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FRICTIONLESS TECHNOLOGY DIFFUSION: THE CASE OF TRACTORS

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Abstract

Empirical evidence suggests that there is a long lag between the time a new technology is introduced and the time at which it is widely adopted. The conventional wisdom is that these observations are inconsistent with the predictions of the frictionless neoclassical model. In this paper we show this to be incorrect. Once the appropriate driving forces are taken into account, the neoclassical model can account for 'slow' adoption. We illustrate this by developing an industry model to study the equilibrium rate of diffusion of tractors in the U.S. between 1910 and 1960.

1 Introduction

Understanding the determinants of the rate at which new technologies are created and adopted is a critical element in the analysis of growth. Even though modeling equilibrium technology creation can be somewhat challenging for standard economic theory, understanding technology adoption should not be. Specifically, once the technology is available, *the adoption decision is equivalent to picking a point on the appropriate isoquant*. Dynamic considerations make this

*University of Wisconsin-Madison & NBER and University of Wisconsin-Madison, respectively. We would like to thank NSF for financial support through our respective grants. We are indebted to William White who generously provided us with a database containing tractor production, technical characteristics and sale prices, and to Paul Rhode who shared with us his data on prices of average tractors and draft horses. We thank Hal Cole, Jeremy Greenwood and seminar participants at UCLA, USC - Marshall School of Business and University of Rochester for their very helpful comments. Naveen Singhal provided excellent research assistance.

calculation more complicated, but they still leave it in the realm of the neoclassical model. A simple minded application of the theory of the firm suggests that profitable innovations should be adopted instantaneously, or with some delay depending on various forms of cost of adjustment.

The evidence on adoption of new technologies seems to contradict this prediction. One of the 'stylized facts' in this literature is that the adoption rate is S-shaped and that it takes a long time until a large fraction of units adopts the new technology. Several studies — e.g. Griliches (1957), Gort and Klepper (1982) and Jovanovic and Lach (1997), among others — have documented the logistic shape of the diffusion curve. Jovanovic and Lach (1997) report that, for a group of 21 innovations, it takes 15 years for its diffusion to go from 10% to 90%, the 10-90 lag. They also cite the results of a study by Grübler (1991) covering 265 innovations who finds that, for most diffusion processes, the 10-90 lag is between 15 and 30 years.[1]

In response to this apparent failure of the simple neoclassical model, a large number of papers have introduced 'frictions' to account for the 'slow' adoption rate. These frictions include, among others, learning-by-doing (e.g. Jovanovic and Lach (1989), Jovanovic and Nyarko (1996), Greenwood and Yorukoglu (1997), Felli and Ortalo-Magné (1997), and Atkeson and Kehoe (2001)), vintage human capital (e.g. Chari and Hopenhayn (1994) and Greenwood and Yorukoglu (1997)), informational barriers and spillovers across firms (e.g. Jovanovic and Macdonald (1994)), resistance on the part of sectoral interests (e.g. Parente and Prescott (1994)), coordination problems (e.g. Shleifer (1986)) and search-type frictions (e.g. Manuelli (2002)).

In this paper we take, in some sense, one step back and revisit the implications of the neoclassical frictionless model for the equilibrium rate of diffusion of a new technology. The application that we consider throughout is another famous case of 'slow' adoption: the farm tractor in American agriculture.[2] Following Lucas (1978), we study an industry model in which managers (farm

[1] There are studies of specific technologies that also support the idea of long lags. Greenwood (1997) reports that the 10-90 lag is 54 years for steam locomotives and 25 years for diesels, Rose and Joskow's (1990) evidence suggest a 10-90 lag of over 25 years for coal-fired steam-electric high preasure (2400 psi) generating units, while Oster's (1982) data show that the 10-90 lag exceeds 20 years for basic oxygen furnaces in steel production. However, not all studies find long lags; using Griliches (1957) estimates, the 10-90 lag ranges from 4 to 12 years for hybrid corn.

[2] Using the fraction of farms that operate a tractor as our measure of diffusion, the 10-90 lag is at least 35 years in the case of tractors.

operators) differ in terms of their skills. We assume that the technology displays constant returns to scale in all factors, including managerial talent. In order to ignore frictions associated with indivisible inputs, we study the case in which there are perfect rental markets for all inputs.[3] Each farm operator maximizes profits choosing the mix of inputs. Given our market structure, this is a static problem. In addition, each manager has to make a discrete location choice: stay and continue farming, or migrate to an urban area and earn urban wages. We assume that migration is costly and, in fact, the cost of migration is the only non-convexity in our setting. The migration decision is dynamic. We take prices and the quality of all inputs as exogenous and we let the model determine the price of one input, land, so as to guarantee that demand equals the available stock.

Our model has three features that influence the diffusion rate: exogenous changes in the price of inputs *other* than the new technology, exogenous changes in the *quality* of the technology, and endogenous *selection* of firms (farm managers) out of the industry. We show that when these three factors are taken into account, the model is very successful at predicting the pattern of diffusion of tractors.

Our work emphasizes that farmers had a choice between a 'new' technology (tractors) and an 'old' technology (horses). However, a simple computation based on prices of tractors and horses cannot explain the observed pattern of diffusion. We study the impact on adoption of the dramatic *decrease in horse prices* and find that it only had a marginal impact on tractor adoption. In the model labor is an input that must be combined with tractors and horses to produce 'traction services'. We find that *changes in wage rates* play a critical role in explaining why the adoption of the tractor was delayed until the 1940s: Only at this time did real wages increase substantially, and this made the horse-technology less attractive.

In addition to the direct effect, the change in real wages has an indirect effect: it affects *migration decisions*. In equilibrium, wage increases induce marginal farm operators to leave the agricultural sector. Equilibrium migration is such that the distribution of skills of the remaining farmers improves over time, and this change also results in higher levels of adoption of tractors.

[3] This, effectively, eliminates the indivisibility at the individual level. Given the scale of the industry that we study, indivisibilities at the aggregate level are not relevant.

Finally, we estimate the *change in the 'quality'* of a tractor using standard hedonic techniques. We find that, even though the amount of 'tractor services' per tractor grew rapidly in the 1920s, the increase was not large enough to induce widespread adoption. Our estimates of tractor quality show a substantial increase in the post World War II period, and this coincides with the era of rapid adoption.

We choose the parameters of the model so that it reproduces several features of the U.S. agricultural sector in 1910. We then use the calibrated model, driven by exogenous changes in prices, to predict the number the tractors (and other variables) for the entire 1910-1960 period. The model is very successful at accounting for the diffusion of the tractor and the demise of the horse. The correlation coefficient between the model's predictions and the data is 0.99 for tractors and 0.98 for horses. We conclude that there is no tension between a frictionless neoclassical model and the rate at which tractors diffused in U.S. agriculture: the reason why diffusion was 'slow' is because it was not cost effective to use tractors more intensively.

In order to ascertain what are the essential features of the model that account for such a good fit, we study several counterfactuals. We analyze versions of the model that keep wages, horse prices and tractor quality fixed at their 1910 levels, and another version that ignores selection of firms (farmers) out of the industry. These alternative specifications fail to match the data in several important dimensions.

The paper is organized as follows. In section 2 we present a brief historical account of the diffusion of the tractor and of the price and quality variables that are the driving forces in our model. Section 3 describes the model, and Section 4 discusses calibration. Sections 5 and 6 present our results, and section 7 offers some concluding comments.

2 Some History

This section presents some evidence the use of tractors and horses by U.S. farmers, on the behavior of wages and employment in the U.S. agricultural sector, and on the changes in the size distribution of farms.

Diffusion of the Tractor. The diffusion of the tractor was not unlike most other technologies, it had a characteristic S-shape. Figure 1 plots the number of tractors on American farms. Diffusion was slow initially. The pace of adoption speeds up after 1940.

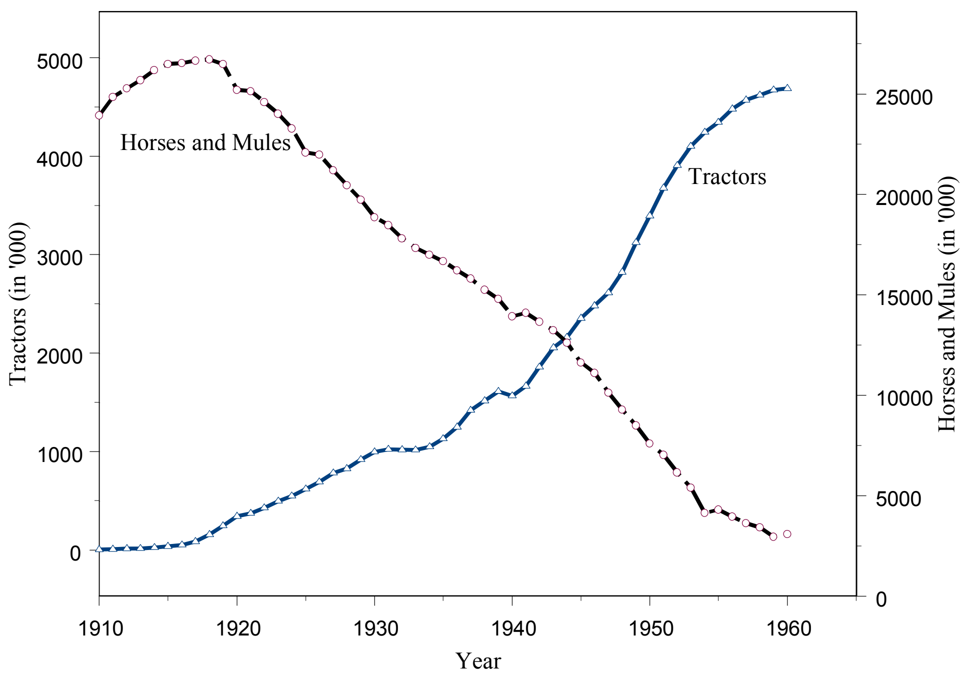

Figure 1: Horses, Mules and Tractors in Farms, 1910-1960

As the tractor made its way into the farms, the stock of horses began to decline. In 1920, there were more than 26 million horses and mules on farms. Thereafter, this stock began to decline and by 1960, there were just about 3 million. While the tractor was primarily responsible for this decline, it should be kept in mind that the automobile was also instrumental in the elimination of the horse technology. As in the case of other technologies, investment in horses —the 'dominated' technology— was positive, even as the stock declined.

Real Prices for Tractors and Horses. Figure 2 plots the real price of a mid-size tractor and a pair of draft horses between 1910 and 1960.[4] Between 1910 and 1920, there is a sharp decline in the price of tractors which is partially reversed in the 1930s. In the 1940s and 1950s prices are lower and comparable to those prevailing in 1920s. One simple conclusion from this evidence is that farmers should have adopted tractors in 1920 at the same rate they did in the 1940s and 1950s. They did not. This observation lies behind the idea that adoption was slow. White (2000) collected data on some tractor characteristics and estimated a quality adjusted price for a tractor. The resulting series —labeled Tractor-quality adjusted in Figure 2— shows a steep decline until the mid-1920s, but very small changes after this. Thus, changes in the price of the technology —in the absence of frictions— appear to be insufficient to explain the pattern of adoption documented in Figure 1.

Farm Real Wage Rates and Employment. Real wages in the agricultural sector remained stagnant from 1910 till about 1930, fell by half between 1930 and 1934, and then doubled between 1940 and 1950. If the tractor was labor-saving, rising wages after 1940 would likely speed up adoption since holding on to the more labor intensive horse technology will be unprofitable. The falling wages in the early thirties might also go towards explaining the reluctance of farmers to switch to tractors during the same time period.

Man-hours on farms remained fairly constant between 1910 and 1930, and then fell by 76% between 1930 and 1970. Farm population decreased dramatically too. In 1910, there were 11.67 million farm workers (full-time equivalents). By 1960, this number had fallen to 5.97 million.

Distribution of Land-Holding Patterns. The distribution of land-holding patterns underwent a significant change between the years 1910 and 1960. The average size of a farm more than doubled, and land in large-sized farms (size above 1000 acres) tripled between 1920 and 1960. Land in mid-size farms (500-999 acres in size) also increased, though the increase was far less spectacular than its larger counterparts. As expected, land in smaller farms of size less than 500 acres decreased, with most of the decline after 1940.

Alternative Explanations. The standard approach to studying the diffusion of the tractor is based on the 'threshold' model originally introduced by David (1966). In its simplest form, the

[4] The data are from Olmstead and Rhode (2001). We thank Paul Rhode for providing the data.

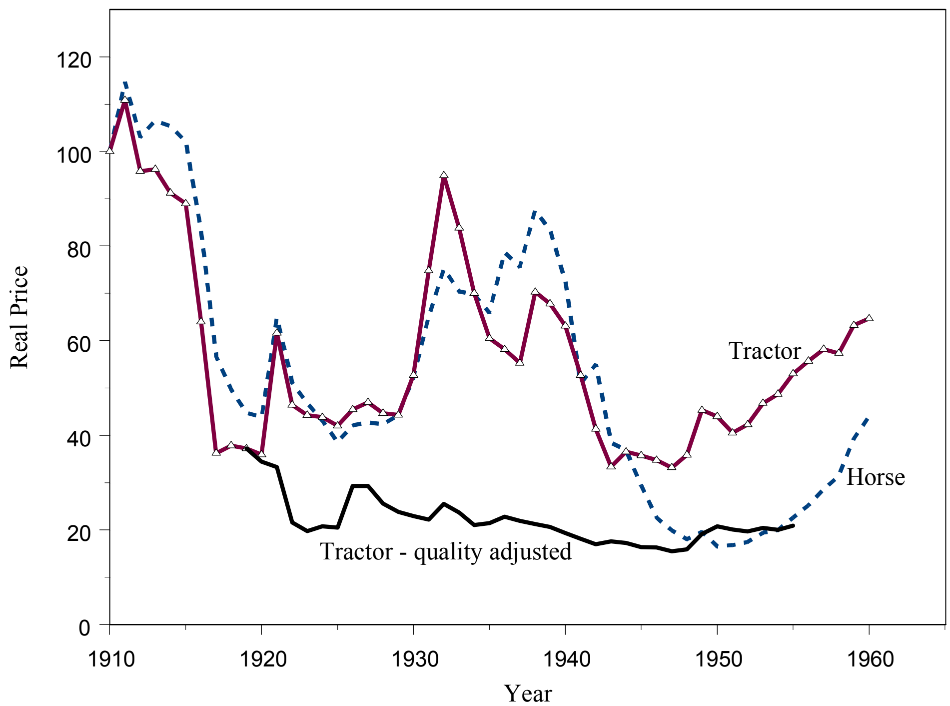

Figure 2: Real Prices for Tractors and Horses. 1910-1960

model takes as given the size of the farm (in acres) and considers the costs of different combinations of horse-drawn and tractor-drawn technologies required to produce a given amount of services. By choosing the cost minimizing technology, the model selects the type (size) of farm that should adopt a tractor. The predictions of the model — given the size distribution of farms — are then compared with the data. These calculations find that in the 1920's and early 1930's, U.S. farmers were too slow to adopt the relatively new tractor technology. Allowing for imperfect capital markets (Clarke (1991)) or introducing uncertainty about the value of output (Lew (2001)) help improve the fit of the model, but not to the point where it is consistent with the evidence.

More recently Olmstead and Rhode (2001) estimate that changes in the price of horses and in the size distribution delayed, to some extent, the adoption of tractors. In their model the size distribution is exogenous. White (2000) emphasizes the role of prices and quality of tractors. Using a hedonic regression, he computes a quality-adjusted price series for tractors. White conjectures that the increase in tractor quality should be taken into account to understand adoption decisions.

3 A Simple Model of Farming and Migration

Our approach is to model technology adoption using a standard profit maximization argument, supplemented by a simple model of migration-choice along the lines of Becker (1964) and Sjaastad (1962). We consider a setting in which farm operators are heterogeneous. Each individual has a level of 'farm organizational ability' or 'skill' denoted by e. The distribution of skills in the population of *potential* farmers is given, and denoted by μ. However, the distribution of skills among *actual* farmers is endogenously determined by the model. In each period, a farmer can either stay (and farm) or migrate to an urban area. To simplify the analysis, we assume that the migration decision is irreversible: once a farmer leaves the rural sector, he cannot return to farming.[5]

If the farmer decides to stay, and operate the farm, he needs to decide how many tractors, horses, acres of land and labor to *rent* in spot markets. We consider the case in which there are *perfect* markets for all inputs. Thus, as is standard in the theory of the firm, indivisibilities at the individual farm level are irrelevant.[6] This implies that our model can be used to predict the total number of tractors but not their distribution across farms.

Each farmer maximizes the present discounted value of utility taking prices as given. If the individual is in the farm sector, he chooses, in every period, the quantity of tractor, horse, land and labor *services* in order to produce agricultural output. The one period profit of a farmer with

[5] Given the relevant values of the cost of migration and the potential gains of reverse migration, we will argue later that this is not as extreme an assumption as it sounds.

[6] Olmstead and Rhode (2001) provide evidence of the prevalence of contract work, i.e. of instances in which a farmer provides 'tractor services' to other farms.

managerial skill level e is given by,

$$\pi_t(q_t, c_t, w_t^F, e) \equiv \max_{k_t, h_t, n_t, a_t} p_{ct} F(k_t, h_t, n_t, a_t, e) - \sum_{\tau=-\infty}^{t} [q_{kt}(\tau) + c_{kt}(\tau)] m_{kt}(\tau) - [q_{ht} + c_{ht}] h_t - w_t^F \bar{n}_t - [q_{at} + c_{at}] a_t,$$

where $F(k_t, h_t, n_t, a_t, e)$ is a standard production function which we assume to be homogeneous of degree one in all inputs, including managerial skill, e,[7] k_t is the demand for *tractor services*, h_t is the demand for *horse services* (which we assume proportional to the stock of horses), $n_t = (n_{ht}, n_{kt}, n_{yt})$ is a vector of labor services corresponding to three potential uses: operating horses, n_{ht}, operating tractors, n_{kt}, or other farms tasks, n_{yt}, and a_t is the demand for land services (which we assume proportional to acreage), and $\bar{n}_t = n_{ht} + n_{kt} + n_{yt}$ is the total demand for labor.

On the cost side, $q_{ht} + c_{ht}$ is the full cost of operating a draft of horses. The term q_{ht} is the rental price of a horse, and c_{ht} includes operating costs (e.g. feed and veterinary services). The term $q_{at} + c_{at}$ is the full cost of using one acre of land, and w_t^F is the cost of one unit of (farm) labor. Effective one period rental prices for horses and land (two durable goods) are given by

$$q_{jt} \equiv p_{jt} - \frac{(1 - \delta_{jt}) p_{jt+1}}{1 + r_{t+1}}, \qquad j = h, a,$$

where δ_{jt} are the relevant depreciation factors, and r_t is the interest rate.

Since we view changes in the quality of tractors as a major factor driving the decision to adopt the technology, we specified the model so that we could capture such variations. Specifically, we assume that tractor services can be provided by tractors of different vintages according to

$$k_t = \sum_{\tau=-\infty}^{t} m_{kt}(\tau) \tilde{k}_t(\tau),$$

where $\tilde{k}_t(\tau)$ is the amount of tractor services provided by a tractor of vintage τ (i.e. built in period τ) at time t, and $m_{kt}(\tau)$ is the number of tractors of vintage τ operated at time t. We assume that the amount of tractor services provided at time t by a tractor of vintage τ is given by,

$$\tilde{k}_t(\tau) \equiv v(x_\tau)(1 - \delta_{k\tau})^{t-\tau},$$

[7] The assumption that F depends on managerial skill, e, follows the work of Lucas (1978). As in Lucas' framework, its main role is to endogenously generate changes in the size distribution of farms.

where $\delta_{k\tau}$ is the depreciation rate of a vintage τ tractor, and $v(x_\tau)$ maps model-specific characteristics, the vector x_τ, into an overall index of tractor 'services' or 'quality.' Thus, our model assumes that the characteristics of a tractor are fixed over its lifetime (i.e. no upgrades), and that tractors depreciate at a rate that is (possibly) vintage specific. The rental price of a tractor is given by

$$q_{kt}(\tau) = p_{kt}(\tau) - \frac{p_{kt+1}(\tau)}{1 + r_{t+1}},$$

where $p_{kt}(\tau)$ is the price at time t of a $t - \tau$ year old tractor, while the term $c_{kt}(\tau)$ captures the variable cost (fuel, repairs) associated with operating one tractor of vintage τ at time t.

The function $\pi_t(q_t, c_t, w_t^F, e)$ captures the payoff in period t to being a farmer. Instead of farming, an individual with skill level e can make an (irreversible) migration decision. If he chooses to migrate to an urban area at time t, he receives a payoff given by

$$V_t^U \equiv \sum_{j=0}^{\infty} R_t(j) w_{t+j}^U - \varphi,$$

where w_{t+j}^U is a measure of the utility associated with working in an urban area at time $t + j$, φ is the fixed cost of migration, and

$$R_t(j) = \begin{cases} 1 & if\ j = 0 \\ \Pi_{s=1}^{j}(1 + r_{t+s})^{-1} & if\ j \geq 1 \end{cases},$$

is the relevant discount factor.

It follows that the utility of an individual with skill e who starts period t in a rural area (i.e. is a potential farmer) satisfies the following Bellman equation

$$V_t(e) = \max \left\{ V_t^U, \pi_t(e) + R_t(1) V_{t+1}(e) \right\}.$$

Given our assumption that F is increasing in e, it follows that $V_t(e)$ is also increasing in e. Moreover, if a farmer with skill level e chooses not to migrate, then all farmers with skill level $e' \geq e$ will not migrate either. Put differently, equilibrium migration is fully described, for each t, by the level of skill of the marginal farmer, e_t^*. Our assumption that the migration decision is irreversible, implies that the equilibrium sequence $\{e_t^*\}$ is non-decreasing.

Optimal choices of inputs and output by a farmer with skill level e are completely summarized by the first order conditions of profit maximization. The resulting demand for input functions for, each e, are denoted by [8]

$$m_t = m(q_t, c_t, w_t^F, e),$$

where $m \in \{k, h, a, \bar{n}\}$ indicates the input type, q_t is a vector of rental prices, and c_t is a vector of operating costs, and w_t^F denotes real wages in the farm sector.

Given that agricultural prices are largely set in world markets, and that domestic and total demand do not coincide, we impose as an equilibrium condition that the demand for land equal the available supply. Thus, land prices are endogenously determined.

3.1 Aggregate Implications

In this section we show how to compute the implications of our simple model for sector-wide aggregates. To this end we need to sum individual factor demands over all possible skill types.

3.1.1 The Number of Farms and Labor in Farms

Let the measure of *potential* farmers be $\bar{N}$. We assume that $\bar{\mathbf{N}} = \int_0^\infty \mu(de)$, for some measure μ. This measure captures the exogenous distribution of skills. let e_t^* be the 'marginal' farmer at time t; then, the number of farmers with ability levels less than or equal to e at time t is $\int_{e_t^*}^e \mu(ds)$, for $e \geq e_t^*$, and 0 for $e < e_t^*$. This distribution is time-varying and endogenously determined. The number of active farmers (and the number of farms)[9] at t is simply $\mathbf{N}_{ft} = \int_{e_t^*}^\infty \mu(de)$.

Let, $\hat{e}_t$ be the value of e that satisfies

$$V_t(\hat{e}_t) \equiv V_t^U.$$

Then, e_t^* evolves according to $e_t^* = \min\left\{e_{t-1}^*, \hat{e}_t\right\}$. This formulation imposes the equilibrium condition that the the marginal farmer be indifferent between migrating or staying, or its identity

[8] To be precise, the demand functions depend on current and future prices. Even though the pure demand decision is static due to our assumption of perfect rental markets, the migration decision implies that future prices influence current demand through their impact on the identity of the farmers who remain in the rural sector, i.e. the level of e.

[9] Our model does not distinguish farms from farm operators.

is unchanged from the previous period. The condition $V_t(\hat{e}_t) = V_t^U$ is not a simple comparison. The reason is that $V_t(e)$ depends on all prices and, in our model, the price of land is determined endogenously (and a function of the distribution μ and the cutoff point e_t^*). Hence, obtaining $\hat{e}_t$ requires the computation of a fixed point at each t.

If each farmer has an endowment of $\hat{n}$ man/year equivalent (including family workers), the total number of man/year equivalent labor provided by farmers is $\hat{n}\mathbf{N}_{ft}$, while the total number of man/year individuals hired is[10]

$$\mathbf{N}_{st} = \int_{e_t^*}^{\infty} \max\left[\bar{n}(q_t, c_t, w_t^F, e) - \hat{n}, 0\right] \mu(de). \tag{1}$$

The ratio of hired to total labor is given by

$$\eta_t = \frac{\mathbf{N}_{st}}{\mathbf{N}_t} = \frac{\int_{e_t^*}^{\infty} \max\left[\bar{n}(q_t, c_t, w_t^F, e) - \hat{n}, 0\right] \mu(de)}{\int_{e_t^*}^{\infty} \bar{n}(q_t, c_t, w_t^F, e)\mu(de)} \tag{2}$$

3.1.2 Tractors, Horses and Land

The aggregate demand for tractor *services* at time t, $\mathbf{K}_t$ is given by

$$\mathbf{K}_t = \int_{e_t^*}^{\infty} k(q_t, c_t, w_t^F, e)\mu(de), \tag{3}$$

while the *number of tractors* purchased at t, m_{kt} is

$$m_{kt} = \frac{\mathbf{K}_t - (1 - \delta_{kt-1})\mathbf{K}_{t-1}}{v(x_t)}. \tag{4}$$

The law of motion for the stock of tractors (in units), $\mathbb{K}_t$, is[11]

$$\mathbb{K}_t = (1 - \delta_{kt-1})\mathbb{K}_{t-1} + m_{kt}. \tag{5}$$

[10] This formulation assumes that if a farmer's demand for labor, $n_t(q_t, c_t, w_t^F, e)$ falls short of his endowment, $\hat{n}$, he can sell the difference in the agricultural labor market. This assumption is the natural analog of the perfect rental markets for tractors, horses, and land.

[11] An alternative measure of the stock of tractors is given by $K_{t+1} = K_t + m_{kt+1} - m_{kt-T}$ were T is the lifetime of a tractor of vintage $t - T$. This alternative formulation assumes that tractors are of the one-hose shay variety and that after T periods they are scrapped.

We assume that horse services are proportional to the stock of horses and, by choice of a constant, we set the proportionality ratio to one. Thus, the aggregate demand for horses is

$$\mathbf{H}_t = \int_{e_t^*}^{\infty} h(q_t, c_t, w_t^F, e)\mu(de). \tag{6}$$

We let the price of land adjust so that the demand for land predicted by the model equals the total supply of agricultural land denoted by $\mathbf{A}_t$. Thus, given wages, agricultural prices and horse and tractor prices, the price of land, p_{at}, adjusts so that

$$\mathbf{A}_t = \int_{e_t^*}^{\infty} a(q_t, c_t, w_t^F, e)\mu(de). \tag{7}$$

3.2 Modeling Tractor Prices

From the point of view of an individual farmer the relevant price of tractor is $q_{kt}(\tau)$: the price of tractor services corresponding to a $t - \tau$ year old tractor. Unfortunately, data on these prices are not available. However, given a model of tractor price formation, it is possible to determine rental prices for all vintages using standard, no-arbitrage, arguments.

As indicated above, we assume that a *new tractor* at time t offers tractor services given by $\tilde{k}_t(t) = v(x_t)$, where $v(x_t)$ is a function that maps the characteristics of a tractor into tractor services. We assume that, at time t, the price of a new tractor is given by

$$p_{kt} = \frac{v(x_t)}{\gamma_{ct}}.$$

In this setting γ_{ct}^{-1} is a measure of markup over the level of quality. If the industry is competitive, it is interpreted as the amount of aggregate consumption required to produce one unit of tractor services using the best available technology x_t.[12] However, if there is imperfect competition, it is a mixture of the cost per unit of quality and a standard markup. For the purposes of understanding tractor adoption we need not distinguish between these two interpretations: any factor —technological change or variation in markups— that affects the cost of tractors will have an impact on the demand for them. In what follows we ignore this distinction, and we label γ_{ct} as productivity in the tractor industry.

[12] We assume that the cost of producing 'older' vectors x_t is such that all firms choose to produce using the newest new technology.

It is possible to show (see the Appendix) that no arbitrage arguments imply that

$$q_{kt}(t) = p_{kt}\left[1 - R_t(1)(1-\delta_{kt})\frac{\gamma_{ct}}{\gamma_{ct+1}}\right] + (1-\Delta_t)C(t+1, T-1), \tag{8}$$

where

$$\begin{aligned} \Delta_t &= \frac{v(x_t)(1-\delta_{kt})}{v(x_{t+1})}, \\ C(t+1, T-1) &\equiv \sum_{j=0}^{T-1} R_{t+1}(j) c_{kt+1+j}, \end{aligned}$$

given that T is the lifetime of a tractor, and c_{kt} is the cost of operating a tractor in period t.

This expression has a simple interpretation. The first term, $1 - R_t(1)(1-\delta_{kt})\frac{\gamma_{ct}}{\gamma_{ct+1}}$, translates the price of a tractor into its flow equivalent. If there were no changes in the unit cost of tractor quality, i.e. $\gamma_{ct} = \gamma_{ct+1}$, this term is just that standard capital cost, $(r_{t+1} + \delta_{kt})/(1 + r_{t+1})$. The second term is the flow equivalent of the present discounted value of the costs of operating a tractor from t to $t + T - 1$, $C(t + 1, T - 1)$. In this case, the adjustment factor, $1 - \Delta_t$, includes more than just depreciation: total costs have to be corrected by the change in the 'quality' of tractors, which is captured by the ratio $\frac{v(x_t)}{v(x_{t+1})}$.

To compute $q_{kt}(t)$ we need to separately identify $v(x_t)$ and γ_{ct}.[13] To this end we specified that the price of a tractor of model m, produced by manufacturer k at time t, p_{mkt}, is given by

$$p_{mkt} = e^{-d_t}\Pi_{j=1}^{N}(x_{jt}^{m})^{\lambda_j} e^{\epsilon_{mt}},$$

where $x_t^m = (x_{1t}^m, x_{2t}^m, ...x_{Nt}^m)$ is a vector of characteristics of a particular model produced at time t, the d_t variables are time dummies, and ϵ_{mt} is a shock. This formulation is consistent with the findings of White (2000).[14] We used data on prices, tractor sales and a large number of characteristics for almost all models of tractors produced between 1919 and 1955 to estimate this equation. In the Appendix, we describe the data and the estimation procedure. Given our

[13] It is clear that all that is needed is that we identify the changes in these quantities.

[14] Formally, we are assuming that the shadow price of the vector of characteristics x_t does not change over time. This is not essential, and the results reported by White (2000), Table 10, can be interpreted as allowing for time-varying shadow prices. Comparing the results in Tables 10 and 11 in White (2000) it does not appear that the extra flexibility is necessary.

estimates of the time dummy, $\hat{d}_t$, and the price of each tractor, $\hat{p}_{mkt}$, we computed our estimate of average quality, $\bar{v}(x_t)$ as

$$\bar{v}(x_t) = \bar{p}_{kt}\hat{\gamma}_{ct},$$

where

$$\begin{aligned} \bar{p}_{kt} &= \sum_m s_{mkt}\hat{p}_{mkt}, \\ \hat{\gamma}_{ct} &= e^{\hat{d}_t}, \end{aligned}$$

with s_{mkt} being the share of model m produced by manufacturer k in total sales at time t. The resulting time-series for $\bar{v}(x_t), \hat{\gamma}_{ct}$ and $\bar{p}_{kt}$ are shown in Figure 3.

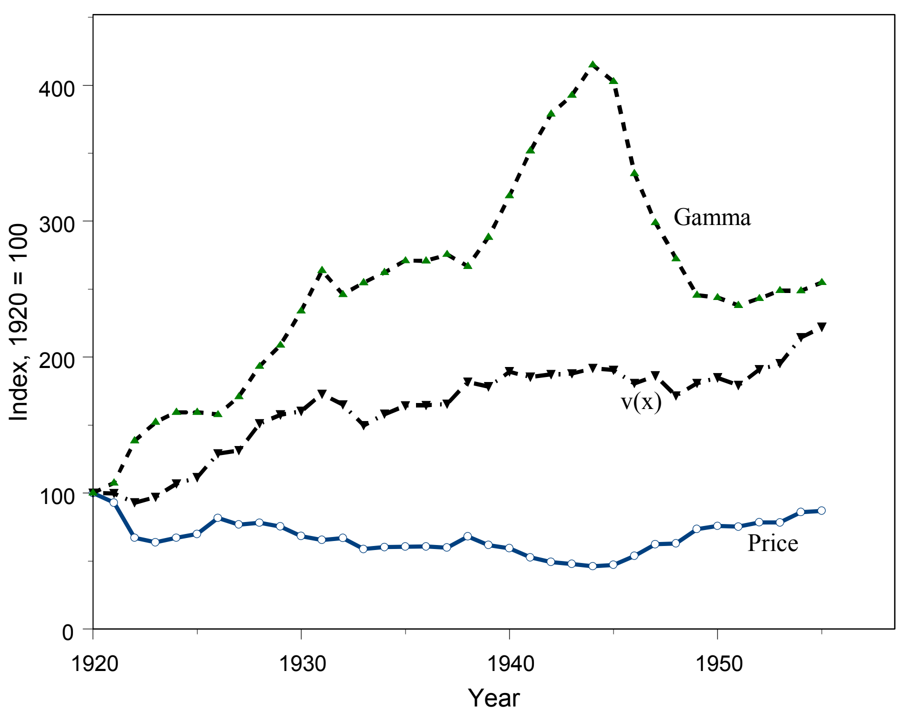

Figure 3: Tractor Prices, Quality and Productivity. 1920-1955. Estimation Results.

Even though the real price of a tractor does not show much of trend after 1920, its components do. Over the whole period our index of quality doubles, and our measure of productivity shows a substantial, but temporary increase in the 1940s, with a return to trend in the 1950s. In the 1920-1955 period γ_{ct} more than doubles. Thus, during this period there were substantial increases in quality and decreases in costs; however, these two factors compensated each other, so that the real price of a tractor shows a modest decrease.

4 Steady States and Calibration

At the steady state all variables are constant. We denote the interest rate by $R = (1+r)^{-1}$. The steady state version of the demand for factors is $m = m(q, c, w^F, e)$ for $m \in \{k, h, a, \bar{n}\}$. To compute steady state aggregates, we use the endogenous distribution of skills of farm operators, which is completely summarized by e^* and μ.

Let the steady state profit flow be denoted $\pi(q, c, w^F, e)$. Then, the no-migration condition in the steady state is

$$\pi(q, c, w^F, e^*) = w^U - \frac{r\varphi}{1+r}.$$

Equilibrium in the land market requires that the appropriate version of (7) hold. Given this, it follows that average farm size , $\bar{a}$, is given by

$$\bar{a} = \frac{\int_{e^*}^{\infty} a(q, c, w^F, e)\mu(de)}{\mathbf{N}_f}. \tag{9}$$

Assuming that there is no change in tractor quality at the steady state, the number of tractors follows from the appropriate version of (4) and (5) it is given by

$$\mathbb{K} = \frac{\int_{e^*}^{\infty} k(q, c, w^F, e)\mu(de)}{v(x)}. \tag{10}$$

The model's prediction for the demand for hired labor, the ratio of hired to total labor and horses follow from the steady state versions of (1), (2), and (6). The model's prediction for total farm output, $\mathbf{Y}$, is

$$\mathbf{Y} = \int_{e^*}^{\infty} y(q, c, w^F, e))\mu(de),$$

where $y(q, c, w^F, e)$ is the value of output of a farm with managerial skill e at the prices corresponding to the steady state.

4.1 Model Specification and Calibration

We consider the following specification of the farm production technology

$$
\begin{aligned}
F^c(y_I, e) &= A_{ct} y_I^{\alpha_c} e^{1-\alpha_c}, \\
y_I &= F^y(z, n_y, a) = z^{\alpha_{zy}} n_y^{\alpha_{ny}} a^{1-\alpha_{zy}-\alpha_{ny}}, \\
z &= F^z(z_k, z_h) = [\alpha_z (z_k)^{-\rho} + (1-\alpha_z) z_h^{-\rho}]^{-1/\rho}, \\
z_k &= F^k(k, n_k) = [\alpha_k k^{-\rho} + (1-\alpha_k) n_k^{-\rho}]^{-1/\rho}, \\
z_h &= F^h(h, n_h) = A_h h^{\alpha_h} n_h^{1-\alpha_h}.
\end{aligned}
$$

This formulation captures the idea that farm output depends on services produced by tractors, z_k, services produced by horses, z_h, labor, n_j, $j = y, h, k$, and managerial skills, e. We take a standard approach and use a Cobb-Douglas formulation except in two cases. We assume that the elasticity of substitution between tractors and labor in the production of tractor services is $1/(1+\rho)$. Since we assume that the elasticity of substitution between horses and labor is one, this formulation allows us to capture potential differential effects of a change in the wage rate upon the choice between tractor and horses. Second, we also assume that basic tractor services, z_k, and horse services, z_h, are combined with elasticity of substitution $1/(1+\rho)$ to produce power services, z.

We specify that exogenous technological evolves according to $A_{ct} = e^{\gamma t}$. The technology is completely specified by 9 parameters: $(\gamma, A_h, \alpha_c, \alpha_{zy}, \alpha_{ny}, \alpha_z, \alpha_k, \alpha_h, \rho)$.

We assume that the distribution μ is log-normal with mean $\hat{\mu}$ and standard deviation σ. In addition to these two parameters, it is necessary to select values for the discount factor β, the cost of migration, φ, the man/year equivalent of a farm family, $\hat{n}$. This is a total of 14 parameters.

We assume that $\beta = 0.96$, and that $\hat{n} = 2$. This last value is equivalent to specifying that the average farm family contributes labor equivalent to 2 workers. Since we could not find reliable estimates of φ we considered initially a value of φ equal to one year of average earnings.[15] We performed some sensitivity analysis and varied φ between a half and one and a half of average

[15] Kennan and Walker (2003) estimate that for high school graduates the cost of moving between urban areas is about $250,000. Thus, our assumption of one year in the baseline case is conservative.

yearly earnings, and our findings remain essentially unchanged.

We take the process $\{A_{ct}\}$ to correspond to total factor productivity. Even though there are estimates of the evolution of TFP for the agricultural sector, it is by no means obvious how to use them. The problem is that, conditional on the model, part of measured TFP changes is due to changes in the quality of tractors, $v(x_t)$, as well as the rate of diffusion of tractors. Thus, in our model, conventionally measured TFP is endogenous. To compute (truly) exogenous TFP we used the following identification assumption: TFP is adjusted so that the model's prediction for the change in output between 1910 and 1960 match the data. This gives us an estimate of γ, which, in this case, is approximately 1.5% per year.

The remaining parameters of the model were picked to minimize the differences between model and data for the year 1910. We used two sets of moments from the 1910 agricultural sector to calibrate the model. The first set of moments corresponds to input shares in agricultural output. The second set of moments is related to properties of the size distribution of farms.

The moments corresponding to input shares are:[16]

- Land share of output.
- Value of horses/output.
- Value of tractors/output.
- Labor share of output.
- Ratio of hired to total labor.

In all cases except land share, the model's predictions are complicated functions of the parameters. The theoretical counterparts of these moments (see the Appendix) are integrals of factor demand functions with respect to the endogenous distribution of farmer's skills.

Since heterogeneity of farmers plays such an important role in our story, we required the model to match as many moments of the distribution of the variable '*acres per farm*' —our measure of firm size— as we could find. To ensure consistency over the 1910-1960 period we restricted

[16] The analogues in the model are in the Appendix.

ourselves to moments for which time series evidence is available in a consistent manner. The best information that we could obtain partitions the data into four bins. It includes information on the number of farms for establishments of 49 acres or less, 50-499 acres, 500-999 acres and 1,000 or more acres. We decided to merge the first two categories, since we suspect that forces other than agricultural prices affect the number of very small farms (less than 49 acres). In addition to this information, we were able to find some moments of the continuous size distribution. Specifically, we have information on average farm size conditional on being in a certain size category.

In order to match the average farm size in 1910, $\bar{a}$, we adjusted total land area ($\mathbf{A}$ in the model). Thus, we used a 'free' parameter to match this statistic.[17] Note, however, that total land area in 1960 is **not** a free parameter. We used data on Land in Farms (from the Historical Statistics) to estimate the supply of land in 1960 —using our units— as

$$\mathbf{A}_{60} = \mathbf{A}_{10} \times \textit{measured change in land in farms}.$$

We are then left with five moments:

- Average acres per farm, conditional on the farm being in the 500-999 acre category, $\bar{a}_{5-10}$.
- Average acres per farm, conditional on the farm being in the 1,000 or more category, $\bar{a}_{10+}$.
- Fraction of land in farms in the 500-999 acre category, s_{5-10}.
- Fraction of land in farms in the 1,000 or more category, s_{10+}.[18]
- The coefficient of variation of 'acres per farm.'

The calibration proceeds as follows. We choose the parameters so that the model —evaluated at the 1910 prices— matches the 10 moments we obtained from the data. Since computing the model's predictions requires a fixed point in the endogenously chosen 'marginal' farmer, e^*, calibration is computationally intensive, and we were unable to match the data exactly. Table 1 spells out the parameters used to calibrate the model.

[17] Alternatively, we could have endogeneized $\bar{N}$, the total mass of potential farmers, to match land. In our numerical exercise we set $\bar{N} = 1$.

[18] Given these statistics the fraction of farms in each category (the 0-499 acres is our residual) can be readily computed.

Parameter	A_h	α_c	α_{zy}	α_{ny}	α_z	α_k	α_h	ρ	$\hat{\mu}$	σ	β	φ	$\hat{n}$	γ
Value	3.7	0.86	0.37	0.4	0.55	0.72	0.6	-0.6	4	1.72	0.96	223	2	.015

Table 1: Calibration

The first two columns of Table 2 present the match between the model and U.S. data for our chosen specification for the year 1910. The match is fairly good in terms of most of the moments. The one exception is the share of horses to output. Relative to the U.S. economy our specification underpredicts the horse output ratio. It is not clear to us what is the reason. It is possible that in 1910 horses were used to produce services not directly related to farming (e.g. transportation), and that our simple model is not well equipped to capture this.

	1910		1960		
Moment	Model	Data	Model	Data	Source
Land - share of output	0.198	0.2	0.198	0.2	Grilliches (1964)
Horses/output ratio	0.174	0.25	0.0066	0.01	Hist.Stat.of U.S.
Tractors/output ratio	0.0030	0.0031	0.133	0.135	Hist.Stat.of U.S.
Labor - share of output	0.47	0.5	0.381	0.401	Lebergott (1964)
C.V. of acres/farm	1.05	1.1	0.99	1.1	Hist.Stat.of U.S.
Labor - Hired/Total	0.27	0.24	0.24	0.26	Hist.Stat.of U.S.
$\bar{a}_{5-10}$	617	646	716	695	Hist.Stat.of U.S.
s_{5-10}	0.1	0.1	0.14	0.12	Hist.Stat.of U.S.
$\bar{a}_{10+}$	3414	3340	3662	3964	Hist.Stat.of U.S.
s_{10+}	0.19	0.19	0.38	0.49	Hist.Stat.of U.S.

Table 2: Match Between Model and Data, 1910 & 1960

The values of the calibrated parameters seem reasonable and, when there is evidence available, fall in the range of estimates from micro studies. Of particular importance for our purposes is the elasticity of substitution between horse services and tractor services. This elasticity —given by $1/(1+\rho)$— is calibrated to be equal to 2.5.[19] The model also does pretty well in matching the

[19] Our preferred value is slightly higher than the value of 1.7 estimated by Kislev and Petersen (1982). However,

size distribution of farms.[20]

5 Steady States Results

We use the model – driven by the exogenous price sequences— to predict the levels of a variety of variables in 1960. We then conduct a number of counterfactual experiments to illustrate the role played by each of our modeling assumptions.

5.1 Baseline Model

The **predictions** of the model for the level of input use in agriculture in 1960 are also presented in the last two columns of Table 2. These predictions were obtained by 'feeding' the 1960 values of the exogenous processes to our baseline model and computing the 1960 steady state equilibrium. Overall, the model performs extremely well in matching most of the moments in 1960. It does a remarkable job capturing the increase in the tractor-output ratio from close to 0 in 1910 to 0.13 in 1960. It also predicts the decline of the horse. The horse-output ratio predicted by the model declines from 0.17 in 1910 to 0.0066 in 1960, slightly less than the observed value of 0.01. Most importantly, the model does get at the 'right' share of tractors in aggregate output, and also shows a reduction in the value of horses relative to output. In terms of labor input, the baseline model is quite successful at predicting the substantial decrease in the share of labor in agriculture (from 0.50 to 0.40 in the data, and 0.47 to 0.38 in the model).

The model predicts that the fraction of land held by the largest farms (1,000+ acres) rises from 19% in 1910 to 38% in 1960. This parallels the observed change from 19% to 49%. The model slightly overpredicts the average size of medium-sized farms (716 acres vs. 695 acres), and it slightly underpredicts the size of the largest farms (3,662 acres vs. 3,964 acres) in 1960. Even

they completely ignored horses and their estimate is likely to be some weighted average of the two elasticites of substitution: labor and capital and labor and horses.

[20] The model produces a continuous distribution of farms (by farm size). We put the distribution in three bins (0-499, 500-999, and 1000+) to match the evidence on acreage. We use this distribution to compute the moments that we report. To compute mean conditional average acreage per farm we use (both for the model and the data) a richer continuous distribution.

though average farm size tripled, both model and data imply that the coefficient of variation of farm size is roughly constant.

As indicated before, we adjusted measured TFP growth so that the increase in total output predicted by the model and the data coincide in 1960. The required increase in TFP was 1.9. By way of comparison, the Historical Statistics reports that overall farm TFP grew by a factor of 2.3. Thus, around 17.4% of the increase in farm TFP between 1910 and 1960 can be accounted for by the diffusion of the tractor, and the steady increase in average quality.

5.2 Sensitivity Analysis

As mentioned before, we view three features of the model as major determinants of the results: selection, changes in prices (both wages and horse prices), and changes in tractor quality. To analyze the *quantitative* role that each of these factors play in generating the results, we recompute the predictions of the model for the year 1960 under alternative scenarios.

5.2.1 Changing the Driving Processes: Constant TFP

Our first set of results takes TFP as given and asks: How well would the model predict tractor adoption in 1960 if the driving processes were held at their 1910 level? This is a simple way of assessing the importance of the included features in generating a good match between model and data.

The results of those counterfactual exercises for the 1960-1910 ratio of tractors and output ratio are summarized in Table 3.[21]

	Baseline	Holding Constant:				U.S. data
		w_{1910}	$v(x_{1910})$	Skill Dist.	p_{h1910}	
Tractors	272	632	139	143	275	280
Output	2.1	5.53	1.58	1.19	2.15	2.1

Table 3: Changing the Driving Processes. No TFP Adjustment. 1960-1910 Ratios.

[21] In the Appendix, we present more detailed data on the predictions of the alternative models for a variety of variables in addition to output.

The first column gives the prediction of the baseline model. Our preferred specification predicts a level of adoption that is fairly close (97%) to that observed in the data (272/280). All the counterfactuals, i.e. the versions of the model in which a specific driving force is fixed, do not perform well. The column labeled w_{1910} reports the prediction of the model when farm wages are (counterfactually) kept at their (relatively low) 1910 values. In this case, the model overpredicts the growth in the number of tractors by over 125%. This is mostly driven by the spectacular increase in agricultural output predicted by the model, which exceeds observed increases by 160%. We view this result as evidence that modeling technology adoption ignoring the effects of changes in other input prices can lead to fairly erroneous conclusions.

The second experiment, reported in the column labeled $v(x_{1910})$, is designed to gauge the role of quality changes in the performance of the model. To this end, we fixed tractor quality at its 1910 level for all periods. In this case, the model predicts that only half as many tractors would be adopted in 1960 relative to the data. A third experiment involved substituting the assumption of equilibrium migration, i.e. the equilibrium condition that migration is driven by a comparison between urban and rural net income, with the assumption that migration was random. Formally, we assume that the distribution of skills in 1960, $\tilde{\mu}$, is now exogenously given by

$$\tilde{\mu}(e) = \frac{\mathbf{N}_{f,60}}{\mathbf{N}_{f,10}} \mu^{1910}(e).$$

This specification 'scales down' the mass of farmers so as to match the decline in the total number of farms between 1910 and 1960, but it assumes that the distribution of skills is unchanged. Ignoring selection results in an underprediction of tractor adoption of the order of 50%.

Finally, we use the model to study what role —if any— the dramatic change in horse prices had on the predictions of the model. Somewhat surprisingly, holding horse prices at their 1910 levels the model still correctly predicts tractor adoption in 1960. As we will argue later, changes in horse prices do not have a large impact on the model's predictions and, hence, were not an important determinant of the speed of diffusion. However, it turns out that the existence of a 'horse technology' is essential since it gives a differential role to changes in real wages.

Thus, a tentative conclusion one draws from this exercise is that all the features that we included in the model, with the possible exception of horses, are quite important to produce a good match to the data. However, some caution needs to be exercised. Recall that in the baseline model we chose TFP —given its endogeneity— so as to match output. Thus, a much stronger 'test' of our baseline specification is to compare its predictions with those of the alternative models where, in each case, TFP is allowed to adjust as to match the growth in agricultural output. We now turn to those results.

5.2.2 Changing the Driving Processes: Adjusted TFP

In this section we describe the predictions of each of the four 'counterfactuals' when TFP is adjusted so that each model matches the observed growth in agricultural output between 1910 and 1960. Of course, given that, by construction, output levels are matched, each model has to be evaluated in terms of its ability to reproduce other moments.

Constant Wages As before, we set rural and urban wages at their 1910 levels and compute the predictions of the model for 1960.[22] The results are presented in the column labeled 'w_{1910}' in Table 4. This version of the model underpredicts both the increase in the number of tractors (193 vs. 280) and the decrease in the number of horses (1/5.11 vs. 1/7.8). As a result, the horse-tractor ratio decreases by a factor of about 1,000 between 1910 and 1960. In the data (and the baseline model) this factor is slightly above 2100. Thus, as suspected, the increase in wages over this period had a substantial impact in driving the substitution between tractors and horses and, hence, influences the speed of diffusion of the tractor. Moreover, this version induces much less migration (there is no reason to migrate since wages are the opportunity cost of farming) and, consequently, severely underpredicts the increase in farm size (1.3 vs. 2.13). This, in turn, leads the model to overpredict the coefficient of variation of 'acres per farm' by approximately a third (1.43 vs. 1.1)

The Importance of Quality-Adjustment Tractor quality increased throughout the period. To quantify the contribution of quality changes to the results, we recomputed the model —with

[22] TFP had to be adjusted by a factor of 1.18 to match the increase in output.

Ratio:1960 to 1910	Baseline	w_{1910}	$v(x_{1910})$	Random	p_{h1910}	U.S. Data
Acres/farm	2.56	1.34	2.74	2.56	2.61	2.13
Stock of Tractors	272	193	184	272	275	280
Stock of Horses	1/7.90	1/5.11	1/4.41	1/7.90	1/8.9	1/7.8
Stock of Labor	1/2.92	1/1.21	1/2.95	1/2.92	1/2.78	1/2.5
Price of Land	0.76	0.74	0.75	0.76	0.77	0.8
Farm Output	2.1	2.1	2.1	2.1	2.1	2.1
Level:1960						
C.V. acres/farm	0.98	1.43	0.93	0.82	0.97	1.1
Hired Lab./Total	0.24	0.21	0.23	0.32	0.23	0.26

Table 4: Changing the Driving Processes: Ajusted TFP

the usual TFP adjustment[23]— assuming that quality remained constant at the 1910 level. A priori it is not obvious in what direction this pushes the results. Increases in the efficiency of new tractors reduces the number of tractors required to perform a given task. On the other hand, higher quality tractors result in lower costs of operations, and this increases the demand for tractor services.

The results of holding $v(x)$ constant at its 1910 level are in column labeled '$v(x_{1910})$' in Table 4. The number of tractors adopted in the absence of quality improvements would have been lower than in the baseline case by about 32%. Similarly the horse-tractor ratio is predicted to decrease by a factor of about 810, which is small when compared to the predictions of the baseline and the observed U.S. values (about 2100). In terms of size distribution, the lack of quality changes increases the effective price of operating a tractor and this, in turn, reduces profits in the farm sector. Lower profits induce more migration and a substantial overprediction of the increase in farm size (3.46 vs. 2.13)

Selective vs. Random Migration It is possible to show (see the Appendix for a formal proof) that if the production function is Cobb-Douglas in managerial skill and other factors (as is the

[23] In this case the adjustment factor was 2.7.

case in our model), *any* aggregate level of input use (and total output) generated by the baseline model can be replicated by the random migration version, given an appropriate adjustment in TFP. The results of this experiment for the 1960-1910 ratio of several variables and the levels of a couple of variables in 1960 are summarized in the column labeled 'Random' in Table 4. Thus, to ascertain the ability of the model to match the data we need to consider moments that depend on the size distribution. Along these lines, the random migration model fails in two dimensions: it underpredicts the coefficient of variation of 'acres per farm' by 25% (0.82 vs. 1.1), and it overpredicts the ratio of hired to total labor. More importantly, it gives counterfactual predictions for some moments of the distribution of farm size. We will discuss this in more detail in the analysis of the transitional dynamics.

The Role of Horse Prices Recent research on the topic of adoption of tractors has emphasized that adjustment of horse prices delayed the diffusion of tractors (see Olmstead and Rhode (2001)). We can use our model to study how would the adoption decision have changed had horse prices not adjusted. The results are in the column 'p_{h1910}' in Table 4. It follows that horse prices did not play a major role. The number of tractors in 1960 would have been very similar to that observed in the data. Of course, the horse-tractor ratio would have been higher, but in every other dimension the model would have performed well.

Should we conclude that explicitly modeling the fact that farmers had a choice between an 'old' and a 'new' technology is an unnecessary feature of the model? No. The reason is simple: had we ignored horse, we would have had only a minor impact from the change in wages, and this would have severely limited the model's ability to match the data. Explicitly modeling the choice of technology is important insofar as it provides a channel through which other, complementary, changes in prices affect the demand for tractors.

5.2.3 Changing the Specification

In this section we present the result of modifying the **specification** of the production function from the one used in the baseline to a 'full' Cobb-Douglas functional form. We also explore the effect of (almost) eliminating the rents that accrue to farm operators. In all cases, we find that the changes substantially worsen the model's ability to match the data.

Elasticity of Substitution Our specification of the technology is such that, at our preferred parameterization, the elasticity of substitution between tractors and labor is 2.5, while the elasticity of substitution between horses and labor is one. These differences in the elasticity of substitution result in variable shares and, more importantly, suggest that wage changes can have an asymmetric effect on the demand of horses and tractors. Specifically, we expect an increase in wages to induce substitution of tractors for horses.

In order to quantitatively assess the importance of this specification, we studied a version of the model in which ρ is set equal to 0. In this case, the production function is Cobb-Douglas in all its variables. As before, we adjusted TFP growth so that the model exactly matches the observed growth in output. TFP was increased by a factor of 1.72. The results for the 1960-1910 ratios and levels in 1960 of some important variables are in the column labeled '$\rho = 0$' in Table 5. It follows that the Cobb-Douglas specification severely underpredicts the diffusion of the tractor. The reason for this result is simple: The Cobb-Douglas functional form implies constant input shares. In the absence of spectacular price decreases in the own price —and tractor prices showed a large, but not spectacular decrease over this period — the model predicts modest increases in the quantities demanded.

Changes in the Share of Managerial Skills The assumption that managerial skills receive a non-zero fraction of total revenue plays a significant role in our model, as it has an impact on migration decisions. We considered a specification that significantly reduces the share of profits that accrue to skill. The results are in the column labeled '$\alpha_c = 0.999$' in Table 5.

The model underpredicts the number of tractors by 13% and, more importantly, predicts a huge increase in average farm size. While in the data, the average acreage per farm increases by a factor of 2.13, this specification predicts an increase four times as large. Of course, this is a direct consequence of the higher elasticity of migration with respect to wages induced by the small share of profits received by farm operators.

Ratio: 1960 to 1910	Baseline	$\rho = 0$	$\alpha_c = 0.999$	U.S. Data
Acres/Farm	2.56	3.44	9.22	2.13
Stock of Tractors	272	4.93	245	280
Stock of Horses	1/7.90	1/1.96	1/8.34	1/7.8
Stock of Labor	1/2.92	1/2.46	1/2.85	1/2.5
Price of Land	0.76	1.31	0.78	0.8
Output	2.1	2.1	2.1	2.1
Level: 1960				
C.V. acres/farm	0.99	1.02	3.47	1.1
Hired Lab./Total	0.24	0.27	0.73	0.26

Table 5: Changing the Production Function

6 Transitional Dynamics: Was Diffusion Too Slow?

The results for 1960 indicate that the model does a reasonable job of matching some of the key features of the data. However, they are silent about the model's ability to account for the *speed* at which the tractor was adopted.

Was diffusion too slow? To answer this question, the entire dynamic path from 1910 to 1960 needs to be computed. To do this, we took the observed path for prices (p_k, p_h, p_c, w, w^F), operating costs (c_k, c_h, c_a) and depreciation rates $(\delta_k, \delta_h, \delta_a)$, and used them as inputs to compute the predictions of the model for the 1910-1960 period.[24] At the same time, we adjusted the time path of TFP so that the model matches the data in terms of the time path of agricultural output, the analog of our steady state procedure. This helps to get the scale right along the entire transition. This exercise is computationally very intensive as it requires solving for the fixed point in the sequence of TFPs from 1910 to 1960, in addition to calculating the equilibrium in the land

[24] We use five-year moving averages for all these sequences. For the years 1910 and 1960, we use actual data (remember that these dates are viewed as steady-states). For all other years, the five year average was constructed as the average of the the year in question, the two years before and the two years. In a sense, using a five year average substitutes for the lack of adjustment costs in the model.

market in each period.

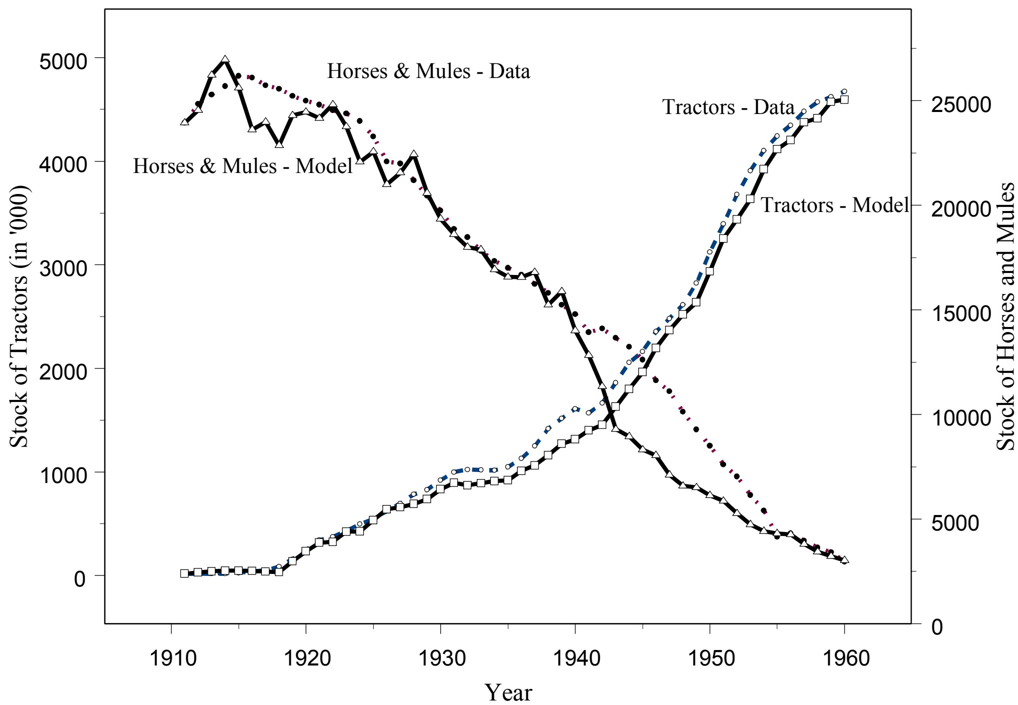

Figure 4: Transitional Dynamics, 1910-1960 - Tractors and Horses

The predictions of the model (and the relevant data for the U.S.) for the number of tractors and horses is depicted in Figure 4. The model does a remarkable job of tracking the actual diffusion of the tractor and the decline of the horse.

To provide a quantitative dimension of goodness of fit, we define a distance between model and data, ϕ, given by

$$\phi = 1 - \frac{d(x,y)}{d(y,\bar{y})},$$

where

$$d(x,y) \equiv \sum_{t=0}^{T} (x_t - y_t)^2,$$

and x is a vector of predictions, while y is a vector (of equal length) that includes observed (U.S.) values. The sample mean of this vector is denoted $\bar{y}$. Thus, ϕ is reminiscent of an R^2, and it seems a natural measure of 'goodness of fit' for models that aim to match transition data. Note that if the model is no better at predicting the data than the sample mean, $\phi = 0$, while if the model fits the data perfectly $\phi = 1$. We computed the measure ϕ for tractors, horses and employment. The results are in Table 6

Series	ϕ	Correlation	Mean (US)	Mean (Model)
Tractors	0.99	0.999	1669	1559
Horses	0.94	0.982	16430	15254
Employment	0.83	0.974	19.75	18.4

Table 6: Measures of Goodness of Fit

A large fraction of the variability around the sample mean for both tractors and horses is accounted for by the model. In addition, the correlation coefficients between the model and the data are very high.

How does the model perform along other dimensions? Figure 5 depicts the model's predictions (and the U.S. values) for employment in the agricultural sector. The model's estimates track the data quite well and, in particular, they capture the sharp decreases in employment in agriculture in the 1940s and 1950s.[25]

Figure 6 plots the predictions of the model for the coefficient of variation of the variable 'acres per farm'. The corresponding population data from the U.S. is included as well. The baseline specification generates estimates of the changes in the coefficient of variation of the variable 'acres per farm' that also follow the time path for the U.S. Interestingly enough, both model and data predict an inverted U-shape pattern, with the period of high migration as a period of high relative inequality in the farm distribution.

Figure 6 also displays the predictions of the random migration model for the coefficient of variation of farm size. Contrary to the data (and the baseline model), ignoring selection results

[25] In the case of employment the corresponding ϕ is 0.84.

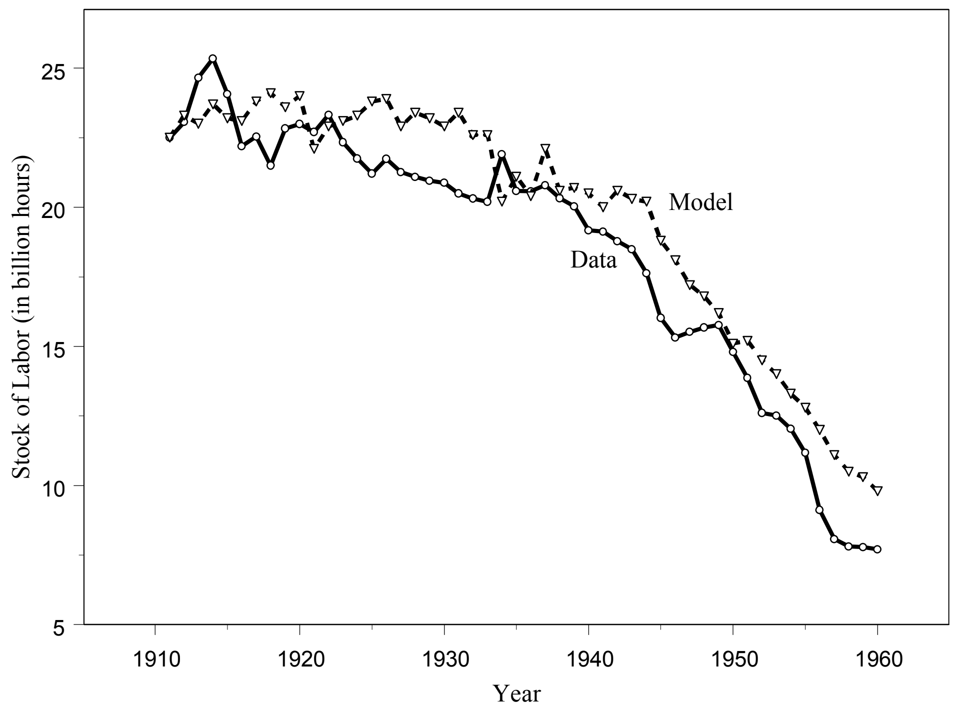

Figure 5: Transitional Dynamics 1910-1960 - Employment in Agriculture

in a downward trend. We view this as additional evidence against this specification.[26]

7 Conclusions

The frictionless neoclassical framework has been used to study a wide variety of phenomena including growth and development. However, the perception that the observed rate at which many new technologies have been adopted is too slow to be consistent with the model, has led to

[26] As shown in the appendix the (continuous) version of the coefficient of risk aversion corresponding to the random migration model is constant over time. Thus, the decreasing trend is driven by the 'discretization' of the variable in our three size categories. Note, however, that this is also how the coefficient of variation corresponding to the U.S. is computed.

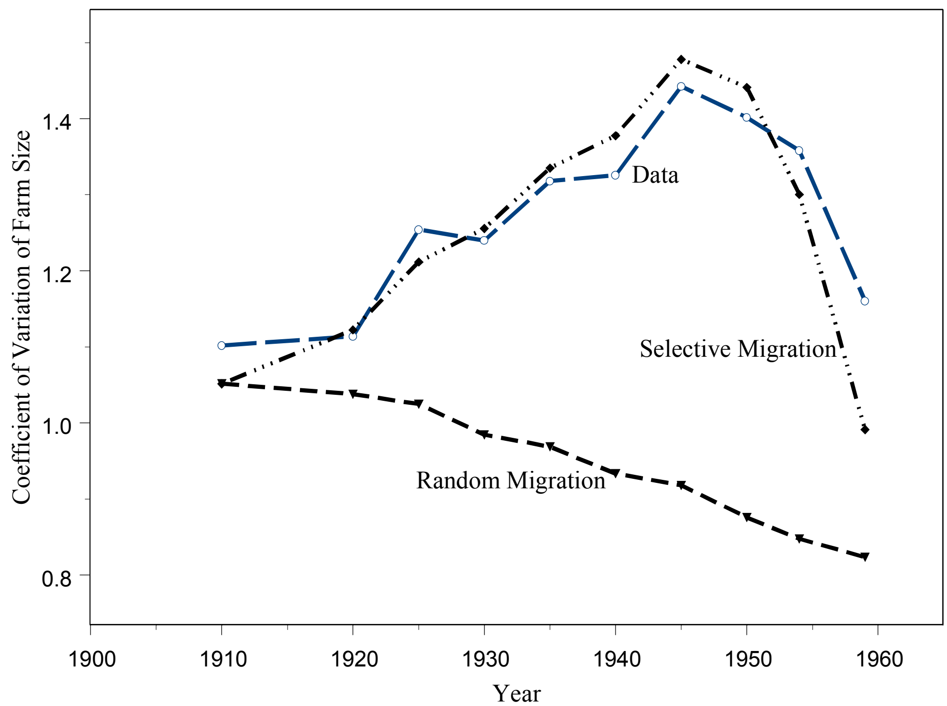

Figure 6: Transitional Dynamics 1910-1960. Coefficient of Variation - Selective vs. Random Migration

the development of alternative frameworks which include some 'frictions.'

In this paper we argue that a careful modeling of the shocks faced by an industry suggests that the neoclassical model can be consistent with 'slow' adoption.[27] Since most models with 'frictions' are such that the equilibrium is not optimal, the choice between standard convex models and the various alternatives has important policy implications. It is clearly an open question how far

[27] More generally, our model suggests that to understand the adoption of a technology in a given sector it may be critical to model developments in another sector. To see this, consider, as in this paper, two technologies that use a given input in different quantities. In this setting shocks to another sector that uses the same input will induce price changes which, in turn, will affect technology choices. Thus, general equilibrium effects can induce slow adoption.

our results can be generalized. However, at the very least, they cast a doubt on the necessity of 'frictions' in accounting for the rate of diffusion of new technologies.

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8 Appendix

8.1 Data Sources

This section details the available data, the sources which they were obtained and what they were used for. The data were used to calibrate the model and to compare the model and the data.

Income and Output

Farm Output: HS Series K 414 - used to compute the endogenous TFP in getting the model to match the data in terms of its predictions for output.

Value of Gross Farm Output: HS Series K 220-239 - used to compute factor shares.

Land

Number of farms: HS Series K4 - used to compute average size of farm

Land in Farms: HS Series K5 and K8 - used to compute average size of farm

Distribution of farm-acreage: HS Series K 162-173 - used to compute the C.V. of average farm size

Labor

Employment in farms (total and hired labor): HS Series K174, K175, K176 - used to compute hired labor as a fraction of total labor.

Man-hours Employed: HS Series K 410 - used to compute Labor's share of income and to compare model and data.

Wage Rates: Rural wages from HS Series K 177; Urban wages from HS Series D 802 - used to compute labor's share of farm income and as an exogenous driving process

Capital - Tractors and Horses

Number of Tractors on farms: HS Series K 184 - used to compute tractor's share of farm income

Number of Horses and Mules on farms: HS Series K 570, K572 - used to compute horses's share of farm income

Depreciation Rates and Operating Costs for Tractors: For the period 1910-1940, we used data from USDA. *Income Parity for Agriculture, Part II.* For the period 1941-1960, we used data from USDA, Agricultural Statistics - used as exogenous driving processes

Horse Prices: Data provided by Paul Rhode - used to compute horses's share of farm income and as an exogenous driving process

Tractor Prices: From 1920 to 1955, we used data from on our hedonic estimations. These estimations were based upon data on tractor prices and characteristics kindly provided to us by William White and augmented to include additional characteristics from Dunning (2000). For the period 1910-1919, and 1956-1960, we extrapolated the data using our estimation and data on average price of tractor available from the USDA, Agricultural Statistics. The exact manner in which this was done is described in the Appendix. This series is used to compute tractor's share of farm income and as an exogenous driving process

8.2 Tractor Prices

8.2.1 Derivation of the User Cost of a Tractor

From

$$p_{kt}(t) = \frac{v(x_t)}{\gamma_{ct}},$$

and given that a vintage-τ tractor at time t provides tractor services equal to $\tilde{k}_t(\tau) \equiv v(x_\tau)(1 - \delta_{k\tau})^{t-\tau}$, the rental price of such a tractor at time t is

$$q_{kt}(\tau) = p_{kt}(\tau) - \frac{p_{kt+1}(\tau)}{1 + r_{t+1}}.$$

This simply says that a $t - \tau$ year old tractor at time t will be a $t - \tau + 1$ year old tractor at $t+1$. This formula has the drawback that it depends on the price of used tractors. Since there are no data available on used tractor prices we now proceed to derive an expression for $q_{kt}(\tau)$ using simple arbitrage arguments. As shown in the text, the optimal choice of $m_{kt}(\tau)$ requires that

$$p_{ct} F_k(t) \tilde{k}_t(\tau) = p_{kt}(\tau) - \frac{p_{kt+1}(\tau)}{1 + r_{t+1}} + c_{kt}(\tau).$$

Iterating forward and denoting by $T(\tau, t)$ the number of periods of useful life that a $t - \tau$ old tractor has left, we get that

$$p_{kt}(\tau) = \sum_{j=0}^{T(\tau,t)} R_t(j) \{ p_{ct+j} F_k(t+j) \tilde{k}_{t+j}(\tau) - c_{kt+j}(\tau) \},$$

where $R_t(j)$ is as defined in the previous section. Using the special structure of $k_t(\tau)$ it follows that

$$p_{kt+1}(\tau) = \gamma_{k\tau} v(x_\tau)(1-\delta_{k\tau})^{t+1-\tau} \sum_{j=0}^{T(\tau,t+1)} R_{t+1}(j) p_{ct+1+j} F_k(t+1+j)(1-\delta_{k\tau})^j - \sum_{j=0}^{T(\tau,t+1)} R_{t+1}(j) c_{kt+1+j}(\tau).$$

To simplify the presentation we assume that two consecutive vintages of tractors have the same depreciation (δ_{kt}) and economic lifetime (T). Moreover, we assume that operating costs vary over time, but are not a function of the vintage, i.e. $c_{kt+j}(\tau) = c_{kt+j}$, for all τ. In this case, using the previous formula for $\tau = t$ (one period old tractors) and $\tau = t+1$ (new tractors) it follows that

$$\begin{aligned} p_{kt+1}(t) &= \frac{v(x_t)(1-\delta_{kt})}{v(x_{t+1})} p_{kt+1}(t+1) - \left[1 - \frac{v(x_t)(1-\delta_{kt})}{v(x_{t+1})}\right] \sum_{j=0}^{T-1} R_{t+1}(j) c_{kt+1+j}(t) \\ &+ \frac{v(x_t)(1-\delta_{kt})}{v(x_{t+1})} R_{t+1}(T) \left[p_{ct+1+T} F_k(t+1+T) v(x_{t+1})(1-\delta_{kt})^T - c_{kt+1+T}\right]. \end{aligned}$$

However, the last term in square brackets must be zero, since a tractor of vintage $t+1$ is optimally scrapped when the marginal product of its remaining tractor services, $p_{ct+1+T} F_k(t+1+T) v(x_{t+1})(1-\delta_{kt})^T$, equals the marginal cost of operating it, c_{kt+1+T}.

Let the present discounted value of the cost of operating a tractor for $T-1$ periods starting at $t+1$ be given by

$$C(t+1, T-1) \equiv \sum_{j=0}^{T-1} R_{t+1}(j) c_{kt+1+j},$$

and the 'effective' depreciation of a vintage t tractor between t and $t+1$ be $\Delta_{t+1} \equiv \frac{v(x_t)(1-\delta_{kt})}{v(x_{t+1})}$. It follows that,

$$q_{kt}(t) = p_{kt}(t) \left[1 - R_t(1)(1-\delta_{kt}) \frac{\gamma_{ct}}{\gamma_{ct+1}}\right] + (1-\Delta_{t+1}) C(t+1, T-1)$$

which is expression (8) in the text.

This formula illustrates the forces at work in determining the rental price of a tractor:

- Increases in the price of a new tractor, $p_{kt}(t)$, increase the cost of operating it. This is the (standard) price effect.

- Periods of anticipated productivity increases —low values of $\frac{\gamma_{ct}}{\gamma_{ct+1}}$— result in increases in the rental price of tractors. This effect is the complete markets analog of the option value of waiting: buyers of a tractor at t know that, due to decreases in the price of new tractors in the future, the value of their used unit will be lower. In order to get compensated for this, they require a higher rental price.
- The term $(1 - \Delta_{t+1})C(t+1, T-1)$ captures the increase in cost *per unit of tractor services* associated with operating a one year old tractor, relative to a new tractor.

8.2.2 Estimation of Tractor Prices

In order to compute the user cost of tractor services, we need to estimate the effect that different factors have upon tractor prices. Our basic specification is

$$\ln p_{mkt} = -d_t + \sum_{j=0}^{N} \lambda_j \ln x_{jt}^m + \epsilon_{mt},$$

where p_{mkt} is the price of a model m tractor produced by manufacturer m at time t, the vector $x_t^m = (x_{1t}^m, x_{2t}^m, ...x_{Nt}^m)$ is a vector of characteristics of a particular model produced at time t, the d_t variables are time dummies, and ϵ_{mt} is a shock that we take to be independent of the x_{jt}^m variables and independent across models and years. We estimated the previous equation by OLS using a sample of 1345 tractor-year pairs covering the period 1920-1955. The basic data comes from two sources. William White very generously shared with us the data he collected which includes prices, sales volume and several technical variables. A description of the sample can be found in his dissertation, White (2000). We complemented White's sample with additional technical information obtained from the Nebraska Tractor Tests covering the 1920-1960 period, as reported by Dunning (1999). The variables in the x vector included technical specifications as well as manufacturer dummies. In the following table we present the point estimates of the technical variables.

In addition, we included 15 manufacturer dummies and 35 time dummies. We selected the variables we used from a larger set, from which we eliminated one of a pair whenever the simple

Variable	Estimate	t	Variable	Estimate	t
FuelCost	0.079	2.73	Row Crop (D)	-0.024	-2.05
Cylinders	0.030	0.61	High Clear (D)	0.008	0.31
Gears	0.108	3.82	Rubber Tires (D)	0.155	5.04
RPM	-0.156	-3.08	Tractor Fuel (D)	-0.153	-2.68
HP	0.573	19.92	Kerosene-Gasoline (D)	0.034	1.55
Plow Speed (Test)	0.111	2.25	Distillate-Gasoline (D)	-0.023	-1.40
Slippage (Test)	-0.021	-2.08	All Fuel (D)	-0.092	-1.70
Length	-0.116	-2.13	Diesel-Gasoline (D)	0.037	1.15
Weight	0.226	7.19	Diesel (D)	0.089	1.85
Speed	-0.055	-1.53	LPG (D)	-0.199	-1.91

Table 7: Regression Results. Point Estimates and t-statistics. A (D) denotes a dummy variable.

correlation coefficient between two variables exceeded 0.80. We experimented using a smaller set of variables as in White (2000), but our estimates of the time dummies were practically identical.

Our data covers the period 1920-1955. However, the period we are interested in studying is 1910-1960. Thus, we need to extend the average price and the time dummies to cover the missing years. The price data for 1910-1919 come from Olmstead and Rhode (2001), and it corresponds to the price of a 'medium tractor'. For the period 1056-1960 we used the price of an average tractor as published by the USDA. Inspection of the pattern of the time dummies —the line labeled Gamma in Figure 3— suggest a fairly non-linear trend. If we exclude the war-time years, it seems as if productivity was relatively constant since the mid 1930s. Thus, for the 1956-1960 period we assume that there was no change in γ_{ct}. The situation is quite different from 1910 to 1920, as this is a period of rapidly falling prices. We estimated the time dummies for the period 1910-1919 from a regression of the time dummies over the 1920-1935 period (before they 'stabilize') on time and time square. Our estimated values imply that most of the drop in tractor prices in this period is due to increases in productivity (more than 80%). This is consistent with the accounts that important changes in the tractor technology did not occurred until the 1920s.

8.3 Calibration

1. *Input moments.* The mapping between observed input ratios and shares and the corresponding objects in the model is given by and their analogues in the model are:

$$\begin{aligned}
\alpha_c(1-\alpha_{zy}-\alpha_{ny}) &= \textit{land share of output},\\
\frac{p_h\int_{e^*}^{\infty} h(q,c,w^F,e)\mu(de)}{\int_{e^*}^{\infty} y^*(q,c,w^F,e))\mu(de)} &= \textit{horses/output ratio},\\
\frac{p_k\int_{e^*}^{\infty} k(q,c,w^F,e)\mu(de)}{\int_{e^*}^{\infty} y^*(q,c,w^F,e))\mu(de)} &= \textit{tractors/output ratio},\\
\frac{w^F\int_{e^*}^{\infty} \bar{n}(q,c,w^F,e)\mu(de)}{\int_{e^*}^{\infty} y^*(q,c,w^F,e))\mu(de)} &= \textit{labor share of output},\\
\frac{\int_{e^*}^{\infty} \max[\bar{n}(q,c,w^F,e)-\hat{n},0]\mu(de)}{\int_{e^*}^{\infty} \bar{n}(q,c,w^F,e)\mu(de)} &= \textit{hired/total labor}
\end{aligned}$$

2. *Size Distribution.* Let $\tilde{e}_k$ be the skill level of a farmer who operates a farm of size $k\times 10^2$. Thus, $\tilde{e}_k$ solves

$$a(q,c,w^F,\tilde{e}_k)=k\times 10^2.$$

Then the average acreage of a farm, conditional on being in, say, the 500-999 acre category is

$$\bar{a}_{5-10}=\frac{\int_{\tilde{e}_5}^{\tilde{e}_{10}} a(q,c,w^F,e)\mu(de)}{\mu(\tilde{e}_{10})-\mu(\tilde{e}_5)},$$

while the share of land in farms in the 500-999 acre category is

$$s_{5-10}=\frac{\int_{\tilde{e}_5}^{\tilde{e}_{10}} a(q,c,w^F,e)\mu(de)}{\int_{e^*}^{\infty} a(q,c,w^F,e)\mu(de)}.$$

Finally, we also matched the second moment of the distribution. For simplicity —and given that the mean is matched by assumption— we chose to match the coefficient of variation. Thus, the model also matches

$$\frac{[\int_{e^*}^{\infty}[a(q,c,w^F,e)-\bar{a}]^2\mu(de)]^{1/2}}{\int_{e^*}^{\infty} a(q,c,w^F,e)\mu(de)} = \textit{C.V. of Farm Land}.$$

8.4 Random vs. Selective Migration

In this section we study a version of the model with random, as opposed to equilibrium, migration. We show that in terms of aggregates like horses, tractors and labor, the two models are observationally equivalent. We also show that they have different predictions for the ratio of hired to total labor.

Let each farmer's production function be given by

$$y(e,k,a) = BF(e,k,a),$$

where F is assumed homogeneous of degree one in all three inputs. We interpret k as tractors, a as land, and e as managerial skill.

The farmer's first order profit maximization conditions are

$$BF_k(1,\kappa,\alpha) = p_k, \tag{11a}$$

$$BF_a(1,\kappa,\alpha) = p_a, \tag{11b}$$

where $\kappa \equiv k/e$, and $\alpha \equiv a/e$ are tractors and land per unit of managerial skill. The system (**??**) determines a pair of input demand functions given by,

$$\kappa = \bar{\kappa}(p_k, p_a, B),$$

$$\alpha = \bar{\alpha}(p_k, p_a, B).$$

These functions are homogeneous of degree 0 in all three arguments.

We assume that there is an initial steady state characterized by a given distribution of skills. To be precise, let's denote the marginal farmer in the initial steady state by e_0^*. The total number of farmers is just $N_{f,0} = \bar{N} - \mu(e_0^*)$.

The problem. Suppose that due to 'some' additional constraint, the identity of the marginal farmer changes to e_S^*. Suppose that the baseline model is successful in matching both the supply of land, $\mathbf{A}$, and a given (but arbitrary) level of output, $\mathbf{Y}$, for some (endogenously determined) price of land, p_a^S, and TFP level, B^S. Then, we want to claim that there exists a value of B, which we will denote, B^R, and a price of land, p_a^R, such that another economy with $e_R^* = e_0^*$, and the number of farmers suitably 'scaled down,' is consistent with exactly the same pair $(\mathbf{A}, \mathbf{Y})$. Moreover, the total demand for tractors are also equal, that is $\mathbf{Y}^S = \mathbf{Y}^R$.

Derivation. It follows from the definition that

$$a(e) = \alpha e,$$

and hence, for the selection model,

$$A^S = \alpha^S \int_{e_S^*}^{\infty} ed\mu(e). \tag{12}$$

To ease notation, let's define the average quality of a manager as

$$\bar{e}_i = \frac{\int_{e_i^*}^{\infty} ed\mu(e)}{\bar{N} - \mu(e_i^*)}.$$

It then follows that

$$A^S = \alpha^S (\bar{N} - \mu(e_S^*))\bar{e}_S. \tag{13}$$

Since the distribution of skills under random migration is given by,

$$\mu_R(e) = \frac{\bar{N} - \mu(e_S^*)}{\bar{N} - \mu(e_0^*)}\mu(e),$$

the analog of (12) is,

$$A^R = \alpha^R \int_{e_0^*}^{\infty} ed\mu_R(e) = \alpha^R (\bar{N} - \mu(e_S^*))\bar{e}_0, \tag{14}$$

where $\bar{e}_0$ is the average quality of a manager in the original steady state. It follows from (13) and (14) that if the random migration model will succeed, then,

$$\alpha^S \bar{e}_S = \alpha^R \bar{e}_0. \tag{15}$$

Since the demand for tractors by a manger with index e is $k(e) = \frac{\kappa}{\alpha} a(e)$, then the prediction of model i for the aggregate stock of tractors is

$$K^i = \frac{\kappa^i}{\alpha^i} A.$$

It follows that $K^S = K^R$ if and only if the ratios $\frac{\kappa}{\alpha}$ are the same in both cases. Thus, given all the values for the selection model, the random migration model generates the same observations if there exist two values, (p_a^R, B^R), such that the following equations hold

$$\bar{\alpha}(p_k, p_a^R, B^R) = \frac{\alpha^S \bar{e}_S}{\bar{e}_0} = \alpha^R \tag{16a}$$

$$\bar{\kappa}(p_k, p_a^R, B^R) = \frac{\kappa^S \bar{e}_S}{\bar{e}_0} = \kappa^R. \tag{16b}$$

Total output predicted by model i is

$$Y^i = B^i F(\kappa^i, \alpha^i, 1)(\bar{N} - \mu(e_S^*))\bar{e}_i.$$

Imposing (16) and that the output level be the same across models we obtain

$$B^R F(\kappa^S, \alpha^S, \frac{\bar{e}_S}{\bar{e}_0}) = B^S F(\kappa^S, \alpha^S, 1). \tag{17}$$

Thus, to illustrate the conditions under which both models are identical, let's take (17) as the *definition* of B^R, and let (16a) *define* p_a^R. Thus, for the given pair (p_a^R, B^R) it is necessary to check that (16b) is satisfied. This last condition (recall that the price p_k is common to both models) is equivalent to,

$$\frac{F_k(\kappa^S, \alpha^S, \frac{\bar{e}_S}{\bar{e}_0})}{F(\kappa^S, \alpha^S, \frac{\bar{e}_S}{\bar{e}_0})} = \frac{F_k(\kappa^S, \alpha^S, 1)}{F(\kappa^S, \alpha^S, 1)}. \tag{18}$$

It is clear that, in general, (18) will not be satisfied and, hence, that the two versions of the model **are not** observationally equivalent. However, in the Cobb-Douglas case (18) holds and the models are observationally equivalent. More precisely, assume that

$$y(e, k, a) = Be^\theta G(k, a)^{1-\theta}, \qquad 0 < \theta < 1.$$

In this case, (18) is

$$\frac{G_k(\kappa^S, \alpha^S)}{G(\kappa^S, \alpha^S)} = \frac{G_k(\kappa^S, \alpha^S)}{G(\kappa^S, \alpha^S)}.$$

In the Cobb-Douglas case (17) implies

$$B^R = B^S \left(\frac{\bar{e}_S}{\bar{e}_0}\right)^\theta.$$

This condition shows the sense in which changes in the average quality of the managers is equivalent to changes in the level of TFP.

Moments of the Distribution of 'Acres/Farm.' The number of acres per farmer of skill e is given by

$$a^i(e) = \alpha^i e.$$

The variance of this random variable is (recall that $\bar{a} = \alpha^i \bar{e}_i$) is

$$\sigma_{a,i}^2 = \frac{(\alpha^i)^2 \int_{e_i^*}^{\infty} (e - \bar{e}_i)^2 d\mu(e)}{\bar{N} - \mu(e_i^*)},$$

while the coefficient of variation is

$$\frac{\sigma_{a,i}}{\bar{a}} = \frac{\left(\frac{\int_{e_i^*}^{\infty} (e - \bar{e}_i)^2 d\mu(e)}{\bar{N} - \mu(e_i^*)} \right)^{1/2}}{\frac{\int_{e_i^*}^{\infty} e d\mu(e)}{\bar{N} - \mu(e_i^*)}}.$$

Thus, for the random migration model, $e_{R,t}^* = e_0^*$ and, hence, the coefficient of variation is constant over time. On the other hand, for the selective migration model, $e_{S,t}^*$ increases over time and, in general, this results in a variable $\frac{\sigma_{a,S}}{\bar{a}}$.

Finally, consider the implication of the two models for the ratio of hired to total workers. Let's assume that both models yield the same prediction about the aggregate number of hours. Let ξ^i be the demand for labor per unit of farm skill e. If total demand for labor coincide, it must be the case that

$$\xi^S \bar{e}_S = \xi^R \bar{e}_R,$$

and that the number of farms who hire workers is given by

$$\begin{cases} 1 - \mu(\breve{e}_S) & \textit{equilibrium} \\ 1 - \mu(\breve{e}_S \frac{\bar{e}_R}{\bar{e}_S}) & \text{rand}\,om \end{cases}$$

where $\xi^S \breve{e}_S = \hat{n}$. Since $\bar{e}_S > \bar{e}_R$ the random migration model overpredicts the fraction of farms with hired workers.

8.5 Counterfactuals: Detailed Results

In this section we present the unadjusted tables corresponding the counterfactuals in the text.

Ratio: 1960 to 1910	Baseline	w_{1910}	U.S. Data
Farm Size	2.56	1	2.13
Stock of Tractors	272	632	280
Stock of Horses	1/7.90	1/1.69	1/7.8
Stock of Labor	1/2.92	1/0.43	1/2.5
Price of Land	0.76	2.24	0.8
Output	2.1	5.53	2.1

Table 8: The Impact of Wages, 1960 versus 1910 (No TFP adjustment)

Ratio: 1960 to 1910	Baseline	$v(x_{1910})$	U.S. Data
Farm Size	2.56	3.43	2.13
Stock of Tractors	272	139	280
Stock of Horses	1/7.90	1/6.21	1/7.8
Stock of Labor	1/2.92	1/4.16	1/2.5
Price of Land	0.76	0.54	0.8
Output	2.1	1.58	2.1

Table 9: The Impact of Quality Adjustment, 1960 versus 1910 (No TFP adjustment)

Ratio: 1960 to 1910	Baseline	Random	U.S. Data
Farm Size	2.56	2.73	2.15
Stock of Tractors	272	143	280
Stock of Horses	1/7.90	1/14.7	1/7.8
Stock of Labor	1/2.92	1/5.9	1/2.5
Price of Land	0.76	0.57	0.8
Farm Output	2.1	1.19	2.1

Table 10: Selective versus Random Migration (No TFP adjustment)

Ratio:1960 to 1910	Baseline	p_{h1910}	U.S. Data
Acres/farm	2.56	2.61	2.13
Stock of Tractors	272	274	280
Stock of Horses	1/7.90	1/8.95	1/7.8
Stock of Labor	1/2.92	1/2.81	1/2.5
Price of Land	0.76	0.77	0.8
Farm Output	2.1	2.093	2.1

Table 11: The impact of Horse Prices (No TFP adjustment)

Ratio: 1960 to 1910	Baseline	$\rho = 0$	U.S. Data
Farm Size	2.56	2.57	2.13
Stock of Tractors	272	6.86	280
Stock of Horses	1/7.90	1/1.43	1/7.8
Stock of Labor	1/2.92	1/1.81	1/2.5
Price of Land	0.76	0.99	0.8
Output	2.1	2.51	2.1

Table 12: $\rho = 0$, 1910 versus 1960 (No TFP adjustment)

Ratio: 1960 to 1910	Baseline	$\alpha_c = 0.999$	U.S. Data
Farm Size	2.56	8.14	2.13
Stock of Tractors	272	221	280
Stock of Horses	1/7.90	1/9.62	1/7.8
Stock of Labor	1/2.92	1/3.34	1/2.5
Price of Land	0.76	0.72	0.8
Output	2.1	1.67	2.1

Table 13: $\alpha_c = 0.999$, 1910 versus 1960 (No TFP adjustment)