

NBER WORKING PAPER SERIES

OPTIMAL INTEREST RATE POLICY IN A SMALL OPEN ECONOMY

Eric Parrado  
Andrés Velasco

Working Paper 8721  
<http://www.nber.org/papers/w8721>

NATIONAL BUREAU OF ECONOMIC RESEARCH  
1050 Massachusetts Avenue  
Cambridge, MA 02138  
January 2002

We thank Andrés Escobar, Luis Oscar Herrera, Paolo Pesenti, Jaume Ventura, and seminar participants at the Econometrics Society Meeting, the Lacea Winter Camp 2001 and NBER-IFM Spring Meetings 2001 for comments. Velasco thanks the Center for International Development at Harvard for generous support. The views expressed herein are those of the authors and not necessarily those of the National Bureau of Economic Research or the IMF.

© 2002 by Eric Parrado and Andrés Velasco. All rights reserved. Short sections of text, not to exceed two paragraphs, may be quoted without explicit permission provided that full credit, including © notice, is given to the source.

Optimal Interest Rate Policy in a Small Open Economy  
Eric Parrado and Andrés Velasco  
NBER Working Paper No. 8721  
January 2002  
JEL No. E52, F41

### ABSTRACT

Using an optimizing model we derive the optimal monetary and exchange rate policy for a small stochastic open economy with imperfect competition and short run price rigidity. The optimal monetary policy has an exact closed-form solution and is obtained using the utility function of the representative home agent as welfare criterion. The optimal policy depends on the source of stochastic disturbances affecting the economy, much as in the literature pioneered by Poole (1970). Optimal monetary policy reacts to domestic and foreign disturbances. If the intertemporal elasticity of substitution in consumption is less than one, as is likely to be the case empirically, the optimal exchange rate policy implies a *dirty float*: interest rate shocks from abroad are met partially by adjusting home interest rates, and partially by allowing the exchange rate to move. This optimal pattern may help rationalize the observed *fear of floating*.

Eric Parrado  
Monetary and Exchange Affairs Department  
International Monetary Fund  
700 19th Street, N.W.  
Washington, DC 20431  
eparrado@imf.org

Andrés Velasco  
Harvard University  
Kennedy School of Government  
79 JFK Street  
Littauer 106  
Cambridge, MA 02138  
and NBER  
andres\_velasco@harvard.edu

## 1. Introduction

After the exchange rate crises of the last decade many small open economies, both rich and poor, have adopted flexible exchange rates in combination with some kind of monetary or interest rate rule. Exactly what such a rule should look like, however, remains very much an open question. In closed economies, inflation-targeting or Taylor-type rules are common, even though the optimality of such rules is yet to be established analytically. In the open economy, host of additional tricky issues turn up. Should monetary policy respond systematically to the nominal or real exchange rate? Equivalently, should the float be completely clean or not?

In the prehistory of international macroeconomics (that is to say, in the 1970s), the answer to most of these questions was clear. The alleged advantage of a flexible exchange rate was that it helped insulate the economy from foreign real shocks. Behind that protective shield, domestic monetary policy could get on with the task of stabilizing the domestic economy. Put in the language of the 1970's literature, floating gave monetary policy a measure of independence so set interest rates in order to attain internal balance, while the real exchange rate did much (if not all) of the work in securing external balance. This is a well known implication of the Mundell-Fleming model. It is also what most textbook treatments prescribe.

But in recent years this conventional wisdom has become less conventional, and (according to some) perhaps also less wise. To begin with, there is the empirical observations that many small countries do not float the way theory suggests they should. For instance, in the recent Asian crisis several countries abandoned pegs to the dollar but then tightened monetary policy in response to adverse shocks, both internal and external, and attempted to fight the depreciation of their currencies. In 1997-98 most Latin American countries also used tight money and high interest rates to prop up their currencies.<sup>1</sup> Gavin, Hausmann, Pagés-Serra, and Stein (1999) and Calvo and Reinhart (2000) have documented this pattern, in what they call the “fear of floating” puzzle.

Theoretically, the benefits of exchange rate flexibility for small open economies have also become a subject of contention. One issue is whether high variability

---

<sup>1</sup>Things have changed more recently, with Chile and Colombia relaxing monetary policy and going for a flexible exchange rate, with the resulting nominal and real depreciation. Note the phenomenon is not universal, however, even among small economies: during the Asian crises Australia and New Zealand allowed their currencies to depreciate sharply and weathered the storm with little cost in terms of domestic output.

in the nominal and real exchange rate (which may also mean high variability in the terms of trade) is harmful to exports or growth. Another is whether floating really provides the kind of monetary independence it is supposed to: high volatility of the nominal and real exchange rate could conceivably cause endemically high domestic interest rates. A related point has to do with the ability of changes in nominal exchange rates to affect relative prices: pass-through may be so large and quick that a depreciation only buys you more inflation. And, even more damning, depreciation could even be contractionary because of negative wealth effects or because of its harmful impact on the balance sheets of domestic banks and firms.

Faced with these important questions, in this paper we adopt a “back to basics” approach. We take a state-of-the-art stochastic macroeconomic model with sticky prices, adapt it to focus on the behavior of a small open economy, and use it to characterize optimal monetary and exchange rate policies. Since our model is built from microfoundations, we can use individual utility functions to evaluate the welfare consequences of policies, with the optimal policy being that which delivers the highest expected utility to domestic residents. In this context we are able to assess how monetary policy should respond to different stochastic disturbances affecting the economy, in an update of the classic literature pioneered by Poole (1970).

It turns out that the 1970s prescriptions are mostly replicated by this vintage 2001 model. In particular, we show that:

- Home interest rates always respond to domestic disturbances: *procyclically* in the case of productivity shocks and *countercyclically* in the case of government spending shocks. In doing so, it replicates the behavior of the economy under flexible prices, and attains the corresponding level of welfare.
- The optimal exchange rate regime is always one of floating, in that the nominal and real exchange rates move in response to shocks from abroad. The direction and size of the optimal responses in home interest rates depend on parameter values, and most crucially on the intertemporal elasticity of substitution in consumption.
- In the empirically relevant case of a small intertemporal elasticity of substitution in consumption, the optimal policy involves a *dirty float* of the exchange rate, with the domestic interest rate partially mimicking changes in world interest rates. This optimal pattern may help rationalize the observed *fear of floating* in some economies.

- This *dirty float* is optimal regardless of whether the foreign central bank follows its optimal policy or any other arbitrary pattern of responses to its own domestic shocks.
- The alternative policy of permanently fixing the exchange rate provokes a welfare loss that is increasing in the variance of both foreign and domestic shocks. This is because fixing imposes two kinds of costs on the domestic economy: the real exchange is no longer available to cushion shocks from abroad, and the interest rate (now endogenous and targeted at maintaining the peg) is no longer available to respond to domestic disturbances.

We carry out the analysis in a model of the “new open economy macroeconomics” tradition brought to life by Obstfeld and Rogoff (1995, 1998). The optimizing, general equilibrium, sticky-price models of this literature lend themselves admirably to the analysis of alternative monetary policy rules. Predictably, there has been a slew of such papers recently.<sup>2</sup> Our model differs from much recent work in the following three dimensions:

- It focuses on a small open economy, while most papers –with the important exception of Galí and Monacelli (2000)– focus on a world economy composed of two countries of comparable size.
- Like Obstfeld and Rogoff (forthcoming) and Corsetti and Pesenti (2001a), but unlike much work in the literature, we obtain closed-form solutions without resorting to log-linear approximations, and we are able to solve explicitly for the first and second moments of the endogenous variables in the model, so we can study the effects of uncertainty on equilibrium variables. Our expressions for welfare are also exact, and can be used quite simply to calculate optimal policies and evaluate alternatives.
- We focus on optimal interest rate policies (so we can afford to be agnostic as to the source of money demand), while most papers have tried to characterize the optimal behavior of the nominal quantity of money.

In order to make the analysis tractable and get closed-form solutions we assume, in line with the literature, some specific functional forms. Moreover, we

---

<sup>2</sup>Aside from those mentioned in the text, a partial list ought to include Monacelli (2001), Parrado (2001), Benigno and Benigno (2000), Ghironi and Rebucci (2000), Obstfeld and Rogoff (2000), and Svensson (2000).

do not introduce credibility or financial fragility considerations, which are surely key in designing optimal monetary and exchange rate policies. We discuss the importance of such omissions in the concluding section.

Perhaps the closest predecessor of this paper is that by Galí and Monacelli (2000), who also study interest rate policies in a small open economy. Yet their framework is different: they work with Calvo staggered prices and are forced to resort to linear approximations to get solutions. Another closely related paper is by Henderson and Kim (1999), who compute exact utilities and optimal monetary policies, but for a closed economy.

The paper is organized as follows. Section 2 contains a description of the theoretical model that includes both domestic and foreign shocks. In section 3 we show how to solve the basic model, while section 4 presents the details of the firm's price setting. Section 5 discusses the structure of monetary policy rules. Section 6 presents a closed-form solution for the welfare function based on the utility function of the representative agent. Section 7 presents the analysis of optimal monetary policies in a benchmark case. Section 8 studies the implications of alternative policy scenarios. The final section suggests extensions and directions for future research.

## 2. The Basic Model

In this model, a home and a foreign economy make up the world. There is measure  $n$  of home agents, each of which has the monopoly in producing a single tradable good. There is measure  $n^*$  of foreign agents, each of which also produces a good under monopoly conditions. Thus,  $n$  and  $n^*$  indicate both the population size and the economic size of each country.

### 2.1. Individual preferences

Home agent  $i$ , who is both a consumer and a producer, has the following utility function

$$U_t^i = E_t \left\{ \sum_{s=t}^{\infty} (1 + \delta)^{-(s-t)} \left[ \frac{(C_s^i)^{1-\rho}}{1-\rho} - \frac{\tilde{K}_s}{1+v} (Y_s^i)^{1+v} \right] \right\}, \quad (2.1)$$

where  $\delta$  is the rate of time preference. The notation  $E_t[x_{t+j}]$  represents the expectation of variable  $x_{t+j}$  conditional on information available at  $t$ .

The variable  $Y^i$  is output produced by home agent  $i$ . We stick to the standard assumption that he has monopoly rights over this variety, so that he is the sole producer of the variety in the world economy. This term captures the disutility the individual experiences from having to produce more output. The stochastic parameter  $\tilde{\kappa}$  represents an inverse productivity shock.

The aggregate real consumption index  $C^i$  for any agent  $i$  is given by

$$C_t^i = \frac{(C_{H,t}^i)^\alpha (C_{F,t}^i)^{1-\alpha}}{\alpha^\alpha (1-\alpha)^{1-\alpha}}, \quad (2.2)$$

where  $C_{H,t}$  and  $C_{F,t}$  are the quantities that home agents consume of domestic and foreign goods, respectively, while  $\alpha$  indicates the share of home agents' consumption of their own good on total consumption.

The two consumption subindexes are symmetric and are defined, as in Dixit and Stiglitz (1977), by

$$C_{H,t} = \left[ \left( \frac{1}{n} \right)^{\frac{1}{\theta}} \int_0^n C_{H,t}^i(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} ; \quad C_{F,t} = \left[ \left( \frac{1}{n^*} \right)^{\frac{1}{\theta}} \int_0^{n^*} C_{F,t}^i(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}, \quad (2.3)$$

where  $C_{H,t}^i(j)$  is the consumption of home variety  $j$  by home agent  $i$  at time  $t$ , and the same for foreign varieties.

Analogously to Obstfeld and Rogoff (1998) and Corsetti and Pesenti (2001a), the elasticity of substitution across goods produced within a country is  $\theta > 1$ , while the elasticity of substitution between the bundles produced by the home economy and the rest of the world is 1.

Rest-of-the-world agents have identical preferences. Foreign values of the corresponding domestic variables will be denoted by an asterisk (\*) and may differ from home variables. Preferences over consumption of goods are symmetric across regions, except that foreign residents have a share  $\alpha^*$  home goods in their consumption basket. The rate of time preference  $\delta$  is the same across countries.

## 2.2. Prices and demand curve facing each monopolist

Home prices indexes for the two preceding consumption baskets, denoted by  $P_{H,t}$  and  $P_{F,t}$  are defined as

$$P_{H,t} = \left[ \frac{1}{n} \int_0^n P_{H,t}(j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}}; \quad P_{F,t} = \left[ \frac{1}{n^*} \int_0^{n^*} P_{F,t}(j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}}, \quad (2.4)$$

where the domestic currency price index for overall real consumption  $C_t$  is given by

$$P_t = P_{H,t}^\alpha P_{F,t}^{1-\alpha}. \quad (2.5)$$

The law of one price holds across all individual goods since agents of the home economy and the foreign economy have identical preferences, so that  $P_{H,t}(j) = S_t P_{H,t}^*(j)$ ,  $\forall j \in [0, n]$ , where  $P_{H,t}(j)$  and  $P_{H,t}^*(j)$  are the prices of home good  $j$  in home and foreign economies, respectively, and  $S_t$  represents the nominal exchange rate. The same relationship obviously holds for foreign goods.

The commodity demand functions resulting from cost minimization are:

$$C_{H,t}(j) = \frac{1}{n} \left[ \frac{P_{H,t}(j)}{P_{H,t}} \right]^{-\theta} C_{H,t}; \quad C_t(j) = \frac{1}{n^*} \left[ \frac{P_{F,t}(j)}{P_{F,t}} \right]^{-\theta} C_{F,t}. \quad (2.6)$$

Using the definition of total consumption (equation (2.2)), one can derive the demand functions for home and foreign goods as:

$$C_{H,t} = \alpha \left[ \frac{P_{H,t}}{P_t} \right]^{-1} C_t; \quad C_{F,t} = (1 - \alpha) \left[ \frac{P_{F,t}}{P_t} \right]^{-1} C_t. \quad (2.7)$$

### 2.3. Asset markets and budget constraints

The typical home agent has access to an internationally tradable bond  $B^*$  denominated in terms of the foreign good and to a domestic bond  $B$  denominated in terms of home money, and held only by domestic residents. His budget constraint, written using the foreign good as the numeraire, is therefore

$$\begin{aligned} & B_t^{*i} - B_{t-1}^{*i} + \frac{B_t^i - B_{t-1}^i}{P_{F,t}} \\ &= r_{t-1}^* B_{t-1}^{*i} + i_{t-1} \frac{B_{t-1}^i}{P_{F,t}} + \frac{P_{H,t}^i Y_{H,t}^i}{P_{F,t}} - \frac{P_{H,t}}{P_{F,t}} T_t^i - \frac{P_{H,t}}{P_{F,t}} C_{H,t}^i - C_{F,t}^i, \end{aligned} \quad (2.8)$$

where  $T_t^i$  are lump-sum taxes levied by the government. Notice that  $r_t^*$ , the real interest rate on the foreign bond, is the ‘‘own’’ rate of return on the foreign good, and  $i_t$  is the nominal return on the domestic bond. We will assume, without loss of generality, that in equilibrium the net supply of this domestic bond is zero.

## 2.4. Fiscal policy

In each country, the government consumes only the local goods, spending equal amounts in each. So, if  $G_t$  is per capita spending, we have

$$G_t = G_t(j). \quad (2.9)$$

The government finances the purchases of such goods with lump-sum taxes. Hence, budget balance requires

$$T_t = \frac{1}{n} \int_0^n T_t^i di = G_t. \quad (2.10)$$

In turn, government expenditures are a stochastic proportion of national output:

$$Y_t - G_t = Y_t e^{-\gamma_t}, \quad (2.11)$$

where  $\gamma_t$  is distributed normally with mean zero and variance  $\sigma_\gamma^2$ .

For the foreign country we have an analogous set of equations. In particular, we assume

$$Y_t^* - G_t^* = Y_t^* e^{-\gamma_t^*}, \quad (2.12)$$

where  $\gamma_t^*$  is distributed normally with mean zero and variance  $\sigma_{\gamma^*}^2$ .

## 3. Solving the Model

### 3.1. The home agent's consumption-savings choice

The representative home agent must choose  $C_{H,t}^i$ ,  $C_{F,t}^i$ ,  $B_t^{*i}$ , and  $B_t^i$  to maximize the objective function (2.1) subject to the consumption basket (2.2) and to the intertemporal budget constraint (2.8). If we let  $\lambda_t^i$  be the corresponding Lagrange multiplier, the first order conditions are:

$$\frac{\alpha}{\frac{P_{H,t}}{P_{F,t}} \frac{C_{H,t}^i}{(C_t^i)^{1-\rho}}} = \lambda_t^i, \quad (3.1)$$

$$\frac{1-\alpha}{\frac{C_{F,t}^i}{(C_t^i)^{1-\rho}}} = \lambda_t^i, \quad (3.2)$$

$$\lambda_t^i = \left( \frac{1+r_t^*}{1+\delta} \right) E_t \{ \lambda_{t+1}^i \}, \quad (3.3)$$

$$\frac{\lambda_t^i}{P_{F,t}} = \left( \frac{1+i_t}{1+\delta} \right) E_t \left\{ \frac{\lambda_{t+1}^i}{P_{F,t+1}} \right\}. \quad (3.4)$$

Combining the first two order conditions (3.1) and (3.2) and using the definition of  $P_t$ , we have

$$\lambda_t^i = \frac{P_{F,t}}{P_t (C_t^i)^\rho}. \quad (3.5)$$

Now combining this last expression with condition (3.3) and imposing symmetry, so that  $i$  superscripts disappear, we have

$$\frac{P_{F,t}}{P_t (C_t)^\rho} = \left( \frac{1+r_t^*}{1+\delta} \right) E_t \left\{ \frac{P_{F,t+1}}{P_{t+1} (C_{t+1})^\rho} \right\}. \quad (3.6)$$

The equivalent expression for the foreign country is

$$\frac{P_{F,t}^*}{P_t^* (C_t^*)^\rho} = \left( \frac{1+r_t^*}{1+\delta} \right) E_t \left\{ \frac{P_{F,t+1}^*}{P_{t+1}^* (C_{t+1}^*)^\rho} \right\}. \quad (3.7)$$

Next turn to the determination of nominal interest rates. Combining expressions (3.4) and (3.5) and again imposing symmetry we have

$$\frac{1}{P_t (C_t)^\rho} = \left( \frac{1+i_t}{1+\delta} \right) E_t \left\{ \frac{1}{P_{t+1} (C_{t+1})^\rho} \right\}. \quad (3.8)$$

Combining equations (3.6) and (3.8) we obtain the Fisher equation linking domestic and nominal interest rates:

$$\frac{1}{P_t Q_t} (1+r_t^*) E_t \left\{ \frac{Q_{t+1}}{(C_{t+1})^\rho} \right\} = (1+i_t) E_t \left\{ \frac{1}{P_{t+1} (C_{t+1})^\rho} \right\}. \quad (3.9)$$

The foreign country also has an equivalent pair of equations:

$$\frac{1}{P_t^* (C_t^*)^\rho} = \left( \frac{1+i_t^*}{1+\delta} \right) E_t \left\{ \frac{1}{P_{t+1}^* (C_{t+1}^*)^\rho} \right\}, \quad (3.10)$$

$$\frac{1}{P_t^* Q_t^*} (1+r_t^*) E_t \left\{ \frac{Q_{t+1}^*}{(C_{t+1}^*)^\rho} \right\} = (1+i_t^*) E_t \left\{ \frac{1}{P_{t+1}^* (C_{t+1}^*)^\rho} \right\}. \quad (3.11)$$

### 3.2. Asymmetries between economies and the current account

So far the two countries are symmetric, except in that the home economy produces measure  $n$  of goods and has a share  $\alpha$  of home goods in its consumption basket, while the foreign economy produces measure  $n^*$  of goods and has a share  $\alpha^*$  home goods in its consumption basket. We now introduce an asymmetry by assuming that the home economy is small relative to the foreign economy, in the sense that share of home goods in foreign consumption is negligible ( $\alpha^* \rightsquigarrow 0$ ).<sup>3</sup> From now on we refer to the home economy as the small open economy (SOE) and to the foreign economy as the rest of the world (ROW).

The assumption of  $\alpha^* \rightsquigarrow 0$  implies the following approximation. The foreign price level can be written as

$$P_t^* = (P_{H,t}^*)^{\alpha^*} (P_{F,t}^*)^{1-\alpha^*} \simeq P_{F,t}^*. \quad (3.12)$$

Next define the real exchange rate  $Q_t = \frac{S_t P_t^*}{P_t}$ . Using this definition and equation (3.12), equations (3.6) and (3.9) become:

$$\frac{Q_t}{(C_t)^\rho} = \left( \frac{1+r_t^*}{1+\delta} \right) E_t \left\{ \frac{Q_{t+1}}{(C_{t+1})^\rho} \right\}, \quad (3.13)$$

and

$$\left( \frac{1}{P_t Q_t} \right) (1+r_t^*) E_t \left\{ \frac{Q_{t+1}}{(C_{t+1})^\rho} \right\} = (1+i_t) E_t \left\{ \frac{1}{P_{t+1} (C_{t+1})^\rho} \right\}. \quad (3.14)$$

Since now  $P_t^* \simeq P_{F,t}^*$ , equations (3.7) and (3.11) become

$$\frac{1}{(C_t^*)^\rho} = \left( \frac{1+r_t^*}{1+\delta} \right) E_t \left\{ \frac{1}{(C_{t+1}^*)^\rho} \right\}, \quad (3.15)$$

and

$$\left( \frac{1}{P_t^*} \right) (1+r_t^*) E_t \left\{ \frac{1}{(C_{t+1}^*)^\rho} \right\} = (1+i_t^*) E_t \left\{ \frac{1}{P_{t+1}^* (C_{t+1}^*)^\rho} \right\}. \quad (3.16)$$

Next consider the current account. As in almost all other papers in this literature, we focus on the case in which the current account is always zero. This can

---

<sup>3</sup>Galí and Monacelli (2000) use the same approximation.

be motivated in a number of ways, the simplest one of which is the existence of complete asset markets.<sup>4</sup> A zero current account for the ROW implies

$$C_t^* = Y_t^* - G_t^*, \quad (3.17)$$

while a zero current account for the SOE implies

$$C_t Q_t^{\frac{1-\alpha}{\alpha}} = Y_t - G_t. \quad (3.18)$$

### 3.3. Log-linear versions

Assuming that the natural logarithms of the exogenous variables are jointly normally distributed,<sup>5</sup> we can express the equilibrium conditions in logs. For the sake of clarity, we define the natural logarithm of any variable  $X$  by  $x$ , and the date  $t - 1$  unconditional variance of  $x_t$ ,  $Var_{t-1}[x_t]$ , by  $\sigma_x^2$ .

Taking logs of equation (3.13) we can express the *real* consumption Euler equation as a function of endogenous variances:

$$\rho(E_t c_{t+1} - c_t) = r_t^* - \delta + (E_t q_{t+1} - q_t) + \frac{1}{2}(\rho^2 \sigma_c^2 + \sigma_q^2 - 2\rho\sigma_{cq}), \quad (3.19)$$

where  $\delta \simeq \log(1 + \delta)$  and  $r_t^* \simeq \log(1 + r_t^*)$ . Next, taking logs of equation (3.15) we obtain

$$\rho(E_t c_{t+1}^* - c_t^*) = -\delta + r_t^* + \frac{1}{2}\rho^2 \sigma_{c^*}^2. \quad (3.20)$$

Now turn to the nominal interest rate equations. Taking logs of (3.14) we have

$$r_t^* + (E_t q_{t+1} - q_t) + \frac{1}{2}(\sigma_q^2 - 2\rho\sigma_{cq}) = i_t - (E_t p_{t+1} - p_t) + \frac{1}{2}(\sigma_p^2 + 2\rho\sigma_{cp}), \quad (3.21)$$

---

<sup>4</sup>We suggest an alternative justification in appendix A.2.

<sup>5</sup>A variable  $X$  is log normally distributed if  $x = \ln(X) \sim N(\mu_x, \sigma_x^2)$ . Thus, if  $\ln(X) = x$  then  $X = e^x$ . In this case  $E[X] = E[e^x] = m(x)$ , where  $m(x)$  is the moment generating function for  $x$  and is given by

$$M(x) = \int_{-\infty}^{\infty} e^x \left[ \frac{1}{\sqrt{2\pi\sigma_x^2}} e^{-\frac{(x-\mu_x)^2}{2\sigma_x^2}} \right] dx.$$

Therefore,

$$E[X] = e^{\mu_x + \frac{1}{2}\sigma_x^2}.$$

where  $i_t \simeq \log(1 + i_t)$ . For the ROW the corresponding equation, obtained from (3.16), is

$$r_t^* = i_t^* - (E_t p_{t+1}^* - p_t^*) + \frac{1}{2} (\sigma_{p^*}^2 + 2\rho\sigma_{c^*p^*}), \quad (3.22)$$

where  $i_t^* \simeq \log(1 + i_t^*)$ .

## 4. Price Setting

Turn now to the problem faced by monopolistic producers in each country, who must set prices. We first revisit the demand functions facing each set of producers, and then solve their respective problems.

### 4.1. Demand functions revisited

Above we had developed expressions for demand for each good in the home economy. We now want to manipulate those expressions, plus the corresponding foreign demand functions, in order to obtain world demand for each particular variety. Appendix A.3 shows that total world demand for each home variety  $j$  can be written as:

$$Y_t^d(j) = \left[ \frac{P_t(j)}{P_{H,t}} \right]^{-\theta} (Y_t - G_t) + G_t. \quad (4.1)$$

This means that, in symmetric equilibrium when  $P_{H,t}(j) = P_{H,t}$ , we have

$$Y_t^d(j) = Y_t. \quad (4.2)$$

Defining aggregate demand as

$$Y_t^d \equiv \frac{1}{n} \int_0^n Y_t^d(j) dj, \quad (4.3)$$

we conclude  $Y_t^d = Y_t$ , as it should be.

### 4.2. The price setters' problem

Home agents set prices for period  $t$  based on period  $t - 1$  information and must satisfy all the demand at the quoted prices. It follows that the problem of home

agent  $i$  in period  $t - 1$  is to choose its price,  $P_{H,t}^i$ , to maximize the expected value of its objective function

$$E_{t-1} \left\{ \left( \frac{1}{1+\delta} \right) \left[ \frac{(C_t^i)^{1-\rho}}{1-\rho} - \frac{\tilde{\kappa}_t}{1+v} (Y_t^i)^{1+v} \right] \right\}. \quad (4.4)$$

Maximization of equation (4.4) is subject to budget constraint (2.8) and demand function (4.1). The first order necessary condition can be rearranged to read

$$E_{t-1} \{ \tilde{\kappa}_t Y_t^{1+v} \} = \frac{\theta - 1}{\theta} E_{t-1} \{ (C_t)^{1-\rho} \}. \quad (4.5)$$

For the ROW an analogous condition holds:

$$E_{t-1} \{ \tilde{\kappa}_t^* (Y_t^*)^{1+v} \} = \frac{\theta - 1}{\theta} E_{t-1} \{ (C_t^*)^{1-\rho} \}. \quad (4.6)$$

### 4.3. Log-linear versions

Taking logs of equation (4.5) we find

$$E_{t-1} y_t + \frac{1+v}{2} \sigma_y^2 + \frac{1}{2(1+v)} \sigma_\kappa^2 = \frac{1}{1+v} \log \left( \frac{\theta - 1}{\theta} \right) + \frac{1-\rho}{1+v} E_{t-1} c_t + \frac{(1-\rho)^2}{2(1+v)} \sigma_c^2, \quad (4.7)$$

where  $E_{t-1} \log \tilde{\kappa}_t = 0$ .

Taking logs of (4.6) we have

$$E_{t-1} y_t^* + \frac{1+v}{2} \sigma_{y^*}^2 + \frac{1}{2(1+v)} \sigma_{\kappa^*}^2 = \frac{1}{1+v} \log \left( \frac{\theta - 1}{\theta} \right) + \frac{1-\rho}{1+v} E_{t-1} c_t^* + \frac{(1-\rho)^2}{2(1+v)} \sigma_{c^*}^2, \quad (4.8)$$

where  $E_{t-1} \log \tilde{\kappa}_t^* = 0$ .

## 5. Specifying Monetary Policy

The alert reader will have noticed that so far money demand has not entered the model. We could avoid introducing money demand explicitly because we describe monetary policy entirely in terms of interest rules. This means that, whatever the shape or form of the money demand function, each central bank lets money supply

adjust endogenously so that a) the nominal interest rate is equal to the chosen rate and b) money demand is satisfied. We now specify the monetary authority reaction functions, which specify the setting of such chosen nominal interest rates at home and abroad.

### 5.1. Policy rule of the SOE

The monetary authority of the small open economy designs an optimal monetary policy. Its policy function is

$$1 + i_t = (1 + i) (P_{H,t})^{\psi_p} (\tilde{\kappa}_t)^{\psi_\kappa} (\tilde{\kappa}_t^*)^{\psi_{\kappa^*}} e^{\psi_\gamma \gamma_t} e^{\psi_{\gamma^*} \gamma_t^*}, \quad (5.1)$$

where the  $\psi_\kappa$  and  $\psi_\gamma$  are the coefficients associated with the domestic shocks while  $\psi_{\kappa^*}$  and  $\psi_{\gamma^*}$  are the coefficients associated with the foreign shocks. We introduce the term  $(P_{H,t})^{\psi_p}$ , where  $\psi_p > 0$ , following Woodford (1999), Henderson and Kim (1999) and others, in order to ensure nominal uniqueness in the equilibrium solution. The formulation implies that the monetary authority raises the nominal interest rate if the log of the home price level is above a target, set to zero as a normalization. Notice also that this rule implies that the authorities' target rate of home inflation is zero.<sup>6</sup> Appendix A.5 shows that under this rule the home price level is determinate –in fact, it is constant and equal to its log target of zero. This means, given the definition of the overall price level  $p_t$ , that we can set the term  $E_t p_{t+1} - p_t = \left(\frac{1-\alpha}{\alpha}\right) (E_t q_{t+1} - q_t)$  in what follows.

Taking logs the policy rule becomes

$$i_t = i + \psi_p p_{H,t} + \psi_\kappa \kappa_t + \psi_{\kappa^*} \kappa_t^* + \psi_\gamma \gamma_t + \psi_{\gamma^*} \gamma_t^*, \quad (5.2)$$

where  $i_t \simeq \log(1 + i_t)$ ,  $i \simeq \log(1 + i)$ ,  $\kappa_t = \log(\tilde{\kappa}_t)$ , and  $\kappa_t^* = \log(\tilde{\kappa}_t^*)$ .

### 5.2. Policy rule of the ROW

The rest of the world also designs an optimal monetary policy. Its rule is given by

$$1 + i_t^* = (1 + i^*) (P_t^*)^{\psi_{p^*}} (\tilde{\kappa}_t^*)^{\psi_{\kappa^*}} e^{\psi_{\gamma^*} \gamma_t^*}. \quad (5.3)$$

As in the case of the small open economy, the term  $(P_t^*)^{\psi_{p^*}}$  ensures that the foreign price level  $P_t^*$  (which, recall, is both the foreign *home* price level and the

---

<sup>6</sup>A non zero rate of inflation could easily be introduced, but it adds nothing to our analysis.

foreign CPI) is both unique and constant. It follows that here the nominal interest rate  $i_{t+1}^*$  is equal to the *ex ante* foreign real interest rate  $r_{t+1}^*$ .

Taking logs the policy rule becomes

$$i_t^* = i^* + \psi_p^* p_t^* + \psi_\kappa^* \kappa_t^* + \psi_\gamma^* \gamma_t^*, \quad (5.4)$$

where  $i_t^* \simeq \log(1 + i_t^*)$  and  $i^* \simeq \log(1 + i^*)$ .

## 6. A Closed-Form Solution

Appendix A.4 shows that both economies, home and rest-of-the-world, have a well defined and unique steady state. In this section we study the behavior of these economies out of long run equilibrium. It turns out that deviations from steady state for all variables of interest can be obtained by means of a simple system of linear equations. For computing that system we use the fact that, since all shocks are temporary, the expectation today of a variable's level tomorrow is invariably the steady state:  $E_{t-1}x_t = x$  for any variable  $x$ , where variables with no time subscripts denote the steady state.

Start with equation (3.19), which can be written

$$c_t - c = -\frac{1}{\rho} (r_t^* - r^*) + \frac{1}{\rho} (q_t - q), \quad (6.1)$$

so that consumption is above its steady state level whenever the real interest rates is below its steady state level, or the real exchange rate is above it. For the ROW, equation (3.20) yields

$$c_t^* - c^* = -\frac{1}{\rho} (r_t^* - r^*). \quad (6.2)$$

Notice that combining (6.1) and (6.2) one obtains

$$c_t - c = (c_t^* - c^*) + \frac{1}{\rho} (q_t - q), \quad (6.3)$$

Next, the balanced current account equation (3.18) gives aggregate demand for the SOE as

$$c_t - c = (y_t - y) - \left( \frac{1 - \alpha}{\alpha} \right) (q_t - q) - \gamma_t, \quad (6.4)$$

while the equivalent expression for the ROW yields

$$c_t^* - c^* = (y_t^* - y^*) - \gamma_t^*. \quad (6.5)$$

The nominal interest rate equation for the SOE can be obtained by re-writing (3.21) in deviations from the steady state, and using the fact that  $p_t - p = \left(\frac{1-\alpha}{\alpha}\right)(q_t - q)$ :

$$r_t^* - r^* = (i_t - i) + \alpha^{-1}(q_t - q), \quad (6.6)$$

while for the ROW this is

$$r_t^* - r^* = i_t^* - i^*. \quad (6.7)$$

The resulting interest rate parity equation is then

$$i_t - i = i_t^* - i^* - \alpha^{-1}(q_t - q), \quad (6.8)$$

Finally, both policy rules can be written as<sup>7</sup>

$$i_t - i = \psi_\kappa \kappa_t + \psi_{\kappa^*} \kappa_t^* + \psi_\gamma \gamma_t + \psi_{\gamma^*} \gamma_t^*, \quad (6.9)$$

$$i_t^* - i^* = \psi_{\kappa^*} \kappa_t^* + \psi_{\gamma^*} \gamma_t^*. \quad (6.10)$$

This completes the description of the system. We have eight independent equations and eight unknowns, so that the system is fully and uniquely determined.

## 7. Computing Optimal Policy

### 7.1. Calculating ex-ante utility

In this section we derive ex-ante utility to get a welfare measure in a closed form. The expected welfare function can be written as

$$E_{t-1} \left\{ \sum_{s=t}^{\infty} (1 + \delta)^{-(s-t)} U_s^i \right\} = \frac{1 + \delta}{\delta} E_{t-1} U_t^i, \quad (7.1)$$

where  $E_{t-1} U_t^i = E_{t-1} \left\{ \frac{(C_t^i)^{1-\rho}}{1-\rho} - \frac{\tilde{\kappa}_t}{1+v} (Y_t^i)^{1+v} \right\}$ .

Using the condition of optimal price setting (4.5) we can write expected utility as simply

---

<sup>7</sup>We have set the terms  $\psi_p^* p_t^*$  and  $\psi_{pH,t}$  equal to zero, since that is the value they take in equilibrium. See appendix A.5.

$$E_{t-1}U_t = \left( \frac{1}{1-\rho} - \frac{1}{1+v} \frac{\theta-1}{\theta} \right) \exp \left[ (1-\rho)c + \frac{1}{2}(1-\rho)^2 \sigma_c^2 \right], \quad (7.2)$$

where we have imposed symmetry and eliminated the  $i$  superscripts.

For the ROW the analogous expression is

$$E_{t-1}U_t^* = \left( \frac{1}{1-\rho} - \frac{1}{1+v} \frac{\theta-1}{\theta} \right) \exp \left[ (1-\rho)c^* + \frac{1}{2}(1-\rho)^2 \sigma_{c^*}^2 \right]. \quad (7.3)$$

## 7.2. Optimal policy: the case of the ROW

We calculate optimal policy maximizing objective function (7.3) subject to the equilibrium conditions of the economy, given by equations (6.2), (6.5), (6.7), and (6.10).

This is done in the following way. From expression (7.3) we see that if we can compute the expected value and the variance of foreign consumption we have a closed-form solution for foreign welfare. Maximizing the resulting expression with respect to the optimal policy coefficients yields the optimal policy rule. That tedious procedure is presented in appendix A.6. The solution is:

$$\begin{aligned} \psi_{\kappa^*}^* &= \frac{\rho}{\rho+v}, \\ \psi_{\gamma^*}^* &= \frac{\rho(1+v)}{\rho+v}. \end{aligned} \quad (7.4)$$

This means that the foreign interest rate rule becomes

$$i_t^* = i^* + \psi_{p^*}^* p_t^* + \frac{\rho}{\rho+v} \kappa_t^* + \frac{\rho(1+v)}{\rho+v} \gamma_t^*, \quad (7.5)$$

so that under the optimal policy steady state foreign consumption (and output) is

$$c^* = \frac{1}{\rho+v} \log \left( \frac{\theta-1}{\theta} \right),$$

and foreign utility is

$$\begin{aligned}
U^* = & \left( \frac{1}{1-\rho} - \frac{1}{1+v} \frac{\theta-1}{\theta} \right) \exp \left\{ \frac{1-\rho}{\rho+v} \log \left( \frac{\theta-1}{\theta} \right) \right\} \\
& \exp \left\{ \frac{1}{2} \left( \frac{1-\rho}{\rho+v} \right)^2 [\sigma_{\kappa^*}^2 + (1+v)^2 \sigma_{\gamma^*}^2] \right\}.
\end{aligned} \tag{7.6}$$

What happens if  $\rho = 1$ ? The policy coefficients that maximize foreign utility are then

$$\begin{aligned}
\psi_{\kappa^*}^* &= \frac{1}{1+v}, \\
\psi_{\gamma^*}^* &= 1,
\end{aligned} \tag{7.7}$$

so that under the optimal policy, foreign steady state consumption is

$$c^* = \frac{1}{1+v} \log \left( \frac{\theta-1}{\theta} \right), \tag{7.8}$$

and foreign utility is

$$U^* = \frac{1}{1+v} \left[ \log \left( \frac{\theta-1}{\theta} \right) - \left( \frac{\theta-1}{\theta} \right) \right]. \tag{7.9}$$

### 7.3. Optimal policy: the case of the SOE

We calculate optimal policy maximizing objective function (7.2) subject to the equilibrium conditions of the economy, given by equations (6.1), (6.2), (6.4), (6.5), (6.6), (6.7), (6.9), and (6.10). The procedure is just as in the case of the ROW, and relies on the fact that, from expression (7.2), if we can compute the expected value and the variance of home consumption we can have a closed-form solution for home welfare. Maximizing the resulting expression (see appendix A.7) one obtains

$$\begin{aligned}
\psi_\kappa &= \frac{\rho}{\rho[1+v(1-\alpha)]+\alpha v}, \\
\psi_\gamma &= \frac{\rho(1+v)}{\rho[1+v(1-\alpha)]+\alpha v}, \\
\psi_{\kappa^*} &= \frac{v(\rho-1)(1-\alpha)}{\rho[1+v(1-\alpha)]+\alpha v} \psi_{\kappa^*}^*, \\
\psi_{\gamma^*} &= \frac{v(\rho-1)(1-\alpha)}{\rho[1+v(1-\alpha)]+\alpha v} \psi_{\gamma^*}^*.
\end{aligned} \tag{7.10}$$

This means that the domestic interest rate rule becomes

$$\begin{aligned}
i_t &= i + \psi_p p_{H,t} + \frac{\rho}{\rho[1+v(1-\alpha)]+\alpha v} \kappa_t + \frac{\rho(1+v)}{\rho[1+v(1-\alpha)]+\alpha v} \gamma_t \\
&\quad + \frac{v(\rho-1)(1-\alpha)}{\rho[1+v(1-\alpha)]+\alpha v} (\psi_{\kappa^*}^* \kappa_t^* + \psi_{\gamma^*}^* \gamma_t^*),
\end{aligned} \tag{7.11}$$

so that under the optimal policy steady state home consumption is

$$c = \frac{1}{\rho+v} \log\left(\frac{\theta-1}{\theta}\right),$$

and home utility is

$$\begin{aligned}
U^{\text{opt}} &= \left( \frac{1}{1-\rho} - \frac{1}{1+v} \frac{\theta-1}{\theta} \right) \exp\left\{ \frac{1-\rho}{\rho+v} \log\left(\frac{\theta-1}{\theta}\right) \right\} \\
&\quad \exp\left\{ \frac{1}{2} \left[ \frac{1-\rho}{\rho[1+v(1-\alpha)]+\alpha v} \right]^2 \alpha^2 [\sigma_\kappa^2 + (1+v)^2 \sigma_\gamma^2] \right\} \\
&\quad \exp\left\{ \frac{1}{2} \left[ \frac{1-\rho}{\rho[1+v(1-\alpha)]+\alpha v} \right]^2 \left[ \frac{(1+v)(1-\alpha)}{\rho+v} \right]^2 \rho^2 [\sigma_{\kappa^*}^2 + (1+v)^2 \sigma_{\gamma^*}^2] \right\}.
\end{aligned} \tag{7.12}$$

If  $\rho = 1$ , the policy coefficients that maximize home utility are

$$\begin{aligned}
\psi_\kappa &= \frac{1}{1+v}, \\
\psi_\gamma &= 1, \\
\psi_{\kappa^*} &= 0, \\
\psi_{\gamma^*} &= 0,
\end{aligned}
\tag{7.13}$$

so that under the optimal policy, domestic steady state consumption is

$$c = \frac{1}{1+v} \log\left(\frac{\theta-1}{\theta}\right), \tag{7.14}$$

and domestic utility is

$$U^{\text{opt}} = \frac{1}{1+v} \left\{ \log \frac{\theta-1}{\theta} - \frac{\theta-1}{\theta} \right\}. \tag{7.15}$$

#### 7.4. Discussion and interpretation

Appendix A.8 computes the equilibrium of our two economies under flexible prices. It turns out that the two optimal interest rate rules computed above cause the fix-price economy to mimic the response of the flex-price economy to real shocks. This means that utility levels are the same regardless of whether prices are sticky or not: a properly designed monetary policy manages to attain the economy’s second best.<sup>8</sup>

Both economies react to their domestic “productivity” disturbances: whenever  $\kappa$  (or  $\kappa^*$ ) is positive, the local central bank raises the local nominal interest rate in order to reduce demand for the home good and hence allow local producers to work less when doing so is particularly irksome. Notice that such monetary policies are *procyclical*: a fall in  $\kappa$  is a positive productivity shock that under flexible prices would elicit greater labor supply; under predetermined prices, the monetary authority instead engineers an expansionary monetary response that has the same effect.<sup>9</sup>

Both economies also react to their “demand” disturbances: whenever government spending  $\gamma$  (or  $\gamma^*$ ) is above its steady state level, the local central bank raises

---

<sup>8</sup>Not the first best, of course, because the monopolistic distortion is still there, and there is nothing monetary policy can do about that.

<sup>9</sup>Obstfeld and Rogoff (forthcoming) make a similar point in the context of a model with two “large” countries.

its nominal interest rate to reduce demand for the local good. Such policies are *countercyclical*: an increase in demand by the government is offset by a cut in demand by the local private sector.

The small open economy also chooses to react to foreign shocks, but only insofar as they are expressed via the foreign real (and nominal) interest rate. That is to say,  $\psi_{\kappa^*} = 0$  if  $\psi_{\kappa^*}^* = 0$ , and  $\psi_{\gamma^*} = 0$  if  $\psi_{\gamma^*}^* = 0$ . Moreover, the optimal SOE policy involves a movement of home interest rates in tandem with foreign rates ( $\psi_{\kappa^*} > 0$  and  $\psi_{\gamma^*} > 0$ ) only if  $\rho$ , the inverse of the intertemporal elasticity of substitution, is larger than one. But if  $\rho < 1$ , home rates move in the opposite direction as foreign rates ( $\psi_{\kappa^*} < 0$  and  $\psi_{\gamma^*} < 0$ ). There is no movement of home interest rates in response to foreign shocks only if  $\rho$  is exactly equal to one.

An intuition for these results can be developed in the following way. Using equation (6.1), (6.4), and (6.6), one can express demand for home output as

$$y_t - y = \gamma_t - \frac{1 + (1 - \alpha)(\rho - 1)}{\rho} (i_t - i) + \frac{(1 - \alpha)(\rho - 1)}{\rho} (i_t^* - i^*). \quad (7.16)$$

Hence, deviations of home output from its steady state level can be written exclusively as a function of the home and foreign interest rate disturbance and the local government demand shock. Notice that the foreign fiscal purchases shock  $\gamma_t^*$  does not enter this expression: it matters for the determination of foreign output, but not of foreign consumption demand.<sup>10</sup>

The sign of the effect of foreign interest rates on domestic demand depends on whether the intertemporal elasticity of substitution  $\rho$  is larger or smaller than one. This is because a fall in foreign interest rates has two effects on domestic aggregate demand: it increases foreign consumption of all goods and it also appreciates the real exchange rate ( $q$  tends to fall) switching demand away from domestic goods. One channel increases demand for home goods, and the other reduces it. If  $\rho > 1$ , as is likely to be the case empirically, a fall in foreign interest rates causes a fall in home demand, calling in turn for a cut in home interest rates to stabilize home demand and output. With unitary elasticity of substitution between home and foreign goods and across time periods ( $\rho = 1$ ), it turns out that the two effects cancel each other, so that shocks to the foreign interest rate have a null impact on foreign demand for home goods.

What kind of an exchange rate regime does the optimal policy imply? If  $\rho$  is exactly one, foreign interest rate shocks call for no reaction by domestic rates,

---

<sup>10</sup>Recall that each government consumes local goods only.

so the policy is a *clean float*. In this case, since the foreign interest rate does not affect home demand, domestic monetary policy can be targeted at offsetting domestic shocks only.

If  $\rho < 1$ , home and foreign interest rates move in opposite directions, so given arbitrage—as it appears in equation (6.8)—nominal and real exchange rate move a great deal in response to foreign financial shocks. This is a case of *love of floating*.

Finally, if  $\rho > 1$ , local rates mimic, but only partially, the movement in foreign rates. This follows from the fact that the term  $\frac{v(\rho-1)(1-\alpha)}{\rho[1+v(1-\alpha)]+\alpha v}$  in the bottom two cells of (7.10), which is the ratio of the movement in domestic and foreign interest rates in response to foreign shocks, is smaller than one. In this case the optimal policy is *dirty float*. This case, which is probably the empirically relevant one, can help rationalize the observed *fear of floating* mentioned in the introduction.

These results can be summarized in the following way. Expressions (6.6) and (6.7) can be rewritten to yield

$$q_t - q = \alpha [(i_t^* - i^*) - (i_t - i)], \quad (7.17)$$

so that, when only foreign shocks occur,

$$\begin{aligned} q_t - q &= \alpha (\psi_{\kappa^*}^* - \psi_{\kappa^*}) \kappa^* + \alpha (\psi_{\gamma^*}^* - \psi_{\gamma^*}) \gamma^*, \\ &= \left\{ \frac{\alpha(v + \rho)}{\rho[1 + v(1 - \alpha)] + \alpha v} \right\} (\psi_{\kappa^*}^* \kappa^* + \psi_{\gamma^*}^* \gamma^*), \end{aligned} \quad (7.18)$$

where the term in curly brackets is positive and decreasing in  $\rho$ . So the larger is  $\rho$ , the smaller is the reaction of the nominal and real exchange rates to the foreign shocks.

## 8. Alternative Policy Scenarios

In this section we study alternative international scenarios in which domestic policy may have to be conducted, and one alternative domestic rule-of-thumb policy: a fixed exchange rate.

### 8.1. No optimal policy abroad

So far we have assumed the ROW central bank follows an optimal policy. What if that is not the case? Does this mean that the home country coefficients are no longer optimal?

We know the optimal foreign policy is given by (7.5). Consider now the following variation on that policy:

$$i_t^* = i^* + \psi_{p^*}^* p_t^* + \phi_{\kappa^*} \frac{\rho}{\rho + \nu} \kappa_t^* + \phi_{\gamma^*} \frac{\rho(1+v)}{\rho + \nu} \gamma_t^*, \quad (8.1)$$

where  $\phi_{\kappa^*}$  and  $\phi_{\gamma^*}$  are any two real numbers. Of course, if  $\phi_{\kappa^*} = \phi_{\gamma^*} = 1$ , then the foreign central bank is following optimal policies. Appendix A.9 shows that in this case the optimal SOE monetary policy is unchanged. The difference between second best and actual home welfare is (see same appendix for details):

$$\begin{aligned} U^{\text{opt}} - U^{\text{nopt}} &= \left( \frac{1}{1-\rho} - \frac{1}{1+v} \frac{\theta-1}{\theta} \right) \exp \left\{ \frac{1-\rho}{\rho+v} \log \left( \frac{\theta-1}{\theta} \right) \right\} \\ &\exp \left\{ \frac{1}{2} \Phi \left[ 2\phi_{\kappa^*} (1 - \phi_{\kappa^*}) - \rho (1 - \phi_{\kappa^*}^2) - v (1 - \phi_{\kappa^*})^2 \right] \sigma_{\kappa^*}^2 \right\} \\ &\exp \left\{ \frac{1}{2} \Phi \left[ 2\phi_{\gamma^*} (1 - \phi_{\gamma^*}) - \rho (1 - \phi_{\gamma^*}^2) - v (1 - \phi_{\gamma^*})^2 \right] (1+v)^2 \sigma_{\gamma^*}^2 \right\} \\ &\exp \left\{ \frac{1}{2} \left[ \frac{\rho(1+v)(1-\alpha)}{\rho[1+v(1-\alpha)] + \alpha v} \right]^2 \left( \frac{1-\rho}{\rho+v} \right)^2 \left[ \sigma_{\kappa^*}^2 + (1+v)^2 \sigma_{\gamma^*}^2 \right] \right\} \\ &- \exp \left\{ \frac{1}{2} \left[ \frac{\rho(1+v)(1-\alpha)}{\rho[1+v(1-\alpha)] + \alpha v} \right]^2 \left( \frac{1-\rho}{\rho+v} \right)^2 \left[ \phi_{\kappa^*}^2 \sigma_{\kappa^*}^2 + (1+v)^2 \phi_{\gamma^*}^2 \sigma_{\gamma^*}^2 \right] \right\}, \end{aligned} \quad (8.2)$$

where  $\Phi = \frac{\rho(\rho-1)(1+v)(1-\alpha)}{[\rho[1+v(1-\alpha)] + \alpha v](\rho+v)^2}$ . Therefore, the loss in utility associated with non-optimal foreign monetary policy is increasing in the variance of foreign shocks, and in the distance between the  $\phi$ 's and their "optimal" level of one.

What is going on here? Two things. The first is that, without the optimal policy in place, foreign output is more variable, and foreign producers protect themselves against that variability (and the cost of possibly having to provide a lot of labor services when it is irksome to do it), by setting higher prices. As a result, steady foreign output drops. Given the fact that  $q = \frac{\alpha\rho}{\alpha+(1-\alpha)\rho} (y - y^*)$ , this means that the steady state real exchange rate depreciates and the terms of trade turn against the home country. This reduces SOE utility.

The second key observation is that there is nothing that the home central bank can do about this unwelcome development. Given that the ROW is large relative to the SOE, home monetary authorities cannot influence output variability abroad. And since foreign demand shocks do not affect home demand (once temporary

movements in the real exchange rate are accounted for), the SOE central bank should not be trying to counteract those either.

## 8.2. A fixed exchange rate

What are the welfare consequences of fixing the exchange rate? Clearly, since in this model a fix is not the welfare-maximizing policy, utility must be lower. But how much lower? What does the difference depend on? These are important questions, since hard exchange rate pegs are alleged to have other virtues: they are simple, credible, and may serve to promote economic and political integration. Those benefits could conceivably more than offset the macroeconomic costs, if the latter are sufficiently low.

Under a fixed exchange rate, SOE monetary policy becomes endogenous, with interest rates adjusting to keep the real exchange rate  $Q_t$  equal to its steady state level  $Q$ . Appendix A.10 shows that in that case the difference between second-best and actual levels of welfare is under fixing:

$$\begin{aligned}
 U^{\text{opt}} - U^{\text{fix}} &= \left( \frac{1}{1-\rho} - \frac{1}{1+v} \frac{\theta-1}{\theta} \right) \exp \left\{ \frac{1-\rho}{\rho+v} \log \left( \frac{\theta-1}{\theta} \right) \right\} \quad (8.3) \\
 &\exp \left\{ -(1-\rho) \frac{\Delta}{2} [\sigma_\kappa^2 + (1+v)^2 \sigma_\gamma^2] \right\} \\
 &\exp \left\{ -(1-\rho) [(1+v)^2 - (1-\rho)^2] \frac{\Delta}{2(\rho+v)^2} \sigma_{\kappa^*}^2 \right\} \\
 &\exp \left\{ -(1-\rho) [(1+v)^2 - (1-\rho)^2] \frac{\Delta(1+v)^2}{2(\rho+v)^2} \sigma_{\gamma^*}^2 \right\},
 \end{aligned}$$

where  $\Delta = \frac{\alpha}{\rho[1+v(1-\alpha)]+\alpha v} > 0$ . Thus, the loss in utility associated with fixing is increasing in the variance not only of foreign shocks, but also domestic shocks. This is because fixing imposes two kinds of costs on the domestic economy: the real exchange is no longer available to cushion shocks from abroad, and the interest rate (now endogenous and targeted at maintaining the peg) is no longer available to respond to domestic disturbances.

## 9. Limitations and extensions

Rather than restating our conclusions, here we discuss their limitations and suggest directions for future research. The omission of all issues having to do with credibility and financial fragility is also important. Dealing with the former is, in principle, easy. Here we have focused on *ex ante* optimal monetary policies, and have therefore swept aside the issues that would arise if government could reoptimize *ex post* (that is, after prices have been set). Because monopolistic competition here renders equilibrium output levels too low, a problem of time inconsistency would occur, possibly leading to inefficiently large movements in the nominal and real exchange rates. Such a “depreciation” bias under floating could be costly, rendering the policy inferior to other simple rules such as credible fixing. But notice: as Corsetti and Pesenti (2001a) and others have stressed, in models such as these a surprise devaluation also entails a surprise drop in the terms of trade. Therefore, myopic policy makers need not be subject to the same inflation/devaluation bias that affects them in the closed economy, and hence time inconsistency problems need not erase the welfare superiority of flexible exchange rates.

Regarding financial fragility and imperfect financial markets, one of us has explored their importance for the design of monetary and exchange rate policies in a small open economy. See Céspedes, Chang, and Velasco (2000 and 2001). In that work we discuss dollarized liabilities and endogenous risk premia arising from costly state-verification, as in Bernanke and Gertler (1989). The bottom line of that work is that such financial imperfections may be very important in determining how severe and costly is the domestic adjustment to adverse external shocks. But flexible exchange rates, even in the presence of dollar liabilities, can help cushion the unwanted effects of such shocks.

## A. Appendices

### A.1. Real exchange rate and prices: some identities

Define the real exchange rate as  $Q_t = \frac{S_t P_t^*}{P_t}$ . The general price level of home and foreign are defined as  $P_t = P_{H,t}^\alpha P_{F,t}^{1-\alpha}$  and  $P_t^* = (P_{H,t}^*)^{\alpha^*} (P_{F,t}^*)^{1-\alpha^*}$ , respectively. Applying natural logs to these four equations and introducing one of the key assumptions of the model –that is assuming that the share of home goods in the consumption basket of the foreign country is negligible– we get the following set of equations:

$$\begin{aligned} q_t &= s_t + p_t^* - p_t, \\ p_t &= \alpha p_{H,t} + (1 - \alpha) p_{F,t}, \\ p_t^* &= \alpha^* P_{H,t}^* + (1 - \alpha^*) p_{F,t}^* \simeq p_{F,t}^*. \end{aligned} \tag{A.1}$$

Combining all four equations we can obtain an expression for the real exchange rate as a function of the terms of trade

$$q_t = \alpha (s_t + p_t^* - p_{H,t}). \tag{A.2}$$

Using the previous relationships we can get the following variances and covariances used in the main text:

$$\begin{aligned} \sigma_p^2 &= (1 - \alpha)^2 \sigma_s^2, \\ \sigma_p^2 &= \left( \frac{1 - \alpha}{\alpha} \right)^2 \sigma_q^2, \\ \sigma_q^2 &= \alpha^2 \sigma_s^2, \\ \sigma_{ps} &= (1 - \alpha) \sigma_s^2, \\ \sigma_{cs} &= \frac{1}{\alpha} \sigma_{cq}, \\ \sigma_{cp} &= (1 - \alpha) \sigma_{cs}, \\ \sigma_{cp} &= \left( \frac{1 - \alpha}{\alpha} \right) \sigma_{cq}. \end{aligned} \tag{A.3}$$

### A.2. Current account

As mentioned in the main text, one way to insure that the current account is always zero is to assume the existence of complete markets. Another one,

sketched in what follows, relies on assuming an extreme asymmetry between the two economies.

Recall that SOE is composed by individuals with measure  $n$ , who make also  $n$  goods, while ROW is composed by individuals with measure  $n^*$ , who make  $n^*$  goods. Clearing in each good market requires

$$n\alpha P_t C_t + n^* \alpha^* (S_t P_t^*) C_t^* = n P_{H,t} (Y_t - G_t), \quad (\text{A.4})$$

$$n(1 - \alpha) P_t C_t + n^* (1 - \alpha^*) (S_t P_t^*) C_t^* = n^* (S_t P_{F,t}^*) (Y_t^* - G_t^*), \quad (\text{A.5})$$

which can be rewritten as

$$\frac{n}{n^*} \alpha P_t C_t + \alpha^* (S_t P_t^*) C_t^* = \frac{n}{n^*} P_{H,t} (Y_t - G_t), \quad (\text{A.6})$$

$$\frac{n}{n^*} (1 - \alpha) P_t C_t + (1 - \alpha^*) (S_t P_t^*) C_t^* = (S_t P_{F,t}^*) (Y_t^* - G_t^*). \quad (\text{A.7})$$

Now introduce two assumptions: i)  $\alpha^* \simeq 0$  and ii)  $\frac{n}{n^*} \simeq 0$ . Using these assumptions together with (A.6) we get

$$P_t^* C_t^* = P_{F,t}^* (Y_t^* - G_t^*). \quad (\text{A.8})$$

Therefore, the ROW current account is zero.

Next, adding the two market clearing conditions and plugging (A.8) in this relation we have an analogous condition for SOE

$$P_t C_t = P_{H,t} (Y_t - G_t), \quad (\text{A.9})$$

so that naturally the home current account is also zero. Finally, using the definitions of the real exchange rate and the domestic price level in this last result yields equation (3.18) in the text.

### A.3. Demand functions

Plugging equation (2.7) into equation (2.6), it follows that the home demand for home and foreign goods is given by

$$C_t^H(j) = \frac{\alpha}{n} \left[ \frac{P_t(j)}{P_{H,t}} \right]^{-\theta} \left[ \frac{P_{H,t}}{P_t} \right]^{-1} C_t \quad j \in [0, n], \quad (\text{A.10})$$

$$C_t^F(j) = \frac{1 - \alpha}{n^*} \left[ \frac{P_t(j)}{P_{F,t}} \right]^{-\theta} \left[ \frac{P_{F,t}}{P_t} \right]^{-1} C_t \quad j \in [0, n^*].$$

There are analogous and symmetric functions for demand originating in the foreign economy

$$C_t^{H^*}(j) = \frac{\alpha^*}{n} \left[ \frac{P_t^*(j)}{P_{H,t}^*} \right]^{-\theta} \left[ \frac{P_{H,t}^*}{P_t^*} \right]^{-1} C_t^* \quad j \in [0, n], \quad (\text{A.11})$$

$$C_t^{F^*}(j) = \frac{1 - \alpha^*}{n^*} \left[ \frac{P_t^*(j)}{P_{F,t}^*} \right]^{-\theta} \left[ \frac{P_{F,t}^*}{P_t^*} \right]^{-1} C_t^* \quad j \in [0, n^*].$$

We are interested in the demand for home goods. Adding up the corresponding expression in (A.10) and (A.11), using weights of  $n$  and  $n^*$  respectively, plus  $G_t(j)$  for government consumption, we get

$$Y_t^d(j) = \alpha \left[ \frac{P_t(j)}{P_{H,t}} \right]^{-\theta} \left[ \frac{P_{H,t}}{P_t} \right]^{-1} C_t + \alpha^* \left( \frac{n^*}{n} \right) \left[ \frac{P_t^*(j)}{P_{H,t}^*} \right]^{-\theta} \left[ \frac{P_{H,t}^*}{P_t^*} \right]^{-1} C_t^* + G_t(j), \quad (\text{A.12})$$

where  $Y_t^d(j) \equiv nC_t^H(j) + n^*C_t^{H^*}(j) + G_t(j)$  is total world demand for good  $j$  produced in the SOE. Next, appealing to the law of one price we get

$$Y_t^d(j) = \left[ \frac{P_t(j)}{P_{H,t}} \right]^{-\theta} \left[ \frac{P_{H,t}}{P_t} \right]^{-1} \left[ \alpha C_t + \alpha^* \left( \frac{n^*}{n} \right) Q_t C_t^* \right] + G_t, \quad (\text{A.13})$$

where we have used the fact that  $G_t(j) = G_t$ . But by market clearing we have

$$\alpha C_t + \alpha^* \left( \frac{n^*}{n} \right) Q_t C_t^* = (Y_t - G_t) \frac{P_{H,t}}{P_t}. \quad (\text{A.14})$$

Plugging this into (A.13) leads to (4.1) in the text.

#### A.4. Steady state

Let variables without time subscripts denote steady state values. In that state, equation (3.19) becomes

$$r^* = \delta - \frac{1}{2} (\rho^2 \sigma_c^2 + \sigma_q^2 - 2\rho\sigma_{cq}), \quad (\text{A.15})$$

where  $\delta \simeq \log(1 + \delta)$  and  $r^* \simeq \log(1 + r^*)$ , while equation (3.20) is

$$r^* = \delta - \frac{1}{2} \rho^2 \sigma_{c^*}^2. \quad (\text{A.16})$$

For the steady state to be well defined it therefore must be the case that

$$\rho^2 \sigma_c^2 + \sigma_q^2 - 2\rho\sigma_{cq} = \rho^2 \sigma_{c^*}^2, \quad (\text{A.17})$$

which follows directly from equation (6.3). For nominal interest rates, the steady state equations are

$$r^* + \frac{1}{2} (\sigma_q^2 - 2\rho\sigma_{cq}) = i + \frac{1}{2} (\sigma_p^2 + 2\rho\sigma_{cp}), \quad (\text{A.18})$$

and

$$r^* = i^* + \frac{1}{2} (\sigma_{p^*}^2 + 2\rho\sigma_{c^*p^*}). \quad (\text{A.19})$$

With sticky prices in the foreign country,  $\sigma_{p^*}^2 = \sigma_{c^*p^*}^2 = 0$ , so that we have

$$i^* = r^* = \delta - \frac{1}{2} \rho^2 \sigma_{c^*}^2. \quad (\text{A.20})$$

And since  $c_t^* = y_t^* - \gamma_t^*$ , it follows that  $\sigma_{c^*}^2 = \sigma_{y^*}^2 + \sigma_{\gamma^*}^2$ , so we have

$$i^* = r^* = \delta - \frac{1}{2} \rho^2 (\sigma_{y^*}^2 + \sigma_{\gamma^*}^2). \quad (\text{A.21})$$

Plugging (A.20) in (A.18), the steady state domestic nominal interest rate must be

$$i = \delta - \frac{1}{2} \rho^2 \sigma_{c^*}^2 + \frac{1}{2} (\sigma_q^2 - 2\rho\sigma_{cq}) - \frac{1}{2} (\sigma_p^2 + 2\rho\sigma_{cp}), \quad (\text{A.22})$$

or

$$i = \delta - \frac{1}{2} (\rho^2 \sigma_c^2 + \sigma_p^2 + 2\rho\sigma_{cp}). \quad (\text{A.23})$$

Again, since  $\sigma_p^2 = \left(\frac{1-\alpha}{\alpha}\right)^2 \sigma_q^2$  and  $\sigma_{cp} = \left(\frac{1-\alpha}{\alpha}\right) \sigma_{cq}$ , we have

$$i = \delta - \frac{1}{2} \left[ \rho^2 \sigma_c^2 + \left(\frac{1-\alpha}{\alpha}\right)^2 \sigma_q^2 + 2 \left(\frac{1-\alpha}{\alpha}\right) \rho \sigma_{cq} \right]. \quad (\text{A.24})$$

Using the fact that  $y_t - \gamma_t = c_t + \left(\frac{1-\alpha}{\alpha}\right) q_t$ , we have that

$$i = \delta - \frac{1}{2} \left[ \rho^2 (\sigma_y^2 + \sigma_\gamma^2) - 2\rho^2 \sigma_{y\gamma} - \left(\frac{1-\alpha}{\alpha}\right)^2 \sigma_q^2 (\rho^2 - 1) - 2\rho \left(\frac{1-\alpha}{\alpha}\right) \sigma_{cq} (\rho - 1) \right]. \quad (\text{A.25})$$

Finally, subtracting (A.25) from (A.21) we get

$$\begin{aligned} i^* - i &= \frac{1}{2} \rho^2 (\sigma_y^2 - \sigma_{y^*}^2) + \frac{1}{2} \rho^2 (\sigma_\gamma^2 - \sigma_{\gamma^*}^2) - \rho^2 (\sigma_{y\gamma} - \sigma_{y^*\gamma^*}) \\ &\quad - \frac{1}{2} \left(\frac{1-\alpha}{\alpha}\right)^2 \sigma_q^2 (\rho^2 - 1) - \rho \left(\frac{1-\alpha}{\alpha}\right) \sigma_{cq} (\rho - 1). \end{aligned} \quad (\text{A.26})$$

Next, turn to the determination of relative prices and consumption in steady state. Equations (4.7) and (4.8) in the text, reproduced and rearranged here for convenience, give the steady state output levels at home and abroad:

$$\begin{aligned} y &= \frac{1}{1+v} \log \left( \frac{\theta - 1}{\theta} \right) - \frac{(1+v)}{2} \sigma_y^2 - \frac{1}{2(1+v)} \sigma_\kappa^2 \\ &\quad + \frac{1-\rho}{1+v} c + \frac{(1-\rho)^2}{2(1+v)} \sigma_c^2, \end{aligned} \quad (\text{A.27})$$

and

$$\begin{aligned} y^* &= \frac{1}{1+v} \log \left( \frac{\theta - 1}{\theta} \right) - \frac{(1+v)}{2} \sigma_{y^*}^2 - \frac{1}{2(1+v)} \sigma_{\kappa^*}^2 \\ &\quad + \frac{1-\rho}{1+v} c^* + \frac{(1-\rho)^2}{2(1+v)} \sigma_{c^*}^2. \end{aligned} \quad (\text{A.28})$$

Subtracting one from the other we find that

$$y - y^* = \frac{1+v}{2} (\sigma_{y^*}^2 - \sigma_y^2) + \frac{1}{2(1+v)} (\sigma_{\kappa^*}^2 - \sigma_\kappa^2) + \frac{1-\rho}{1+v} (c - c^*) + \frac{(1-\rho)^2}{2(1+v)} (\sigma_c^2 - \sigma_{c^*}^2). \quad (\text{A.29})$$

Next, using the fact that  $c - c^* = \frac{1}{\rho}q$  and  $y = c + \left(\frac{1-\alpha}{\alpha}\right)q$ , we have

$$q = \frac{\alpha\rho}{\rho - \alpha(\rho - 1)} (y - y^*). \quad (\text{A.30})$$

Plugging this result in (A.29) we arrive at

$$q = \Delta\rho(1+v) \left[ \frac{1+v}{2} (\sigma_{y^*}^2 - \sigma_y^2) + \frac{1}{2(1+v)} (\sigma_{\kappa^*}^2 - \sigma_\kappa^2) - \frac{1}{2(1+v)} \left(\frac{1-\rho}{\rho}\right)^2 (\sigma_q^2 - 2\rho\sigma_{cq}) \right]. \quad (\text{A.31})$$

where  $\Delta = \frac{\alpha}{\rho[1+v(1-\alpha)]+\alpha v} > 0$ . Finally, compute  $c = y^* + \frac{1}{\rho}q$ . Using (A.28) and (A.31) we arrive at

$$c = \frac{1}{v+\rho} \log\left(\frac{\theta-1}{\theta}\right) - \frac{(1+v)^2}{2} \left[ \Delta\sigma_y^2 + \left(\frac{1}{v+\rho} - \Delta\right) \sigma_{y^*}^2 \right] - \frac{1}{2} \left[ \Delta\sigma_\kappa^2 + \left(\frac{1}{v+\rho} - \Delta\right) \sigma_{\kappa^*}^2 \right] + \frac{1}{2} (1-\rho)^2 \left[ \Delta\sigma_c^2 + \left(\frac{1}{v+\rho} - \Delta\right) \sigma_{c^*}^2 \right]. \quad (\text{A.32})$$

### A.5. Pinning down the home price level

Combining equations (3.19) in the text and (A.15) we get

$$c_t - c = -\frac{1}{\rho} (r_t^* - r^*) + \frac{1}{\rho} (q_t - q). \quad (\text{A.33})$$

Now subtract (A.18) from (3.21) in the text to obtain

$$(r_t^* - r^*) + (E_t q_{t+1} - q_t) = (i_t - i) - (E_t p_{t+1} - p_t). \quad (\text{A.34})$$

But from the definitions of  $p$  and  $q$  it must be the case that

$$E_t p_{t+1} - p_t = (E_t p_{H,t+1} - p_{H,t}) - \left( \frac{1-\alpha}{\alpha} \right) (E_t q_{t+1} - q_t). \quad (\text{A.35})$$

Combining these last two equations and recalling that  $E_t q_{t+1} = q$  we have

$$(r_t^* - r^*) - \left( \frac{1}{\alpha} \right) (q_t - q) = (i_t - i) - \{E_t p_{H,t+1} - p_{H,t}\}. \quad (\text{A.36})$$

Next recall that the monetary policy rule is

$$i_t - i = \psi_\kappa \kappa_t + \psi_{\kappa^*}^* \kappa_t^* + \psi_\gamma \gamma_t + \psi_{\gamma^*}^* \gamma_t^* + \psi_P p_{H,t}.$$

Using this plus (6.1) and (6.4) in (A.36) and rearranging we have

$$\begin{aligned} E_t p_{H,t+1} &= (1 + \psi_P) p_{H,t} + \psi_\kappa \kappa_t + \psi_{\kappa^*}^* \kappa_t^* + (\psi_\gamma - \rho) \gamma_t + \psi_{\gamma^*}^* \gamma_t^* \quad (\text{A.37}) \\ &\quad + \rho (y_t - y) + (1 - \rho) \left( \frac{1-\alpha}{\alpha} \right) (q_t - q). \end{aligned}$$

Solving this difference equation forward and ruling out bubbles we have

$$\begin{aligned} p_{H,t} &= E_t \sum_{s=0}^{\infty} \left\{ \rho (y_{t+s} - y) + (1 - \rho) \left( \frac{1-\alpha}{\alpha} \right) (q_{t+s} - q) \right\} (1 + \psi_P)^{-s} \quad (\text{A.38}) \\ &\quad + E_t \sum_{s=0}^{\infty} \left\{ \psi_\kappa \kappa_{t+s} + \psi_{\kappa^*}^* \kappa_{t+s}^* + (\psi_\gamma - \rho) \gamma_{t+s} + \psi_{\gamma^*}^* \gamma_{t+s}^* \right\} (1 + \psi_P)^{-s}. \end{aligned}$$

Taking expectations of this equation as of  $t-1$ , recalling that all shocks have zero mean and that therefore home output and the real exchange rate are expected to be at its steady state value in all future periods, this expression becomes

$$E_{t-1} p_{H,t} = 0. \quad (\text{A.39})$$

Notice finally that since  $p_{H,t}$  is pre-set as of  $t-1$ ,  $E_{t-1} p_{H,t} = p_{H,t} = 0$ .

Next we can calculate  $E_t p_{H,t+1}$ . Leading (A.38) by one period we have

$$\begin{aligned} p_{H,t+1} &= E_{t+1} \sum_{s=0}^{\infty} \left\{ \rho (y_{t+1+s} - y) + (1 - \rho) \left( \frac{1-\alpha}{\alpha} \right) (q_{t+1+s} - q) \right\} (1 + \psi_P)^{-s} \quad (\text{A.40}) \\ &\quad + E_{t+1} \sum_{s=0}^{\infty} \left\{ \psi_\kappa \kappa_{t+1+s} + \psi_{\kappa^*}^* \kappa_{t+1+s}^* + (\psi_\gamma - \rho) \gamma_{t+1+s} + \psi_{\gamma^*}^* \gamma_{t+1+s}^* \right\} (1 + \psi_P)^{-s}. \end{aligned}$$

Taking expectations of this expression as of  $t$  we have  $E_t p_{H,t+1} = 0$ . We conclude that  $E_t p_{H,t+1} - p_{H,t} = 0$  for all  $t$ . An analogous computation must be carried out to pin down the foreign price level.

## A.6. Optimal policy in the ROW

Start by taking the logs of equation (4.6)

$$(1+v)y^* + \frac{1}{2}(1+v)^2\sigma_{y^*}^2 + \frac{1}{2}\sigma_{\kappa^*}^2 + (1+v)\sigma_{\kappa^*,y^*} = \log \frac{\theta-1}{\theta} + (1-\rho)c^* + \frac{1}{2}(1-\rho)^2\sigma_{c^*}^2. \quad (\text{A.41})$$

Plugging the log version of equation (2.12) into the previous equation we can express expected consumption as a function of endogenous variances

$$\begin{aligned} c^* = & -\frac{1}{2(\rho+v)}\sigma_{\kappa^*}^2 - \frac{1(1+v)^2}{2(\rho+v)}\sigma_{\gamma^*}^2 - \frac{(1+v)^2}{(\rho+v)}\sigma_{c^*,\gamma^*} - \frac{(1+v)}{(\rho+v)}\sigma_{\kappa^*,y^*} \\ & + \frac{1}{(\rho+v)}\log \frac{\theta-1}{\theta} + \frac{1}{2(\rho+v)}[(1-\rho)^2 - (1+v)^2]\sigma_{c^*}^2. \end{aligned} \quad (\text{A.42})$$

Combining the ROW policy rule (5.4) with equation (6.2) we get

$$c_t^* - c^* = -\frac{1}{\rho}(\psi_{\kappa^*}^*\kappa_t^* + \psi_{\gamma^*}^*\gamma_t^*). \quad (\text{A.43})$$

Using (A.43) we can compute all the endogenous variances in terms of exogenous variances only:

$$\begin{aligned} \sigma_{c^*,\gamma^*} &= -\frac{1}{\rho}\psi_{\gamma^*}^*\sigma_{\gamma^*}^2, \\ \sigma_{\kappa^*,y^*} &= -\frac{1}{\rho}\psi_{\kappa^*}^*\sigma_{\kappa^*}^2, \end{aligned} \quad (\text{A.44})$$

$$\sigma_{c^*}^2 = \left(\frac{1}{\rho}\psi_{\kappa^*}^*\right)^2\sigma_{\kappa^*}^2 + \left(\frac{1}{\rho}\psi_{\gamma^*}^*\right)^2\sigma_{\gamma^*}^2. \quad (\text{A.45})$$

Therefore, we can also compute expected consumption only in terms of exogenous variances

$$\begin{aligned}
c^* &= \frac{1}{\rho + v} \log \left( \frac{\theta - 1}{\theta} \right) \tag{A.46} \\
&\quad - \frac{1}{2(\rho + v)} \left\{ 1 - 2(1 + v) \frac{1}{\rho} \psi_{\kappa^*}^* - [(1 - \rho)^2 - (1 + v)^2] \left( \frac{1}{\rho} \psi_{\kappa^*}^* \right)^2 \right\} \sigma_{\kappa^*}^2 \\
&\quad - \frac{1}{2(\rho + v)} \left\{ (1 + v)^2 - 2(1 + v)^2 \frac{1}{\rho} \psi_{\gamma^*}^* - [(1 - \rho)^2 - (1 + v)^2] \left( \frac{1}{\rho} \psi_{\gamma^*}^* \right)^2 \right\} \sigma_{\gamma^*}^2.
\end{aligned}$$

Plugging both the variance of consumption (A.45) and the expected consumption (A.46) into expected utility (7.3), and then maximizing, we get the policy coefficients in (7.4) in the text.

### A.7. Optimal policy in the SOE

Start by taking the logs of equation (4.5)

$$(1 + v) y + \frac{1}{2} (1 + v)^2 \sigma_y^2 + \frac{1}{2} \sigma_{\kappa}^2 + (1 + v) \sigma_{\kappa, y} = \log \frac{\theta - 1}{\theta} + (1 - \rho) c + \frac{1}{2} (1 - \rho)^2 \sigma_c^2. \tag{A.47}$$

Plugging the log version of equation (2.11) into the previous equation we can express expected consumption as a function of endogenous variances:

$$\begin{aligned}
c &= \frac{\rho(1 - \alpha)(1 + v)}{\rho(1 + v) + \alpha v(1 - \rho)} c^* \tag{A.48} \\
&\quad - \frac{1}{2} \Delta (1 + v)^2 \left\{ -2 \left[ \frac{\alpha + \rho(1 - \alpha)}{\alpha} \right] \left( \rho \frac{1 - \alpha}{\alpha} \right)^2 \sigma_{c^*}^2 + \sigma_{\gamma}^2 \right. \\
&\quad \left. - 2 \left[ \frac{\alpha + \rho(1 - \alpha)}{\alpha} \right] \left( \rho \frac{1 - \alpha}{\alpha} \right) \sigma_{c, \gamma} + 2 \left[ \frac{\alpha + \rho(1 - \alpha)}{\alpha} \right] \sigma_{c, \gamma} - 2 \left( \rho \frac{1 - \alpha}{\alpha} \right) \sigma_{c^*, \gamma} \right\} \\
&\quad - \frac{1}{2} \Delta \sigma_{\kappa}^2 - (1 + v) \Delta \sigma_{\kappa, y} + \Delta \log \frac{\theta - 1}{\theta} + \frac{1}{2} \Delta \left\{ (1 - \rho)^2 - (1 + v)^2 \left[ \frac{\alpha + \rho(1 - \alpha)}{\alpha} \right]^2 \right\} \sigma_c^2.
\end{aligned}$$

Next, combining equations (6.1), (6.3), and (6.6) we have

$$c_t - c = -\frac{\alpha}{\rho} (i_t - i) + (1 - \alpha) (c_t^* - c^*), \tag{A.49}$$

where  $c_t^* - c^*$  is given by equation (A.43). Therefore,

$$c_t - c = -\frac{\alpha}{\rho} (i_t - i) - \frac{1 - \alpha}{\rho} (r_t^* - r^*). \quad (\text{A.50})$$

Finally, using both policy rules we get

$$c_t - c = -\frac{\alpha}{\rho} (\psi_\kappa \kappa + \psi_\gamma \gamma_t) - [\alpha \psi_{\kappa^*} + (1 - \alpha) \psi_{\kappa^*}^*] \frac{\kappa^*}{\rho} - [\alpha \psi_{\gamma^*} + (1 - \alpha) \psi_{\gamma^*}^*] \frac{\gamma_t^*}{\rho}. \quad (\text{A.51})$$

Now we can compute all the endogenous variances in terms of exogenous variances only:

$$\begin{aligned} \sigma_{c,\gamma} &= -\frac{\alpha}{\rho} \psi_\gamma \sigma_\gamma^2, & (\text{A.52}) \\ \sigma_{\kappa,y} &= -\frac{\alpha + \rho(1 - \alpha)}{\rho} \psi_\kappa \sigma_\kappa^2, \\ \sigma_{c^*}^2 &= \left(\frac{1}{\rho} \psi_{\kappa^*}^*\right)^2 \sigma_{\kappa^*}^2 + \left(\frac{1}{\rho} \psi_{\gamma^*}^*\right)^2 \sigma_{\gamma^*}^2, \\ \sigma_{c,c^*} &= \left(\frac{1}{\rho}\right)^2 \psi_{\kappa^*}^* [\alpha \psi_{\kappa^*} + (1 - \alpha) \psi_{\kappa^*}^*] \sigma_{\kappa^*}^2 + \left(\frac{1}{\rho}\right)^2 \psi_{\gamma^*}^* [\alpha \psi_{\gamma^*} + (1 - \alpha) \psi_{\gamma^*}^*] \sigma_{\gamma^*}^2, \\ \sigma_{\gamma,c^*} &= 0, \end{aligned}$$

$$\begin{aligned} \sigma_c^2 &= \left(\frac{\alpha}{\rho} \psi_\kappa\right)^2 \sigma_\kappa^2 + [\alpha \psi_{\kappa^*} + (1 - \alpha) \psi_{\kappa^*}^*]^2 \left(\frac{1}{\rho}\right)^2 \sigma_{\kappa^*}^2 & (\text{A.53}) \\ &+ \left(\frac{\alpha}{\rho} \psi_\gamma\right)^2 \sigma_\gamma^2 + [\alpha \psi_{\gamma^*} + (1 - \alpha) \psi_{\gamma^*}^*]^2 \left(\frac{1}{\rho}\right)^2 \sigma_{\gamma^*}^2. \end{aligned}$$

Therefore, we can also compute expected consumption only in terms of exogenous variances

$$\begin{aligned}
c = & \Delta \frac{\rho(1-\alpha)(1+v)}{\alpha} \left[ \frac{1}{\rho+v} \log\left(\frac{\theta-1}{\theta}\right) - \frac{1}{2(\rho+v)} \sigma_{\kappa^*}^2 - \frac{1}{2} \frac{(1+v)^2}{(\rho+v)} \sigma_{\gamma^*}^2 \right. \\
& \left. + \frac{(1+v)^2}{(\rho+v)} \frac{1}{\rho} \psi_{\gamma^*}^* \sigma_{\gamma^*}^2 + \frac{(1+v)}{(\rho+v)} \frac{1}{\rho} \psi_{\kappa^*}^* \sigma_{\kappa^*}^2 \right] \tag{A.54} \\
& + \Delta \frac{\rho(1-\alpha)(1+v)}{\alpha} \left\{ \frac{1}{2(\rho+v)} [(1-\rho)^2 - (1+v)^2] \left[ \left(\frac{1}{\rho} \psi_{\kappa^*}^*\right)^2 \sigma_{\kappa^*}^2 + \left(\frac{1}{\rho} \psi_{\gamma^*}^*\right)^2 \sigma_{\gamma^*}^2 \right] \right\} \\
& - \frac{1}{2} \Delta (1+v)^2 \left(\rho \frac{1-\alpha}{\alpha}\right)^2 \left[ \left(\frac{1}{\rho} \psi_{\kappa^*}^*\right)^2 \sigma_{\kappa^*}^2 + \left(\frac{1}{\rho} \psi_{\gamma^*}^*\right)^2 \sigma_{\gamma^*}^2 \right] - \frac{1}{2} \Delta (1+v)^2 \sigma_{\gamma}^2 \\
& + \Delta (1+v)^2 \left[ \frac{\alpha + \rho(1-\alpha)}{\alpha} \right] \left(\rho \frac{1-\alpha}{\alpha}\right) \left[ \begin{aligned} & \left(\frac{1}{\rho}\right)^2 \psi_{\kappa^*}^* [\alpha \psi_{\kappa^*} + (1-\alpha) \psi_{\kappa^*}^*] \sigma_{\kappa^*}^2 \\ & + \left(\frac{1}{\rho}\right)^2 \psi_{\gamma^*}^* [\alpha \psi_{\gamma^*} + (1-\alpha) \psi_{\gamma^*}^*] \sigma_{\gamma^*}^2 \end{aligned} \right] \\
& + v(1+v)^2 \left[ \frac{\alpha + \rho(1-\alpha)}{\alpha} \right] \frac{\alpha}{\rho} \psi_{\gamma} \sigma_{\gamma}^2 - \frac{1}{2} \Delta \sigma_{\kappa}^2 + (1+v) \Delta \left[ \frac{\alpha + \rho(1-\alpha)}{\rho} \psi_{\kappa} \sigma_{\kappa}^2 \right] \\
& + \Delta \log \frac{\theta-1}{\theta} + \frac{1}{2} \Delta \left\{ (1-\rho)^2 - (1+v)^2 \left[ \frac{\alpha + \rho(1-\alpha)}{\alpha} \right]^2 \right\} \\
& \left[ \begin{aligned} & \left(\frac{\alpha}{\rho} \psi_{\kappa}\right)^2 \sigma_{\kappa}^2 + \left(\frac{\alpha}{\rho} \psi_{\gamma}\right)^2 \sigma_{\gamma}^2 + [\alpha \psi_{\kappa} + (1-\alpha) \psi_{\kappa}^*]^2 \left(\frac{1}{\rho}\right)^2 \sigma_{\kappa^*}^2 \\ & + [\alpha \psi_{\gamma} + (1-\alpha) \psi_{\gamma}^*]^2 \left(\frac{1}{\rho}\right)^2 \sigma_{\gamma^*}^2 \end{aligned} \right].
\end{aligned}$$

Plugging both the variance of consumption (A.53) and the expected consumption (A.54) into expected utility (7.2) and maximizing we get the policy coefficients in (7.10) in the text.

### A.8. The flex-price solution

With flexible prices, price-setting equations (4.5) and (4.6) must hold, but not just in expectational form, so that we have

$$\tilde{\kappa}_t (Y_t)^{1+v} = \frac{\theta-1}{\theta} (C_t)^{1-\rho}, \tag{A.55}$$

and

$$\tilde{\kappa}_t^* (Y_t^*)^{1+v} = \frac{\theta-1}{\theta} (C_t^*)^{1-\rho}. \tag{A.56}$$

Taking logs we have

$$y_t = \frac{1}{1+v} \log\left(\frac{\theta-1}{\theta}\right) + \frac{1-\rho}{1+v} c_t - \frac{1}{1+v} \kappa_t, \quad (\text{A.57})$$

$$y_t^* = \frac{1}{1+v} \log\left(\frac{\theta-1}{\theta}\right) + \frac{1-\rho}{1+v} c_t^* - \frac{1}{1+v} \kappa_t^*. \quad (\text{A.58})$$

Using  $c_t^* = y_t^* - \gamma_t^*$  we arrive at:

$$c_t^* = \frac{1}{\rho+v} \log\left(\frac{\theta-1}{\theta}\right) - \frac{1}{\rho+v} \kappa_t^* - \frac{1+v}{v+\rho} \gamma_t^*. \quad (\text{A.59})$$

Next, using  $c_t = c_t^* + \frac{1}{\rho} q_t$  and  $c_t + \left(\frac{1-\alpha}{\alpha}\right) q_t = y_t - \gamma_t$  we have

$$c_t = \Delta \log\left(\frac{\theta-1}{\theta}\right) - \Delta \kappa_t + \rho \Delta (1+v) \left(\frac{1-\alpha}{\alpha}\right) c_t^* - \Delta (1+v) \gamma_t. \quad (\text{A.60})$$

Plugging (A.59) into (A.60) we get

$$\begin{aligned} c_t &= \frac{1}{\rho+v} \log\left(\frac{\theta-1}{\theta}\right) - \Delta \kappa_t - \Delta (1+v) \gamma_t \\ &\quad + \rho \Delta (1+v) \left(\frac{1-\alpha}{\alpha}\right) \left[ -\frac{1}{\rho+v} \kappa_t^* - \frac{1+v}{\rho+v} \gamma_t^* \right], \end{aligned} \quad (\text{A.61})$$

so that

$$E_{t-1} c_t^* = c^* = \frac{1}{\rho+v} \log\left(\frac{\theta-1}{\theta}\right), \quad (\text{A.62})$$

and

$$E_{t-1} c_t = c = \frac{1}{\rho+v} \log\left(\frac{\theta-1}{\theta}\right). \quad (\text{A.63})$$

Subtracting (A.63) from (A.61) we obtain

$$\begin{aligned} c_t - c &= -\Delta \kappa_t - \Delta (1+v) \gamma_t \\ &\quad - \rho (1+v) \left(\frac{1-\alpha}{\alpha}\right) \frac{1}{\rho+v} \Delta \kappa_t^* - \rho (1+v) \left(\frac{1-\alpha}{\alpha}\right) \frac{1+v}{\rho+v} \Delta \gamma_t^*, \end{aligned} \quad (\text{A.64})$$

while subtracting (A.62) from (A.59) yields

$$c_t^* - c^* = -\frac{1}{\rho+v} \kappa_t^* - \frac{1+v}{\rho+v} \gamma_t^*. \quad (\text{A.65})$$

Using (A.64) and (A.65) we can compute the respective variances of home and foreign consumption

$$\begin{aligned}\sigma_c^2 &= \Delta^2 \sigma_\kappa^2 + \Delta^2 (1+v)^2 \sigma_\gamma^2 \\ &\quad + \rho^2 \left( \frac{1-\alpha}{\alpha} \right)^2 \left( \frac{1+v}{\rho+v} \right)^2 \Delta^2 [\sigma_{\kappa^*}^2 + (1+v)^2 \sigma_{\gamma^*}^2], \\ \sigma_{c^*}^2 &= \left( \frac{1}{\rho+v} \right)^2 \sigma_{\kappa^*}^2 + \left( \frac{1+v}{\rho+v} \right)^2 \sigma_{\gamma^*}^2.\end{aligned}\tag{A.66}$$

Do the optimal policies under sticky prices mimic the behavior of the flexible price economy? Start with the easier case of the ROW. Taking logs of (7.5) we get the optimal policy rule

$$i_t^* - i^* = \frac{\rho}{\rho+v} \kappa_t^* + \frac{\rho(1+v)}{\rho+v} \gamma_t^*.\tag{A.67}$$

Using this and (6.7) into equation (6.2) in the text we get (A.65) exactly. Hence, movements of consumption are the same as in under flexible prices.

Next, do the same for the SOE. The optimal policy (7.11) in logs turns out to be:

$$\begin{aligned}i_t - i &= \frac{\rho}{\rho[1+v(1-\alpha)] + \alpha v} \kappa_t + \frac{\rho(1+v)}{\rho[1+v(1-\alpha)] + \alpha v} \gamma_t \\ &\quad + \frac{v(\rho-1)(1-\alpha)}{\rho[1+v(1-\alpha)] + \alpha v} \frac{\rho}{v+\rho} \kappa_t^* + \frac{v(\rho-1)(1-\alpha)}{\rho[1+v(1-\alpha)] + \alpha v} \frac{\rho}{v+\rho} \gamma_t^*.\end{aligned}\tag{A.68}$$

Combining (A.64), (A.65), and (A.68) we have

$$c_t - c = -\frac{\alpha}{\rho} (i_t - i) - \frac{1-\alpha}{\rho} (i_t^* - i^*).\tag{A.69}$$

Finally, plugging (6.6) in the text into this last expression we get

$$\begin{aligned}c_t - c &= -\frac{\alpha}{\rho[1+v(1-\alpha)] + \alpha v} \kappa_t - \frac{\alpha}{\rho[1+v(1-\alpha)] + \alpha v} (1+v) \gamma_t \\ &\quad - \frac{(1+v)(1-\alpha)}{\rho[1+v(1-\alpha)] + \alpha v} \frac{\rho}{v+\rho} \kappa_t^* - \frac{(1+v)(1-\alpha)}{\rho[1+v(1-\alpha)] + \alpha v} \frac{\rho(1+v)}{v+\rho} \gamma_t^*.\end{aligned}\tag{A.70}$$

We conclude that in the SOE, too, fixed-price movements in consumption mimic the movements in a flex-price economy if the optimal interest rule is used.

What about utility? Expressions (7.2) and (7.3) for welfare still hold, regardless of whether prices are fixed or flexible. Using (A.63) and (A.62) in those one can check that utility under flexible prices is the same as utility with fixed prices (equations (7.6) and (7.12) in the main text) and the optimal policy rule.

### A.9. No optimal policy abroad

Under the arbitrary policies, steady state foreign output (or consumption), earlier given in (A.46), becomes

$$\begin{aligned}
c^* &= \frac{1}{\rho + v} \log \left( \frac{\theta - 1}{\theta} \right) \\
&\quad - \frac{1}{2(\rho + v)} \left\{ 1 - 2 \frac{1+v}{\rho} \phi_{\kappa^*} - [(1-\rho)^2 - (1+v)^2] \left( \frac{1}{\rho} \phi_{\kappa^*} \right)^2 \right\} \sigma_{\kappa^*}^2 \\
&\quad - \frac{1}{2(\rho + v)} \left\{ (1+v)^2 \left( 1 - 2 \frac{1}{\rho} \phi_{\gamma^*} \right) - [(1-\rho)^2 - (1+v)^2] \left( \frac{1}{\rho} \phi_{\gamma^*} \right)^2 \right\} \sigma_{\gamma^*}^2.
\end{aligned} \tag{A.71}$$

Plugging this equation into equation (A.54) we obtain home steady state consumption, and then home utility as a function of both home and foreign policy coefficients. Thus, maximizing expected utility with respect to the home policy coefficients we get the same result described in equation (7.11) above. With that policy, the utility level of the representative home is

$$\begin{aligned}
U^{\text{nopt}} &= \left( \frac{1}{1-\rho} - \frac{1}{1+v} \frac{\theta-1}{\theta} \right) \exp \left\{ \frac{1-\rho}{\rho+v} \log \left( \frac{\theta-1}{\theta} \right) \right\} \\
&\quad \exp \left\{ \frac{1}{2} \Phi \left[ 2\phi_{\kappa^*} (1 - \phi_{\kappa^*}) - \rho (1 - \phi_{\kappa^*}^2) - v (1 - \phi_{\kappa^*})^2 \right] \sigma_{\kappa^*}^2 \right\} \\
&\quad \exp \left\{ \frac{1}{2} \Phi \left[ 2\phi_{\gamma^*} (1 - \phi_{\gamma^*}) - \rho (1 - \phi_{\gamma^*}^2) - v (1 - \phi_{\gamma^*})^2 \right] \sigma_{\gamma^*}^2 (1+v)^2 \right\} \\
&\quad \exp \left\{ \frac{1}{2} \left[ \frac{\alpha(1-\rho)}{\rho[1+v(1-\alpha)] + \alpha v} \right]^2 [\sigma_{\kappa^*}^2 + (1+v)^2 \sigma_{\gamma^*}^2] \right\} \\
&\quad \exp \left\{ \frac{1}{2} \left[ \frac{\rho(1+v)(1-\alpha)}{\rho[1+v(1-\alpha)] + \alpha v} \right]^2 \left( \frac{1-\rho}{\rho+v} \right)^2 [\phi_{\kappa^*}^2 \sigma_{\kappa^*}^2 + (1+v)^2 \phi_{\gamma^*}^2 \sigma_{\gamma^*}^2] \right\}.
\end{aligned} \tag{A.72}$$

where  $\Phi = \frac{\rho(\rho-1)(1+v)(1-\alpha)}{[\rho[1+v(1-\alpha)]+\alpha v](\rho+v)^2}$ . Subtracting (A.72) from (7.15) one gets expression (8.2) in the text.

### A.10. A fixed exchange rate

Combining equation (6.3) with equation (6.4) we get domestic goods market equilibrium as

$$y_t - y - \gamma_t = c_t - c = c_t^* - c^*. \quad (\text{A.73})$$

In turn, temporary deviations in foreign consumption, as always, are given by

$$c_t^* - c^* = -\frac{1}{\rho}(r_t^* - r^*) = -\frac{1}{\rho}(i_t^* - i^*). \quad (\text{A.74})$$

If, for simplicity, we assume that the ROW central bank follows its optimal policy, (A.73) and (A.74) together yield

$$y_t - y = \gamma_t - \frac{1}{\rho+v}\kappa_t^* - \frac{1+v}{\rho+v}\gamma_t^*, \quad (\text{A.75})$$

and

$$c_t - c = -\frac{1}{\rho+v}\kappa_t^* - \frac{1+v}{\rho+v}\gamma_t^*. \quad (\text{A.76})$$

Substituting these equations into (4.5) we obtain

$$\begin{aligned} (1+v)y &= \log\left(\frac{\theta-1}{\theta}\right) + (1-\rho)c - \frac{\sigma_\kappa^2}{2} - \frac{(1+v)^2}{2}\sigma_\gamma^2 \\ &\quad - \frac{1}{2(\rho+v)^2} [(1+v)^2 - (1-\rho)^2] \sigma_{\kappa^*}^2 \\ &\quad - \frac{(1+v)^2}{2(\rho+v)^2} [(1+v)^2 - (1-\rho)^2] \sigma_{\gamma^*}^2. \end{aligned} \quad (\text{A.77})$$

Clearly, since the foreign central bank follows optimal policies, ROW steady state income is  $c^* = \frac{1}{\rho+v} \log\left(\frac{\theta-1}{\theta}\right)$ . Using this and (A.77) in  $y = \frac{\alpha+\rho(1-\alpha)}{\alpha}c - \frac{\rho(1-\alpha)}{\alpha}c^*$  we obtain

$$\begin{aligned}
c &= \frac{1}{\rho+v} \log\left(\frac{\theta-1}{\theta}\right) - \Delta \frac{1}{2} [\sigma_{\kappa}^2 + (1+v)^2 \sigma_{\gamma}^2] & (A.78) \\
&\quad - \frac{\Delta}{2(\rho+v)^2} [(1+v)^2 - (1-\rho)^2] \sigma_{\kappa^*}^2 \\
&\quad - \Delta \frac{(1+v)^2}{2(\rho+v)^2} [(1+v)^2 - (1-\rho)^2] \sigma_{\gamma^*}^2.
\end{aligned}$$

Thus, home welfare, using (7.2) turns out to be

$$\begin{aligned}
U^{\text{fix}} &= \left( \frac{1}{1-\rho} - \frac{1}{1+v} \frac{\theta-1}{\theta} \right) \exp \left\{ \frac{1-\rho}{\rho+v} \log\left(\frac{\theta-1}{\theta}\right) \right\} & (A.79) \\
&\quad \exp \left\{ -\Delta \frac{1-\rho}{2} [\sigma_{\kappa}^2 + (1+v)^2 \sigma_{\gamma}^2] \right\} \\
&\quad \exp \left\{ -(1-\rho) [(1+v)^2 - (1-\rho)^2] \frac{\Delta}{2(\rho+v)^2} \sigma_{\kappa^*}^2 \right\} \\
&\quad \exp \left\{ -(1-\rho) [(1+v)^2 - (1-\rho)^2] \frac{\Delta(1+v)^2}{2(\rho+v)^2} \sigma_{\gamma^*}^2 \right\} \\
&\quad \exp \left\{ \frac{1}{2} \left[ \frac{1-\rho}{\rho[1+v(1-\alpha)] + \alpha v} \right]^2 \alpha^2 [\sigma_{\kappa}^2 + (1+v)^2 \sigma_{\gamma}^2] \right\} \\
&\quad \exp \left\{ \frac{1}{2} \left[ \frac{1-\rho}{\rho[1+v(1-\alpha)] + \alpha v} \right]^2 \left[ \frac{(1+v)(1-\alpha)}{\rho+v} \right]^2 \rho^2 [\sigma_{\kappa^*}^2 + (1+v)^2 \sigma_{\gamma^*}^2] \right\}.
\end{aligned}$$

Finally, subtracting (A.79) from (7.15) one gets expression (8.3) in the text.

## References

- [1] Benigno, Gianluca, and Paolo Benigno (2000), “Price Stability as a Nash Equilibrium in Monetary Open-Economy Models,” mimeo, New York University.
- [2] Bernanke, Ben, and Mark Gertler (1989), “Agency costs, Net worth, and Business Fluctuations,” *American Economic Review*, Vol. 79, No. 1, pp. 14-31.
- [3] Calvo, Guillermo, and Carmen Reinhart (2000), “Fear of Floating,” mimeo, University of Maryland.
- [4] Céspedes, Luis F., Roberto Chang, and Andrés Velasco (2000), “Balance Sheets and Exchange Rate Policy,” mimeo, Harvard University.
- [5] Céspedes, Luis F., Roberto Chang, and Andrés Velasco (2001), “Dollarization of Liabilities, Exchange Rates, and Credible Monetary Policies,” paper prepared for the NBER conference on Crisis Prevention, Islamorada, Florida, January 2001.
- [6] Corsetti, Giancarlo, and Paolo Pesenti (2001a), “Welfare and Macroeconomic Interdependence,” *The Quarterly Journal of Economics*, Vol. 116, No. 2, pp. 421-445.
- [7] Corsetti, Giancarlo, and Paolo Pesenti (2001b), “International Dimensions of Optimal Monetary Policy,” mimeo, FRB of New York.
- [8] Dixit, Avinash, and Joseph Stiglitz (1977), “Monopolistic Competition and Optimum Product Diversity,” *American Economic Review*, Vol. 67, No. 3, pp. 297-308.
- [9] Flood, Robert P. (1979) “Capital Mobility and the Choice of Exchange Rate System,” *International Economic Review*, Vol. 20, No. 2, pp. 405-416.
- [10] Galí, Jordi, and Tommaso Monacelli (2000), “Optimal Monetary Policy and Exchange Rate Volatility in a Small Open Economy,” mimeo, Boston College.
- [11] Gavin, Michael, Ricardo Hausmann, Carmen Pagés-Serra, and Ernesto Stein (1999), “Financial Turmoil and Choice of Exchange Rate Regime,” mimeo, IADB.

- [12] Ghironi, Fabio, and Alessandro Rebucci (2000), "Monetary Rules for Emerging Market Economies," mimeo, FRB of New York.
- [13] Henderson, Dale W., and Jinill Kim (1999), "Exact Utilities under Alternative Monetary Rules in a Simple Macro Model with Optimizing agents," in A. Razin, A. Rose, and P. Isard (eds.), *International Finance and Financial Crises: Essays in Honor of Robert P. Flood, Jr.* Washington and Boston: IMF and Kluwer Academic Publishers.
- [14] Lane, Phillip R. (2001), "The New Open Economy Macroeconomics: A Survey," *Journal of International Economics*, Vol. 54, No. 2, pp. 235-266.
- [15] Monacelli, Tommaso (2001), "Relinquishing Monetary Independence," mimeo, Boston College.
- [16] Obstfeld, Maurice, and Kenneth Rogoff (1995), "Exchange Rate Dynamics Redux," *Journal of Political Economy*, Vol. 103, No. 3, pp. 624-660.
- [17] Obstfeld, Maurice, and Kenneth Rogoff (1996), *Foundations of International Macroeconomics*, Cambridge: MIT Press.
- [18] Obstfeld, Maurice, and Kenneth Rogoff (1998), "Risk and Exchange Rates," NBER, WP #6694.
- [19] Obstfeld, Maurice, and Kenneth Rogoff (2000), "New Directions for Stochastic Open Economy Models," *Journal of International Economics*, Vol. 50, No. 1, pp. 117-153.
- [20] Obstfeld, Maurice, and Kenneth Rogoff (2002), "Global Implications of Self-Oriented National Monetary Rules" forthcoming in *The Quarterly Journal of Economics*.
- [21] Parrado, Eric (2001), "Inflation Targeting and Exchange Rate Rules in an Open Economy," mimeo, IMF.
- [22] Poole, William (1970), "Optimal Choice of Monetary Policy Instruments in a Simple Stochastic Macro Model," *The Quarterly Journal of Economics*, Vol. 84, No.2, pp. 197-216.
- [23] Rotemberg, Julio J., and Michael Woodford (1998), "An Optimization Based Econometric Framework for the Evaluation of Monetary Policy," in *NBER Macroeconomics Annual 1997*, Cambridge: MIT Press, pp. 297-346.

- [24] Rotemberg, Julio J., and Michael Woodford (1999), “Interest-Rate Rules in an Estimated Sticky Price Model,” in J. Taylor (ed.), *Monetary Policy Rules*, Chicago: The University of Chicago Press, pp. 57-119.
- [25] Svensson, Lars E.O. (2000), “Open Economy Inflation Targeting,” *Journal of International Economics*, Vol. 50, No. 1, pp. 155-183.
- [26] Woodford, Michael (1999), “Interest and Prices,” Chapters 2 and 4. Manuscript, Princeton University.