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### ABSTRACT

What are the economic effects of an interest rate cut when an economy is in the midst of a financial crisis? Under what conditions will a cut stimulate output and employment, and raise welfare? Under what conditions will a cut have the opposite effects? We answer these questions in a general class of open economy models, where a financial crisis is modeled as a time when collateral constraints are suddenly binding. We find that when there are frictions in adjusting the level of output in the traded good sector and in adjusting the rate at which that output can be used in other parts of the economy, then a cut in the interest rate is most likely to result in a welfare-reducing fall in output and employment. When these frictions are absent, a cut in the interest rate improves asset positions and promotes a welfare-increasing economic expansion.

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# 1. Introduction

In recent years there has been considerable controversy over the appropriate monetary policy in the aftermath of a financial crisis. Some argue that the central bank should raise domestic interest rates to defend the currency and halt the flight of capital. Others argue that interest rate reductions are called for. They note that a country that has just experienced a financial crisis is typically sliding into a steep recession. They appeal to the widespread view that in developed economies like the US, central banks typically respond to situations like this by reducing interest rates. These authors urge the same medicine for emerging market economies in the wake of a financial crisis. They argue that to raise interest rates at such a time is a mistake, and is likely to make a bad situation even worse. One expositor of this view, Paul Krugman (1999, pp.103-105), puts it this way:

“But when financial disaster struck Asia, the policies those countries followed in response were almost exactly the reverse of what the United States does in the face of a slump. Fiscal austerity was the order of the day; interest rates were increased, often to punitive levels....Why did these extremely clever men advocate policies for emerging market economies that would have been regarded as completely perverse if applied at home?”

We describe a framework that allows us to articulate the two views just described. The framework has two building blocks. First, we assume that to carry out production, firms require domestic working capital to hire labor and international working capital to purchase an imported intermediate input. Second, we adopt the asset market frictions formalized in the limited participation model as analyzed in Lucas (1990), Fuerst (1992) and Christiano and Eichenbaum (1992, 1995). The limited participation assumption has the consequence that an expansionary monetary action makes the domestic banking system relatively liquid and induces firms to hire more labor. To the extent that the imported intermediate input complements labor, the interest rate drop leads to the increased use of this factor too. This is in the spirit of the traditional liquidity channel emphasized in the closed economy literature, which stresses the positive effects of an interest rate cut on output. So, absent other considerations, the model rationalizes the Krugman view outlined above.

Our model has an additional feature which may be particularly relevant during a crisis. We suppose that a crisis is a time when international loans must be collateralized by physical assets such as land and capital, and that this restriction is binding. To understand how collateral affects the monetary transmission mechanism in our model, it is useful to consider a simplified version of our collateral constraint expressed in units of the foreign currency:

$$\frac{Q}{S}K \geq R^*z + B.$$

Here,  $B$  represents the stock of long-term external debt;  $z$  represents short-term external borrowing to finance a foreign intermediate input;  $R^*$  represents the associated interest rate;  $K$  represents domestic physical assets like land and capital;  $Q$  is the value (in domestic currency units) of a unit of  $K$ ; and  $S$  represents the nominal exchange rate. We suppose that under normal conditions, the collateral constraint is not binding, while it suddenly binds with the onset of a crisis. This may be because in normal times, output in addition to land and capital

is acceptable as collateral. Then, in a crisis the fraction of domestic assets accepted as collateral by foreigners suddenly falls.<sup>1</sup> In any case, in our analysis we model the imposition of a binding collateral constraint as an exogenous, unforeseen event.<sup>2</sup>

We then compare the ensuing transition path of the economy under two scenarios. In the benchmark scenario, the monetary authority does not adjust policy in response to the collateral shock. In the alternative scenario, the monetary authority reduces the domestic rate of interest relative to what it is in the benchmark scenario. We find that in the benchmark scenario, output and employment are low during the transition to the new steady state. The shadow-cost of international debt,  $B$ , is higher while the collateral constraint is binding, and the economy responds by increasing the current account and paying down the debt. In the new steady state the debt is reduced to the point where the collateral constraint is marginally nonbinding. That is, the collateral constraint is satisfied as an equality, but with a zero multiplier.

Although the transition path after a collateral shock is of independent interest because it captures key features of actual economies in the aftermath of a crisis, it is not the central focus of our analysis. Our key objective is to understand the impact on the transition of a cut in the interest rate. We study this by comparing the dynamic equilibrium of the economy under the benchmark and alternative scenarios. We now briefly describe the results. In doing so, we make use of the fact that  $R^*$  and  $K$  are held fixed throughout the paper. We also find it convenient in summarizing the results here to ignore the impact of the interest rate cut on  $B$ .<sup>3</sup> Finally, in describing the intuition for the results we make use of our numerical finding that whenever there is a monetary policy-induced cut in the interest rate, there is a depreciation of the currency, i.e., a jump in  $S$ . Using these observations and the collateral constraint evaluated at equality, it is easy to see why it is that for some versions of our model an interest rate cut produces a contraction, and for others it produces an expansion.

The contraction outcome is perhaps the easiest to understand. When  $S$  jumps, the left side of the collateral constraint falls. Supposing that  $Q$  does not jump very much, this means that the right side must be reduced, i.e.,  $z$  must fall. Our assumption that the imported intermediate

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<sup>1</sup>Our characterization of a crisis as a time when collateral constraints suddenly bind is not unprecedented. For example, Caballero (2000, p. 5) states that a crisis is a time of “...sudden loss in the international appeal of a country’s assets.” He also states that a (p.4) “crisis is a time when (a) a significant fraction of firms or economic agents are in need of financing to either repay debt or implement new investments needed to save high return projects – I will refer to these agents as ‘distressed firms’ – and (b) on net, the economy as a whole needs substantial external resources but does not have enough assets and commitment to obtain them.”

<sup>2</sup>In some respects our framework resembles a reduced form representation of the environment considered in Albuquerque and Hopenhayn (1997) and further developed in Cooley, Marimon and Quadrini (2001) and Monge (2001). There, an investment project requires an initial fixed investment, followed by a sequence of expenditures to make the investment project productive. The papers in this literature derive the optimal dynamic contract between the entrepreneur and a bank, as well as a sequential decentralization. In the latter, the initial fixed investment is financed by long term debt that resembles our  $B$ , and the sequence of expenditures is financed by working capital loans with the entrepreneur being restricted by a collateral constraint that resembles the one we adopt. This literature suggests a variety of factors that could cause collateral constraints to suddenly become binding. For example, if there is a shock that causes the court system to be overwhelmed by bankruptcy filings and other business in a recession, collateral constraints could suddenly bind because lenders now understand that the default option is more attractive to the marginal entrepreneur who wishes to borrow.

<sup>3</sup>As noted earlier in the introduction, in the full analysis reported in the body of the paper,  $B$  is treated as a variable that moves endogenously over time.

good is important in domestic employment and production, ensures that a recession follows. In this outcome, the currency mismatch between assets and liabilities in the collateral constraint plays the central role.

That an expansion outcome is possible is also easy to see. If the nominal interest rate cut succeeds in reducing the real interest rate used to discount future flows, then asset prices,  $Q$ , may in fact jump a substantial amount. Indeed, in closed economy settings when there are no currency mismatches in balance sheets, it is often considered the ‘natural’ outcome that a cut in the interest rate lifts asset prices and improves balance sheets. If the rise in  $Q$  is sufficiently strong to offset the nominal depreciation, then the left side of the collateral constraint is increased by the interest rate cut. In this case, there is room in the collateral constraint for  $z$  to go up, and for domestic production to rise.

The above discussion suggests that the contraction outcome is most likely in economies where an increase in  $z$  does not lead to a substantial increase in  $Q$ , the value of productive capital and land. Two features promote this possibility in our model environment. The first occurs if increases in  $z$  encounter strong decreasing returns in production, and complementary factors of production cannot be brought in to offset this. The second occurs if there is little substitutability between traded and nontraded goods in the production of final goods. By inhibiting the ability of the economy to effectively exploit increases in  $z$ , these two features reduce the likelihood that an increase in  $z$  is associated with a substantial rise in  $Q$ . We find that when these frictions are not present, then an interest rate cut tends to be associated with the expansion outcome.

The role of asset prices in propagating shocks is a topic that is of independent interest. The existing literature focuses on the role of asset prices in magnifying and propagating the effects of shocks.<sup>4</sup> We obtain the magnification effect here too, in the version of the model that implies the expansion outcome. In that model, the response of output and employment to an interest rate cut is the same sign and stronger than what it is when the collateral constraint is ignored altogether. Interestingly, in the version of the model that implies the contraction outcome, the collateral constraint actually has the effect of changing the sign of the economy’s response.

The organization of the paper is as follows. The next section presents our general model. Section three presents a version of the model simplified by the assumption that the stock of external long-term debt is held constant. The advantage of this simplification is that the model can be studied analytically. The insights that are obtained from this are useful for understanding the more relevant version of the model, in which the long-term external debt is determined endogenously. Numerical methods are used to study this version of the model in the third section of the paper. The final section concludes.

## 2. The Model

We adopt a standard traded-good/non-traded good small open economy model. The model has households, firms, a financial intermediary, and a domestic monetary authority.

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<sup>4</sup>For recent papers on closed economy models that emphasize the role of asset prices in magnifying and propagating shocks, see Bernanke, Gertler and Gilchrist (1999) and Carlstrom and Fuerst (1997, 2000) and the literature that they cite. For open economy models that assign an important role to asset prices, see Mendoza and Smith (2000) and the literature they cite.

## 2.1. Households

There is a representative household, which derives utility from consumption,  $c_t$ , and leisure as follows:

$$\sum_{t=0}^{\infty} \beta^t u(c_t, L_t), \quad (2.1)$$

where  $L_t$  denotes labor. We adopt the following specification of utility:

$$u(c, L) = \frac{\left[ c - \frac{\psi_0}{1+\psi} L^{1+\psi} \right]^{1-\sigma}}{1-\sigma}. \quad (2.2)$$

The household begins the period with a stock of liquid assets,  $\tilde{M}_t$ . Of this, it deposits  $D_t$  with the financial intermediary, and the rest,  $\tilde{M}_t - D_t$ , is allocated to consumption expenditures. The cash constraint that the household faces on its consumption expenditures is:

$$P_t c_t \leq W_t L_t + \tilde{M}_t - D_t, \quad (2.3)$$

where  $W_t$  denotes the money wage rate and  $P_t$  denotes the price level.

The household also faces a flow budget constraint governing the evolution of its assets:

$$\tilde{M}_{t+1} = R_t(D_t + X_t) + P_t^T \pi_t + [W_t L_t + \tilde{M}_t - D_t - P_t c_t]. \quad (2.4)$$

Here,  $R_t$  denotes the gross domestic rate of interest,  $\pi_t$  denotes lump-sum dividend payments received from firms, and  $X_t$  is a liquidity injection from the monetary authority. Also,  $\pi_t$  is measured in units of traded goods, and  $P_t^T$  is the domestic currency price of traded goods. The term on the right of the equality reflects the household's sources of liquid assets at the beginning of period  $t + 1$ : interest earnings on deposits and on the liquidity injection, profits and any cash that may be left unspent in the period  $t$  goods market.

The household maximizes (2.1) subject to (2.3)-(2.4), and the following timing constraint. A given period's deposit decision is made before that period's liquidity injection is realized, while all other decisions are made afterward. The Euler equation associated with the labor decision is:

$$\psi_0 L_t^\psi = \frac{W_t}{P_t}. \quad (2.5)$$

We refer to this as the labor supply equation. The intertemporal Euler equation associated with the deposit decision is:

$$u_{c,t} = \beta R_t u_{c,t+1} \frac{P_t}{P_{t+1}}. \quad (2.6)$$

## 2.2. Firms

There are two types of representative, competitive firms. The first produces the final consumption good,  $c$ , purchased by households. Final goods production requires tradeable and non-tradeable intermediate goods which are produced by the second type of representative firm. We now discuss these two types of firms.

### 2.2.1. Final Good Firms

The production function of the final good firms is:

$$c = \left\{ \left[ (1 - \gamma) c^T \right]^{\frac{\eta-1}{\eta}} + \left[ \gamma c^N \right]^{\frac{\eta-1}{\eta}} \right\}^{\frac{\eta}{\eta-1}}, \quad 1 \geq \eta \geq 0, \quad 0 < \gamma < 1, \quad (2.7)$$

where  $c^T$  and  $c^N$  denote quantities of tradeable and non-tradeable intermediate inputs, respectively. One interpretation is that these firms are retailers that package traded and non-traded intermediate goods into a final consumption good. Here,  $\eta$  denotes the elasticity of substitution in production between the two intermediate inputs. For later purposes, it is useful to note that as  $\eta \rightarrow 0$ ,

$$c = \min \left\{ (1 - \gamma) c^T, \gamma c^N \right\}.$$

As noted in the introduction, this specification of the technology for producing final goods increases the likelihood that the economy will contract when there is a cut in the domestic rate of interest.

Let  $P^T$  and  $P^N$  denote the prices of traded and non-traded goods. Zero profits and efficiency imply that the price of  $c$ ,  $P$ , and these input prices have the following relationship:

$$p = \left[ \left( \frac{1}{1 - \gamma} \right)^{1-\eta} + \left( \frac{P^N}{\gamma} \right)^{1-\eta} \right]^{\frac{1}{1-\eta}}, \quad p = \frac{P}{P^T}. \quad (2.8)$$

For  $\eta \neq 1$ , efficiency also dictates:

$$p^N = \frac{\gamma}{1 - \gamma} \left( \frac{(1 - \gamma) c^T}{\gamma c^N} \right)^{\frac{1}{\eta}}, \quad p^N = \frac{P^N}{P^T}. \quad (2.9)$$

When  $\eta = 0$ , this expression is replaced by  $(1 - \gamma) c^T = \gamma c^N$ . The object,  $P$ , in the model corresponds to the model's 'consumer price index', denominated in units of the domestic currency. The object,  $p$ , is the consumer price index denominated in units of the traded good.

### 2.2.2. Intermediate Inputs

A single representative firm produces the traded and non-traded intermediate inputs. That firm manages three types of debt, two of which are short-term. The firm borrows at the beginning of the period to finance its wage bill and to purchase a foreign input, and repays these loans at the end of the period. In addition, the firm holds the outstanding stock of external (net) indebtedness,  $B_t$ . In terms of assets, the firm owns all the economy's physical capital. This specification of the firm allows us to abstract from problems associated with the poor distribution of collateral among firms, that is emphasized by Caballero and Krishnamurthy (2001).

The firm's optimization problem is:

$$\max \sum_{t=0}^{\infty} \beta^t \Lambda_{t+1} \pi_t, \quad (2.10)$$

where

$$\pi_t = p_t^N y_t^N + y_t^T - w_t R_t L_t - R^* z_t - r^* B_t + (B_{t+1} - B_t), \quad (2.11)$$

denotes dividends, denominated in units of traded goods. Here,  $w_t = W_t/P_t^T$  is the wage rate, denominated in units of the traded good. Also,  $B_t$  is the stock of external debt at the beginning of period  $t$ , denominated in units of the traded good;  $R^*$  is the gross rate of interest (fixed in units of the traded good) on loans for the purpose of purchasing  $z_t$ ; and  $r^*$  is the net rate of interest (again, fixed in terms of the traded good) on the outstanding stock of external debt. The price,  $\Lambda_{t+1}$ , is taken parametrically by firms. In equilibrium, it is the multiplier on  $\pi_t$  in the (Lagrangian representation of the) household problem:<sup>5</sup>

$$\begin{aligned} \Lambda_{t+1} &= \frac{u_{c,t+1}}{p_{t+1}} \frac{P_t^T}{P_{t+1}^T} \beta \\ &= \frac{u_{c,t+1}}{p_{t+1}} \frac{p_t^T}{p_{t+1}^T} \frac{1}{1+x_t} \beta, \end{aligned} \quad (2.12)$$

where

$$p_t^T = \frac{P_t^T}{M_t}.$$

Here,  $M_t$  is the aggregate stock of money at the beginning of period  $t$ , which evolves according

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<sup>5</sup>The intuition underlying (2.12) is straightforward. The object  $\Lambda_{t+1}$  in (2.12), is the marginal utility of one unit of dividends, denominated in traded goods, transferred by the firm to the household at the end of period  $t$ . This corresponds to  $P_t^T \pi_t$  units of domestic currency. The households can use this currency in period  $t+1$  to purchase  $P_t^T \pi_t / P_{t+1}$  units of the consumption good. The value, in period  $t$ , of these units of consumption goods is  $\beta u_{c,t+1} P_t^T \pi_t / P_{t+1}$ , or  $\beta u_{c,t+1} P_t^T \pi_t / (p_{t+1} P_{t+1}^T)$ , where  $u_{c,t}$  is the marginal utility of consumption. This is the first expression in (2.12).



to:

$$\frac{M_{t+1}}{M_t} = 1 + x_t. \quad (2.13)$$

With one exception, we adopt the convention that a price expressed in lower case indicates the price has been scaled by the price of traded goods. The exception,  $p_t^T$ , is the domestic currency price of traded goods, scaled by the beginning of period stock of money. Alternatively,  $p_t^T$  is the inverse of a measure of real balances.

The firm production functions are:

$$\begin{aligned} y^T &= \left\{ \theta [\mu_1 V]^{\frac{\xi-1}{\xi}} + (1-\theta) [\mu_2 z]^{\frac{\xi-1}{\xi}} \right\}^{\frac{\xi}{\xi-1}}, \\ V &= A (K^T)^\nu (L^T)^{1-\nu}, \\ y^N &= (K^N)^\alpha (L^N)^{1-\alpha}, \end{aligned} \quad (2.14)$$

where  $\xi$  is the elasticity of substitution between value-added in the traded good sector,  $V_t$ , and the imported intermediate good,  $z_t$ . In the production functions,  $K^T$  and  $K^N$  denote capital in the traded and non-traded good sectors, respectively. They are owned by the representative intermediate input firm. We keep the stock of capital fixed throughout the analysis. It does not depreciate and there exists no technology for making it bigger.

Our specification of technology is designed to encompass a variety of cases. In one, there is no substitutability between  $z$  and  $V$  in production, i.e.,  $\xi = 0$ , so that

$$y^T = \min \{ \mu_1 V, \mu_2 z \}. \quad (2.15)$$

An optimizing producer sets  $V = (1/\mu_1)y^T$  and  $z = (1/\mu_2)y^T$ , so that the share of value-added,  $V$ , in total output,  $y^T$ , is  $1/\mu_1$  and the share of imported intermediate inputs in total output is  $1/\mu_2$ . We impose that these shares sum to unity. In another specification,  $z$  is the only variable factor of production and occurs when  $\xi = \mu_1 = \mu_2 = \nu = 1$ :

$$y^T = (AK^T)^\theta z^{1-\theta}. \quad (2.16)$$

Later, we shall see that (2.15) is associated with the expansion outcome, the outcome in which a cut in the interest rate produces an increase in output and employment. This is because, in (2.15) a fall in productivity can be avoided when  $z$  increases as long as  $V$  is increased simultaneously. In the case of (2.16), there are no complementary factors that can be adjusted to overcome the fall in productivity associated with an increase in  $z$ , when  $\theta > 0$ . As explained in the introduction, this production function is associated with the contraction outcome, that is, one in which a cut in the interest rate produces a fall in output and employment.

We impose the following restriction on borrowing:

$$\frac{B_{t+1}}{(1+r^*)^t} \rightarrow 0, \text{ as } t \rightarrow \infty. \quad (2.17)$$

We suppose that international financial markets impose that this limit cannot be positive. That it cannot be negative is an implication of firm optimality.

The firm's problem at time  $t$  is to maximize (2.10) by choice of  $B_{t+j+1}$ ,  $y_{t+j}^N$ ,  $y_{t+j}^T$ ,  $z_{t+j}$ ,  $L_{t+j}^T$  and  $L_{t+j}^N$ ,  $j = 0, 1, 2, \dots$ , subject to the various constraints just described. In addition, the firm takes all prices and rates of return as given and beyond its control. The firm also takes the initial stock of debt,  $B_t$ , as given. This completes the description of the firm problem in the pre-crisis version of the model, when collateral constraints are ignored.

The crisis brings on the imposition of the following collateral constraint:

$$\tau^N q_t^N K^N + \tau^T q_t^T K^T \geq R^* z_t + (1+r^*)B_t + w_t R_t L_t, \quad (2.18)$$

where  $L_t \equiv L_t^T + L_t^N$ . Here,  $q^i$ ,  $i = N, T$  denote the value (in units of the traded good) of a unit of capital in the non-traded and traded good sectors, respectively. Also,  $\tau^i$  denotes the fraction of these stocks accepted as collateral by international creditors. The left side of (2.18) is the total value of collateral, and the right side is the payout value of the firm's debt. It is the total amount that the firm would have to pay, to completely eliminate all its debt by the end of period  $t$ . Before the crisis, firms ignore (2.18), and assign a zero probability that it will be implemented. With the coming of the crisis, firms believe that (2.18) must be satisfied in every period henceforth, and do not entertain the possibility that it will be removed.

The equilibrium value of the asset prices,  $q_t^i$ ,  $i = N, T$ , is the amount that a potential firm would be willing to pay in period  $t$ , in units of the traded good, to acquire a unit of capital and start production in period  $t$ . We let  $\lambda_t \geq 0$  denote the multiplier on the collateral constraint ( $= 0$  in the pre-crisis period) in firm problem. Then,  $q_t^i$  is the derivative of the Lagrangian representation of the firm's problem with respect to  $K_t^i$ :

$$q_t^i = VMP_{k,t}^i + \lambda_t \tau^i q_t^i + \frac{\beta}{\Lambda_{t+1}} \sum_{j=1}^{\infty} \beta^{j-1} \Lambda_{t+1+j} \left\{ VMP_{k,t+j}^i + \lambda_{t+j} \tau^i q_{t+j}^i \right\} \quad (2.19)$$

or,

$$q_t^i = \frac{VMP_{k,t}^i + \beta \frac{\Lambda_{t+2}}{\Lambda_{t+1}} q_{t+1}^i}{1 - \lambda_t \tau^i}, \quad i = N, T. \quad (2.20)$$

Here,  $VMP_{k,t}^i$  denotes the period  $t$  value (in terms of traded goods) marginal product of capital in sector  $i$ . With our assumptions on technology, these are:

$$VMP_{k,t}^N = \alpha p_t^N \frac{y_t^N}{K^N},$$

$$VMP_{k,t}^T = \begin{cases} \left(\frac{y_t^T}{\mu_1 V_t}\right)^{\frac{1}{\xi}} \theta \nu \frac{\mu_1 V_t}{K^T}, & \xi \neq 0 \\ \nu \frac{\mu_1 V_t}{K^T} \left[1 - \frac{(1+\lambda_t)R^*}{\mu_2}\right], & \xi = 0 \end{cases}.$$

When  $\lambda_t \equiv 0$ , (2.19) is just the standard asset pricing equation. It is the present discounted value of the value of the marginal physical product of capital. When the collateral constraint is binding, so that  $\lambda_t$  is positive, then  $q_t^i$  is greater than this. This reflects that in this case capital is not only useful in production, but also for relieving the collateral constraint. In our model capital is never actually traded, since all firms are identical. However, if there were trade, then the price of capital would be  $q_t^i$ . If a firm were to default on its credit obligations, the notion is that foreign creditors could compel the sale of its physical assets in a domestic market for capital. The price,  $q_t^i$ , is how much traded goods a domestic resident is willing to pay for a unit of capital. Foreign creditors would receive those goods in the event of a default. We assume that with these consequences for default, default never occurs in equilibrium.

We now derive the Euler equations of the firm. Differentiating the date 0 Lagrangian representation of the firm problem with respect to  $B_{t+1}$ :

$$1 = \beta \frac{\Lambda_{t+2}}{\Lambda_{t+1}} (1 + r^*) (1 + \lambda_{t+1}), \quad t = 0, 1, 2, \dots \quad (2.21)$$

Following standard practice with small open economy models, we assume  $\beta(1 + r^*) = 1$ , so that<sup>6</sup>

$$\Lambda_{t+1} = \Lambda_{t+2} (1 + \lambda_{t+1}), \quad t = 0, 1, 2, \dots \quad (2.22)$$

A high value for  $\lambda$ , which occurs when the collateral constraint is binding, raises the effective rate of interest on debt. The interpretation is that when  $\lambda$  is large, then the debt has an additional cost, beyond the direct interest cost. This cost reflects that when the firm raises  $B_{t+1}$  in period  $t$ , it not only incurs an additional interest charge in period  $t + 1$ , but it is also further tightens its collateral constraint in that period. This has a cost because, via the collateral constraint, the extra debt inhibits the firm's ability to acquire working capital in period  $t + 1$ . Thus, when  $\lambda$  is high, there is an additional incentive for firms to reduce  $\pi$  and 'save' by paying down the external debt. Although the firm's actual interest rate on external debt taken on in period  $t$  is  $1 + r^*$ , its 'effective' interest rate is  $(1 + r^*) (1 + \lambda_{t+1})$ .

The firm's first order conditions for labor in the non-traded and traded sectors, and for  $z$  are, when  $\xi \neq 0$ :

$$(1 - \alpha) p_t^N \frac{y_t^N}{L_t^N} = w_t (1 + \lambda_t) R_t \quad (2.23)$$

$$\left(\frac{y_t^T}{\mu_1 V_t}\right)^{\frac{1}{\xi}} \theta (1 - \nu) \frac{\mu_1 V_t}{L_t^T} = w_t (1 + \lambda_t) R_t \quad (2.24)$$

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<sup>6</sup>See, for example, Obstfeld and Rogoff (1997).

$$\left(\frac{y_t^T}{\mu_2 z_t}\right)^{\frac{1}{\xi}} (1 - \theta) \mu_2 = (1 + \lambda_t) R^* \quad (2.25)$$

The presence of  $R_t$  on the right side of (2.23)-(2.24) reflects that to hire labor, firms must borrow cash in advance in the domestic money market, at the gross interest rate,  $R_t$ . When the collateral constraint is binding, then the effective interest rate is higher than  $R_t$ . The gross interest rate on short term foreign loans,  $R^*$ , appears on the right of (2.25) because firms must borrow foreign funds in advance to acquire  $z_t$ . Note that the effective foreign interest rate is higher than the actual interest rate when the collateral constraint is binding.

When  $\xi = 0$ , then of course (2.23) still holds, but (2.24) and (2.25) are replaced by:

$$(1 - \nu) \frac{\mu_1 V_t}{L_t^T} \left[ 1 - \frac{(1 + \lambda_t) R^*}{\mu_2} \right] = w_t (1 + \lambda_t) R_t \quad (2.26)$$

$$\mu_1 V_t = \mu_2 z_t \quad (2.27)$$

Ignoring the term in square brackets in (2.26), this is just the marginal product of  $L^T$  in producing  $\mu_1 V_t$ . The term in square brackets reflects that expansions in  $y^T$  also requires an increase in  $z$ .

### 2.3. Financial Intermediary and Monetary Authority

The financial intermediary takes domestic currency deposits,  $D_t$ , from the household at the beginning of period  $t$ . In addition, it receives the liquidity transfer,  $X_t = x_t M_t$ , from the monetary authority.<sup>7</sup> It then lends all its domestic funds to firms who use it to finance their employment working capital requirements,  $W_t L_t$ . Clearing in the money market requires  $D_t + X_t = W_t L_t$ , or, after scaling by the aggregate money stock,

$$d_t + x_t = w_t p_t^T L_t, \quad (2.28)$$

where  $d_t = D_t/M_t$ .

The monetary authority in our model simply injects funds into the financial intermediary. Its period  $t$  decision is taken after the household has selected a value for  $D_t$ , and before all other variables in the economy are determined. This is the standard assumption in the limited participation literature. It is interpreted as reflecting a sluggishness in the response of household portfolio decisions to changes in market variables. With this assumption, a value of  $x_t$  that deviates from what households expected at the time  $D_t$  was set produces an immediate reaction

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<sup>7</sup>In practice, injections of liquidity do not occur in the form of lump sum transfers, as they do in our model. It is easy to show that our formulation is equivalent to an alternative, in which the injection occurs as a result of an open market purchase of government bonds which are owned by the household, but held by the financial intermediary. We do not adopt this interpretation in our formal model in order to conserve on notation.

by firms and the financial intermediary but not, in the first instance, by households. The name, ‘limited participation’, derives from this feature, namely that not all agents react immediately to (or, ‘participate in’) a monetary shock. As a result of this timing assumption, many models exhibit the following behavior in equilibrium. An unexpectedly high value of  $x_t$  swells the supply of funds in the financial sector, since  $D_t$  on the left side of (2.28) cannot fall in response to a positive  $x_t$  shock. To get firms to absorb the increase in funds, a fall in the equilibrium rate of interest is required. When that fall does occur, they borrow the increased funds and use them to hire more labor and produce more output.

We abstract from all other aspects of government finance. The only policy variable of the government is  $x_t$ .

## 2.4. Equilibrium

We consider a perfect foresight, sequence-of-market equilibrium concept. In particular, it is a sequence of prices and quantities having the properties: (i) for each date, the quantities solve the household and firm problems, given the prices, and (ii) the labor, goods and domestic money markets clear.

Clearing in the money market requires that (2.28) hold and that actual money balances,  $M_t$ , equal desired money balances,  $\tilde{M}_t$ . Combining this with the household’s cash constraint, (2.3), we obtain the equilibrium cash constraint:

$$p_t^T p_t c_t = 1 + x_t. \quad (2.29)$$

According to this, the total, end of period stock of money must equal the value of final output,  $c_t$ . Market clearing in the traded good sector requires:

$$y_t^T - R^* z_t - r^* B_t - c_t^T = -(B_{t+1} - B_t). \quad (2.30)$$

The left side of this expression is the current account of the balance of payments, i.e., total production of traded goods, net of foreign interest payments, net of domestic consumption. The right side of (2.30) is the change in net foreign assets. Equation (2.30) reflects our assumption that external borrowing to finance the intermediate good,  $z_t$ , is fully paid back at the end of the period. That is, this borrowing resembles short-term trade credit. Note, however, that this is not a binding constraint on the firm, since our setup permits the firm to finance these repayments using long term debt. Market clearing in the non-traded good sector requires:

$$y_t^N = c_t^N. \quad (2.31)$$

It is instructive to study this model’s implications for interest parity. Combining the house-

hold and firm intertemporal conditions, (2.6) and (2.21), with (2.12), we obtain

$$R_{t+1} = (1 + r^*) \frac{P_{t+1}^T}{P_t^T} (1 + \lambda_{t+1}), \quad t = 0, 1, 2, \dots \quad (2.32)$$

On the right hand side, of this expression,  $(1 + r^*)P_{t+1}^T/P_t^T$  is the rate of interest on external debt, expressed in domestic currency units. Expression (2.32) with  $\lambda = 0$  is the usual interest rate parity relation. When  $\lambda > 0$ , there is a collateral premium on the domestic rate of interest. Expression (2.32) highlights our implicit assumption that foreign and domestic markets for loanable funds are isolated, at least in times when the collateral constraint is binding. When  $\lambda > 0$ , so that the domestic interest rate exceeds the foreign rate, lenders of foreign currency would prefer to exchange their currency for domestic currency and lend in the domestic currency market. Similarly, firms borrowing domestic funds for the purpose of paying their wage bill would prefer to borrow in the foreign currency market and convert the proceeds into domestic currency. That  $\lambda > 0$  is possible in equilibrium reflects that we rule out this type of cross-border borrowing and lending.<sup>8</sup>

As an empirical proposition, interest rate parity does poorly. In response to this, researchers often introduce exogenously a term like our  $\lambda$  in (2.32). In conventional practice,  $\lambda$  is interpreted as reflecting a risk premium. Our setup may provide an alternative interpretation.

Details about computing equilibrium for this model are reported in the appendix.

### 3. Qualitative Analysis of the Equilibrium

Our full model is not analytically tractable and so to understand its implications for the questions we ask requires numerical simulation. However, in the special case in which long-term external debt is constant, it is possible to obtain analytic results, at least locally. This is the case considered in this section. The next section considers the case where the debt is a choice variable. We identify a set of sufficient conditions which guarantee that a cut in the domestic rate of interest is contractionary. Under these assumptions,  $z$  is the only variable factor of production in the production of traded goods and it is subject to diminishing returns; traded and non-traded goods are not very substitutable in the production of final goods; and the size of the external debt is small. The assumptions that the elasticity of substitution between traded and non-traded goods is low and that the debt is low appears to be crucial to the result. That is, it is possible to construct examples where a combination of the other assumptions does not hold and where an interest rate cut still produces a recession. However, in the examples considered below, a modest degree of substitution between traded and non-traded goods and a modest amount of external debt always has the consequence that an interest rate cut produces an expansion.

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<sup>8</sup>Our market-segmentation assumption may capture what actually happens in the aftermath of a financial crisis. Domestic residents may be fearful of borrowing in foreign markets because of concerns about exchange risk (hedging markets tend to become very illiquid at times like this). Similarly, foreign residents may not want to lend in domestic markets. While our market-segmentation assumption may be plausible, the factors that justify it are not present in our model.

In the first subsection below, we describe the nature of the monetary experiments analyzed here. The second subsection identifies a particular version of our model for which we have analytic results. That section also explains why our strategy of characterizing monetary policy in terms of the interest rate simplifies the technical analysis of the model, while entailing no loss of generality. The third subsection investigates the properties of that model, and of deviations from that model.

### 3.1. The Nature of the Policy Experiment

In our analysis, we compare two equilibria, for  $t = 0, 1, 2, \dots$ . In both, the collateral constraint is binding in each date. In each case, we characterize monetary policy by the choice of the nominal interest rate,  $R_t$ , in the domestic money market. In the *baseline equilibrium*,  $R_t$  is held constant,  $R_t = R_s$ , in each period. Our restriction that the current account is always zero guarantees that the relative prices and quantities in this equilibrium are time-invariant. In the *policy intervention equilibrium*, the monetary authority unexpectedly implements a one-time drop in the interest rate in  $t = 0$ , i.e.,  $R_0 < R_s$ ,  $R_t = R_s$  for  $t \geq 1$ . This drop has a non-neutral impact on allocations because of our assumption - taken from the literature on the limited participation models of money - about the timing of actions by different agents during the period. At the beginning of the period, the household makes a deposit decision. Then, the monetary authority takes its action and after that all the other period  $t$  variables are determined. We assume that at the beginning of period  $t = 0$ , when the household makes its deposit decision, it expects  $R_t = R_s$  for  $t \geq 0$ . At the beginning of period  $t = 1, 2, \dots$  the household expects  $R_t = R_s$  despite the fact that its expectation was violated in period  $t = 0$ .

Given the assumptions of our model, the relative prices and quantities in the baseline and policy intervention equilibria are identical in  $t \geq 1$ , but they differ in  $t = 0$ . Our analysis focuses on this difference in period 0. In particular, we investigate what conditions guarantee that output and employment in  $t = 0$  for the policy intervention equilibrium are lower than they are in the baseline equilibrium. Because they are time invariant, we refer to values of relative prices and quantities in  $t \geq 0$  in the baseline equilibrium, and  $t > 0$  in the policy intervention equilibrium as their *steady state* values. Because of the simplicity of these equilibria, the analysis has a static flavor. It only involves comparing the steady state relative prices and quantities with the  $t = 0$  values of the variables in the policy intervention equilibrium.

### 3.2. A Simplified Model

Throughout this section, we assume  $B_{t+1} \equiv B_t$ . In addition, we assume that  $z$  is essential in production of the traded good, and that labor cannot be adjusted in that sector. We capture this with the specification,  $\xi = \nu = \mu_1 = \mu_2 = 1$ , so that the traded goods production function is given by (2.16). For simplicity, we also exclude the wage bill from the collateral constraint:

$$\tau^N q^N K^N + \tau^T q^T K^T \geq R^* z + (1 + r^*)B. \quad (3.1)$$

With these simplifications, we can analyze the response of the variables at date 0 to the

$t = 0$  cut in the domestic rate of interest as the intersection of two curves – each one involving the endogenous variables,  $p^N$  and  $L$ , and the exogenous policy variable,  $R$  (when there is no risk of confusion, we drop time subscripts). The first curve summarizes equilibrium in the labor market, and so we refer to it as the LM (‘Labor Market’) curve. The other curve, because it incorporates restrictions from the asset market, is called the AM (‘Asset Market’) curve. We now discuss these in turn. The simplicity of the analysis reflects in part the fact that we characterize policy in terms of the interest rate, rather than the money supply. The last subsection below shows that this involves no loss of generality, since there is always a money growth rate that can support any interest rate policy, as long as  $R > 1$ .

### 3.2.1. Labor Market

Equating the household and non-traded good firm Euler equations for labor, (2.5) and (2.23), we obtain:<sup>9</sup>

$$RL^{\psi+\alpha} = \frac{p^N(1-\alpha)(K^N)^\alpha}{\psi_0 p}. \quad (3.2)$$

In this expression, it is understood that  $p$  is the simple function of  $p^N$  given in (2.8). As noted above, we think of  $R$  as an exogenous variable. So, this expression characterizes the relationship between  $L$  and  $p^N$  imposed by equilibrium in the labor market. It is easy to see that this LM equation is positively sloped when graphed with  $p^N$  on the vertical axis and  $L$  on the horizontal. A higher  $p^N$  is consistent with a higher  $L$  because it shifts the labor demand curve to the right, while leaving the location of labor supply unchanged.<sup>10</sup> It is also easy to see that a fall in  $R$  shifts the LM equation to the right. This reflects that a fall in  $R$  shifts labor demand to the right and this results in an increase in equilibrium  $L$  for a fixed level of  $p^N$ .

### 3.2.2. Asset Market

We now turn to the AM equation. This is constructed by combining the production functions in both sectors, (2.14), the first order condition for the intermediate input, (2.25), the  $p^N$  equation, (2.9), and the collateral constraint, (3.1), under the assumption that it is binding. Substitute the expression for asset prices, (2.19), into the collateral constraint, (3.1), evaluated with an equality and assume that  $\tau^N = \tau^T = \tau$  to obtain:

$$\frac{\tau}{1-\lambda\tau}[\theta y^T + \alpha p^N y^N + \Omega pc] = R^*z + (1+r^*)B, \quad (3.3)$$

where  $\Omega = \frac{\beta}{p_s c_s}(q_s^N K^N + q_s^T K^T)$  is a constant. Absence of a time subscript indicates  $t = 0$ , and the subscript,  $s$ , denotes steady state. Here, we have used the fact,  $\Lambda_2/\Lambda_1 = pc/p_s c_s$ . The first

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<sup>9</sup>The absence of a multiplier in (3.2) reflects that we now drop the wage bill from the collateral constraint.

<sup>10</sup>Following convention, we think of labor supply and demand as corresponding to the Euler equations, (2.5) and (2.23). We think of these relationships in a diagram with  $W/P$  on the vertical axis and  $L$  on the horizontal.



two terms in the left hand side of the collateral constraint are the value of the marginal product of capital at  $t = 0$  ( $VM P_K^i$ ), multiplied by the respective capital stocks. The third is the present discounted value of future cash flows. Using the zero profit condition on final consumption good firms,  $pc = c^T + p^N c^N$ , we can write current spending in terms of non-tradeables as

$$pc = \left\{ 1 + \left[ \frac{(1-\gamma)p^N}{\gamma} \right]^{\eta-1} \right\} p^N c^N. \quad (3.4)$$

Substituting this into (3.3), our expression for the collateral constraint reduces to:

$$\begin{aligned} & \frac{\tau}{1-\lambda\tau} \left\{ \theta y^T + \left[ \alpha + \Omega \left( 1 + \left[ \frac{(1-\gamma)p^N}{\gamma} \right]^{\eta-1} \right) \right] p^N y^N \right\} \\ & = R^* z + (1+r^*)B \end{aligned} \quad (3.5)$$

Equilibrium in the goods market yields the following expression for  $p^N$ :

$$p^N = \left( \frac{1-\gamma}{\gamma} \right)^{\frac{1-\eta}{\eta}} \left( \frac{c^T}{c^N} \right)^{\frac{1}{\eta}} = \left( \frac{1-\gamma}{\gamma} \right)^{\frac{1-\eta}{\eta}} \left( \frac{A (K^T)^\theta z^{1-\theta} - R^* z - r^* B}{(K^N)^\alpha L^{1-\alpha}} \right)^{\frac{1}{\eta}}. \quad (3.6)$$

Finally, take into account the first order condition for  $z$ :

$$(1-\theta)A (K^T)^\theta z^{-\theta} = (1+\lambda)R^*. \quad (3.7)$$

Equations (3.5), (3.6), and (3.7) represent three equations in the four unknowns,  $\lambda$ ,  $z$ ,  $p^N$  and  $L$ . The third defines  $\lambda$  as a function of  $z$  and the second defines  $z$  as a function of  $p^N$  (it is single-valued as long as  $\lambda \geq 0$ ) and  $L$ . So, the three equations can be used to define a relationship between  $p^N$  and  $L$  alone. This relationship is what we call the AM curve.

It is clear that the slope of the AM curve is essential in determining whether an interest rate cut is expansionary or contractionary. For example, if it is downward sloped, then a shift right in the LM curve induced by a cut in the interest rate drives  $L$  up and  $p^N$  down. The contractionary case results when the AM curve is positively sloped and cuts the LM curve from below. In general, it is not possible to say what the slope of the AM curve is. We shall see in the next subsection that for particular parameter configurations, it is possible to determine the slope.

Finally, we find it useful to define the version of the AM curve that holds when the collateral

constraint is not binding.<sup>11</sup> In this case, finite  $z$  requires  $\theta > 0$ . When the collateral constraint is not binding, we lose one equation, (3.5), and one variable,  $\lambda$ , from our system. As a result, the AM curve is defined simply by (3.6) and (3.7) with  $\lambda = 0$ . It is trivial to see that in this case, the AM curve is definitely downward sloped.

### 3.2.3. Equilibrium

As the previous discussion indicates, to construct the AM curve it is necessary to first compute the values of the variables in the baseline equilibrium (i.e., the steady state values of the variables). This is a straightforward exercise, which is discussed in the appendix. In the numerical experiments reported in this paper, we always found that the steady state of the model is unique.

In the remainder of this subsection we verify that for a given period 0 interest rate,  $R$ , the values of  $p^N$ ,  $L$  defined by the intersection of the AM and LM curves correspond to a policy intervention equilibrium. By this we mean that, given such values of  $p^N$  and  $L$ , values for  $p$ ,  $c^N$ ,  $c^T$ ,  $c$ ,  $w$ ,  $\lambda$ ,  $z$ ,  $y^T$ ,  $y^N$ ,  $q^T$ ,  $q^N$ ,  $p^T$ , and  $x$  can be found which satisfy all the equilibrium conditions for  $t = 0$ . Verifying that this is true for all but the last two variables is straightforward. For example,  $p$  can be constructed from  $p^N$  using (2.8),  $c^N$  can be constructed from the non-traded good production function, and so on.

We now briefly discuss the construction of  $p^T$  and  $x$ . Divide the money market clearing condition, (2.28), by the equilibrium cash constraint, (2.29), to obtain:

$$\begin{aligned} \frac{d+x}{1+x} &= \frac{wL}{pc} = \frac{p^N(1-\alpha)c^N L}{pRLc} = \frac{1-\alpha p^N c^N}{R pc} \\ &= \frac{1-\alpha}{R \left[ 1 + \left( \frac{(1-\gamma)p^N}{\gamma} \right)^{\eta-1} \right]}, \end{aligned}$$

after using (2.23) and (3.4). Since  $d$  is predetermined at its steady state value, this expression can be used to deduce  $x$ . Obviously, there is always an  $x$  that satisfies this expression, for any  $R > 1$ .<sup>12</sup> Whether a cut in  $R$  requires that the monetary authority increases or decreases  $x$  depends upon the response of  $p^N$ . We can then determine  $p^T$  from (2.29).

Finally, we use a standard argument to deduce the nominal exchange rate from  $p^T$ . We assume purchasing power parity in foreign and domestic traded goods. Then, taking the initial stock of money and the foreign price level as predetermined, we can interpret variations in  $p^T$  as reflecting movements in the nominal exchange rate.

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<sup>11</sup>The AM curve in this case is a bit of a misnomer, since asset prices do not appear.

<sup>12</sup>We only consider equilibria with  $R > 1$ . Accordingly, in our calculations we impose that the cash in advance constraint is always binding.

### 3.3. Effects of an Interest Rate Cut

In this section, we examine the response of equilibrium outcomes at  $t = 0$  to an interest rate cut. Consider first the case when the collateral constraint is not binding. As noted above, in this case the AM curve is downward sloping. From this we conclude:

**Proposition 1** If the collateral constraint is not binding, then a cut in  $R$  produces a rise in  $L$ , a fall in  $p^N$ , and no change in  $z$ .

The monetary transmission mechanism underlying this result corresponds to the standard mechanism emphasized in the literature on the limited participation model of money. A cut in  $R$  reduces the cost of hiring labor, and so results in an expansion in employment and a rise in the production of non-traded goods. The cut in the interest rate produces a fall in the marginal cost of producing non-traded goods, relative to the marginal cost of producing traded goods, and this results in the fall in  $p^N$ . The central bank engineers the cut in  $R$  by producing a suitable move in  $x$ .

We now turn to the case when the collateral constraint is binding in both the baseline and policy intervention equilibria. We begin with the case,  $\theta = 0$ , when  $z$  is the only factor of production in the traded good sector. In this case, a cut in  $R$  is always expansionary. When  $\theta = 0$ , substitution of (3.6) and (3.7) into (3.5) results in the following analytic representation of the AM curve:

$$\begin{aligned} & \left[ \left( \frac{\lambda\tau}{1-\lambda\tau} \right) \Omega - 1 \right] \left( \frac{1-\gamma}{\gamma} \right)^{\eta-1} \\ & = \left\{ \frac{[r^* + \lambda(1+r^*)] B}{(p^N)^\eta y^N} - \left( \frac{\lambda\tau}{1-\lambda\tau} \right) (\alpha + \Omega) (p^N)^{1-\eta} \right\}. \end{aligned} \quad (3.8)$$

In addition, it is evident from (3.6) that when  $\theta = 0$ ,  $z$  is an increasing function of  $(p^N)^\eta y^N$ . Finally, as long as  $A > R^*$ ,  $\lambda$  is a positive constant.

Note first that when  $B = 0$ , (3.8) pins down a unique value for  $p^N$ , so that the AM equation is horizontal. In this case, a cut in  $R$  produces a rise in  $L$  and no change in  $p^N$  or  $z$ . The intuition for this is simple, and can be seen by inspecting (3.5) and (3.6). Note that, when  $B = \theta = 0$  two things happen. First, an equiproportional rise in  $z$  and  $y^N$  produces no change in  $p^N$ . This is because with  $B = \theta = 0$  there are no diminishing returns as  $c^T$  increases with  $z$ . Second, for fixed  $p^N$ , an equiproportional increases in  $y^N$  and  $z$  produces equiproportional increases in the left and right side of the collateral constraint. Under these circumstances, the collateral constraint simply does not get in the way of the type of expansion in output associated with an interest rate cut that occurs when the collateral constraint is nonbinding. On the contrary, the collateral constraint amplifies the response of employment to an interest rate shock by preventing the decline in  $p^N$  that Proposition 1 says would occur in the absence of that constraint.

When  $B > 0$  then both proportionality results cited in the previous paragraph fail, and the AM curve is no longer horizontal. For example, there are now diminishing returns in transforming additional  $z$  into extra  $c^T$ . With  $B > 0$  the AM curve has a negative slope,

according to (3.8).<sup>13</sup><sup>14</sup> Loosely, a rise in  $B$  produces a clockwise rotation in the AM curve. As a result, a cut in  $R$  generates a rise in  $L$  and a fall in  $p^N$  when  $B > 0$ . Equation (3.8) also shows that  $(p^N)^\eta y^N$  rises with the cut in  $R$  for  $0 \leq \eta < 1$ . This implies that the cut in  $R$  generates a rise in  $z$ . We summarize these findings in a proposition:

- Proposition 2** (i) When  $\theta = B = 0$ ,  $A > R^*$ , a cut in  $R$  produces a rise in  $L$  and  $z$ , and no change in  $p^N$ .  
(ii) When  $\theta = 0$ ,  $B > 0$  and  $A > R^*$ , a cut in  $R$  produces a rise in  $L$  and  $z$ , and a fall in  $p^N$ .

We conclude from this discussion that when  $\theta = 0$ , our simple environment cannot rationalize the notion that an interest rate cut produces a recession.

We now turn to the case,  $\theta > 0$ . Suppose first that  $\eta = 1$ . From (3.6), we see that  $z$  can be expressed as a function of  $p^N y^N$ .<sup>15</sup> According to (3.7)  $\lambda$  is a function of  $z$ , and, hence of  $p^N y^N$ . Substituting these results into (3.5), we conclude that when  $\theta > 0$  and  $\eta = 1$ , the AM curve pins down  $p^N y^N$ . In particular, the curve is downward-sloping. As a result, a cut in  $R$  produces a rise in  $L$  and a fall in  $p^N$ . Because  $p^N y^N$  remains unchanged, it follows that  $z$  does not change. The AM curve and the LM curves before and after the cut in the interest rate are displayed in Figure 1.<sup>16</sup> We summarize this finding as follows:

- Proposition 3** When  $\theta > 0$  and  $\eta = 1$ , then a cut in  $R$  produces a rise in  $L$ , a fall in  $p^N$ , and no change in  $z$ .

We have not been able to obtain analytic results for  $0 \leq \eta < 1$ , when  $\theta > 0$ . However, when we linearize the AM curve about steady state we find, for  $\eta = 0$ :<sup>17</sup>

$$\frac{dp^N}{dL} = \frac{p^N}{L} \frac{\left\{ \theta y^T - (1 - \lambda\tau) [r^* + \lambda(1 + r^*)] B \right\} (1 - \alpha)}{\tau \lambda (\alpha + \Omega) p^N y^N}.$$

Note that when  $B = 0$ , this expression is definitely positive. If, in addition, the slope is steeper than the slope of the LM curve, we know that with a small cut in the interest rate, there is a

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<sup>13</sup>Equation (3.8) suggests the possibility that when  $\eta > 1$  and large enough, then the AM curve may be positively sloped with  $B > 0$ , perhaps even steeper than the LM curve. The latter case is the one that is required for a cut in  $R$  to generate a recession. We have not considered this case because we view the case,  $\eta > 1$ , as empirically implausible. Still, analysis of this case may yield insights into the nature of our model, and we plan to do this in future drafts.

<sup>14</sup>The slope of the AM curve is given by:

$$\frac{dp^N}{dL} = - \frac{[\gamma + \lambda(1 + r^*)] B}{\eta [\gamma + \lambda(1 + r^*)] B / p^N + (1 - \eta) y^N \lambda \tau (\alpha + \Omega) / (1 - \lambda\tau)} \frac{1 - \alpha}{L}.$$

<sup>15</sup>This requires that the function mapping  $z$  into  $A(K^T)^\theta z^{1-\theta} - R^*z - r^*B$  be invertible. It is invertible, given that we restrict  $z$  to those values that satisfy (3.7) with  $\lambda \geq 0$ .

<sup>16</sup>The parameter values used in this figure are:  $\beta = 1/1.05$ ,  $\alpha = 0.25$ ,  $\theta = 0.6$ ,  $x_s = 0.06$ ,  $\psi_0 = 0.3$ ,  $K^N = K^T = 1$ ,  $A = 1.9$ ,  $R^* = 1 + r^* = 1.05$ ,  $\tau = 0.01$ ,  $B = 0$ ,  $\eta = 0.9$ .

<sup>17</sup>See the appendix for a derivation.

period 0 set of equilibrium allocations in which  $L$  and  $p^N$  are both lower. In this case,  $z$  must fall too. We have constructed numerical examples with  $B = \eta = 0$ , in which the AM curve indeed does cut the LM curve from below and a cut in the interest rate does generate a drop in  $L$  and  $z$ . In these examples, we verified numerically that there is a unique intersection to the LM and AM curves. Figure 2 displays the AM and LM curves for one example with  $\eta = 0$ .<sup>18</sup>

So far, we have found the following. We have an example with diminishing returns in the production of traded goods, zero elasticity of substitution between traded and non-traded goods, and low external debt, in which a cut in  $R$  induces a fall in output. However, we find that substantial deviation from any one of these assumptions reverses the result.

Consider, for example, the parameter,  $\eta$ . We found that when it was increased to about 0.2, then an interest rate cut leads to an expansion in  $L$ . However,  $z$  still falls in this case. It falls enough so that  $GDP$  falls too, when measured in base year prices. When  $\eta$  was increased to 0.3, then  $GDP$  actually rises.<sup>19</sup>

Consider the effect of raising  $B$ . The preceding discussion suggests the possibility that increasing the debt could rotate the AM curve clockwise from a position with positive slope to one with a negative slope. In numerical experiments we have found that this is indeed the case. Figure 3 displays the results of one such experiment. It corresponds to the model economy underlying Figure 2, except that  $B$  has been increased to 0.1, or 27 percent of  $GDP$ .<sup>20</sup> We find it intriguing that the addition of substantial amount of external debt can convert a situation from one in which an interest rate cut results in a contraction, into one in which it results in an expansion. The economic interpretation of this finding deserves further exploration.

We have also explored more basic perturbations on the production function, by changing  $\mu_1$ ,  $\mu_2$ ,  $\nu$ ,  $\xi$  from their values of unity in the above examples. One consistent result we found is that reductions in  $\nu$ , which opens up a role for variable labor in the production of  $y^T$ , moves the system in the direction of the result that a drop in  $R$  produces an expansion in the economy.<sup>21</sup> By reducing the costs associated with diminishing labor productivity of reallocating labor across sectors, dropping  $\nu$  seems to help support assets values and prevent a tightening of the collateral constraint in the wake of a cut in  $R$ . This seems to operate in two ways. First, a reallocation of resources away from the non-traded good sector and towards the traded good sector limits the fall in  $p^N$  after a cut in  $R$ . Other things the same, this supports asset values in that sector. Second, the allocation of labor towards the traded good sector pushes up asset values there by raising the productivity of capital in that sector. Although we did find values for  $\mu_1$ ,  $\mu_2$ ,  $\nu$ ,  $\xi$  that imply a large reduction in output and employment after an interest rate cut, the reduction

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<sup>18</sup>The parameter values underlying this example are:  $\beta = 1/1.05$ ,  $\alpha = 0.25$ ,  $\theta = 0.6$ ,  $x_s = 0.06$ ,  $\psi = 1$ ,  $\psi_0 = 0.3$ ,  $K^N = K^T = 1$ ,  $A = 1.9$ ,  $R^* = 1 + r^* = 1.05$ ,  $\tau = 0.01$ ,  $\gamma = 0.5$ ,  $B = 0$ . When,  $\eta = 0$ , we obtain the following steady state properties for this model:  $L_s = 0.604$ ,  $p_s^N c_s^N / c_s^T = 1.459$ ,  $\lambda = 0.796$ . We defined  $GDP$  as  $p_s^N c^N + c^T + r^* B$ . In this example, we found that this quantity drops 6 percent with a 4 percentage point cut in  $R$ .

<sup>19</sup>When the example of the previous footnote was modified so that  $\eta = 0.2, 0.3$  the percent change in base year  $GDP$  induced by a 4 percentage point drop in  $R$  is  $-0.12$  and  $0.3$  respectively.

<sup>20</sup>We did an experiment using the parameter values from the previous footnote. We set  $\eta = 0$  and  $B = 0.4$ . In the steady state, this implies a debt to GDP ratio of 0.60, or 60 percent. We found that  $L$  and  $z$  rise 1.5 and 3.1 percent, respectively, with a 4 percentage point drop in  $R$ . With  $B = 0$ ,  $L$  and  $z$  both *drop* by 7.9 and 23 percent, respectively with the same drop in  $R$ .

<sup>21</sup>For example,  $\nu$  was reduced from unity to 0.85, then a four percentage point cut in  $R$  produces an 0.04 percent jump in GDP and an 0.87 percent jump in total employment. Recall from a previous footnote that when  $\nu = 1$ , then there is a 7.94 percent drop in employment and a 6 percent drop in GDP.

in output and employment was converted into an expansion with the introduction of a modest amount of substitutability between  $c^N$  and  $c^T$  and a modest amount of external debt.<sup>22</sup>

## 4. Quantitative Analysis

In this section we study versions of our model in which the external debt is endogenous. We saw in the previous section how the implications of a model for the effects of a domestic interest rate cut are sensitive to assumptions. To further clarify the nature of this sensitivity, this section analyzes two versions of our model: one that rationalizes the view that an interest rate cut reduces output and utility and another that rationalizes the opposite view.

The nature of the monetary experiment is similar to the one studied in the previous section. There is a benchmark analysis, in which monetary policy is treated as constant and the economy is confronted with a binding collateral constraint. When the debt is endogenous, the economy responds to this situation by running a current account surplus until the debt is reduced to the point where the collateral constraint is marginally non-binding. At this point, the current account drops to zero and the economy is in a steady state. During the transition, output is low because the binding collateral constraint inhibits borrowing. This scenario is depicted in Figure 4.

We analyze the impact on the transition path of a cut in the nominal rate of interest implemented by the monetary authority. As in the previous section, the policy intervention has a non-neutral impact because it can affect the degree of liquidity in domestic financial markets. This in turn reflects our specification that monetary actions occur at a point in time when the household's deposit decision is a predetermined variable.<sup>23</sup> We adopt the following timing. The first thing that happens in period 0 is that the collateral constraint is imposed, and is believed to be in place forever. Second, households make their deposit decision under the assumption that the economy is in the no policy reaction equilibrium. Third, the monetary action is taken. The timing in this experiment is depicted in Figure 5. Although we have not verified this, we suspect that the outcome of our analysis is not sensitive to the nature of policy in the no policy reaction equilibrium. That is, the difference between what happens to the economy in the benchmark scenario and the policy intervention scenario is qualitatively unaffected by the nature of the benchmark scenario.

The impact on the transition path of the interest rate cut is very different for the two economies that we consider. The two economies differ in the way they model production in the traded good sector. In one, labor plays no role and output is the Cobb-Douglas function of  $z$  and  $K^T$  only given in (2.16). In the other, labor is used in the traded good sector. In this case, the production function is given by the Leontief specification in (2.15), with value-added,  $V$ , given by the specification in (2.14). In each model economy, the production function in the non-traded sector corresponds to the specification in (2.14). Also, in each model production of

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<sup>22</sup>For example, with  $\mu_1 = 1$ ,  $\mu_2 = 2.1$ ,  $\xi = 0.7$ ,  $\nu = 0.85$ ,  $B = 0$ , a four percentage point cut in  $R$  produces a 15 percent drop in total employment and a 14 percent drop in real GDP. When  $B$  is then raised to 0.1 (so that the debt to GDP ratio in the steady state is 0.18) and  $\eta$  is increased to 0.3, then total employment rises 2 percent and GDP rises by 1 percent, with a four percentage point interest rate cut.

<sup>23</sup>It is only predetermined for one period. After that, the deposit decision is free to respond.

the consumption good involves zero elasticity of substitution between traded and non-traded goods. Finally, preferences in the two economies are the same.

Consistent with the analysis of the previous section, we find that the model without labor in the traded good production function has the implication that output contracts, foreign capital inflows dry up and welfare falls with a cut in the domestic rate of interest. The other model implies that an economic expansion follows a cut in the interest rate.

The following section discusses the parameter values used for the two models. The section after that presents and discusses the numerical simulation results.

#### 4.1. Parameter Values and Steady State

The parameter values for the two versions of our model are displayed in Table 1. Consider first the parameter values for the version in which labor enters in the production of tradables (see left side of Table 1). These were chosen to replicate several stylized features of recent crises countries, in particular, Thailand and Korea.

The share of tradables in total production for Korea, assuming that tradables correspond to the non-service sectors, was approximately one third before the crisis.<sup>24</sup> Combining this assumption with estimates of labor shares from Young (1995), we estimate shares of capital for the tradable and nontradable sectors in Korea to be respectively 0.48 and 0.21.<sup>25</sup> Uribe (1995) and Rebelo and Vegh (1995) estimate the same shares to be 0.52 and 0.37. We adopt values that are close to both these point estimates by specifying  $\nu = 0.50$  and  $\alpha = 0.36$ . There is conflicting evidence on the appropriate value of  $\sigma$ . For example, Reinhart and Vegh (1995) estimate the elasticity of intertemporal substitution in consumption for Argentina to be equal to 0.2. Higher values are used in macroeconomic studies. In our analysis, we set  $\sigma = 1$ . We take the foreign interest rate to be equal to 6 percent and we assume and we assume that  $\beta = 1/1.06$ . We also assume a money growth rate,  $x$ , of 6 percent to obtain a nominal domestic interest rate of 12.3 percent, roughly in line with the experience of Korea and Thailand in the years before the crises. We set  $\psi = 3$ , implying a labor supply elasticity of  $1/3$ . This is in between the elasticity used in standard business cycle models, and the elasticity often reported in empirical analyses of labor supply.

The parameters,  $\mu_1$  and  $\mu_2$ , in the production technology were chosen to reproduce the ratio of imported intermediate inputs in manufacturing to manufacturing value-added in Korea for the year, 1995. In that year, this ratio is 0.40, or,  $z/V = 0.40$ .<sup>26</sup> This, together with the facts,

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<sup>24</sup>According to Bank of Korea (1996), Table 20-1, pages 198-199, total Korean GDP in 1995 was 352 trillion won and value-added in the tradable sector (agriculture, mining and quarrying, and manufacturing) was 118 trillion won.

<sup>25</sup>According to Young (1995), Table VII, page 660, the share of labor in South Korean GDP in the period 1966-1990 was 0.703. The corresponding figure for manufacturing was 0.521. To obtain the share of labor in the non-manufacturing sector, we solved  $0.33 \times 0.521 + 0.67 \times x = 0.703$ , for  $x$ . The result is  $x = 0.793$ . For the purpose of these estimates, we identify tradables with the manufacturing sector and non-tradables with the rest.

<sup>26</sup>This ratio was obtained as follows. Table 4 of Bank of Korea (1998), reports that the value of total intermediate inputs was 69 percent of gross output in manufacturing in 1995. Thus, value added in manufacturing was 31 percent of gross output. Table 13 reports that the ratio of imported intermediate inputs to total inputs in manufacturing was 18 percent in 1995, or 12.4 ( $= 0.18 \times 0.69$ ) percent of gross output. Our result is obtained as the ratio,  $12.4/31.0 = 0.40$ .

$\mu_1 V = \mu_2 z$  and  $1/\mu_1 + 1/\mu_2 = 1$ , implies  $\mu_1 = 1.40$  and  $\mu_2 = 1.4/0.4$ .

We chose  $\tau$  and the stock of debt in the initial steady state equilibrium so that the initial and final debt to output ratio correspond roughly to the experience of Korea and Thailand. Korea's (Thailand's) external debt started at 33% of GDP by end-1997 (60.3%) and is forecasted to be at 26.8% of GDP (51% of GDP) and the end of the year 2000. Based on these observations, we aimed to parameterize the models so that the model economy starts in the range, 30-60%, and then drops by an amount in the range of 8 - 10 percentage points.

We base our calibration of the relative size of  $K^T$  and  $K^N$  just prior to the 1997 Asian crisis on Korean data. To our knowledge, there do not exist direct, published estimates of sectoral Korean capital for that year. Instead, we followed two strategies. The first is the one in Fernandez de Cordoba and Kehoe (1998). Using the definition of the share of capital income in value added, and assuming the rental rate of capital is the same in all sectors,

$$K^j = \frac{s^j K}{s} y^j, \quad j = T, N.$$

Here,  $s^j$  is the share of capital income in value-added,  $y^j$ , in sector  $j$ ,  $j = T, N$ . These shares, including the aggregate share,  $s$ , were taken from Young, as discussed above. We obtained the aggregate capital to aggregate output ratio,  $K/y$ , from Summers and Heston (1991). Table II, page 353, reports that this is  $16,659/12,275 = 1.36$  in 1985. We estimate output for the tradable and nontradable sectors in 1995 using data from the Bank of Korea.<sup>27</sup> Pursuing these calculations, we find that  $K^T$  is 259,330 trillion won and  $K^N$  is 220,861 trillion won. To convert into units of account in the model, we normalize  $K^T = 1$  and set  $K^N = 0.85$ .

The second strategy for estimating the relative size of  $K^T$  and  $K^N$  in 1995 for Korea uses the ratio of sectoral investment. We obtained annual sectoral investment for the years 1990-1995 from the OECD.<sup>28</sup> Consistent with our definition of sectoral GDP, we compute investment in tradables using the sum of investment in manufacturing, mining and agriculture. We compute investment in nontradables as total gross fixed capital formation minus tradables investment. The average of the ratio of investment in nontradables to investment in tradables over our sample is 2.2. If we normalize  $K^T = 1$ , this suggests  $K^N = 2.2$ . The two estimation strategies just described produce very different estimates of the relative size of  $K^T$  and  $K^N$ . Each rests on a different set of assumptions that are, at best, crude approximations. The first strategy assumes the rental rate of capital in the two sectors is the same. In our model, and most likely in reality too, there is no such requirement since capital is fixed in place. The second strategy assumes the capital stock is growing at the same rate in each sector and that depreciation is the same.<sup>29</sup> Again, this is at best an approximation.

We decided to go with the larger estimate of the relative size of  $K^N$ . In particular, we

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<sup>27</sup>See the footnote prior to the previous one for details.

<sup>28</sup>The data were obtained from the web at <http://www.sourceoecd.org>.

<sup>29</sup>Implicitly, we assume the following capital accumulation technology for sector  $i$ :  $K_{i,t+1} = (1 - \delta)K_{i,t} + I_{i,t}$ . If the growth rate of the capital stock in each sector is  $\mu$ , then we have

$$[\mu - (1 - \delta)] K_{i,t} = I_{i,t},$$

so that the ratio of investment in two sectors equals the ratio of the capital stock across the same two sectors.



set  $K^T = 1$ ,  $K^N = 2$ . We did this because there is some direct evidence to support this - the investment data just described - and to promote the ability of the model to match the relatively large size of the nontraded sector suggested by the work of Burstein, Eichenbaum and Rebelo (2001). They argue that the ratio of the value of  $c^N$  to the value of  $c^T$  is in the range of 3 – 4 for emerging market economies.<sup>30</sup>

The parameter values we chose for the version of the model in which labor does not appear in the traded good production function are reported in the right side of Table 1. A difference from the parameters in the left is that the labor supply elasticity is higher for this version of the model. We stress this version’s implication that a cut in  $R$  generates a fall in output, and we presume that a lower labor supply elasticity would only have made this contraction worse.

The steady state properties of the no-collateral constraint version of the model are meant to capture the pre-crisis situation, and these are reported in Table 2. The collateral constraint is imposed in period 0, and the economy eventually converges to the new steady state, one in which the collateral constraint is not binding. The properties of that steady state are reported in Table 3.

To evaluate the plausibility of our model parameterization, it is useful to consider data sectoral employment. Bank of Korea estimates suggest that the ratio of employment in non-tradables to employment in tradables averaged 1.52 over the period, 1991-1995.<sup>31</sup> We also obtained sectoral employment from the OECD for 1997. Here, we found that the percent of total employment in agriculture and manufacturing was 42.3 while the percent in services was 57.7. The ratio of the two suggests the ratio of employment in nontradables to tradables was 1.38. Both these figures suggest that our ‘labor in the traded good sector’ version of the model overstates the amount of labor in the nontraded good sector. This result is largely driven by the relatively high consumption of nontraded goods used in the calibration of the model.

In comparing tables 3 and 4, one other feature is worth noting. In the model in the left side of these tables, the debt to GDP ratio falls about 10 percentage points of total output, which corresponds roughly to Thailand’s experience (see above). In the model on the right, the debt to GDP ratio falls about 26 percentage points of output, which is rather large. Overall, we view the parameter values as forming a reasonable basis for carrying out the exercises that interest us.

## 4.2. Baseline Scenario

We now consider the dynamic effects of the imposition of the collateral constraint, when monetary policy takes the form of a constant money growth rate throughout the transition to the new steady state. Figures 6a-6b shows the variables of the model in equilibrium, as the economy transits from the high initial debt to the lower level of debt in the steady state where the collateral constraint is marginally non-binding. The results in Figures 6a-6b pertain to the economy in which labor is used in the traded good sector. Note that firms respond to the

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<sup>30</sup>We include what Burstein, Eichenbaum and Rebelo (2001) model as the ‘distribution sector’ in our non-traded good sector.

<sup>31</sup>This is an average over quarterly ratios. The denominator has employment in the tradable sector, which we measure as the sum of employment in agriculture, mining and manufacturing. The numerator has employment in the nontradable sector. We measure this as total employment minus employment in the tradable sector. These Bank of Korea data were obtained from the IMF’s edss data base.

enormous 70 percentage point jump in the shadow cost of foreign borrowing by paying off the external debt. The current account jumps from zero to 4 percent of aggregate output in the period that the collateral constraint is imposed. The current account remains above 1 percent of aggregate output for roughly 3 years. The imposition of the collateral constraint generates a general cutback in borrowing for working capital purposes. But, this shows up primarily in a reduction in domestic borrowing for labor. There is actually a small rise in borrowing to finance imports of  $z$ , because the great emphasis placed on paying off the long-term international debt.<sup>32</sup> Total employment drops roughly 10 percent, but this drop is experienced by the nontraded sector while the traded sector experiences a rise in employment. Total output in the traded good sector actually expands, but domestic consumption of both traded and nontraded goods falls.<sup>33</sup> The overall slowdown in economic activity contributes to a fall in asset values, as the marginal physical product of assets decline. There is a small drop in the domestic rate of interest, presumably as the demand for funds falls with the reduction in output.

Finally, the nominal exchange rate exhibits an immediate 58 percent depreciation followed by a long period of appreciation as the exchange rate returns to its original position (see  $P^T$ ). Interestingly, the 58 percent depreciation has a relatively small impact on  $P$ , the consumer price index. The inflation rate in period 0 is roughly 20 percent, which is only 14 percentage points higher than what it would have been in the absence of the collateral shock. There are two reasons for this. First, traded goods make up only about one-third of the consumer price index. Second, the relative price of non-traded goods,  $p^N$ , falls by about 38 percent in the period of the collateral constraint, so that the domestic currency price of non-traded goods rises by only about 20 percent.

Figures 7a-7b display the corresponding results for the model in which labor cannot be used in the traded sector. With one exception, the effects are qualitatively similar to what we saw in the previous figures. The exception is that now imports of the intermediate good fall substantially. The main difference in the results has to do with magnitudes. The effects tend to be larger because the debt in this version of the model is much higher in the initial steady state than it is in the version just discussed. Thus, the shadow cost of borrowing jumps 648 percent in the period of the imposition of the collateral constraint. Associated with this there is a major reduction in employment and imports of the intermediate good. Capital inflows (to finance  $z$ ) display the ‘sudden stop’ feature emphasized by Calvo (1998). Finally, the collateral constraint triggers an immediate real and nominal depreciation that overshoots.

We take it that these characteristics of our models correspond reasonably well, at least qualitatively, with what actually happened after the 1997 financial crises in several Asian countries (see Boorman, *et. al.* (2000).) On this basis, we feel justified in using these models to study the effects of a cut in domestic interest rates in the wake of a financial crisis. We turn to this now.

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<sup>32</sup>The numerical experiments we have run suggests that a fall in total borrowing is a robust response to the imposition of collateral constraint. For some parameterizations, this implies a rise in  $z$ , and for others, a fall.

<sup>33</sup>These numbers are not reported in the figures.

### 4.3. The Effect of an Interest Rate Cut

We now suppose that in period 0, after the collateral constraint has been imposed, the monetary authority temporarily deviates from its constant money growth path. It does so by doing whatever is necessary with the money supply to obtain a given reduction in the period 0 rate of interest. This policy action, which is unanticipated, is executed after the household has made its deposit decision. Agents expect, correctly, that the monetary authority will revert to its constant money growth path in  $t \geq 1$ . This one-time change policy has no impact on the ultimate steady state to which the economy is headed. It only affects the nature of the transition path. We ask what it does to the economic variables along that path, and whether things are made better or worse in a welfare sense.

Results regarding the contemporaneous impact of the policy intervention are reported in Table 4.<sup>34</sup> The cut in the interest rate is 9 percentage points in each model economy. It is accomplished by a one-time change in money growth in period 0, after which the steady state money growth rate of 6 percent is resumed. In explaining what happens in the two models, we adopt the approach taken in the introduction, which centers the discussion on the collateral constraint, (3.1), which we reproduce here:

$$\tau q^N K^N + \tau q^T K^T = R^* z + (1 + r^*) B.$$

Here, we have not included the wage bill, which we abstract from for purposes of this discussion (though, not in the computational experiments). In both model economies, the cut in the interest rate generates a nominal depreciation (see Table 4).<sup>35</sup> Other things the same, this makes the left side, the asset side, of the collateral constraint fall. To see this, recall that the collateral constraint is measured in units of traded goods. So if only  $P^T$  rose, and no other price - when measured in domestic currency units - or quantity changed, the asset side would fall, requiring a fall in  $z$ . There is another price effect that may have a similar impact on the collateral constraint. In particular, the cut in the domestic rate of interest, by having a relatively large impact on marginal costs in the non-traded good sector, may cause the relative price of goods produced in that sector,  $p^N$ , to fall. This has a further depressive effect on the asset side of the collateral constraint, because  $p^N$  is used to value the productivity of the assets in the non-traded good sector.<sup>36</sup> If this were the whole story, then the interest rate cut leaves us with a mismatch between the asset and liability sides of the collateral constraint, which could only be resolved by reducing capital inflows through a cut in  $z$ .

But, this is not the whole story. Inspection of the collateral constraint reveals another option: one could in principle *increase*  $z$ . Of course, this has the wrong effect on the liability side of the collateral constraint. However, this problem is somewhat alleviated if the external

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<sup>34</sup>We have not been able to establish formally the uniqueness properties of the baseline equilibrium and the policy intervention equilibrium. To build confidence that we do have uniqueness, we searched numerically for other equilibria and never found any.

<sup>35</sup>This corresponds to the traditional view of monetary policy in open economies. See Mussa (2000) for evidence from episodes of emerging market crises that seem to provide support for the traditional view.

<sup>36</sup>Recall that  $p^N$  enters in the value of the marginal product of capital in the non-traded good sector (see (2.19).)

debt is very large.<sup>37</sup> In this case, the percentage increase in the liability side of the collateral constraint associated with a given rise in  $z$  is small. What about the impact of a rise in  $z$  on the asset side of the collateral constraint? In general - and in our models specifically - one expects the increased use of  $z$  to raise the value of the economy's assets by raising their marginal physical product. However, this channel is not strong enough if  $z$  is subject to strongly diminishing returns. In this case, the rise in asset values associated with a rise in  $z$  is small and likely to be dominated by the rise in liabilities.<sup>38</sup>

This is the situation in the model where labor does not enter the traded good sector. In that model, the equilibrium response of  $z$  to a cut in  $R$  involves a reduction in  $z$ , not an increase. This reduction in  $z$  sets into motion additional forces in our model which keep it falling. In particular, the lack of substitutability between traded and non-traded goods in the production of final consumption goods has the consequence that a fall in  $z$  reduces demand for the non-traded good, so that employment there falls. This has the effect of further reducing asset values, aggravating the assets and liability mismatch in the collateral constraint. The effects on asset prices can be seen in Table 4, which shows that  $q^T$  rises by a very small amount, and is dominated in the collateral constraint by the fall in  $q^N$ . Hence, the value of assets as a whole falls.

The situation is different in the model where labor does play a role in the traded good sector. Now the option of restoring equality to the collateral constraint by increasing  $z$  is a greater possibility. This is because an infusion of labor into the traded good sector can work against the diminishing returns associated with an increase in  $z$ . As a result, a rise in  $z$  could in principle raise the asset side of the collateral constraint by more than the liability side. The tables indicate that this is precisely what happens in the model in which labor enters the traded good sector.

We calculated the present discounted value of utility from period 0 on, for our baseline scenario and for the scenario in which the monetary authority responds by cutting the rate of interest. We did this for each of our two models. Note that utility in the steady state to which the economy converges after the collateral constraint is imposed is higher than utility in the pre-crisis steady state. This reflects the wealth effects of the reduced level of debt in the collateral-constrained steady state. In the case of the model with labor in the traded good sector, the present discounted utility in the equilibrium with the interest rate cut is higher than what it is when the monetary authority does not react. Utility falls with the interest rate cut in the other model.

Figures 8a-8b display the impact of the interest rate cut on the whole dynamic path, for the version of our model in which labor can be used in the traded goods sector. We see that the cut in  $R$  lifts up asset values, relaxes the collateral constraint (note how  $\lambda$  falls), and stimulates

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<sup>37</sup>In this context, our analysis of the previous section is relevant. There we presented evidence that suggests: (i) if a model is to rationalize the notion that an interest rate cut generates a recession, then the AM curve must be positively sloped and cut the LM curve from below, and (ii) increasing the external debt rotates the AM curve clockwise. Conditions (i) and (ii) suggest that if an economy with low debt produces a recession with an interest rate cut, then the recession will be smaller or it may even turn into a boom for a model in which the external debt is higher.

<sup>38</sup>There is another channel that could in principle be operative. A cut in the domestic interest rate, by increasing the supply of nontraded goods (recall, the interest rate cut reduces the marginal cost of those goods), raises the demand for traded goods, to the extent that these complement with nontraded goods. If so, then the shadow cost of the collateral constraint is likely to increase and this can have the effect of raising asset prices. This effect does not appear to be strong in the particular numerical examples displayed here.

output and employment. Note that the economy takes advantage of the boom to raise the current account even further, accelerating the transition to the new steady state. Interestingly, the depreciation of the currency now generates an almost equal increase in domestic inflation. The reason for this is that  $p^N$  rises with this policy. Presumably, this reflects a wealth effect, which leads to greater purchases of non-tradable goods and, hence, in their marginal cost. This stands in interesting contrast with the results displayed in Figure 6b. There, the depreciation of the currency is associated with a contraction, producing a negative wealth effect, which - by damping  $p^N$  - causes the resulting general inflation to be a lot smaller than the rise in the price of traded goods.

Figures 9a-9b display the impact of the interest rate cut in the model in which labor does not appear in the traded goods production function. The results here could not be more different from what we saw in Figure 8a-8b. Now, the collateral constraint tightens, employment and output drop, asset prices fall, there is a depreciation in the nominal exchange rate, together with a much smaller rise in inflation, presumably due to the negative wealth effect associated with the fall in  $R$ .

We now briefly discuss the welfare consequences of the cut in interest rates in this model. In our model, absent the collateral constraint, the frictions associated with  $R > 1$  imply that reducing the rate of interest is always desirable. In our environment, the optimality of the Friedman rule - and, hence, of cutting the interest rate - is not so obvious because of the presence of the collateral constraint. Table 5 reports the utility levels associated with the various equilibria. First, notice that utility in the pre-crisis steady state is always lower than what it is in the new steady state. This simply reflects that in the latter, there is less external debt and so foreigners are imposing fewer ‘taxes’ in the form of interest rate payments. Note too, that utility in the transition to the new steady state is lower than utility in the old steady state: although people are happier in the new steady state, if they were given the choice whether to stay in the old equilibrium or transit to the new one, they would prefer the old equilibrium.

Finally, consider the welfare calculations associated with the central question that interests us. We can evaluate the impact on utility of the cut in the interest rate by comparing the discounted utility of the paths with and without the monetary policy interventions. Here, we find that utility goes down in the model with no labor in the traded good sector. In this model, the transition to the new steady state is made harder by the interest rate cut. In the other model, the transition is made easier.

## 5. Conclusion

We analyzed a small open economy model in which firms require two types of working capital: domestic currency to hire domestic inputs and foreign currency to finance imports of an intermediate input. We adopt a reduced form model of a financial crisis, and ask what is the economic impact of a cut in the domestic rate of interest at such a time. We model a financial crisis as a time when collateral constraints on borrowing are imposed and are binding. Our notion of a ‘financial crisis’ corresponds to what some might think of as a ‘credit crunch’.

In our model, application of binding collateral constraints causes the economy to run a current account surplus and bring its debt down to the steady state in which the collateral constraint is marginally non-binding. During the transition, the collateral constraint limits the

amount of borrowing that firms can do, and so leads to a reduction in output and employment. In addition, asset values fall with the slowdown in activity, and real and nominal exchange rates depreciate and overshoot with the onset of the crisis. These features of the transition dynamics in our model correspond - at least qualitatively - with what was observed in the Asian crises that began in late 1997. We believe that this justifies taking our reduced form model of a financial crisis seriously, as a laboratory for studying the economic effects of a cut in the domestic interest rate in the aftermath of a financial crisis.

To understand our analysis of the effects of the interest rate cut, it is sufficient to keep in mind firms' collateral constraint: the requirement that the value of their assets be no less than the value of their liabilities. We model the former as consisting of productive assets such as land and capital in the domestic economy. Also, most of firms' liabilities take the form of international debt. Our framework captures the tensions emphasized in the literature that are created by operation of this collateral constraint.

First, an interest rate cut engineered by the central bank produces a nominal exchange rate depreciation in our model. Other things the same, this tightens the collateral constraint by producing a fall in the value of the domestic assets of the firm, while not affecting the value of international liabilities. This effect arises from the widely discussed mismatch in the currency denomination of assets and liabilities. This effect could be compounded if in addition to a nominal depreciation, there is also a real depreciation. Second, an interest rate cut can also alleviate the collateral constraint by pushing up asset values.

We find that the first scenario - the one in which currency mismatch problems cause an interest rate cut to produce a contraction - is more likely when there are limitations in how flexibly the economy can exploit an increase in the quantity of the intermediate good. The second scenario - the one in which an interest rate cut produces an expansion by inflating asset values - is more likely when these limitations are not present. We conclude that resolving the debate over the effects of an interest rate cut in the aftermath of a financial crisis requires understanding how much short-run flexibility there is in the economy. We suspect that there is relatively little such flexibility, at least in the short run, so that the contraction scenario may be the most plausible one.

## A. Appendix

In this technical appendix we discuss various issues raised in the text. The first subsection discusses the computation of the steady state in the version of the model of section 3 in which the collateral constraint is binding. The second subsection derives the linearization formulas used in the local analysis in section 3. The third subsection discusses the solution of the version of our model analyzed in section 4, in which the current account is not constrained to be zero.

### A.1. Steady State in the Model of Section 3

For convenience, we repeat some of the equations of the model here:

$$RL^{\psi+\alpha} = \frac{p^N(1-\alpha)(K^N)^\alpha}{\psi_0 p}. \quad (\text{A.1})$$

The collateral constraint is:

$$\frac{\tau^N \alpha p^N (K^N)^\alpha L^{1-\alpha}}{1 - \lambda \tau^N - \beta} + \frac{\tau^T \theta A (K^T)^\theta z^{1-\theta}}{1 - \lambda \tau^T - \beta} = R^* z + (1 + r^*) B. \quad (\text{A.2})$$

The first order necessary condition for  $z$  in the traded good sector is:

$$(1 - \theta) A (K^T)^\theta z^{1-\theta} = (1 + \lambda) z R^*. \quad (\text{A.3})$$

The price equation is:

$$p^N = \frac{1 - \gamma}{\gamma} \left( \frac{A (K^T)^\theta z^{1-\theta} - R^* z - r^* B}{(K^N)^\alpha L^{1-\alpha}} \right)^{\frac{1}{\eta}}. \quad (\text{A.4})$$

When  $\eta = 0$ , we replace (A.4) with

$$(K^N)^\alpha L^{1-\alpha} = A (K^T)^\theta z^{1-\theta} - R^* z - r^* B. \quad (\text{A.5})$$

The unknowns are  $L$ ,  $p^N$ ,  $z$ ,  $\lambda$ .

We now discuss how to find the steady state when  $\theta > 0$  and  $\eta = 0$ . [the case,  $\eta > 0$  will be added later]. We use the equations, (A.1)-(A.3) and (A.5) to define a mapping from  $z$  to  $z'$ , whose fixed point corresponds to an equilibrium. Rewrite (A.2) as follows:

$$p^N = \frac{(1 - \lambda \tau^N - \beta) \left[ R^* z + (1 + r^*) B - \frac{\tau^T \theta A (K^T)^\theta z^{1-\theta}}{1 - \lambda \tau^T - \beta} \right]}{\tau^N \alpha (K^N)^\alpha L^{1-\alpha}}, \quad (\text{A.6})$$

Combining (A.5) and (A.3), we obtain (the discussion below assumes  $B = 0$ , which will be

fixed later):

$$\begin{aligned}
(K^N)^\alpha L^{1-\alpha} &= z \left[ A (K^T)^\theta z^{-\theta} - R^* \right] \\
&= z \left[ \frac{1+\lambda}{1-\theta} R^* - R^* \right] \\
&= \frac{\lambda+\theta}{1-\theta} R^* z
\end{aligned} \tag{A.7}$$

or,

$$L = \left[ \frac{\lambda+\theta}{1-\theta} \frac{R^* z}{(K^N)^\alpha} \right]^{\frac{1}{1-\alpha}}. \tag{A.8}$$

Note that (A.3) defines a mapping from  $z$  to  $\lambda$ . Taking this and (A.8) into account, (A.6) defines a mapping from  $z$  to  $p^N$  :

$$p^N = f(z), \tag{A.9}$$

where

$$f(z) = \left[ \frac{1-\lambda\tau^N - \beta}{1-\lambda\tau^T - \beta} \right] \left[ \frac{1-\theta}{\lambda+\theta} \right] \frac{\left[ (1-\lambda\tau^T - \beta) - \tau^T(1+\lambda)\theta/(1-\theta) \right]}{\tau^N \alpha}. \tag{A.10}$$

Here, it is understood that  $\lambda$  is the function of  $z$  implied by (A.3).

Solving (A.1) for  $L$  :

$$L = \left\{ \frac{(1-\alpha)(K^N)^\alpha}{R\psi_0 \left[ \frac{1}{p^N} + 1 \right]} \right\}^{\frac{1}{\psi+\alpha}}.$$

Combining this with (A.7), we obtain:

$$\left[ \frac{\lambda+\theta}{1-\theta} \frac{R^* z}{(K^N)^\alpha} \right]^{\frac{1}{1-\alpha}} = \left[ \frac{(1-\alpha)(K^N)^\alpha}{R\psi_0 \left( \frac{1}{p^N} + 1 \right)} \right]^{\frac{1}{\psi+\alpha}}.$$

Solve this for  $z$  :

$$\begin{aligned}
z &= \frac{1-\theta}{\lambda+\theta} \frac{(K^N)^\alpha}{R^*} \left[ \frac{(1-\alpha)(K^N)^\alpha}{R\psi_0 \left( \frac{1}{p^N} + 1 \right)} \right]^{\frac{1-\alpha}{\psi+\alpha}} \\
&= g(p^N, \lambda),
\end{aligned} \tag{A.11}$$



say. We can use this and  $f$  in (A.9) to define a mapping from  $z$  into itself:

$$z' = g(p^N, \lambda) = g(f(z), \lambda(z)) = h(z),$$

say, where  $\lambda(z)$  summarizes (A.3). It is easy to see that  $h$  is an increasing function of  $z$  as long as  $\tau^N = \tau^T$ .

To actually find the fixed point, if it exists, it is useful to be able to restrict the set of candidate equilibrium values of  $z$ . We know that we must have  $\lambda \geq 0$ ,  $(1 - \lambda\tau^N - \beta) \geq 0$ ,  $(1 - \lambda\tau^T - \beta) \geq 0$ ,  $p^N \geq 0$ . The first of these implies an upper bound on  $z$ , and the others imply a lower bound. (These conditions imply  $L \geq 0$ , so we don't list that separately.) From (A.9) we see that  $p^N \geq 0$  requires:

$$\lambda \leq \frac{1 - \theta}{\tau^T} \left[ 1 - \beta - \tau^T \frac{\theta}{1 - \theta} \right] = \frac{(1 - \theta)(1 - \beta)}{\tau^T} - \theta$$

This places a lower bound on  $z$ :

$$z \geq \left\{ \frac{R^*}{(1 - \theta)A(K^T)^\theta} \left[ \frac{(1 - \theta)(1 - \beta)}{\tau^T} - \theta + 1 \right] \right\}^{\frac{-1}{\theta}}.$$

We want this lower bound (of course!) to be less than the upper bound on  $z$  implied by  $\lambda \geq 0$ . This places a restriction on the parameters:

$$\theta \leq \frac{(1 - \theta)(1 - \beta)}{\tau^T},$$

or,

$$\tau^T \leq \frac{(1 - \theta)(1 - \beta)}{\theta}.$$

This is a pretty tight upper bound on  $\tau^T$ .

The value of a unit of capital in the non-traded and traded good sectors is, respectively:

$$q^N = \frac{\alpha p^N (K^N)^{\alpha-1} L^{1-\alpha}}{1 - \lambda\tau^N - \beta}, \quad q^T = \frac{\theta A (K^T)^{\theta-1} z^{1-\theta}}{1 - \lambda\tau^T - \beta}.$$

## A.2. Linearization of the Model in Section 3

We derive provide formulas for linearizing the model of section 3 about its steady state. Define the percent deviation of a variable from its steady state value as  $\hat{x} = dx/x$ .

Linearizing the *LM* curve around the steady state we obtain:

$$\hat{R} + (\psi + \alpha)\hat{L} = \hat{p}^N - \hat{p}.$$

or

$$\hat{R}_t + (\psi + \alpha)\hat{L}_t = \left[ 1 - \left( \frac{p\gamma}{p^N} \right)^{\eta-1} \right] \hat{p}_t^N,$$

where we have used the fact that

$$\hat{p} = \left( \frac{p\gamma}{p^N} \right)^{\eta-1} \hat{p}^N$$

Finally, rearrange to obtain

$$\hat{p}_t^N = \frac{\hat{R}_t + (\psi + \alpha)\hat{L}_t}{1 - \left( \frac{p\gamma}{p^N} \right)^{\eta-1}}, \quad (\text{A.12})$$

where

$$1 - \left( \frac{p\gamma}{p^N} \right)^{\eta-1} \geq 0,$$

after evaluating  $p$  in terms of  $p^N$ .

To linearize the AM curve, begin with the  $p^N$  equation:

$$\hat{p}^N = -\frac{1 - \alpha}{\eta} \hat{L} + \frac{1}{\eta} \hat{c}^T$$

From the resource constraint we obtain:

$$dc^T = \left[ (1 - \theta)A (K^T)^\theta z^{-\theta} - R^* \right] dz,$$

so that

$$\hat{c}^T = \frac{\left[ (1 - \theta)A (K^T)^\theta z^{-\theta} - R^* \right]}{c^T} z \hat{z},$$

where  $\hat{z} = dz/z$ . Then,

$$\hat{p}^N = -\frac{1-\alpha}{\eta}\hat{L} + \frac{1}{\eta}\left[(1-\theta)A(K^T)^\theta z^{-\theta} - R^*\right]\frac{z}{c^T}\hat{z},$$

or

$$\hat{p}^N = -\frac{1-\alpha}{\eta}\hat{L} + \left(\frac{\lambda R^* z}{\eta c^T}\right)\hat{z}.$$

We can now obtain  $\hat{z}$  as a function of  $\hat{p}^N$  as follows:

$$\hat{z} = \frac{\frac{1-\alpha}{\eta}\hat{L} + \hat{p}^N}{\frac{\lambda R^* z}{\eta c^T}} = \frac{c^T}{\lambda R^* z}\left[(1-\alpha)\hat{L} + \eta\hat{p}^N\right]. \quad (\text{A.13})$$

From the first order condition for labor in the traded goods sector,  $(1-\theta)A(K^T)^\theta z^{-\theta} = (1+\lambda)R^*$ , we obtain

$$\begin{aligned} -\theta\hat{z} &= \widehat{(1+\lambda)} = \frac{(1+\lambda_t) - (1+\lambda)}{(1+\lambda)} = \frac{\lambda_t - \lambda}{1+\lambda} \\ &= \hat{\lambda}\frac{\lambda}{1+\lambda}. \end{aligned} \quad (\text{A.14})$$

Totally differentiating the expression for the binding collateral constraint with respect to  $L$ ,  $\lambda$ ,  $z$ , and  $p^N$ , we obtain:

$$\begin{aligned} &\frac{\tau}{1-\lambda\tau}R^*z\lambda\hat{\lambda} \\ &+ \left(\frac{\tau}{1-\lambda\tau}\right)\left(\alpha + \Omega\left[1 + \left[\frac{(1-\gamma)p^N}{\gamma}\right]^{\eta-1}\right]\right)(1-\alpha)p^N(K^N)^\alpha L^{1-\alpha}\hat{L} \\ &+ \left(\frac{\tau}{1-\lambda\tau}\right)\{\alpha p^N(K^N)^\alpha L^{1-\alpha} \\ &+ p^N(K^N)^\alpha L^{1-\alpha}\Omega\left(\left[1 + \left[\frac{(1-\gamma)p^N}{\gamma}\right]^{\eta-1}\right] + (\eta-1)\left[\frac{(1-\gamma)p^N}{\gamma}\right]^{\eta-1}\right)\}\hat{p}^N \\ &= \left\{R^*z - \frac{(1-\theta)\tau\theta A(K^T)^\theta z^{1-\theta}}{1-\lambda\tau}\right\}\hat{z} \end{aligned}$$

or,

$$\begin{aligned}
& \frac{\tau}{1-\lambda\tau} R^* z \lambda \hat{\lambda} \tag{A.15} \\
& + \left( \frac{\tau}{1-\lambda\tau} \right) \left( \alpha + \Omega \left[ 1 + \left[ \frac{(1-\gamma)p^N}{\gamma} \right]^{\eta-1} \right] \right) (1-\alpha)p^N (K^N)^\alpha L^{1-\alpha} \hat{L} \\
& + \left( \frac{\tau}{1-\lambda\tau} \right) \left\{ p^N (K^N)^\alpha L^{1-\alpha} \left[ \alpha + \Omega \left( \left[ 1 + \eta \left[ \frac{(1-\gamma)p^N}{\gamma} \right]^{\eta-1} \right) \right] \right] \right\} \hat{p}^N \\
& = \left\{ R^* z - \frac{(1-\theta)\tau\theta A (K^T)^\theta z^{1-\theta}}{1-\lambda\tau} \right\} \hat{z}
\end{aligned}$$

Let's simplify the expression on  $(1-\alpha)\hat{L}$ . According to the collateral constraint:

$$\frac{\tau}{1-\lambda\tau} [\theta A (K^T)^\theta z^{1-\theta} + \left( \alpha + \Omega \left\{ 1 + \left[ \frac{(1-\gamma)p^N}{\gamma} \right]^{\eta-1} \right\} \right) p^N (K^N)^\alpha L^{1-\alpha}] = R^* z + (1+r^*)B$$

so that,

$$\begin{aligned}
& \frac{\tau\lambda}{1-\lambda\tau} \left( \alpha + \Omega \left\{ 1 + \left[ \frac{(1-\gamma)p^N}{\gamma} \right]^{\eta-1} \right\} \right) p^N y^N \\
& = \lambda R^* z + \lambda(1+r^*)B - \frac{\tau\lambda}{1-\lambda\tau} \theta y^T \\
& = (1-\theta)y^T - R^* z - \frac{\tau\lambda}{1-\lambda\tau} \theta y^T + \lambda(1+r^*)B \\
& = y^T - \theta y^T \frac{1}{1-\lambda\tau} - R^* z + \lambda(1+r^*)B \\
& = c^T + r^* B - \frac{\theta y^T}{1-\lambda\tau} + \lambda(1+r^*)B \\
& = c^T - \frac{\theta y^T}{1-\lambda\tau} + [r^* + \lambda(1+r^*)] B
\end{aligned}$$

Now consider the term on  $\hat{p}^N$ .

$$\eta c^T - \frac{\tau\lambda}{1-\lambda\tau} \left( \alpha + \Omega \left\{ 1 + \eta \left[ \frac{(1-\gamma)p^N}{\gamma} \right]^{\eta-1} \right\} \right) p^N y^N$$

$$\begin{aligned}
&= c^T - \frac{\tau\lambda}{1-\lambda\tau} \left( \alpha + \Omega \left\{ 1 + \left[ \frac{(1-\gamma)p^N}{\gamma} \right]^{\eta-1} \right\} \right) p^N y^N \\
&\quad + (\eta-1) \left( c^T - \frac{\tau\lambda}{1-\lambda\tau} \Omega \left[ \frac{(1-\gamma)p^N}{\gamma} \right]^{\eta-1} p^N y^N \right) \\
&= \frac{\theta y^T}{1-\lambda\tau} - [r^* + \lambda(1+r^*)] B + (\eta-1) \left( c^T - \frac{\tau\lambda}{1-\lambda\tau} \Omega \left[ \frac{(1-\gamma)p^N}{\gamma} \right]^{\eta-1} p^N y^N \right).
\end{aligned}$$

But,

$$\begin{aligned}
&c^T - \frac{\tau\lambda}{1-\lambda\tau} \Omega \left[ \frac{(1-\gamma)p^N}{\gamma} \right]^{\eta-1} p^N y^N \\
&= y^T - R^* z - r^* B - \frac{\tau\lambda}{1-\lambda\tau} \Omega \left[ \frac{(1-\gamma)p^N}{\gamma} \right]^{\eta-1} p^N y^N \\
&= y^T - (1+\lambda)R^* z + \lambda[R^* z + (1+r^*) B] \\
&\quad - \lambda(1+r^*) B - r^* B - \frac{\tau\lambda}{1-\lambda\tau} \Omega \left[ \frac{(1-\gamma)p^N}{\gamma} \right]^{\eta-1} p^N y^N \\
&= y^T + \frac{\tau\lambda}{1-\lambda\tau} [\theta A (K^T)^\theta z^{1-\theta} + \left( \alpha + \Omega \left\{ 1 + \left[ \frac{(1-\gamma)p^N}{\gamma} \right]^{\eta-1} \right\} \right) p^N y^N] \\
&\quad - \frac{\tau\lambda}{1-\lambda\tau} \Omega \left[ \frac{(1-\gamma)p^N}{\gamma} \right]^{\eta-1} p^N y^N - (1+\lambda)R^* z - [r^* + \lambda(1+r^*)] B \\
&= y^T - (1-\theta)y^T + \frac{\tau\lambda}{1-\lambda\tau} [\theta A (K^T)^\theta z^{1-\theta} + (\alpha + \Omega) p^N y^N] - [r^* + \lambda(1+r^*)] B \\
&= \theta y^T + \frac{\tau\lambda}{1-\lambda\tau} [\theta y^T + (\alpha + \Omega) p^N y^N] - [r^* + \lambda(1+r^*)] B \\
&= \frac{\theta y^T}{1-\lambda\tau} + \frac{\tau\lambda}{1-\lambda\tau} (\alpha + \Omega) p^N y^N - [r^* + \lambda(1+r^*)] B
\end{aligned}$$

So:

$$\begin{aligned}
&\frac{\tau\lambda}{1-\lambda\tau} \left( \alpha + \Omega \left\{ 1 + \eta \left[ \frac{(1-\gamma)p^N}{\gamma} \right]^{\eta-1} \right\} \right) p^N y^N \\
&= \eta c^T - \left\{ \frac{\theta y^T}{1-\lambda\tau} - [r^* + \lambda(1+r^*)] B + (\eta-1) \left( c^T - \frac{\tau\lambda}{1-\lambda\tau} \Omega \left[ \frac{(1-\gamma)p^N}{\gamma} \right]^{\eta-1} p^N y^N \right) \right\} \\
&= \eta c^T - \left\{ \frac{\theta y^T}{1-\lambda\tau} - [r^* + \lambda(1+r^*)] B + \right.
\end{aligned}$$

$$\begin{aligned}
& (\eta - 1) \left( \frac{\theta y^T}{1 - \lambda\tau} + \frac{\tau\lambda}{1 - \lambda\tau} (\alpha + \Omega) p^N y^N - [r^* + \lambda(1 + r^*)] B \right) \} \\
= & \eta c^T - \left\{ \frac{\theta y^T}{1 - \lambda\tau} - \eta [r^* + \lambda(1 + r^*)] B + (\eta - 1) \left( \frac{\theta y^T}{1 - \lambda\tau} + \frac{\tau\lambda}{1 - \lambda\tau} (\alpha + \Omega) p^N y^N \right) \right\}
\end{aligned}$$

Substituting the above expression into (A.15):

$$\begin{aligned}
& \frac{\tau}{1 - \lambda\tau} R^* z \lambda \hat{\lambda} \\
& + \frac{1}{\lambda} \left[ c^T - \frac{\theta y^T}{1 - \lambda\tau} + [r^* + \lambda(1 + r^*)] B \right] (1 - \alpha) \hat{L} \\
& + \frac{1}{\lambda} \left[ \eta c^T - \eta \frac{\theta y^T}{1 - \lambda\tau} - (\eta - 1) \frac{\tau\lambda}{1 - \lambda\tau} (\alpha + \Omega) p^N y^N + \eta [r^* + \lambda(1 + r^*)] B \right] \hat{p}^N \\
= & \left\{ R^* z - \frac{(1 - \theta) \tau \theta A (K^T)^\theta z^{1-\theta}}{1 - \lambda\tau} \right\} \hat{z}
\end{aligned}$$

Substituting out for  $\hat{\lambda}$  in terms of  $\hat{z}$  from (A.14) and for  $\hat{z}$  in terms of  $\hat{L}$  and  $\hat{p}^N$  from (A.13), we obtain:

$$\begin{aligned}
& - \frac{\tau}{1 - \lambda\tau} R^* z \theta (1 + \lambda) \frac{c^T}{\lambda R^* z} [(1 - \alpha) \hat{L} + \eta \hat{p}^N] \\
& + \frac{1}{\lambda} \left[ c^T - \frac{\theta y^T}{1 - \lambda\tau} + [r^* + \lambda(1 + r^*)] B \right] (1 - \alpha) \hat{L} \\
& + \frac{1}{\lambda} \left[ \eta c^T - \eta \frac{\theta y^T}{1 - \lambda\tau} - (\eta - 1) \frac{\tau\lambda}{1 - \lambda\tau} (\alpha + \Omega) p^N y^N + \eta [r^* + \lambda(1 + r^*)] B \right] \hat{p}^N \\
= & \left\{ R^* z - \frac{(1 - \theta) \tau \theta A (K^T)^\theta z^{1-\theta}}{1 - \lambda\tau} \right\} \frac{c^T}{\lambda R^* z} [(1 - \alpha) \hat{L} + \eta \hat{p}^N]
\end{aligned}$$

Collecting terms and simplifying (using the first order necessary condition for  $z$  and the collateral constraint):

$$\begin{aligned}
& \frac{1}{\lambda} \left[ c^T - \frac{\theta y^T}{1 - \lambda\tau} + [r^* + \lambda(1 + r^*)] B \right] (1 - \alpha) \hat{L} \\
& + \frac{1}{\lambda} \left[ \eta c^T - \eta \frac{\theta y^T}{1 - \lambda\tau} - (\eta - 1) \frac{\tau\lambda}{1 - \lambda\tau} (\alpha + \Omega) p^N y^N + \eta [r^* + \lambda(1 + r^*)] B \right] \hat{p}^N \\
= & \left\{ R^* z + \frac{\tau}{1 - \lambda\tau} R^* z \theta (1 + \lambda) - \left( \frac{\tau}{1 - \lambda\tau} \right) [\theta (1 + \lambda) R^* z] \right\}
\end{aligned}$$

$$\begin{aligned}
& \times \frac{c^T}{\lambda R^* z} [(1 - \alpha) \hat{L} + \eta \hat{p}^N] \\
& = \frac{c^T}{\lambda} [(1 - \alpha) \hat{L} + \eta \hat{p}^N]
\end{aligned}$$

or

$$\begin{aligned}
& \left[ \frac{1}{\lambda} \left( c^T - \frac{\theta y^T}{1 - \lambda \tau} \right) - \frac{c^T}{\lambda} + \frac{[r^* + \lambda(1 + r^*)] B}{\lambda} \right] (1 - \alpha) \hat{L} \\
& + \left[ \frac{1}{\lambda} \left( \eta c^T - \eta \frac{\theta y^T}{1 - \lambda \tau} - (\eta - 1) \frac{\tau \lambda}{1 - \lambda \tau} (\alpha + \Omega) p^N y^N \right) - \eta \frac{c^T}{\lambda} + \frac{\eta [r^* + \lambda(1 + r^*)] B}{\lambda} \right] \hat{p}^N \\
& = 0
\end{aligned}$$

Multiply by  $\lambda$ , cancel terms, as appropriate, multiply by  $1 - \lambda \tau$ , to get:

$$\begin{aligned}
& \left( -\eta \theta y^T - (\eta - 1) \tau \lambda (\alpha + \Omega) p^N y^N + (1 - \lambda \tau) \eta [r^* + \lambda(1 + r^*)] B \right) \hat{p}^N \\
& = \left\{ \theta y^T - (1 - \lambda \tau) [r^* + \lambda(1 + r^*)] B \right\} (1 - \alpha) \hat{L}.
\end{aligned}$$

or,

$$\frac{\hat{p}^N}{\hat{L}} = \frac{-\left\{ \theta y^T - (1 - \lambda \tau) [r^* + \lambda(1 + r^*)] B \right\} (1 - \alpha)}{\eta \theta y^T + (\eta - 1) \tau \lambda (\alpha + \Omega) p^N y^N - (1 - \lambda \tau) \eta [r^* + \lambda(1 + r^*)] B} = s^{AM}, \quad (\text{A.16})$$

say, where  $s^{AM}$  is the slope of the  $AM$  curve. From this expression we can see that for the Cobb-Douglas case ( $\eta = 1$ ), the slope is definitely negative:

$$\frac{\hat{p}^N}{\hat{L}} = -(1 - \alpha) < 0$$

So, with perfect substitutability, the  $AM$  curve slopes downward, and a cut in  $R$  must produce a fall in  $p^N$  and a rise in  $L$ . With  $\eta = 0$ :

$$\frac{\hat{p}^N}{\hat{L}} = \frac{\left\{ \theta y^T - (1 - \lambda \tau) [r^* + \lambda(1 + r^*)] B \right\} (1 - \alpha)}{\tau \lambda (\alpha + \Omega) p^N y^N}.$$

Note that when  $B = 0$ , this expression is definitely positive. It cannot be signed when  $B > 0$ .

By substituting out for  $\hat{p}^N$  in (A.16) from the LM curve, (A.12), we find:

$$\frac{\hat{R}}{1 - \left(\frac{p^N}{\gamma p}\right)^{1-\eta}} + s^{LM} \hat{L} = s^{AM} \hat{L},$$

where

$$s^{LM} = \frac{\psi + \alpha}{1 - \left(\frac{p^N}{\gamma p}\right)^{1-\eta}}.$$

Then,

$$\frac{\hat{L}}{\hat{R}} = \frac{1}{(s^{AM} - s^{LM}) \left(1 - \left(\frac{p^N}{\gamma p}\right)^{1-\eta}\right)}.$$

So, for a cut in  $R$ ,  $\hat{R} < 0$ , to produce an equilibrium drop in employment,  $\hat{L} < 0$ , we need  $s^{AM} > s^{LM}$ . Since the latter is positive, this requires that  $s^{AM}$  be more than just positive. It must be big enough. The condition is:

$$\frac{\left\{\theta y^T - (1 - \lambda\tau) [r^* + \lambda(1 + r^*)] B\right\} (1 - \alpha)}{\eta\theta y^T + (\eta - 1)\tau\lambda(\alpha + \Omega)p^N y^N - (1 - \lambda\tau)\eta [r^* + \lambda(1 + r^*)] B} > \frac{\psi + \alpha}{1 - \left(\frac{p^N}{\gamma p}\right)^{1-\eta}}.$$

### A.3. Solving the Main Model

We stress the case where there is substitution between  $c^N$  and  $c^T$ , and between  $z_t$  and  $V_t$ . The case of no substitutability involves obvious modifications on the discussion below and so is not included. A technical manuscript which covers this case in detail is available on request.

We begin with a discussion of the steady state in which all real variables and relative prices with time subscripts are constant. We consider two steady states. The pre-crisis steady state is one in which the initial level of debt, say  $B_0$ , violates (2.18). We define the post-crisis steady state as one in which the initial level of debt, say  $B_\infty$ , has the property that the collateral constraint is satisfied as an equality. We then discuss the computation of the dynamic path taking the economy from first steady state to the second. We do this under two scenarios. In our baseline scenario government policy, defined here in terms of money growth, is constant. In the alternative scenario the money growth rate is adjusted to hit a particular target interest rate in period 0, while money growth is returned to the money growth rate is changed in the period when the collateral constraint is imposed.

#### A.3.1. Steady State

To understand how these steady states are computed, it is useful to note that - subject to feasibility - corresponding to any initial level of debt there is a unique steady state when the



collateral constraint, (2.18), is ignored. To find the post crisis steady state, we simply alter the initial debt until the collateral constraint is satisfied as an equality. Then,  $B_0$  is selected as a number bigger than  $B_\infty$ , to be consistent with data as discussed in the text.

The following discussion explains how we find the steady state corresponding to an arbitrary initial value of the debt.

The steady state interest rate,  $R$ , is determined from (2.6), (2.13), and the fact that  $p_t = P_t/M_t$  is constant in steady state:

$$R = \frac{1+x}{\beta}.$$

From here on, we treat  $R$  as a known quantity.

Rewriting the firm's first order condition for  $z$  :

$$y^T = \mu_2 z \left( \frac{R^*}{(1-\theta)\mu_2} \right)^\xi.$$

The resource constraint in the traded good sector is:

$$y^T - c^T - R^* z = r^* B.$$

Combining this with the previous expression,

$$\left[ \left( \frac{1}{1-\theta} \right)^\xi \left( \frac{R^*}{\mu_2} \right)^{\xi-1} - 1 \right] R^* z = c^T + r^* B. \quad (\text{A.17})$$

Here is an algorithm for finding the steady state which involves a nonlinear search in the single variable,  $c^T$ .

Suppose  $c^T$  is given. Then,  $z$  can be computed from (A.17).  $L^T$  may then be obtained from:

$$z = \frac{\mu_1 V}{\mu_2} \left[ \frac{1}{\theta} \left( \frac{R^*}{(1-\theta)\mu_2} \right)^{\xi-1} - \left( \frac{1-\theta}{\theta} \right) \right]^{\frac{-\xi}{\xi-1}} \quad (\text{A.18})$$

Given  $L^T$ ,  $L^N$  may be obtained by combining the price equation:

$$p^N = \frac{\gamma}{1-\gamma} \left( \frac{(1-\gamma)c^T}{\gamma c^N} \right)^{\frac{1}{\eta}} \quad (\text{A.19})$$

and equality of  $VMP_L$ 's across the two sectors:

$$(1 - \alpha)p^N (K^N)^\alpha (L^N)^{-\alpha} = \left\{ \frac{1}{\theta} - \left( \frac{1 - \theta}{\theta} \right) \left( \frac{R^*}{(1 - \theta)\mu_2} \right)^{1 - \xi} \right\}^{\frac{1}{1 - \xi}} \theta(1 - \nu)\mu_1 A (K^T)^\nu (L^T)^{-\nu}.$$

Substitute the former into the latter, to obtain:

$$\begin{aligned} & (1 - \alpha) \frac{\gamma}{1 - \gamma} \left( \frac{(1 - \gamma)c^T}{\gamma(K^N)^\alpha (L^N)^{1 - \alpha}} \right)^{\frac{1}{\eta}} (K^N)^\alpha (L^N)^{-\alpha} \\ &= \left\{ \frac{1}{\theta} - \left( \frac{1 - \theta}{\theta} \right) \left( \frac{R^*}{(1 - \theta)\mu_2} \right)^{1 - \xi} \right\}^{\frac{1}{1 - \xi}} \theta(1 - \nu)\mu_1 A (K^T)^\nu (L^T)^{-\nu}, \end{aligned}$$

or

$$(L^N)^{-[\frac{1}{\eta}(1 - \alpha) + \alpha]} = D, \tag{A.20}$$

where

$$D = \frac{\left\{ \frac{1}{\theta} - \left( \frac{1 - \theta}{\theta} \right) \left( \frac{R^*}{(1 - \theta)\mu_2} \right)^{1 - \xi} \right\}^{\frac{1}{1 - \xi}} \theta(1 - \nu)\mu_1 A (K^T)^\nu (L^T)^{-\nu}}{(1 - \alpha) \frac{\gamma}{1 - \gamma} \left( \frac{(1 - \gamma)c^T}{\gamma(K^N)^\alpha} \right)^{\frac{1}{\eta}} (K^N)^\alpha}$$

With  $L^N$ ,  $c^T$  in hand, compute  $p^N$  from the price equation, (A.19). Finally, assess whether labor supply equals labor demand in the traded good sector:

$$\frac{1}{R} \left\{ \frac{1}{\theta} - \left( \frac{1 - \theta}{\theta} \right) \left( \frac{R^*}{(1 - \theta)\mu_2} \right)^{1 - \xi} \right\}^{\frac{1}{1 - \xi}} \theta(1 - \nu)\mu_1 A (K^T)^\nu (L^T)^{-\nu} = \psi_0 (L^T + L^N)^\psi p. \tag{A.21}$$

Adjust  $c^T$  until (A.21) holds exactly. The five equations, (A.17)-(A.21), can be used in this way to pin down the five variables,  $L^N$ ,  $L^T$ ,  $p^N$ ,  $c^T$ ,  $z$ .

The variables,  $p$  and  $c$ , may be obtained from (2.8) and (2.7). Then, obtain  $p^T$  from

$$pp^T c = 1 + x.$$

The wage rate comes from

$$w = \psi_0 L^\psi p, \quad L = L^T + L^N$$

We now obtain the steady state value of  $d$ , the ratio of deposits to the beginning of period

money stock. Dividing (2.3) by  $P^T$  :

$$pc = wL + \frac{1}{p^T} - \frac{d}{p^T},$$

so that

$$d = p^T [wL - pc] + 1.$$

We require  $0 \leq d \leq 1$ .

Now we go for the asset values. From (2.20),

$$q^i = VMP_K^i + \frac{q^i}{1 + r^*},$$

so that

$$q^i = \frac{1 + r^*}{r^*} VMP_K^i, \quad i = N, T,$$

where

$$\begin{aligned} VMP_K^N &= p^N \frac{\alpha y^N}{K^N}, \\ VMP_K^T &= \left[ \frac{y^T}{\mu_1 V} \right]^{\frac{1}{\xi}} \frac{\theta \nu \mu_1 V}{K^T}. \end{aligned}$$

The value of collateral is, in units of the traded good,

$$q^N K^N + q^T K^T$$

This completes the discussion of the steady state.

### A.3.2. The Transition Path

We imagine that in date 0 the economy has an initial debt level of  $B_0$ . At this level of debt the collateral constraint is binding. In the baseline equilibrium, money growth is kept constant. That is,  $x_t = x$  for  $t = 0, 1, 2, \dots$ . We compute the equilibrium path of the economy to the new steady state where the debt level is  $B_\infty$ . In the second equilibrium,  $x_0 \neq x$ , but  $x_t = x$  for  $t = 1, 2, \dots$ . In this equilibrium, the monetary adjustment is unanticipated in the sense that when households make their deposit decision in the beginning of period 0 they do so under the assumption that they are in the baseline equilibrium. As noted above, they do not adjust this

decision when it turns out that  $x_0 \neq x$ .

We first consider the computation of the baseline equilibrium. We then discuss the computation of the equilibrium in which there is a monetary intervention. The basic strategy is based on solving a system of non-linear equations in the Lagrange multipliers on the collateral constraint. For a given set of Lagrange multipliers, we compute a sequence of candidate allocations and prices, imposing the following conditions: (i) quantities and prices eventually end up in the new steady state; (ii) the initial level of debt is  $B_0$ ; and (iii) all equilibrium conditions except the collateral constraint are imposed. We then evaluate the collateral constraint at each date. We adjust the Lagrange multipliers until it is satisfied. If the multipliers turn out to violate non-negativity, then we conclude there is no equilibrium.

**Baseline Scenario** At date 0,  $B_0$  is given. We want to compute an equilibrium set of sequences,

$$q_t^T, q_t^N, c_t^T, c_t^N, L_t^T, L_t^N, p_t^T, p_t^N, R_t, w_t, z_t, B_{t+1}, \lambda_t, \quad t = 0, 1, 2, 3, \dots,$$

for a given sequence of  $x_t$ 's. These 13 sequences must satisfy 13 equilibrium conditions. These are the two equations defining the  $q$ 's (2.20); the firm's intertemporal Euler equation (2.22), and its three intra-temporal Euler equations, (2.23), (2.24), (2.25); the marginal condition relating  $p^N$  to the consumption goods, (2.9); the resource constraint in the traded and non-traded good sectors, (2.30) and (2.31); the collateral constraint, (2.18); the household's intra- and inter-temporal Euler equations, (2.5), (2.6); and finally, the cash in advance constraint, (2.29).<sup>39</sup>

We seek an equilibrium which converges asymptotically to the steady state where the debt is  $B_\infty$  and the collateral constraint is marginally non-binding. This means that all the above sequences converge to their values in the steady state equilibrium whose computation is discussed in the previous section.

Here is our strategy for accomplishing this. We assume that the system arrives in a steady state in period  $T + 2$  (in practice, we found that  $T = 10$  works well.) We specify exogenously (below, we explain in detail how this is done), a sequence,  $\lambda \equiv (\lambda_0, \lambda_1, \dots, \lambda_{T+1})$ , with  $\lambda_t = 0$  for  $t \geq T + 2$ . Also,  $\Lambda_{T+2} = \Lambda_s$ , where the subscript, 's', means steady state. Similarly, all the other 13 variables are assumed to be in the new steady state for  $t \geq T + 2$ . The value of  $T$  that we used in the calculations satisfies the property that the economy has for all practical purposes achieved convergence to the new steady state before  $T + 2$ .

The idea is to vary  $\lambda$  until the collateral constraint,

$$\tau^N q_t^N K^N + \tau^T q_t^T K^T - R^* z_t - (1 + r^*) B_t = 0$$

is satisfied for  $t = 0, 1, \dots, T + 1$ . (This equation is satisfied by construction for  $t \geq T + 2$ .) To do this, we need to compute a mapping from  $\lambda$  to the  $q_t^i$ 's, the  $z_t$ 's and the  $B_t$ 's.

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<sup>39</sup>Equilibrium also requires that the limiting condition, (2.17), be satisfied. We can verify that this is satisfied ex post, when we have found a set of Lagrange multipliers which produce allocations where the collateral constraint is satisfied in each period.

First, we set up a mapping:

$$\left(\Lambda_{t+1}, p_t^T\right) \rightarrow \left(c_t^N, c_t^T, p_t^N, w_t, R_t, L_t^N, L_t^T, \Lambda_t, p_{t-1}^T\right), \quad (\text{A.22})$$

starting with  $t = T + 2$  and ending with  $t = 1$ . We then handle  $t = 0$  separately.

**Dates,  $t \geq 1$**  The object,  $\Lambda_t$  is obtained using (2.22):

$$\Lambda_t = \Lambda_{t+1}(1 + \lambda_t),$$

which is an equation that is available for  $t = 1, \dots, T + 2$ . Then, we make use of (2.12) to solve for  $p_{t-1}^T$ :

$$\Lambda_t = \frac{u_{c,t} p_{t-1}^T}{p_t} \frac{1}{p_t^T} \frac{1}{1 + x_{t-1}} \beta, \quad (\text{A.23})$$

which is available for  $t = 1, \dots, T + 2$ . To solve this, we require the other variables first. We do this using our equilibrium conditions and the given  $\Lambda_t, p_t^T$ . We find these variables by setting up a one-dimensional search for  $L_t^N$ . So, suppose that in addition to  $(\Lambda_t, \lambda_t, p_t^T)$ , we have  $L_t^N$ . Then, from our assumptions about technology,  $c_t^N = (K^N)^\alpha (L_t^N)^{1-\alpha}$ . Given  $L_t^N$  and  $c_t^N$ , the following two equations can be solved for  $p_t^N$  and  $c_t^T$ :

$$\begin{aligned} p_t^N &= \frac{\gamma}{1 - \gamma} \left( \frac{(1 - \gamma) c_t^T}{\gamma c_t^N} \right)^{\frac{1}{\eta}} \\ p_t p_t^T c_t &= 1 + x_t, \end{aligned} \quad (\text{A.24})$$

where

$$p = \left[ \left( \frac{1}{1 - \gamma} \right)^{1-\eta} + \left( \frac{p^N}{\gamma} \right)^{1-\eta} \right]^{\frac{1}{1-\eta}} \quad (\text{A.25})$$

$$c = \left\{ [(1 - \gamma) c^T]^{\frac{\eta-1}{\eta}} + [\gamma c^N]^{\frac{\eta-1}{\eta}} \right\}^{\frac{\eta}{\eta-1}}. \quad (\text{A.26})$$

This can be treated as a one dimensional search problem in  $c_t^T$  alone.

We now have  $L^N, p^N, p, c^N$  and  $c^T$  in hand. The next step is to find  $z$  and  $L^T$ . One equation

that is useful for this purpose is the first order condition for  $z$  :

$$\left(\frac{y^T}{\mu_2 z}\right)^{\frac{1}{\xi}} \mu_2 = \left[\frac{1+\lambda}{1-\theta}\right] R^*.$$

Since

$$\frac{y^T}{z\mu_2} = \left\{ \theta \left[\frac{\mu_1 V}{\mu_2 z}\right]^{\frac{\xi-1}{\xi}} + 1 - \theta \right\}^{\frac{\xi}{\xi-1}}, \quad V = A (K^T)^\nu (L^T)^{1-\nu},$$

we have

$$\frac{\mu_1 V}{\mu_2 z} = \left[ \frac{1}{\theta} \left( \frac{(1+\lambda)R^*}{(1-\theta)\mu_2} \right)^{\xi-1} - \left( \frac{1-\theta}{\theta} \right) \right]^{\frac{\xi}{\xi-1}}. \quad (\text{A.27})$$

We require

$$\frac{\mu_2 - (1-\theta)^{\frac{\xi}{1-\xi}} R^*}{(1-\theta)^{\frac{\xi}{1-\xi}} R^*} > \lambda,$$

which guarantees that, as long as  $0 \leq \xi \leq 1$ , the object in braces is positive. This is necessary, for  $z$  and  $V$  to be positive. This is a restriction we place on  $\lambda$ .

Equation (A.27) involves two unknowns,  $z$  and  $L^T$ . We need another equation to pin these two variables down. Before obtaining these, it is useful to work on the expression for the marginal product of labor in the traded good sector:

$$VMP_L^T = \left(\frac{y^T}{\mu_1 V}\right)^{\frac{1}{\xi}} \theta(1-\nu)\mu_1 \frac{V_t}{L_t^T},$$

or,

$$VMP_L^T = \left\{ \frac{1}{\theta} - \left(\frac{1-\theta}{\theta}\right) \left(\frac{(1+\lambda)R^*}{(1-\theta)\mu_2}\right)^{1-\xi} \right\}^{\frac{1}{1-\xi}} \theta(1-\nu)\mu_1 A (K^T)^\nu (L^T)^{-\nu}. \quad (\text{A.28})$$

Equating the  $VMP_L$ 's in each sector:

$$(1-\alpha)p^N \frac{y^N}{L^N} = \left[ \frac{1}{\theta} - \left(\frac{1-\theta}{\theta}\right) \left(\frac{(1+\lambda)R^*}{(1-\theta)\mu_2}\right)^{1-\xi} \right]^{\frac{1}{1-\xi}} \theta(1-\nu)\mu_1 A (K^T)^\nu (L^T)^{-\nu}.$$

This can be solved for  $L^T$ :

$$L^T = \left(L^N\right)^{\frac{\alpha}{\nu}} \left\{ \frac{\left[ \frac{1}{\theta} - \left(\frac{1-\theta}{\theta}\right) \left(\frac{(1+\lambda)R^*}{(1-\theta)\mu_2}\right)^{1-\xi} \right]^{\frac{1}{1-\xi}} \theta(1-\nu)\mu_1 A \left(K^T\right)^\nu}{(1-\alpha)p^N \left(K^N\right)^\alpha} \right\}^{\frac{1}{\nu}}. \quad (\text{A.29})$$

Then, setting  $V = A \left(K^T\right)^\nu \left(L^T\right)^{1-\nu}$ , we obtain  $z$  from (A.27):

$$z = \frac{\mu_1 V}{\mu_2} \left\{ \frac{1}{\theta} \left[ \frac{(1+\lambda)R^*}{(1-\theta)\mu_2} \right]^{\xi-1} - \left(\frac{1-\theta}{\theta}\right) \right\}^{\frac{-\xi}{\xi-1}}. \quad (\text{A.30})$$

Compute

$$u_{c,t} = \left( c_t - \frac{\psi_0}{1+\psi} \left( L_t^T + L_t^N \right)^{1+\psi} \right)^{-\sigma}. \quad (\text{A.31})$$

Now, it is possible to solve for  $p_{t-1}^T$  using (A.23). But, we are not done yet, because we started with a guess for  $L_t^N$ .

From the labor supply equation, (2.5),

$$w = p\psi_0 \left( L^T + L^N \right)^\psi. \quad (\text{A.32})$$

From (2.32) we obtain  $R_t$ :

$$\beta R_t = (1 + x_{t-1}) (1 + \lambda_t) (p_t^T / p_{t-1}^T), \quad (\text{A.33})$$

$t = 1, 2, \dots$ . Evaluating the product of  $R$  obtained from here and  $w$  obtained from (A.32), we can evaluate (2.24):

$$f(L_t^N; \Lambda_t, \lambda_t, p_t^T) \equiv (1 - \alpha) p_t^N \frac{y_t^N}{L_t^N} - w_t R_t. \quad (\text{A.34})$$

The idea is to adjust  $L_t^N$  until  $f(L_t^N; \Lambda_t, \lambda_t, p_t^T) = 0$ .

**Date**  $t = 0$  We now have in hand,

$$\left( c_t^N, c_t^T, p_t^N, w_t, R_t, L_t^N, L_t^T, \Lambda_t, p_{t-1}^T \right), \text{ for } t = 1, \dots, T + 2.$$

Next, we seek  $(c_t^N, c_t^T, p_t^N, w_t, R_t, L_t^N, L_t^T)$  for  $t = 0$ . Note that we do not have  $\Lambda_0$ , since (??) is not available for  $t = 0$ . This means that we cannot find  $L_0^N$  by setting  $f = 0$ , in (A.34) as we do for  $t = 1, \dots, T + 1$ . We replace equation (A.33) by

$$\frac{u_{c,0}}{p_0 p_0^T} = \frac{\beta R_0 u_{c,1}}{p_1 p_1^T (1 + x_0)}. \quad (\text{A.35})$$

Then, we solve for  $L_0^N$  as follows. Fix  $L_0^N$ . Solve for  $p_0^N, p_0, c_0^N, c_0^T, c_0$  using the iterative algorithm described around (A.24). Then, compute  $L_0^T$  using (A.29),  $z_0$  using (A.30),  $u_{c,0}$  using (A.31), and  $w_0$  using (A.32). Solve for  $w_0 R_0$  using

$$w_0 R_0 = (1 - \alpha) p_0^N \left( \frac{K^N}{L_0^N} \right)^\alpha,$$

and compute  $R_0$  from  $w_0 R_0 / w_0$ . Finally, evaluate

$$g(L_0^N) = \frac{u_{c,0}}{p_0 p_0^T} - \frac{u_{c,1}}{p_1 p_1^T} \frac{\beta R_0}{1 + x_0}.$$

Adjust  $L_0^N$  until  $g(L_0^N) = 0$ . Another way to write the  $g$  function substitutes based on  $pp^T c = 1 + x$ ,

$$g(L_0^N) = \frac{u_{c,0} c_0}{1 + x_0} - \frac{u_{c,1} c_1}{1 + x_1} \frac{\beta R_0}{1 + x_0},$$

or,

$$g(L_0^N) = u_{c,0} c_0 - u_{c,1} c_1 \frac{\beta R_0}{1 + x_1},$$

The next step is to evaluate the  $q_t^i$ 's and the  $B_t$ 's. The  $q_t^i$ 's can be solved recursively from (2.20). The  $B_t$ 's can be obtained by simulating (2.30) forward, for the fixed  $B_0$ .

In practice, we found that the following parameterization of the  $\lambda$ 's works well. We let  $\lambda_t$  for  $t = 0, 1, \dots, N - 1$ ,  $N < T$  be free parameters and we set  $\lambda_t$  for  $N < t < T + 2$  by linear interpolation and imposing  $\lambda_{T+2} = 0$ . We chose the free  $\lambda$ 's to enforce exactly the collateral constraints in periods  $t = 0, 1, 2, \dots, N - 1$  and  $T$ . The adequacy of this computational strategy can be evaluated ex post by evaluating the collateral constraints for dates with  $t \neq T$  and  $t \notin \{0, 1, \dots, N\}$ . We found that this procedure works well for  $T = 10$  and  $N = 6$ .

**Surprise Scenario** We now suppose that  $d_0$  is set according to the equilibrium in the previous subsection. In reflection of this, we drop the household's dynamic first order condition, (A.35), from consideration in period 0. The computational strategy for finding the equilibrium in this scenario is essentially identical to what we described in the previous subsection, apart from the obvious changes that must be made to for handling period 0.



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Table 1: Parameter Values, Two Models Used in Analysis							
Labor in Traded Good Sector				No Labor In Traded Good Sector			
$\beta$	0.94	$\gamma$	0.27	$\beta$	0.94	$\gamma$	0.33
$\psi$	3.00	$R$	1.12	$\psi$	1	$R$	1.12
$R^*$	1.06	$r^*$	0.06	$R^*$	1.06	$r^*$	0.06
$\alpha$	0.36	$K^N$	2.00	$\alpha$	0.36	$K^N$	11.25
$\nu$	0.50	$K^T$	1.00	$\nu$	1	$K^T$	5.
$\mu_1$	1.4	$\mu_2$	3.50	$\mu_1$	1	$\mu_2$	1
$\tau$	0.185	$\theta$	NA	$\tau$	0.11	$\theta$	0.6
$\psi_0$	4.59	$\sigma$	1	$\psi_0$	0.0036	$\sigma$	2
$A$	0.87	$\xi$	0.	$A$	3.2	$\xi$	1

Table 2: Steady State for Two Models, Ignoring Collateral Constraint							
Labor in Traded Good Sector				No Labor In Traded Good Sector			
$L$	0.54	$z$	0.13	$L$	26.	$z$	6.8
$L^T$	0.13	$L^N$	0.40	$L^T$	NA	$L^N$	26
$c^T$	0.27	$c^N$	0.72	$c^T$	9.69	$c^N$	19.4
$w$	1.05	$V$	0.32	$w$	0.44	$V$	NA
$\frac{p^N c^N}{c^T}$	2.79	$y^T$	0.44	$\frac{p^N c^N}{c^T}$	2.12	$y^T$	18.1
$p^N$	1.04	$p^T$	1.05	$p^N$	1.06	$p^T$	0.035
$q^T$	2.73	$q^N$	2.37	$q^T$	38.46	$q^N$	11.63
$B$	0.70	$\frac{B}{p^N c^N + y^T - R^* z}$	0.66	$B$	20	$\frac{B}{p^N c^N + y^T - R^* z}$	0.66

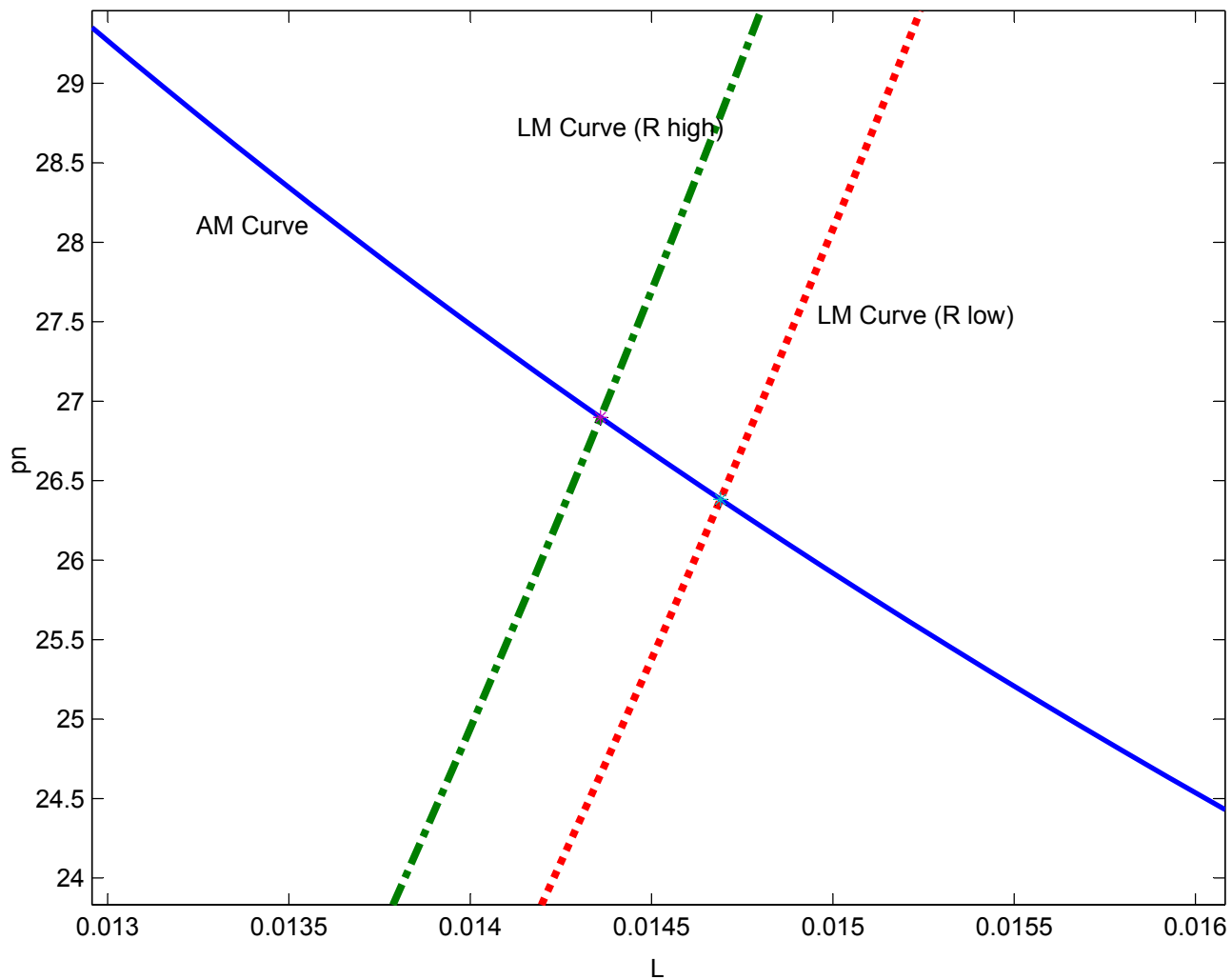
Table 3: Steady State for Two Models, Respecting Collateral Constraint							
Labor in Traded Good Sector				No Labor In Traded Good Sector			
$L$	0.54	$z$	0.12	$L$	28.	$z$	6.85
$L^T$	0.13	$L^N$	0.41	$L^T$	NA	$L^N$	28
$c^T$	0.27	$c^N$	0.72	$c^T$	10.01	$c^N$	20.05
$w$	1.05	$V$	0.32	$w$	0.44	$V$	NA
$\frac{p^N c^N}{c^T}$	2.85	$y^T$	0.44	$\frac{p^N c^N}{c^T}$	2.69	$y^T$	18.1
$p^N$	1.06	$p^T$	1.03	$p^N$	1.34	$p^T$	0.029
$q^T$	2.68	$q^N$	2.43	$q^T$	38.46	$q^N$	15.21
$B$	0.59	$\frac{B}{p^N c^N + y^T - R^* z}$	0.55	$B$	14.62	$\frac{B}{p^N c^N + y^T - R^* z}$	0.40

Table 4: Contemporaneous Effect of Cut in $R$ at Date 0 (Relative to constant $x$ path)		
Change in:	Labor in Traded Good Sector	No Labor In Traded Good Sector
$R$	-0.09	-0.09
$\lambda$	-0.014	0.09
Percent Change in:		
$L$	1.75	-0.43
$z$	1.12	-2.04
$L^T$	2.24	NA
$L^N$	1.53	-0.43
$c^T, c^N$	0.98	-0.27
$W/P^T$	9.25	-4.53
$W/P$	12.85	4.13
Current Account	2.78	-3.89
$p^T$	3.50	4.57
Real Exchange Rate	1.00	1.04
$p^N$	0.31	-8.16
$q^T$	0.62	0.10
$q^N$	0.61	-0.40
$VMP_k^T$	2.00	-0.82
$VMP_k^N$	1.29	-8.41
$P$	3.70	0.26

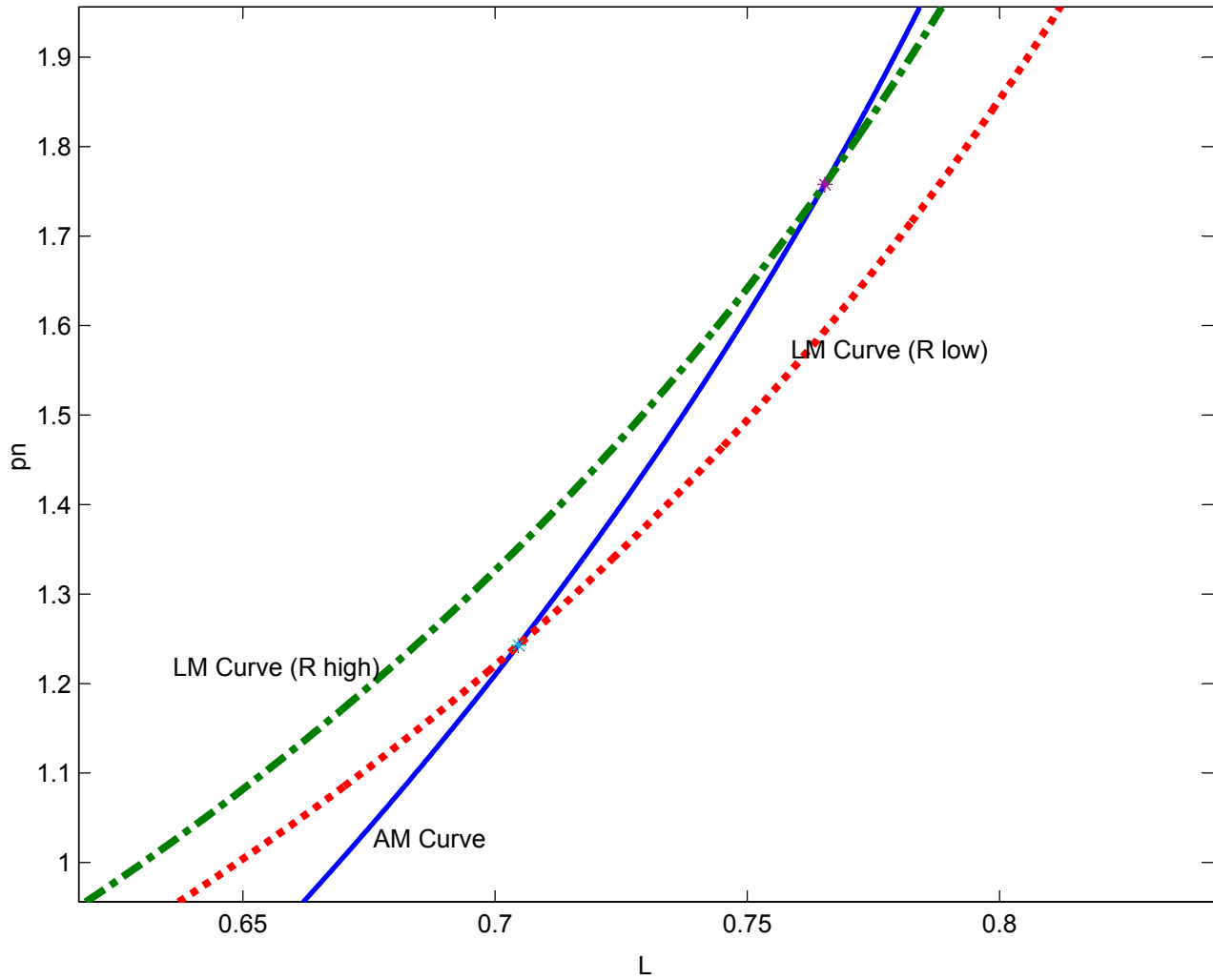
Table 5: Welfare Analysis		
Equilibrium	Present Discounted Value of Utility	
	Labor in	No Labor in
	Traded Good Sector	Traded Good Sector
Pre-crisis steady state	-30.64399	-3.39104
Collateral-constrained steady state	-30.51503	-3.33955
Baseline transition path, $t \geq 0$	-30.68844	-3.94647
Path, $t \geq 0$ , with interest rate cut	-30.68228	-3.95733



**Figure 1: The Effect of an Interest Cut with  $\eta = 0.9$  and No Debt**



**Figure 2: The Effect of an Interest Cut with  $\eta = 0$  and No Debt**



**Figure 3: The Effect of an Interest Cut with  $\eta = 0$  and Modest Debt**

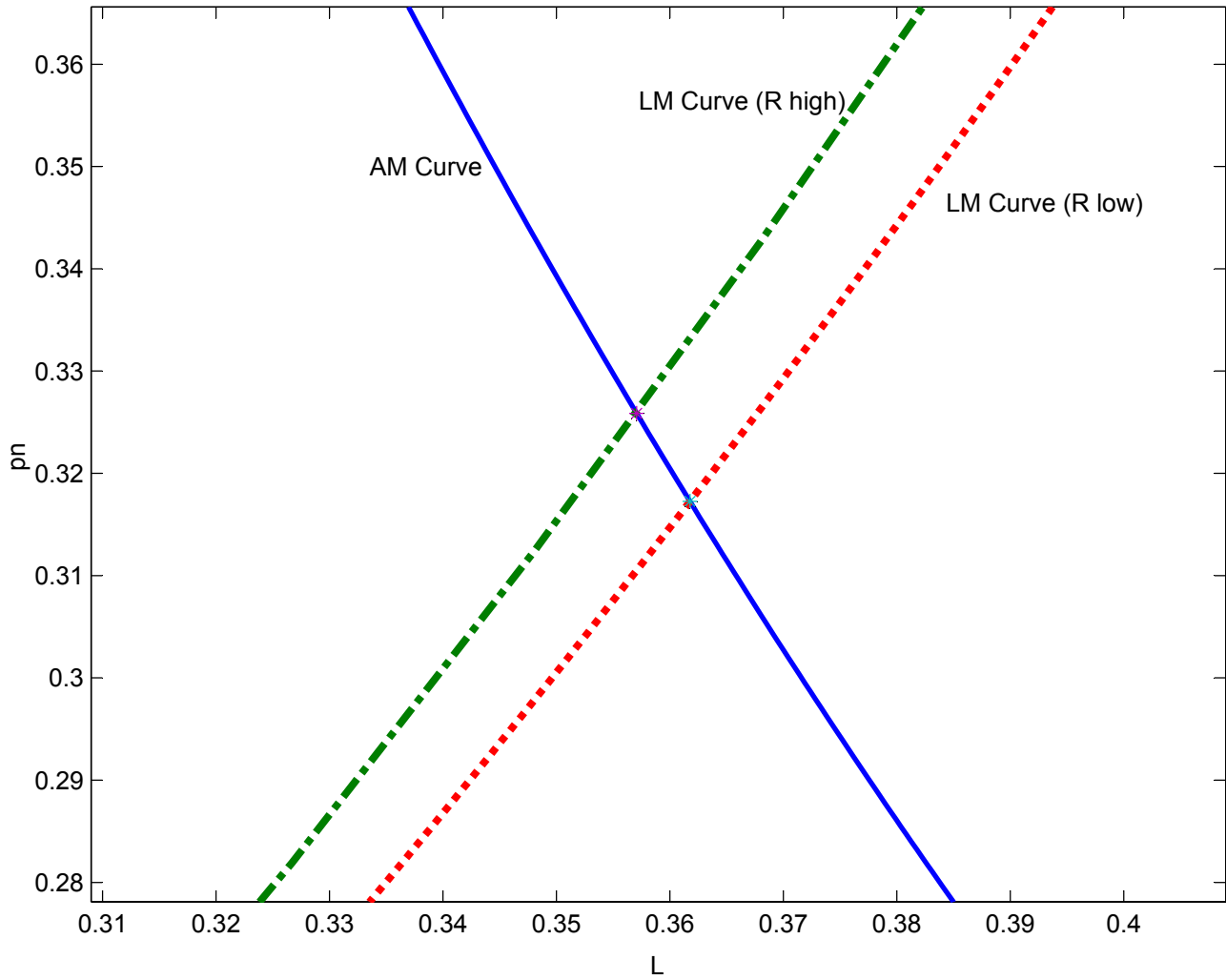


Figure 4:  
Response of Debt and Output to Collateral Shock In  
the Absence of Monetary Policy Response

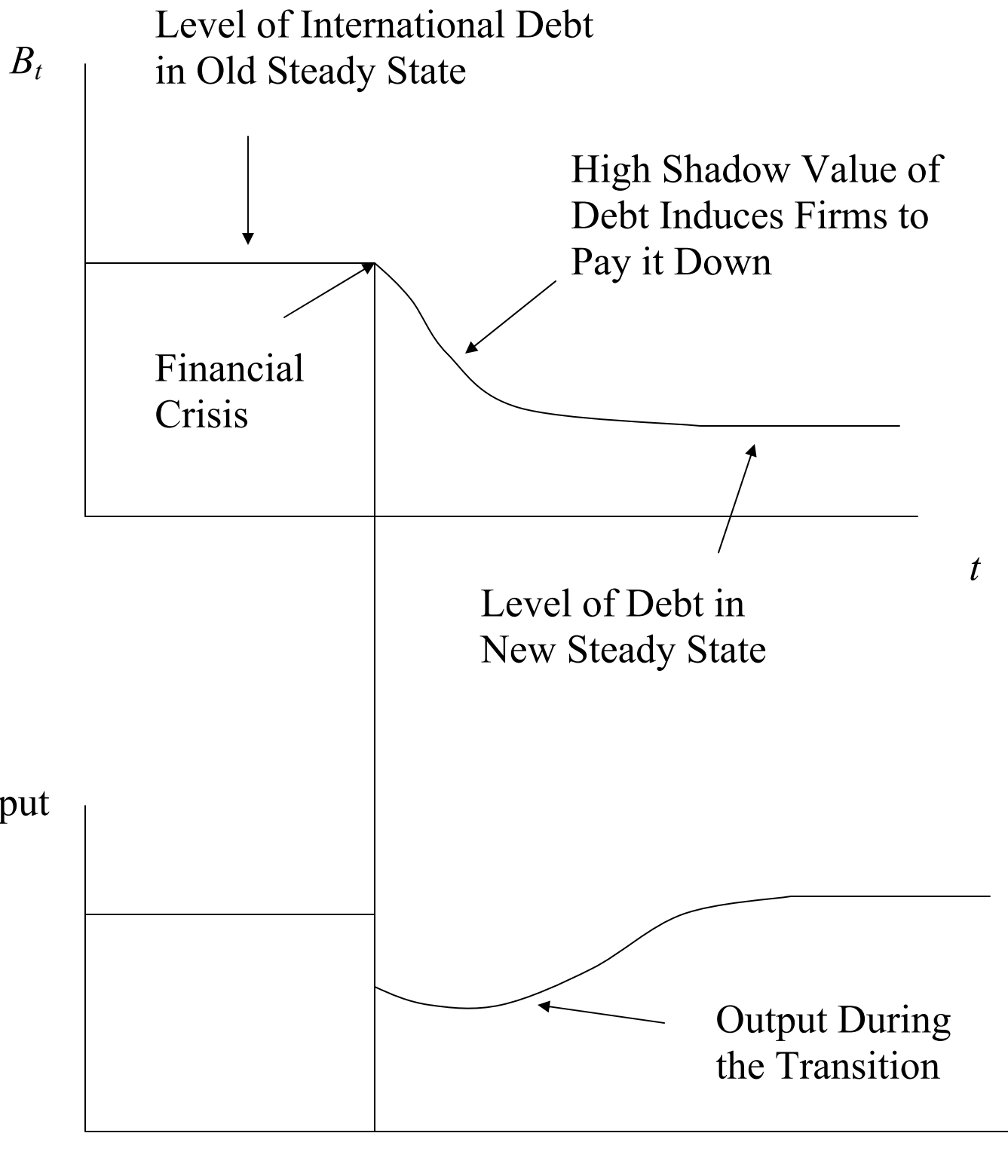


Figure 5: Model Timing

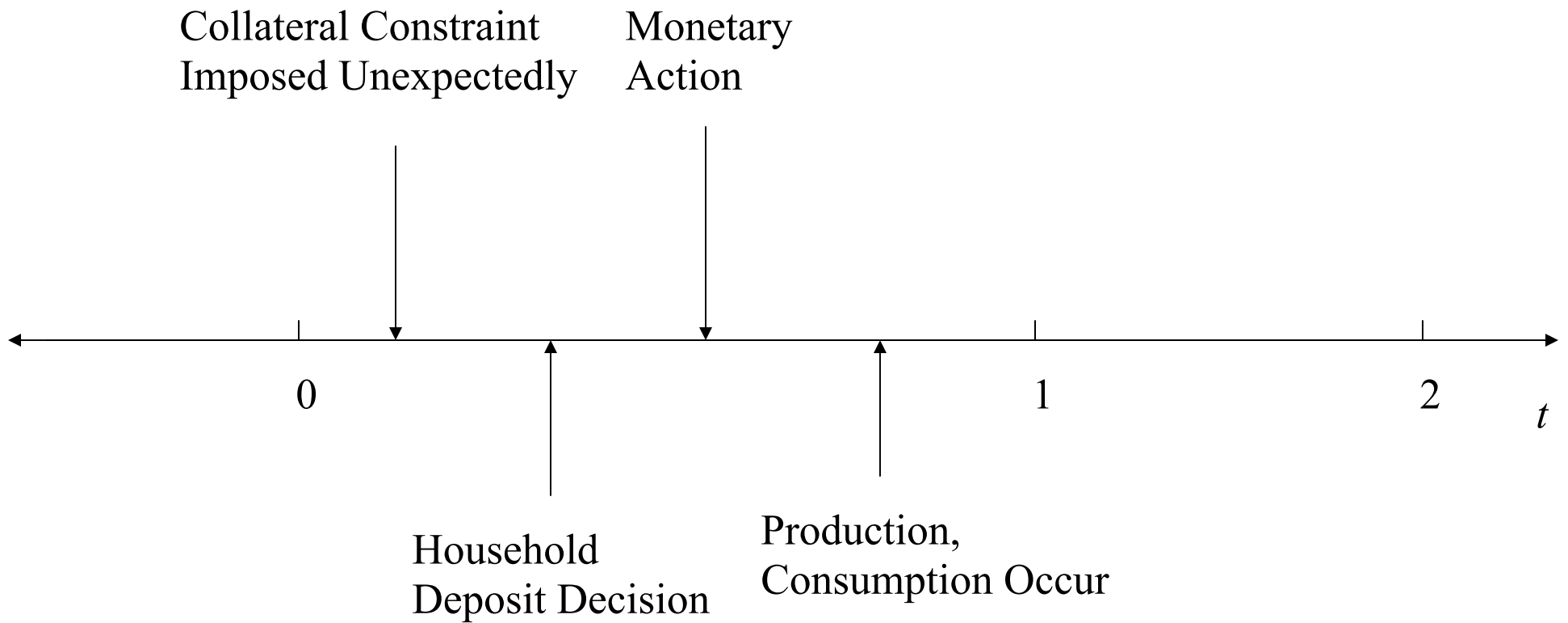
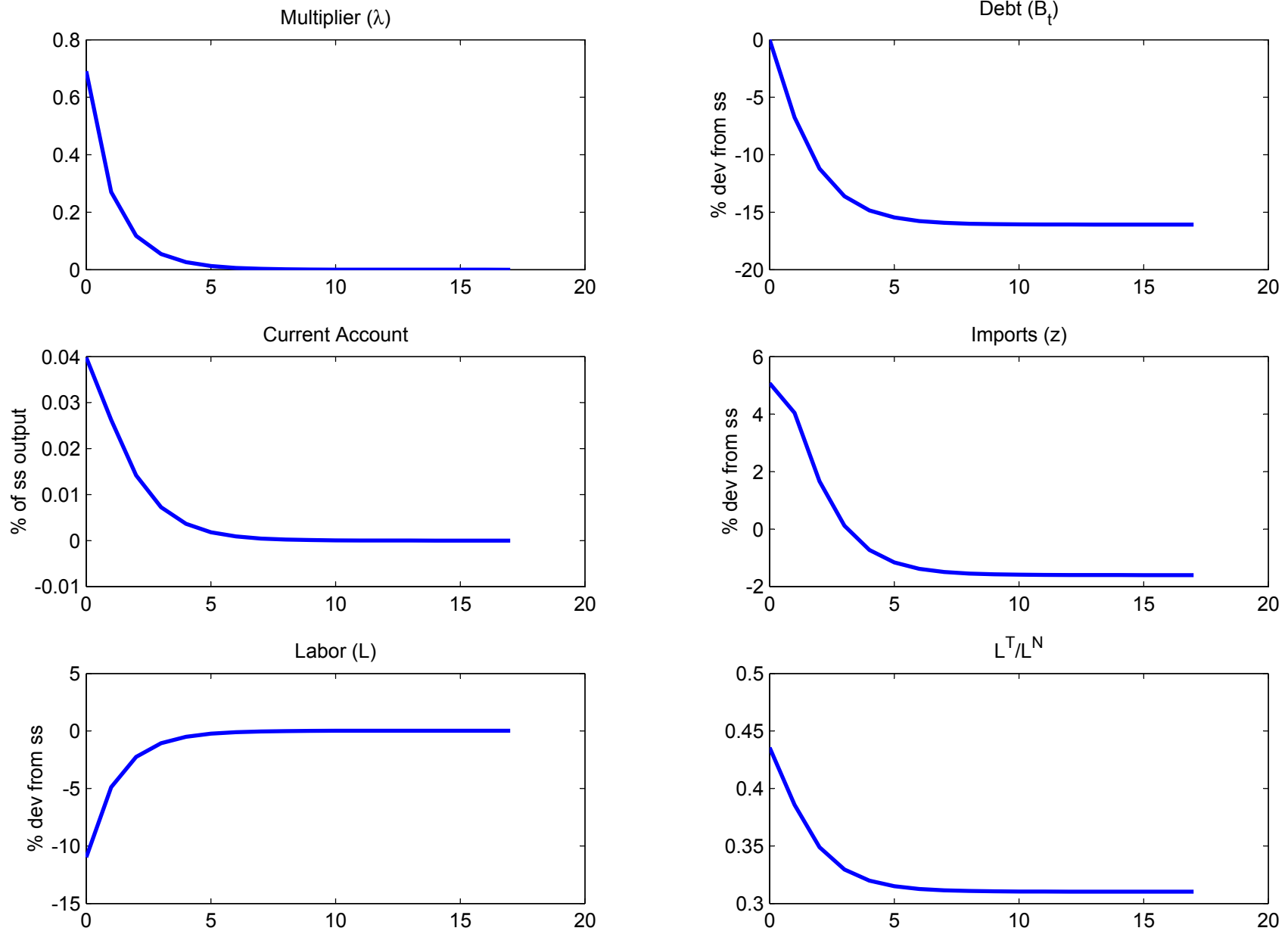
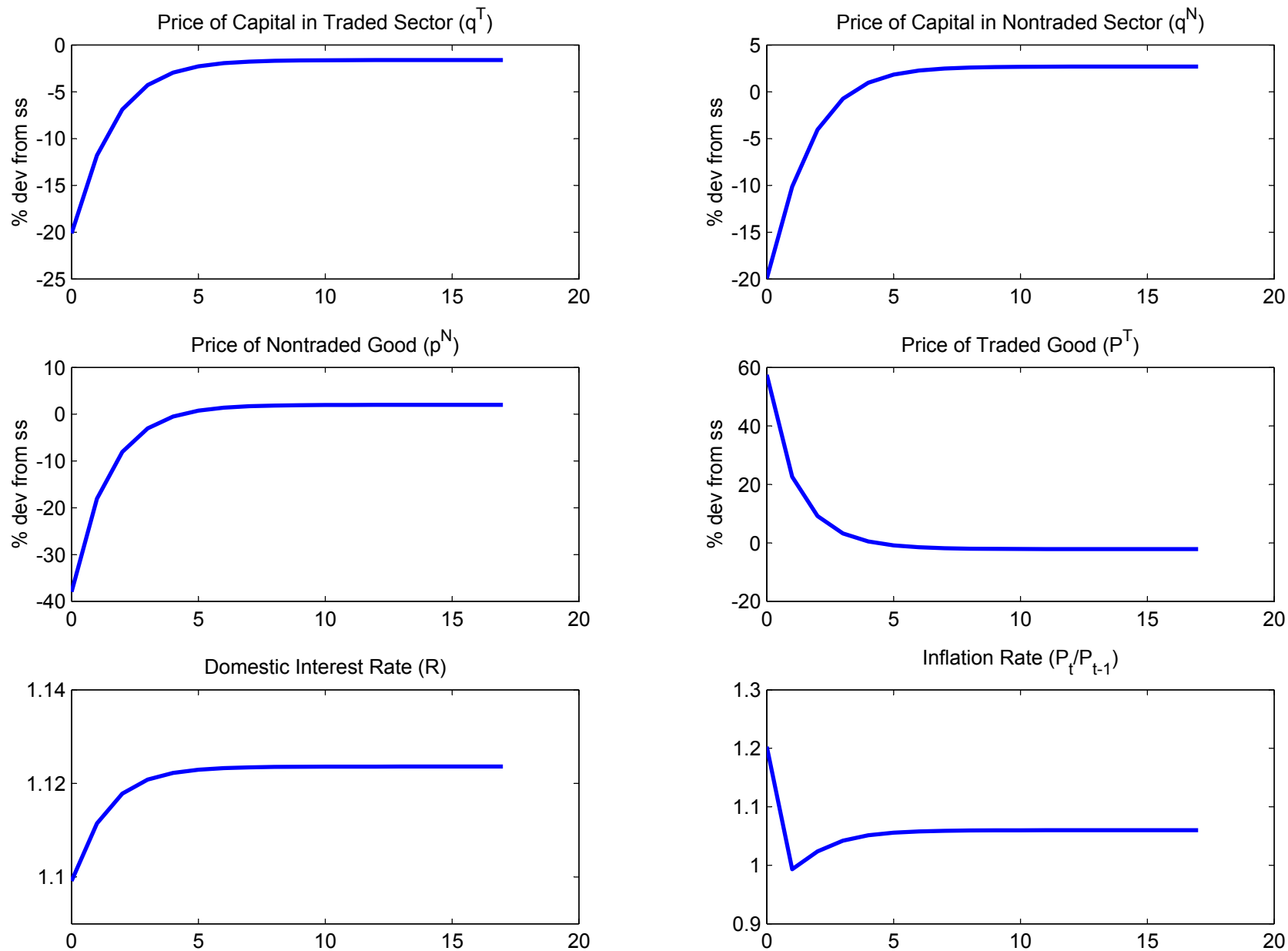


Figure 6a: Transition to Lower Debt In Aftermath of Crisis, No Policy Response, Expansion Scenario Model



Notes: % dev from ss - Percent Deviation from Pre-Crisis Steady State  
 % of ss output - Percent of Pre-Crisis Gross Output

Figure 6b: Transition to Lower Debt In Aftermath of Crisis, No Policy Response, Expansion Scenario Model, (Cont'd)



Note: % dev from ss - Percent Deviation from Pre-Crisis Steady State

Figure 7a: Transition to Lower Debt In Aftermath of Crisis, No Policy Response, Contraction Scenario Model

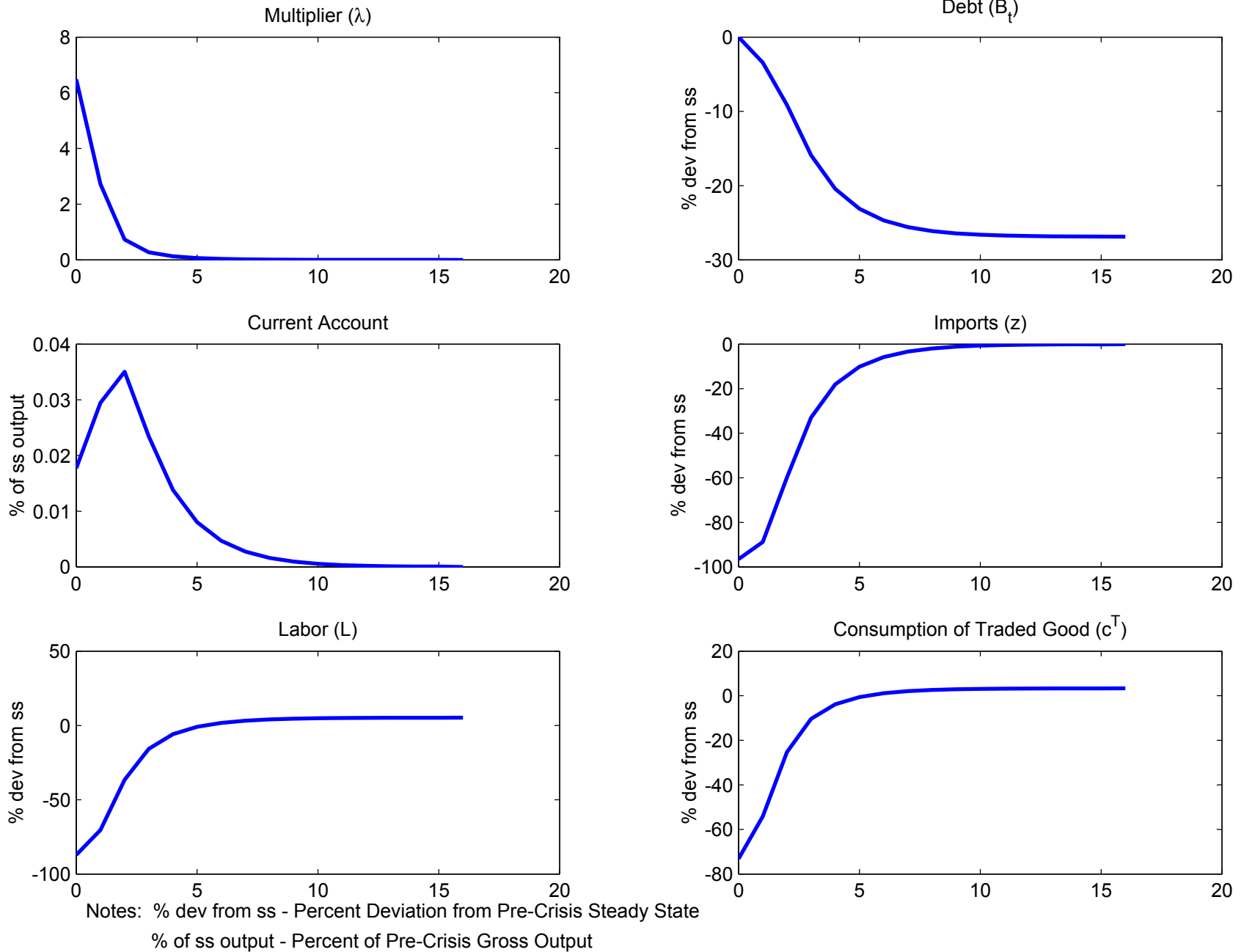
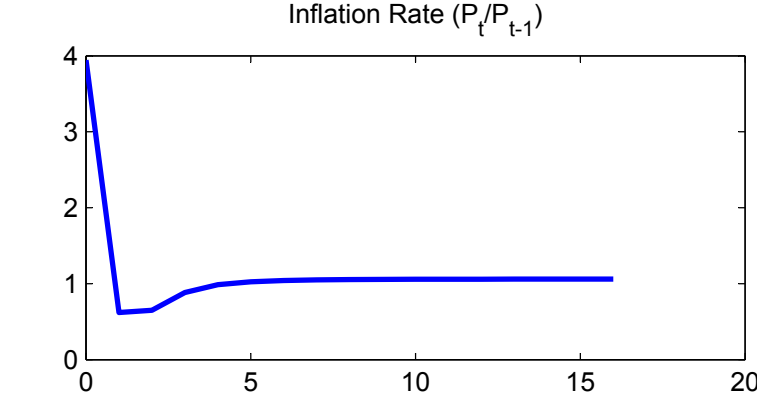
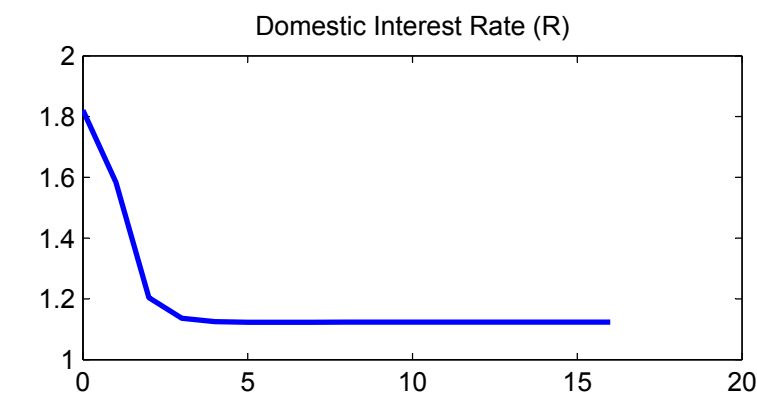
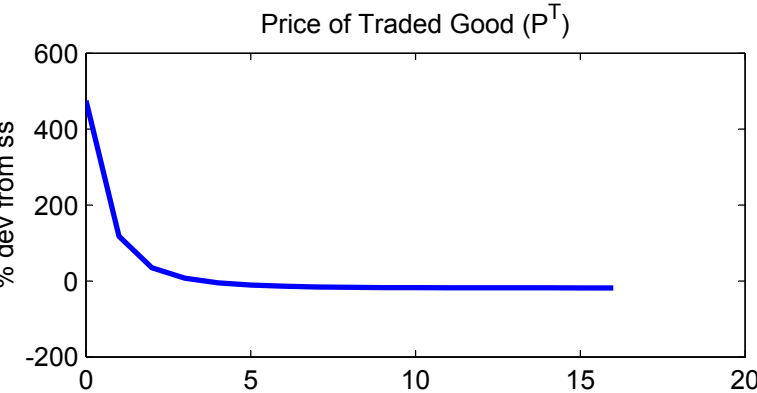
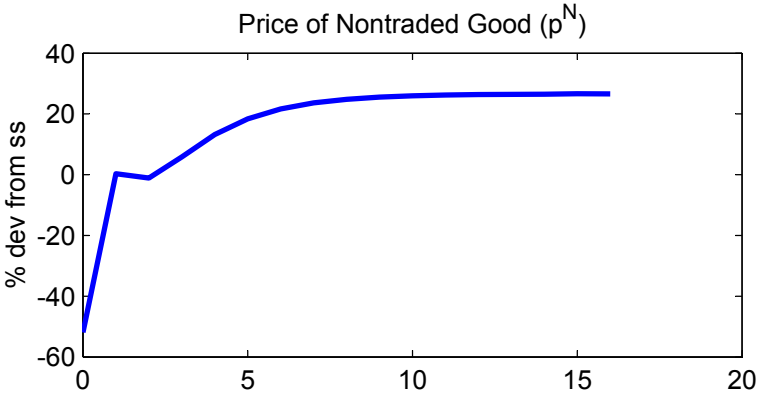
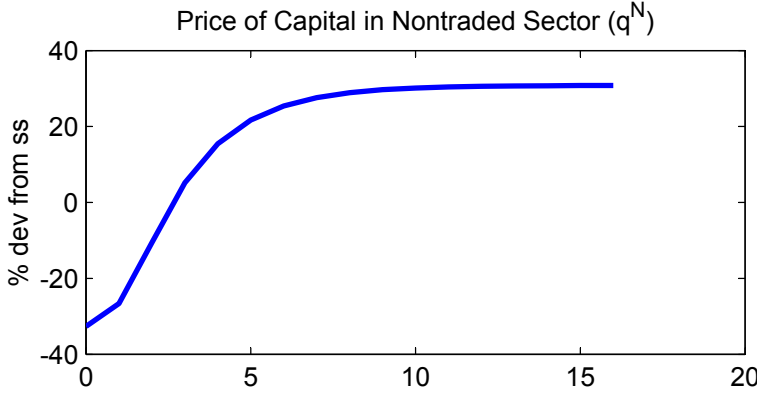
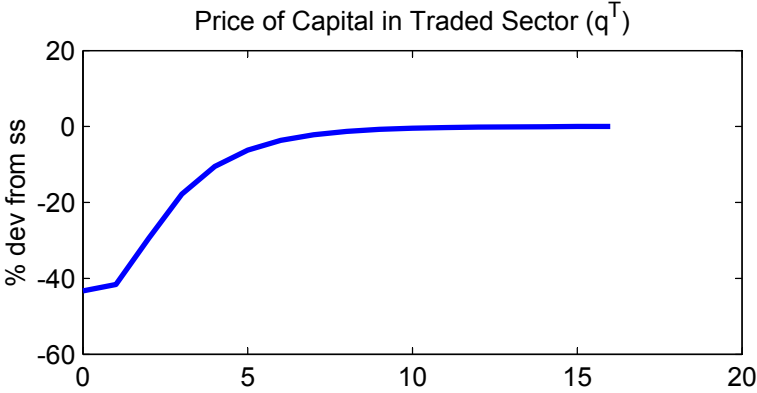


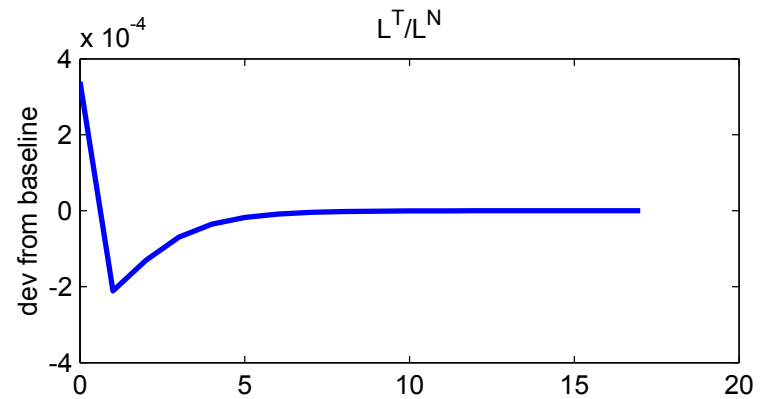
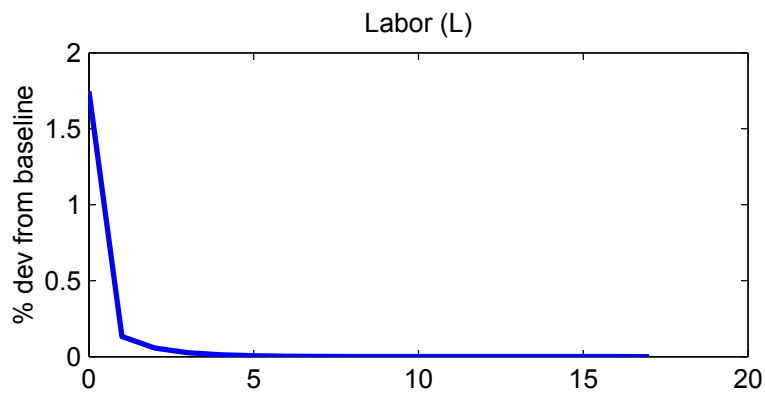
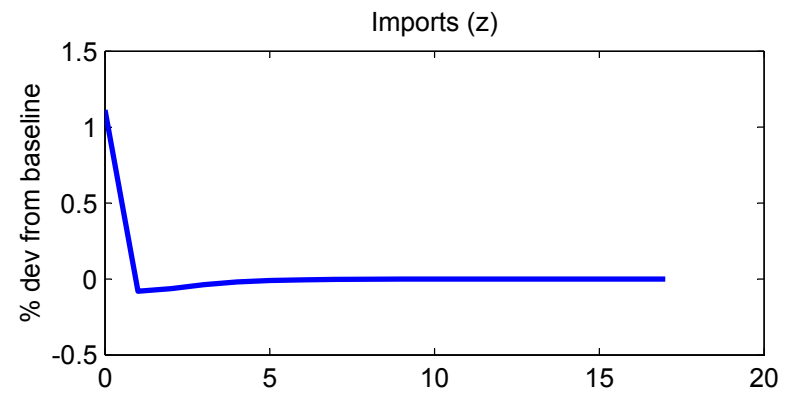
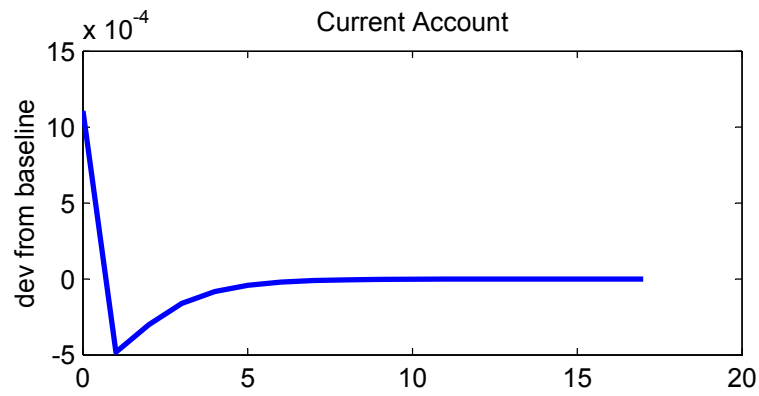
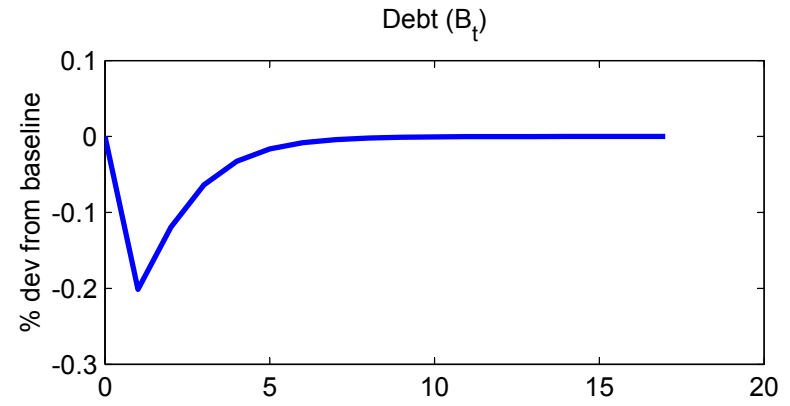
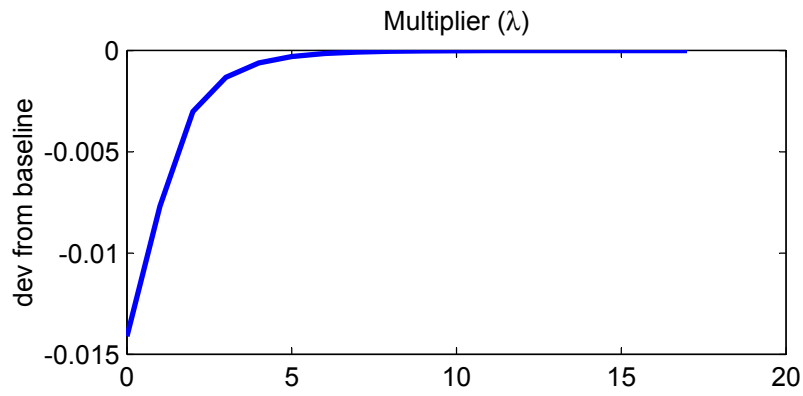


Figure 7b: Transition to Lower Debt In Aftermath of Crisis, No Policy Response, Contraction Scenario Model, (Cont'd)



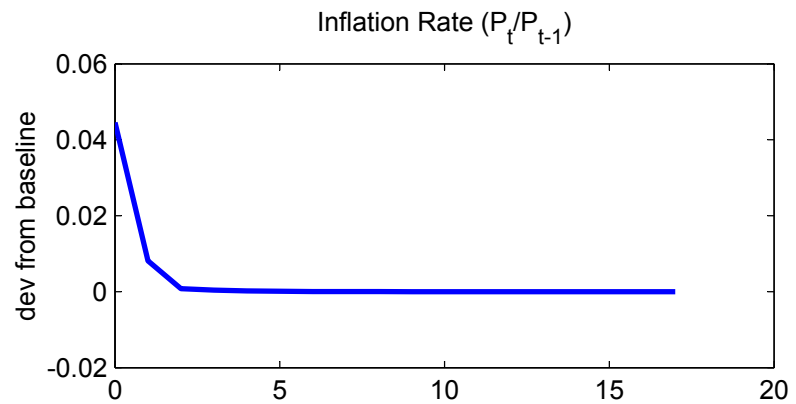
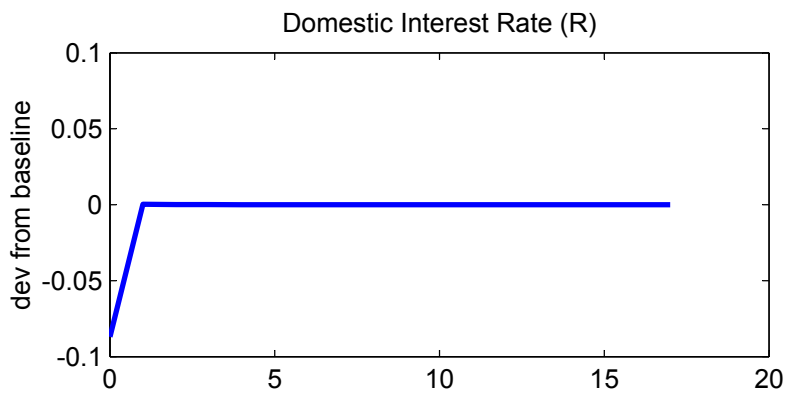
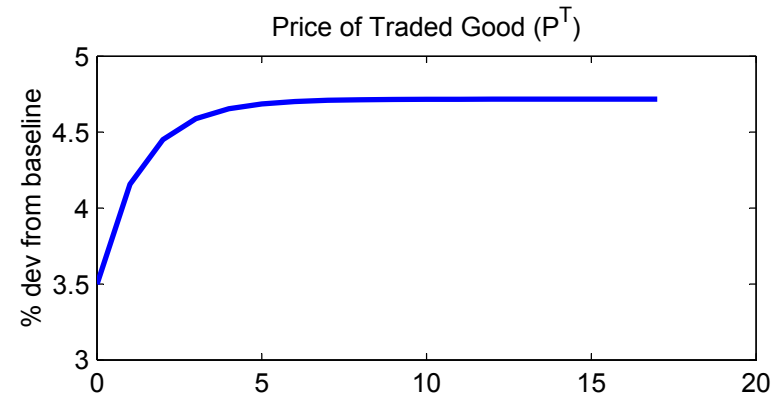
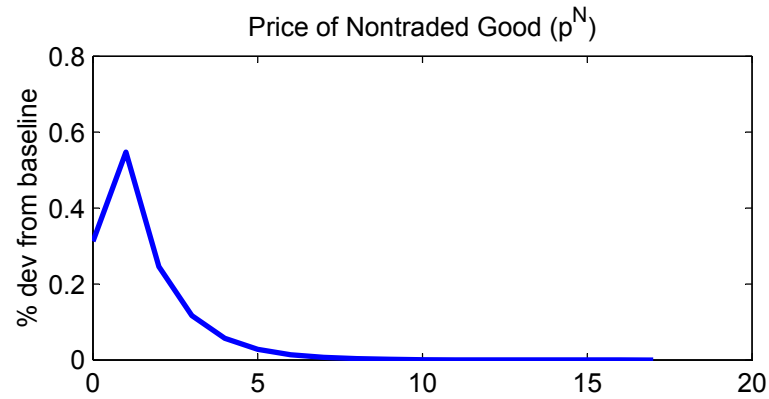
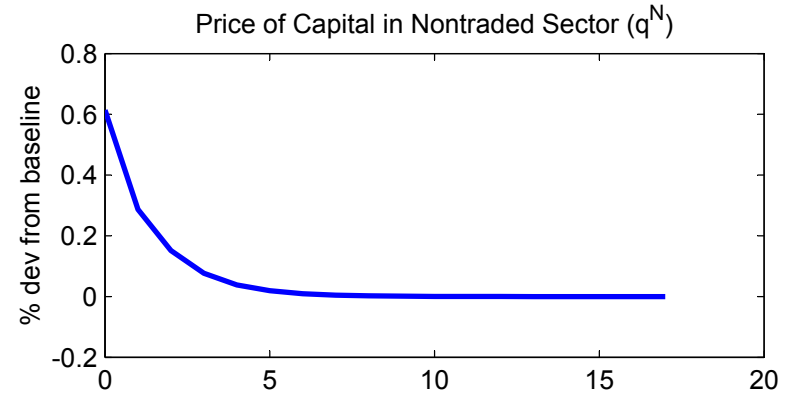
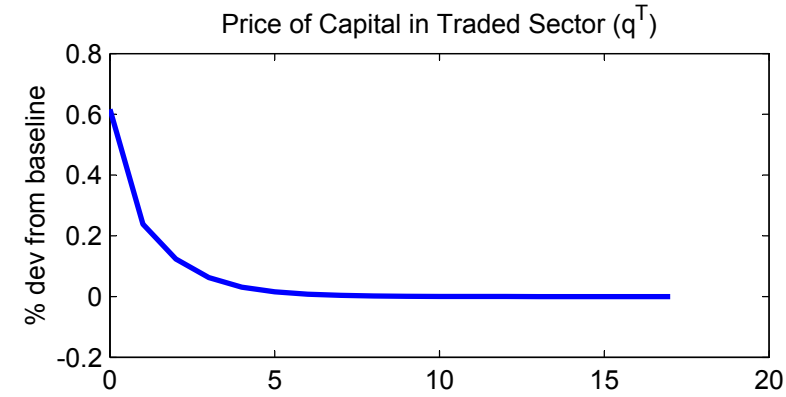
Note: % dev from ss - Percent Deviation from Pre-Crisis Steady State

Figure 8a: Effect on Transition of Policy Cut in Interest Rate, Expansion Scenario Model



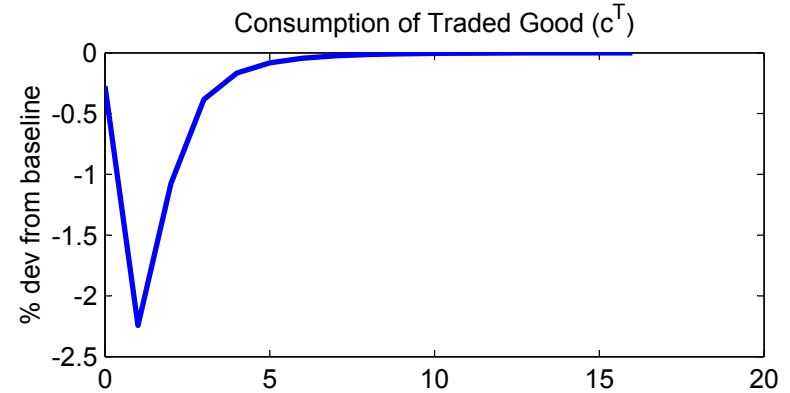
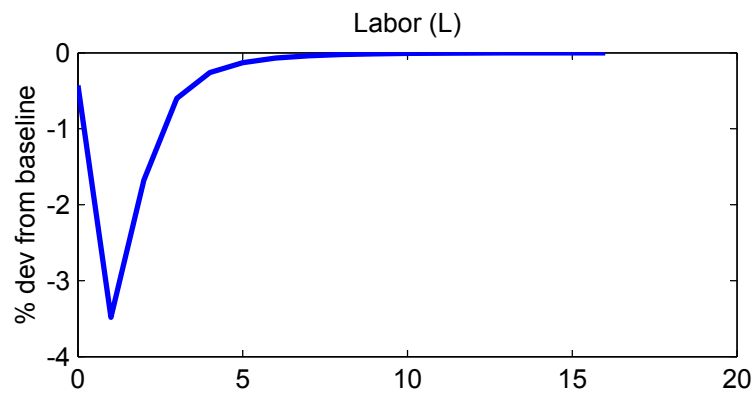
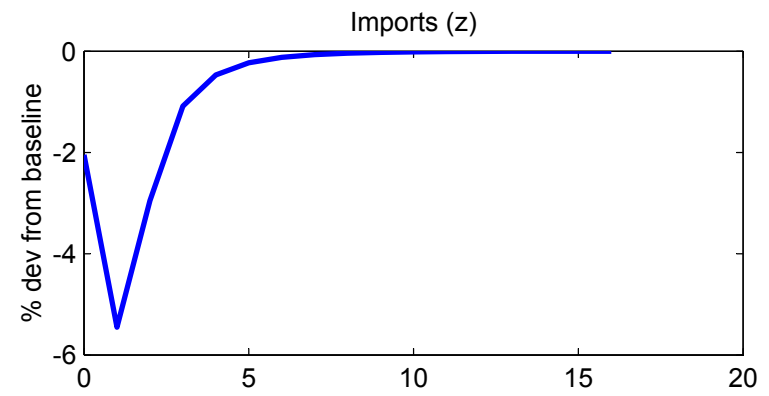
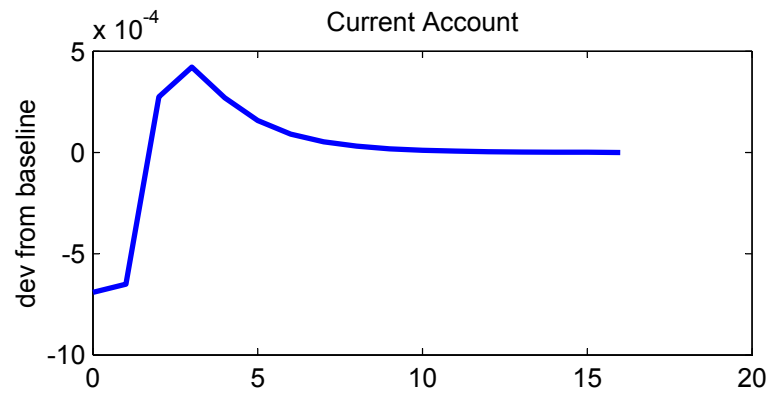
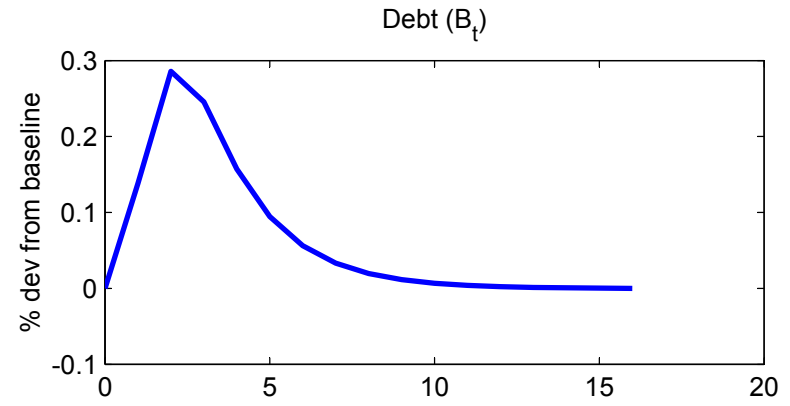
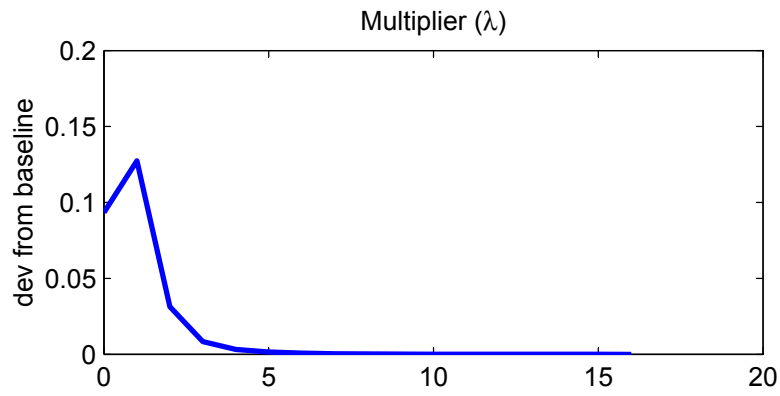
Notes: % dev from baseline - Percent Deviation from Baseline Path of Constant Money Growth  
 % dev from baseline - Deviation from Baseline Path (Percentage Points)

Figure 8b: Effect on Transition of Policy Cut in Interest Rate, Expansion Scenario Model (Cont'd)



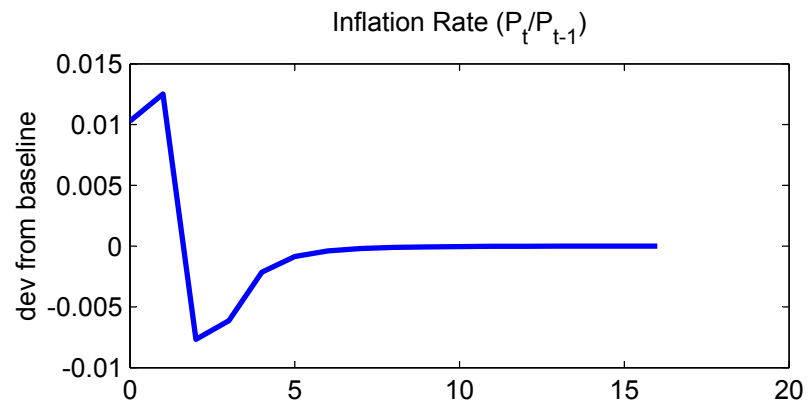
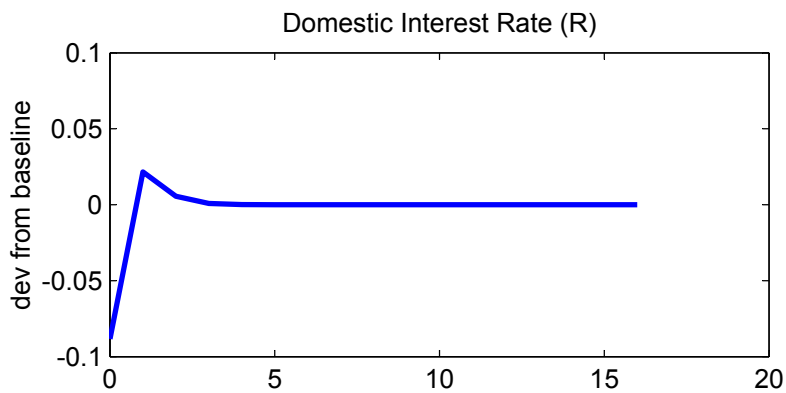
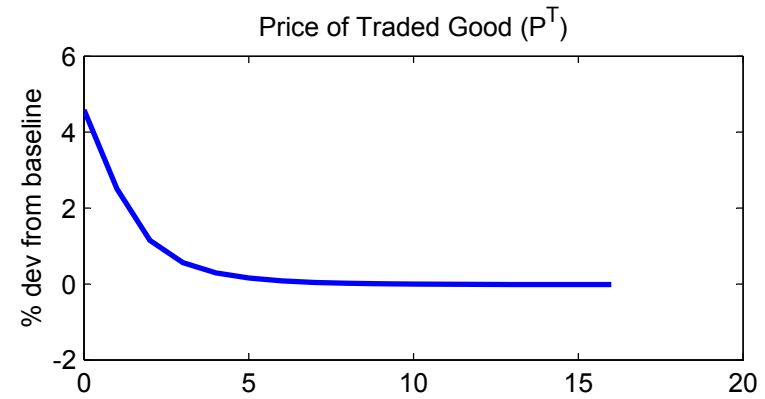
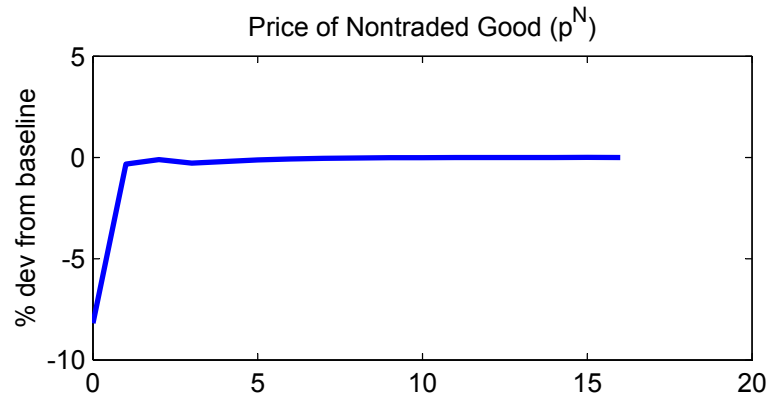
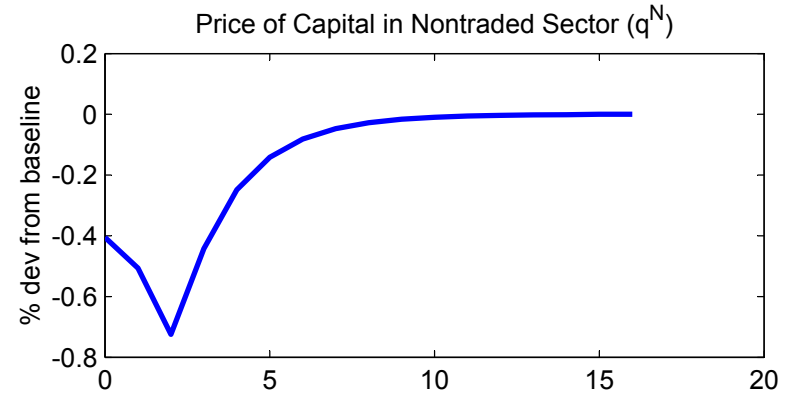
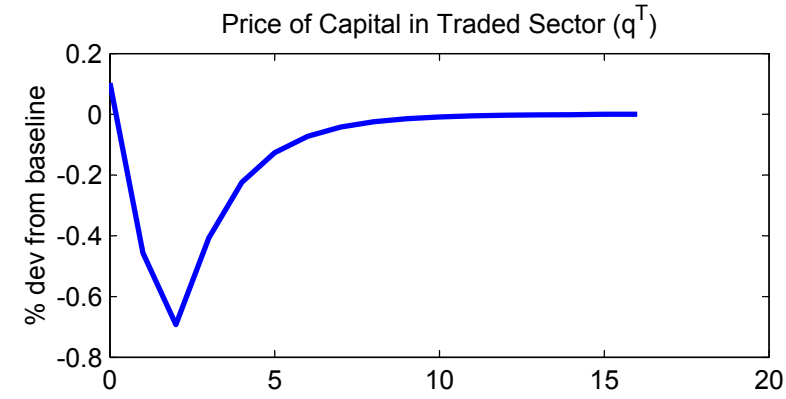
Notes: % dev from baseline - Percent Deviation from Baseline Path of Constant Money Growth  
 % dev from baseline - Deviation from Baseline Path (Percentage Points)

Figure 9a: Effect on Transition of Policy Cut in Interest Rate, Contraction Scenario Model



Notes: % dev from baseline - Percent Deviation from Baseline Path of Constant Money Growth  
 % dev from baseline - Deviation from Baseline Path (Percentage Points)

Figure 9b: Effect on Transition of Policy Cut in Interest Rate, Expansion Scenario Model (Cont'd)



Notes: % dev from baseline - Percent Deviation from Baseline Path of Constant Money Growth  
 % dev from baseline - Deviation from Baseline Path (Percentage Points)