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RISK AVERSION, LIABILITY RULES, AND SAFETY

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ABSTRACT

This paper investigates the performance of liability rules in two-party stochastic externality problems where negotiations are feasible and side payments are based on the realized level of externalities. Results show that an increase in polluter liability does not necessarily increase safety or efficiency in cases where the polluter is risk neutral. Complete polluter liability is found to yield Pareto optimality. When either party is risk averse, an increase in polluter liability may sometimes reduce safety and efficiency. If the polluter is risk neutral and the victim is risk averse, Pareto optimality is only achieved by assigning full liability on the polluter, i.e. giving the victim complete property rights to a clean environment. If the polluter is risk averse and the victim is risk neutral, no level of polluter liability is optimal. In this case, optimality can only be achieved through a contract on abatement activities, such that the risk-averse polluter receives a guaranteed payment regardless of the stochastic outcome.

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I. Introduction

It is generally agreed that, when externalities exist, the competitive price mechanism does not yield a Pareto-optimal resource allocation. The Pigouvian tax has been the traditionally prescribed remedy to external pollution problems (see Baumol and Oates; and Mishan). In a seminal paper, however, Ronald Coase described an alternative means to control externalities --one especially relevant for cases where the number of decision makers involved is small. Coase claimed that a competitive system with well-defined property right assignments, perfect information, and zero transaction costs would attain Pareto optimality through a process of voluntary bargaining and side payments. In his words: "It is necessary to know whether the damaging business is liable for damage caused since without the establishment of this initial delimitation of rights there can be no market transaction to transfer and recombine them. But the ultimate result (which maximizes the value of production) is independent of the legal position if the pricing system is assumed to work without cost" (see Coase [1960, p. 8]).

Coase's work has spawned a wide body of literature in both the economics of law and the environment (e.g. Farrell; DeSerpa; Medema). With rare exception, these literatures have examined externalities in a deterministic setting. In a practical sense, however, pollution problems are often stochastic with externalities depending on random forces such as human error, weather, or other natural phenomena. Indeed, many of the cases described in the seminal externality literature were stochastic in nature. For example, the foundry-laundry case discussed by Pigou described a situation where pollution occurs only when the wind is blowing in a certain direction. Similarly, Stigler described a case where contamination of a stream kills fish depending on flow rates affecting dissolution and dispersion. Moreover, some of the most spectacular environmental disasters – the nuclear meltdown at Chernobyl and the plant explosion at Bhopal – have been the result of random events, not consistent and predictable ones.

This stochasticity is important for two reasons. First, when pollution is stochastic, the victim cannot necessarily infer polluter action, thus inhibiting their ability to contract on pollution abatement or prevention activities. The departure from the assumption of determinism makes information asymmetries between polluter and victim more likely. In this case, victims may be limited to contracts on realized damages or be forced to engage in expensive monitoring that may allow them to contract on the polluter activities that contribute to victim damages. Second, stochastic pollution problems mean at least one agent involved in the Coasean bargain will have a stochastic income stream, suggesting an important role for risk preferences. Given wide variations in the wealth and size of agents that generate and suffer from pollution, agents are likely to differ in their ability and willingness to shoulder risk. These heterogeneous risk preferences will, of course, manifest themselves in equilibrium contracts.

This latter issue of risk preferences has received some attention in the economics literature. Greenwood and Ingene constructed a model where both parties have full information and therefore are able to employ nonstochastic side-payment schemes, which depend on variables that affect the distribution of pollution (in their case, these variables are the output levels of both parties). They demonstrate that the Coase result – the invariance of pollution levels to the assignment of property rights – holds if both parties are risk neutral, but does not under risk aversion. Graff Zivin and Small expand upon this single-case result by deriving a contract curve that describes all possible equilibria as function of the initial allocation of property rights, agents relative bargaining power, and risk preferences. They demonstrate that the Coase theorem obtains only if both agents have the same coefficient of absolute risk aversion. They also show that the ability to hedge environmental risks can restore the Coase result.

Our analysis of liability rules in two-party stochastic externality problems when negotiations are feasible differs from the existing literature in several distinct directions. First, the focus of this paper is not on the attainment of the Coase result, per se, but on identifying the conditions under which changes in the assignment of liability will increase, maintain, or decrease safety. Second, unlike most of the literature on liability rules and externalities, this paper does not limit the analysis to the two extreme cases, when the polluter is either fully liable for damage or not liable at all, but rather investigates the performance of the continuum of liability rules between these two extremes.[1] The analysis is done by abstracting to the case of a unilateral stochastic externality that occurs either at a fixed level or does not occur at all such that the polluter's activities affect only the probability and not the level of pollution. The modeling of the stochastic externality is thus similar to that in Spense.

Lastly, we assume that information is asymmetric. Agents cannot contract directly on firm activities, such as input use, abatement activities, or output levels. Instead, this paper considers side-payment schemes based on realized damages, which are observable by both parties. Rather than bribing the polluter to reduce output or increase abatement, as in Greenwood and Ingene and Graff Zivin and Small, the polluter receives a side payment from the victim for every period with no externality.

In this regard, our model can be viewed as a departure from the Coasean world of perfect information and similar in nature to the literature on the optimal design of liability rules. In most of those models, however, the agency relationship is generally viewed as one between the government

[1]We note, however, that the spirit of such a continuum was captured in the analysis by Graff Zivin and Small through their use of the bargaining power concept.

and the firms causing injuries, where both parties are assumed risk neutral and polluters and victims do not negotiate directly with one another (e.g. Shavell, 1980; Polinsky). The work that does address liability and incentives in the presence of risk aversion assumes symmetric information between agents (see Shavell, 1987). Extensions that examine asymmetries include some recent work on vicarious liability, which analyzes agency problems within the firm, i.e. where firm management is the principal and the agents are employees engaged in illegal activities (Shavell, 1997; Privileggi et al.). Like other principal-agent models in the economic literature on contracting (e.g. Stiglitz; Hart; Lazear and Moore), the principal is always assumed to be risk-neutral. The model developed differs from these in that we examine the realistic case of a risk-averse principal, the pollutee. Moreover, given our focus on infrequent but quite damaging events, the stochastic structure of our model and, in turn, the modeling approach undertaken is rather different. This yields rather distinct results to a slightly different set of questions.

The results show that, when both parties are risk neutral, the equilibrium distribution of pollution will be identical under a wide range of liability rules. This continuum includes the case of no polluter liability and cases of partial polluter liability, which result in positive side payments from pollutee to polluter whenever pollution does not occur. However, as polluter liability becomes large enough to crowd out pollutee side payments, increases in polluter liability yield increases in safety. Only complete polluter liability consistently yields socially optimal levels of safety. When the polluter is risk neutral but the pollutee is risk averse, complete polluter liability is again the only arrangement that always produces socially optimal safety levels. When the polluter is risk averse and the pollutee is risk neutral, optimality cannot be achieved with a contract on realized damages. This occurs, in part, because any contract that can be struck on damages will leave the polluter bearing some risk. In this case, optimality can only be achieved through a contract on abatement activities, such that the risk-averse polluter receives a guaranteed payment regardless of the stochastic outcome.

These results are especially important because many stochastic environmental problems occur at the intersection between public/corporate interests and small/local interests. The former are likely to be risk neutral, while the latter are likely to be risk averse. When polluters are large entities and victims are local inhabitants, polluters should be held fully accountable for all damages. On the other hand, when polluters are small and victims are large, policy makers may prefer to invest in monitoring efforts that allow contracts to be set on production and abatement activities. Indeed these results may help explain why, in practice, we see corporations held liable for damages they inflict on their neighbors, while farmers engaged in noxious activities are held subject to regulations on input use.

II. The Model

Suppose a region includes a polluter and a pollutee. The polluter produces output and generates pollution. The amount of pollution generated as a byproduct in the production process is a dichotomous random variable, Y, which takes on values of zero or one with

$$q \equiv \Pr(Y=0);\ 1-q \equiv \Pr(Y=1)\,.$$

The term q is called "safety." Each level of safety requires some ongoing pollution abatement activities. The cost of attaining a certain safety level q is given by the cost function, $c(q)$, for which marginal cost is increasing ($c' > 0$, $c'' > 0$), and $c(0) = 0$. The cost of safety includes the direct cost of pollution abatement activities and losses due to a reduction in output relative to the case where no attention is given to safety. The polluter's short-run profit (or quasi rent), when the pollution problem is ignored, is equal to *a* dollars per period. The introduction of safety measures thus reduces short-run profit to $a - c(q)$.

The pollutee's short-run profit (or quasi rent) is affected by the occurrence of pollution. When pollution does not occur, the pollutee's profit is equal to *b* dollars per period. When pollution occurs, the pollutee's profit is reduced by *R* dollars and is equal to $b - R$.

Both agents operate subject to a liability rule $L(\alpha)$ imposed externally. The parameter α is a fraction of the pollutee's loss (in periods when pollution occurs) which is paid by the polluter, i.e., the polluter compensates the pollutee with αR dollars whenever pollution occurs. Thus, full polluter liability holds when $\alpha = 1$, full pollutee liability (no polluter liability) holds when $\alpha = 0$, and partial polluter liability corresponds to $0 < \alpha < 1$.

Since the behavior of the polluter affects the pollutee, the latter may use side payments to motivate the polluter to increase safety. Assuming that the pollutee cannot monitor the polluter's activities (or, alternatively, monitoring costs are extremely high), the use of side payments based on the actual levels of the polluter's output and abatement activities (and thus the actual level of *q*) is not practical. Instead, a side-payment agreement based on a variable the pollutee can observe--namely, the level of pollution--is possible. Whenever pollution does not occur, the pollutee will pay the polluter a bribe of *S* dollars. This arrangement is the most natural since it is simple and easily enforceable.

Both parties are assumed to maximize the expected utility of their income. The polluter chooses his output and safety level for a given liability rule and a given bribe offer. The pollutee has to determine the size of bribe offered, given the liability rule and the polluter’s reaction function (which denotes safety as a function of the bribe) that is assumed to be known by the pollutee. The decisions of both parties, taken simultaneously, determine the equilibrium level of safety and the distribution of profit, given the liability rule.

III. The Case of Risk Neutrality of Both Parties

When both parties are risk neutral, both maximize their expected income. Thus, for a given liability rule and bribe, the polluter's decision problem is

(1) $$\max_q \; q\,[a - c(q) + S] + (1 - q)\,[a - c(q) - \alpha R].$$

Assuming that the problem has an internal solution, the first-order condition for optimality is

(2) $$R\alpha + S - c'(q) = 0.$$

For a risk-neutral polluter, safety is a function of the sum of the liability payment and the bribe. Denote this sum by $Z = \alpha R + S$ and call it the polluter's gain from a safe period. The polluter's gain reflects the increase in the polluter's earnings during an unpolluted period relative to a polluted period. Condition (2) states that optimal safety is determined by equating the marginal cost of safety to the polluter's gain from a safe period. Using (2), it is possible to construct the safety function $q(Z)$ which denotes safety as a function of polluter gain from a safe period. Total differentiation of (2) shows that the safety function is positively sloped with

(3) $$q_Z = c_{qq}^{-1} > 0.$$

The safety function of the polluter is assumed to be known by the pollutee, and the latter uses it in his decision making process to determine the level of the bribe he should offer. The pollutee's profit is b - S in a period without pollution and $b - R\,(1 - \alpha)$ when pollution occurs. Thus, the expected profit of the pollutee is $q(Z)\,(b - S) + [1 - q(Z)]\,[b - (1 - \alpha)\,R]$. The expected profit can be expressed as a function of the polluter's gain from a safe period as follows:

(4) $$L(Z) = b - (1 - \alpha)R + q(Z)(R - Z).$$

Thus, for a given liability rule, the optimization problem of the pollutee becomes

(5) $$\max_Z L(Z)$$

subject to $Z \geq \alpha R$. From the pollutee's expected profit formulation in (4), one deduces that maximizing behavior requires that the bribe be smaller than the pollutee's loss when pollution occurs, i.e.,

(6) $$R(1-\alpha) - S = R - Z > 0.$$

In case of complete pollutee liability ($\alpha = 0$), the expected profit of the pollutee when no bribe is paid (when $S = 0$) is equal to the expected profit when a bribe of R dollars is paid [which is $b - R$ since $q(0) = 0$]; when the bribe obeys (6), expected profit is higher than $b - R$. Thus, there is an optimal bribe for which $0 < Z < R$.

At the optimal solution,

(7) $$L_Z(\alpha) \equiv q_Z(R - Z) - q(Z) = 0.$$

To interpret this condition, note that a marginal increase in the bribe increases safety by q_Z, and the pollutee earns $R - Z$ dollars more when pollution does not occur; thus, the pollutee's marginal gain from bribing is $q_Z(R - Z)$. On the other hand, an increased bribe reduces the pollutee's gain in a safe period; thus, the pollutee's marginal cost associated with the bribe is equal to $q(Z)$. Condition (7) thus indicates that optimal bribery occurs when the marginal gain for the pollutee from the bribe is equal to his marginal loss. By dividing both terms in (7) by the average expected gain from the bribe, $[q(Z) \cdot (R - Z)/Z]$, one finds the optimality condition,

(8) $$\eta(Z) = \frac{Z}{R - Z},$$

where $\eta(Z) = q_Z \cdot Z / q(Z)$ is the elasticity of the safety function. By the properties of the cost function, this elasticity is a positive bounded function of Z. The polluter's gain from a safe period per unit of the pollutee's gain from a safe period, $Z/(R - Z)$, is an increasing function of Z which converges to infinity as Z converges to R.

Both functions are depicted in Figure 1. When Z is small, the marginal gain from bribing is larger than the marginal cost of bribing; thus, for small Z, $\eta(Z)$ is above $Z/(R - Z)$. As Z is increased, both curves become closer and intersect at A where condition (8) holds. The polluter's gain from a safe period at the intersection point is S_0, which is the bribe paid under complete pollutee liability. The elasticity of safety curve intersects $Z/(R - Z)$ at A from above. It is assumed that the properties of the cost function ensure that A is a unique intersection.[2] To the right of A, the elasticity curve is below $Z/(R - Z)$ reflecting the supposition that, for $Z > S_0$, the marginal gain from the bribe is smaller than the marginal loss and that any increase in the bribe when $Z > S_0$ will reduce the pollutee's expected profit.

In cases with partial pollutee liability ($\alpha > 0$), the feasible set of Z's is smaller than under complete pollutee liability since the polluter's gain from a safe period is at least αR. However, the marginal gain and loss of bribing are the same for any α. Therefore, the curves $\eta(Z)$ and $Z/(R - Z)$ can be used to find the optimal bribe and safety as functions of the liability rule.

A comparison of outcomes for two liability rules can be demonstrated using Figure I. In the first case the liability payment is smaller than the bribe under complete pollutee liability, i.e., $Z_1 = \alpha R < S_0$. In the second case the liability payment is greater than S_0, i.e., $Z_2 = \alpha R > S_0$. In the first case the pollutee maximizes his expected profits by paying a bribe that raises the polluter's gain from a safe period to S_0. Thus, the bribe for $\alpha = Z_1 / R$ is $S_0 - \alpha R$, and safety is the same as if

[2] It is very reasonable that $\eta(Z)$ intersects $Z/(R - Z)$ only once. At A, the slope of η_A is smaller than that of $Z/(R - Z)$ which means that, by (7), $q_{ZZ}(R - S_0) - 2q_Z < 0$ or $-c_{qqq}(R - S_0) - 2c_{qq}^2 < 0$. To have another intersection

$\alpha = 0$. In the second case the polluter's price of safety is greater than S_0 and, since the marginal gain from a bribe is smaller than the marginal loss for $Z > S_0$, any bribe will reduce the pollutee's expected profit. Thus, for $\alpha = Z_2 / R$, the bribe is zero. These two results can be generalized to yield:

PROPOSITION 1: When both the polluter and pollutee are risk neutral, (a) the optimal bribe is a decreasing linear function of the polluter liability share α for $\alpha \le S_0 / R$ with intercept S_0 and slope $-R$; for $\alpha \ge S_0 / R$, optimality implies no bribe, i.e.,

$$S(\alpha) = \begin{cases} S_0 - R\alpha & 0 \le \alpha \le S_0/R \\ 0 & \alpha \ge S_0/R, \end{cases} \tag{9}$$

and (b) safety is constant at $q = q(S_0)$ for $0 \le \alpha \le S_0 / R$ and is increasing in α according to $q = q(\alpha R)$ for $S_0 / R < \alpha \le 1$.

Proposition 1 demonstrates that, for cases of uncertain pollution where bribe payments are conditioned on the actual levels of the externality (and not on the polluter activity levels), different liability rules do not necessarily yield the same safety levels. Note, however, that there is a wide range of rules (where polluter liability is relatively small, $\alpha < S_0 / R$) which yields constant safety levels. In this case, changes in polluter liability completely crowd out the bribe such that the gain from a safe period, and thus safety, is unchanged. In cases of higher polluter liability, an increase in polluter liability will increase safety.

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requires that $-c_{qqq} < -2c_{qq}^2 /(R - Z)$ for some $Z > S_0$. It is impossible to have a cost curve with $c_{qq} > 0$ with a large negative third derivative. A sufficient condition for uniqueness is that $c_{qqq} > -2c_{qq}^2 /(R - Z)$.

Figures IIa and IIb depict the results of Proposition 1 graphically. Example: Consider the case of a homogeneous cost function,

(10) $$c(q) = Aq^m, \qquad m > 1.$$

The safety function associated with *c(q)* is

(11) $$q(Z) = \left(\frac{Z}{mA}\right)^{1/(m-1)}.$$

and the isoelastic cost function has resulted in an isoelastic safety function with elasticity $(m - 1)^{-1}$. For $\alpha = 0$, the optimal bribe is

(12) $$S_0 = \frac{R}{m}.$$

By Proposition 1, the optimal bribe and safety levels are functions of the liability rules of the form:

(13) $$S(\alpha) = \begin{cases} R/m\,(1 - m\alpha) & 0 \le \alpha \le 1/m \\ 0 & \text{otherwise} \end{cases}$$

(14) $$q = \begin{cases} \left(R/m^2 A\right)^{1/(m-1)} & 0 \le \alpha < 1/\text{m} \\ \left(\alpha R/mA\right)^{1/(m-1)} & 1/\text{m} \le \alpha \le 1. \end{cases}$$

As condition (13) and (14) indicate, less convex cost functions (with smaller elasticities of safety) have larger segments of liability rules that yield constant safety levels (and positive bribe payments). However, when changes in liability rules affect safety ($\alpha > 1/m$), safety levels are more sensitive to changes in the polluter's liability share when the elasticity of the cost function is lower. Generally, cost functions with lower elasticities result in larger bribes (when bribes are paid) and larger safety levels.

IV. Pollutee Risk Aversion

As in the previous section, suppose the polluter is risk neutral, but now consider the pollutee to be risk averse with a well-behaved utility function, u, defined on profits with $u' > 0$ and $u'' < 0$. For a given liability rule and bribe, the expected utility of the pollutee is given by $q(Z) \cdot u(b\text{-}S) + [1\text{-}q(Z)] \cdot u[b\text{-}(1\text{-}\alpha)R]$. Using the definition, $Z = \alpha R + S$, the expected utility of the pollutee can be written as the following function of α and Z:

(15) $$\tilde{L}(Z,\alpha) = q(Z) \cdot u(b + \alpha R - Z) + [1 - q(Z)] \cdot u(b + \alpha R - R) .$$

The decision problem of the pollutee thus becomes

(16) $$\max_Z \quad \tilde{L}(Z,\alpha)$$

subject to $Z \geq \alpha R$. Also, the expected profit function in (15) can be rewritten as

(17) $$\tilde{L}(Z,\alpha) = q(Z) \cdot \left[u(b + \alpha R - Z) - u(b + \alpha R - R)\right] + u(b + \alpha R - R) .$$

In the context of (17), it is clear that, as in the previous case, optimizing behavior will result in $Z < R$ which means that the bribe will be smaller than the pollutee's loss when pollution occurs.

To find the behavior of the optimal bribe, differentiate $\tilde{L}(Z,\alpha)$ with respect to Z to obtain

(18) $$\tilde{L}_Z(Z,\alpha) = q_Z\left[u(b + \alpha R - Z) - u(b + \alpha R - R)\right] - q(Z) \cdot u'(b + \alpha R - Z) = 0$$

To interpret this condition, note that the marginal gain from bribing is equal to the marginal increase in safety times the difference in the utility derived in periods with and without pollution. The marginal loss from bribing is equal to the marginal reduction in the utility of income from the increased bribe times the probability of paying the bribe. Hence, (18) implies that, again, the optimal bribe equates the pollutee's marginal gain and loss from bribing.

Multiplying the differences between the marginal gain and loss from bribing by a positive value, $Z / \left\{q(Z) \cdot [u(b + \alpha R - Z) - u(b + \alpha R - R)]\right\}$, results in

(19) $$\eta(Z) - Z \cdot h(Z,\alpha)$$

where $h(Z,\alpha) = u'(b+\alpha R - Z)/[u(b+\alpha R - Z) - u(b+\alpha R - R)]$. Thus, when the marginal gain from bribing exceeds the marginal loss, the expression in (19) is positive and vice versa.

To examine the expression in (19), the elasticity of safety and $Z \cdot h(Z,\alpha)$ are depicted as functions of *Z* in Figure III; $Z \cdot h(Z,\alpha)$ is an increasing function of *Z*, which rises from zero and approaches infinity as *Z* approaches *R*. The functions $\eta(Z)$ and $Z \cdot h(Z,\alpha)$ intersect at *B* where the polluter's gain from a safe period is Z_1. Clearly, the value of Z_1 depends on α. For a given α, the marginal impact of a bribe on the pollutee's expected utility is positive for $Z < Z_1(\alpha)$ and negative for $Z > Z_1(\alpha)$. Thus, for those liability rules with a liability payment smaller than Z_1, a positive bribe will be paid to raise the polluter's price of a safe period to Z_1 [i.e., $S(\alpha) = Z_1(\alpha) - \alpha R$ if $R\alpha < Z_1(\alpha)$] while, in cases where the liability payment exceeds Z_1, no bribe will be paid [i.e., $S(\alpha) = 0$ if $\alpha R < Z_1(\alpha)$].

Recall that, when a bribe is paid by a risk-neutral pollutee, the polluter's gain from a safe period is equal to S_0 and is determined by the intersection of the elasticity of safety curve with $Z/(R-Z)$ (denoted by point A in Figure III). To compare S_0 with $Z_1(\alpha)$, it is important to compare $Z/(R-Z)$ with $Z \cdot h(Z,\alpha)$. The difference between the latter two is given by

(20) $$\begin{aligned} \frac{Z}{R-Z} - Z \cdot h(Z,\alpha) &= Z\left[\frac{1}{R-Z} - \frac{u'(b+\alpha R - Z)}{u(b+\alpha R - Z) - u(b+\alpha R - R)}\right] \\ &= \frac{Z[u(b+\alpha R - Z) - u(b+\alpha R - R) - u'(b+\alpha R - Z)\cdot(R-Z)]}{(R-Z)[u(b+\alpha R - Z) - u(b+\alpha R - R)]}. \end{aligned}$$

By risk aversion, the expression in (20) is positive implying that $Z/(R-Z)$ is greater than $Z \cdot h(Z,\alpha)$. Hence, the intersection of $Z/(R-Z)$ with the elasticity curve (point A) is to the left of the intersection of $Z \cdot h(Z,\alpha)$ with the elasticity curve (point B) and $Z_1(\alpha) > S_0$ for all α. These results indicate that, for liability rules that result in positive bribes, a risk-averse pollutee will pay a greater bribe and gain higher levels of safety than a risk-neutral pollutee.

To analyze the behavior of the risk-averse pollutee in more detail, first consider the behavior of $h(Z,\alpha)^{-1}$,

$$h(Z,\alpha)^{-1} = \frac{u(b+\alpha R - Z) - u(b+\alpha R - R)}{u'(b+\alpha R - Z)} \tag{21}$$

Approximating the denominator of (21) with the first two elements of its Taylor-series expansion yields,

$$h(Z,\alpha)^{-1} \approx (R-Z)\left[1 + R_A(b-\alpha R - Z)\frac{R-Z}{2}\right], \tag{22}$$

where $R_A(\cdot) \equiv -u''(\cdot)/u'(\cdot)$ is the Arrow-Pratt measure of absolute risk aversion.

The formula in (22) indicates that $Z \cdot h(Z,\alpha)$ is increasing in liability for a pollutee with decreasing absolute risk aversion, is constant for cases of constant absolute risk aversion, and is decreasing for cases of increasing absolute risk aversion. Applying these results to the graphical presentation of Figure III implies that, for liability rules with positive bribes, the polluter's gain from a safe period and the safety level rise (fall) with the polluter's liability share when pollutees have increasing (decreasing) absolute risk aversion. A pollutee with constant risk aversion has constant safety with respect to liability rules with positive bribes.

These results can be explained intuitively as follows. When a pollutee has decreasing absolute risk aversion, his behavior tends toward that of a risk-neutral polluter as he gets a fixed increase in income. An increase in α is such an increase; since safety under risk aversion is higher than under risk neutrality (when $S > 0$), an increase in α will reduce the safety of a pollutee with decreasing absolute risk aversion.

Results relating to the polluter's gain from a safe period can lead to the necessary additional results relating to the behavior of the bribe for various liability rules. For a pollutee with decreasing absolute risk aversion, an increase in the polluter's liability results in a reduction in the bribe for two reasons. First, it causes a reduction in the polluter's gain from a safe period; second, it increases the liability payment. The bribe level is the difference between the polluter's gain from a safe period and the liability payment. In the case of constant absolute risk aversion, the bribe is decreasing for the second reason only. In the case of increasing absolute risk aversion, an increase in the polluter's liability share raises the polluter's gain from a safe period and, hence, leads to an increase in the bribe; but the increase in the liability payment itself causes a reduction in the bribe. Hence, it is not clear that the bribe is always decreasing with an increase in the polluter's liability share. However, this is the probable result in most cases since the bribe is positive in the case of complete pollutee liability and is zero for complete polluter liability (and possibly for other cases with high polluter liability).

The results of this section are summarized by:

PROPOSITION 2: Suppose that the polluter is risk neutral. Then (a) a larger bribe is paid with a risk-averse pollutee than with a risk-neutral pollutee. For a pollutee with decreasing absolute risk aversion, the bribe function is decreasing with the polluter's liability share (to the point of no bribe), and the slope is smaller than -*R*. For constant absolute risk aversion, the bribe function is linear with slope -*R*; and for increasing absolute risk aversion, the slope of the bribe function is larger than -*R*, i.e.,

$$(23) \qquad \frac{dS}{d\alpha}\begin{cases} < -R & \\ = -R & \text{when} \\ > -R & \end{cases} \begin{matrix} \mathrm{R}'_{\mathrm{A}} < 0 \\ \mathrm{R}'_{\mathrm{A}} = 0 \\ \mathrm{R}'_{\mathrm{A}} > 0. \end{matrix}$$

(b) When the bribe is paid, more safety is attained under pollutee risk aversion than with a risk-neutral pollutee; the same safety is attained when no bribe is paid. In cases of a positive bribe, optimal safety is decreasing with the polluter's liability share for pollutees with decreasing absolute risk aversion. Safety is constant under constant absolute risk aversion and is increasing for increasing absolute risk aversion, i.e., for $S > 0$:

(24)
$$\frac{dq}{d\alpha}\begin{cases} < 0 & \\ = 0 & \text{when} \\ > 0 & \end{cases} \begin{matrix} R'_A < 0 \\ R'_A = 0 \\ R'_A > 0. \end{matrix}$$

The results of Proposition 2 are depicted in Figures IV and V.

V. The Case of a Risk-Averse Polluter

The case analyzed in this section is the antipode of the case in the previous section. Here the pollutee is assumed to be risk neutral, while the polluter is assumed to be risk averse. Since the pollutee is risk neutral, the analysis in Section 3 indicates that a positive bribe is paid if the elasticity of the safety curve intersects $Z/(R-Z)$ at a level of Z larger than the polluter's liability payment. The bribe is the difference between the polluter gain from a safe period ($Z = \alpha R + S$) and his liability payment. No bribe is paid when the intersection is at a level of Z where $Z < \alpha R$. The safety level is determined by the safety function of the risk-averse polluter, given the polluter's gain from a safe period and his liability share.

Thus, to understand the outcome for a risk-averse polluter, the properties of his safety function must be analyzed. Suppose the polluter has a twice differentiable utility function $u(\pi)$ defined on profits (quasi rents) with $u' > 0$ and $u'' < 0$. Profit during a safe period is π_1; and during a period with pollution, profit is π_2 where

(25) $$\begin{aligned}\pi_1 &= a - c(q) + S \\ \pi_2 &= a - c(q) - \alpha R.\end{aligned}$$

Introducing the definition of *Z* into (25) yields

(26) $$\begin{aligned}\pi_1 &= a - \alpha R - c(q) + Z \\ \pi_2 &= a - \alpha R - c(q).\end{aligned}$$

The decision problem of the polluter thus becomes

(27) $$\max_q \quad E(u) = q \cdot u(1) + (1-q) \cdot u(2)$$

where $u(i) = u(\pi_i), \quad i = 1,2$.

Suppose that a solution exists satisfying the first-order condition for optimality,

(28) $$c_q = \frac{u(1) - u(2)}{Eu'}.$$

The second-order condition for optimality is

(29) $$M = -2c_q\left[u'(1) - u'(2)\right] - E(u')c_{qq} + c_q^2 E(u'') < 0.$$

The optimality conditions (28) and (29) imply that the safety level attained by a risk-averse polluter does not depend only on the sum of the bribe and liability payment (polluter's gain from a safe period) but also on its composition since the marginal utility of income is different depending on the occurrence of pollution. The safety function for risk aversion will be denoted by $\tilde{q}(Z,\alpha)$. It is interesting to compare $\tilde{q}$ with *q(Z)*, the safety function under risk neutrality.

Condition (28) indicates that, under risk aversion, the optimal safety level for a given (Z, α) combination is determined by equating the marginal cost of safety and the polluter's marginal benefit from safety (in monetary terms). The marginal benefit from safety is equal to the increase in utility in an unpolluted period (over a polluted period) translated into monetary terms through division by the average marginal utility of income.

If for a given (α, Z) combination the marginal benefit from safety (of a risk-averse polluter) is higher than the polluter's gain from a safe period, it follows from (2) and (28) that the safety level attained by a risk-averse polluter is higher than the safety level attained by a risk-neutral polluter (since marginal cost is increasing). Under risk aversion, the sign of

$$\text{(30)} \qquad \frac{u(1)-u(2)}{q\cdot u'(1)+(1-q)\cdot u'(2)}-Z=\frac{1}{Eu'}\left[u(1)-u(2)-q\cdot u'(1)\cdot Z-(1-q)\cdot u'(2)\cdot Z\right]$$

is negative when q is near zero and is positive when q is near one. Thus, one concludes:

LEMMA 1: For a given combination of liability payments and bribes, risk neutrality leads to lower safety than risk aversion in cases where safety is sufficiently high under risk aversion, but risk neutrality leads to greater safety in problems where safety is sufficiently low under risk aversion, i.e.,

$$\text{(31)} \qquad \begin{array}{lll} q(Z)>\tilde{q}(Z,\alpha) & \text{if} & \tilde{q}(Z,\alpha)\rightarrow 0 \\ q(Z)<\tilde{q}(Z,\alpha) & \text{if} & \tilde{q}(Z,\alpha)\rightarrow 1. \end{array}$$

The behavior described in Lemma 1 can be explained by the fact that a risk-averse firm gives up expected profits to reduce financial uncertainty. Since complete certainty occurs only when q is zero or one, uncertainty is reduced by adjustment of q toward either zero or one. Thus, for a given policy, optimal safety under risk aversion is below (above) optimal safety under risk neutrality in problems with low (high) safety.

To understand the behavior of the safety function under risk aversion, differentiate (28) with respect to q, α, and Z to find

$$\text{(32)} \qquad \tilde{q}_Z=\frac{u''(1)\cdot c_q\cdot q-u'(1)}{M}$$

and

$$\tilde{q}_{\alpha} = R \cdot \frac{u'(1) - u'(2) - c_q \cdot E(u'')}{M}. \tag{33}$$

From (32), safety is increasing with the polluter's gain from safe periods; hence, the elasticity of safety with respect to *Z* is always positive.

The effects of a change in the liability rule on the safety function are not conclusive. However, introducing (28) in (33) yields

$$\tilde{q}_{\alpha} = R \cdot \frac{u(1) - u(2)}{M} \left[\frac{u'(1) - u'(2)}{u(1) - u(2)} \cdot \frac{E(u'')}{E(u')} \right]. \tag{34}$$

When absolute risk aversion is decreasing, $R'_A < 0$ (in which case $u''' > 0$), then

$$\begin{aligned} & u''(1) \cdot Z > u'(1) - u'(2) > u''(2) \cdot Z \\ & u'(1) \cdot Z < u(1) - u(2) < u'(2) \cdot Z \end{aligned} \tag{35}$$

for $Z > 0$ and $0 \le q \le 1$ which implies

$$\frac{u''(1)}{u'(1)} > \frac{u'(1) - u'(2)}{u(1) - u(2)} > \frac{u''(2)}{u'(2)}. \tag{36}$$

It is easy to verify from (34) and (36) that, for a polluter with decreasing absolute risk aversion, the safety function is increasing in liability (for a given *Z*) when safety is high [$\tilde{q}(Z,\alpha) \to 1$] and decreasing in liability when safety is low [$\tilde{q}(Z,\alpha) \to 0$].[3] This result follows logically from the lemma. Since risk aversion results in more (less) safety than risk neutrality when $q \to 1$ ($q \to 0$) and an increase in *a* holding *Z* unchanged reduces the fixed profit of the polluter, the increase in *a* will increase the risk aversion of a polluter with decreasing absolute risk aversion; thus,

$$\begin{aligned} & \tilde{q}_{\alpha} > 0 \quad \text{if} \quad q \to 1 \\ & \tilde{q}_{\alpha} < 0 \quad \text{if} \quad q \to 0. \end{aligned} \tag{37}$$

[3] The properties of such a safety function are discussed in greater detail in Just and Zilberman.

A heuristic graphical description of the behavior of the safety functions under risk neutrality and risk aversion (with $\alpha = 1$) is presented in Figure VI. There is a critical level of the polluter's gain from a safe period, say, Z^+ at which the two safety functions intersect. When the polluter's gain from a safe period is below Z^+, the safety function under risk neutrality is higher; whereas, when the polluter's gain from a safe period is above Z^+, the safety function under risk aversion is higher. For a polluter with decreasing absolute risk aversion, the safety function for $0 \le \alpha < 1$ is between the safety function for $\alpha = 1$ and that of a risk-neutral polluter (not shown).

To determine the bribe payment, the (risk-neutral) pollutee uses the safety function $\tilde{q}(\alpha, Z)$ of the risk-averse polluter. The final outcome can be derived using the analysis of Section 3 (where both polluter and pollutee are risk neutral) and by replacing the safety function *q(Z)* with $\tilde{q}(\alpha, Z)$. The level of bribe paid for any liability rule is determined by the intersection of the elasticity of safety function [$\tilde{\eta}(Z,\alpha) = \tilde{q}_Z(Z,\alpha) \cdot Z / \tilde{q}(Z,\alpha)$] with $Z/(R-Z)$. If at the intersection point the polluter's gain from a safe period is lower than the polluter's liability payment ($Z < \alpha R$), no bribe will be paid. Otherwise, the bribe is equal to the difference between the two ($S = Z - \alpha R$).

Using the same arguments as in Section 3, it can be easily verified that a positive bribe is paid under complete pollutee liability ($\alpha = 0$), and no bribe is paid under complete polluter liability ($\alpha = 1$). Moreover, bribes apparently are paid in cases where the polluters' liability shares are relatively low, whereas no bribes will be paid when the polluters' liability shares are relatively high.

Unlike the case of a risk-neutral polluter, liability rules with positive bribes do not yield the same safety level when the polluter is risk averse. The reason for the difference is that, for a risk-averse polluter, different liability rules result in different safety functions. The relative complexity of the safety function under risk aversion prevents obtaining further results for liability rules with bribe payments. However, the properties of the safety function for risk-averse polluters derived previously are useful for analysis of the outcomes for liability rules for which no bribes are paid. For these cases, the safety obtained by a risk-averse polluter is equal to $\tilde{q}(\alpha R, \alpha)$, and the safety for a risk-neutral polluter is $q(\alpha R)$. Using Lemma 1, one can thus deduce:

PROPOSITION 3: For liability rules with no bribe payments, risk aversion on the part of the polluter leads to lower safety than risk neutrality if safety under risk aversion is sufficiently high; risk neutrality leads to higher safety when the safety under risk aversion is sufficiently low.

VI. Conclusions and Welfare Implications

If both a polluter and pollutee belong to an economy possessing a well functioning price system (prices reflect social valuations) with the exception of the externality between the two, then efficiency can be attained if joint expected profits of the polluter and the pollutee can be maximized without imposing risk on any risk averter. The optimality problem of this paper is thus

(38) $$\max_q \quad a - c(q) + b - (1-q) \cdot R .$$

Assuming that an internal solution exists, the first-order condition which holds at the optimal safety level q^* is

(39) $$c_q(q^*) = R .$$

Efficiency implies that the marginal cost of safety should be equal to the (marginal) damage of pollution. Using the safety function for a risk-neutral polluter, efficiency is thus attained if the safety level is $q^* = q(R)$ and no risk is carried by a risk-averse agent.

This result and Propositions 1 and 2 indicate that, when the polluter is risk neutral, only complete polluter liability always results in efficient allocation when bribes are based on actual damage. Furthermore, when the polluter is risk averse, even complete polluter liability does not yield efficiency. That is, the safety level obtained by a risk-averse polluter under complete polluter liability is not necessarily equal to the efficient safety level. When safety is low, a risk-averse polluter tends to reduce safety beyond the efficient safety level (even with full liability); thus, none of the other liability rules will obtain the efficient safety level. When safety is high, a risk-averse polluter tends to increase safety above the efficient safety level. In this case, the efficient safety level might be attained by some partial liability rules, but even then the outcome is not efficient since a risk-averse agent--the polluter--is carrying risk. Thus, when the polluter is risk averse, it is impossible to obtain efficiency by assignment of a liability rule. In this case, optimality could be achieved if contracts could be struck on verifiable polluter activities, which would shift risk bearing from the polluter to the pollutee. It should also be noted that if fair insurance were provided as a means of transferring risk from the polluter, then efficiency would be possible even if the polluter were liable for stochastic damages.[4]

The convexity of the cost function implies that (excluding risk-bearing costs) outcomes with safety levels which are closer to $q*$ are more efficient. Proposition 1 indicates that, despite the superiority of full polluter liability, there is a range of liability rules ($\alpha < 1/m$) where increases in polluter liability will not increase safety but will have only equity effects if both parties are risk neutral. When the pollutee is risk averse, the effect of increases in polluter liability on safety will depend on the relationship between the degree of risk aversion and pollutee wealth. If the polluter is risk averse, an increase in polluter liability may result in outcomes that are smaller and farther from the optimal level, $q*$.

[4] A more detailed discussion of insurance and efficient Coasian risk sharing is contained in Graff Zivin and Small.

Most environmental pollution problems occur in a stochastic environment where firm activities (e.g. prevention and abatement activities) are not easily monitored and agents are often risk averse. Agent risk aversion will depend, in part, on firm size and access to financial and insurance markets that help the firm diversify and hedge risk. If large, well connected firms are viewed as (nearly) risk neutral and small firms with limited access to financial management tools are viewed as risk averse, then the results of this paper can easily be translated into practical policy suggestions. When both polluter and pollutee are large, liability rules are of little consequence – agents will bargain to efficient outcomes. When the polluter is large and the pollutee is small, polluters should face complete liability for damages. When the polluter is small and the pollutee is large, all liability rules will be non-optimal and efforts to contract on verifiable polluter activities, rather than realized damages, will be preferred. Interestingly, these policy prescriptions appear consistent with some current forms of regulation. For example, large corporations are generally held liable for damages to the environment, while small producers, like farmers or dry cleaners, are generally subject to regulations on input use.

The analysis presented here is short run in nature. The pollution damage is assumed to be unaffected by the pollutee's actions. Future work should examine possible (moral hazard) increases in the pollutee's output resulting in increased pollution damage when the polluter is fully liable. In such a world, the general lessons from our analysis should remain the same; only Pareto optimality will never be achievable. The potential for moral hazard will necessitate some risk bearing by the pollutee, much like the potential for moral hazard in insurance markets forces insurers to introduce deductibles and co-payments such that plans only partially cover insured risks. Note that if the pollutee behavior were observable and contractible, optimality might still be achieved through a combination of a pollutee negligence standard with residual polluter liability. Additional work might also consider a continuum of pollution levels rather than an all or none process. Nevertheless, our results point to some necessary conditions in any stochastic pollution problem where continuous monitoring of safety precautions, is impractical.

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Figure I: Optimal Safety Under Risk Neutrality

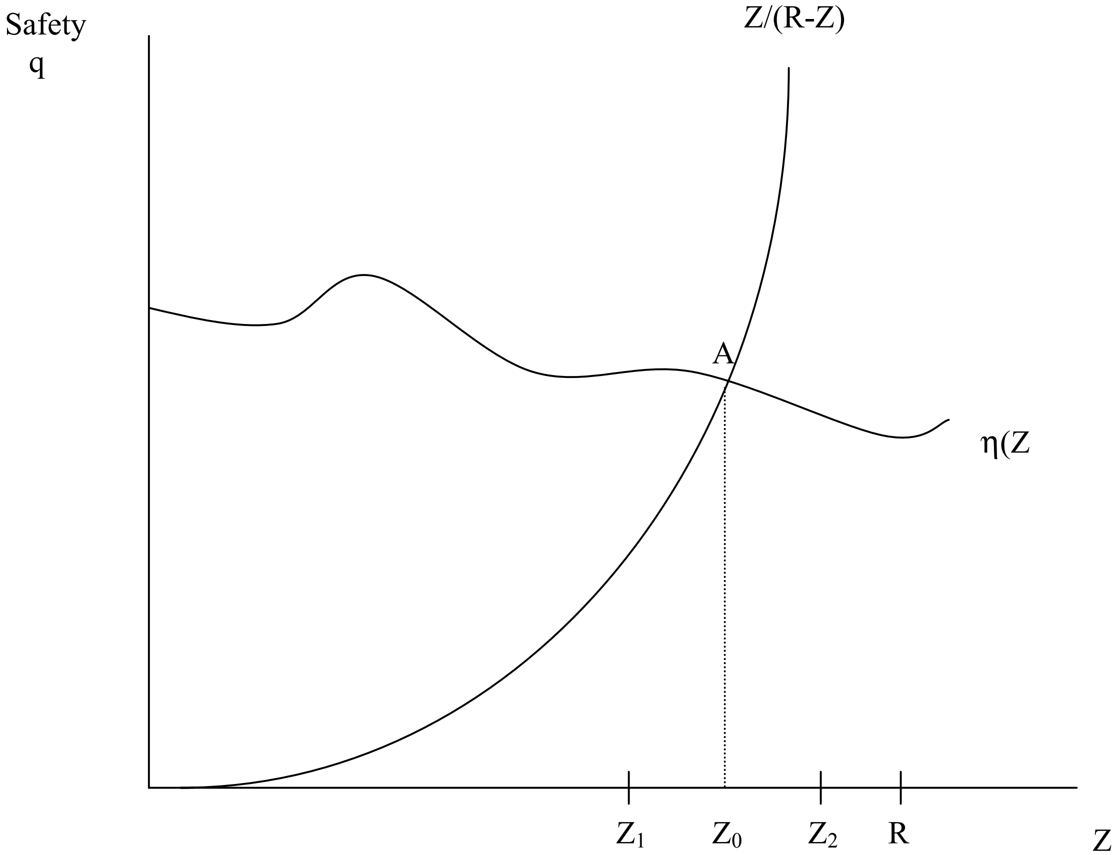

Figure IIa: Bribe as a Function of the Polluter Liability Share Under Risk Neutrality

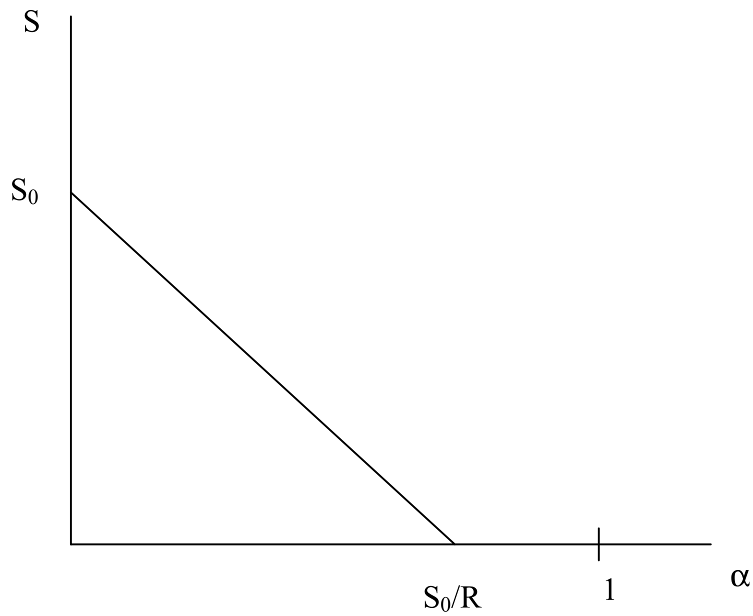

Figure IIb: Safety as a Function of the Polluter Liability Share Under Risk Neutrality

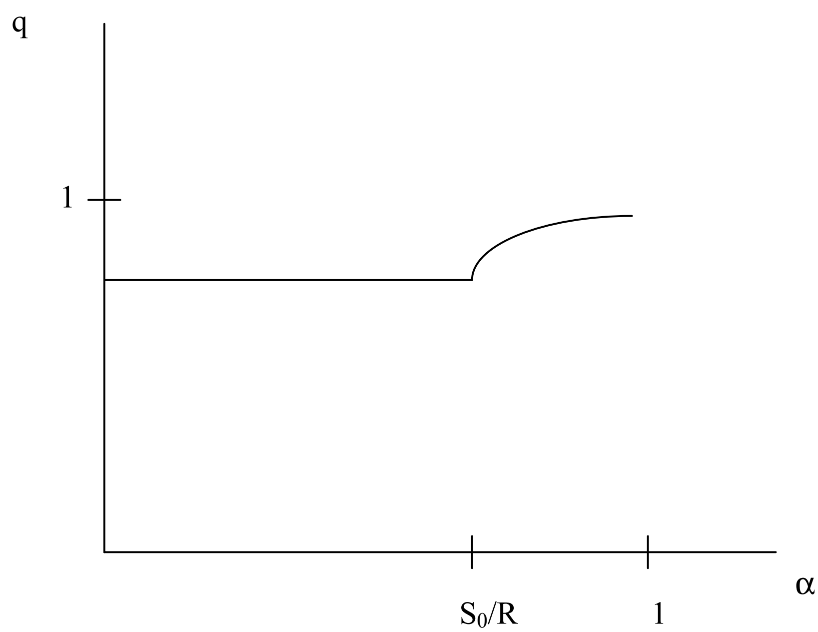

Figure III: Optimal Safety Under Pollutee Risk Aversion

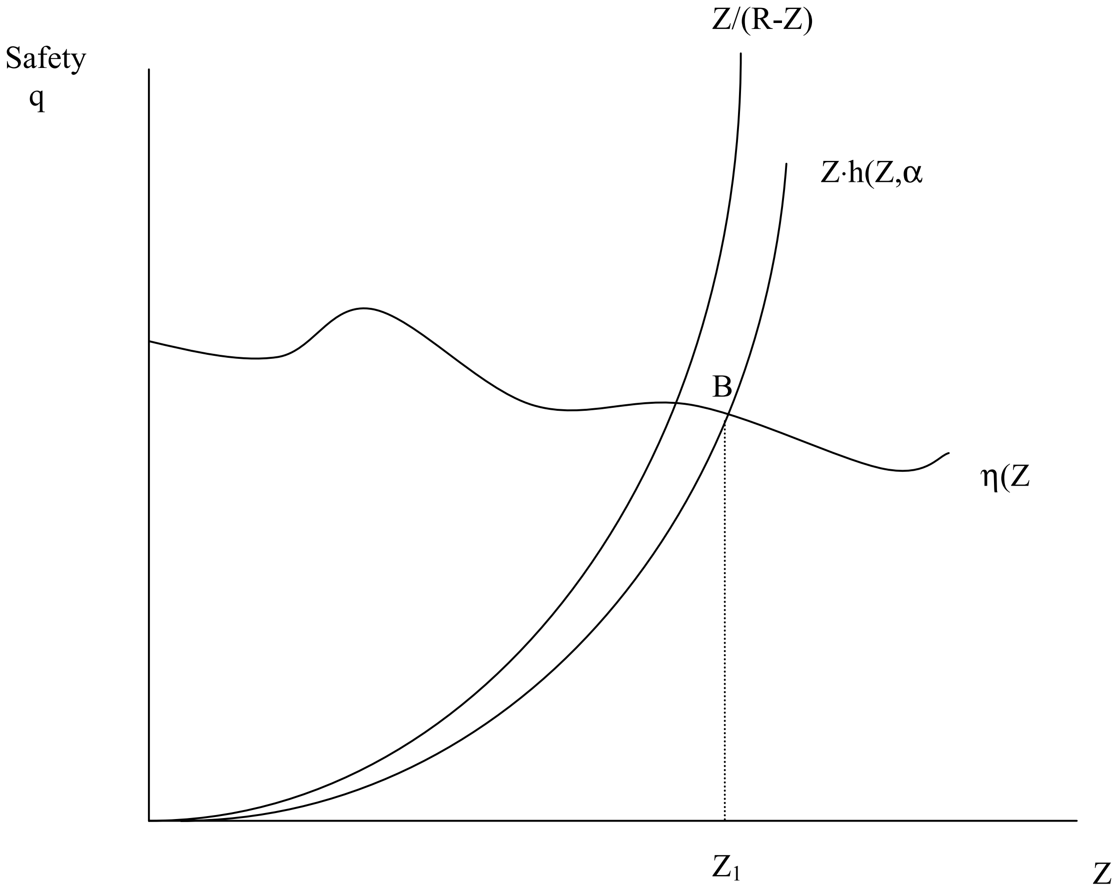

Figure IV: Bribe as a Function of the Polluter's Liability Share When the Pollutee is Risk Averse

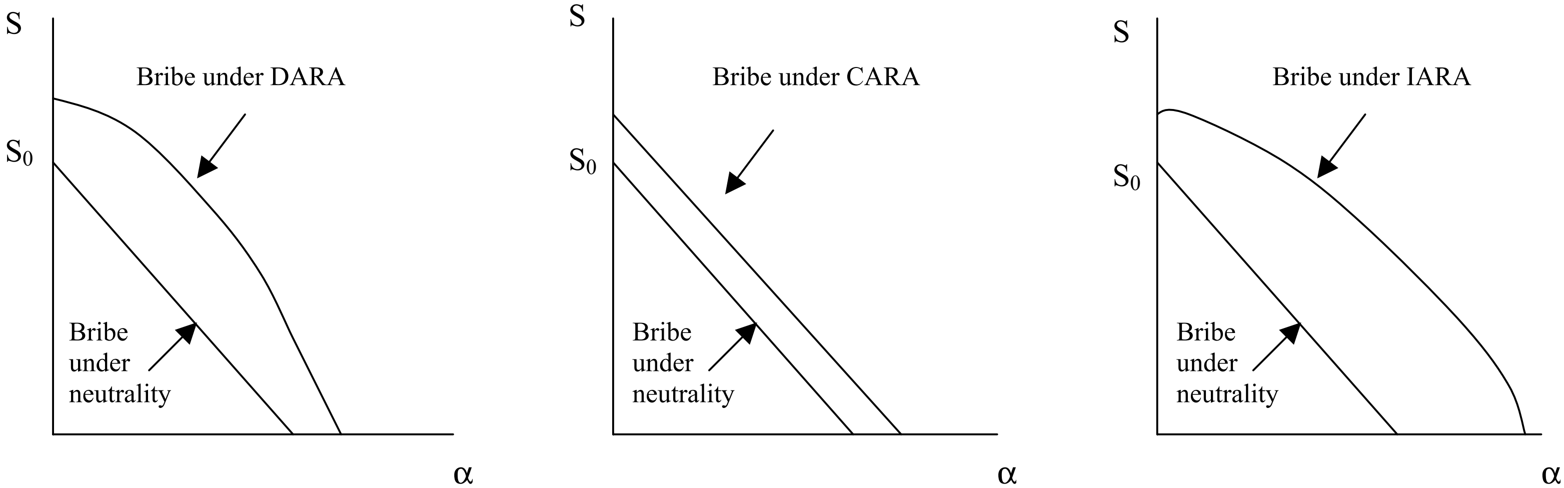

Figure IVa Figure IVb Figure IVc

Figure V: Safety as a Function of the Polluter's Liability Share When the Pollutee is Risk Averse

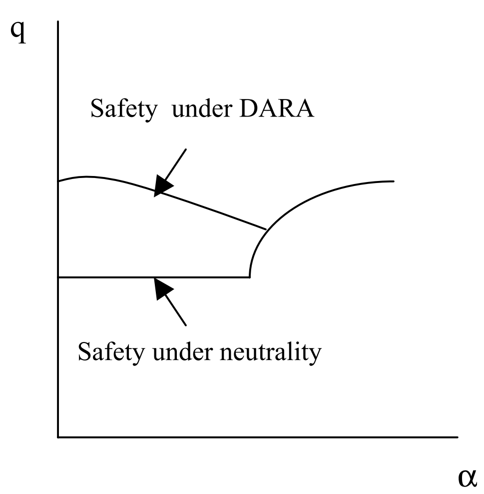

Figure Va

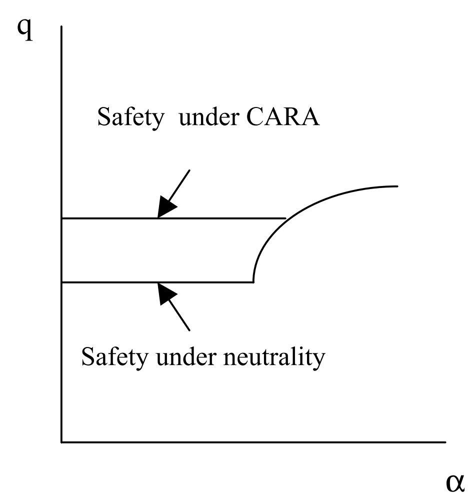

Figure Vb

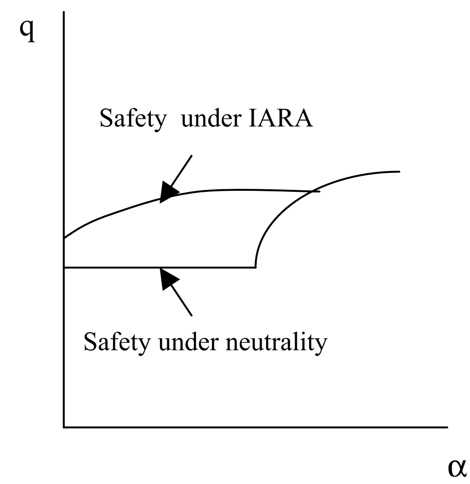

Figure Vc

Figure VI: Safety Under Risk Neutrality and
Under Polluter Risk Aversion