HPC for Structural Estimation

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Outline

I am going to talk about what I know.

- Structural estimation is a broad field that uses many methods for many kinds of models.
- I work only in one corner of this very large field.
- Full solution estimation of dynamic models of the firm.

High performance computing is extremely useful for full solution methods

- One simple estimation can (now) be done on a workstation.
- An entire paper is hard to do on a single workstation.
- High performance computing has allowed me to do things with papers that I could never have done otherwise.

Outline

The models I solve look like this

$$V(w, z) = \max_{w'} \pi(w, w', z) + \beta \int V(w', z') dq(z', z)$$

- $V(\cdot)$ is the value of the firm.
- A prime means tomorrow. No prime means today.
- w is a vector of endogenous state variables.
- z is a vector of exogenous state variables that follow a Markov process.
- $q(z' \mid z)$ is the Markov transition function.
- β is a discount factor

The solution is easy to parallelize

- Discretize (w, z) into a finite number of feasible points.

$$\{\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_N\}, \qquad \{\tilde{z}_1, \tilde{z}_2, \ldots, \tilde{z}_M\}$$

- Use
    - value function iteration (slow, reliable)
    - policy function iteration (faster, less reliable)
    - polynomial approximations to V (faster, squirrelly)

  to solve the model

- Farm out the solution for each $(\tilde{w}_i, \tilde{z}_j)$ tuple out to a different thread on a workstation.

- Use OpenMP (intuitive) or MPI (unintuitive and usually faster)

Shared memory and unshared memory parallelization

- OpenMP is a set of compiler directives that make loops operate in parallel.
  - All of the instances of the loop can share variables in memory.
- MPI is a method of parallelization that does not require shared memory.
  - An entire section of code runs as many identical copies that are utterly independent of each other.
  - You can send info back and forth as necessary.
  - Hence, the name: Message Passing Interface.

Simulated minimum distance is the tool I use for estimation

- Compute statistics in actual data.

$$n^{-1} \sum_{i=1}^{n} h\left(x_i\right)$$

- Solve a model and simulate data from the model.
- Compute the exact same statistics in the simulated data.

$$S^{-1} \sum_{s=1}^{S} h\left(y_{is}\left(b\right)\right)$$

- Simulated data are a function of the model parameters, b.
- Try to get the two sets of statistics as close as possible by choosing the model parameters.

More formally, ...

- Define

$$g_n(b) = n^{-1} \sum_{i=1}^{n} \left[h(x_i) - S^{-1} \sum_{s=1}^{S} h(y_{is}(b)) \right].$$

- The simulated moments estimator of b is then defined as the solution to the minimization of

$$\hat{b} = \arg\min_{b} Q(b, n) \equiv g_n(b)' \hat{W}_n g_n(b),$$

- $\hat{W}_n$ is a positive definite matrix.

I have tried many different minimization algorithms

- Cannot use gradient based methods.
- Need to rely on heuristic methods
  - Multistart Nelder Meade (can be parallelized, unreliable)
  - Simulated Annealing (cannot be parallelized, slow, reliable)
  - Differential Evolution (can be parallelized)
  - Particle Swarm (can be parallelized)
- The last two algorithms are useful on a many node cluster.
  - Very useful for models that take longer to solve: Michaels, Page, and Whited (2018), Gao, Whited, and Zhang (2018)

Outline

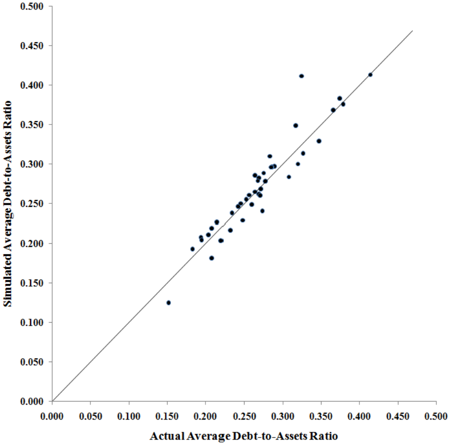
Hennessy and Whited (2005, 2007)

- I have been using distributed computing (though **not** high performance) since 2003.
- I used a multi-start Nelder Meade algorithm on a bunch unused PCs in the UW-Madison plasma physics lab.

- That was inefficient but the only thing available.

DeAngelo, DeAngelo, and Whited (2011)

- A kind PhD student gave me access to his account on his local HPC cluster.
- I produced this.



Cross-sectional heterogeneity

- The models I use are models of a single economic agent or of an industry with limited heterogeneity.
- It makes no sense to estimate these models on data generated by many extremely heterogeneous firms.
- I have found access to HPC clusters to be invaluable for examining cross-sectional heterogeneity.
  - Nikolov and Whited (2014)
  - Warusawitharna and Whited (2016)
  - Li, Whited, and Wu (2016)

Outline

All of the econometric estimators I use are based on the analogy principle

- GMM, M-estimators, Minimum distance estimators are all examples
- While they are asymptotically efficient and consistent
- They can have terrible finite sample properties
  - Arellano and Bond (1991), Altonji and Segal (1996), Hansen, Heaton, and Yaron (1996)
- Very few evaluations of the finite sample properties of the simulation counterparts of these estimators — SMM, SMD
  - Michaelides and Ng (2000), Eisenhauer, Heckman, and Mosso (2015)

Bazdresch, Kahn, and Whited (2018)

- And nothing for the class of simulated minimum distance estimators used in corporate finance.
- We evaluated the finite sample performance of these estimators on data generated from models of the firm.
- How do you do a Monte Carlo study of an estimator that can take days to converge?
  - Use a relatively simple model.
  - And . . .

The finite sample properties of these estimators are surprisingly good!

- Estimators of the parameters have low RMSE and bias.
- Optimal weight matrices help with this **a lot**.
- Specification tests have excellent power to detect model misspecification.

Outline

1. Introduction
2. HPC for Estimation
3. Examples
4. Finite Sample Performance
5. Conclusion

HPC is becoming more and more widespread

- Basic operating knowledge of Linux
- Knowledge of a fast language: C++, Fortran, Julia
- Learn parallelization paradigms
- Not as intimidating as it looks

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