**Memorandum**

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**TO:** Joint Committee on Taxation Blue Ribbon Advisory Panel

**SUBJECT:** Consumption, Labor Supply, and Savings in the MEG Model

**FROM:** Staff of the Joint Committee on Taxation

**DATE:** October 3, 2002

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This paper shows the derivation of the general form of the consumption and labor supply functions used in MEG model. The determination of consumption may be varied by making utility either a function of consumption alone, or a function of consumption and leisure. If utility is modeled as a function of consumption only, then labor supply is modeled using an ad hoc labor supply function. If leisure is included in the utility function then the labor supply and consumption functions are derived jointly from the utility maximizing framework. This paper also shows the differences in consumption, saving and labor supply responses between these two approaches to modeling the labor supply decision. Finally, the parameter values used to calibrate the MEG model and the fundamental equations of the model are presented so that the important behavioral responses and other steady state properties of the model can be discussed in full.

## The Consumption Function in MEG

The aggregate consumption function in the MEG Model, which is derived from the consumer's utility maximization, is specificied in a form that is commonly found in the macro-econometric time-series literature:

(1) $$C_t = b_1 NW_{t-1} + b_2 YA_t(1-\bar{m}_t) + b_3 YT_t + b_4 YL_t(1-\bar{\tau}_t)$$

where $C_t$ is real consumption in period $t$, $NW_{t-1}$ is real net worth at the end of the previous period, $YA_t$ is real net asset income, $\bar{m}_t$ is the average tax rate on asset income, $YT_t$ is (non-taxed) real transfer income, $YL_t$ is real labor income, and $\bar{\tau}_t$ is the average tax rate on labor income. The marginal propensities to consume (MPCs) out of income and wealth are $b_1$, $b_2$, $b_3$, and $b_4$.[1]

Empirical research suggests that $b_1$ and $b_2$ are between 0.04 and 0.07, $b_3$ is around unity, and $b_4$ is between 0.5 and 0.7.[2] The values used to calibrate the MEG model are within these ranges. While, often in the empirical literature the marginal propensities to consume out of income and wealth are estimated as fixed parameters, in the MEG model the MPCs are not constant but allowed to change as the parameters that determine the consumer's consumption and labor supply change. As shown in equations (A29), (A31), and(A35) in the appendix, these MPCs depend on the real after-tax rate of return, the rate of time preference, the average and marginal tax rates, both the inter and intra-temporal substitution elasticities, the intensity factors in the utility function, the expected rate of growth in the consumer's real wage, and the age, life

[1] A description of the lifetime utility function and lifetime budget constraint from which the consumption function is derived is in the appendix.

[2] See Ando, Albert, and Franco Modigliani, "The 'Life Cycle' Hypothesis of Saving: Aggregate Implications and Tests," American Economic Review, vol. 53 (March 1963), pp. 55-84, and Brayton, Flint and Peter Tinsley, eds. "A Guide to the FRB/US: A Macroeconomic Model of the United States," Finance and Economics Discussion Working Paper42, Board of Governors of the Federal Reserve System, 1996.

expectancy, and retirement age of the representative consumer. Note that the exact form of the MPCs varies depending on which version of the labor supply function is used in the model.

The personal saving rate can be derived from the consumption function and the distribution of personal disposable income between labor income, asset income, and transfer income. As shown in equation (A47) in the appendix, the "long-run" saving rate is described by the equation:

(2) $$s_{y,j} = \frac{g\left[1 - b_4 - w(T)_t\left(b_3 - b_4\right)\right]}{g + b_1 + \left(b_2 - b_4\right)\left(r_t\left(1 - m_t\right) - p_t\right)}$$

where $r_t$ is the pre-tax nominal interest rate, $m_t$ is the marginal tax rate on asset income, $p_t$ is the inflation rate, and the term $w(T)_t$ is the share of transfer income in disposable income. This term can be written as:

$$w(T)_t = \frac{k_t}{(1-a)(1-\bar{t}_t) + a(1-\bar{m}_t)\dfrac{r_t - p_t}{r_t - p_t + d} + k_t}$$

where $a$ is capital's share of nominal output, $d$ is the depreciation rate on capital, $k_t$ is the exogenous ratio of transfers to gross output, and $g$ is the steady state growth rate. Thus, the "long-run" saving rate is a function of the MPCs on wealth and income as well as real after-tax rate of return and the share of transfer income in disposable income.

**The Labor Supply Functions in MEG**

The MEG model provides a choice between two approaches to modeling labor supply. The first approach determines labor supply using an equation of the labor force participation rate. In this version, labor supply depends on the real wage, the average and marginal tax rates, and exogenous parameters that determine the income elasticity of the labor force participation rate with respect to the average after-tax real wage and the substitution elasticity of the labor force participation rate with respect to the marginal after-tax real wage. The general form of the labor force participation rate is given by:

(3) $$LFPR = W^{\frac{e^I - e^S}{1-e^I}} \left[1-\bar{t}_t\right]^{\frac{e^I}{1-e^I}} \left[1-t_t\right]^{\frac{e^S}{1-e^I}}$$

where $W$ is the real pre-tax wage received by households, $e^I$ is the income elasticity of labor force participation with respect to average after-tax income received by households, $e^S$ is the substitution elasticity of the labor force participation rate with respect to the marginal after-tax labor income received by households, $\bar{t}_t$ is the average tax rate on income, and $t_t$ is the average marginal tax rate on labor income. In this specification the effect of a change in average and marginal tax rates on labor income will depend directly on the assumed values of the income and substitution elasticities of labor force participation with respect to the after-tax wage rate. Labor augmenting technological growth will affect the labor force participation rate if the income and substitution elasticities are not equal. This implies that unless the income and substitution elasticities are equal the labor force participation rate would have a trend.

There have been many empirical analyses of the responses of labor supply to changes in after-tax wages.[3] Studies have found that there is a difference in responsiveness between men and women, and between low-income workers and other workers. For this reason, the labor force participation equation is calculated as the income-weighted average of the labor force participation rate of four groups of taxpayers[4]:

(4) $$LFPR = \sum_{i=1}^{4} n_i \left[ W^{\frac{e^I - e^S}{1-e^I}} \left[1-\bar{t}_t\right]^{\frac{e^I}{1-e^I}} \left[1-t_t\right]^{\frac{e^S}{1-e^I}} \right]$$

where $n_i$ is the weighting parameter for group $i$ and is calculated as wage income in group $i$ divided by the sum of wage income in all the groups. The four groups are: (1) low-income primary earners, (2) other primary earners, (3) low income secondary earners, and (4) other secondary earners.[5] Given this specification, $e_i^I$ is the income elasticity of labor force participation for group $i$, $e_i^S$ is the substitution elasticity of the labor force participation rate for group $i$, $\bar{t}_{ti}$ is the average tax rate on labor income for group $i$, and $t_{ti}$ is the average marginal tax rate on labor income for group $i$.

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[3] For a recent review of this literature, see Jane G. Gravelle, "Labor Supply Response and Dynamic Scoring," Congressional Research Service Memorandum, August 21, 2002. See also Frank S. Russell, "Labor Supply and Taxes," Congressional Budget Office Memorandum, January, 1996.

[4] While the literature tends to focus on the difference in participation rates between men and women, two observations have led us to change this distinction to differences between primary earners and secondary earners. The first is that the primary "story" generally advanced to explain gender differences in participation focuses on the primary v. secondary nature of the wages contributed to the households between the two groups. The second is that secular trends in study results seem to indicate men's labor force participation becoming slightly more elastic, and women's slightly less over a period that coincides with a general increase in women's roles as primary earners.

[5] The disaggregated labor supply equations are assigned portions of the total labor force based on their share of the wages earned by the total labor force. This approach was chosen under the assumption that the groups' share of total wages is a proxy for the groups' contributions to output. In addition, the total wages of each group, multiplied

The second approach uses the labor supply function that corresponds to the case in which consumption and labor supply are jointly determined. In this specification, utility is a function of consumption and leisure, instead of consumption only. Labor supply, $L_{y,j}$, can be written in the form

$$(5) \qquad L_{y,j} = H - E_{y,j} = H - \left(\frac{a_E}{a_C}\right)\left(\frac{C_{y,j}}{W_{y,j}}\right)[1 - t_t]^{-s_2}$$

where $H$ is the fixed endowment of hours to allocate between labor and leisure, $E_{y,a}$ is time devoted to non-taxed leisure, $a_E$ and $a_C$ are intensity parameters with respect to leisure and consumption from a constant elasticity of scale utility function, $C_{y,a}$ is the real value of consumption, $W_{y,a}$ is the pre-tax real wage, and $s_2$ is the intra-temporal elasticity of substitution.

In this specification, labor supply is constant in the steady state since pre-tax income does not appear in the long run labor supply equation (A50) in the appendix. However, labor supply does depend on average and marginal tax rates. Conceptually, an income elasticity for a change in the average tax rate can be calculated as

$$(6) \qquad \frac{\partial L_{y,j}}{\partial \left(1\text{-}\bar{t}_t\right)} \frac{1\text{-}\bar{t}_t}{L_{y,j}}$$

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by the average tax rate for that group, is the best measure of tax revenues generated by their participation. This approach effectively assumes that the relative wage rate does not change between groups.

while a substitution elasticity for a change in the marginal tax rate can be calculated as

(7) $$\frac{\partial L_{y,j}}{\partial\left(1-t_{t}\right)}\frac{1-t_{t}}{L_{y,j}}.$$

The latter computation allows the personal saving rate to depend on the marginal tax rate on labor income. (Note that the term $C_{y,a}$ in equation (5) depends on the marginal tax rate, $t_{t}$.) Labor supply is also a function of the real after-tax rate of return. This result is problematic since casual observation and empirical testing indicate that individuals do not adjust their work hours very much, if at all, in response to a change in the rate of return.

**Calibration of the MEG Model**

The MEG model is calibrated so that in the initial period the steady state equilibrium matches aggregate quarterly data on the U.S. economy. Consumption and saving are calculated as if the age profile of the population is steady over time, and thus, the representative consumer is always the same age but born in successive years. Assuming that some of the growth in the real wage and real transfer income (*g*) is unrelated to the age of the consumer because it is macroeconomic in nature (i.e., economy-wide technical change or capital deepening) and that for given tax rates and the rate of return, the MPCs are also fixed, it follows that in the steady state all incomes, net worth, and consumption grow at the rate *g* while leisure (and labor supply) are stationary.

The consumption function in equation (1) is calibrated to match consumption parameters in the macroeconomic empirical literature. Following is a sample calculation for a representative consumer aged 45, with a life expectancy of 75, who retires at age 62. The nominal pre-tax rate of return is .08, the tax rate on asset income is 0.25, and the inflation rate is 0.02. The rate of time preference is 0.015, the inter-temporal substitution elasticity is 0.25, and the intra-temporal substitution elasticity is 0.8. The distribution parameter for leisure (consumption) is 0.65 (0.35). The tax rate on labor is 0.25, and both the real wage and real transfers per capita are expected to grow at a 2% annual rate. Under these assumptions, the calculated MPCs are .044 (net worth), .044 (asset income), 1.08 (transfer income), and .69 (labor income). These are very similar to the values currently used in the macro-econometric literature.

**The Personal Saving Rate**

In equation (2), the relevant definitions of income, consumption (as opposed to consumer expenditures), and saving include durable goods as an investment yielding imputed flows of income and consumption. Hence, personal saving (and the saving rate) should be thought of not as the National Income and Product Account (NIPA) concept, but as the Flow-of-Funds measure that has been adjusted to incorporate the proper treatment of consumers' durable goods. From 1952 though 1989, this measure of the saving rate averaged about 13.5 percent. Since then it has fallen to around five percent, likely because of the increase in the ratio of (equity) wealth to income, which, as recent months have demonstrated, might not be a steady-state phenomenon. Over the entire period from 1952 to 2001, the average saving rate is about 12.25 percent. The steady-state saving rate used to calibrate MEG is considerably higher than the current NIPA personal saving rate.  For the parameters used in the earlier example, and using the current estimated value of $w(T)_t = .12$, the steady-state value of the personal saving rate is 12.5 percent, almost exactly the secular average.

The elasticity of the long-run saving rate with respect to changes in the after-tax rate of return, $i_t = r_t(1 - m_t) - p_t$, measures the sensitivity of long run personal saving to changes in the after-tax interest rate.  For the representative consumer and the same parameter values used in calculations above, a one percentage point increase in the real after-tax rate of return raises the personal saving rate by 0.6 percentage points.  (This translates to an elasticity of 0.29).  The inter-temporal substitution elasticity, $s_1$, is the key parameter controlling this response.  As $s_1$ rises, the response quickly escalates.  For example, with $s_1 = 1$ (which also required an increase in the rate of time preference, $r$, to 0.033 to calibrate to reasonable values for the MPCs), the personal saving rate increases by 6.1 percentage points in response to one percentage point increase in the after-tax rate of return (or an elasticity of 1.9).  Changes in the intra-temporal substitution elasticity have very little impact on the responsiveness of the saving rate to changes in the rate of return.

**Labor Supply**

In this specification of the labor supply, the income and substitution parameters assigned for (1) low-income primary earners, (2) other primary earners, (3) low income secondary earners, and (4) other secondary earners are as follows:  (1) -0.1 and 0.2; for (2) -0.1 and 0.1; for (3)  -0.3 and 0.6; and for (4) -0.2 and 0.8.  This decomposition allows different types of tax proposals to affect the total labor supply response in different ways, depending on whether they change average and marginal rates by the same amount, and how they affect different groups of people. According to our calculations based on income and wage data from the Joint Committee on Taxation's individual tax model, the income-weighted average of the income parameters is -0.13; the wage-weighted average of the substitution for parameters is 0.18. Note that these composite parameters imply, on average, a net labor supply elasticity of .05 for a tax change that affects each of these groups equivalently. However, there are few tax changes - not even an across the board rate cut - that affects each of these groups equivalently.

In the alternative specification, the labor supply elasticity is determined by structural parameters such as the intratemporal elasticity of substitution, instead of being explicitly defined as in the reduced-form labor supply equation. In this case, the labor supply elasticity measures the percentage change in labor supply in response to a change in the after-tax wage. Note however, that this only an approximation of the labor supply elasticity, since the wage rate and labor supply are determined endogenously in the model.

If leisure is included in the utility function, then labor supply depends upon the real after tax rate of return. For example, given the initial calibration described above, a one percent increase in the rate of return raises labor supply by approximately 0.01 percent. This response derives from the fact that the higher rate of return raises the saving rate (i.e., lowers the consumption rate), which, through equation (A6), implies less leisure (i.e., more labor.)

For the parameters used above, decreasing the marginal tax rate on labor and asset income by one percent increases labor supply by approximately 0.32 percent. This implies a substitution elasticity of about -0.30. This is determined primarily by the intra-temporal substitution elasticity. If intratemporal elasticity of substitution equals zero, $s_2 = 0$, then the substitution elasticity of labor supply is approximately -0.01 (since the rate of return affects the individuals labor supply.) If intratemporal elasticity of substitution equals one, $s_2 = 1$, the substitution elasticity is approximately 0.4. For the parameters used above, decreasing the average tax rate on labor and asset income by one percent reduces labor supply by approximately 0.31 percent. The effects of a general reduction in the marginal and average tax rates by one percent increases labor supply by approximately 0.01 percent.

## Technical Appendix

### Lifetime Utility Function

For a consumer born in year *y*, of age *a*, with a life expectancy of *L*, and who retires at age *R*, the consumer's lifetime utility is:

$$\text{(A1)} \qquad SLU_y = \frac{s_1}{s_1 - 1} \sum_{a=0}^{L} \frac{U_{\text{y,a}}^{(s_1 - 1)/s_1}}{(1 + r)^{a}}$$

where $U_{y,a}$ is the utility function, $r$ is the consumer's pure real rate of time preference, and $s_1$ is the intertemporal elasticity of substitution . Within a given year, the utility function is separable in consumption and leisure (or home production):

$$\text{(A2)} \qquad U_{\text{y,a}} = \left[ a_C^{1/s_2} C_{\text{y,a}}^{(s_2 - 1)/s_2} + a_E^{1/s_2} \left( W_{y,a} E_{y,a} \right)^{(s_2 - 1)/s_2} \right]^{s_2/(s_2 - 1)}$$

where $C_{y,a}$ is the real value of consumption, $E_{y,a}$ is time devoted to non-taxed leisure valued at the pre-tax real wage, $W_{y,a}$, $a_C$ and $a_E$ are intensity parameters, and $s_2$ is the intratemporal elasticity of substitution between consumption and leisure.

The treatment here assumes that increases in labor productivity render the consumer's time more efficient whether that time is applied to labor or to leisure. Consequently, while taxes can alter the terms of substitution between labor and leisure, productivity gains do not. This assumption renders it possible to derive a steady state with a stationary allocation of aggregate hours between work and leisure for a given age-distribution of the population.

**Lifetime Budget Constraint**

It is assumed that the consumer is born and dies with no wealth, so that the present discounted value of nominal lifetime consumption just exhausts the present discounted values of nominal lifetime after-tax labor income and nominal transfer income. The consumer has a fixed endowment of hours, $H$, to allocate between labor and leisure in each year until retirement. $H$ is devoted entirely to leisure during retirement. Finally, it is assumed that the consumer receives a stream of transfer income that lasts the remainder of the consumer's life. The lifetime budget constraint can be written as:

$$\sum_{a=0}^{L} P_{\mathrm{y+a}} C_{\mathrm{y,a}} \frac{1+r_y\left(1-m_y\right)}{\prod_{s=0}^{a}\left(1+r_{y+s}\left(1-m_{y+s}\right)\right)} = \sum_{a=0}^{R}\left[\left(H-E_{\mathrm{y,a}}\right) \; W\$_{\mathrm{y,a}} - TL\$_{\mathrm{y,a}}\right]\frac{1+r_y\left(1-m_y\right)}{\prod_{s=0}^{a}\left(1+r_{y+s}\left(1-m_{y+s}\right)\right)}$$

(A3)

$$+\sum_{a=0}^{L} YT\$_{\mathrm{y,a}} \frac{1+r_y\left(1-m_y\right)}{\prod_{s=0}^{a}\left(1+r_{y+s}\left(1-m_{y+s}\right)\right)}$$

where $P_{y+a}$ is the price of consumption in year $y+a$, $r_{y+a}$ is the nominal pre-tax rate of return, $m_{y+a}$ is the (marginal equals average) tax rate on asset income, $W\$_{y,a}$ is the consumer's nominal pre-tax wage rate, $YT\$_{y,a}$ is the consumer's nominal transfer income, and $TL\$_{y,a}$ is nominal taxes collected on labor income. $TL\$_{y,a}$ is a function of nominal labor income:

(A4) $$TL\$_{y,a} = G_{y+a}[(H - E_{y,a})W\$_{y,a}].$$

Note that macroeconomic variables and tax policy that are not specific to the consumer ( $P_{y+a}$ , $r_{y+a}$ , $\overline{m}_{y+a}$ , $G_{y+a}$ ) depend only on time, which is indexed by the consumer's year of birth. The consumer's wage rate, transfer income, consumption and leisure depend both on time and the consumer's age. (For future reference, we adopt the notation that when the mnemonic for a dollar-denominated variable has a *$* as a suffix, it is a nominal magnitude; otherwise it is real. For example, $YT_{y,a}$ is the consumer's real transfer income.)

**First-Order Conditions**

The consumer maximizes lifetime utility (A1) subject to the lifetime budget constraint (A3). The first-order conditions (FOCs) of the constrained maximization are:

(A5)

$$\frac{\left[a_C^{1/a_2} C_{y,a}^{(a_2-1)/a_2} + a_E^{1/a_2}\left(W_{\text{y,a}} E_{\text{y,a}}\right)^{(a_2-1)/a_2}\right]^{\frac{a_2-a_1}{a_1(1-a_2)}} a_C^{1/s_2} C_{y,a}^{-1/s_2}}{(1+r)^a} = \frac{l\,P_{y+a}}{\prod_{s=0}^{a}\left(1+r_{y+s}\left(1-m_{y+s}\right)\right)}$$

$$\frac{\left[a_C^{1/a_2} C_{y,a}^{(a_2-1)/a_2} + a_E^{1/a_2}\left(W_{\text{y,a}} E_{\text{y,a}}\right)^{(a_2-1)/a_2}\right]^{\frac{a_2-a_1}{a_1(1-a_2)}} a_E^{1/s_2}\left(W_{\text{y,a}} E_{\text{y,a}}\right)^{-1/s_2} W_{\text{y,a}}}{(1+r)^a} = \frac{l\,W\$_{\text{y,a}}\left(1-t_{y+a}\right)}{\prod_{s=0}^{a}\left(1+r_{y+s}\left(1-m_{y+s}\right)\right)}$$

where $l$ is a Lagrange multiplier and $t_{y+a} = G'_{y+a}$ is the marginal tax rate on labor income. These FOCs can be manipulated in a variety of ways that prove useful in deriving the consumption function.

**Intra-temporal Relationship Between Consumption and Leisure**

The intra-temporal relationship between leisure and consumption is derived by taking the ratio of the first-order conditions. Re-arranging terms and defining the pre-tax real wage as $W_{y,a} = \frac{W\$_{y,a}}{P_{y+a}}$ leads to:

(A6)
$$E_{\text{y,a}} W_{\text{y,a}} = C_{\text{y,a}}\left(\frac{a_E}{a_C}\right)\left[1-t_{y+a}\right]^{-s_2}.$$

Note that if, for a particular consumer born in year *y*, consumption and the after-tax real wage change proportionately as the consumer's age, *a*, rises, then that consumer's allocation of hours between leisure and labor is stationary throughout life. As shown below, stationary labor supply for an individual consumer is a special case that is unlikely to obtain. Over time, however, if productivity raises the real wage and consumption proportionately for consumers of any age, then leisure taken by a consumer at any age is stationary. Then, for a given age distribution of the population, aggregate leisure and labor are stationary. As shown below, this condition is satisfied under the assumptions adopted here.

### Optimal Time Path of Consumption

To derive the optimal time path of consumption, first substitute the intra-temporal relationship (A6) between consumption and leisure into the first-order condition (A5) for $C_{y,a}$:

(A7)
$$\frac{\left[ a_C^{1/s_2} C_{y,a}^{(s_2-1)/s_2} + a_E^{1/s_2}\left[\left(\frac{a_E}{a_C}\right)\left(1-t_{y+a}\right)^{-s_2} C_{y,a}\right]^{\frac{s_2-1}{s_2}}\right]^{\frac{s_1-s_2}{s_1(s_2-1)}} a_C^{1/s_2} C_{y,a}^{-1/s_2}}{(1+r)^a} = \frac{l P_{y+a}}{\prod_{s=0}^{a}\left(1+r_{y+s}\left(1-m_{y+s}\right)\right)}.$$

After collecting terms:

(A8)
$$\frac{C_{y,a}^{-1/s_1} a_C^{\frac{s_1-1}{s_1(s_2-1)}} \left[1+\left(\frac{a_E}{a_C}\right)\left[\left(1-t_{y+a}\right)\right]^{1-s_2}\right]^{\frac{s_1-s_2}{s_1(s_2-1)}}}{(1+r)^a} = \frac{l P_{y+a}}{\prod_{s=0}^{a}\left(1+r_{y+s}\left(1-m_{y+s}\right)\right)}$$

Taking the ratio of consumption in two adjacent years gives:

(A9) $$\frac{C_{y,a}}{C_{y,a-1}} = \left[\frac{1+r_{y+a}\left(1-m_{y+a}\right)}{\left(1+r\right)\left(1+p_{y+a}\right)}\right]^{s_1}\left[\frac{F_{y,a}}{F_{y,a-1}}\right]^{\frac{s_2-s_1}{1-s_2}}$$

where the inflation rate is defined by $1+p_{y+a} = \frac{P_{y+a}}{P_{y+a-1}}$ and

(A10) $$F_{y,a} = 1+\left(\frac{a_E}{a_C}\right)\left(1-t_{y+a}\right)^{1-s_2}.$$

Expression (A9) implies that, given the lifetime budget constraint, consumption could rise or fall with the consumer's age. Note, however, that for a given marginal tax rate on labor income, $F_{y,a}/F_{y,a-1} = 1$. Hence, (A9) can be approximated by:

(A11) $$\dot{C}_{y,a} \approx s_1\left(r_{y+a}\left(1-m_{y+a}\right)-p_{y+a}-r\right)$$

where $\dot{C}_{y,a}$ is the growth rate of consumption. From (A11), it follows that if the real after-tax rate of return exceeds the rate of pure time preference, consumption tends to rise as the consumer ages as long as there is some inter-temporal substitution. For typical values of these parameters, consumption rises as the consumer ages.

**Optimal Time Path for Leisure**

To derive the optimal time path of leisure, form the growth of (A6) and then substitute in the optimal time path for consumption (A9):

(A12) $$\frac{E_{y,a}}{E_{y,a-1}} = \left[\frac{1+r_{y+a}\left(1-m_{y+a}\right)}{\left(1+r\right)\left(1+p_{y+a}\right)}\right]^{s_1}\left[\frac{F_{y,a}}{F_{y,a-1}}\right]^{\frac{s_2-s_1}{1-s_2}}\left[\frac{W_{\mathrm{y,a-1}}}{W_{\mathrm{y,a}}}\right]\left[\frac{1-t_{y+a}}{1-t_{y+a-1}}\right]^{1-s_2}$$

Again noting that, for unchanged taxes, $F_{y,a}$ is constant leads to:

(A13) $$\dot{E}_{y,a} \approx s_1\left(r_{y+a}\left(1-m_{y+a}\right)-p_{y+a}-r\right)-\dot{W}_{y,a}.$$

Equation (A13) reveals that leisure (labor) could either rise (fall) or fall (rise) with the consumer's age. This is true for a range of reasonable parameters.

**Manipulation of the Lifetime Budget Constraint**

Assuming the consumer's current age is $j$, $0 < j < R < L$, the lifetime budget constraint can be re-organized as:

$$\sum_{a=j}^{L} P_{y+a} C_{y,a} \frac{1+r_y\left(1-m_y\right)}{\prod_{s=0}^{a}\left(1+r_{y+s}\left(1-m_{y+s}\right)\right)}$$

$$\text{(A14)} = \sum_{a=0}^{j-1}\left(\left(H-E_{y,a}\right) \; W\$_{y,a}\left(1-\bar{t}_{y+a}\right)+YT\$_{y,a}-P_{y+a}C_{y,a}\right)\frac{1+r_y\left(1-m_y\right)}{\prod_{s=0}^{a}\left(1+r_{y+s}\left(1-m_{y+s}\right)\right)}$$

$$+\sum_{a=j}^{L} YT\$_{y,a}\frac{1+r_y\left(1-m_y\right)}{\prod_{s=0}^{a}\left(1+r_{y+s}\left(1-m_{y+s}\right)\right)}+\sum_{a=j}^{R}\left(H-E_{y,a}\right) \; W\$_{y,a}\left(1-\bar{t}_{y+a}\right)\frac{1+r_y\left(1-m_y\right)}{\prod_{s=0}^{a}\left(1+r_{y+s}\left(1-m_{y+s}\right)\right)}$$

where $\bar{t}_{y+a}=\dfrac{TL\$_{y,a}}{\left(H-E_{y,a}\right)W\$_{y,a}}$ is the average tax rate on labor income. Next, multiply expression

(A14) through by $\dfrac{\prod_{s=0}^{j}\left(1+r_{y+s}\left(1-m_{y+s}\right)\right)}{1+r_y\left(1-m_y\right)}$ and re-arrange terms:

(A15)

$$\sum_{a=j}^{L} P_{y+a} C_{y,a} \frac{1+r_{y+j}\left(1-m_{y+j}\right)}{\prod_{s=j}^{a}\left(1+r_{y+s}\left(1-m_{y+s}\right)\right)}$$

$$=\left(1+r_{y+j}\left(1-m_{y+j}\right)\right) \quad \sum_{a=0}^{j-1}\left(\left(H-E_{y,a}\right) \; W\$_{y,a}\left(1-\bar{t}_{y+a}\right)+YT\$_{y,a}-P_{y+a}C_{y,a}\right)\frac{\prod_{s=0}^{j-1}\left(1+r_{y+s}\left(1-m_{y+s}\right)\right)}{\prod_{s=0}^{a}\left(1+r_{y+s}\left(1-m_{y+s}\right)\right)}$$

$$+\sum_{a=j}^{L} YT\$_{y,a}\frac{1+r_{y+j}\left(1-m_{y+j}\right)}{\prod_{s=j}^{a}\left(1+r_{y+s}\left(1-m_{y+s}\right)\right)}+\sum_{a=j}^{R}\left(H-E_{y,a}\right) \; W\$_{y,a}\left(1-\bar{t}_{y+a}\right)\frac{1+r_{y+j}\left(1-m_{y+j}\right)}{\prod_{s=j}^{a}\left(1+r_{y+s}\left(1-m_{y+s}\right)\right)}+$$

Consider first term on the right-hand side of (A15). The term inside the summation is the accumulation of the consumer's financial net worth through the end of year *j-1*. Hence, the lifetime budget constraint can be re-written as:

(A16)
$$\sum_{a=j}^{L} P_{y+a} C_{y,a} \frac{1+r_{y+j}\left(1-m_{y+j}\right)}{\prod_{s=j}^{a}\left(1+r_{y+s}\left(1-m_{y+s}\right)\right)} = NW\$_{y,j-1}\left(1+r_{y+j}\left(1-m_{y+j}\right)\right)$$
$$+\sum_{a=j}^{L} YT\$_{a} \frac{1+r_{y+j}\left(1-m_{y+j}\right)}{\prod_{s=j}^{a}\left(1+r_{y+s}\left(1-m_{y+s}\right)\right)} + \sum_{a=j}^{R}\left(H-E_{y,a}\right) W\$_{y,a}\left(1-\overline{t}_{y+a}\right)\frac{1+r_{y+j}\left(1-m_{y+j}\right)}{\prod_{s=j}^{a}\left(1+r_{y+s}\left(1-m_{y+s}\right)\right)}.$$

Manipulation with price levels leads to:

(A17)
$$\sum_{a=j}^{L} C_{y,a}\left(\frac{P_{y+a}}{P_{y+j}}\right)\frac{1+r_{y+j}\left(1-m_{y+j}\right)}{\prod_{s=j}^{a}\left(1+r_{y+s}\left(1-m_{y+s}\right)\right)} = \frac{NW\$_{y,j-1}}{P_{y+j-1}}\left(\frac{P_{y+j-1}}{P_{y+j}}\right)\left(1+r_{y+j}\left(1-m_{y+j}\right)\right)$$
$$+\sum_{a=j}^{L}\left(\frac{YT\$_{y,a}}{P_{y+a}}\right)\left(\frac{P_{y+a}}{P_{y+j}}\right)\frac{1+r_{y+j}\left(1-m_{y+j}\right)}{\prod_{s=j}^{a}\left(1+r_{y+s}\left(1-m_{y+s}\right)\right)}$$
$$+\sum_{a=j}^{R}\left(H-E_{y,a}\right)\left(\frac{W\$_{y,a}\left(1-\overline{t}_{y+a}\right)}{P_{y+a}}\right)\left(\frac{P_{y+a}}{P_{y+j}}\right)\frac{1+r_{y+j}\left(1-m_{y+j}\right)}{\prod_{s=j}^{a}\left(1+r_{y+s}\left(1-m_{y+s}\right)\right)}.$$

Substitute into (A17) the definitions of the inflation rate, the pre-tax real wage, real incomes, and real net worth. Also, note that real disposable asset income can be defined as

$\left(1-\overline{m}_{y+j}\right)YA_{y,j} = NW_{y,j-1}\left[\frac{1+r_{y+j}\left(1-m_{y+j}\right)}{1+p_{y+j}}-1\right]$, where $\overline{m}_{y+j}$ is the average personal tax rate on real personal asset income. Then:

$$\sum_{a=j}^{L} C_{y,a}\frac{\prod_{s=j}^{a}\left(1+p_{y+s}\right)}{1+p_{y+j}}\frac{1+r_{y+j}\left(1-m_{y+j}\right)}{\prod_{s=j}^{a}\left(1+r_{y+s}\left(1-m_{y+s}\right)\right)} = NW_{y,j-1}+YA_{y,j}\left(1-\overline{m}_{y+j}\right)$$

(A18) $$+\sum_{a=j}^{L} YT_{y,a}\frac{\prod_{s=j}^{a}\left(1+p_{y+s}\right)}{1+p_{y+j}}\frac{1+r_{y+j}\left(1-m_{y+j}\right)}{\prod_{s=j}^{a}\left(1+r_{y+s}\left(1-m_{y+s}\right)\right)}$$

$$= \sum_{a=j}^{R}\left(H-E_{y,a}\right)\left[W_{y,a}\left(1-\overline{t}_{y+a}\right)\right]\frac{\prod_{s=j}^{a}\left(1+p_{y+s}\right)}{1+p_{y+j}}\frac{1+r_{y+j}\left(1-m_{y+j}\right)}{\prod_{s=j}^{a}\left(1+r_{y+s}\left(1-m_{y+s}\right)\right)}$$

Future values of the pre-tax rate of return, the inflation rate, and tax rates are determined using myopic expectations which implies:

(A19) $$r_{y+a} = r_{y+j}, p_{y+a} = p_{y+j}, m_{y+a} = m_{y+j}, \overline{m}_{y+a} = \overline{m}_{y+j}, t_{y+a} = t_{y+j}, \overline{t}_{y+a} = \overline{t}_{y+j} \quad a > j$$

Substitute (A19) into (A18) and, for expositional simplicity, set $y+j = t$:

$$\sum_{a=j}^{L} C_{\text{y,a}} \left[ \frac{1+p_t}{1+r_t(1-m_t)} \right]^{a-j} = NW_{\text{y,j-1}} + YA_{y,j}(1-\overline{m}_t) +$$

(A20)

$$+\sum_{a=j}^{L} YT_{y,a} \left[ \frac{1+p_t}{1+r_t(1-m_t)} \right]^{a-j} + (1-\overline{t}_t) \sum_{a=j}^{R} \left( H - E_{y,a} \right) \; W_{y,a} \left[ \frac{1+p_t}{1+r_t(1-m_t)} \right]^{a-j}$$

The interpretation of (A20) is straightforward: for a consumer of age *j* at time *t*, the present discounted value of real consumption over his or her remaining lifetime equals real financial net worth plus real after-tax asset income plus the present discounted value of remaining real after-tax labor and transfer income.

In a simpler analysis without leisure in the utility function, the consumption function would be derived next by substituting the optimal time path of consumption into the left hand side of (A20), assuming leisure (and hence labor supply) is fixed so that labor income grows at the same rate as the real wage, and then solving for current consumption in terms of net worth, current incomes, and expected growth in transfers and the real wage. Here, however, a complicating factor is that future values of labor income depend on future values of leisure, which in turn depend on future values of the real wage. In order to eliminate these dependencies from the ultimate form of the consumption function, some further manipulations of the lifetime budget constraint are required. To begin, collect on to the left hand side all the terms in consumption and leisure:

(A21)

$$\sum_{a=j}^{L}\left[C_{y,a}+E_{y,a}W_{y,a}\left(1-\bar{t}_{t}\right)\right]\left[\frac{1+p_{t}}{1+r_{t}\left(1-m_{t}\right)}\right]^{a-j}=NW_{\mathrm{y,j\text{-}1}}+YA_{y,j}\left(1-\bar{m}_{t}\right)$$

$$+\sum_{a=j}^{L}YT_{y,a}\left[\frac{1+p_{t}}{1+r_{t}\left(1-m_{t}\right)}\right]^{a-j}+H\left(1-\bar{t}_{t}\right)\sum_{a=j}^{R}W_{y,a}\left[\frac{1+p_{t}}{1+r_{t}\left(1-m_{t}\right)}\right]^{a-j}$$

Substituting (A6) into the left hand side of (A21) and collecting terms in $C_{y,a}$:

(A22)

$$\sum_{a=j}^{L}C_{\mathrm{y,a}}\left[1+\frac{a_{E}}{a_{C}}\left(1-t_{t}\right)^{-s_{2}}\left(1-\bar{t}_{t}\right)\right]\left[\frac{1+p_{t}}{1+r_{t}\left(1-m_{t}\right)}\right]^{a-j}=NW_{\mathrm{y,j\text{-}1}}+YA_{\mathrm{y,j}}\left(1-\bar{m}_{t}\right)$$

$$+\sum_{a=j}^{L}YT_{y,a}\left[\frac{1+p_{t}}{1+r_{t}\left(1-m_{t}\right)}\right]^{a-j}+H\left(1-\bar{t}_{t}\right)\sum_{a=j}^{R}W_{y,a}\left[\frac{1+p_{t}}{1+r_{t}\left(1-m_{t}\right)}\right]^{a-j}.$$

Making use of the assumption that expectations of taxes are myopic, and restating (A22) in terms of the current levels of consumption, the pre-tax real wage, real transfer income, and the respective factors for cumulative future growth:

(A23)

$$\left[1+\frac{a_{E}}{a_{C}}\left(1-t_{t}\right)^{-s_{2}}\left(1-\bar{t}_{t}\right)\right]C_{\mathrm{y,j}}\sum_{a=j}^{L}\left(\frac{C_{\mathrm{y,a}}}{C_{\mathrm{y,j}}}\right)\left[\frac{1+p_{t}}{1+r_{t}\left(1-m_{t}\right)}\right]^{a-j}=NW_{\mathrm{y,j\text{-}1}}+YA_{\mathrm{y,j}}\left(1-\bar{m}_{t}\right)+$$

$$=YT_{\mathrm{y,j}}\sum_{a=j}^{L}\left(\frac{YT_{\mathrm{y,a}}}{YT_{\mathrm{y,j}}}\right)\left[\frac{1+p_{t}}{1+r_{t}\left(1-m_{t}\right)}\right]^{a-j}+H\left(1-\bar{t}_{t}\right)\mathrm{W}_{\mathrm{y,j}}\sum_{a=j}^{R}\left(\frac{W_{\mathrm{y,a}}}{W_{\mathrm{y,j}}}\right)\left[\frac{1+p_{t}}{1+r_{t}\left(1-m_{t}\right)}\right]^{a-j}.$$

It is assumed that the consumer expects the real wage to grow at an average rate of *gw* over the remainder of his or her life working life. This growth can result either from macroeconomic factors, like technical advance or economy wide capital deepening, or from age-dependent factors specific to the individual, like experience or investment in human capital. It need not equal the real growth rate of aggregate productivity. Assuming that transfer income is expected to grow at the rate *gyt* over the remainder of the consumer's life implies:

$$\left[1+\frac{a_E}{a_C}(1-t_t)^{-s_2}(1-\bar{t}_t)\right]C_{\mathrm{y,j}}\sum_{a=j}^{L}\left(\frac{C_{y,a}}{C_{y,j}}\right)\left[\frac{1+p_t}{1+r_t(1-m_t)}\right]^{a-j} = NW_{\mathrm{y,j\text{-}1}}+YA_{\mathrm{y,j}}(1-\bar{m}_t)$$

(A24)

$$= YT_{\mathrm{y,j}}\sum_{a=j}^{L}[1+gyt]^{a-j}\left[\frac{1+p_t}{1+r_t(1-m_t)}\right]^{a-j} + H(1-\bar{t}_t)\,\mathrm{W}_{\mathrm{y,j}}\sum_{a=j}^{R}[1+gw]^{a-j}\left[\frac{1+p_t}{1+r_t(1-m_t)}\right]^{a-j}.$$

Each term on the right hand side of (A24) is either exogenous to the maximization problem, or is predetermined by the consumer's previous saving decisions.

**Deriving the Consumption Function**

Now the consumption function can be derived by substituting the optimal time path for consumption into (A24) and the solving the resulting expression for current consumption. First, substitute (A9) into (A24)

$$\left[1+\frac{a_E}{a_C}(1-t_t)^{-s_2}(1-\bar{t}_t)\right]C_{y,j}\sum_{a=j}^{L}\left[\frac{1+p_t}{1+r_t(1-m_t)}\right]^{(a-j)}\left[\frac{1+r_t(1-m_t)}{(1+r)\ (1+p_t)}\right]^{s_1(a-j)}\left[\frac{F_{y,a}}{F_{y,j}}\right]^{\frac{s_2-s_1}{1-s_2}}=$$

(A25) $NW_{y,j-1}+YA_{y,j}(1-\bar{m}_t)+YT_{y,j}\sum_{a=j}^{L}(1+gt)^{a-j}\left[\frac{1+p_t}{1+r_t(1-m_t)}\right]^{a-j}$

$$+(1-t_t)YL_{y,j}\sum_{a=j}^{L}(1-gw)^{a-j}\left[\frac{1+p_t}{1+r_t(1-m_t)}\right]^{a-j}.$$

Under myopic expectations of taxes, $t_a=t_j$ for $a>j$. It then follows immediately from (A9) that for $a>j$, $F_{y,a}/F_{y,a-1}=1$. Next define the consumption discount factor, $d_t$, as

(A26) $$d_t=\left[\frac{1+p_t}{1+r_t(1-m_t)}\right]\left[\frac{1+r_t(1-m_t)}{(1+r)(1+p_t)}\right]^{s_1}.$$

Note that the discount factor depends only on the real after-tax interest rate, the inflation rate, and the rate of time preference. The summation of $d_t$ over the consumer's remaining lifetime is:

(A27) $$\sum_{a=j}^{L}d_t^{a-j}=\frac{1-d_t^{L-j+1}}{1-d_t}=\mathrm{G}(d,L)_{t,j}.$$

The summation of the discount factor depends on the age of the consumer, but not the year in which the consumer was born. Next define the wage discount factor as:

(A28)
$$q_t = \frac{(1+p_t)(1+gw_t)}{1+r_t(1-m_t)}$$

and the summation of $q_t$ over the consumer's remaining working career as:

(A29)
$$\sum_{a=j}^{R} q_{\mathrm{t}}^{a-j} = \frac{1-q_t^{R-j+1}}{1-q_t} = \mathrm{G}(q, R)_{\mathrm{t,j}}.$$

Finally, define the transfer income discount factor as:

(A30)
$$g_t = \frac{(1+p_t)(1+gyt_t)}{1+r_t(1-m_t)}$$

and the summation of $g_t$ over the consumer remaining lifetime as:

(A31)
$$\sum_{a=j}^{L} g_{\mathrm{t}}^{a-j} = \frac{1-g_t^{L-j+1}}{1-g_t} = \mathrm{G}(g, L)_{\mathrm{t,j}}.$$

Substituting (A27), (A29), and (A31) into (A25) and re-arranging terms yields:

(A32)
$$\begin{aligned} C_{\mathrm{y,j}} + E_{\mathrm{y,j}} W_{\mathrm{y,j}} (1-\bar{t}_t) = & \left( \frac{1}{\mathrm{G}(d, L)_{t,j}} \right) NW_{\mathrm{y,j-1}} + \left( \frac{1}{\mathrm{G}(d, L)_{t,j}} \right) YA_{\mathrm{y,j}} (1-\bar{m}_t) \\ & + \left( \frac{\mathrm{G}(g, L)_{t,j}}{\mathrm{G}(d, L)_{t,j}} \right) YT_{\mathrm{y,j}} + \left( \frac{\mathrm{G}(q, R)_{t,j}}{\mathrm{G}(d, L)_{t,j}} \right) H (1-\bar{t}_t) \quad W_{\mathrm{y,j}} \end{aligned}$$

Subtracting $\left(\frac{G(q,R)_{t,j}}{G(d,L)_{t,j}}\right)E_{y,j}W_{y,j}(1-\bar{t}_t)$ from both sides of (A32) and collecting terms yields:

$$C_{y,j}+E_{y,j}W_{y,j}(1-\bar{t}_t)\left(1-\frac{G(q,R)_{t,j}}{G(d,L)_{t,j}}\right)=\left(\frac{1}{G(d,L)_{t,j}}\right)NW_{y,j-1}+\left(\frac{1}{G(d,L)_{t,j}}\right)YA_{y,j}(1-\bar{m}_t)+$$

(A33)

$$+\left(\frac{G(g,L)_{t,j}}{G(d,L)_{t,j}}\right)YT_{y,j}+\left(\frac{G(q,L)_{t,j}}{G(d,L)_{t,j}}\right)(H-E_{y,j})\ W_{y,j}(1-\bar{t}_t).$$

Note that $(H-E_{y,j})W_{y,j}$ is current pre-tax real labor income, $YL_{y,j}$. Substituting (A6) into the left hand side of (A33) and collect terms in $C_{y,j}$:

$$C_{y,j}\left\{1+\frac{a_E}{a_C}(1-t_t)^{-s_2}(1-\bar{t}_t)\left(1-\frac{G(q,R)_{t,j}}{G(d,L)_{t,j}}\right)\right\}=\left(\frac{1}{G(d,L)_{t,j}}\right)NW_{y,j-1}$$

(A34)

$$+\left(\frac{1}{G(d,L)_{t,j}}\right)YA_{y,j}(1-\bar{m}_t)+\left(\frac{G(g,R)_{t,j}}{G(d,L)_{t,j}}\right)YT_{y,j}+\left(\frac{G(q,R)_{t,j}}{G(d,L)_{t,j}}\right)(1-\bar{t}_t)YL_{y,j}.$$

Finally, define $h_{t,j}=1+\frac{a_E}{a_C}(1-t_t)^{-s_2}(1-\bar{t}_t)\left(1-\frac{G(q,R)_{t,j}}{G(d,L)_{t,j}}\right)$, collect terms, and then solve for the current value of consumption:

$$C_{y,j} = \left( \frac{1}{h_{t,j} \quad G(d,L)_{t,j}} \right) NW_{y,j-1} + \left( \frac{1}{h_{t,j} \quad G(d,L)_{t,j}} \right) YA_{y,j} (1 - \bar{m}_t)$$

(A35)

$$+ \left( \frac{G(g,L)_{t,j}}{h_{t,j} \quad G(d,L)_{t,j}} \right) YT_{y,j} + \left( \frac{G(q,R)_{t,j}}{h_{t,j} \quad G(d,L)_{t,j}} \right) (1 - \bar{t}_t) YL_{y,j}.$$

This is the desired form of the consumption function with the MPCs defined as:

$$b_1 = \left( \frac{1}{h_{t,j} \quad G(d,L)_{t,j}} \right), \qquad b_2 = \left( \frac{1}{h_{t,j} \quad G(d,L)_{t,j}} \right),$$

$$b_3 = \left( \frac{G(g,L)_{t,j}}{h_{t,j} \quad G(d,L)_{t,j}} \right), \qquad b_4 = \left( \frac{G(q,R)_{t,j}}{h_{t,j} \quad G(d,L)_{t,j}} \right).$$

The MPCs depend on the real after-tax rate of return, the rate of time preference, the average and marginal tax rates, both the inter and intra-temporal substitution elasticities, the intensity factors in the utility function, the expected rate of growth in the consumer's real wage, the age of the consumer, life expectancy, and retirement age. If leisure is not included in the utility function, $a_E = 0$ and $h_{t,j} = 1$; otherwise the form of the consumption function is not effected.

**Personal Disposable Income**

To consider income and substitution effects for the personal saving rate and for labor supply we need to model the distribution of personal disposable income between labor income, asset income, and transfer income. This requires bringing a production function into the analysis and specifying how government sets transfer payments. Using a Cobb-Douglas production function with labor-augmenting technical change under a neutral corporate income tax, gross output is:

$$Q_t = AK_t^a\left((1+g)^t L_t\right)^{1-a} \tag{A36}$$

where A is a scalar, $a$ is "capital's share" of nominal output (assuming factor markets are competitive) and $g$ is the rate of technical advance. If the relative price of consumption goods is 1, then real disposable labor income is:

$$(1-\bar{t}_t)YL_t = (1-\bar{t}_t)(1-a)Q_t. \tag{A37}$$

Real disposable asset income is:

$$(1-\bar{m}_t)YA_t = (1-\bar{m}_t)\left(aQ_t - \bar{d}K_t\right) = (1-\bar{m}_t)(r_t - p_t)K_t = (1-\bar{m}_t)aQ_t\left(\frac{r_t - p_t}{r_t - p_t + \bar{d}}\right) \tag{A38}$$

where $\bar{d}$ is the fixed depreciation rate on capital. Assume government sets transfers as an exogenous share, $k_t$, of gross output. Then real personal disposable income can be written as:

$$YPD_t = \left[(1-a)(1-\bar{t}_t) + a(1-\bar{m}_t)\frac{r_t - p_t}{r_t - p_t + d} + k_t\right]Q_t. \tag{A39}$$

The share of transfers in disposable income is:

(A40) $$w(T)_t = \frac{k_t}{(1-a)(1-\bar{t}_t) + a(1-\bar{m}_t)\frac{r_t - p_t}{r_t - p_t + d} + k_t}$$

and the share of labor income in disposable income is:

(A41) $$w(L)_t = \frac{(1-a)(1-\bar{t}_t)}{(1-a)(1-\bar{t}_t) + a(1-\bar{m}_t)\frac{r_t - p_t}{r_t - p_t + d} + k_t}.$$

**Deriving the Personal Saving Rate**

Re-writing (36) with the MPCs redefined condenses the exposition to:

(A42) $$C_{y,j} = b_1 NW_{y,j-1} + b_2 YA_{y,j}(1-\bar{m}_t) + b_3 YT_{y,j} + b_4 YL_{y,j}(1-\bar{t}_t).$$

Real personal disposable income is:

(A43) $$YPD_{y,j} = YA_{y,j}(1-\bar{m}_t) + YT_{y,j} + YL_{y,j}(1-\bar{t}_t).$$

Note that this definition treats undistributed corporate profits as part of personal income. Real personal saving is:

(A44) $$S_{\mathrm{y,j}} = YPD_{\mathrm{y,j}} - C_{\mathrm{y,j}}\ .$$

The short-run saving rate is calculated using the recently observed value of real financial net worth. Note that $YA_{y,j}\left(1-\overline{m}_t\right)=NW_{y,j-1}\left(r_t\left(1-m_t\right)-p_t\right)$. Then (A43) through (A44) can be combined to yield:

(A45) $$s_{\mathrm{y,j}} = \frac{S_{\mathrm{y,j}}}{YPD_{\mathrm{y,j}}} = 1 - \frac{C_{\mathrm{y,j}}}{YPD_{\mathrm{y,j}}} = 1 - \left(\frac{NW_{\mathrm{y,j-1}}}{YPD_{\mathrm{y,j}}}\right)\left[b_1 + \left(b_2 - b_4\right)\left(r_t\left(1-m_t\right)-p_t\right)\right]$$
$$-b_4 - w\left(T\right)_t\left(b_3 - b_4\right).$$

This short-run saving rate is a function of all the underlying parameters as well as the ratio of wealth to income. A long-run saving rate can be derived by observing that real net worth at the end of the previous year is the accumulation of past real saving. If all variables, including saving, grow at the steady-state rate *g*, then:

(A46) $$\frac{NW_{\mathrm{y,j-1}}}{YPD_{\mathrm{y,j}}} = \frac{1}{g}\left(\frac{S_{\mathrm{y,j}}}{YPD_{\mathrm{y,j}}}\right) = \frac{s_{\mathrm{y,j}}}{g}\ .$$

Substituting (A46) into (A45) and solving for $s_{y,j}$:

(A47) $$s_{\mathrm{y,j}} = \frac{g\left[1 - b_4 - w\left(T\right)_t\left(b_3 - b_4\right)\right]}{g + b_1 + \left(b_2 - b_4\right)\left(r_t\left(1-m_t\right)-p_t\right)}\ .$$

**Deriving the Labor Supply Function**

From (A6), current labor supply for a consumer of age *j* is:

(A48)
$$L_{y,j} = H - E_{y,j} = H - \left(\frac{a_E}{a_C}\right)\left(\frac{C_{y,j}}{W_{y,j}}\right)[1-t_t]^{-s_2}.$$

Substituting into this the expression for the long run saving rate leads to, after some manipulation:

(A49)
$$L_{y,j} = H - E_{y,j} = H - \left(\frac{a_E}{a_C}\right)(1-s_{y,j})\left(\frac{YPD_{y,j}}{L_{y,j}W_{y,j}(1-\bar{t}_t)}\right)L_{y,j}(1-\bar{t}_t)(1-t_t)^{-s_2}$$

Utilizing the definition of labor's share of disposable income, (A49) can be solved for labor supply as:

(A50)
$$L_{y,j} = \frac{H}{1+\left(\frac{a_E}{a_C}\right)\left(\frac{1-s_{y,j}}{w(L)_t}\right)[1-\bar{t}_t][1-t_t]^{-s_2}}.$$