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INTERNATIONAL TRADE AND THE ENVIRONMENT:  
A FRAMEWORK FOR ANALYSIS

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**ABSTRACT**

This paper sets out a general equilibrium pollution and trade model to provide a framework for examination of the trade and environment debate. The model contains as special cases a canonical pollution haven model as well as the standard Heckscher-Ohlin-Samuelson factor endowments model. We draw quite heavily from trade theory, but develop a simple pollution demand and supply system featuring marginal abatement cost and marginal damage schedules familiar to environmental economists. We have intentionally kept the model simple to facilitate extensions examining the environmental consequences of growth, the impact of trade liberalization, and strategic interaction between countries.

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# **International Trade and the Environment:**

## **A Framework for Analysis**

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### **1.1 Globalization and the Trade vs. Environment debate**

“It was the best of times, it was the worst of times”. This line, written by Charles Dickens over 100 years ago, captures the present day divide between supporters and critics of globalization. Over this last decade, North America and much of Europe enjoyed its longest peacetime expansion, unemployment rates hit historic lows, and real income growth in much of the developing world soared. To many these are the fruits of globalization. But this same decade saw little progress in addressing climate change, a decline in fish and forest stocks, and by some measures rising inequality in the world distribution of income. To many others these are the costs of globalization.

Debates over “globalization” have been going on for some time. But nowhere has the divide between the two views of globalization been more apparent than in recent discussions concerning trade liberalization and the environment.

For the last ten years environmentalists and the trade policy community have squared off over the environmental consequences of liberalized trade. This debate was fueled by negotiations over the North American Free Trade Agreement and the Uruguay round of GATT negotiations, both of which occurred at a time when concerns over global warming, species

extinction and industrial pollution were rising. The debate was intensified by the creation of the World Trade Organization (WTO) and proposals for future rounds of trade negotiations. Trade negotiators saw the WTO as a step forward because of its improved dispute settlement procedures and because it closed loopholes in previous trade agreements. Environmentalists, however, were disturbed by the intrusion of trade agreements into areas previously thought of as in the realm of purely domestic policy, such as subsidies and product standards.

Some well-publicized trade disputes have involved the right of governments to restrict imports for environmental reasons, as was the case with genetically modified food. To the trade policy community, these types of disputes underscore the need to expand the scope of the international rules-based trading system to ensure that governments do not use restrictive domestic policy to unfairly restrict imports. To environmentalists, this represents a dangerous loss of sovereignty, as they fear that local government policies will be constrained by international trade tribunals. Perhaps not surprisingly, then, an attempt to initiate a new round of multilateral trade negotiations in Seattle became a flashpoint for growing unrest with globalization.

In our reading of the literature on trade and the environment, we have often found the ratio of rhetoric to results astonishingly high. This is an area of public policy debate sorely in need of guidance from further theory and empirical work. A first step towards a resolution requires us to define terms, develop theory useful to the discussion, and devise methods that may help in the estimation of key empirical magnitudes. The first purpose of this paper is to contribute to this effort. A second purpose of this paper is to move two academic communities closer together – researchers in the fields of international trade and those in environmental economics.

Accordingly this paper develops “tools” useful to an examination of the trade and environment debate, and because the debate straddles two fields – environmental economics and international trade – we develop basic concepts from each field. We apologize at the outset for being pedantic at times, but our objective is to ensure readers from either field can follow and extend our analysis.

## **1.2 The Model**

We develop a simple general equilibrium model that may provide a foundation for further analysis of trade and environmental policy. In this paper we introduce this general equilibrium pollution and trade model, acquaint readers with its workings, and define the Scale, Composition and Technique effects.

The model we develop is deliberately simple. Despite this, it contains as one special case the seminal Heckscher-Ohlin model of International Trade, and as another, a version of our Pollution Haven model [Copeland and Taylor (1994)]. Therefore, both factor endowments and pollution regulations play a role in determining relative prices and hence comparative advantage. This ensures the model has sufficient richness to address the issues at hand.

While much of environmental economics makes use of partial equilibrium analysis, we need a general equilibrium approach to examine the interaction of trading economies. By the end of this paper we hope to have constructed, in a relatively transparent way, a simple general equilibrium pollution demand and supply system determining equilibrium pollution as a function of world prices, endowments, technology and preferences. Environmental economists would refer to our constructs as general equilibrium marginal abatement cost and marginal damage schedules, and this is what they are. This system can then be used to examine the

environmental consequences of growth and trade liberalization. Therefore, it is important the reader understands what underlies its construction.

It is easy to lose the forest for the trees in a paper with over 70 equations. And while the paper contains many derivations and diagrams, the reader's understanding of this paper's content relies on grasping the four major conceptual steps of our construction. Before we launch into the specifics it may be useful to spell them out here.

The first step is to understand why we can treat pollution as an input into the production of goods when in fact it is a joint (and undesirable) output. This is discussed in Section 1.3 where we define the joint production technology and introduce abatement. This leads to a discussion of potential versus net output, and a formulation explicitly linking pollution abatement costs to emissions per unit output. Having pollution as an input facilitates our use of national income or GNP functions in comparative static exercises, and frees us from the less familiar joint output approach.

The second step is to understand how we construct the general equilibrium demands for pollution. Once we treat pollution as an input we can then ask, for a given price of pollution emissions, what would the private sector choose to emit? The answer to this question gives us the general equilibrium direct demand for pollution. Since this approach follows from the private sector being quoted a price to pollute, it follows most readily from a formulation with pollution taxes. Similarly, for any given level of allowed emissions, we can ask what is the marginal value of one more unit of emissions? This gives us the general equilibrium inverse demand for pollution. Again since this approach follows from the private sector being presented with a limit on emissions, it follows most readily from a formulation with emission permits.

Because policy is efficient and there are no other distortions, the choice of a pollution

tax or an allowable emissions limit is inconsequential to the results. And the two representations of demand contain the same information. Despite this fact, it should come as no surprise to the reader that in some cases it is more convenient to use the direct demand and in others cases, the indirect demand. The direct demand is useful because pollution emissions are written as explicit function of the pollution tax. The indirect demand is useful because it directly represents the marginal benefit of pollution. Therefore, the second conceptual step is understanding the derivation of these two representations of pollution demand, and recognizing their equivalence.

While treating pollution as an input makes its inclusion into general equilibrium analysis easier, researchers in this field must often weigh the relative merits of the *Pollution Haven Hypothesis* and the *Factor Endowment Hypothesis*. This means we have to allow relative prices to reflect the abundance of two primary factors – call them capital and labor – and yet maintain a role for pollution regulations to matter as well. We need to do this in a tractable manner if our framework is to remain useful for examining trade and trade policy. To do so, we make two key assumptions: we assume the abatement activity employs factors in the same manner as does production of the dirty good; and we assume a specific form for the abatement production function.

With these two assumptions our three-factor model simplifies tremendously. For example, if we hold emissions per unit output in the dirty industry constant, our model inherits all the comparative static properties of the Heckscher-Ohlin model. Specifically, as we show in Section 1.4, the Stolper-Samuelson theorem holds: an increase in the relative price of the dirty good raises the real return to capital and lowers it to labor. As well, the Rybczinski theorem holds as well: therefore, an increase in capital raises the output of the capital-intensive dirty good and lowers the output of the labor-intensive clean good.

The third conceptual step is for the reader to recognize that the Rybczinski theorem and the Stolper-Samuleson theorem carry over to determine the properties of both the inverse and direct demands for pollution. So for example the direct demand has several useful properties that follow from the Rybczinski theorem, and the inverse demand exhibits a Stolper-Samuleson like property with respect to goods prices.

Finally we need the reader to understand how we combine the government's policy rule (mapping income, prices and pollution levels into pollution policy) with the private sector's demand for pollution to solve for the equilibrium level of pollution and its price. While this is similar to the partial equilibrium equating of marginal damage and marginal abatement cost, in general equilibrium there are several complications. To deal with these complications in a tractable manner it proves useful to employ national income or GNP functions. Therefore we review for the reader the concept of a national income or GNP function and then apply it in an environmental economics context.

A GNP function captures all of the production side behavior in our economy and being a maximum value function it has several useful properties. The benefit of this formulation is that it is more general than our specific model, and in many cases easier to deal with. Rather than differentiating zero profit conditions and resource constraints we can exploit the properties of the national income function. When ambiguities arise, we can and will appeal to our specific formulation to resolve them. It is then that it becomes necessary to understand how our model is related to the standard Heckscher-Ohlin model of international trade. Understanding the construction and use of GNP functions is the last conceptual step in this paper.

Once we have grasped how to model pollution as an input, developed a model with familiar comparative static properties, and represented the entire supply side via a national

income function we define the *Scale*, *Composition* and *Technique* effects in Section 1.5. We then demonstrate how changes in pollution caused by shocks to the economy can be decomposed into these three effects. Researchers will find this decomposition useful for the examination of trade liberalization and growth. Antweiler, et al. (2001) provides preliminary estimates of their magnitudes.

We complete this paper in Section 1.6 by determining the efficient level of pollution using our pollution supply-and-demand framework. Our pollution supply and demand can be interpreted as general equilibrium marginal damage and marginal abatement cost schedules, and so this section clearly links our approach to standard textbook treatments of pollution in environmental economics.

### 1.3 Technology

We start by considering a small open economy that faces fixed world prices. At least two goods are needed for trade to occur, and for trade to be interesting the two goods should differ in pollution intensity. Consequently, we assume the economy produces two goods, X and Y. Good X generates pollution during its production, and good Y does not. We let good Y be the numeraire (so that  $p_Y = 1$ ), and denote the relative price of good X by  $p$ .

There are two primary factors, capital and labor (K and L), with market returns  $r$  and  $w$ . Both factors are inelastically supplied.<sup>1</sup> X is capital intensive and Y is labor intensive. This

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<sup>1</sup> Recently, one branch of the environmental literature [the double dividend literature – see Fullerton and Metcalf (1998) for a review] has focussed on models with endogenous labor supply in order to analyze the

means that for any  $w$  and  $r$ , the capital/labor ratio in  $X$  is higher than in  $Y$ :

$$\frac{K_x}{L_x} > \frac{K_y}{L_y}. \quad (1.1)$$

We assume the capital intensive sector is also the polluting sector. For industrial pollution, this is consistent with the evidence.<sup>2</sup>

To keep things simple, we assume that pollution from any given firm harms consumers but does not affect productivity in other firms. As well, we rule out pollution generated during consumption.

Both goods are produced with a constant returns to scale technology. The production function for good  $Y$  is

$$y = H(K_y, L_y). \quad (1.2)$$

We assume that  $H$  is increasing and strictly concave in inputs.

The  $X$  industry jointly produces two outputs – good  $X$  and emissions  $Z$ . However, abatement is possible, and so emission intensity is a choice variable. To capture the possibility of abatement very simply, suppose that a firm can allocate an endogenous fraction  $\theta$  of its inputs to abatement activity. Increases in  $\theta$  reduce pollution, but at the cost of diverting primary factors from  $X$  production. The joint production technology is given by:

$$x = (1-\theta)F(K_x, L_x), \quad (1.3)$$

$$z = \phi(\theta)F(K_x, L_x), \quad (1.4)$$

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interaction between pollution taxes and distortionary labor taxes. As our focus is on trade policy we follow the standard international trade literature and treat labor supply as exogenous.

<sup>2</sup> For example, see the evidence cited in Antweiler, Copeland and Taylor (2001).

where  $F$  is increasing, concave and linearly homogeneous,  $0 \leq \theta \leq 1$ ,  $\phi(0) = 1$ ,  $\phi(1) = 0$ , and  $d\phi/d\theta < 0$ . We discuss the origins of  $\phi$  in detail below.

If  $\theta = 0$ , there is no abatement and by choice of units, each unit of output generates one unit of pollution. We can think of  $F(K_x, L_x)$  as potential output; this is the output of  $X$  that would be generated if there were no pollution abatement. That is, without abatement activity, we have:

$$x = F(K_x, L_x), \quad (1.5)$$

$$z = x. \quad (1.6)$$

If firms choose  $\theta > 0$ , then some resources are allocated towards abatement. If a vector  $(K_x, L_x)$  of inputs is allocated to the  $X$  sector, then  $\theta K_x$  units of capital and  $\theta L_x$  units of labor are allocated to abatement.<sup>3</sup> Equivalently, we can think of the firm as producing a gross or *potential output* of  $F(K_x, L_x)$ , and using a fraction  $\theta$  of this as an input for abatement. This leaves the firm with a *net output*  $(1-\theta)F(K_x, L_x)$  which is available for consumption and export.

It is convenient for expository purposes to put a little more structure on (1.4); hence we adopt the following functional form for abatement:

$$f(q) = (1 - q)^{1/a} \quad (1.7)$$

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<sup>3</sup> We are assuming the abatement technology uses the same factor intensity as the production of the final good  $X$ . This is a simple way to capture the notion that abatement is costly, but avoids the complexity of modelling three activities (each with different factor intensities) in a general equilibrium model.

where  $0 < \alpha < 1$ . Using (1.3)-(1.7), we can eliminate  $\theta$  and invert the joint production technology to obtain:

$$x = z^\alpha [F(K_x, L_x)]^{1-\alpha}, \quad (1.8)$$

which is valid for  $z \leq F$ , because  $\theta \geq 0$ . That is, although pollution is a joint output, we can equivalently treat it as an input.<sup>4</sup> This allows us to make use of familiar tools, such as isoquants and unit cost functions. One can think of pollution  $Z$  as the use of "environmental services", as the firm must dispose of its emissions in the environment. Alternatively, if we treat  $Z$  explicitly as pollution emissions, then we can think of the firm as requiring  $Z$  pollution permits in order to produce.

To understand the construction of (1.8) it is useful to unbundle abatement. To do so we need to distinguish between pollution produced,  $z^P$ , and pollution emitted,  $z$ . Recall production of  $F$  produces pollution in direct proportion when there is no abatement; therefore  $z^P = F$ . But if abatement occurs we define pollution emissions  $z$ , as the difference between pollution produced and pollution abated. Denoting the quantity abated by  $A$ , we have:

$$z = z^P - A$$

Abatement is like any other activity the firm undertakes in the  $X$  industry. The quantity abated depends on the amount of resources allocated to abatement, which we denote  $x^A$ , and the

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<sup>4</sup> This generalizes the model in Copeland and Taylor (1994) to allow for two primary factors. Separability ensures the marginal rate of substitution between capital and labor is not affected by pollution taxes or quotas. This will allow us to use simple diagrams to illustrate much of our analysis. The unitary elasticity of substitution assumption implicit in (1.8) simplifies the algebra. Much of our work will generalize to the case where  $x = \Phi[z, F(K_x, L_x)]$ , with both  $F$  and  $\Phi$  being linearly homogeneous. But we have opted the simpler (albeit more restrictive) specification for clarity.

amount of pollution potentially abated,  $z^P$ . Assuming abatement is a constant returns activity then allows us to conduct the following transformations:

$$\begin{aligned}
 z &= z^P - A(z^P, x^A) \\
 &= z^P - z^P A(1, x^A / z^P) \\
 &= (1 - a(\mathbf{q}))F(K_x, L_x) \\
 &= \mathbf{f}(\mathbf{q})F(K_x, L_x)
 \end{aligned} \tag{1.9}$$

where the first line is a definition, the second follows from CRS in abatement, the third by introducing the definition  $a(\theta) = A(1, \theta)$ , and the fourth by relabeling to match (1.4).

The particular form we adopted for  $\phi$  in (1.7) then corresponds to a particular abatement production function,  $A$ . Our choice in (1.7) has two benefits. First, it ensures we obtain the neat expression (1.8). This in turn requires the share of pollution taxes in the value of net output be constant. This aids in calculations as it did in Copeland and Taylor (1994). Second, it ensures the first unit of abatement has a bounded marginal product. This feature makes zero abatement on the part of firms optimal for low pollution taxes. This seems sensible, and in fact this feature of our technology was exploited by Stokey (1998) in explaining the Environmental Kuznet's Curve.

To understand the relationship between net output, potential output and the resources allocated to abatement we depict in Figure 2.1 isoquants for two different levels of net output in the X sector. The higher isoquant (labeled  $X_1$ ) corresponds to higher output. An isoquant illustrates the trade-off between “inputs” of potential output, denoted by  $F$ , and pollution emissions, denoted by  $Z$ , for a constant amount of net output. The constant returns to scale assumption implies all isoquants have the same shape: higher isoquants are radial blow-ups of lower isoquants.

At point A on the isoquant for  $X_1$ , no abatement is undertaken and pollution is proportional to output.<sup>5</sup> This corresponds to  $\theta = 0$  in (1.3) and (1.4). Similarly, other points along the dashed ray through the origin correspond to the no-abatement points on other isoquants.

As we move down along an isoquant, pollution falls because firms allocate resources to abatement. To maintain a constant level of net output, the inputs into production as measured by  $F$  must increase as the pollution level falls.

### 1.3.1 Cost minimization

In a competitive market, firms choose production techniques to minimize their cost of production. Because there is pollution, production costs depend on the regulatory regime. If there is no regulation, then there is no incentive to abate, and firms choose a point like A in Figure 2.1. If there is regulation, the firm's problem is more complex: it must satisfy constraints imposed by the regulator as well as those coming from the market.

Our model can incorporate a variety of regulatory approaches. For example, in some jurisdictions, governments impose emission intensity restrictions. We could capture this regulation as a constraint that emissions per unit output not exceed some target. In other cases, governments charge an emission tax, which is a fee per unit of emissions released into the environment. And in other cases, firms must purchase emission permits if they want to pollute.

We assume here that firms have to pay a fee for each unit of emissions that they

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<sup>5</sup> Recall we have chosen units to make the factor of proportionality, equal to one.

generate. The fee can either be an emissions tax, or it can be the market price for a pollution permit. We make this assumption in part because of its simplicity, but also because it ensures that the government's pollution target is implemented efficiently.<sup>6</sup>

Our focus is on the larger issue of how trade liberalization affects the environment, and we want to ensure that our results are not confused with side issues arising from the inefficient implementation of a pollution target. Notice we are not requiring the pollution target itself be efficient.

Let us suppose then that firms face a price  $\tau$  for each unit of emissions they generate. Given the price of emissions  $\tau$ , and the prices of capital and labor ( $r$  and  $w$ ), firms are now faced with a standard cost minimization problem. Moreover, because of the separability of our production function, we can break the firm's problem into two steps: first minimizing the cost of producing potential output  $F$ ; and then finding the most efficient way to combine  $F$  with environmental services to produce net output  $X$ .

First, the firm can find the minimum cost of producing a unit of  $F$  (potential output). Because of constant returns to scale, a unit cost function for  $F$  exists, which we denote by  $c^F$ . That is, the firm has only to determine the most efficient techniques to produce one unit of  $F$ , because by constant returns to scale, multiple units are produced by simply scaling up

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<sup>6</sup> A restriction on emissions per unit of output is not an efficient way to implement a pollution target – it can be shown to be equivalent to an emissions tax combined with an output subsidy. The output subsidy component of the policy leads to inefficiently high output. The problem is that if a firm is told to satisfy a restriction on emissions per unit of output, it can satisfy the regulation by either reducing emissions or by increasing output. In fact, in some cases, such a policy can lead to an increase in overall pollution. The policy can be rendered efficient if it is accompanied by an output tax, in which case it becomes equivalent to an emissions tax. In some strategic trade policy contexts, a government may actually want to subsidize

production. The unit cost function for F can be found by solving the following problem:

$$c^F(w, r) = \min_{\{k, l\}} \{rk + wl : F(k, l) = 1\}. \quad (1.10)$$

The firm chooses the combination of capital and labor that allows it to produce a unit of potential output at lowest cost. The total cost of producing more than one unit of F is just  $c^F(w, r)F$ .

Next, the firm can determine how much abatement activity to undertake, by finding the unit cost function for net output, which we denote by  $c^x$ . Again, by constant returns to scale, it suffices to find the efficient production techniques for one unit. The firm weighs emissions charges against the cost of foregone potential output to determine the most cost-effective techniques of production. Formally, the firm solves the following cost minimization problem:

$$c^x(w, r, \tau) = \min_{\{z, F\}} \{\tau z + c^F(w, r)F : z^a F^{1-a} = 1\}. \quad (1.11)$$

The solution is illustrated in Figure 2.2. The unit isoquant for net output of X is illustrated. The isocost line has slope  $-c^F/\tau$ , which is the relative cost of the two inputs (potential output and environmental services) used to produce net output X. The cost-minimizing choice of emissions and primary factor inputs ( $F_0, Z_0$ ) is at point B.

To solve for the optimal level of emissions per unit of net output at a point like B, we can solve the problem (1.11), and rearrange the first order conditions to obtain:

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output, and if production subsidies are illegal under trade rules, a devious choice of seemingly inefficient pollution instruments can actually be to a country's advantage. But that is an issue for a future paper.

$$\frac{z}{F} \frac{(1-a)}{a} = \frac{c^F}{t}. \quad (1.12)$$

Because (1.8) is linearly homogenous, we must also have:

$$px = c^F F + tz \quad (1.13)$$

Therefore, using (1.13) and (1.12) we can solve for pollution emissions per unit of net output, which we denote by  $e$ :<sup>7</sup>

$$e \equiv \frac{z}{x} = \frac{ap}{t} \leq 1. \quad (1.14)$$

The emission intensity falls as pollution taxes rise because emissions become more expensive. The emission intensity rises when the price of the polluting good rises because the resources used in abatement have become more valuable.

The interior optimum depicted in Figure 2.2 is not assured though.<sup>8</sup> As the emission tax falls, the isocost line in Figure 2.2 gets steeper; and for a sufficiently low emission tax, the firm will find that it is not cost-effective to abate at all and will choose point A in the diagram. To determine the conditions under which this occurs, define  $\tau^*$  as the pollution tax that leaves a firm indifferent to abating or not. When there is no abatement at all,  $z = x = F$  and  $e = 1$ . Evaluating (1.14) at this point yields:

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<sup>7</sup> Those familiar with the properties of Cobb-Douglas production functions can obtain (1.14) more quickly by noting from (1.8) that at an interior solution, the share of emission charges in the total cost of production of X must be  $\alpha$ ; that is,  $\tau z / px = \alpha$ . Rearranging yields (1.14).

<sup>8</sup> It would be assured if we adopted a formulation where the marginal product of abatement approached infinity with zero abatement as we did in Antweiler, Copeland and Taylor (2001).

$$t^* = ap. \quad (1.15)$$

For any pollution tax above  $\tau^*$ , the firm actively abates, (1.14) is relevant and emissions per unit of output,  $e$ , are less than 1. For any pollution tax below  $\tau^*$ , the firm chooses not to abate and emissions per unit of output are equal to 1: pollution is simply proportional to output.

Once we have found the emissions intensity, the economy's overall quantity of pollution emissions is simply

$$z = ex. \quad (1.16)$$

Our main interest is in the level of pollution, and so the next step is to show how the economy's aggregate output of  $X$  is determined.

### 1.3.2 Net and Potential Production Possibilities

The simplest way to illustrate the determination of output in a general equilibrium model is with the aid of the production frontier. However, because pollution is endogenous, the production frontier in our model is three dimensional; that is, if we think of pollution and  $X$  as joint outputs, we have three goods:  $X$ ,  $Y$  and  $Z$ . Alternatively, if we interpret pollution as an input, then the feasible production of  $X$  and  $Y$  varies with the level of  $Z$ , and so there is no unique relation between  $X$  and  $Y$ .

However, by distinguishing between potential and net output, and exploiting some of the structure of our technology, we can illustrate the market equilibrium in a two-dimensional diagram, as indicated in Figure 2.3.

First we can draw the production frontier for potential output. This indicates the maximum amount of potential output  $F$  in the  $X$  industry that can be produced for any level of  $Y$ , given factor endowments and technologies. That is, the potential output frontier illustrates the production possibilities for the economy if no abatement is undertaken. In Figure 2.3, this is the outermost curve labeled Potential Frontier.

As well, we can draw a conditional (net) frontier, relating the maximum level of net output  $X$  that can be produced for a given output of  $Y$  and for a given emission intensity  $e$ . In Figure 2.3, we have drawn one such net frontier (for a particular level of  $e$ ) – this is the innermost curve labeled Net Frontier. All net frontiers lie inside the potential frontier because some resources are used for abatement unless the economy is specialized in  $Y$ .

For a given emission intensity  $e$ , we can derive the corresponding net frontier from the potential frontier as follows. Substituting (1.16) into (1.8) and rearranging yields

$$x = e^{\alpha/(1-\alpha)} F(K_x, L_x) \quad (1.17)$$

Recalling that  $e \leq 1$ , (1.17) implies that net output is a fixed fraction of potential output. We can alternatively obtain a simple relation between net and potential output by referring to (1.3):

$$x = (1 - \theta)F(K_x, L_x), \quad (1.18)$$

where recall that  $\theta$  is the fraction of resources allocated to abatement. Again, this shows that net output is a fixed fraction of potential output for a given emission intensity. Using (1.18), it is easy to derive the net frontier graphically from the potential frontier: given any  $Y$ , net output  $X$  is obtained by shifting in the potential output  $F$  by a fraction  $1 - \theta$ .

Finally, it is useful to combine (1.17) and (1.18) to obtain a relation between  $e$  and  $\theta$  for future reference.

$$e = (1 - q)^{(1-a)/a}. \quad (1.19)$$

Since  $\theta$  is the fraction of value-added used in abatement, (1.19) neatly links pollution abatement costs to emissions per unit of output. Lower emissions per unit output come at the expense of higher pollution abatement costs.

#### 1.4 Equilibrium along the Net and Potential Production Frontiers

We can now exploit our two frontiers to illustrate the equilibrium levels of output and pollution for a given market goods price  $p$  and pollution emission charge  $\tau$ . Suppose that  $\tau$  is large enough so that firms allocate some resources to abatement.<sup>9</sup> Consider the profit maximization problem for a firm in the  $X$  sector. Profits  $\pi^X$  for a firm producing  $X$  are given by revenue, less payments to all labor and capital employed, and pollution charges:

$$\mathbf{p}^x = px - wL_x - rK_x - tz. \quad (1.20)$$

But using (1.16) we can eliminate  $z$  from (1.20):

$$\mathbf{p}^x = p(1 - te)x - wL_x - rK_x. \quad (1.21)$$

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<sup>9</sup> If  $\tau$  is so low that firms do not abate, then the net and gross frontiers coincide. Firms in  $X$  then receive  $p(1 - \tau)$  per unit of  $X$  produced.

As long as the pollution tax is high enough so that abatement occurs, we can exploit the results of the firm's cost minimization problem by using (1.14) to eliminate  $e$  from (1.21). This yields:

$$\mathbf{p}^x = p(1 - \mathbf{a})x - wL_x - rK_x. \quad (1.22)$$

We can therefore think of the firm as choosing  $x$  subject to the technology (1.17), while facing a producer price of  $q = p(1 - \alpha)$  for net output. In aggregate firms' behavior leads to a production point along the net frontier where the absolute value of the slope of the net frontier is equal to the producer price  $q \equiv p(1 - \mathbf{a})$ . This is point A in Figure 2.3. Pollution can then be determined in the bottom half of the diagram, by recalling that  $z = ex$ . This yields point D in the diagram, which corresponds to a pollution level of  $Z_0$ .

Alternatively, we can depict the equilibrium along the potential frontier. In the aggregate, the behavior of firms will place the economy on the potential frontier. Knowing that some of the potential output will be reallocated to abatement, and using (1.18), we can rewrite (1.22) as

$$\mathbf{p}^x = q^F F - wL_x - rK_x \quad (1.23)$$

where  $q^F$  is the producer price a firm obtains for producing one unit of potential output  $F$ :

$$q^F = p(1 - \mathbf{a})e^{\mathbf{a}/(1-\mathbf{a})} = p(1 - \mathbf{a})(1 - \mathbf{q}). \quad (1.24)$$

Hence, referring to (1.23), we can think of the firm's profit maximization problem entirely in terms of a decision about how much gross output to produce, providing that we use

the correct producer price  $q^F$ . Given that good Y is the numeraire, the relative producer price of F is  $q^F$ , and so the market outcome is at point B. This is where the absolute value of the slope of the potential frontier is equal to  $q^F$ . At this point, producers receive an effective price for gross output of  $q^F = p(1 - \alpha)(1 - \theta)$ . This is less than p because only a fraction  $1 - \theta$  of output is available for sale outside the firm (the rest is used for abatement), and of that only a fraction  $(1 - \alpha)$  remains after pollution taxes are paid.

Finally, we can also illustrate equilibrium pollution by combining (1.16) and (1.17) to obtain:

$$z = e^{1/(1-a)} F. \quad (1.25)$$

For a given emission intensity, pollution is directly proportional to gross output, and this relation is plotted in the bottom half of the diagram. Consequently, once F is determined by  $q^F$  at point B, we can drop down to point C to determine pollution.

#### 1.4.1 Uses of the Two Frontiers

The two frontiers can be used to clarify the distinction between production of final goods and production for abatement. In Figure 2.3, Point A represents the production point on the net frontier. This is the quantity of X actually available for consumption or trade. The distance BA represents that portion of gross X production used in the abatement process; and hence the ratio of BA to gross production B represents the share of production in the dirty industry allocated to abatement; that is  $\theta$ .

As well, note that potential output (represented by the outer frontier) is determined by technology and endowments alone and is independent of the level of pollution. This is a useful reminder that an economy cannot pollute itself to prosperity. It can at best abate nothing and produce the maximum level of outputs consistent with its endowments and technology. This seems like a simple point, but often when authors adopt a formulation where pollution is an input it appears that generating more pollution generates more income. In fact, all a society can do is decide how to divide its potential output across two aggregate goods – pollution prevention and real consumption.

Improvements in the technology for producing goods shifts out the gross and net frontiers uniformly, but improvement in the abatement technology alone shifts the net frontier outwards leaving the gross frontier unaffected. For example, an improvement in abatement technology shifts the net frontier towards the gross frontier. The gross frontier is unaffected, but  $q^F$  must rise because with a constant product price and fixed tax,  $\theta$  falls. The economy moves towards the dirty good industry.

And finally note that if emissions per unit of output are held fixed, then changes in potential and net output mimic one another. For example, if factor endowments change then both potential output and net output change, but the ratio of the two would remain the same.

#### 1.4.2 Equilibrium using algebra

We can also use algebra to determine the production side equilibrium. We will specify the equilibrium conditions in terms of gross output. There are two sorts of equilibrium conditions for a competitive small open economy: free entry conditions and full employment

conditions.<sup>10</sup> With free entry, if both sectors are active, we must have zero profits in each sector. This implies that price must equal unit cost. We previously derived the unit cost function for F in (1.10). Similarly, the unit cost function for Y is

$$c^Y(w, r) = \min_{\{k, l\}} \{rk + wl : H(k, l) = 1\}. \quad (1.26)$$

The free entry conditions are therefore:

$$c^F(w, r) = q^F, \quad (1.27)$$

$$c^Y(w, r) = 1. \quad (1.28)$$

In each sector, the producer price must equal the unit cost when there is positive production. These two conditions jointly determined factor prices (w,r).

The full employment conditions simply require that the demand for each of the primary factors equal supply. The factor demands can be determined from the cost functions with the aid of Shephard's Lemma.<sup>11</sup> For example, in sector Y, the amount of labor required to produce one unit of Y ( which we denote  $a_{LY}$ ) is obtained by taking the derivative of the unit cost function with respect to the wage:

$$a_{LY}(w, r) = \frac{\partial c^Y(w, r)}{\partial w}.$$

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<sup>10</sup> The reader who wants more details on analyzing simple general equilibrium trade models is referred to Dixit and Norman (1980) and Woodland (1982).

Total labor demand in sector Y is therefore  $a_{LY}Y$ , which is the total output of Y multiplied by the unit labor requirement. Other factor demands are determined analogously, and recalling that factor supplies are exogenous, we may therefore write the full employment conditions as:

$$\begin{aligned} a_{LF}(w,r)F + a_{LY}(w,r)Y &= L \\ a_{KF}(w,r)F + a_{KY}(w,r)Y &= K \end{aligned} \tag{1.29}$$

where  $a_{Li} \equiv \partial c^i / \partial w$ , and  $a_{Ki} \equiv \partial c^i / \partial r$ .

It is now very important to recognize that our system of endogenous variables resembles the standard 2-sector Heckscher-Ohlin model. Specifically, the system of equations (1.27) - (1.28) can be solved for equilibrium factor prices  $(w,r)$  as a function of  $q^F$  alone. With factor prices then determined, (1.29) solves for outputs  $(Y,F)$  as functions of  $K$  and  $L$ . Net output of  $X$  can then be determined by (1.17), and total pollution can be obtained from either (1.16) or (1.25).

This solution method follows since  $q^F$  is given by (1.24) and it depends on both  $e$  – which is endogenous – and world prices  $p$  – which are not. Therefore, our system has the property that for a given emission per unit output,  $e$ , we can solve for all remaining endogenous variables. The emission intensity is determined by (1.14). The emission price is of course a policy choice and we will discuss its determination later.

### 1.4.3 Comparative statics

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<sup>11</sup> See Dixit and Norman (1980) and Woodland (1982) for derivation of Shepherd's Lemma and its application to the analysis of simple general equilibrium trade models.

The system (1.27) - (1.29) looks very much like the standard two-sector competitive trade model – and indeed it is. The only difference is that the producer price  $q^F$  differs from the market price to take into account pollution taxes and abatement. This is very useful, because it means that for given pollution taxes (or given emission intensities), the model inherits the standard properties of the Heckscher-Ohlin model of international trade.

First, the Stolper-Samuelson Theorem holds: an increase in the producer price of a good increases the real return to the factor used intensively in the production of that good, and lowers the real return to the other factor. To see this, note that as long as the economy is diversified in production, factor prices are determined by (1.27)-(1.28). This is illustrated in Figure 2.4. The zero profit conditions for F and Y have been illustrated (there are two zero profit curves for F illustrated in the diagram, corresponding to two different levels of  $q^F$ ). The zero profit curves are level curves of the cost function, and so are downward sloping (along a curve of constant cost, an increase in  $w$  requires a fall in  $r$  to keep costs constant); and they are convex because cost functions are concave in input prices. Moreover, the absolute value of the slope of the zero profit condition is the capital / labor ratio. To see this, consider the zero profit curve for Y in the figure. Along the curve, we have:

$$\left. \frac{dw}{dr} \right|_{c^Y} = - \frac{\partial c^Y / \partial r}{\partial c^Y / \partial w} = - \frac{k_Y}{l_Y},$$

where the final result follows from using Shepherd's Lemma again. Because we have assumed that X is capital intensive relative to Y (recall (1.1)), the zero profit curve for F is steeper than that for Y. The initial equilibrium factor prices are  $(w_0, r_0)$ .

Now suppose that  $q^F$  rises from  $q_0^F$  to  $q_1^F$ . This would happen either if  $p$  rises or if the

government lowers the pollution tax. Then the zero profit curve for F shifts out and we can see that an increase in the producer price of F causes  $r$  to rise and  $w$  to fall. That is, a reduction in the pollution tax will raise the return to the factor which the polluting sector uses intensively (capital) and reduce the return to the factor used intensively in the rest of the economy (labor).

Next, the Rybczinski Theorem holds for a given emission intensity. An increase in the endowment of capital increases the output of the capital intensive good (X or F) and reduces the output of the labor intensive good (Y). To see this, first note that as long as the economy is diversified, changes in factor endowments have no effect on factor prices, because for a diversified economy, factor prices are completely determined by (1.27)-(1.28) which is not affected by factor endowment changes. Consequently, the effect of endowment changes can be obtained from (1.29), treating factor prices as constant. Adjustment to endowment changes takes place entirely via changes in output, not via changes in factor prices.

For constant factor prices, the equations in (1.29) are linear, as illustrated in Figure 2.5. Because Y is labor intensive, the full employment condition for labor (the curve labelled "L") is flatter than that for capital (two such curves are illustrated - labelled " $K_0$ " and " $K_1$ "). The initial outputs are  $(F_0, Y_0)$ . Suppose the endowment of capital rises. Then the full employment condition for capital shifts out from  $K_0$  to  $K_1$ . This increases the output of F (and hence also X), and reduces the output of Y. The intuition for this is that as the capital intensive sector expands, it requires labor to be used in conjunction with the new capital. But this labor must be drawn from sector Y, so Y contracts (which in turn free up even more capital to reinforce the expansion of the X sector).

The Rybczinski theorem will be important in helping us understand the incidence of pollution across the world. It implies, for example, that holding the emission intensity and goods

prices constant, a capital inflow will stimulate the polluting industry and lead to a contraction of the clean industry.

Before moving on to analyze the comparative statics of the equilibrium, it is worth noting that we could reformulate the equilibrium conditions in terms of net output. If we divide both sides of (1.27) by  $e^{a/(1-a)}$ , we obtain:

$$\frac{c^F(w,r)}{e^{a/(1-a)}} = p(1-a), \quad (1.30)$$

$$c^Y(w,r) = 1. \quad (1.31)$$

The right hand side of (1.30) is the producer price of net output, taking into account pollution tax payments, and the right hand side of (1.30) is the unit cost of producing X, less payments for pollution. Hence (1.30) is simply the zero profit condition for X producers.

And similarly, the full employment conditions can be written in terms of the net output of X:

$$\begin{aligned} L &= a_{LX}(w,r)X + a_{LY}(w,r)Y \\ K &= a_{KX}(w,r)X + a_{KY}(w,r)Y \end{aligned} \quad (1.32)$$

which is equivalent to (1.29) once we note that:

$$\begin{aligned} a_{LX}(w,r) &= a_{LF}(w,r) / e^{a/(1-a)} \\ a_{KX}(w,r) &= a_{KF}(w,r) / e^{a/(1-a)} \end{aligned} \quad (1.33)$$

Because the system (1.30)- (1.33) is a transformation of the system (1.27)- (1.29), the two are equivalent, and either one may be used to analyze the equilibrium.

#### 1.4.4 Consumers

We assume there are  $N$  identical consumers in the economy. Each consumer cares about both consumption and environmental quality. We assume that pollution is harmful to consumers and that it is a pure public bad (all consumers experience the same level of pollution). The utility function of a typical consumer is given by

$$U(x, y, z) = u(x, y) - h(z) \quad (1.34)$$

where  $u(x, y)$  is increasing, homothetic and concave; and  $h$  is increasing and convex.

For simplicity, we have assumed that preferences over consumption goods are homothetic and that the utility function is strongly separable with respect to consumption goods and environmental quality.

Homotheticity is a standard assumption in the international trade literature, and it helps in two ways. First, it ensures that we can write the indirect utility function as simply an increasing function of real income : nominal income divided by a price index.<sup>12</sup> Thus it allows us to simplify our decision problem through a form of aggregation. The other benefit of assuming homotheticity is that it ensures that the relative demand for goods is unaffected by income levels. This is very helpful because it allows us to explain trade patterns as functions of regulation

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<sup>12</sup> To see this note, a homothetic function is an increasing transform of a function homogenous of degree 1; that is,  $u = g(f(x, y))$  where  $f$  is homogenous of degree 1 and  $g$  is increasing. If  $f$  is homogenous of degree 1, then demands are  $x = a(p)I$ , and  $y = b(p)I$  and using linear homogeneity we can write  $u = g(I f(a(p), b(p)))$ . Define  $k(p) = f^{-1}(\cdot)$ , then we have  $u = g(I/k(p))$  where  $k(p)$  is the price index specific to the function  $f$ , and utility is an increasing function of real income so defined.

differences and relative costs alone.<sup>13</sup>

The strong separability assumption means that the marginal rate of substitution between X and Y is not affected by the level of environmental quality and it also limits the extent to which goods prices can affect the demand for environmental quality.

Each consumer maximizes utility, treating pollution, prices and per-capita income, I, as given. This yields an indirect utility function of the following form:

$$V(p, I, z) = v(I / \mathbf{b}(p)) - h(z) \quad (1.35)$$

The function v is the indirect utility function dual to u(x,y), and  $\beta$  is a price index. The function v is increasing and concave because of the structure we imposed on u. Note that it is our homotheticity assumption that has allowed us to write indirect utility as a function of real income, defined as:

$$R = \frac{I}{\mathbf{b}(p)}. \quad (1.36)$$

#### 1.4.5 National Income and Revenue Functions

National and per capita income will play a key role in our analysis. As we saw above, consumer utility depends on income; and this means that a consumer's demand for both

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<sup>13</sup> Without homotheticity over goods consumption, the relative demand for goods would vary with income: rich and poor countries would have different spending patterns, and trade would depend on the interaction between factor endowments, regulation, and income-induced differences in national spending patterns. This would complicate our model, and distract from our main goals, but the interested reader should be able to extend the analysis to allow for this added motive for trade.

consumption goods and environmental quality will depend on income. This, in turn, will mean that efficient environmental policy will depend on consumer income.

Because we have a general equilibrium model, income is endogenous. Income is the value of payments to all factors, including any pollution charges; or equivalently, it is the value of net goods production. Income will therefore depend on what the economy produces, as well as on goods prices and environmental policy. That is, the determination of income requires that we solve the entire general equilibrium of the economy.

Fortunately, there is a simpler way to obtain an expression for the economy's income that avoids having to resolve the entire model every time we want to undertake a comparative static exercise. As has become standard in the international trade literature, we can exploit the fact that the private sector of a perfectly competitive economy maximizes the value of national income.<sup>14</sup> That is, we can represent national income as the solution to an optimization problem.<sup>15</sup> The implied maximum value function will then have a number of very nice properties that will help us in our analysis. An added benefit of this approach is that we can define a national income function for a very general economy, and so in some cases it will allow us to easily generalize our results beyond the confines of the simple technology we have assumed for our economy. But as well, we will be able to exploit the structure imposed by our technology assumptions to get clean results to build intuition.

Let us start then by assuming a somewhat more general technology than we have used so far. Let  $T(K,L,Z)$  be a two-dimensional convex production possibility set with constant

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<sup>14</sup> It is at this point that our assumption that pollution does not cause production externalities is exploited.

returns to scale. That is,  $T$  is the set of all combinations of net outputs  $(X,Y)$  that can be produced given the primary factor endowments  $K$  and  $L$ , and for a given level of pollution  $Z$ . The production technology we specified in (1.2)-(1.8) is an example of such a technology. Now define the maximum value function  $G$  as follows:

$$G(p^x, p^y, K, L, z) = \max_{\{x,y\}} \{p^x x + p^y y : (x, y) \in T(K, L, z)\} \quad (1.37)$$

The function  $G$ , which we call the *national income function*, tells us the value of national income at world prices, for any level of pollution and factor endowment, given the underlying technology. One can show that the first order conditions for the solution to the maximum problem in (1.37) are exactly the same as the equilibrium conditions for our competitive economy.<sup>16</sup>

We have treated aggregate pollution as exogenous in (1.37) and hence we will show how to make it endogenous below. Also note that although we suppress the price of  $Y$  throughout most of our analysis because we are treating it as the numeraire, we have included  $p^Y$  in the above for clarity. In most of our applications, we will set  $p^X = p$ , and  $p^Y = 1$ , and with slight abuse of notation will refer to the national income function as  $G(p,K,L,z)$ , where the role of the price of  $Y$  is suppressed.

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<sup>15</sup> See Woodland (1982) and Dixit and Norman (1980) for a detailed analysis of the national income function (sometimes called the revenue function). Copeland (1994) applies the national income function to economies with pollution.

<sup>16</sup> Woodland (1982) does this explicitly. The interested reader should demonstrate that the first order conditions for (1.37) are equivalent to the equilibrium conditions already spelled out in terms of either gross

The national income function has a number of very useful properties, many of which follow from its being a maximum value function. These are discussed in detail in Woodland (1982), and here we simply point out those properties that will be of particular relevance.

First, Hotelling's Lemma holds; that is, outputs can be recovered by differentiating with respect to goods prices:

$$\frac{\partial G(p^x, p^y, K, L, z)}{\partial p^x} = x, \quad \frac{\partial G(p^x, p^y, K, L, z)}{\partial p^y} = y. \quad (1.38)$$

This follows from the envelope theorem.

Next, the returns to capital and labor can be found by differentiating with respect to the relevant factor endowment:

$$\frac{\partial G(p^x, p^y, K, L, z)}{\partial K} = r, \quad \frac{\partial G(p^x, p^y, K, L, z)}{\partial L} = w. \quad (1.39)$$

The intuition for this is straightforward. Suppose the economy acquires an extra unit of capital. The derivative  $\partial G / \partial K$  tells us how much national income rises because of the extra unit of capital. But this must be the value of the marginal product of capital, which in a competitive market is equal to the market return to capital. Similarly, an extra worker earns the value of his or her marginal product, which is the wage.

#### 1.4.6 General equilibrium marginal abatement cost

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or net output. To do so, it proves useful to solve for the cost function in (1.11) explicitly and then isolate

Perhaps the most useful property for our purposes is that if we apply the same logic as above, then the derivative of the national income function with respect to pollution emissions is equal to the price the firms have to pay for the right to pollute:

$$\frac{\partial G(p^x, p^y, K, L, z)}{\partial z} = t. \quad (1.40)$$

The intuition is the same as above: if the private sector is allowed to release one more unit of emissions, national income will rise by the value of the marginal product of emissions, which in a competitive market is equal to the price paid by the firm for the right to pollute. If we think of environmental services as an input, then the logic is exactly the same as that we exploited in discussing factor returns above.

The expression  $\partial G / \partial z$  can be interpreted as a general equilibrium *marginal abatement cost*. If we think of reducing emissions  $z$ , then the fall in national income due to a drop in allowable emissions is just  $\partial G / \partial z$ ; that is, it measures the cost to the economy of adjusting to a lower emission target. Reduced emissions will be achieved by the private sector in two ways: by investing more in abatement activity, and by producing less of the dirty good X and more of the clean good Y. In the latter case, the cost to the economy of abatement is the cost of moving along the production frontier from X towards Y. With either a market for emission permits or a pollution tax, the private sector will choose the most efficient combination of these two strategies. The derivative  $\partial G / \partial z$  measures the cost to the economy of reducing emissions when the emission reduction is achieved at lowest possible cost.

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$c^F(w,r)$ .

Another interpretation of the result in (1.40) then is simply that the pollution charge paid by the firm will equal the marginal abatement cost. This is a familiar result from environmental economics, although there it is usually presented in a partial equilibrium framework.

#### 1.4.7 More properties

Because  $G$  is a maximum value function, it has an important curvature property: it is convex in prices. The economic interpretation of this is that output supplies slope upwards.

That is:

$$\frac{\partial^2 G}{\partial p^x{}^2} = \frac{\partial x}{\partial p^x} \geq 0, \quad \frac{\partial^2 G}{\partial p^y{}^2} = \frac{\partial y}{\partial p^y} \geq 0. \quad (1.41)$$

And because of constant returns to scale,  $G$  is concave in endowments:

$$\frac{\partial^2 G}{\partial K^2} = \frac{\partial r}{\partial K} \leq 0, \quad \frac{\partial^2 G}{\partial L^2} = \frac{\partial w}{\partial L} \leq 0, \quad \frac{\partial^2 G}{\partial z^2} = \frac{\partial t}{\partial z} \leq 0. \quad (1.42)$$

That is, inverse demands for all factors slope downwards. Holding all other endowments fixed, increasing the supply of, say, labor, will typically reduce (or more generally, will not increase) the value of its marginal product. Most relevant to us is the last result in (1.42): it says that the general equilibrium marginal abatement cost curve slopes down.

And finally,  $G$  has a couple of homogeneity properties. First, it is homogenous of degree 1 in prices; that is:

$$G(\mathbf{I}p^x, \mathbf{I}p^y, K, L, z) = \mathbf{I}G(p^x, p^y, K, L, z) \quad \text{for } \mathbf{I} > 0 \quad (1.43)$$

This just says that doubling all goods prices doubles national income, but has no effect on production decisions. Second,  $G$  is homogeneous of degree one in endowments. Doubling all endowments, but leaving prices unchanged just scales up the economy.

$$G(p^x, p^y, \mathbf{1}K, \mathbf{1}L, \mathbf{1}z) = \mathbf{1}G(p^x, p^y, K, L, z) \text{ for } \mathbf{1} > 0$$

This follows from constant returns to scale.

#### 1.4.8. Endogenous Pollution with a National Income Function

It might appear that a potential limitation of the usefulness of (1.37) is that the emission level  $z$  is treated as exogenous when solving the optimization problem. This is fine if the government regulates pollution using an emission permit system. In that case the government specifies the overall supply of pollution permits, and the private sector through its competitive behavior ends up maximizing the value of national income, given the fixed available aggregate supply of emission permits. However, if there is no regulation, or if there is a fixed pollution tax in place, then we do not want to treat  $z$  as exogenous. It turns out there are two ways of defining a national income function with endogenous pollution; one involves using (1.37) but treating  $z$  as endogenous; and the other approach involves defining a different national income function. Because both approaches have their uses, we will do both.

First, we can exploit (1.40). If  $Z$  is exogenous, then (1.40) gives us the equilibrium market price of an emissions permit. But suppose instead that there is an exogenous pollution tax  $\tau$  and that  $z$  is determined by the market response to the tax. Then if we treat  $\tau$  as fixed,

(1.40) determines  $z$  endogenously. Referring to Figure 2.6, we can think either of a fixed  $z$  determining  $\tau$ , or a fixed  $\tau$  determining  $z$ .

An alternative and perhaps more direct approach is to revert to the original interpretation of our technology as a joint production technology producing three outputs (X,Y,Z). And suppose that there is an exogenous pollution tax  $\tau$ . Then standard competitive economic theory tells us that the private sector will maximize the value of output given the prices of X, Y, and Z. The only slight twist to keep in mind here is that the price of pollution is negative from the point of view of firms, because they must pay a tax on emissions. Therefore, we can define:<sup>17</sup>

$$\tilde{G}(p^x, p^y, \mathbf{t}, K, L) = \max_{\{x, y, z\}} \{p^x x + p^y y - \mathbf{t}z : (x, y) \in T(K, L, z)\}. \quad (1.44)$$

This is the value of net revenue generated by the private sector; however, national income also includes pollution tax revenue. Consequently, total national income is:

$$I = \tilde{G}(p, \mathbf{t}, K, L) + \mathbf{t}z.$$

But notice that when we add pollution tax revenue to  $\tilde{G}$ , we are left with:

$$I = px + y,$$

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<sup>17</sup> This is the approach taken in Copeland (1994).

and so it should not surprise the reader that if we treated the equilibrium pollution  $Z_o$  that solved our problem (1.44), and confronted the economy with a fixed number of pollution permits  $Z_o$ , then we would have:

$$G(p, K, L, Z_o) = \tilde{G}(p, \tau, K, L) + \tau Z_o.$$

The function  $\tilde{G}$  satisfies all the same properties as  $G$  that we outlined above, with the exception of (1.40) because it is a function of  $\tau$  instead of  $Z$ . Instead, we have the following envelope property, which is an application of Hotelling's Lemma:

$$\frac{\partial \tilde{G}(p, \tau, K, L)}{\partial \tau} = -Z(p, \tau, K, L). \quad (1.45)$$

That is, we can obtain the derived demand for pollution by differentiating the national income function  $\tilde{G}$  with respect to the pollution tax. Moreover, because  $\tilde{G}$  is convex in all prices (including  $\tau$ ), we have  $\tilde{G}_{\tau\tau} \geq 0$ , which implies:

$$\frac{\partial Z}{\partial \tau} \leq 0.$$

The derived demand for the right to pollute is decreasing in the pollution tax.

Finally, (1.45) and (1.40) are related. The former describes the derived demand for pollution; the latter is the inverse demand. Both describe the same curve in Figure 2.6.

## 1.5 Scale, Technique and Composition Effects

Because the linkages between the economy and the environment are both subtle and

complex, it is useful to decompose changes in pollution into three fundamental forces: scale, composition and technique effects. This approach was used by Grossman and Krueger (1993) to interpret the empirical evidence in their influential study of the potential effects of NAFTA on the environment, and we have found it useful to help clarify both theoretical and empirical analysis. It is particularly useful in comparing the effects of different types of shocks to the economy. For example, both trade liberalization and capital accumulation tend to raise the productive capacity of the economy (this will lead to a scale effect in each case), but they may stimulate very different types of economic activity (their composition effects will differ). Moreover, because they both raise income and because environmental quality is a normal good, both types of changes could lead the government to tighten environmental policy (which will lead to a technique effect). By breaking the effects of policy changes into scale, technique, and composition effects, we can clarify how different types of shocks have both common and divergent effects on the economy. Moreover, as shown in Antweiler, Copeland and Taylor (2001) this approach can also help us disentangle the effects of different types of shocks empirically.

### 1.5.1 Definitions

In this section, we will define the scale, technique and composition effects, and give some examples to illustrate how to employ them.

Trade and growth both stimulate economic activity, and therefore both increase the economy's scale. To be more precise, we need a measure of the scale of the economy; that is, we need an index of output. There many ways to create such a quantity index, but for

simplicity, we will use the value of net output at a given level of world prices as our measure of the economy's scale. Our measure of scale,  $S$ , is defined as

$$S = p_x^o x + p_y^o y \quad (1.46)$$

where  $p_x^o$  and  $p_y^o$  denote the base-period level of world prices; that is, the level of world prices prior to any shocks that we analyze. If world prices change, we continue to construct  $S$  using the old (initial) world prices. This is so that scale will not change simply because of a change in valuation. But of course if the *outputs* of  $x$  and  $y$  change in response to world price changes, then our measure of scale will change – we evaluate the new outputs at the old prices.

Given this definition of scale, and choosing units to set base-period prices to unity (that is, we set  $p_x^o = p_y^o = 1$ ), we now use (1.46) to write pollution as

$$z = ex = e\mathbf{j}_x S \quad (1.47)$$

where  $\phi_x = p_x^o x / S = x / S$  is the value share of net output of  $x$  in total output evaluated at base-period prices. Hence pollution emissions depend on the emissions intensity of production,  $e$ , the importance of the dirty good industry in the economy,  $\phi_x$ , and the scale of the economy,  $S$ .

Taking logs and totally differentiating yields our decomposition:

$$\hat{z} = \hat{S} + \hat{\phi}_x + \hat{e}, \quad (1.48)$$

where  $\hat{z} = dz/z$ , etc.

The first term is the **scale effect**. It measures the increase in pollution that would be generated if the economy were simply scaled up, holding constant the mix of goods produced and production techniques. As an example, if there were constant returns to scale and all of the

endowments of the economy grew by 10%, and if there were no change in relative prices or emissions intensities, then we should expect to see a 10% increase in pollution.

The second term is the **composition effect** as captured by the change in the share of the dirty good in national income. If we hold the scale of the economy and emissions intensities constant, then an economy that devotes more of its resources to producing the polluting good will pollute more.

Finally, we have the **technique effect**, captured by the last term in (1.48). Holding all else constant, a reduction in the emissions intensity will reduce pollution.

Let us illustrate these concepts using some diagrams. We will work through several examples both to illustrate how the model works, and to show how different sources of economic growth affect pollution in different ways. Because it is cumbersome to illustrate shifts in both net and gross frontiers, we will focus on the net frontier throughout.

### 1.5.2 The Scale Effect: Balanced growth

To isolate the scale effect it is useful to assume that the emissions intensity is held fixed. This would be the case for example if the government had a fixed pollution tax. To start we illustrate in Figure 2.7 an initial equilibrium with point A indicating the initial output point (on the net frontier) with producers receiving  $p(1-\alpha)$  per unit of net output. In the lower panel of the figure we graph a pollution emissions function  $z = ex$  with a given fixed emission intensity of  $e_0$ . Given the initial production point, A, the initial level of pollution is  $z_a$ . Suppose we scale up the economy by increasing each of the endowments by an equal percentage. Because of constant

returns to scale, the new production frontier is just a radial expansion of the old one. The new production point is at point B, which must be on the same ray through the origin as A. Pollution has increased from  $z_a$  to  $z_b$  and this increase represents the pure scale effect.

Referring to (1.48) we see there is no technique effect because we have held policy constant by assumption; and there is no composition effect because both the X and Y sectors expand equally. Therefore, we conclude balanced growth in endowments in the presence of a fixed emission intensity will raise pollution via a pure scale effect.

### 1.5.3 The Composition Effect: Capital Accumulation

Next consider the composition effect. To do so we again fix the emissions intensity, and now consider a change in only the endowment of capital. The consequence of this change for both pollution and outputs is illustrated in Figure 2.8. In this case, the outward shift of the production frontier is skewed towards the X-axis, because industry X is capital intensive. At a constant producer price  $p(1-\alpha)$ , production in our economy moves from point A to point C. We know from the Rybczinski theorem that the economy produces more X and less Y at C than at A.

Both scale and composition effects are operative; and we now illustrate how to decompose the movement from A to C into these two effects. The line denoted  $P_0$  measures the value of the initial output at our base-period world prices; this is the initial scale of the economy at point A. This line is steeper than the producer price line because of pollution policy (and possibly also because of trade barriers). For any movement along the  $P_0$ , the scale of the economy is constant. We therefore decompose the total change in the economy into a

movement from A to B, and from B to C.

The movement from A to B is a pure composition effect, because we have hypothetically held the scale of the economy constant, and found the pure effect of increasing the share of X in the economy. This composition effect yields an increase in pollution from  $z_a$  to  $z_b$ . Note this is positive, because X is the dirty industry.

Next, the movement from B to C is the pure scale effect - it is the effect on pollution of increasing the scale of the economy, while holding the composition of output fixed. That is, along a ray through the origin and through point B, the composition of production is constant. Pollution rises from  $z_b$  to  $z_c$  via the scale effect.<sup>18</sup>

With capital accumulation, both the scale and composition effects are positive, and therefore the net effect is to raise pollution.

If we instead were to consider growth in the endowment of labor, we would also obtain a positive scale effect, but pollution would fall via the composition effect. We know from the Rybczinski theorem that an increase in the supply of labor will raise the output of the clean good Y and lower the output of the dirty good. Therefore, the composition effect of labor accumulation has an opposite effect to that of capital accumulation.

Summing up, the composition effect is positive if a shock to the economy leads it to produce a basket of goods that is more pollution intensive on average than it did previously. In the model above, this is a simple observation, but in more general models this basic result still

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<sup>18</sup> One could alternatively consider the scale effect first, and then the composition effect. As with income and substitution effects, when we consider a discrete change, the magnitude (but not the direction) of the effects will depend on the order in which they are constructed.

holds true.<sup>19</sup>

#### 1.5.4 The Technique Effect: a Change in Emission Intensity

To examine the technique effect we now consider the effects of a change in pollution policy. Suppose there is an exogenous increase in the pollution emissions tax. From (1.22) we know the producer price for net output is unaffected by this change, but from (1.14) the emissions intensity has to fall. As a result, the net frontier must shift in as more resources are allocated to abatement.<sup>20</sup> The effects of this exogenous policy change are illustrated in Figure 2.9. Initially, the economy is at point A, pollution is  $z_a$  and emissions per unit of output are  $e_o$ . An increase in the pollution tax increases abatement activity and hence reduces emissions per unit of output ( $e$  falls to  $e_1$ ). The pollution function in the lower part of the diagram shifts up (for any level of  $x$  output, there is less pollution). Holding output at A, pollution falls from  $z_a$  to  $z_1$ . This is the technique effect: a higher pollution tax leads to cleaner production techniques; and, holding the scale and composition of output fixed, this lowers pollution emissions.

The policy change also has two other effects. Note that because the production frontier rotates inward, the final equilibrium is at point C. This movement is comprised of a scale effect (A to B) leading to the further drop in pollution from  $z_1$  to  $z_b$ , and the composition effect (B to C) reducing pollution even more to  $z_c$ . There is a fall in the scale of output because of increased abatement (which consumes resources). There is a composition effect because the resource cost of further abatement affects the dirty industry disproportionately. As a result, the

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<sup>19</sup> See for example the discussion of composition effects in Copeland and Taylor (1994).

opportunity cost of producing X rises and with constant prices, producers shift towards the clean good Y.

Tightening up pollution policy therefore reduces pollution via three effects: cleaner techniques, lower scale of output, and a shift in the composition of economic activity towards the cleaner good.

## 1.6 Endogenous Pollution Policy

So far, we have analyzed the equilibrium of the economy under the assumption that pollution policy is exogenous. In general, we expect pollution policy to be endogenous. In particular, we expect that changes in per capita income will lead to an increase in the demand for environmental quality, and, if governments are responsive, this may lead to a tightening up of pollution regulations. Both trade and growth affect per-capita income, and we need to account for possible endogenous policy responses when analyzing their effects on the environment. As well, endogenous policy differences across countries can themselves be a cause of international trade. This is the well-known *Pollution Haven Hypothesis*, and we will need to understand how pollution policy varies with the economic characteristics of a country in order to fully analyze it.

There are many ways to model endogenous environmental policy. One is to assume that the government is responsive to the preferences of consumers and provides efficient policy. Another is to assume governments respond to interest group pressure along the lines of Grossman and Helpman (1994). Here we will assume policy is efficient. This choice is made

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<sup>20</sup> The potential frontier remains in place.

because the major theories in this area focus on cross-country differences in either income or factor endowments as a basis for trade, whereas much of the recent political economy literature has focused on explaining within country but across industry variation in protection. As well, efficient policy has played a central role in the literature and all of the political economy approaches tend to build on the analytics of the efficient policy approach. It will also serve as a useful base case to compare with other scenarios.

The efficient level of pollution is determined by weighing the benefits of pollution against the costs. As we showed earlier, it is helpful to treat pollution (or environmental services) as an input used by producers. Moreover, as we have shown pollution is an input with a variable aggregate supply. A standard tool for analyzing input markets is the demand and supply diagram; and we find that it is useful to use such a diagram to illustrate the equilibrium level of pollution. The demand for pollution is a derived demand, as firms in the X sector derive benefits from securing the right to pollute. The "supply" of pollution reflects the policy regime. When pollution policy is optimal, the supply of pollution reflects the aggregate willingness to allow environmental damage. The interaction between these demand and supply side factors determines the equilibrium level of pollution.

### 1.6.1 The Demand for Pollution

We have already described many features of the private sector's demand for pollution. In our previous analysis we fixed the emission intensity for many of our results, and this requires us to fix the pollution tax or permit price  $\tau$ . For example, when we found higher pollution levels with either balanced factor growth (the scale effect example) or capital accumulation (the

composition effect example), implicit in our analysis was an outward shift in pollution demand by the private sector in both these cases. Similarly, when we found a negative relationship between lower emission intensities and pollution, this in effect illustrated that our pollution demand curve has negative slope. Higher pollution taxes lower pollution. Therefore our earlier exercises were very simple characterizations of the private sector's derived demand for pollution.

At this point we want to be a little more explicit about the properties of pollution demand, as well as to introduce a convenient diagram. Starting with a general technology, recall from the properties of the national income function that the inverse demand for pollution is given by<sup>21</sup>

$$t = G_z(p, K, L, z). \quad (1.49)$$

This defines an implicit function  $z = Z(\tau, p, K, L)$ . We can differentiate and solve for the slope of the pollution demand curve:

$$\frac{dz}{dt} = \frac{1}{G_{zz}} \leq 0 \quad (1.50)$$

The slope of the derived demand for pollution is non-positive because  $G$  is concave.<sup>22</sup>

We can say more about pollution demand by recalling pollution is determined by the emission intensity and the output of  $x$ . This yields a direct derived demand for the right to pollute as a function of the pollution tax  $\tau$ , factor endowments and the price of  $X$ .

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<sup>21</sup> As we indicated earlier, we will suppress the role of the price of  $p^y$  in  $G$  because we have set  $p^y = 1$ .

<sup>22</sup> For general technology, it is possible that pollution demand may have flat regions. For example, the model presented in Copeland and Taylor (2000) exhibits this property.

$$z = e(p/\mathbf{t})x(p, \mathbf{t}, K, L) \quad (1.51)$$

where  $e$  is defined in (1.14) and we have made use of (1.44) to write output as a function of goods prices, taxes and endowments. The slope of the general equilibrium pollution demand is given by

$$\frac{\partial z}{\partial \mathbf{t}} = e_{\mathbf{t}}x + ex_{\mathbf{t}} < 0 \quad (1.52)$$

and hence while (1.50) tells us pollution demand slopes downward, (1.52) identifies the two mechanisms at work in creating the negative slope.

The derived demand for pollution is illustrated in Figure 2.10. Pollution demand slopes down for two reasons: first, higher pollution taxes make abatement more profitable, thereby reducing the emissions intensity of production. This is the technique effect captured by the first term in (1.52). In discrete form this would represent a movement from  $z_4$  to  $z_1$  in Figure 2.9. Second, with greater abatement efforts resources are drawn away from production of final goods and services and this causes the output of  $x$  to fall as producers exit the  $x$  industry and move into  $y$ . This change is due to both scale and composition effects. And again in discrete terms this is the movement from  $z_1$  to  $z_c$  in Figure 2.9.

Pollution demand shifts in response to changes in factor endowments and goods prices. An increase in the endowment of capital shifts the demand for pollution to the right. To see this, first recall from the production side equilibrium conditions that output  $X$  (or  $F$ ) is only a function of  $\tau$  through its effect on emissions per unit output. Therefore we could write the direct demand to reflect this:

$$z = e(p/\mathbf{t})x(p, e(p/\mathbf{t}), K, L) \quad (1.53)$$

Then differentiate with respect to K to obtain:

$$\frac{dz}{dK} = e(p/\mathbf{t}) \frac{dx(p, e(p/\mathbf{t}), K, L)}{dK} > 0 \quad (1.54)$$

For a given pollution tax and goods price, an increase in K has no effect on the emission intensity. Therefore, the effect of capital accumulation on the demand for pollution depends on the response of the output of X. But we can now invoke the Rybczinski theorem since e is held fixed when we take this derivative. And hence capital accumulation stimulates output of the capital-intensive dirty good X, and so the demand for pollution rises (the private sector will want to pollute more for any given  $\tau$ ). We illustrate this shift in Figure 2.10. For the initial pollution tax  $\tau_0$ , pollution demand rises from  $z_a$  to  $z_c$ . This is exactly the same as the increase in pollution illustrated in Figure 2.8 when we considered an increase in capital.

We could alternatively have differentiated (1.49) set  $d\tau = 0$  and solved for the resulting change in z from the change in K. This would have required us to sign the cross derivative  $G_{ZK}$  which a priori is uncertain. As well, using the direct demand we can also employ *Jones magnification effect* that capital intensive output rises more than proportionately with capital. Therefore, when calculating quantity responses in pollution demand, using the direct demand is more convenient.

In contrast to the case of capital, an increase in the endowment of labor shifts pollution demand to the left:

$$\frac{dz}{dL} = e(p/\mathbf{t}) \frac{dx(p, e(p/\mathbf{t}), K, L)}{dL} < 0. \quad (1.55)$$

This again follows from the Rybczinski Theorem.

Finally, an increase in the price of the dirty good shifts pollution demand to the right because abatement becomes relatively more expensive, and because factors are drawn into the now more attractive dirty good industry:

$$\frac{dz}{dp} = x \frac{de(p/\mathbf{t})}{dp} + e(p/\mathbf{t}) \frac{dx(p, e(p/\mathbf{t}), K, L)}{dp} > 0 \quad (1.56)$$

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As discussed earlier, the pollution demand curve can be thought of as a general equilibrium marginal abatement cost curve - it measures the opportunity cost to the economy of reduced pollution emissions. Capital accumulation and increases in the dirty good price raise marginal abatement costs and increases in the endowment of labor reduce marginal abatement costs.

To examine how the marginal benefit of polluting has changed with the price of the dirty good, we may want to employ the inverse demand however. In this case we would find:

$$\frac{\partial \mathbf{t}}{\partial p} = G_{zp}(p, K, L, z) > 0$$

and because of the structure imposed by our model, we can use (1.53) to show that:

$$\frac{\partial \mathbf{t}}{\partial p} \frac{p}{\mathbf{t}} = \frac{G_{zp}p}{G_z} > 1 \quad (1.57)$$

and hence the marginal benefit of polluting rises more than proportionately from an increase in the price of the dirty good. And consequently, for applications where we are interested in price responses, employing the inverse demand curve is often convenient.

The pollution demand curve in (1.49) can be thought of as a marginal benefit of polluting curve. With this interpretation, we have just shown how changes in endowments and goods prices affect the marginal benefit of polluting. An increase in the price of the dirty good increases the marginal benefit of polluting because the value of the marginal product of emissions is higher. Capital accumulation increases the marginal benefit of polluting because a more capital abundant country is relatively more productive in the dirty industry. And labor accumulation reduces the marginal benefit of polluting because it makes the economy more productive in the clean industry.

### 1.6.2 Marginal Damage and the "Supply" of Pollution

Let us now find the optimal pollution policy. The demand for pollution as captured by (1.49) measures the marginal benefit of polluting. To determine the optimal pollution policy, we need to balance this against the marginal damage from polluting. Because we have assumed all consumers are identical, the government finds the optimal policy by choosing the pollution level to maximize the utility of a representative consumer subject to production possibilities and private sector behavior.

We start by formulating the government's problem with general preferences and technology:

$$\text{Max}_z \{V(p, I, z) \text{ s.t. } I = G(p, K, L, z) / N\} \quad (1.58)$$

where  $V$  is the indirect utility function of a typical consumer. All  $N$  consumers are identical and so each receives the same income.<sup>23</sup> The first order condition for the choice of pollution is:

$$V_p \frac{dp}{dz} + V_I \frac{dI}{dz} + V_z = 0. \quad (1.59)$$

An increase in pollution will affect goods prices, income and environmental damage, and each of these affects the consumer. If we divide both sides of (1.59) by  $V_I$ , rearrange, and use Roy's identity, we have:

$$-D^x \frac{dp}{dz} + \frac{dI}{dz} = -\frac{V_z}{V_I}. \quad (1.60)$$

where  $D^x$  is the demand for  $X$  by a typical consumer. The term on the right hand side of (1.60) is the marginal rate of substitution between emissions and income; in other words it measures the typical consumer's willingness to pay for reduced emissions. In the environmental literature, this is referred to as "marginal damage". We denote this by  $MD$ , and hence we define:

$$MD \equiv -\frac{V_z}{V_I}. \quad (1.61)$$

To further simplify (1.60), use the constraint in (1.58) and the properties of the national income function to obtain:

$$\frac{dI}{dz} = \frac{G_p \frac{dp}{dz} + G_z}{N} = \frac{x \frac{dp}{dz} + t}{N} \quad (1.62)$$

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<sup>23</sup> The government may use either pollution permits or emission taxes. In either case, any revenue accruing

Substituting (1.62) and (1.61) into (1.60) yields:

$$t = N \bullet MD + m \frac{dp}{dz} . \quad (1.63)$$

where  $m = N \bullet D^x - x$  is imports of X (if X is exported, then  $m < 0$ ). The condition (1.63) says that the government should choose pollution so that the emissions price faced by the private sector is equal to aggregate marginal damage plus a term that reflects the terms of trade effect induced by changes in pollution.

To interpret (1.63), let us first assume there is no international trade, so that  $m = 0$ . In this case, (1.63) reduces to:

$$t = N \bullet MD . \quad (1.64)$$

Recall that environmental quality is a pure public good (or equivalently, pollution is a pure public bad). The condition (1.64) is simply the Samuelson rule for public goods provision: the government chooses pollution so that firms face an emissions price which is equal to the sum of the marginal damages across all consumers. Notice that the direct effect of pollution on goods prices drops out of the rule for optimal policy if the country does not trade. The reason for this is that although an increase in pollution lowers prices to consumers, it also lowers producer prices and hence lowers income. Without international trade and with an efficient domestic market, these two effects exactly offset each other.

Now introduce international trade in goods. Then the world price  $p$  of dirty goods is determined by global demand and supply for the dirty good. If Home changes its emissions

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to the government is embodied in  $G$  as a return to  $z$  and is rebated to consumers in lump sum.

policy, this will affect Home's demand and supply for the dirty good, and therefore global demand and supply will be affected. If Home is small in world markets, however, this effect will be small, and Home's domestic emissions policy will have a negligible effect on world prices. That is, if Home is small, it is essentially a price taker in world markets. In the international trade literature, the *small country assumption* is that domestic policies have no effect on world prices. For many, if not most, countries in the world, this is a realistic assumption. Under the small country assumption,  $dp/dz = 0$ , and so (1.63) again reduces to the simple Samuelson rule in (1.64). That is, when a small country is open to international trade, its efficient pollution policy is simply to internalize the pollution externality and ensure that firms face an emissions charge that is equal to the aggregate marginal damage.

If the Home country is large, then changes in its emission policy may affect world goods prices. This is because an increase in its allowable emissions will stimulate the supply of the dirty good, and if the Home country is sufficiently big, this can push down the world price of the dirty good. Because changes in world prices affect the price at which Home buys and sells goods from foreigners, this effect shows up in the optimal policy rule. That is, if a country is large, then it has market power; and one way to exploit this power is to use pollution policy to try to manipulate world prices.

If Home imports the dirty good and the world price of the dirty good falls, Home's terms of trade improve. That is, an increase in domestic emissions can yield an added benefit to home by reducing the world price of the good that it imports. This means that a dirty good importer that is large in world markets has a strategic incentive to set the domestic price of emissions somewhat below marginal damage in order to manipulate the world price of dirty goods. In contrast, a dirty good exporter is worse off if the price of dirty goods falls, because

the price at which it sells its exports drops. Such a country has a strategic incentive to set the domestic price of emissions somewhat above marginal damage in order to reduce the world supply of dirty goods and thereby get a better price for exports.

The strategic manipulation of pollution policy to affect world prices has played an important role in the policy literature. In such cases, the government uses pollution policy both to target pollution and as an instrument of international trade policy. This raises a number of complicated issues, such as why a government with market power doesn't use some other instrument (such as a trade barrier or other more direct taxes and subsidies) to manipulate world prices. In fact, if a government is unrestricted in its choice of policy instruments, the first best policy is to use trade policy to exploit its global market power and use environmental policy solely to internalize externalities, so we again obtain (1.64).<sup>24</sup> As well, if pollution policy is set at the regional or local level, then even in a large country, the individual regulator may not perceive any market power. At this point we do not want to focus on these and other issues of strategic trade policy, and so we will simply assume that countries (or regulators) are small in world markets.

With our small country assumption then, the government's optimal pollution policy is given by (1.64). At this point it is instructive to exploit some of the structure (homotheticity and separability) we have put on preferences. Referring to (1.35), we can write (1.64) as:

$$\mathbf{t} = N \bullet [-V_z / V_I] = N \bullet \frac{\mathbf{b}(p)h'(z)}{v'(R)} = N \bullet MD(p, R, z) \quad (1.65)$$

where recall that  $R = I/\beta(p)$  denotes real income. The assumption of homotheticity in goods consumption allows us to write marginal damage as a function of real income, goods prices and emissions:  $MD = MD(p,R,z)$ .

Next, we can use the national income function to substitute for real income and rewrite (1.65) as:

$$t = N \bullet MD\left[p, \frac{G(p, K, L, z)}{N\mathbf{b}(p)}, z\right] \quad (1.66)$$

We can think of (1.66) as the government's general equilibrium supply curve for pollution. It reflects the country's willingness to allow pollution. The pollution supply curve is upward sloping:

$$\frac{\partial MD}{\partial z} = t \left[ \frac{h''}{h'} - t \frac{v''}{v' N\mathbf{b}} \right] > 0 \quad (1.67)$$

where we have used  $G_z = \tau$ . The sign of (1.67) follows in our case from the convexity of  $h$  and concavity of  $u$ , but more generally marginal damage slopes upwards because increases in pollution tend to make environmental quality scarce relative to consumption. Hence a diminishing marginal rate of substitution between consumption and environmental quality yields the result.

The pollution supply curve also shifts with changes in prices or real income. Consider an increase in real income, holding prices constant. From (1.65) we obtain:

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<sup>24</sup> However, if a country signs a free trade agreement, it is restricted in its use of trade policy. The government is then forced to look for second best trade policy instruments. The question then is whether environmental policy is an attractive candidate for such a role.

$$MD_R = -bh'(z) / v'(R). \quad (1.68)$$

If  $v$  is concave, then  $MD_R$  is positive. Marginal damage is increasing in real income because environmental quality is a normal good. If  $v$  is linear, then real income gains have no effect on marginal damage.

It is worth considering a simple example to illustrate how pollution policy depends on income. Assume utility takes the following form:

$$V(p, I, z) = \ln\left(\frac{I}{b(p)}\right) - gz.$$

Then (1.65) becomes:

$$t = b(p)R = gI.$$

That is, the efficient price of emissions is directly proportional to aggregate income in this example.

Finally, consider a change in relative prices, holding real income constant. This is a pure substitution effect. From (1.65) it is easy to obtain that marginal damage shifts up with an increase in  $p$ . Recall that  $\beta(p)$  is rising in  $p$ . As  $p$  rises, goods get more expensive relative to environmental quality. At the margin, environmental quality is now more highly valued and the willingness of citizens to supply pollution falls.

### 1.6.3 Market Equilibrium

The equilibrium level of pollution is determined by the interaction between the pollution demand curve and society's willingness to tolerate pollution as captured by supply:

$$G_z(p, K, L, z) = N \bullet MD(p, \frac{G(p, K, L, z)}{N \mathbf{b}(p)}, z) . \quad (1.69)$$

This is illustrated in Figure 2.11. The efficient level of pollution  $z_0$  is determined by the intersection of the pollution supply and pollution demand curves. To implement this efficient level of pollution the government can employ either a pollution tax  $\tau_0$  or the issue  $z_0$  marketable permits which would yield an equilibrium permit price  $\tau_0$ . Any equilibrium that can be implemented with a tax can also be implemented with a permit system.

This figure also makes clear the link between our general equilibrium model and the standard treatment of optimal emissions in partial equilibrium models in environmental economics textbooks. As we noted earlier, pollution demand can be interpreted as the general equilibrium marginal abatement costs, and pollution "supply" is simply marginal damage. That is, optimal pollution emissions are determined by equating marginal abatement costs to marginal damage. The difference is that in our framework, the choice of emissions  $z$  also simultaneously determines national income  $G(p, K, L, z)$  and all factor prices, such as wages and the return to capital are fully endogenous. Moreover, our structure allows for an examination of how shocks to the economy as a whole, such as capital accumulation or trade liberalization, will affect pollution via their effects on marginal abatement costs or marginal damage.

## 1.7 Conclusion

This paper has set out a general equilibrium pollution and trade model to provide a framework for future analysis of the trade and environment debate. We have drawn quite heavily from trade theory, but in the end have developed a simple pollution demand and supply system featuring marginal abatement cost and marginal damage schedules. These constructs should be familiar to environmental economists. We have intentionally kept the model quite simple, as this should allow others to extend our analysis to consider the environmental consequences of growth, the impact of trade liberalization, and strategic interaction across countries.<sup>25</sup>

The framework also provides, as special cases, canonical models capturing both the *Pollution Haven Hypothesis* and the *Factor Endowment Hypothesis*. And as demonstrated in Antweiler et al. (2001), it yields a parsimonious reduced form suitable for empirical estimation and hypothesis testing.

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<sup>25</sup> We have, of course, imposed some assumptions that limit our analysis in some directions. In particular, two very important issues that we have not mentioned are the role of production externalities, and the presence of international spillovers in pollution. For an examination of these issues see Copeland and Taylor (1995), Brander and Taylor (1997), and Copeland and Taylor (1999).

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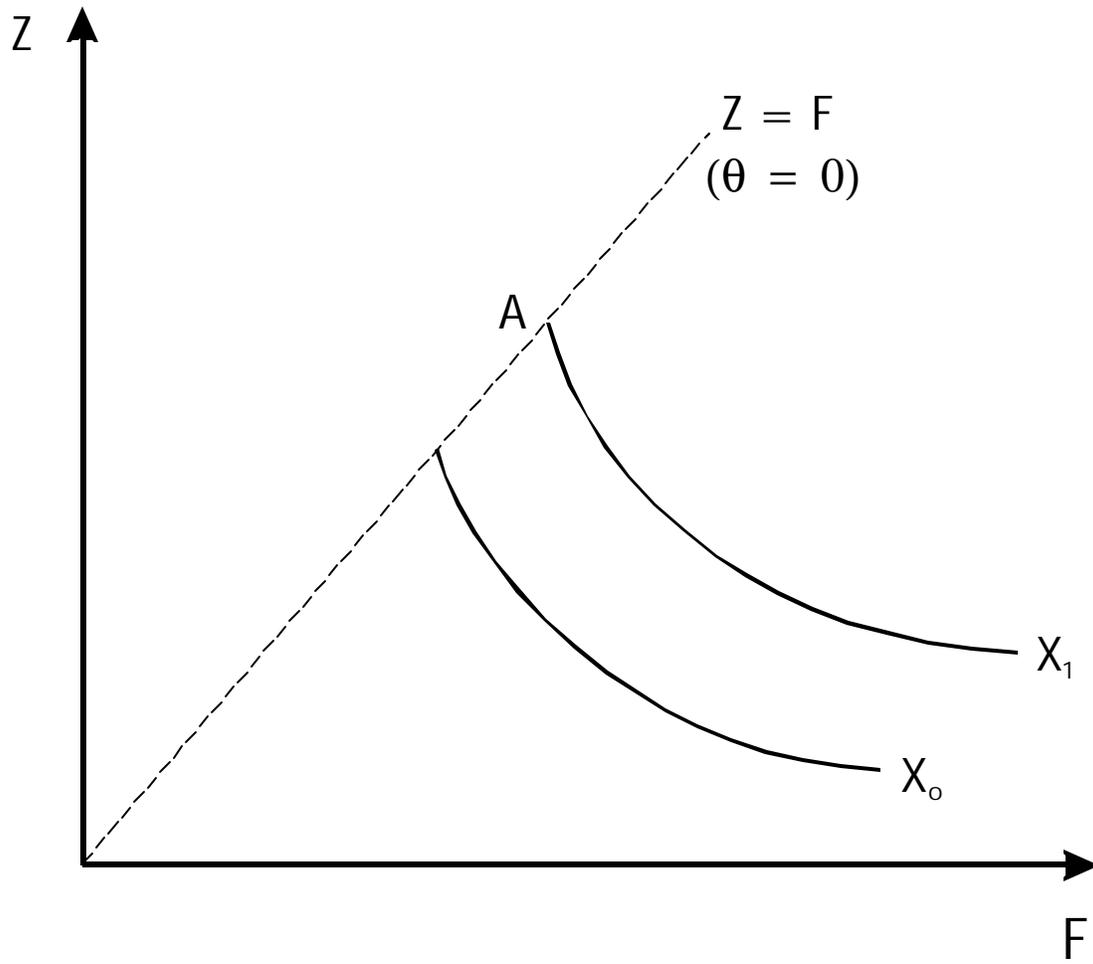


Figure 2.1 Isoquants for the X industry

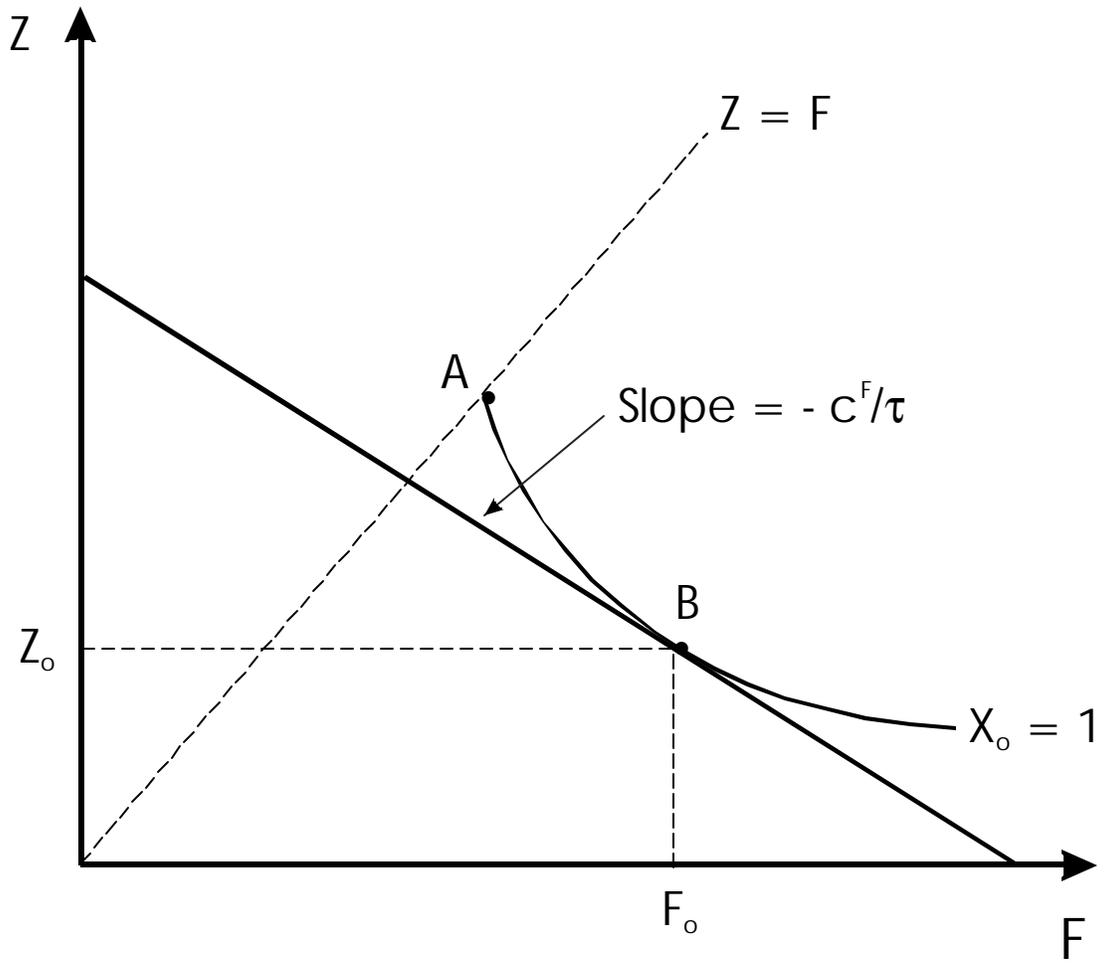


Figure 2.2 Cost minimization in the X industry

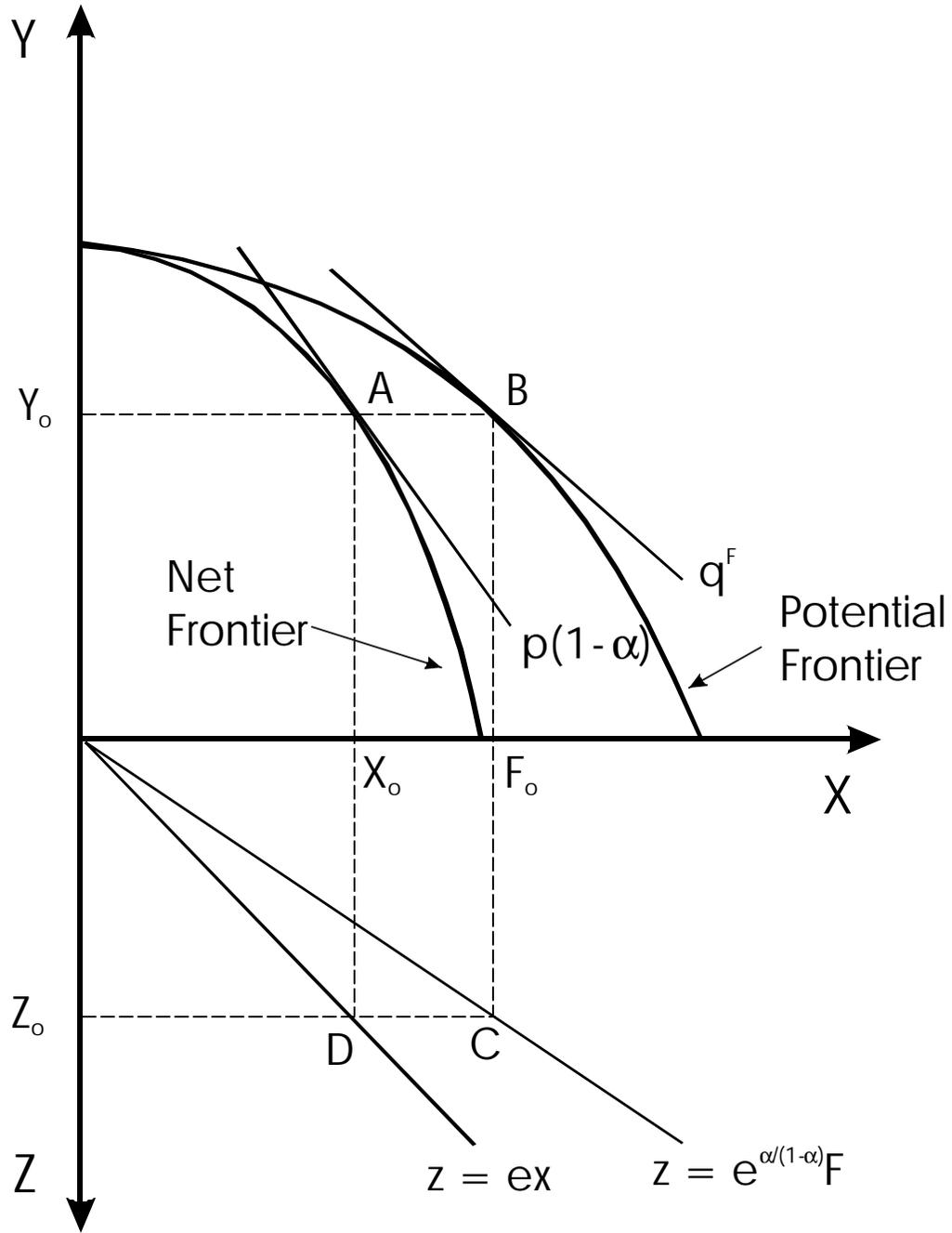


Figure 2. 3 Potential and Net production frontiers

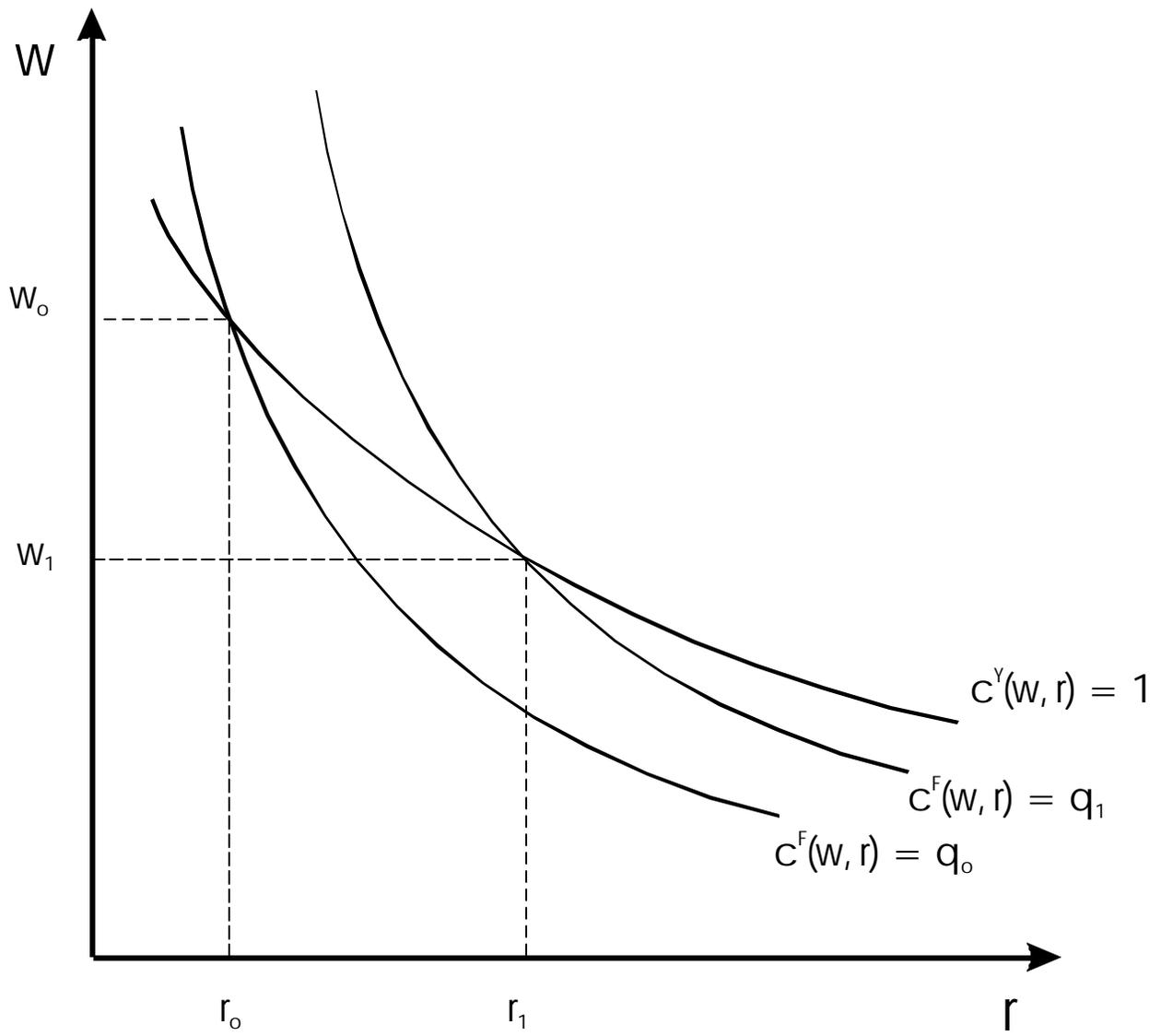


Figure 2. 4 Producer prices and factor prices

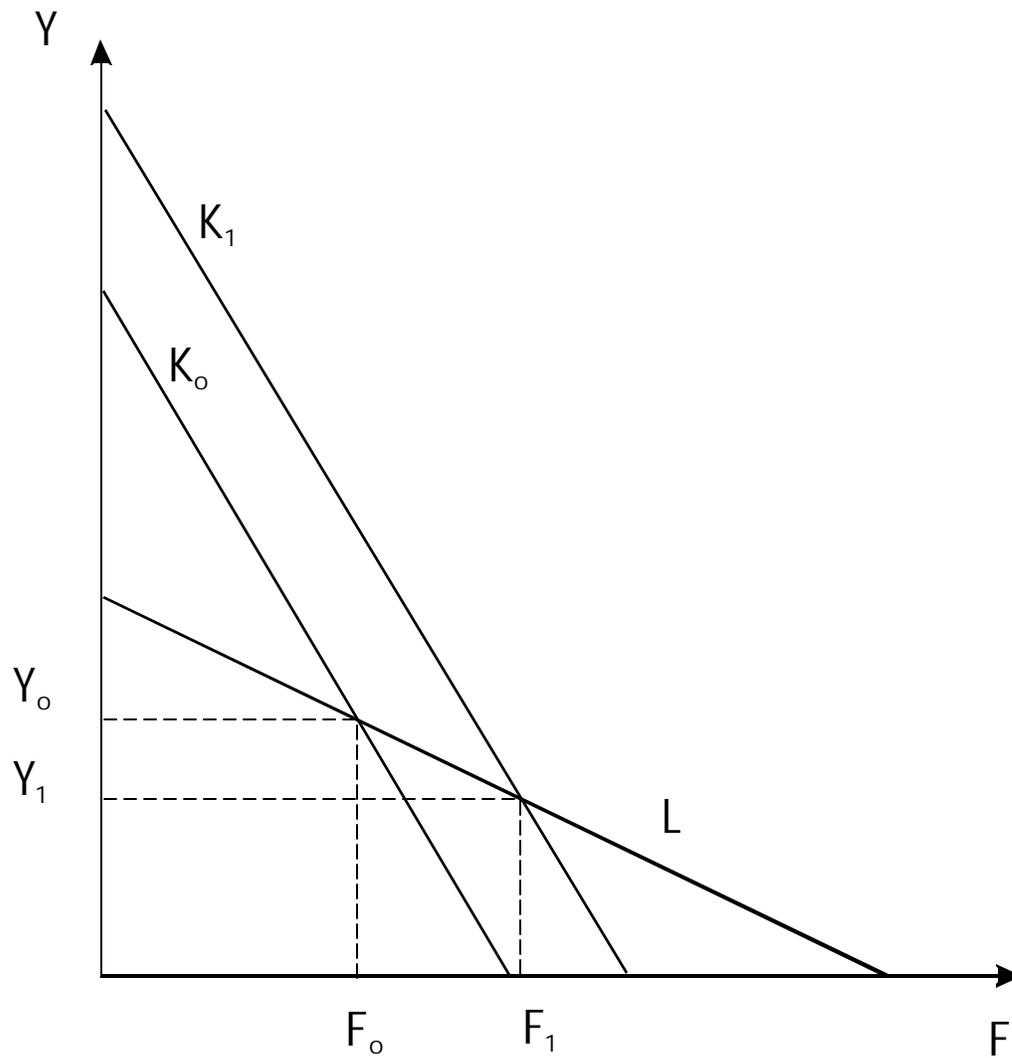


Figure 2.5 Effect of endowment changes on outputs

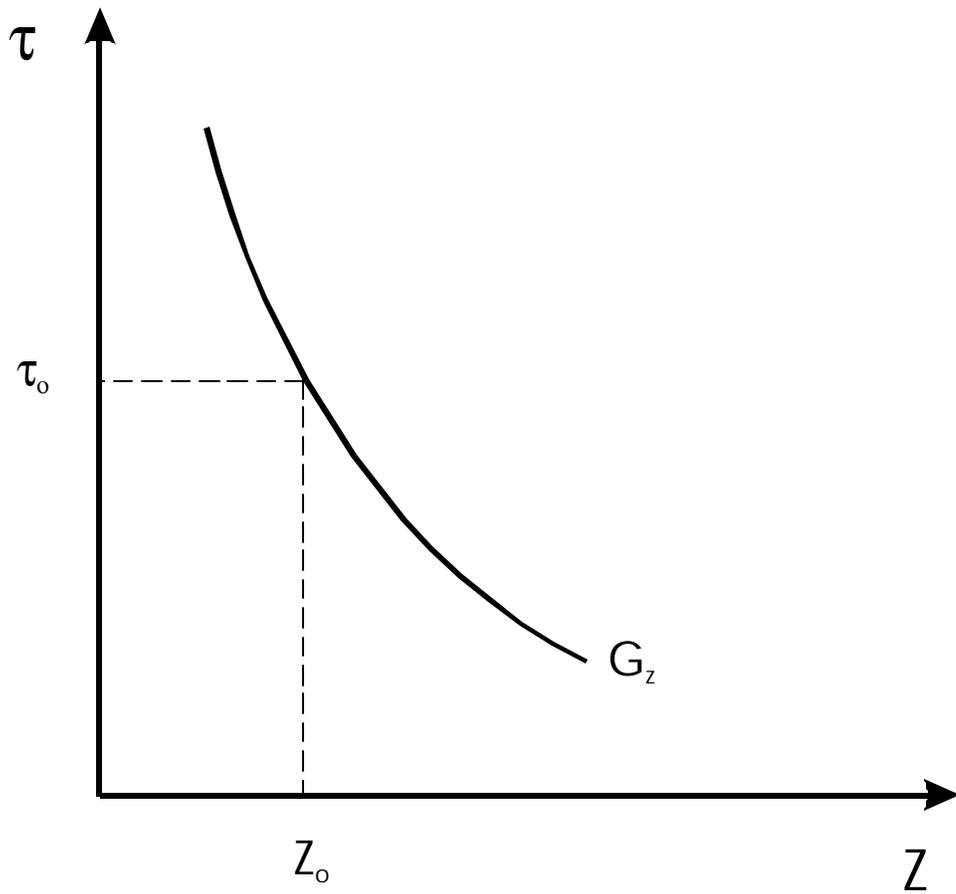


Figure 2.6 Pollution demand

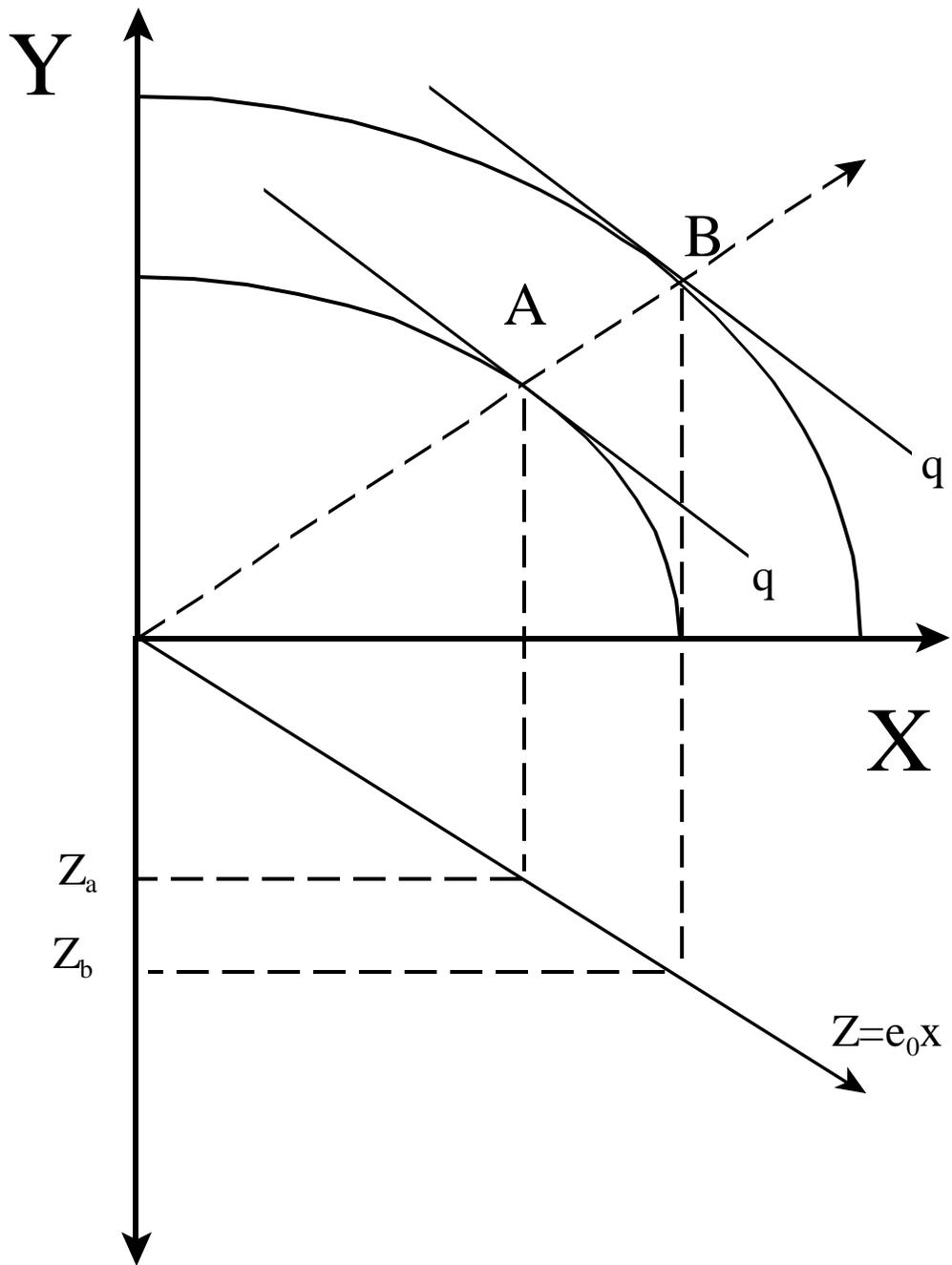


Figure 2.7 The Scale Effect

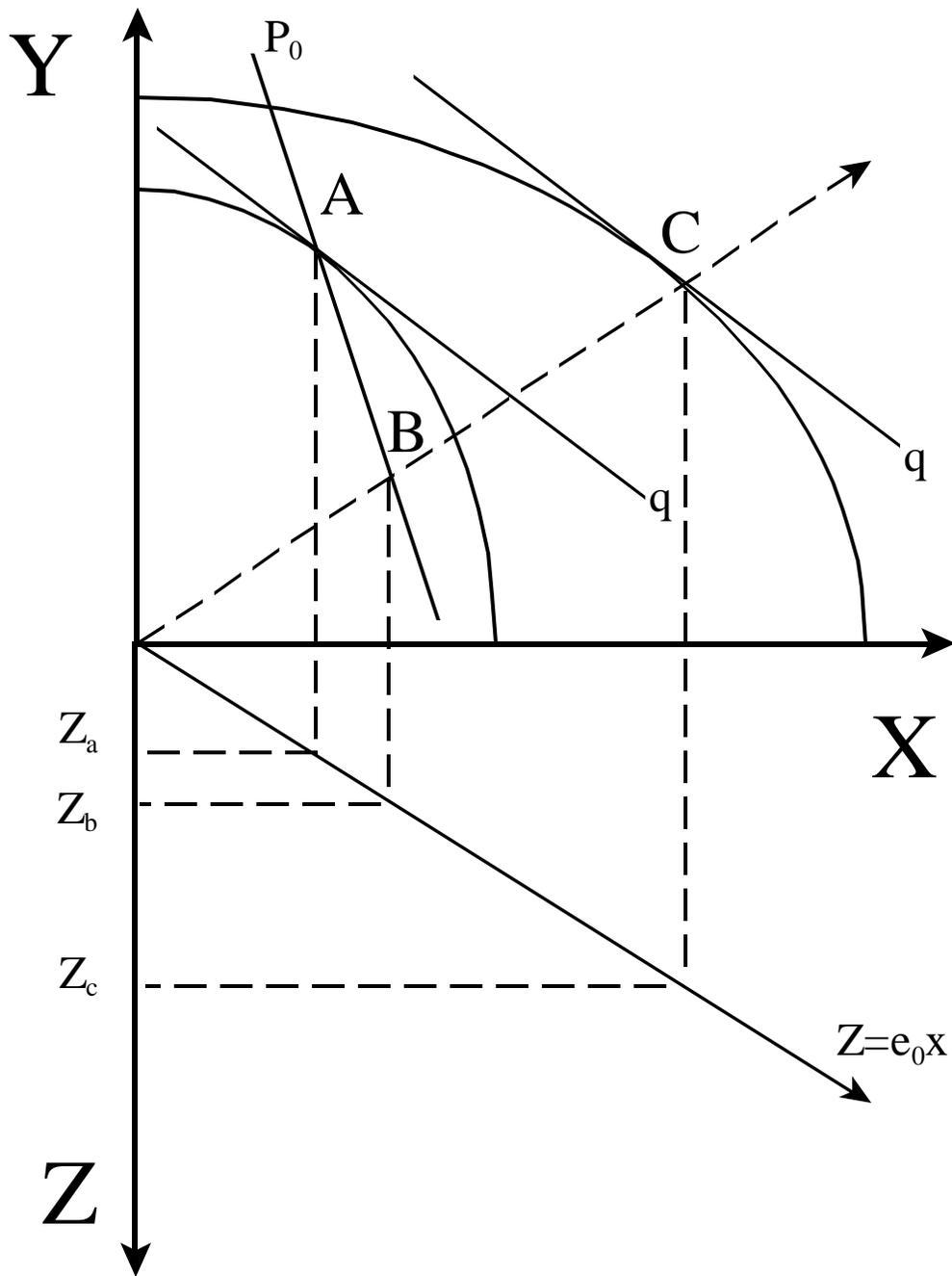


Figure 2.8 The Composition Effect

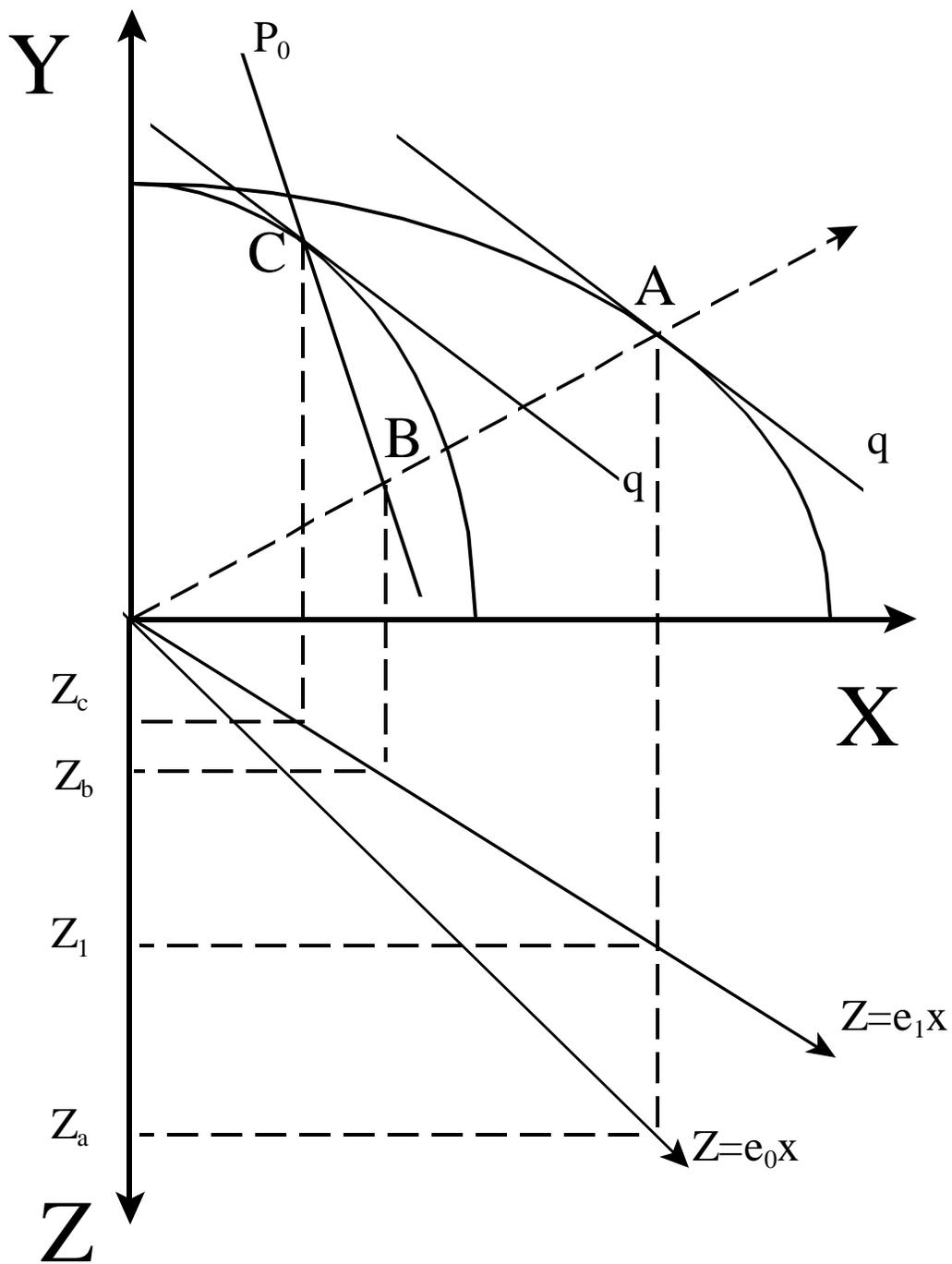


Figure 2.9 The Technique Effect

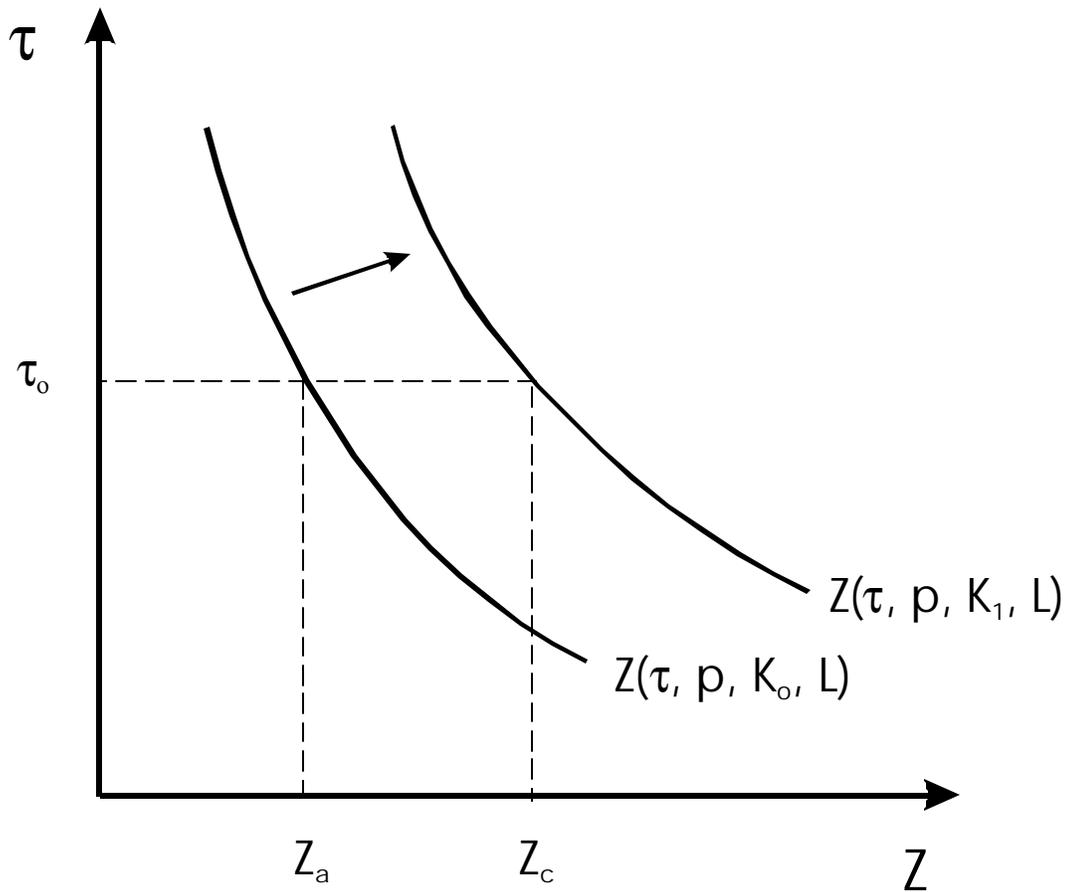


Figure 2.10 Capital accumulation

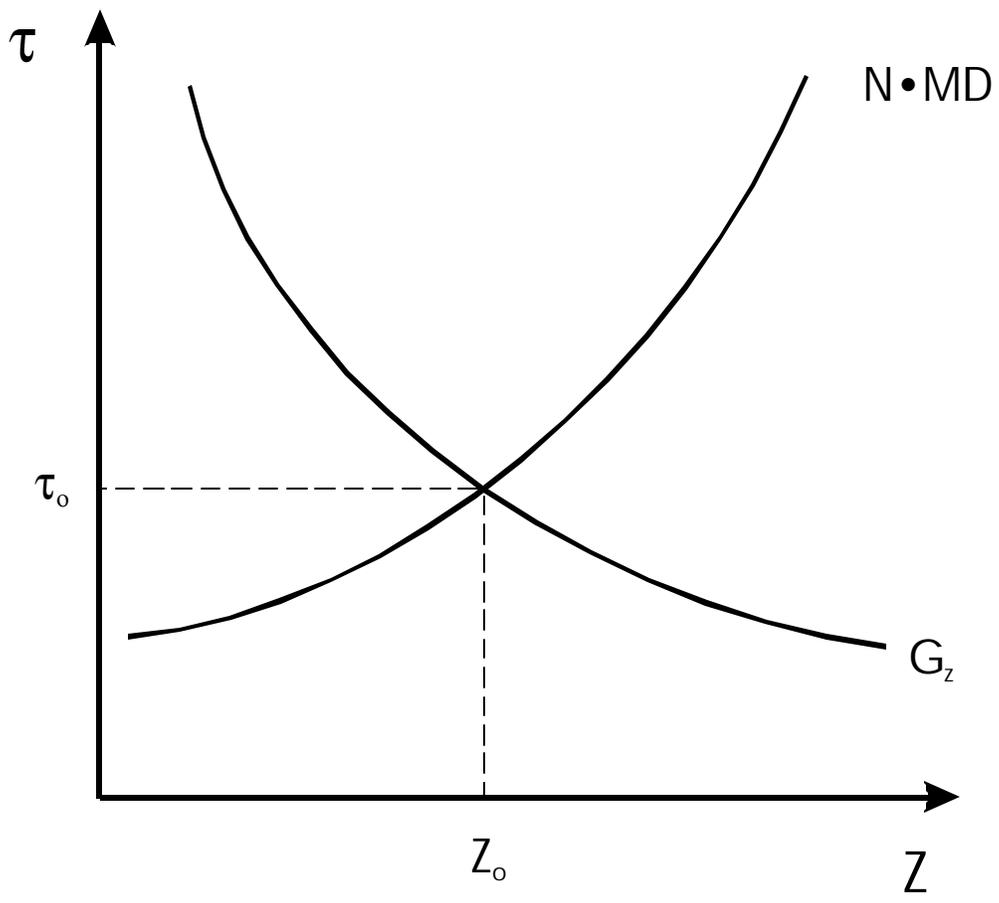


Figure 2.11 Optimal Pollution