

NBER WORKING PAPER SERIES

BUILDING THE IPO ORDER BOOK:
UNDERPRICING AND PARTICIPATION
LIMITS WITH COSTLY INFORMATION

Ann E. Sherman
Sheridan Titman

Working Paper 7786
<http://www.nber.org/papers/w7786>

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
July 2000

We wish to thank Bill Wilhelm, Pegaret Pichler, Jay Ritter, Larry Benveniste, an anonymous referee and seminar participants at the University of Minnesota and at the American Finance Association 2000 Annual Meeting for helpful comments. Much of this work was done while one or both authors were at Hong Kong University of Science and Technology, the support of which is gratefully acknowledged. The views expressed herein are those of the authors and not necessarily those of the National Bureau of Economic Research.

© 2000 by Ann E. Sherman and Sheridan Titman. All rights reserved. Short sections of text, not to exceed two paragraphs, may be quoted without explicit permission provided that full credit, including © notice, is given to the source.

Building the IPO Order Book: Underpricing and Participation Limits
With Costly Information

Ann E. Sherman and Sheridan Titman

NBER Working Paper No. 7786

July 2000

JEL No. G2

ABSTRACT

This paper examines the book building mechanism for marketing initial public offerings. We present a model where the underwriter selects a group of investors along with a pricing and allocation mechanism in a way that maximizes the information generated during the process of going public at a minimum cost. Unlike previous models, we take into account the moral hazard problem that is faced by investors when evaluation is costly. Our results suggest that for firms with the most to gain from accurate pricing, the number of investors participating in the offering is larger, and underpricing will be greater. When the demand for accuracy is relatively low, the expected amount of underpricing exactly offsets the investors' costs of acquiring information. However, when the demand for accuracy is high, the expected amount of underpricing can exceed the cost of information and investors can earn rents.

Ann E. Sherman
Department of Finance
University of Minnesota
321 19th Avenue South, Rm. 3-122
Minneapolis, MN 55455

Sheridan Titman
Department of Finance
University of Texas
Austin, TX 78712
and NBER
titman@mail.utexas.edu

1. Introduction

Initial public offerings (IPOs) are marketed to investors in a number of different ways. An underwriter can offer the shares to the market after collecting only limited information, following certain "fairness" rules that give everyone an opportunity to participate in the offering.¹ This method had been used historically in most IPOs outside of the U.S. and is examined in articles by Rock (1986), Beatty and Ritter (1986), Chowdhry and Sherman (1996a and 1996b), Brennan and Franks (1997) and Benveniste and Busaba (1997). In addition, various auction mechanisms have been used to sell IPOs. While these mechanisms are widely used, the largest IPOs in the U.S. and increasingly around the world are marketed in what is known as a book building process, which we investigate in this paper.²

The book building process, which was first examined in the academic literature by Benveniste and Spindt (1989) and Benveniste and Wilhelm (1990), can be described as consisting of the following three steps:³ The investment bank first decides which investors will be invited to evaluate and perhaps buy the issue. Second, investors evaluate the issue and provide the investment bank with preliminary indications of their demand for the issue. Third, the investment bank prices the issue and allocates shares to investors, generally allocating more shares to investors who indicate higher levels of demand.

The book building method is somewhat controversial because, in contrast to the other methods, underwriters generally exclude investors from the bidding process. This practice of

¹ Anyone is allowed to apply for shares in the open offer (also known as the fixed price, public offer or universal offer) method, and orders are sometimes allocated pro rata. More often, small orders are favored over large orders. Chowdhry and Sherman (1996b) show that favoring small investors reduces the winner's curse faced by the uninformed, thus reducing the amount of underpricing needed.

² Loughran, Ritter and Rydqvist (1994) were the first to examine international patterns in the usage of various methods. More recently, Sherman (2000b) documents the increased use of book building, the continuing popularity of open offer (primarily in smaller markets) and the relative rarity of auctions. Ljungqvist, Jenkinson and Wilhelm (2000) also document the increased use of book building. Book building and open offer are frequently combined in hybrid offerings where book building is used to set the price and to allocate shares to institutional investors, while open offer is used to allocate shares to retail investors. Another recent development is the sale of shares in three U.S. IPOs through Vickrey auctions (i.e., sealed bid, uniform price auctions, sometimes mistakenly referred to as Dutch auctions).

³ The book building method has recently attracted a substantial amount of attention in the academic literature. Recent papers include Maksimovic and Pichler (1997), Stoughton and Zechner (1998), Biais and Faugeron Crouzet (1999), Mello and Parsons (1998), Chen and Ritter (1999), Cornelli and Goldreich (1999), and Derrien and Womack (1999) and Sherman (2000a).

excluding investors is especially controversial because the shares are underpriced on average, giving at least the appearance that the book building process provides a convenient mechanism for underwriters to reward their favorite clients. The Benveniste and Spindt and Benveniste and Wilhelm articles suggest that there is an economic motivation for underpricing IPOs; specifically, investment banks need to underprice IPOs to effectively extract information from their investors. However, these papers do not explain why IPOs are marketed only to an exclusive group of investors. Indeed, in these models, exclusivity is costly since the investment bank can lower the level of underpricing by marketing the IPO to a larger group of informed investors.

Our model of the book building process, which extends Benveniste and Spindt and Benveniste and Wilhelm, provides an explanation for why investment banks limit the number of investors that are invited to participate in an IPO. The model is based on two premises: The first is that collecting information is costly, which is a fairly standard and intuitive assumption. The second is that there are economic benefits associated with generating as much information as possible, so that the IPOs are priced accurately in the secondary market. This second assumption is consistent with the observation that investment banks pay top salaries for star analysts and do other things outside of the book building process that help them accurately price IPOs.

The benefits of pricing accuracy can be justified in a number of ways. First, as discussed in Sherman (1992), Benveniste, Busaba, and Wilhelm (1997), Subrahmanyam and Titman (1999) and Maksimovic and Pichler (1997), there may be feedback from market demand to investment choices. Of course, the entrepreneurs in these models prefer having their firms' shares sell for the highest possible price. However, the cost associated with having undervalued stock (e.g., being forced to abandon positive NPV projects) is greater than the benefits associated with having overvalued stock. This implies that, *ex ante*, the entrepreneur would like a mechanism for taking his firm public that prices the firm as accurately as possible.

The better investment choices that arise as a result of more accurate pricing provide just one of many motivations for why investment banks value accurate pricing. In addition, Titman and Trueman (1986) suggest that investment banks will attract higher quality clients if they maintain a reputation for accurate pricing. Their model suggests that the high quality firms will pay a premium for a more accurate underwriter who can better certify that the firm is indeed high

quality. Understandably, the lower quality firm is not willing to pay a premium for a more accurate assessment of their lower value. Accurate pricing of the initial offering may also be valued for liquidity reasons. When there is a substantial amount of private information that is not incorporated into a stock's initial price, the stock is likely to be less liquid, which could adversely affect its value. Finally, it should be noted that investment banks could be sued if they substantially misprice a new issue.

As Benveniste and Spindt and others have emphasized, the investment banks pricing policy affects the incentives of investors to truthfully reveal their private information. In particular, if investment bankers do not sufficiently underprice those new issues with the most favorable information, investors will tend to under-report their information (i.e. they will downplay the value of the issue in an attempt to lower the issue price). When information is costly for investors to obtain, there is a second rationale for underpricing new issues. Specifically, the good new issues (i.e., those with the most favorable private information) must be underpriced sufficiently to induce investors to collect information. As we show, in equilibrium, the information collection constraint, rather than the truth-telling constraint, is the binding constraint that determines pricing policy. If the pricing policy provides sufficient incentive for investors to collect information, they will also have sufficient incentive to reveal the information.

To understand this, note that the investment bank in our model collects more information if more investors participate, and that the only cost of attracting more participants is that the issue may need to be underpriced more. What this means, is that if the truth-telling constraint binds, that additional investors can be induced to participate without requiring additional underpricing. As we will show, this implies that, in equilibrium, the investment bank will always choose an investor pool large enough that the information purchase constraint is binding.

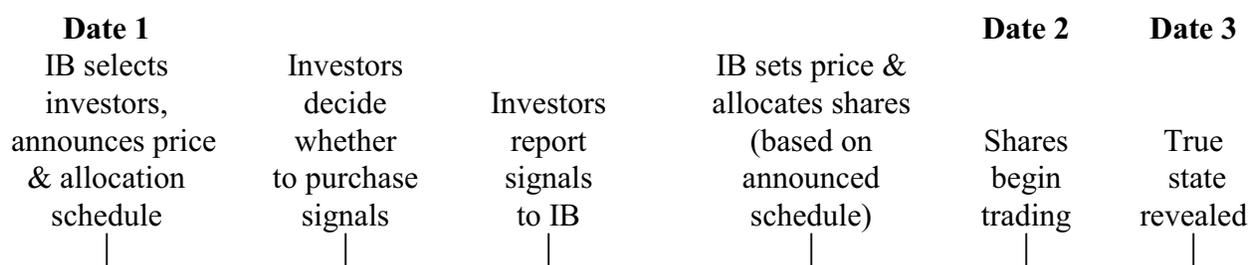
We also show that when there is little need for accurate pricing, few investors will be invited to participate in the offering, little underpricing will be needed, and the expected gain from underpricing will exactly offset the investors' cost of information. When there is a strong need for accurate pricing, more informed investors will be invited to participate in the offering. If enough investors are chosen, the informed investors will earn rents, i.e., the average underpricing will exceed their information costs. In addition, we show that exogenous limits on investor participation may lead to uninformed investors earning excess returns.

The rest of the paper is organized as follows. Section 2 describes the economic setting. Section 3 discusses the various incentive constraints needed to induce investors to purchase and accurately report information, and describes the underwriter's optimization problem. Section 4 describes the equilibrium, which is illustrated by a simple numerical example in section 5. Comparative statics are presented in section 6, and section 7 examines the effect of limits on investor participation. Section 8 discusses the circumstances under which investors earn abnormal returns, and section 9 presents our conclusions.

2. The Model

Our model assumes that there are no conflicts of interest between the underwriter, who will be pricing and marketing the issue, and the entrepreneur, who has chosen to take his firm public. Both prefer a high issue price but also value price accuracy for the reasons discussed in the introduction.

The model consists of three dates. In the first date, the investment bank (IB) announces the price and allocation schedule and selects the investors who will participate in the offering. The investors decide whether to purchase a signal and choose whether or not to reveal the signal to the IB. The IB then sets the offering price, based on the information provided by investors. At date 2 the shares begin trading, and at date 3 the state of the economy is revealed and the firm is liquidated. Below is a summary of the timing in this model.



2.1 The issuer

The issuer requires a fixed amount of capital to initiate an investment project, and it plans to sell a fixed number of shares, N . Holding N fixed allows us to avoid accounting for the different levels of dilution that would occur if the number of shares sold varied. If the

entrepreneur raises more than is needed to fund the investment (because the price was higher than anticipated), the excess will be paid to the original shareholders as a dividend.

We assume that the market price at date two, the initial trading date, reflects the information gathered by the investment bank (through the price and allocations chosen), and that the true state, good (g), bad (b) or uncertain (u), is revealed and incorporated in market prices in period three. On date three, the value per share given state j , $j \in \{g, b, u\}$, is s^j . The value of the additional information is such that $s^g = s^u + \alpha$ and $s^b = s^u - \alpha$, where $0 < \alpha \leq s^u$.

2.2 The investors and their signals

To distribute and price the issue, the underwriter enlists the help of H risk neutral investors, where H is chosen by the underwriter. These investors have access to capital as well as information. As a departure from the existing literature, we assume that the investors must purchase their information, paying c dollars for a signal that may be good, bad or uninformative (neutral). Good or bad signals should be interpreted as favorable or unfavorable information that the underwriter did not detect on his own. To simplify our analysis we will assume that there may exist either favorable or unfavorable undetected information, but not both. This means that all investors who receive an informative signal receive the same signal.⁵ However, it is possible that some investors receive an informative signal while others receive an uninformative or neutral signal. This information environment is consistent with Benveniste and Wilhelm (1990).

Investors have an alternative investment with an expected return of r , which to simplify notation is set equal to 0. In another departure from the existing literature, our model does not

⁵ One way to interpret the signal is as fundamental, verifiable information that has been overlooked by the investment bank during its due diligence evaluation. Another interpretation is that the investors are providing feedback on current market demand. However, demand for an issue depends on more than pure liquidity - fund managers do not buy a stock simply because they have some spare cash. Demand depends on the value that investors expect the market to place on the stock, now and in the future.

Therefore, a third interpretation of the information gathered here is that it has to do not with the mere verification of objective facts but with how to interpret those facts and to forecast the future of the firm. Valuation is a complex process, involving evaluating the capabilities of the management and forecasting the future of the firm, the industry and the economy. Experienced fund managers who have read the prospectus, heard the sales presentation and questioned the management of the company may in the end know "more" than the investment bankers about the market value of the issue.

Last, one should note that the issue price and number of shares sold depends on the feedback from investors during book building. IPOs have even been canceled as a result of the response to a road show. Therefore, the underwriters involved obviously feel that they are getting some sort of information from the process, however one wants to interpret that information.

impose binding wealth constraints on the investors.⁶ As long as the return is sufficient, investors will purchase as many shares as the underwriter allocates to them. In section 7 we consider the effects of adding a wealth constraint.

2.3 Allocations, prices and probabilities

In addition to selecting the number of investors, the underwriter prices the issue and decides how many shares to allocate to each investor. The allocation to each investor may depend on both the signal reported by that particular investor and the signals reported by other investors (i.e. on h , the number of investors out of H that reported either g or b). The following notation is for all $j \in \{g, b, u\}$ and all $k \in \{g, b\}$:

- s_j = average issue price when state j is reported (recall that s^j is the share value in state j);
- $q_{j,k,h}$ = allocation to an investor who reports j when k is reported by h of H investors;
- $q_{j,k}$ = expected allocation to an investor who reports j when at least one investor reports k ;
- $q_{k,k}^*$ = expected allocation to an investor who reports k , when at least one *other* investor *also* reports k (this average excludes either $q_{g,g,1}$ or $q_{b,b,1}$);
- q_u = allocation per investor when all investors report u .

Last, we must specify the probabilities. Let π be the probability that there exists relevant information that was overlooked by the underwriter. We assume that the information that was overlooked is equally likely to be good or bad. With probability $(1-\pi)$ there is no additional information for the investors to discover.

Let π_i be the conditional probability that investor i receives an informative signal, either good or bad (conditional on the existence of overlooked information). The unconditional probability that investor i will receive a good (or bad) signal is $\pi_i\pi/2$. The unconditional probability of state g occurring and of h of the H investors receiving the signal g is $P(g,h)$ ⁷. Since the probabilities are symmetric, $P(b,h) = P(g,h)$. The unconditional probability that none of the H investors receive informative signals is $P(\cdot,0) = P(g,0) + P(b,0) + (1 - \pi)$.

The conditional probability that h of $H-1$ other investors will receive a good signal, given that one investor has received a good signal is $P'(g,h)$, and again $P'(b,h) = P'(g,h)$. Finally, given

⁶ The institutional investors in IPOs manage large amounts of money, relative to the size of the IPOs. Thus liquidity constraints are unlikely to be the primary determinant of their allocations.

⁷ The formulas for $P(g,h)$ and $P'(g,h)$ are given in Benveniste and Wilhelm(1990) and Sherman(2000a).

that one investor fails to receive an informative signal, the conditional probability that no other investor will receive an informative signal is

$$P'(u,0) = \frac{1}{1 - \pi\pi_i} \left[1 - \pi + \frac{\pi^2}{2} (1 - \pi_i) (P'(g,0) + P'(b,0)) \right]$$

3. Incentive Constraints and the Underwriter's Objective Function

There are two main types of incentive constraints for investors in this model. The investment bank needs to give investors the incentive both to buy the information and to report it accurately. Section 3.1 examines the information reporting or truth-telling constraints, while section 3.2 examines the information collection constraints. Section 3.3 gives the investment bank's maximization problem.

3.1 Information reporting constraints

As part of its book building strategy the underwriter must design an allocation and pricing schedule that elicits accurate information from investors. Since the investment bank will use the reported information to price the issue, the pricing and allocation strategy must counteract investor incentives to withhold favorable information that will lead to a higher issue price. We will be considering Nash equilibria where, conditioned on the underwriter's strategy, investors have an incentive to truthfully reveal their information, given their expectation that other investors will also report information accurately.

There will typically be multiple solutions to the underwriter's problem in this model, with many sets of allocations and prices that elicit truthful revelation and give the underwriter the same expected utility. From both the investors' and our point of view, many of these multiple equilibria are effectively the same. To eliminate extraneous equilibria, we make the following assumptions:

1. The underwriter allocates zero shares to all investors who reveal conflicting signals. For example, if one investor reports g and another investor reports b , the underwriter knows that someone is lying, although it cannot tell who, so it allocates zero shares to both. Although these conflicting reports will not exist within the equilibrium, this out of equilibrium assumption is needed to specify the equilibrium.
2. Investors who provide the same signals receive the same allocations.

3. The price per share in states g and b do not depend on h, the number of investors who report an informative signal.

Because investors are risk neutral, the issuer has no incentive to violate assumptions 2 or 3. Varying either allocations or prices based on h would not change the expected returns that investors require; it would simply increase the variance of investor returns. An exception to this is when there are binding participation limits on investors. As we show in section 7, the underwriter can lessen the effects of certain exogenous investment limits by making prices a function of h.⁸

Let $R(j,k)$ be the expected profit to an investor who receives signal j and reports signal k. In equilibrium, investors are induced to report their information truthfully, which implies that the following truth-telling constraints (described in more detail in Appendix A) must be satisfied:

$$R(j,j) \geq R(j,k) \text{ for all } j, k \in \{g, b, u\} \quad (1)$$

It should be noted that the cost of acquiring information does not affect the information reporting conditions, since it is a sunk cost by the time the investor decides what signal to report. On the other hand, whether or not the investor plans to accurately report her information certainly affects her incentive to buy a signal. After all, if the investor planned to report u (or g or b) regardless of the actual signal, then there would be no reason to buy a signal.

3.2 Information collection constraints

In addition to the truth-telling conditions, a constraint is needed to guarantee that investors choose to acquire information. The condition is that buying and reporting a signal offers at least as high an expected profit as not purchasing a signal and falsely reporting u:

$$(\pi_i \pi / 2) \{R(g,g) + R(b,b)\} + (1 - \pi_i \pi) R(u,u) \geq R(\cdot, u) + c .$$

where $R(\cdot, u)$ is the expected return to an investor that reports u without observing a signal. This expression can be re-written as

⁸ Similarly, because investors are risk neutral and because they make their decisions without knowing h, we can look at the average allocations in each situation without solving explicitly for the price and allocation for each investor given every possible h. Additional information on how the underwriter will set individual allocations is available on request from the authors.

$$\frac{\pi}{2} \{ (s^g - s_g) [\pi_i q_{g,g} - (1 - \pi (1 - \pi_i)) (1 - P'(g,0)) q_{u,g}] + (s^b - s_b) [\pi_i q_{b,b} - (1 - \pi (1 - \pi_i)) (1 - P'(g,0)) q_{u,b}] - (1 - \pi (1 - \pi_i)) (P'(g,0) + P'(b,0)) (s^u - s_u) q_u \} \geq c \quad (2)$$

In addition, buying a signal and reporting it must be at least as good as saving the cost c and falsely reporting either g or b (or not participating at all). As long as the truth-telling constraints are satisfied, however, the return to falsely reporting g or b (or to not participating) will never be higher than the return to falsely reporting u .

3.3 The Underwriter's Objective

The underwriter has nine main choice variables - the number of investors to include, three prices (one for each state) and five allocations, including one each for investors who report g or b and three for investors who report u (one for each state). These variables are chosen to maximize a utility function that is a separable function of the accuracy of the initial aftermarket price and the expected amount that the shares must be underpriced. In particular, the underwriter chooses H , the number of investors invited to participate in the offering, by trading off the increase in accuracy associated with a larger group of investors against a corresponding increase in the expected amount of underpricing. To illustrate this tradeoff, we first solve the underwriter's maximization problem, as given below:

$$\begin{aligned} \text{Max} & \quad \{ ((\pi/2) - P(g,0)) s_g + ((\pi/2) - P(b,0)) s_b \\ & \quad + P(\cdot, 0) s_u - f(P(b,0)) \} \\ \text{Variables} & \quad q_u, q_{b,b}, q_{g,g}, \\ & \quad q_{u,b}, q_{u,g}, H \\ & \quad s_u, s_b, s_g \end{aligned}$$

Subject to equations (1), (2) and the following constraints that guarantee that N shares are sold regardless of the state:

$$\begin{aligned} h q_{j,j,h} + (H-h) q_{u,j,h} &= N, \text{ for all } 0 \leq h \leq H, j \in \{g, b\}; \\ H q_u &= N. \end{aligned}$$

The underwriter is maximizing the expected price per share, minus a term that reflects the possibility of price inaccuracy. The function $f(P(b,0))$ is included in the utility function to

capture the underwriter's desire for greater pricing accuracy. $P(b,0)$ is the probability that a bad (or equivalently a good) state exists that has not been discovered. We assume that $f(P(b,0)) > 0$ for all $P(b,0) \in \{0, \pi/2\}$, $f'(\cdot) > 0$, $f''(\cdot) > 0$, $f(0) = 0$ and $f(\pi/2)$ is sufficiently large, relative to the cost of the information, that the underwriter will always choose to induce at least one investor to purchase a signal.⁹

4. Defining the Equilibrium

The following analysis characterizes a Nash equilibrium in which each investor selected by the underwriter optimally purchases information and truthfully reveals the information to the underwriter. As we will show, this equilibrium requires that the underwriter use specific pricing and allocation rules that are optimal within the context of our model. The general properties of the equilibrium are described in Proposition 1, the proof of which is in Appendix B.

Proposition 1: Given our assumptions, the solution to this model will have the following properties:

- Good (state g) issues will always be underpriced;
- Neutral (state u) or bad (state b) issues will be underpriced only for high levels of H , when the underwriter places a high value on price accuracy;
- For a sufficiently high H , all issues will be underpriced;
- Allocations to informed investors of shares in hot issues will be higher for those investors that express a stronger interest ($q_{g,g} > q_{u,g}$).

5. A Numerical Example

The equilibrium pricing and allocation strategy described in Proposition 1 can be illustrated with a relatively simple numerical example. The example assumes the following values: $\pi = \pi_i = .5$; $c = 1$; $s^u = 1$; $s^g = 1.5$; $s^b = 0.5$; $N = 100$. The underwriter in this example

⁹ If the underwriter and the issuer did not value price accuracy, then the optimal solution would be to allow an unlimited number of investors to participate in the offering. Each investor would be expected to receive such a small number of shares that it would not be cost-effective to evaluate the issue. Therefore the shares would be priced at their expected value minus some arbitrarily small but positive epsilon.

must choose the number of investors participating in the offer and come up with a pricing and allocation schedule that induces the investors to acquire information and reveal it truthfully.¹⁰

5.1 Example with $H = 2$

First consider the case where there are only two investors. Given a good signal, shares are worth 1.5 each. If only one investor reports a good signal, he receives all 100 shares. If both investors report good signals, each gets 50 shares. Given that one investor receives a good signal, the chance that the other investor also receives a good signal is $\pi_i = 50\%$. Thus the expected allocation for an investor who receives and reports a good signal is $0.5 * 100 + 0.5 * 50 = 75$. The unconditional probability that an investor will get a good signal is $\pi_i\pi/2 = .125$. The expected profit per share is $1.5 - s_g$.

To give investors the incentive to purchase signals, the shares must be underpriced sufficiently in state g to compensate for the cost of the signals. Thus, the probability of getting a good signal times the profit per share times the expected number of shares, both conditional on getting a good signal, must be at least as big as 1, the cost of the signal, or $0.125 * (1.5 - s_g) * 75 \geq 1$,¹¹ which simplifies to $s_g \leq 1.393$. If this constraint binds, underpricing in state g will be 7.1%.

However, we also need to confirm that the investor has the incentive to accurately report her information, given the above pricing and allocation rule. The investor will not falsely report b after observing g or u because it would lead to an allocation of zero (or, if others also report b , to an allocation of fairly priced state b shares). She will not report g after observing u or b , or report u after observing b , because such overreporting would lead to overpricing.

Finally, we must verify that the investor will not report u after observing g . If other investors report g , then an investor who falsely reports u will receive zero shares. If none of the other investors report g , then each investor receives $N/H = 50$ shares which are underpriced by 0.5 per share (since they are worth 1.5 but are priced at 1.0). Thus the expected return to falsely

Thus we would see few or no restrictions on investor access to shares and essentially no underpricing.

¹⁰ As we saw in section 3.1, there are many information reporting constraints that must be satisfied. For this example, however, we will only discuss the conditions that may be binding. The reader can easily verify that the other conditions have been satisfied.

¹¹ This is the information collection constraint, (2), which was given in section 3.2. The constraint is greatly simplified because $s^b - s_b = s^u - s_u = q_{u,g} = 0$.

reporting u after observing g is $0.5 * (1.5 - 1.0) * 50 = 12.5$. On the other hand, if the investor truthfully reports g , her expected allocation is 75 shares that are underpriced by $(1.5 - s_g)$ per share. Thus, to induce the investor to accurately report her information, s_g must satisfy $(1.5 - s_g) * 75 \geq 12.5$ ¹², or $s_g \leq 1.333$. Since this is lower than the 1.393 from the information purchase restriction, this constraint binds and g shares are underpriced by 11.1%.

In this case, the expected amount that investors realize from underpricing exceeds the cost of acquiring information, implying that investors earn rents. The underwriter will thus have to restrict participation in the offer, due to excess demand. However, as we show below, the underwriter can both increase price accuracy and reduce the amount of underpricing by including more investors. Therefore $H = 2$ is not an equilibrium solution for this example.

5.2 Example with $H = 3$

We now examine how the presence of a third participant affects the incentives of investors to purchase and reveal information. The method for solving this example is the same as that shown for $H = 2$.¹³ Inducing investors to purchase information when $H = 3$ requires that $s_g \leq 1.363$, while inducing investors to accurately report information requires that $s_g \leq 1.429$. Clearly the information purchase constraint binds, so $s_g = 1.363$ and shares are underpriced by 9.1%.

Having more investors always increases pricing accuracy, since each additional investor generates an additional signal. With only two investors, the probability that an informative state is not discovered is $\pi (1 - \pi_i)^2 = 12.5\%$. With three investors, this probability drops to $\pi (1 - \pi_i)^3 = 6.25\%$. Hence, adding a third investor cuts the risk of not discovering the true state in half.

In this example, price accuracy increases *and* underpricing decreases as we go from two to three investors, showing that $H = 2$ is not optimal. The reason that underpricing decreases when we add a third investor is that, when $H = 2$, the truth telling constraint is binding and this constraint is easier to satisfy when the number of investors increases. However, as the number of investors increases, the information collection constraint, which is more difficult to satisfy with higher H , becomes the binding constraint.¹⁴ The general result, illustrated by the preceding example, is stated in the following proposition and proved in Appendix B:

¹² This is the equivalent of equation (A1) from the information reporting constraints in Appendix A.

¹³ Additional details on solving the examples are available on request from the authors.

¹⁴ For instance, with four rather than three investors, underpricing must increase to 11.4% to satisfy the

Proposition 2: For low H , underpricing may decrease as H increases, whereas, for sufficiently high H , underpricing always increases as H increases. In equilibrium, the underwriter will always select an investor pool large enough that underpricing increases if H is increased further.

5.3 Example with $H = 13$

Proposition 2 is important because past work on book building has focused on the truth-telling constraint against under-reporting hot issues (i.e. against withholding good information). Proposition 2 shows that, when information collection is endogenous and costly (and when the moral hazard problem of investors is recognized), this constraint that has received so much attention will not bind. In contrast, the truth-telling constraints against over-reporting (i.e. against exaggerating the quality of the issue in order to receive a larger allocation) have often been overlooked in the past, but we show that they play a major role in determining which shares are underpriced.

The key over-reporting constraint in this model is (A4), which guarantees that investors who observe u will not falsely report g . When the issuer places a high value on information and thus chooses a sufficiently high H , condition (A4) cannot be satisfied unless: i) $q_{u,g} > 0$; ii) state u shares are underpriced; and/or iii) state b shares are underpriced.

In our example, this occurs at $H = 13$. With only 12 investors, the price of state g shares is 1.02, which means underpricing of 32.0% and a price for g shares just barely above 1.0, the value of u shares. Equation (A4) does not bind at $H = 12$, but it does bind at $H = 13$. With these parameters, the solution at $H = 13$ is to underprice g shares by 32.7%, u shares by 0.7% and b shares by 5.8%.¹⁵

information collection constraint, whereas underpricing of only 2.2% is required to induce truth-telling. With four investors, the probability that they all miss an informative state is 3.125%, only half of what it was with three investors, demonstrating the trade-off between pricing accuracy and underpricing. The investment banker can more accurately price an issue if a larger number of informed investors participate in the offering. However, with more investors the issue must be underpriced more on average. In equilibrium, there is a cost associated with more accurate pricing.

¹⁵ If we allow "fractional" investors, there is an extremely narrow range for H , from 12.026 to 12.029, where underpricing of only u and not b shares is optimal. As one might guess, there is a limit to how high H can go. In this example, $H = 25$ is still feasible (with 100% underpricing of b shares), but at $H =$

When H is sufficiently high, investors will earn excess returns. With $H = 13$, investors earn an expected excess return of 0.0371 in addition to the information cost of 1. For $H \leq 12$ the expected return exactly equals the cost of acquiring information, so that investors are indifferent about whether or not to participate in the offer. Since investors earn excess returns when H is high, the underwriter will have to limit participation in the offering. This leads us to the following proposition, which is proved in Appendix B.

Proposition 3: When the underwriter places a sufficiently high value on information, informed investors will earn economic rents and participation in the IPO will be rationed.

6. Comparative Statics

The example developed in the last section illustrates the importance of costly information in selecting the optimal size of the investor pool, as well as the importance of making the size of the investor pool endogenous. If information is free ($c = 0$), then constraint (2) never binds. Thus the information reporting constraints determine underpricing, and underpricing decreases as H increases. The optimal H would be infinite, meaning that underwriters would not limit the number of investors in an IPO.

In reality, becoming informed is costly and underwriters regularly limit the number of investors that have access to new issues. As we show in the following propositions, which are proved in Appendix B, the number of investors given access to a new issue depends on the cost of collecting information and the value of that information:

Proposition 4: As the cost of buying a signal, c , decreases:

- the size of the investor pool increases;
- underpricing of hot issues (where state g is revealed) decreases.

Proposition 5: As the value of the information (i.e. π , the probability that a good or bad state exists) or the value that the underwriter places on pricing accuracy (i.e., $f'(P(b,0))$), increases:

- the size of the investor pool increases;

26, there is no set of non-negative prices that satisfies all of the constraints. This limit on H is due to a feasibility constraint on how much the underwriter can offer.

- average underpricing increases.

These propositions state that, as information becomes either more valuable or less expensive, the underwriter purchases more of it (H increases). As the size of the investor pool increases, expected underpricing must increase to cover the cost of the extra signal, since expected total underpricing must equal the total cost of the information. However, underpricing of hot (state g) issues actually decreases as information becomes less expensive and it may also decrease as information becomes more valuable.

Proposition 6: As the accuracy of the information (i.e. π_i , the probability that the signal will be informative) increases:

- the size of the investor pool can either increase or decrease;
- underpricing decreases.

The second result, that underpricing decreases, arises because it is easier to convince investors to purchase a signal when that signal becomes more accurate. The first result is ambiguous, since there is both an “income” and a “substitution” effect to an increase in signal accuracy. When information is very accurate (high π_i), the number of participants is small because extra investors are not needed to get a high degree of price accuracy. When the accuracy of each signal is low, on the other hand, more signals are needed to have a reasonable chance of discovering bad (or good) issuers. However, if accuracy is too low, signals will simply not be worth purchasing. At very low accuracy levels, an increase in accuracy may actually lead to an increase in the number of investors, particularly if π and $f'(\cdot)$ are low also (i.e. if the value of information is small and the preference for greater price accuracy increases only at a slow rate).

7. The effects of participation limits

This section considers the effects of limits on investor participation. So far, we have not imposed any fixed limits on the number of shares that an investor may purchase, or on the number of investors participating in the offering. We will now consider two types of limits, either a minimum number of investors or a maximum allocation per investor. More specifically:

- I. There must be at least H^* investors purchasing a minimum of ϵ shares each; or
- II. There is a maximum number of shares that can be allocated per investor, q^{\max} .

The first limit is imposed in some countries to ensure broad participation by investors. For instance, in Australia there must be at least 500 shareholders each holding shares worth at least AUD 2,000. The second limit might exist because of either cash constraints or limits in the mandate of a mutual or pension fund (e.g., some funds require that no more than $x\%$ of their portfolio go into any one stock). Alternatively, the limit may be due to a law requiring investors to report their holdings when they have more than, say, 5% of total shares, or to make a general offer if they hold more than, say, 35%. Finally, note that either of these constraints may arise because of corporate control considerations.

Assuming that the restrictions are binding, the size of the investor pool must increase. As H increases, underpricing will increase also. The underwriter is trying to convince more investors to purchase signals and will have to offer a lower price to get them to do so. Thus we have the following result, which is proved in Appendix B:

Proposition 7: Under either type of restriction on investor participation, both expected underpricing and the size of the investor pool will increase.

In addition to increasing underpricing, these restrictions make it necessary for the underwriter to allocate at least some shares in state g issues to investors that report u . In order to minimize the cost of doing this, the underwriter will shift underpricing to just a few special cases where virtually everyone reports a good signal.¹⁶ Good issues will be fairly priced whenever $q_{u,g,h} > 0$, with underpricing concentrated in the cases where $q_{u,g,h} = 0$. Thus, we would expect IPO initial returns to be highly skewed, with most returns relatively low but with a few hot issues having enormous price jumps.

A skewed distribution pattern of initial returns can result from either type of participation restriction. However, other distribution effects on initial returns will depend on which type of restriction occurs. The first type of limit (a minimum number of investors) is more likely to

¹⁶ In section 3.1, we made some simplifying assumptions to get rid of the uninteresting, extraneous equilibria. In this section we are relaxing assumption 3, that prices do not depend on h , since making s_g a function of h now makes a significant difference in the solution.

affect small issues, implying that underpricing will be greater for small than for large issues. The second type of limit (a maximum allocation per investor) is more likely to affect large issues, implying the opposite size pattern for initial returns. Where the limits occur for reasons of corporate control, the pattern will only be evident for firms that have reason to fear loss of control (for instance, firms where there is no controlling shareholder).

An alternative way of satisfying the above constraints is to allocate shares to uninformed investors.¹⁷ In Benveniste and Wilhelm's model, uninformed investors are useful because they increase the underwriter's ability to induce truth-telling from informed investors. This occurs because the underwriter has the option to distribute shares to the uninformed when the informed give less favorable signals. In our model, there is no incentive to do this because, in equilibrium, it is the information collection constraint that binds.

Nevertheless, if there are binding participation restrictions, uninformed investors do indeed become useful. The restrictions force H to increase, which leads to inefficient levels of information collection and to excessive underpricing. Being able to satisfy the restrictions partly through the addition of uninformed investors allows the underwriter to limit the increase in underpricing and information gathering. However, the uninformed investors only help the underwriter up to a point, since replacing too many of the informed with uninformed will mean giving excess returns to the new investors without receiving any additional information in exchange. The underwriter will always benefit from inviting at least one uninformed investor to participate in the offering, but may or may not choose to add more than one investor. This is discussed in the following proposition, which is proved in Appendix B.

Proposition 8: Given limits on investor participation, the underwriter will always be able to reduce underpricing by replacing at least one informed investor with an uninformed investor.

¹⁷ U.S. firm commitment underwriters allocate IPO shares to both institutional and retail investors. One rationale for including retail investors is that the retail brokerage offices provide information about the retail demand for an issue which is valuable for pricing the issue. A second interpretation is that the retail investors are uninformed and are included to broaden the distribution of the shares for regulatory or corporate control reasons. Much IPO research assumes that retail investors are uninformed.

8. When Do Investors Earn Economic Rents?

In the previous literature, e.g., Benveniste and Spindt (1989) and Benveniste and Wilhelm (1990), investors are endowed with information and thus enjoy rents whenever they are allocated underpriced shares. In our model, which assumes that information collection is costly, this need not be the case since the cost of collecting information can offset an investor's gain from underpricing. As we show, in some cases the expected level of underpricing exactly offsets the cost of information collection, which implies that there are no rents associated with being selected by the underwriter to participate in a new issue. However, in other cases, the level of underpricing will exceed the information costs. Our model suggests two reasons why this may be the case. The first relates to the issuer's desire for price accuracy. The second has to do with the participation restrictions described in the previous section.

First, excess returns are more likely if the issuer places a high value on price accuracy. When the demand for accuracy is sufficiently great, offering a return large enough to compensate for information costs requires that shares be underpriced in neutral and/or bad states as well as good ones, as shown in Proposition 3. In these cases, there will be rents associated with being invited to participate in an IPO.

In a relevant empirical study, Hanley (1993) found that issues priced within their initial filing range had an average initial return of 10.0%, while those priced strictly below the bottom of the price range listed in their prospectus had a positive but statistically insignificant average return of 0.6%. For IPOs priced strictly above their initial filing range, the average initial return was 20.7%. The differences in initial returns between these three categories was significant at the 1% level. Given the 10.0% initial return for "medium" IPOs (i.e. neither hot nor cold, the equivalent of state u issues in our model), our model implies that informed investors are indeed earning excess returns on average for US IPOs.

Another consequence of a strong desire for price accuracy is that $q_{u,g}$ may be positive. For H sufficiently low that (A4) does not bind, it is optimal to set $q_{u,g} = 0$, meaning that informed investors receive shares in hot IPOs only if they express a strong preference for them. In our model, $q_{u,g} > 0$ is another sign (besides the underpricing of u and/or b issues) that (A4) is binding. Thus, if we see informed investors receiving at least some shares in all offerings by an

underwriter, our model implies that those investors are indeed earning economic rents. Such a pattern is shown in Hanley and Wilhelm (1985) and James (1997).¹⁸

The second reason why investors might earn excess returns is because of the participation restrictions discussed in the last section. If firms require a broad shareholder base, either because of listing requirements or for corporate control reasons, we expect to see more underpricing. Informed investors will still receive just enough to compensate for their evaluation costs (unless the desire for price accuracy is very high), but uninformed investors may receive abnormal returns. As shown in the proof of Proposition 8, uninformed investors receive excess returns only as a response to the first type of participation restriction, a minimum number of investors, and not in response to a limit on the maximum allocation per investor.

Thus, under some circumstances, governments could add restrictions to the IPO distribution process, or to listing or corporate control regulations, that would lead to some "average investors" receiving abnormally high returns. However, these uninformed investors are brought in only if they are sufficiently cheap, which essentially means only if they can be given small allocations. If the binding participation restriction required each investor to hold a large number of shares, the issuer would choose informed rather than uninformed investors, because informed investors contribute information in return for their shares. Thus, the uninformed may sometimes get a high rate of return per share, but they never get large total returns.

A restriction on the minimum number of investors is a common listing requirement for stock exchanges. In the US (unlike many countries), firms are legally allowed to "go public" without listing on an exchange. Nevertheless, many US firms that are going public plan to list immediately on a specific exchange, so listing requirements may influence the IPO allocation process. The New York Stock Exchange, the American Stock Exchange and NASDAQ all

¹⁸ Hanley and Wilhelm document allocation patterns for US firm commitment IPOs by a major underwriter. This type of published study is rare, because investment banks prefer to keep their data confidential. They find that the institutional investors on the underwriter's list typically take a share in every IPO by that underwriter (as long as the issue matches the strategy of the fund). James looks at 13f filings of institutional investors in the first quarter of each new issue to see which institutional investors have shares. He finds evidence that investment banks form stable coalitions with groups of investors who participate regularly in that underwriter's issues. Hanley and Wilhelm found first quarter 13f holdings to be highly correlated with initial allocations, although there may be some bias. Benveniste, Erdal and Wilhelm (1999) and Krigman, Shaw and Womack (1999) use TAQ data to show that institutional investors have a tendency to flip or sell cold issues in the first day of trading. If anything, this would bias James' study against finding steady investor groups, yet such groups are still apparent.

require listing firms to have at least a certain minimum number of investors each owning at least one round lot.¹⁹ Since a round lot is a relatively small allocation, compared to the average holdings of an institutional investor, it would be relatively cheap to find some uninformed investors (perhaps some politicians, athletes or other celebrities) and allocate them 100 shares each simply to round out the investor lists.

To look for evidence of this, one could examine firms that are planning to list on an exchange and are at or close to the minimum investor number. If the allocation figures for these firms show a cluster of allocations at the minimum level, with perhaps a large gap between those minimum allocations and the allocations to other investors, this would indicate that those small investors are getting excess returns, and that the level of underpricing has been driven up by the listing requirement.

9. Conclusion

This paper explores the book building process which is increasingly being used to distribute IPOs throughout the world. By endogenizing the information acquisition process, we have been able to examine how the underwriter selects the investors that participate in the offering and how the price and allocation schemes are determined. We demonstrate the trade off that the underwriter faces when increasing the size of the investor pool. If more investors are invited to participate, the issue can be priced more accurately. However, increased participation increases underpricing.

When information is costless, the optimal number of participating investors is infinite and underpricing approaches zero. When information is costly, the level of underpricing is determined by the desire for information. For firms with the most to gain from accurate pricing, more investors will be invited to participate in the offering and underpricing will be greater.

In some cases, the expected level of underpricing exactly compensates for the costs of acquiring information, leaving investors indifferent about whether or not to participate in IPOs. In other cases, however, investors realize excess returns from participating in new issues, and

¹⁹ A firm that wants to list on the NYSE must have at least 2,000 round lot holders (or a lower number of investors plus certain minimum trading volume levels, but trading volume alternatives presumably would not be available for an IPO that wanted to list immediately). Nasdaq requires at least 400 round lot holders for a National Market listing and 300 round lot holders for a SmallCap Market listing. The

thus participation must be rationed. This will occur when the investment bank wants a large number of investors to participate in the offering, either because it wants better information or because of regulatory or corporate governance reasons.

Empirical evidence in James (1997) suggests that underwriters form investor groups that regularly participate in the underwriters' IPOs. These investor groups are consistent with the sets of participants described in our model. In addition, anecdotal evidence suggests that underwriters may be rationing not only shares in select hot issues, but also entrance into these groups, implying that at least part of the underpricing can be viewed as rents associated with buying IPOs. While we explain why these rents may arise in competitive markets, additional empirical work is needed to gauge the importance of the information and incentive issues raised in this paper.

Our analysis provides potentially testable empirical implications on the determinants of IPO underpricing. In particular, the model suggests that firms with a high need for price accuracy are likely to be the most underpriced. These firms might include riskier companies, smaller firms with shares that are likely to be more thinly traded, and firms that expect to have significant capital needs in the future. In addition, some firms' operations might be particularly vulnerable to aftermarket price volatility, which could cause them to lose clients or make it more difficult for them to attract top employees. These firms will be willing to pay for greater price discovery in the IPO process, since this is likely to lead to less volatility in the aftermarket.

Studying the book building method in detail is important because of the growing international trend towards its use (documented in Sherman (2000b)). However, this method is often unpopular with the public. There is an appearance that the system is less fair because it leaves out small investors. As a result, many countries that are beginning to allow book building are retaining or adding restrictions that force part of the shares in each IPO to be allocated to small investors. We show in this paper that these constraints are not costless - they increase the level of underpricing, thereby increasing the cost of going public.

References

- Beatty, Randolph P. and Jay R. Ritter, 1986, Investment Banking, Reputation, and the Underpricing of Initial Public Offerings, *Journal of Financial Economics* **15**, 213-232.
- Benveniste, Lawrence and Walid Busaba, 1997, Bookbuilding vs. Fixed Price: An Analysis of Competing Strategies for Marketing IPOs, *Journal of Financial & Quantitative Analysis* **32**, 383-403.
- Benveniste, Lawrence M., Walid Busaba, and William J. Wilhelm, 1997, Information Externalities in Primary Equity Markets, Unpublished, paper, Boston College.
- Benveniste, Lawrence, Sina Erdal and William Wilhelm, 1999, Who Benefits from Secondary Market Price Stabilization of IPOs? *Journal of Banking and Finance*, forthcoming.
- Benveniste, Lawrence M. and Paul A. Spindt, 1989, How Investment Bankers Determine the Offer Price and Allocation of New Issues, *Journal of Financial Economics* **24**, 343-361.
- Benveniste, Lawrence M. and William J. Wilhelm, 1990, A Comparative Analysis of IPO Proceeds under Alternative Regulatory Regimes, *Journal of Financial Economics* **28**, 173-207.
- Biais, Bruno and Anne Marie Faugeron Crouzet, 1999, IPO Auctions, Unpublished paper, Universite de Toulouse.
- Brennan, Michael and Julian Franks, 1997, Underpricing, Ownership and Control in Initial Public Offerings of Equity Securities in the UK, *Journal of Financial Economics* **45**, 391-413.
- Chen, Hsuan-Chi and Jay R. Ritter, 1999, The Seven Percent Solution, *Journal of Finance*, forthcoming.
- Chowdhry, Bhagwan and Ann E. Sherman, 1996a, International Differences in Oversubscription and Underpricing of Initial Public Offerings, *Journal of Corporate Finance* **2**, 359-381.
- Chowdhry, Bhagwan and Ann E. Sherman, 1996b, The Winner's Curse and International Methods of Allocating Initial Public Offerings, *Pacific-Basin Finance Journal* **4**, 15-30.
- Cornelli, Francesca and David Goldreich, 1999, Book Building and Strategic Allocation, Unpublished paper, London Business School.
- Derrien, Francois and Kent L. Womack, 1999, Auctions vs. Book-Building and the Control of Underpricing in Hot IPO Markets, Unpublished paper, Dartmouth College.
- Hanley, Kathleen Weiss, 1993, The Underpricing of Initial Public Offerings and the Partial Adjustment Phenomenon, *Journal of Financial Economics* **34**, 231-250.
- Hanley, Kathleen Weiss and William J. Wilhelm, Jr., 1995, Evidence on the Strategic Allocation of Initial Public Offerings, *Journal of Financial Economics* **37**, 239-257.

- James, Kevin, 1997, Do Large Underwriters Form Investor Coalitions?: Evidence From 13F Data, forthcoming *Journal of Finance*.
- Krigman, Laurie, Wayne Shaw and Kent Womack, 1999, The Persistence of IPO Mispricing and the Predictive Power of Flipping, *Journal of Finance* **54**, No. 3, 1015-1044.
- Ljungvist, Alexander, Tim Jenkinson and William J. Wilhelm Jr., 2000, Has the Introduction of Book Building Increased the Efficiency of International IPOs?, Unpublished paper, Stern School of Business, New York University.
- Loughran, Tim; Jay R. Ritter and Kristian Rydqvist, 1994, Initial Public Offerings: International Insights, *Pacific-Basin Finance Journal* **2**, 165-199.
- Maksimovic, Vojislav and Pegaret Pichler, 1997, Private versus Public Offerings: Optimal Selling Mechanisms with Adverse Selection, Unpublished paper, University of Maryland.
- Mello, Antonio and John Parsons, 1998, Going Public and the Ownership Structure of the Firm, *Journal of Financial Economics* **49**, 79-109.
- Novshek, William, 1993, *Mathematics for Economists*, Academic Press, San Diego.
- Rock, Kevin F., 1986, Why New Issues are Underpriced, *Journal of Financial Economics* **15**, 187-212.
- Sherman, Ann E., 1992, The Pricing of Best Efforts Initial Public Offerings, *Journal of Finance* **47**, 781-790.
- Sherman, Ann E., 2000a, IPOs and Long Term Relationships: An Advantage of Book Building, *Review of Financial Studies*, forthcoming.
- Sherman, Ann E., 2000b, Global Trends in IPO Methods: Book Building vs. Auctions, Unpublished paper, University of Minnesota and Notre Dame University.
- Stoughton, Neal and Josef Zechner, 1998, IPO-Mechanisms, Monitoring and Ownership Structure, *Journal of Financial Economics* **49**, 45-77.
- Stoughton, Neal, Kit Pong Wong and Josef Zechner, 1998, IPOs and Product Quality, Unpublished paper, University of California-Irvine.
- Subrahmanyam, Avinidhar and Sheridan Titman, 1999, The Going Public Decision and the Development of Financial Markets, *Journal of Finance* **54**.
- Titman, Sheridan and Brett Trueman, 1986, Information Quality and the Valuation of New Issues, *Journal of Accounting and Economics* **8**, 159-172.

Appendices

Appendix A: Information reporting or truth-telling constraints.

The full set of six truth-telling or information reporting constraints are given below, expanding equation (1). These conditions are discussed in Section 3.1.

$\mathbf{R(g,g) \geq R(g,u):}$

$$P'(g,0) \{ (s^g - s_g) (q_{g,g,1}) - (s^g - s_u) (q_u) \} + (1 - P'(g,0)) \{ (s^g - s_g) (q_{g,g}^* - q_{u,g}) \} \geq 0 \quad (\text{A1})$$

$\mathbf{R(u,u) \geq R(u,b):}$

$$\frac{\pi^2 (1 - \pi_i)}{2 (1 - \pi\pi_i)} \{ (1 - P'(b,0)) (s^b - s_b) (q_{u,b} - q_{b,b}^*) + (1 - P'(g,0)) \{ (s^g - s_g) q_{u,g} \} \} \\ + P'(u,0) \{ (s^u - s_u) q_u - (s^u - s_b) q_{b,b,1} \} \geq 0 \quad (\text{A2})$$

$\mathbf{R(g,g) \geq R(g,b):}$

$$P'(g,0) \{ (s^g - s_g) q_{g,g,1} - (s^g - s_b) q_{b,b,1} \} + (1 - P'(g,0)) \{ (s^g - s_g) q_{g,g}^* \} \geq 0 \quad (\text{A3})$$

$\mathbf{R(u,u) \geq R(u,g):}$

$$\frac{\pi^2 (1 - \pi_i)}{2 (1 - \pi\pi_i)} \{ (1 - P'(g,0)) (s^g - s_g) (q_{u,g} - q_{g,g}^*) + (1 - P'(b,0)) (s^b - s_b) q_{u,b} \} \\ + P'(u,0) \{ (s^u - s_u) q_u - (s^u - s_g) q_{g,g,1} \} \geq 0 \quad (\text{A4})$$

$\mathbf{R(b,b) \geq R(b,u):}$

$$P'(b,0) \{ (s^b - s_b) q_{b,b,1} - (s^b - s_u) q_u \} + (1 - P'(b,0)) \{ (s^b - s_b) (q_{b,b}^* - q_{u,b}) \} \geq 0 \quad (\text{A5})$$

$\mathbf{R(b,b) \geq R(b,g):}$

$$P'(b,0) \{ (s^b - s_b) q_{b,b,1} - (s^b - s_g) q_{g,g,1} \} + (1 - P'(b,0)) \{ (s^b - s_b) q_{b,b}^* \} \geq 0 \quad (\text{A6})$$

Three of these conditions, equations (A4), (A5) and (A6), limit the amount of underpricing in the model, since they all involve reporting better information than has been received - saying good if bad or uncertain, or saying uncertain if bad. Falsely reporting good information may raise the price of the issue but will also bring a larger allocation. If

underpricing is excessive, investors will find it worthwhile to raise the price per share by reporting a “better” signal, since the better report would allow the investor to get more of the still-underpriced shares.

The other three constraints, equations (A1), (A2) and (A3), all involve underreporting - falsely reporting bad information or withholding good information. To prevent this, there must be at least a minimum level of underpricing, and allocations must be larger when better information is reported. In practice, the two constraints most likely to bind are (A1) and (A4). Equation (A1) provides conditions that keep investors from withholding good information by reporting u or “no information” when they receive the signal g. Because it plays a major role in the solution of this model, we will explain the terms:

$R(g,g) \geq R(g,u)$: An investor who receives signal g but reports u will get q_u underpriced shares (worth s^g but priced at s_u) if, with probability $P'(g,0)$, no other investor gets a good signal. If, instead of lying, the investor had reported g, the shares would have been less underpriced ($s^g - s_g$ rather than $s^g - s_u$), but the investor would have received $q_{g,g,1}$ shares rather than q_u . If, with probability $1 - P'(g,0)$, at least one other investor reports signal g, then the price will be the same whether or not the first investor lies, but the first investor’s allocation of shares will on average be $q_{u,g}$ if she reports u but $q_{g,g}^*$ if she reports g. Thus, to induce the investor to truthfully report g, the advantage of possible greater underpricing if the investor lies must be offset by a lower expected allocation.

Appendix B: Proofs of Propositions

Proof of Proposition 1

With 3 states, there are 8 possible underpricing combinations: I. no underpricing; II. u only; III. b only; IV. u and b but not g; V. g only; VI. g and u but not b; VII. g and b but not u; VIII. underpricing in all states - g, u and b.

Given our assumptions, outcome I will not be chosen by the issuer. II. is not feasible because (2) could not be satisfied for any $q_u > 0$. III is not feasible: satisfying (2) would require $q_{b,b} > q_{u,b}$ (since $1 - \pi(1 - \pi_i) > \pi_i$); (A3) would require $q_{b,b,1} = 0$, which implies $q_{b,b}^* > q_{b,b}$, and (A2) would require $q_{u,b} > q_{b,b}^*$, which violates (2) and/or (A3). Similarly, IV is not feasible because (2) would require $q_{b,b} > q_{u,b}$ and (A1) would require $q_u = 0$, violating (A2).

Thus, the only feasible outcomes are V - VIII, all of which involve underpricing of g shares. Next, to show that V is feasible: (A2), (A3), (A5) and (A6) won't bind ((A2) requires $q_{b,b,1} = 0$). This leaves (2), (A1) and (A4). (2) requires $q_{g,g} > q_{u,g}$. (A1) can be satisfied for sufficiently high H, $q_{g,g}$ and $s^g - s_g$. (A4) will bind for sufficiently high H, placing limits on $q_{u,g}$ and $s^g - s_g$. Note that $q_{u,g} = 0$ unless (A4) binds.

For sufficiently low H such that (A4) does not bind, V is optimal. VII is not optimal for a low H because, to satisfy (A2) with $s^b > s_b$, requires either $q_{u,g} > 0$ or $q_{u,b} \geq q_{b,b}^*$, either of which would make it more difficult to satisfy (2), leading to higher underpricing for the same H (a suboptimal outcome). Next, V dominates VI and VIII for low H because $s^u > s_u$ makes it more expensive to satisfy (2) without any corresponding loosening of another binding restriction (because H is low enough that (A4) does not bind).

For a higher H, either V, VI, VII or VIII may be optimal, depending on the parameters. VI is more likely for low π , while VII is more likely for high π . A higher π means a higher probability of extreme values, either high or low, for the IPO shares.

Last, VIII is always optimal for sufficiently high H. With VII, it is not possible to lower $q_{u,g}$ to zero and still satisfy (A2). Once the lower limit on $q_{u,g}$ has been reached with $s^u = s_u$, adding underpricing of u shares will allow $q_{u,g}$ to be further lowered while satisfying (A2), making the underpricing of g shares more efficient. With VI, the direct effect of $s^u > s_u$ is to make (2) harder to satisfy. However, this effect is small for high H, because $P'(g,0)$ is small. Underpricing u makes it easier to satisfy (A4), allowing $q_{u,g}$ to be lower, which will more than offset the direct effect on (2) (for sufficiently large H) and thus lower underpricing. However, if s_u gets too low, (A5) cannot be satisfied unless $s^b > s_b$.

Proof of Proposition 2

First, when equation (2) is binding, $\partial(s^g - s_g)/\partial H$ is positive. We examine the case where all underpricing occurs in state g, but the case where u and/or b shares are also underpriced is a straightforward extension. If equation (2) is simplified and rearranged, with equilibrium terms substituted out, we get:

$$M \equiv \frac{2cH}{\pi N[1 - (1 - \pi_i)^H]} - (s^g - s_g) = 0.$$

We can see that $\partial M / \partial (s^g - s_g) = -1 < 0$; and

$$\frac{\partial M}{\partial H} = \frac{2c}{\pi N[1 - (1 - \pi_i)^H]} - \frac{2cH\pi_i(1 - \pi_i)^H}{\pi N[1 - (1 - \pi_i)^H]^2} = \frac{2c}{\pi N[1 - (1 - \pi_i)^H]} \left(1 - \frac{H\pi_i(1 - \pi_i)^H}{1 - (1 - \pi_i)^H} \right)$$

After rearranging the last term, we can see that this is positive if $1 > (1 - \pi_i H)(1 - \pi_i)^H$. This condition is satisfied for $H = 1$, and the right hand side decreases as H increases. Therefore, when (2) is binding,

$$\frac{\partial(s^g - s_g)}{\partial H} = - \frac{\frac{\partial M}{\partial H}}{\frac{\partial M}{\partial(s^g - s_g)}} > 0.$$

Next, when (A1) is binding, $\partial(s^g - s_g)/\partial H$ is negative. The proof of this is similar to the proof shown above for the case where (2) is binding, and this result is shown for a similar environment in Benveniste and Wilhelm.

Equations (A1) and (2) both give minimum levels for underpricing of g issues, $s^g - s_g$. These partial derivative results show that (A1) is more likely to bind for a lower H (since the minimum underpricing that it requires is highest when H is low), while (2) is more likely to bind for a high H . Last, the underwriter will always choose an H such that (2) rather than (A1) is the binding constraint, since the higher H leads to both less underpricing and greater price accuracy.

Proof of Proposition 3

To satisfy the information collection constraints, each investor's expected return to participating must be better than the best alternative. In equilibrium, the best alternative would be to keep the cost of the signal, pretend to evaluate and report "u" or uncertain. In this case, the investor's expected return is equal to the cost of the signal, c , plus the expected excess return to "blindly" reporting u , $R(\cdot, u)$. This second term is:

$$R(\cdot, u) = (\pi/2)(1-P'(g,0))\{(s^g - s_g)q_{u,g}\} + (\pi/2)(1-P'(b,0))\{(s^b - s_b)q_{u,b}\} + P'(u,0)\{(s^u - s_u)q_u\}.$$

When (A4) does not bind, this term is zero, since $q_{u,g} = 0$, $s^b = s_b$ and $s^u = s_u$. However, if (A4) binds, then at least one of the following will be true: $q_{u,g} > 0$, $s^b > s_b$ or $s^u > s_u$. In this case, $R(\cdot, u)$ is strictly positive, and thus the expected return to investors must be strictly greater than c .

Proofs of Proposition 4 - 6

We will examine the case in which the issuer chooses a sufficiently low H that (A4) does not bind. It is straightforward to extend these results to the other cases. Because the underwriter plans to sell N shares regardless of the state, $q_{u,g}$ and $q_{u,b}$ are functions of $q_{g,g}$ and $q_{b,b}$. Also, $q_{u,g} = 0$ when (A4) does not bind. From (A2), $q_{b,b,1} = 0$. Last, using the facts that $P(g,0) = P(b,0)$, $2s^u = s^g + s^b$, $s_u = s^u$ and $s_b = s^b$, the first order condition (FOC) for H reduces to $s^g - s_g = f'(P(b,0))$. Thus, we have a system of two equations, (2) and the FOC for H , and two remaining unknowns, s_g and H . After substitutions, these equations become:

$$F_1: \quad \frac{2cH}{\pi N [1 - (1 - \pi_i)^H]} - (s^g - s_g) = 0.$$

$$F_2: \quad f' \left(\frac{\pi}{2} (1 - \pi_i)^H \right) - (s^g - s_g) = 0.$$

Since the equations F_1 and F_2 are both differentiable and the determinant of the Jacobian is non-zero (as we will show), the Implicit Function Theorem can be used as long as the parameters are such that a solution exists. Thus we can use Cramer's Rule to get the comparative statics for this two-equation system. The determinant of the Jacobian is clearly negative and thus non-zero, as required by the Implicit Function Theorem. $|J_{x,y}|$ is the determinant of the matrix where the column of partial derivatives for x is replaced with the column of partial derivatives for y (where x is the endogenous variable and y is an exogenous variable).

To get the partial derivatives, we use Cramer's Rule: $\partial x / \partial y = - |J_{x,y}| / |J|$. The minus sign arises because we have a system of the form $Ax + b = 0$, rather than the usual $Ax = b$ (see Novshek, 1993, p. 173). The comparative statics are fairly straight-forward, except for the "mixed" cases discussed in section 6. Details are available from the authors.

Proof of Proposition 7 -

First, H will have to increase under either of these restrictions. Under I, H must equal $H^* + 1 > H'$ so that we can still have $q_{b,b,1} = 0$. Under II, we again need to be able to set $q_{b,b,1} = 0$ to satisfy equation (A2), so $H = N/q^{\max} + 1 > H'$ (recall that H' is the unconstrained optimal H).

Second, the underwriter will have to allocate some shares in g issues to investors that report u (i.e., $q_{u,g,h} > 0$ for at least some h), in order to satisfy the participation limits. However, the underwriter can adjust allocations and prices based on h to satisfy (A1)-(A6) without

allocating *underpriced* shares to investors that report u . The guideline will be that $s_{g,h} < s^g$ (underpricing) only for $h = H^* + 1$ or $h \geq N/q^{\max}$, in which case $q_{u,g,h} = 0$.

The first order condition for H will no longer bind, since we no longer have an interior optimum. Equation (2) will once again determine underpricing. It was shown in the proof of Proposition 2 that underpricing must increase when H increases, as long as (2) is binding. Therefore, since H has increased, the equilibrium underpricing must increase also. In extreme cases, the underwriter will be forced to allocate some underpriced shares in good issues to investors that report u , and the overall level of underpricing will have to increase even more.

Proof of Proposition 8 -

Under either type of investor restriction, adding one uninformed investor to the syndicate will allow the underwriter to lower the number of informed investors (to H^* under I and to N/q^{\max} under II), while still allowing the underwriter to set $q_{b,b,1} = 0$ and sell all N shares. There is essentially no cost to having one uninformed investor, since that investor will only be allocated fairly priced (state u or b) shares and will not get the underpriced state g shares. Since H was above the optimal level before, costlessly lowering it toward the unconstrained optimum clearly increases the utility of the underwriter.

However, if more than one informed investor is replaced by an uninformed investor, it will become necessary to allocate some underpriced shares to an uninformed investor. Under restriction I, the number of informed investors that would be replaced by uninformed will depend primarily on the size of ε . If ε is very small, the cost of allocating only ε underpriced shares to each uninformed investor is relatively low. Thus it might be worth adding uninformed investors in order to reduce the number of “extra” informed investors.

Under restriction II, there is no advantage to adding more than one uninformed investor, because informed investors are already getting the maximum possible number of underpriced shares, q^{\max} . If informed investor allocations do not change, then the amount of required underpricing will not change (as can be seen from equation (2) with $q_{g,g}$ replaced by q^{\max}). By replacing informed with uninformed investors, the underwriter loses the additional information that would have been gained from the informed investors while getting nothing in return.