Average \$OIB of portfolio #9 (past winners) - Average \$OIB of portfolio #2 (past losers) (unit:\$1,000)					
Portfolio Formation Time	Post 30 minutes	Post 60 minutes	Post 90 minutes	Post 120 minutes	Post 150 minutes	Post 180 minutes
30 minutes	306.57 (8.69)	456.60 (8.98)	561.07 (8.68)	666.06 (8.28)	860.43 (8.70)	945.87 (8.20)
60 minutes	401.32 (9.09)	686.37 (10.32)	964.11 (10.49)	1216.87 (10.98)	1492.19 (10.91)	1693.18 (10.18)
90 minutes	359.51 (6.91)	784.92 (9.82)	988.57 (9.41)	1134.59 (9.10)	1459.92 (9.34)	1736.77 (9.07)

Table 3b: The Incremental $OIB Difference from Post 30-Minute to Post 180-Minute

* $OIB difference is the difference between the equally-weighted dollar-amount order imbalance of the past winners (portfolio #9) and that of the past losers (portfolio #2). Trades are signed using the Lee and Ready (1991) algorithm. Incremental $OIB difference is the $OIB difference during each 30-minute interval after the portfolio formation. The sample period is from Jan 07th to May 17th, 2002. The corresponding t-statistics are included in the parenthesis below the value of $OIB difference.

	$OIB Difference = Average $OIB of portfolio #9 (past winners) - Average $OIB of portfolio #2 (past losers) (unit:$1,000)					
Portfolio Formation Time	Post 0~30 minutes	Post 30~60 minutes	Post 60~90 minutes	Post 90~120 minutes	Post 120~150 minutes	Post 150~180 minutes
30 minutes	306.57 (5.59)	150.04 (5.21)	104.46 (3.28)	104.99 (2.87)	194.37 (4.71)	85.43 (2.32)
60 minutes	401.32 (9.09)	285.05 (6.95)	277.74 (5.63)	252.76 (5.56)	275.32 (4.77)	200.99 (3.18)
90 minutes	359.51 (6.91)	425.42 (7.91)	203.65 (3.68)	146.02 (2.81)	325.33 (4.92)	276.85 (3.27)

Table 4a: QSPR% (Quoted Percentage Spread) Difference:
Portfolio (#9+#2) - Portfolio (#6+#5)

* QSPR% (Quoted Percentage Spread) is the quoted bid-ask spread divided by the mid-point of the quote in percent. QSPR% difference is the sum of equally-weighted QSPR% of the past winners (portfolio #9) and that of the past losers (portfolio #2) minus the sum of equally-weighted QSPR% of the portfolios with middle ranking (portfolios #5 and #6). QSPR% difference is measured during each 30-minute interval after the portfolio formation. The sample period is from Jan 07th to May 17th, 2002. The corresponding t-statistics are included in the parenthesis below the value of QSPR% difference.

	QSPR% Difference: (Portfolio (#9+#2) - Portfolio (#6+#5))					
Portfolio Formation Time	Post 0~30 minutes	Post 30~60 minutes	Post 60~90 minutes	Post 90~120 minutes	Post 120~150 minutes	Post 150~180 minutes
30 minutes	0.0294% (32.18)	0.0243% (25.75)	0.0219% (21.99)	0.0209% (20.06)	0.0197% (18.27)	0.0197% (17.94)
60 minutes	0.0291% (23.80)	0.0247% (17.79)	0.0215% (14.25)	0.0207% (13.07)	0.0202% (13.68)	0.0206% (12.72)
90 minutes	0.0269% (15.22)	0.0244% (14.48)	0.0207% (11.28)	0.0194% (10.29)	0.0193% (9.68)	0.0180% (10.49)

Table 4b: Change of QSPR% (Quoted Percentage Spread) Difference:
Portfolio (#9+#2) - Portfolio (#6+#5)

* QSPR% (Quoted Percentage Spread) is the quoted bid-ask spread divided by the mid-point of the quote in percent. QSPR% difference is the sum of equally-weighted QSPR% of the past winners (portfolio #9) and that of the past losers (portfolio #2) minus the sum of equally-weighted QSPR% of the portfolios with middle ranking (portfolios #5 and #6). QSPR% difference is measured during each 30-minute interval after the portfolio formation. The change of QSPR% difference is the QSPR% difference during the current 30-minute interval minus that of the pervious 30-minute interval. The sample period is from Jan 07th to May 17th, 2002. The corresponding t-statistics are included in the parenthesis below the value of QSPR% difference.

	QSPR% Difference: (Portfolio (#9+#2) - Portfolio (#6+#5))					
Portfolio Formation Time		Post 30~60 minutes - Post 0~30 minutes	Post 60~90 minutes - Post 30~60 minutes	Post 90~120 minutes - Post 60~90 minutes	Post 120~150 minutes - Post 90~120 minutes	Post 150~180 minutes - Post 120~150 minutes
30 minutes		-0.0052% (-5.19)	-0.0023% (-1.93)	-0.0011% (-0.98)	-0.0012% (-1.77)	0.0000% (-0.04)
60 minutes		-0.0044% (-4.21)	-0.0033% (-2.69)	-0.0008% (-0.60)	-0.0005% (-0.35)	0.0004% (0.27)
90 minutes		-0.0025% (-1.92)	-0.0037% (-2.48)	-0.0013% (-0.76)	-0.0001% (-0.07)	-0.0013% (-0.86)

Table 5a: ESPR% (Effective Percentage Spread) Difference:
Portfolio (#9+#2) - Portfolio (#6+#5)

* ESPR% (Effective Percentage Spread) is the effective bid-ask spread, which is the difference between the execution price and the mid-point of the prevailing bid-ask quote, divided by the mid-point of the quote in percent. ESPR% difference is the sum of equally-weighted ESPR% of the past winners (portfolio #9) and that of the past losers (portfolio #2) minus the sum of equally-weighted ESPR% of the portfolios with middle ranking (portfolios #5 and #6). ESPR% difference is measured during each 30-minute interval after the portfolio formation. The sample period is from Jan 07th to May 17th, 2002. The corresponding t-statistics are included in the parenthesis below the value of ESPR% difference.

	ESPR% Difference: (Portfolio (#9+#2) - Portfolio (#6+#5))					
Portfolio Formation Time	Post 0~30 minutes	Post 30~60 minutes	Post 60~90 minutes	Post 90~120 minutes	Post 120~150 minutes	Post 150~180 minutes
30 minutes	0.0207% (31.87)	0.0172% (25.07)	0.0157% (22.15)	0.0145% (19.66)	0.0129% (16.90)	0.0123% (15.95)
60 minutes	0.0208% (23.42)	0.0175% (17.18)	0.0151% (14.00)	0.0145% (12.89)	0.0136% (12.86)	0.0134% (12.08)
90 minutes	0.0192% (15.47)	0.0173% (14.39)	0.0142% (11.42)	0.0135% (10.01)	0.0133% (9.51)	0.0125% (10.41)

Table 5b: Change of ESPR% (Effective Percentage Spread) Difference: Portfolio (#9+#2) - Portfolio (#6+#5)

* ESPR% (Effective Percentage Spread) is the effective bid-ask spread, which is the difference between the execution price and the mid-point of the prevailing bid-ask quote, divided by the mid-point of the quote in percent. ESPR% difference is the sum of equally-weighted ESPR% of the past winners (portfolio #9) and that of the past losers (portfolio #2) minus the sum of equally-weighted ESPR% of the portfolios with middle ranking (portfolios #5 and #6). ESPR% difference is measured during each 30-minute interval after the portfolio formation. The change of ESPR% difference is the ESPR% difference during the current 30-minute interval minus that of the pervious 30-minute interval. The sample period is from Jan 07th to May 17th, 2002. The corresponding t-statistics are included in the parenthesis below the value of ESPR% difference.

	ESPR% Difference: (Portfolio (#9+#2) - Portfolio (#6+#5))					
Portfolio Formation Time		Post 30~60 minutes - Post 0~30 minutes	Post 60~90 minutes - Post 30~60 minutes	Post 90~120 minutes - Post 60~90 minutes	Post 120~150 minutes - Post 90~120 minutes	Post 150~180 minutes - Post 120~150 minutes
30 minutes		-0.0036% (-6.46)	-0.0015% (-2.49)	-0.0012% (-1.80)	-0.0015% (-1.11)	-0.0006% (-0.78)
60 minutes		-0.0034% (-4.13)	-0.0024% (-2.59)	-0.0006% (-0.64)	-0.0009% (-0.86)	-0.0002% (-0.17)
90 minutes		-0.0019% (-1.96)	-0.0031% (-2.98)	-0.0007% (-0.58)	-0.0001% (-0.09)	-0.0008% (-0.75)

Table 6a: The Cumulative Return Differences from Post 30-Minute to Post 180-Minute

(Subperiod Results)

* The sample firms are sorted in ascending order on the basis of their returns in the past 30, 60, and 90 minutes. Ten decile portfolios are formed based on these rankings. The extreme portfolios (portfolio #1 and portfolio #10) are excluded. The return difference is the return of past winners (portfolio #9) minus the return of past losers (portfolio #2). The portfolio return is the equally-weighted average of the returns of the stocks contained. Cumulative return difference is the return difference from the portfolio formation to the end of each post-return observation period. The corresponding t-statistics are included in the parenthesis below the value of return difference

Bull Trend: Feb 04 to March 19, 2002,						
	Return Difference = Return of portfolio #9 (past winners) - Return of portfolio #2 (past losers) (unit:0.01%)					
Portfolio Formation Time	Post 30 minutes	Post 60 minutes	Post 90 minutes	Post 120 minutes	Post 150 minutes	Post 180 minutes
30 minutes	1.43 (1.47)	1.54 (1.21)	2.41 (1.55)	2.91 (1.76)	1.96 (1.04)	1.27 (0.63)
60 minutes	2.05 (1.78)	2.82 (1.76)	5.37 (2.68)	5.56 (2.39)	5.45 (2.09)	3.64 (1.39)
90 minutes	1.52 (1.09)	4.44 (2.34)	6.27 (2.51)	3.97 (1.33)	3.42 (1.14)	0.13 (0.04)

Bear Trend: March 19 to May 10, 2002,						
	Return Difference = Return of portfolio #9 (past winners) - Return of portfolio #2 (past losers) (unit:0.01%)					
Portfolio Formation Time	Post 30 minutes	Post 60 minutes	Post 90 minutes	Post 120 minutes	Post 150 minutes	Post 180 minutes
30 minutes	2.12 (2.74)	4.27 (4.06)	5.23 (4.04)	6.18 (4.42)	5.36 (3.54)	4.31 (2.82)
60 minutes	3.73 (3.40)	5.58 (3.61)	8.15 (4.55)	8.83 (4.49)	8.23 (3.80)	7.00 (2.92)
90 minutes	1.37 (1.31)	6.40 (3.95)	6.23 (2.95)	5.27 (2.29)	5.09 (1.86)	1.25 (0.42)

Table 6b: The Incremental Return Differences from Post 30-Minute to Post 180-Minute

(Subperiod Results)

* The sample firms are sorted in ascending order on the basis of their returns in the past 30, 60, and 90 minutes. Ten decile portfolios are formed based on these rankings. The extreme portfolios (portfolio #1 and portfolio #10) are excluded. The return difference is the return of past winners (portfolio #9) minus the return of past losers (portfolio #2). The portfolio return is the equally-weighted average of the returns of the stocks contained. Incremental return difference is the return difference during each 30-minute interval after the portfolio formation. The sample period is from Jan 07th to May 17th, 2002. The corresponding t-statistics are included in the parenthesis below the value of return difference

Bull Trend: Feb 04 to March 19, 2002,						
	Return Difference = Return of portfolio #9 (past winners) - Return of portfolio #2 (past losers) (unit:0.01%)					
Portfolio Formation Time	Post 0~30 minutes	Post 30~60 minutes	Post 60~90 minutes	Post 90~120 minutes	Post 120~150 minutes	Post 150~180 minutes
30 minutes	1.43 (1.47)	0.11 (0.14)	0.86 (1.05)	0.50 (0.71)	-0.96 (-1.16)	-0.68 (-0.87)
60 minutes	2.05 (1.78)	0.77 (0.77)	2.55 (2.49)	0.19 (0.15)	-0.12 (-0.09)	-1.80 (-1.43)
90 minutes	1.52 (1.09)	2.92 (1.91)	1.83 (1.21)	-2.30 (-1.72)	-0.55 (-0.37)	-3.29 (-2.35)
Bear Trend: March 19 to May 10, 2002,						
	Return Difference = Return of portfolio #9 (past winners) - Return of portfolio #2 (past losers) (unit:0.01%)					
Portfolio Formation Time	Post 0~30 minutes	Post 30~60 minutes	Post 60~90 minutes	Post 90~120 minutes	Post 120~150 minutes	Post 150~180 minutes
30 minutes	2.12 (2.74)	2.15 (3.17)	0.97 (1.50)	0.95 (1.58)	-0.82 (-1.43)	-1.05 (-1.77)
60 minutes	3.73 (3.40)	1.86 (1.74)	2.57 (2.91)	0.68 (0.79)	-0.60 (-0.71)	-1.23 (-1.27)
90 minutes	1.37 (1.31)	5.03 (3.96)	-0.17 (-0.17)	-0.96 (-0.95)	-0.18 (-0.14)	-3.84 (-3.53)

Table 7a: The Cumulative $OIB Difference from Post 30-Minute to Post 180-Minute (Subperiod Results)

* $OIB difference is the difference between the equally-weighted dollar-amount order imbalance of the past winners (portfolio #9) and that of the past losers (portfolio #2). Trades are signed using the Lee and Ready (1991) algorithm. Cumulative $OIB difference is the $OIB difference from the portfolio formation to the end of each post-return observation period. The corresponding t-statistics are included in the parenthesis below the value of $OIB difference.

Bull Trend: Feb 04 to March 19, 2002,						
	$OIB Difference = Average $OIB of portfolio #9 (past winners) - Average $OIB of portfolio #2 (past losers) (unit:$1,000)					
Portfolio Formation Time	Post 30 minutes	Post 60 minutes	Post 90 minutes	Post 120 minutes	Post 150 minutes	Post 180 minutes
30 minutes	365.05	500.59	665.99	762.23	975.38	1131.23
	(6.01)	(5.69)	(5.78)	(5.50)	(5.70)	(5.44)
60 minutes	458.18	801.22	1059.62	1430.50	1838.57	2072.70
	(5.37)	(6.78)	(6.44)	(8.96)	(7.04)	(6.99)
90 minutes	501.67	861.41	1162.96	1307.24	1554.69	1827.97
	(5.74)	(6.24)	(6.00)	(5.44)	(5.10)	(4.94)
Bear Trend: March 19 to May 10, 2002,						
	$OIB Difference = Average $OIB of portfolio #9 (past winners) - Average $OIB of portfolio #2 (past losers) (unit:$1,000)					
Portfolio Formation Time	Post 30 minutes	Post 60 minutes	Post 90 minutes	Post 120 minutes	Post 150 minutes	Post 180 minutes
30 minutes	261.15	400.66	509.75	594.27	796.67	832.17
	(4.94)	(5.34)	(5.45)	(4.85)	(5.14)	(4.74)
60 minutes	346.49	603.83	919.18	1095.94	1274.91	1482.82
	(5.54)	(5.96)	(6.66)	(6.65)	(6.41)	(6.62)
90 minutes	293.43	771.69	890.63	1002.86	1294.36	1448.61
	(4.18)	(6.36)	(5.62)	(5.78)	(6.36)	(5.94)

Table 7b: The Incremental $OIB Difference from Post 30-Minute to Post 180-Minute (Subperiod Results)

* $OIB difference is the difference between the equally-weighted dollar-amount order imbalance of the past winners (portfolio #9) and that of the past losers (portfolio #2). Trades are signed using the Lee and Ready (1991) algorithm. Incremental $OIB difference is the $OIB difference during each 30-minute interval after the portfolio formation. The corresponding t-statistics are included in the parenthesis below the value of $OIB difference.

Bull Trend: Feb 04 to March 19, 2002,						
	$OIB Difference = Average $OIB of portfolio #9 (past winners) - Average $OIB of portfolio #2 (past losers) (unit:$1,000)					
Portfolio Formation Time	Post 0~30 minutes	Post 30~60 minutes	Post 60~90 minutes	Post 90~120 minutes	Post 120~150 minutes	Post 150~180 minutes
30 minutes	365.05	135.55	165.40	96.24	213.15	155.85
	(6.01)	(2.54)	(3.13)	(1.78)	(3.55)	(2.32)
60 minutes	458.18	343.03	258.41	370.88	408.07	234.13
	(5.37)	(4.87)	(3.36)	(5.13)	(3.89)	(1.72)
90 minutes	501.67	359.73	301.55	144.28	247.46	273.28
	(5.74)	(3.85)	(3.23)	(1.37)	(2.23)	(1.71)

Bear Trend: March 19 to May 10, 2002,						
	$OIB Difference = Average $OIB of portfolio #9 (past winners) - Average $OIB of portfolio #2 (past losers) (unit:$1,000)					
Portfolio Formation Time	Post 0~30 minutes	Post 30~60 minutes	Post 60~90 minutes	Post 90~120 minutes	Post 120~150 minutes	Post 150~180 minutes
30 minutes	261.15	139.51	109.10	84.52	202.40	35.49
	(6.01)	(3.35)	(2.29)	(1.26)	(2.55)	(0.62)
60 minutes	346.49	257.34	315.35	176.76	178.98	207.90
	(5.54)	(4.03)	(3.76)	(2.82)	(1.90)	(2.46)
90 minutes	293.43	478.25	118.95	112.23	291.50	154.25
	(5.74)	(5.40)	(1. 38)	(1.60)	(2.64)	(1.31)

Table 8a: QSPR% (Quoted Percentage Spread) Difference:
Portfolio (#9+#2) - Portfolio (#6+#5)

(Subperiod Results)

* QSPR% (Quoted Percentage Spread) is the quoted bid-ask spread divided by the mid-point of the quote in percent. QSPR% difference is the sum of equally-weighted QSPR% of the past winners (portfolio #9) and that of the past losers (portfolio #2) minus the sum of equally-weighted QSPR% of the portfolios with middle ranking (portfolios #5 and #6). QSPR% difference is measured during each 30-minute interval after the portfolio formation. The corresponding t-statistics are included in the parenthesis below the value of QSPR% difference.

Bull Trend: Feb 04 to March 19, 2002,						
	QSPR% Difference: (Portfolio (#9+#2) - Portfolio (#6+#5))					
Portfolio Formation Time	Post 0~30 minutes	Post 30~60 minutes	Post 60~90 minutes	Post 90~120 minutes	Post 120~150 minutes	Post 150~180 minutes
30 minutes	0.0313% (18.10)	0.0261% (14.98)	0.0246% (13.01)	0.0232% (11.08)	0.0203% (9.15)	0.0207% (9.90)
60 minutes	0.0319% (15.24)	0.0251% (9.11)	0.0246% (8.07)	0.0223% (8.49)	0.0212% (7.16)	0.0210% (6.76)
90 minutes	0.0275% (8.64)	0.0253% (8.26)	0.0230% (7.72)	0.0217% (6.45)	0.0204% (5.83)	0.0198% (6.55)

Bear Trend: March 19 to May 10, 2002,						
	QSPR% Difference: (Portfolio (#9+#2) - Portfolio (#6+#5))					
Portfolio Formation Time	Post 0~30 minutes	Post 30~60 minutes	Post 60~90 minutes	Post 90~120 minutes	Post 120~150 minutes	Post 150~180 minutes
30 minutes	0.0266% (21.76)	0.0211% (16.53)	0.0182% (13.64)	0.0172% (12.91)	0.0169% (12.21)	0.0166% (11.06)
60 minutes	0.0236% (13.18)	0.0201% (11.83)	0.0165% (8.22)	0.0166% (7.95)	0.0166% (8.41)	0.0160% (7.09)
90 minutes	0.0198% (8.86)	0.0190% (8.06)	0.0151% (5.52)	0.0143% (5.48)	0.0143% (5.54)	0.0137% (5.36)

Table 8b: Change of QSPR% (Quoted Percentage Spread) Difference:
Portfolio (#9+#2) - Portfolio (#6+#5) (Subperiod Results)

QSPR% (Quoted Percentage Spread) is the quoted bid-ask spread divided by the mid-point of the quote in percent. QSPR% difference is the sum of equally-weighted QSPR% of the past winners (portfolio #9) and that of the past losers (portfolio #2) minus the sum of equally-weighted QSPR% of the portfolios with middle ranking (portfolios #5 and #6). QSPR% difference is measured during each 30-minute interval after the portfolio formation. The change of QSPR% difference is the QSPR% difference during the current 30-minute interval minus that of the pervious 30-minute interval. The corresponding t-statistics are included in the parenthesis below the value of QSPR% difference.

Bull Trend: Feb 04 to March 19, 2002,						
	QSPR% Difference: (Portfolio (#9+#2) - Portfolio (#6+#5))					
Portfolio Formation Time		Post 30~60 minutes - Post 0~30 minutes	Post 60~90 minutes - Post 30~60 minutes	Post 90~120 minutes - Post 60~90 minutes	Post 120~150 minutes - Post 90~120 minutes	Post 150~180 minutes - Post 120~150 minutes
30 minutes		-0.0052% (-3.89)	-0.0015% (-1.00)	-0.0014% (-0.81)	-0.0029% (-1.77)	0.0004% (0.20)
60 minutes		-0.0067% (-3.37)	-0.0005% (-0.24)	-0.0022% (-1.07)	-0.0011% (-0.43)	-0.0002% (-0.07)
90 minutes		-0.0022% (-0.95)	-0.0024% (-0.82)	-0.0012% (-0.40)	-0.0013% (-0.39)	-0.0006% (-0.25)

Bear Trend: March 19 to May 10, 2002,						
	QSPR% Difference: (Portfolio (#9+#2) - Portfolio (#6+#5))					
Portfolio Formation Time		Post 30~60 minutes - Post 0~30 minutes	Post 60~90 minutes - Post 30~60 minutes	Post 90~120 minutes - Post 60~90 minutes	Post 120~150 minutes - Post 90~120 minutes	Post 150~180 minutes - Post 120~150 minutes
30 minutes		-0.0055% (-5.70)	-0.0029% (-2.91)	-0.0011% (-0.93)	-0.0002% (-0.21)	-0.0003% (-0.25)
60 minutes		-0.0035% (-2.56)	-0.0036% (-2.25)	0.0002% (0.10)	-0.0001% (-0.03)	-0.0006% (-0.36)
90 minutes		-0.0008% (-0.44)	-0.0040% (-2.38)	-0.0007% (-0.38)	-0.0001% (-0.03)	-0.0005% (-0.27)

Table 9a: ESPR% (Effective Percentage Spread) Difference:
Portfolio (#9+#2) - Portfolio (#6+#5) (Subperiod Results)

* ESPR% (Effective Percentage Spread) is the effective bid-ask spread, which is the difference between the execution price and the mid-point of the prevailing bid-ask quote, divided by the mid-point of the quote in percent. ESPR% difference is the sum of equally-weighted ESPR% of the past winners (portfolio #9) and that of the past losers (portfolio #2) minus the sum of equally-weighted ESPR% of the portfolios with middle ranking (portfolios #5 and #6). ESPR% difference is measured during each 30-minute interval after the portfolio formation. The corresponding t-statistics are included in the parenthesis below the value of ESPR% difference.

Bull Trend: Feb 04 to March 19, 2002,						
	ESPR% Difference: (Portfolio (#9+#2) - Portfolio (#6+#5))					
Portfolio Formation Time	Post 0~30 minutes	Post 30~60 minutes	Post 60~90 minutes	Post 90~120 minutes	Post 120~150 minutes	Post 150~180 minutes
30 minutes	0.0215% (17.47)	0.0184% (14.52)	0.0172% (12.99)	0.0160% (11.10)	0.0132% (8.55)	0.0129% (9.17)
60 minutes	0.0230% (14.93)	0.0178% (8.93)	0.0168% (7.80)	0.0157% (8.58)	0.0142% (6.89)	0.0134% (6.51)
90 minutes	0.0198% (9.39)	0.0176% (7.92)	0.0157% (7.44)	0.0148% (6.02)	0.0138% (5.89)	0.0134% (6.56)

Bear Trend: March 19 to May 10, 2002,						
	ESPR% Difference: (Portfolio (#9+#2) - Portfolio (#6+#5))					
Portfolio Formation Time	Post 0~30 minutes	Post 30~60 minutes	Post 60~90 minutes	Post 90~120 minutes	Post 120~150 minutes	Post 150~180 minutes
30 minutes	0.0190% (21.83)	0.0151% (15.83)	0.0131% (13.44)	0.0121% (12.73)	0.0112% (10.90)	0.0103% (9.42)
60 minutes	0.0168% (10.16)	0.0138% (10.84)	0.0119% (8.02)	0.0117% (7.63)	0.0110% (7.56)	0.0105% (6.46)
90 minutes	0.0141% (8.67)	0.0139% (8.32)	0.0102% (8.02)	0.0099% (5.68)	0.0095% (5.38)	0.0098% (5.43)

Table 9b: Change of ESPR% (Effective Percentage Spread) Difference: Portfolio (#9+#2) - Portfolio (#6+#5) (Subperiod Results)

* ESPR% (Effective Percentage Spread) is the effective bid-ask spread, which is the difference between the execution price and the mid-point of the prevailing bid-ask quote, divided by the mid-point of the quote in percent. ESPR% difference is the sum of equally-weighted ESPR% of the past winners (portfolio #9) and that of the past losers (portfolio #2) minus the sum of equally-weighted ESPR% of the portfolios with middle ranking (portfolios #5 and #6). ESPR% difference is measured during each 30-minute interval after the portfolio formation. The change of ESPR% difference is the ESPR% difference during the current 30-minute interval minus that of the pervious 30-minute interval. The corresponding t-statistics are included in the parenthesis below the value of ESPR% difference.

Bull Trend: Feb 04 to March 19, 2002,						
	ESPR% Difference: (Portfolio (#9+#2) - Portfolio (#6+#5))					
Portfolio Formation Time		Post 30~60 minutes - Post 0~30 minutes	Post 60~90 minutes - Post 30~60 minutes	Post 90~120 minutes - Post 60~90 minutes	Post 120~150 minutes - Post 90~120 minutes	Post 150~180 minutes - Post 120~150 minutes
30 minutes		-0.0032% (-3.02)	-0.0011% (-0.96)	-0.0012% (-0.93)	-0.0028% (-1.98)	-0.0003% (-0.22)
60 minutes		-0.0052% (-3.53)	-0.0010% (-0.58)	-0.0010% (-0.68)	-0.0015% (-0.77)	-0.0008% (-0.44)
90 minutes		-0.0022% (-1.30)	-0.0019% (-0.88)	-0.0009% (-0.37)	-0.0010% (-0.41)	-0.0004% (-0.21)

Bear Trend: March 19 to May 10, 2002,						
	ESPR% Difference: (Portfolio (#9+#2) - Portfolio (#6+#5))					
Portfolio Formation Time		Post 30~60 minutes - Post 0~30 minutes	Post 60~90 minutes - Post 30~60 minutes	Post 90~120 minutes - Post 60~90 minutes	Post 120~150 minutes - Post 90~120 minutes	Post 150~180 minutes - Post 120~150 minutes
30 minutes		-0.0040% (-5.23)	-0.0020% (-2.55)	-0.0010% (-1.15)	-0.0009% (-1.03)	-0.0009% (-0.96)
60 minutes		-0.0030% (-2.57)	-0.0020% (-1.47)	-0.0002% (-0.12)	-0.0007% (-0.52)	-0.0005% (-0.39)
90 minutes		-0.0001% (-0.09)	-0.0037% (-3.14)	-0.0003% (-0.20)	-0.0004% (-0.25)	0.0003% (0.21)