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ABSTRACT

This paper examines the specification errors of several asset pricing models using the methodology of Hansen and Jagannathan (1997) and a common data set. The models are the CAPM, the Consumption CAPM, the Jagannathan and Wang (1996) conditional CAPM, the Campbell (1996) dynamic asset pricing model, the Cochrane (1996) production-based model, and the Fama-French (1993) three-factor and five-factor models. We use returns on the Fama-French twenty-five portfolios sorted by size and book-to-market ratio and the risk-free rate as our test assets. The sample is 1952 to 1997. We allow the parameters of the models' pricing kernels to fluctuate with the business cycle which we measure in two ways. One uses the Hodrick-Prescott (1997) filter applied to either industrial production for monthly models or real GNP for quarterly models. The second approach for quarterly models uses the consumption-wealth measure developed by Lettau and Ludvigson (1999). While we cannot reject correct pricing for Campbell's model, a stability test indicates that the parameters may not be stable. None of the models correctly prices returns that are scaled by the term premium.

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1 Introduction

Throughout the 1970's and 1980's, financial economists investigated the pricing implications of the capital asset pricing model (CAPM) developed by Sharpe (1964) and Lintner (1965). The well-known prediction of the CAPM is that the expected excess return on an asset equals the covariance of the return on the asset with the return on the market portfolio times the market price of risk, which is the ratio of the expected excess return on the market portfolio to the variance of the return on the market portfolio. The expected return prediction of the CAPM can equivalently be stated as the beta of the asset times the expected excess return on the market portfolio, where the beta is the covariance of the asset's return with the return on the market portfolio divided by the variance of the market return.

As empirical research began to uncover a number of expected-return anomalies that the CAPM could not explain, Roll (1977) argued that the model was not testable. Because investors and firms assessing their costs of capital want to know the determinants of expected returns, empirical research continued, but it was necessarily conducted under the recognition that the tests involve a joint hypothesis on the model and the choice of the market portfolio. Even before the anomalies began to accumulate, theorists such as Merton (1973) noted that the CAPM is a static model, and they developed intertemporal models that demonstrated how covariances of returns with variables other than the market return could influence expected returns if the consumption and investment opportunity sets of investors vary over time. By examining the solution to dynamic portfolio optimization problems, Hansen and Singleton (1982) developed an empirical consumption-based capital asset pricing model (CCAPM) in which an expected return depends on the covariance of the return with the marginal utility of consumption.

The empirical failure of the CCAPM and the theoretical appeal of the Merton logic led Campbell (1993, 1996) to develop a dynamic asset pricing model in which an expected return depends on the covariances of the return with the market portfolio and with the innovation in the present discounted value of future expected market returns. In the Campbell model, anything that forecasts market returns becomes a risk factor for asset returns.

Jagannathan and Wang (1996) noted that it is possible for the CAPM to hold as a conditional model of expected returns with conditional betas, but the unconditional model would be more complicated since betas could vary over time. They developed an empirical model of this beta-premium sensitivity by taking a stand on the nature of the predictability of market returns.

Cochrane (1996) responded to the failure of the CCAPM by noting that the production side of the economy also must satisfy dynamic Euler equations. This logic led him to develop the implications of a production-based asset pricing model in which covariances of asset returns with macroeconomic measures of investment are important risk factors.

Finally, the empirical failure of the CAPM and the theoretical appeal of multi-factor models led Fama and French (1992, 1993, 1995, 1996) to develop a three-factor model. It is fair to say that this new model, or some extended variant of it, is now the workhorse for risk adjustment in academic circles.

The variety of the above models and the alternative data sets on which they have been tested pose a severe difficulty for someone who is trying to understand if any of these models is a reasonable replacement for the CAPM. The purpose of this paper is to compare these models on a common data set. We do this using the methodology proposed by Hansen and Jagannathan (1997), who develop a distance metric we call the HJ-distance. The Hansen-Jagannathan (1997) methodology begins with the recognition that the absence of arbitrage opportunities implies the existence of a common pricing kernel or stochastic discount factor that prices all assets. The HJ-distance measures the distance between the implied pricing proxy of each model and the true pricing kernel. It can also be interpreted as the normalized maximum pricing error of the model for portfolios formed from that set of assets. If the model is correct, the HJ-distance is zero, and there are no pricing errors.[2] We test whether HJ-distance equals zero using the statistical test developed in Jagannathan and Wang (1996). Although the measurement of HJ-distance solves a Generalized Method of Moments (GMM) problem, it is not the optimal GMM of Hansen (1982). We also report results from optimal GMM tests of the models.

Because there is considerable evidence that expected returns fluctuate over time, we want to allow for time-varying prices of risks. We do this by allowing the parameters of the models to fluctuate with the business cycle. We measure the business cycle in two ways. One uses the Hodrick-Prescott (1997) filter applied to either industrial production for monthly models or real GNP for quarterly models. The second approach for quarterly models uses the consumption-wealth measure developed by Lettau and Ludvigson (1999). Also, because Loughran (1997) and Daniel and

[2] Glasserman and Jin (1998) provide an alternative way of comparing models of stochastic discount factors (SDF) by examining the physical probability measures of asset prices and the implied measures of the SDF's.

Titman (1997) argue that return characteristics are different in January than outside of January, we use a January dummy variable to allow the parameters of the models to differ across this month and the other months.

Both HJ-distance and optimal GMM assume that the parameters of the model are stable over time. If a model is misspecified because its parameters are not stable, it may nevertheless pass the test of HJ-distance equal zero, but it would not predict well out of sample. This situation can characterize both conditional and unconditional models. Ghysels (1998) finds that using conditioning variables to improve asset pricing models may actually worsen their performance out-of-sample because of parameter instability. We therefore follow Ghysels (1998) who uses the supLM test developed by Andrews(1993) to investigate instability in parameters.

The common returns that we require each of the models to price are the returns on the twenty-five portfolios constructed by Fama and French (1993) in which firms are sorted by the market value of their equity (size) and the ratio of the book values of their equities to the market values of their equities (the book-to-market ratio). We use returns in excess of the Treasury bill return, and we also require the models to price the Treasury bill return. The sample period is 1952 to 1997 with either monthly or quarterly data.

Because asset pricing involves conditional expectations, any variable that is in the investors' information set can be used to condition returns. We use this insight to provide a robustness check on the models. The one variable that we use to condition returns is the term spread between the yields on long-term and short-term government bonds.

The paper is organized as follows. The next section provides a discussion of the econometric aspects of the paper including the derivations of HJ-distance, and the test that HJ-distance equals zero. Section 3 discusses the data and the parameterization of the different models. Section 4 contains the empirical results. Section 5 provides concluding remarks.

2 HJ-distance and Conditional Asset Pricing Models

2.1 Model Setup

Assume we have n assets to be priced. It is well-known that in the absence of arbitrage opportunities there exists a set M of stochastic pricing kernels m which price every asset correctly. That is,

$$E_t(m_{t+1}R_{j,t+1}) = p_j, \forall\ j, t > 0, \forall\ m_{t+1} \in M_{t+1}, \tag{1}$$

where m_{t+1} is the stochastic pricing kernel at time $t+1$, M_{t+1} is the set of correct pricing kernels, $R_{j,t+1}$ is the return for portfolio j at time $t+1$, and the price for return $R_{j,t+1}$ at time t is p_j. If $R_{j,t+1}$ is a gross return for a portfolio, then $p_j = 1$; if $R_{j,t+1}$ is an excess return for a portfolio, then $p_j = 0$. Because equation (1) holds conditioned on the information set at t, denoted Φ_t, by the law of iterated expectations the unconditional version of equation (1) is

$$E(m_{t+1}R_{j,t+1}) = p_j, \forall\ j, t > 0, \forall\ m_{t+1} \in M_{t+1}. \tag{2}$$

We use equation (2) to estimate and test the various asset-pricing models.

As Hansen and Jagannathan (1997) note, an asset pricing model provides a pricing proxy, y_{t+1}. If the model is true, then $y_{t+1} \in M_{t+1}$. We will examine models in which the pricing proxy y_{t+1} is a linear function of a constant and a vector of variable factors, f_{t+1}. Define $F'_{t+1} = [1, f'_{t+1}]$, and define the vector of parameters $b' = [b_0, b'_1]$. Then the pricing proxy is

$$y_{t+1} = b' F_{t+1} = b_0 + b'_1 f_{t+1}, \tag{3}$$

where F_{t+1} is the $k \times 1$ factor vector, and b is the $k \times 1$ coefficient vector. The parameter vector b provides the information of whether one factor is an important determinant of the pricing kernel. For ease of presentation, we drop the time subscript when it is not necessary for clarity of presentation.

Cochrane (1996) notes that if the model is true, equation (2) holds for all n assets with y_{t+1} substituted for m_{t+1}. Then, if p is the $n \times 1$ vector of p_j's, the pricing model has an equivalent representation in terms of multivariate betas and prices of risks:

$$E(R) = R^0 p + \beta' \Lambda, \tag{4}$$

where

$$R^0 = \frac{1}{E(y)} = \frac{1}{E(b'F)}, \tag{5}$$

$$\beta = cov(f, f')^{-1} cov(f, R'), \tag{6}$$

and

$$\Lambda = -R^0 cov(f, f')b_1. \tag{7}$$

In equation (4), R^0 is the unconditional riskfree rate or the zero-beta rate, the β's are the projections of the returns onto the factors, and the Λ's are the prices of beta risks. All of the parameters can be calculated once we know b. To answer whether the jth factor significantly influences the expected returns on a particular set of portfolios, we must assess whether the corresponding Λ_j is significantly different from zero. Notice $\Lambda_j = 0$ does not mean $b_{1,j} = 0$, and vice versa. Only when $cov(f, f')$ is diagonal are the two statements equivalent. The derivations and proofs of these statements can be found in Cochrane (1996).

In discussing prices of factor risks, one must be clear about whether it is beta risk or covariance risk. Campbell (1996), for example, uses covariance decomposition of equation (2) to write

$$E(R) = R^0 p - R^0 cov(m, R). \tag{8}$$

By substituting y_{t+1} for m_{t+1} in equation (2), we have

$$\begin{aligned} E(R) &= R^0 - R^0 cov(R, f')b_1 \\ &= R^0 + \sum_{j=1}^{k} q_j cov(f_j, R). \end{aligned} \tag{9}$$

Thus, the price of the jth covariance risk is

$$q_j = -R^0 b_{1,j}. \tag{10}$$

Since R^0 is not very different from 1, we do not report statistics for q_j.

2.2 HJ-distance

Hansen and Jagannathan (1997) note that when the asset pricing model is true, $y \in M$, but if the model is false, $y \notin M$. Thus, for false models there is a strictly positive distance between y and M. Hansen and Jagannathan (1997) define the distance, which we call HJ-distance, as

$$\delta = \min_{m \in L^2} \|y - m\|, \text{where } E(mR) = p, \tag{11}$$

and $\|x\| = \sqrt{E(x^2)}$.[3] The problem defined in equation (11) can be rewritten as the following Lagrangian minimization problem:

$$\delta^2 = \min_{m \in L^2} \sup_{\lambda \in R^n} \left\{ E\left(y-m\right)^2 + 2\lambda' \left[E\left(mR\right) - p\right] \right\}. \tag{12}$$

The value of δ is the minimum distance from the pricing proxy y to the set of true pricing kernels M. Let $\widetilde{m}$ and $\widetilde{\lambda}$ be the solution to equation (12). One can think of $y - \widetilde{m}$ as the minimal adjustment to y to make it a true pricing kernel. Hansen and Jagannathan (1997) solve equation (12) to find

$$y - \widetilde{m} = \widetilde{\lambda}' R, \tag{13}$$

where

$$\widetilde{\lambda} = E\left(RR'\right)^{-1} E(yR - p). \tag{14}$$

Thus, the HJ-distance is

$$\delta = \|y - \widetilde{m}\| = \left\|\widetilde{\lambda}' R\right\| = \left[\widetilde{\lambda}' E(RR')\widetilde{\lambda}\right]^{1/2}.$$

Substituting for the value of $\widetilde{\lambda}$ from equation (14) gives

$$\delta = \left[E(yR-p)' E\left(RR'\right)^{-1} E(yR-p)\right]^{1/2}. \tag{15}$$

By solving the conjugate problem to equation (11), Hansen and Jagannathan (1997) also provide an important alternative interpretation to δ. It is the maximum pricing error for the set of asset payoffs with norm equal to one. With n basic assets, R, the maximum pricing error δ is achieved by a portfolio of those assets with weights θ, where $\|\theta' R\| = 1$. After simplification, θ is given by

$$\theta = \frac{1}{\delta} E\left(RR'\right)^{-1} E(yR-p) = \frac{1}{\delta}\widetilde{\lambda}. \tag{16}$$

Hansen and Jagannathan (1997) note that $\widehat{b}$, the estimate of b, can be chosen to minimize δ. To see the relation of this problem to a standard Generalized Method of Moment(GMM) problem, define the pricing error vector $g = E(yR - p)$, and its sample counterpart

$$g_T = \frac{1}{T}\sum_{t=1}^{T} R_t y_t - p, \tag{17}$$

[3] Hansen and Jagannathan (1997) also consider a distance measure in which m is required to be strictly positive. If the problem is solved without the constraint and $y_{t+1} > 0$ for all t, the two solutions coincide. In their empirical analysis, Hansen and Jagannathan (1997) find this additional restriction does not make a big difference.

and let $W = E(RR')^{-1}$. Then, by squaring equation (15), $\widehat{b}$ can be chosen as

$$\widehat{b} = \arg\min \delta^2 = \arg\min g_T' W g_T. \tag{18}$$

While equation (18) is a standard GMM problem, it is not the optimal GMM of Hansen (1982) which uses as the weighting matrix a consistent estimator of

$$W^* \equiv \left[T \cdot var(g_T)\right]^{-1}. \tag{19}$$

Hansen (1982) demonstrates that W^* is optimal in the sense that the estimates $\widehat{b}$ have the smallest asymptotic covariance. In general, the optimal weighting matrix assigns big weights to assets with small variances in their pricing errors, and it assigns small weights to assets with large variances of their pricing errors. It is obvious that W^* changes with different models. This makes it unsuitable for the task of making comparisons among competing models. The alternative weighting matrix of Hansen and Jagannathan (1997), $W = E(RR')^{-1}$, is invariant across competing asset-pricing models. Using a common weighting matrix allows us to have a uniform measure of performance across models for a common set of portfolios. The only assumption needed is that W is non-singular. Cochrane (1996) argues that $E(RR')$ may be nearly singular in which case the inversion is problematic, but as we discuss later, we did not encounter inversion problems.

A big advantage of linear factor models is that they can be solved analytically. To demonstrate the solution, we first introduce some notation. Let the sample moment of the pricing errors, as in equation (17), be

$$g_T(b) = \frac{1}{T}\sum_{t=1}^{T} R_t\,(b'F_t) - p, \tag{20}$$

let the gradient with respect to the parameters be

$$D_T = \frac{\partial g_T}{\partial b} = \frac{1}{T}\sum_{t=1}^{T} R_t F_t', \tag{21}$$

and let the inverse of the estimated second moment matrix of the returns be

$$W_T = \left(\frac{1}{T}\sum_{t=1}^{T} R_t R_t'\right)^{-1}. \tag{22}$$

Also, define

$$W_T^* = S_T^{-1}, \tag{23}$$

where S_T is a consistent estimate of $var\left[T \cdot g_T(b)\right]$. The analytical solution for $\widehat{b}$ from the first order condition of equation (18) is given by

$$\widehat{b} = (D_T' W_T D_T)^{-1}(D_T' W_T p). \tag{24}$$

From Hansen (1982), the asymptotic variance of $\widehat{b}$ is

$$var(\widehat{b}) = \frac{1}{T}(D_T' W_T D_T)^{-1} D_T' W_T S_T W_T D_T (D_T' W_T D_T)^{-1}. \tag{25}$$

For optimal GMM, equation (25) reduces to

$$var(\widehat{b}) = \frac{1}{T}(D_T' S_T^{-1} D_T)^{-1}. \tag{26}$$

One purpose of this paper is to determine whether any of our candidate models of the stochastic discount factor has an HJ-distance equal zero. We construct our test statistics following Theorem 3 in Jagannathan and Wang (1996). The distribution of δ is not standard under the assumption that the true δ equals zero. Jagannathan and Wang (1996) demonstrate that the distribution of $T\delta^2$ involves a weighted sum of $n-k$ $\chi^2(1)$ statistics, where n is the number of assets and k is the number of estimated parameters. The weights are the $n-k$ non-zero eigenvalues of

$$A = S_T^{\frac{1}{2}} W_T^{\frac{1}{2}\prime} [I_n - W_T^{\frac{1}{2}} D_T (D_T' W_T D_T)^{-1} D_T' W_T^{\frac{1}{2}\prime}] W_T^{\frac{1}{2}} S_T^{\frac{1}{2}\prime}. \tag{27}$$

In equation (27), $S_T^{\frac{1}{2}}$ and $W_T^{\frac{1}{2}}$ are the upper-triangular matrices from the Cholesky decompositions of S_T and W_T, and I_n is the n-dimensional identity matrix. It can be demonstrated that A has exactly $n-k$ nonzero eigenvalues, which are positive and are denoted by $\theta_1, ..., \theta_{n-k}$. Then, the asymptotic sampling distribution of the HJ-distance is

$$T\delta^2 \xrightarrow{d} \sum_{j=1}^{n-k} \theta_j \upsilon_j \text{ as } T \to \infty, \tag{28}$$

where $\upsilon_1, ..., \upsilon_{n-k}$ are independent $\chi^2(1)$ random variables. We simulate the statistics 10,000 times to determine the p-value for the estimated HJ-distance.

We also consider additional model diagnostics. The covariance matrix of the pricing errors for the model is

$$
\begin{aligned}
& var\left[g_T\left(\widehat{b}\right)\right] \\
= \; & \frac{1}{T}\left[I_n - D_T(D_T'W_TD_T)^{-1}D_T'W_T\right]S_T\left[I_n - D_T(D_T'W_TD_T)^{-1}D_T'W_T\right]'.
\end{aligned} \tag{29}
$$

Thus, we can construct a Wald test statistic for the null hypothesis that $g_T = 0$ as

$$
g_T'\left(\widehat{b}\right) var\left[g_T\left(\widehat{b}\right)\right]^{-1} g_T\left(\widehat{b}\right) \xrightarrow{d} \chi^2(n-k). \tag{30}
$$

Since $var\left[g_T\left(\widehat{b}\right)\right]$ only has rank $n-k$, we use the pseudo inverse, following Cochrane (1996) footnote 6. When we do optimal GMM, this Wald test reduces to the well-known J-test, with

$$
\begin{aligned}
J \;=\; & g_T'\left(\widehat{b}\right) var\left[g_T\left(\widehat{b}\right)\right]^{-1} g_T\left(\widehat{b}\right) = Tg_T'\left(\widehat{b}\right)W_T^*g_T\left(\widehat{b}\right) \\
& \xrightarrow{d} \chi^2(n-k).
\end{aligned} \tag{31}
$$

From equation (14) the covariance matrix of the Lagrange multipliers is

$$
var(\widetilde{\lambda}) = W_T var\left[g_T\left(\widehat{b}\right)\right] W_T. \tag{32}
$$

Since the maximum pricing error δ is achieved by $\theta' R$ with $\theta = \widetilde{\lambda}/\delta$, we can examine the importance of individual assets to the pricing error by examining the null hypothesis $\widetilde{\lambda}_j = 0$.

Finally, it is important to distinguish which pricing errors are under discussion. We defined the pricing errors of the models in equation (20). It is the sample average for the differences in prices when we use y to price R minus the correct prices which should be zero for an excess return and one for a gross return. As in other research, we can also define average return errors as

$$
\begin{aligned}
\pi \;=\; & \overline{R} - E(R) = \frac{1}{T}\sum_{t=1}^{T} R_t - R^0[p_n - cov(y,R)] \\
=\; & R^0 g_T(\widehat{b}).
\end{aligned} \tag{33}
$$

To avoid confusion, we refer to $g_T(\widehat{b})$ as model errors, and π as the pricing errors of the basic assets. Since R^0 differs across models, the two do not provide the same information. We look at $g_T(\widehat{b})$ mainly for details associated directly with δ. We examine π to compare pricing errors for the basic assets across models.

2.3 Conditional Models and Stability Tests

Examining the unconditional implications of linear factor models has two inherent problems. One is that only unconditional risk premiums are estimated. The second is that the models force prices of fundamental risks to be constant across business cycles. Cochrane (1996), Ferson and Harvey (1999), and others try to solve these two problems by using macroeconomic variables as conditioning variables. In equation (3), all parameters in b are constant. To allow them to vary with some element z_t in Φ_t, we write

$$\begin{aligned} y_{t+1} &= b'(z_t)F_{t+1} \\ &= (b_{0,1} + b_{0,2}z_t) + \left[b'_{1,1} + (b_{1,2}z_t)'\right] F_{t+1} \\ &= b_{0,1} + b_{0,2}z_t + b'_{1,1}F_{t+1} + b'_{1,2}(F_{t+1}z_t). \end{aligned} \tag{34}$$

The last equal sign demonstrates Cochrane's (1996) point, scaling the prices of factors is equivalent to scaling the factors.

If prices of risks fluctuate over the business cycle, we can capture this effect by using variables that are associated with business cycles. There are three requirements for macroeconomic variables to be legitimate instruments. First, they must be included in the time t information set. Second, they should summarize the status of the business cycle. Third, since the number of the parameters increases geometrically with the number of conditioning variables, which can make the estimates unreliable, we can only allow one conditioning variable each time. Because the previous literature has focused on both monthly and quarterly horizons, we would like a similar conditioning variable for each horizon.

Daniel and Torous (1995) find that the cyclical element in industrial production (IP) is predictive for common stock returns. We adopt their use of IP as one instrument for the monthly models. For quarterly models, we use the cyclical component of real GNP. Because the cyclical components are not observable, we derive both series by using the Hodrick-Prescott (1997) filter applied recursively. We will elaborate on the construction of our data in the next section.

Lettau and Ludvigson (1999a) provide an alternative to these output-based measures of the business cycle. Lettau and Ludvigson (1999a) demonstrate that the cyclical element in the log consumption-aggregate wealth ratio (CAY) is strongly predictive for excess stock returns. This argument is consistent with the CCAPM. Lettau and Ludvigson (1999b) test the CCAPM and the CAPM using CAY as a conditioning variable. In their cross-sectional test conditioning with CAY

substantially improves the performance of the models. We also include CAY as a conditioning variable for the quarterly models.

Loughran (1997) and Daniel and Titman (1997) argue that the B/M effect in stock returns is largely driven by a January effect, that is, the B/M effect is not present at other times of the year. The basic assets we use are the Fama and French twenty-five portfolios which are constructed precisely to incorporate the B/M and size effects. We use a January dummy variable (JAN) to allow prices of risks to differ between January and other months of the year.

Another important issue is the stability of the model's parameters. Conditional models are attractive because unconditional models may not adequately capture time-varying risk premiums. But, this approach is not costless. If the conditional version is correctly specified and captures the dynamics in risk premiums, it will outperform the unconditional models. However, if the model's implied time-varying risk premiums are inherently misspecified because we choose the wrong conditioning variable, this false model may still appear to work well in small samples since it uses additional degrees of freedom. Ghysels (1998) finds that conditional models are fragile and may have bigger pricing errors than unconditional models.

If the model is correctly specified, parameter stability is not a problem. We use the supLM test of Andrews (1993) to see whether there are structural shifts in the parameters. The null hypothesis is there are no structural shifts. Andrews (1993) argues that the supLM test is powerful against the alternative of a single structural break at an unknown time. He also argues that even if this is not the most interesting alternative hypothesis, it provides a reasonable test of parameter stability. The LM statistics are evaluated at 5% increments between 20% and 80% of the sample, and the largest is the supLM statistics. The distribution for the supLM statistic is presented in Table 1 of Andrews (1993).

To keep the estimation tractable, we use the twenty-six portfolios as the basic assets to be priced. We also investigate whether the model is robust to a different set of assets by adopting Cochrane's approach of scaling returns. Cochrane (1996) notes that conditioning information can be used to scale returns as implied by equation (1). These scaled returns can be interpreted as the returns to managed portfolios. The portfolio manager changes the weight of each portfolio according to the signal he observes from the conditioning variable. To illustrate, we multiply both sides of equation (1) by any variable $x_t \in \Phi_t$ to get

$$E_t(m_{t+1}R_{j,t+1})x_t = x_t p_j, \ \forall\ j, t > 0, \ \forall\ x_t \in \Phi_t. \tag{35}$$

By the law of iterated expectation, we have

$$E(m_{t+1}R_{j,t+1}x_t) = E(x_t p_j),\ \forall\ j, t > 0, \forall\ x_t \in \Phi_t. \tag{36}$$

Equation (40) provides the orthogonality conditions for scaled returns. If the model is robust to changes in the underlying assets, it should be able to price the new assets correctly. That is, if the model can price non-scaled returns R, under the null hypothesis that the parameters are not asset-sensitive, the model should be able to price scaled returns Rx as well.

We first calculate parameter estimates from optimal GMM using the twenty-six returns as

$$\widehat{b} = \arg\min g_T(R)'W^* g_T(R). \tag{37}$$

Then, under the null that $\widehat{b}$ is the true parameter, the set of scaled returns Rx should be correctly priced with $\widehat{b}$. We calculate the new J statistics as

$$J = g_T(Rx, \widehat{b})' var[g_T(Rx, \widehat{b})]^{-1} g_T(Rx, \widehat{b}), \tag{38}$$

where

$$g_T(Rx, \widehat{b}) = \frac{1}{T}\sum_{t=1}^{T-1}(R_{t+1}x_t)\left(\widehat{b}' F_{t+1}\right) - px_t, \tag{39}$$

The J-statistic is distributed as a $\chi^2(n)$ under the null. The degrees of freedom are n because we have n orthogonality conditions, and we do not estimate any additional parameters. The same argument applies to HJ-distance. With the new orthogonality conditions for scaled returns, we need to calculate the new δ and the distribution of $T\delta^2$. Since the first stage estimates by optimal GMM are not very different from those obtained from HJ-distance estimation, we choose to use the estimates from optimal GMM to calculate new HJ-distance for the new scaled assets.

3 Data

Unless otherwise indicated, all data are from CRSP. For the monthly models, the sample period is 1952:01 to 1997:12, for 552 total observations. For the quarterly models, the sample is from 1953:01 to 1997:04, for 180 total observations. We begin in 1953:01 because CAY is only available after 1953:01.

3.1 The Portfolio Returns

Our basic assets are the twenty-five excess returns on the portfolios sorted by size and book-to-market ratio that are calculated as in Fama and French (1993).[4] Excess returns are constructed by subtracting the T-bill rate, and our twenty-sixth asset is the gross return on the T-bill. The previous literature finds that the twenty-five B/M and size portfolios are very hard to price correctly because they incorporate both size premiums and value premiums. We require the models to price these excess equity returns and the riskfree rate, as well.

Portfolios are numbered 11 to 55, where the first number refers to the size quintile and the second number refers to the B/M quintile. For example, 11 is the portfolio of the smallest firms with the lowest B/M, while 55 is the portfolio with the largest firms and highest B/M. Table 1 provides summary statistics for the twenty-five portfolios for the sample period 1952:01 to 1997:12. It is similar to Table 2 of Fama and French (1993), which involves a shorter sample period from 1963:01 to 1991:12. For our longer sample, most average returns are larger, except for the low B/M firms. Since the standard errors are smaller, the t-statistics are larger except for the low B/M firms.

As demonstrated in section 2, the weighting matrix for the calculation of HJ-distance depends only on the assets and is the same for different models. The weighting matrix is not the same when we use conditioning information to scale returns. Hence, we have four weighting matrices: monthly non-scaled returns, monthly scaled returns, quarterly non-scaled returns, and quarterly scaled returns. Because our main results are derived from monthly and quarterly non-scaled returns, we focus primarily on these two cases. Since

$$W = E(R'R)^{-1},$$

we first want to demonstrate that $E(R'R)$ is non-singular. The condition numbers of the two matrices of sample second moments are 13548 and 7851 for monthly and quarterly returns, respectively[5]. This indicates that inversion of the matrices should be well behaved.

Cochrane (1996) argues that one can transform the weighting matrix using eigenvalue decomposition such that $W = \Gamma Q \Gamma'$ where Γ is an orthonormal matrix with the eigenvectors of W on its columns, and Q is a diagonal matrix of eigenvalues. Then, the HJ-distance problem in equation (15) can be rewritten as

[4] We thank Ken French for providing the data.

[5] For monthly scaled returns, the condition number is 10264; for quarterly scaled returns, the condition number is 5238.

$$\delta = \left[E(yR - p_n)' \Gamma Q \Gamma' E(yR - p_n) \right]^{1/2}.$$

The elements of the jth column in Γ can be interpreted as weights that are assigned to individual portfolios for the jth eigenvalue in Q. If there are a few large eigenvalues of W with eigenvectors that place large weights on only a few portfolios, the GMM problem may be choosing parameters that are associated only with a few portfolios. Because W does not change across models, it is not unfair to ask the competing models to price the same portfolios. But, we do want the structure of the weighting matrix to be reasonable. Figure 1 demonstrates which particular portfolios receive the largest weights for the largest two eigenvalues of the weighting matrices. The weights are standardized to sum to one. Figure 1 demonstrates that no particular portfolio receives an unusually large weight.

3.2 Conditioning Variables

3.2.1 Conditioning Variables to Scale Factors

We use five variables to capture movements in the prices of risks over the business cycle. For the monthly models, the cyclical part of the natural logarithm of the industrial production index is one conditioning variable. The industrial production index is from the Citibase monthly dataset. The series is available from January 1947 to April 1999 . We use the Hodrick-Prescott (1997) filter on the first five years to initialize the cyclical series. The smoothing parameter is set to be 6400. Thus the first element of our cycle is 1951:12. We then use the procedure recursively on all available data to find the subsequent elements for the cyclical series. This method guarantees that each element is in the time t information set. Panel A of Figure 2 gives the log industrial production index and the cyclical element IP we use.

As mentioned above, in monthly models we also scale the factors with a January dummy, JAN, that takes the value 1 for each January and is 0 otherwise. For quarterly models, JAN takes the value 1 for the first quarter and is 0 otherwise.

For the quarterly models, we also scale the factors with the cyclical component of real GNP. The data are also from the Citibase quarterly dataset (beginning in 1946:01). We use the recursive Hodrick-Prescott (1997) filter with the smoothing parameter equal to 1600. Because GNP is not announced until the following quarter, we lag GNP twice to make sure it is in the time t information set. Alternatively, Lettau and Ludvigson (1999) develop another conditioning variable, the change in the consumption-wealth ratio, CAY.[6] The CAY series is lagged one period to be a legitimate instrumental variable. Panels B and C of Figure 2 present the dynamics of GNP and CAY. The cyclical components of the GNP and CAY series are not particularly highly correlated. The contemporaneous correlation is -0.0441, and the cross correlations indicate that CAY leads GNP by 3 to 4 quarters, as theory predicts consumption should lead income.

Table 2 provides some information on the predictive power of the three conditioning variables except JAN. We use the conditioning variables to estimate the next period return on the value-weighted market return. All of the three conditioning variables have significant predictive power. The explained part of returns is small, as anticipated. With monthly data the R^2 for IP is 1%, and with quarterly data it is 3% for GNP, and 11% for CAY.

3.2.2 Conditioning Variable to Scale Returns

We only use one series as the conditioning variable for scaled returns. It is the term premium, calculated as the difference between the 30-year government bond yield and the 1-year government bond yield. The data are from CRSP, which provides a monthly index. We construct the quarterly series by using the end-of-quarter observations.

3.3 The Asset Pricing Models

We evaluate eight asset-pricing models. The simplest model incorporates only a constant in the SDF, and it is called the Null model. The Null model is used as a benchmark. With only a constant factor present, the distance between y and $\widetilde{m}$ is $\delta = \min_{m \in M} std(m)$. Thus, we can interpret the HJ-distance as the standard deviation for the least volatile element in M. In the conditional case, the Null model has two factors, the constant and the conditional *cycle*. Thus, the conditional Null model determines whether the movement in the cycle is an important pricing factor.

[6] The data are obtained from Ludvigson's website: http://www.ny.frb.org/rmaghome/economist/ludvigson.html. CAY is calculated as $CAY_t = c_t - wa_t - (1-w)y_t$, where c_t is consumption, a_t is asset wealth, y_t is labor income, and w is the weight of asset wealth in total wealth. w is estimated by OLS using all observations. Because of the cointegration relationship between c_t, a_t and y_t, the sample estimate ($\widehat{w}$) for w is said to be superconsistent. Lettau and Ludvigson (1999) argue that $\widehat{w}$ can therefore be treated as if it is the true parameter. Thus $\widehat{CAY}_t$, as a function of $\widehat{w}$, can be treated as if it is in time t information even though $\widehat{w}$ is estimated using all observations, and when using $\widehat{CAY}_t$ in estimation there is no need to adjust the standard errors for the sampling variability in $\widehat{w}$.

The second model is the CAPM. The model SDF has two factors, a constant, and the excess return on the market portfolio. We use the excess return on the value-weighted CRSP index over the one month risk free rate R_{VW}, as a proxy for the market excess return. For the quarterly model, we compound the monthly market returns to produce quarterly returns and subtract the three month interest rate. In the conditional model of the SDF, there are 4 factors: the constant, the *cycle*, R_{VW} and $R_{VW} \cdot cycle$.

The third model is a linearized CCAPM. The original CCAPM is non-linear and requires a particular form for the utility function. Rather than develop nonlinear models of marginal utility, we simply use consumption growth, Δc, as the factor. We use the growth rate in real nondurables consumption from Citibase. The unconditional model of the SDF has two factors, the constant and Δc. The conditional model has four factors: the constant, the *cycle*, Δc, and $\Delta c \cdot cycle$.

The fourth model is the conditional CAPM developed by Jagannathan and Wang (1996)(hereafter the JW model). This model is derived from the assumption that the CAPM holds as a conditional model and that the return on the market is predictable with the default premium, R_{PREM}, which is the difference between the yield on *baa* and *aaa* corporate bonds from the Board of Governors of the Federal Reserve. The JW model's unconditional form involves two betas. One is the original market-beta. The other beta incorporates variation in the market beta, which Jagannathan and Wang (1996) call beta-premium sensitivity. Beta premium sensitivity is captured by variation in the default premium. R_{PREM} measures the instability of the market beta over the business cycle. Jagannathan and Wang (1996) also argue that the value-weighted index is an inadequate proxy for the market return. They include labor income growth, R_{LBR}, as an additional factor reflecting a return to human capital.[7] There are four factors in the JW model, a constant, R_{VW}, R_{LBR} and R_{PREM}. We construct the data as described in Jagannathan and Wang (1996) for monthly models. For the quarterly model, R_{LBR} is calculated as the quarterly growth rate in labor income, and R_{PREM} is constructed by selecting the third observation in each quarter. Although the JW model is already an unconditional version of a conditional model, we also estimate our conditional version which implies a total of eight factors in the model SDF.

The fifth model is a linear version of Campbell's (1996) log-linear asset pricing model. Camp-

[7] Jagannathan and Wang (1996) measure labor income growth as $R_{LBR,t} = \frac{L_{t-1}+L_{t-2}}{L_{t-2}+L_{t-3}}$, where L is labor income per capita calculated as the difference between personal income and dividend income per capita. The data are obtained from Citibase. Jagannathan and Wang(1996) use a two-month average to "minimize the influence of measurement errors".

bell (1996) develops an intertemporal asset pricing model that allows for changes in investment opportunities. Factors are determined by their ability to predict the return on the market. As in Jagannathan and Wang (1996), Campbell (1996) argues that labor income is an important additional factor to fully reflect investor's wealth. However, the labor income factor, LBR, is constructed as the monthly growth rate in real labor income (from Citibase). The other three factors are the following: the dividend yield on R_{VW}, DIV; the relative bill rate, RTB, calculated as the difference between the 1-month T-bill rate and its 1-year backward moving average; and the yield spread between long and short-term government bonds TRM, which we constructed as the difference in yields between 30-year government bond and 1-year government bond. In total, there are six factors in the SDF for this model: the constant, R_{VW}, LBR, DIV, RTB and TRM. In Campbell (1996), the pricing proxy is actually defined as $y = \exp(-F'b)$ and there are constraints across the parameters. Here we simply put the six factors into a linear SDF model, $y = F'b$. For the conditional models, we have twelve factors in total.

The sixth model is a linearized version of Cochrane's (1996) production based asset pricing model. Cochrane (1996) argues that returns should be well priced by the investment return, which is a complicated function of the investment-capital ratio and several parameters. But, Cochrane (1996) finds that the investment growth rate performs equally well, and we adopt the investment growth rate model instead of the investment return model. The factors are the growth rate on real non-residential investment, GNR, and the growth rate on real residential investment, GR. Both original series are from Citibase. The model has three factors in the unconditional model, a constant, GNR, and GR. The conditional Cochrane model has six factors. The data are from Citibase. Since we only have quarterly data for real investment, we do not compute a monthly model in this case.

The above six models are all based on explicit economic theories. We also consider two empirical asset pricing models. They are called "empirical" because their key pricing factors are derived from the data. The seventh model is the Fama-French (1993) three factor model (hereafter the FF3 model). The first factor is the excess return on the market portfolio, R_{VW}, as calculated above. To mimic the risk factors in returns related to size and B/M ratio, Fama and French (1993) first sort all stocks into two size portfolios, *small* and *big*, they also sort all stocks into three B/M portfolios, *high*, *medium* and *low*. Factor SMB (small minus big) is constructed as the difference in returns on *small* and *big*, thus it captures risk related to size. Factor HML (high minus low) is constructed as the difference in returns on *high* and *low*, thus it captures risk related to B/M ratio. The unconditional model of the SDF has four factors: constant, R_{VW}, SMB, and HML. We construct quarterly factors by compounding the monthly factors. There are eight factors in the conditional model.

The eighth model is the Fama-French (1993) five-factor model in which they add a term-structure factor and a default-premium factor to their three factor model (hereafter the FF5 model). Fama and French (1993) use the difference between the yield on a thirty-year bond and the yield on the one-month bill as a term structure factor, that is, $TERM$. Default risk, R_{PREM}, is proxied by the difference between the yields on *baa* and *aaa* corporate bonds (as in JW). We construct quarterly data by compounding the monthly R_{VW}, SMB and HML, and we use the third observation of each quarter for $TERM$ and R_{PREM}. The conditional model has twelve factors.

4 Empirical Results

4.1 Basic Model Diagnostics

The basic model diagnostics are presented in the seven panels of Table 3. The first row of each panel reports the HJ-distance (δ) estimates. The second row provides the p-values of the test $\delta = 0$ as in equation (28). The third row contains the standard errors for the HJ-distance estimates calculated under the null hypothesis that the true distance is not equal to zero as in equation (45) of Hansen and Jagannathan (1997). These allow an assessment of the precision with which δ is estimated. The fourth row reports the p-values of the Wald tests that the pricing errors are all zero as in equation (30). The fifth row reports the p-values of the J-statistics from optimal GMM estimates of the models. The sixth row presents the values of the supLM test, and the seventh row provides the p-values for these tests from Table 1 of Andrews (1993). The eighth row reports the number of parameters.

Recall that the HJ-distance has two interpretations. It is the distance between the true SDF and the model's implied SDF, and it is the maximum pricing error for any portfolio formed from the basic assets with norm of the payoff on the portfolio equal to one. Since the second moment of the payoff equals one, and because the mean of the payoff must be less than the second moment, the true price of the payoff must be less than one if the expected return is to be greater than one. Thus, the maximum pricing error understates what the percentage pricing error would be.

Interpretation of the HJ-distance estimates in finite samples is hampered by the fact that zero is on the boundary of the parameter space. Even if the null hypothesis is true, we expect in finite samples that the estimated HJ-distance will be positive. Of course, if the p-values of the test statistics are well behaved, false rejections of the null hypothesis only occur the correct percentage of the time.

The Monte Carlo experiments conducted by Ahn and Gadarowski (1999) indicate that the expected value of the HJ-distance for a three factor model can be quite large and depends on the number of assets and the number of time periods. From Table 1 of Ahn and Gadarowski (1999) with 25 returns, we find average HJ-distances of 0.393 for 160 observations, 0.260 for 330 observations and 0.174 for 700 observations. Hence, by extrapolating to our monthly sample of 552 observations, we should not be surprised to see an HJ-distance equal to 0.21, even though a model is true. Similarly, for a quarterly sample of 180 observations, we should not be surprised to see an HJ-distance equal to 0.38, even though the model is true.

Ahn and Gadarowski (1999) also investigate the empirical size of the test that HJ-distance equals zero. For 25 assets they find that 5.5% of their experiments exceed the 1% critical value with 160 observations, 2.5% are greater with 330 observations, and 1.5% are greater with 700 observations. Thus, for our sample sizes, the monthly model appears to be close to having the correct size of the test if a three-factor model is true, while the rejection rates for the quarterly model appear to be too high.

Monthly Models. The first two rows of Panel A in Table 3 indicate that the Null model, the CAPM, the CCAPM, the JW model, and the FF3 model all have HJ-distances that are larger than 0.32. The p-values for the tests that these distances are zero are all less than 0.0001. The standard errors of the HJ-distances, calculated under the hypothesis that a model is false, are all about 0.05. The fourth and fifth rows report the Wald tests of whether the pricing errors on the twenty-six original portfolios are jointly zero when evaluated at either the parameters that minimize the HJ-distance or the parameters from optimal GMM, respectively. Generally, we find little disagreement between these tests, and in panel A of Table 3 we find five out of the six models are all rejected at the 0.001 level of significance or smaller. Campbell's model achieves the smallest HJ-distance, and the p-value of the test $\delta = 0$ indicates we cannot reject correct pricing. Thus the model captures the size and B/M effects and also prices the riskfree rate. It is noticeable that the same model also passes the J-test of optimal GMM. Unfortunately, Campbell's model does not have stable parameters. It fails the supLM test severely.

The HJ-distance of the FF5 model is smaller than that of the FF3 model. As one might suspect, this difference comes from the fact that the T-bill rate is hard for the FF3 model to price because the FF3 model only includes equity pricing factors. To evaluate this conjecture, we did a test which only used the twenty-five size and B/M portfolios. There were only small differences between the FF3 model and the FF5 model in that test. Even for the FF5 model, the point estimate of HJ-distance is still around 0.30. If we subtract the bias in the statistic of 0.21, we can conclude that the maximum pricing error is around 0.11.

Panel B of Table 3 reports the results when the factors are scaled by cycle(IP). We find the magnitudes of HJ-distances all shrink significantly by approximately 10% except for the Null model. The p-value's for the test of HJ-distance equal zero are now near 5%. We test whether the conditioning information is statistically significant with a Wald test on the joint hypothesis that the parameters for all scaled factors equal zero. For the CAPM, the CCAPM and the JW model, the p-value's are smaller than 0.02, which means the scaling variable IP significantly captures time-varying behavior of risks. Using cycle(IP) reduces HJ-distance for all models, and Campbell's model achieves the smallest distance. Scaled factors also improve the supLM statistics, though none of the models pass both the test of HJ-distance equal zero and the supLM test. It is notable that the CAPM with scaled factors marginally passes both the test of HJ-distance equal zero and the optimal GMM test. Again, all results from minimizing HJ-distance are similar to what we find from the optimal GMM approach.

The fact that scaled factor models have smaller HJ-distances than non-scaled factors models comes from two sources. First, the conditioning information reduces the pricing errors by allowing the prices of risks to vary with the business cycle. Second, by doubling the number of parameters, a scaled factor model uses additional degrees of freedom in the minimization problem and is better able to fit the data. This better fit may be spurious, though, as small-sample biases may worsen. In the next section, we will examine the details of individual models.

According to Loughran (1997), the January effect explains a substantial part of the B/M effect. When we allow only for a January dummy variable in addition to the constant term of the SDF's, there are very few changes compared to the results in Panel A of Table 3. These results are not reported to save space. Panel C of Table 3 reports results with all factors scaled by JAN. This effectively separates the January observations from the non-January observations by allowing different factor prices in January. For the Null model, the Wald statistic for the test that the conditional parameter equals zero in the Null model is 0.0001, which demonstrates the importance of a January effect. Allowing for a January conditioning variable improves the point estimates of HJ-distance for all the models. Nevertheless, p-values of the J statistics indicated that the CAPM, the CCAPM, and the FF3 models are still rejected at the 0.05 level of significance. The most dramatic improvement is in the JW model which now passes all of the tests except the stability test. The Wald test on the importance of the scaled factors indicates their joint significance. There is a slight improvement in the performance of the FF3 model although the joint test of the significance of the scaled factors has a p-value of 0.15. The FF5 model and Campbell's model already do reasonably well with non-scaled factors. Scaling all the factors in these models with a January dummy does not appear to add any important factors since the p-values of the Wald tests are both quite large.

Quarterly Models. The previous literature typically reports either monthly or quarterly models. Some models, such as Cochrane's (1996) model, can only be applied to quarterly data because of data constraints. In this section we investigate the performance of the models with quarterly data. Several issues arise. First, time aggregation may worsen the fit between the factors and the models by smoothing the factors[8]. Second, market imperfections that cause short-term deviations from the models may be lessened because the returns are cumulated. Third, as noted above, the small-sample performance of any model deteriorates with a smaller number of observations. The first and third effects suggest the performance of the models with quarterly data deteriorates, while the second factor allows for improvement.

Panel D provides the summary results for the eight quarterly models, the seven previously investigated plus Cochrane's (1996) model. Although the point estimates of the HJ-distances are much larger for the quarterly models than the monthly models, recall from our discussion of Ahn and Gadarowski (1999) that values like 0.38 are to be expected in these sample sizes even if the model is true. Nevertheless, the quarterly HJ-distances generally exceed the average of the Ahn and Gadarowski (1999) figures by more than the monthly estimates exceed the corresponding average from the Monte Carlo experiments. For example, the monthly FF3 model has an HJ-distance of

[8]This logic leads Cochrane (1996) to time average monthly returns in constructing quarterly returns. While we construct the quarterly returns from the compound monthly returns as $R_{t+1} + R_{t+2} + R_{t+3}$, Cochrane (1996) uses $\frac{1}{3}R_{t+1} + \frac{2}{3}R_{t+2} + R_{t+3} + \frac{2}{3}R_{t+4} + \frac{1}{3}R_{t+5}$.

0.323 and the Monte Carlo average is approximately 0.21 for a difference of 0.113. At the quarterly sampling interval we find a difference of 0.537-0.38=0.157.

While the p-values of the tests that HJ-distance equals zero are all less that 0.0370, recall also that in this sample size the asymptotic p-values probably understate the probability of a type I error as Ahn and Gadarowski (1999) find that 15.7% of their empirical experiments exceed the 0.05 asymptotic critical value in samples of 160 observations. Hence, it seems reasonable to conclude that the evidence against the JW model, the FF5 model, and Campbell's model is not particularly strong. Unfortunately these three models all fail the parameter stability test.

In Panel E, we scale all factors by the lagged cyclical component of GNP. Including the conditioning information reduces the magnitude of HJ-distance by 5-10%. Two models, the FF3 model and Cochrane's, now pass the test of HJ-distance equal zero and the supLM test. Once again the HJ-distance tests are consistent with the results from optimal GMM. The test that all $\widehat{b}$'s for scaled factors equal zero indicates scaling with GNP does not significantly improve the performance of the models. One should keep in mind, though, this is a joint test which may overshadow the significance of individual parameters.

An alternative quarterly scaling variable is CAY from Lettau and Ludvigson (1999). They find that scaling with CAY greatly improves the performance of the CCAPM in pricing the excess returns on the twenty-five FF portfolios over a sample period 1963 to 1997. However, for our sample of 1953 to 1998, CAY does not produce a noticeable improvement for the CCAPM. The scaled model fails both the test of HJ-distance equal zero and the optimal GMM test. None of the models scaled by CAY passes both the test of HJ-distance equal zero and the supLM test.

Panel G provides results when all the factors are scaled by JAN. For the quarterly models, JAN takes the value 1 for the first quarter of each year, and 0 otherwise. The first thing to note is scaling all factors with JAN reduces the magnitude of the HJ-distance for all models. The JW model, the FF5 model and Campbell's model all have p-values for the test of HJ-distance equal zero above 80%. Surprisingly, the FF3 model does not pass the HJ-distance test and the J test. This is because the scaled factor model is still unable to price the small growth firms. Cochrane's model passes both the test of HJ-distance equal zero and the supLM test. More details for this model are provided in the section on successful models.

Correlations of Adjustment to Pricing Proxies. If the adjustments to two models as calculated in equation (13) are highly correlated, we know the pricing element lacking in one model is also left out of the other model.

Panel A of Table 4 reports the correlations of the adjustments between the monthly models. The Null model is the benchmark in the first grid. Those models that have a high correlation with it would be less likely to pass the HJ-distance test and the optimal GMM test. The CAPM, CCAPM and the JW model all have high correlation (over 0.90) with the Null model, and from Table 3, we know they all fail the two tests. The FF3 model, the FF5 model and Campbell's model have relatively low correlations with the Null model, and as we already know, the last one passes both tests. Still there are differences between the three models. Since the FF3 model is nested in the FF5 model, they have a correlation coefficient of 0.91. The difference comes from the macro variables in the FF5 model. Both the FF5 model and Campbell's model include the term premium, and they have a correlation of 0.82.

By adding conditioning information, the correlations between the necessary adjustments to the models to make them equal the true SDF are reduced. From the first column of Table 3, we find the adjustments to the CAPM, CCAPM, and the JW model are now correlated with the Null model at 0.75-0.85 level, and the adjustments to the FF3 model, the FF5 model and Campbell's model are correlated with the Null model at 0.6-0.7 level. Thus, conditioning information aids in explaining time-varying risks and changes the pattern of the adjustments.

Panel B of Table 4 reports the information for quarterly models, which is similar to what we have for monthly models. One should notice that although the correlations between models are different, the numbers are big(above 0.50). This means either those models share the same problem or the statistics suffer from small sample biases. Once again, since HJ-distance will be positive in any model in a finite sample, correlations of pricing errors will be positive and possibly quite large even if a model is true.

4.2 Model Errors and Pricing Errors for Non-scaled Factor Models

Additional information on the performance of the models is available by examining the model errors and the Lagrange multipliers which are the components of δ. To check whether conditioning information improves the performance of a model, we first need to understand the performance of the original non-scaled factor model. The average model errors with their standard errors are presented in Figure 3. Since monthly unconditional model errors share very similar patterns with the quarterly model errors, we only present monthly model errors g_T as defined in equation (20).

For Cochrane's model, we report quarterly model errors.

The model errors for the Null model range from 1.15% per month for portfolio 25 to -0.01% for the T-bill rate. The B/M effect is very evident in the Figure as in each quintile, higher B/M portfolios have larger pricing errors. There is less dispersion in the pricing errors across the five B/M portfolios as size increases. The model under-estimates the returns on all portfolios except the T-bill rate.

From Panel B, the CAPM correctly prices the largest size portfolios, but it tends to under-estimate returns on high B/M portfolios and to over-estimate returns on low B/M portfolios. The model error is between -0.50% per month and 0.45% per month.

The CCAPM is presented in Panel C. It has a pattern very similar to the Null model, which is consistent with the high correlation between the adjustments $y - \widetilde{m} = \widetilde{\lambda}' R$ of the Null model and CCAPM.

The JW model is presented in Panel D of Figure 3. It has a very similar pattern to the CAPM except the over-estimation for low B/M portfolios is slightly smaller.

Panel E reports the pattern for Campbell's pricing errors. The model considerably attenuates the B/M effect. The average errors range from -0.28% to 0.30%. Part of the ability of the model to pass the test of HJ-distance equal zero arises from its increased standard errors relative to the CAPM. Although δ can be compared across models, the p-values of the tests are not comparable because they are based on the eigenvalues of A in equation (27) which depends on the pricing factors, the variance of pricing errors and the number of parameters.

Panel F presents the pricing errors in Cochrane's model which share the same magnitude and pattern as the quarterly CAPM. There is a distinct B/M effect as in the monthly CAPM.

The FF3 model is presented in Panel G. The additional two factors SMB and HML dampen the size effect and the B/M effect. Now there is no particular pattern for the model errors. They are scattered around the zero axis. The FF3 model over-predicts the average returns for both the smallest firms and the largest firms, but especially the small growth stocks (low B/M ratio).

The FF5 model has a similar pattern to the FF3 model, except it reduces the pricing errors slightly.

All models share one common characteristic, they do not misprice the T-bill rate. Model errors for the T-bill rate are always around zero.

4.3 Interesting Models

Since we have 21 monthly models and 32 quarterly models, we are unable to display parameter estimates for all of them, but we report results for "interesting models". Our definition of "interesting" is the model at least marginally passes the test of HJ-distance equal zero at the 1% marginal level of significance and the scaling parameters for scaled factor models are jointly significant at the 5% level. As we observed in the previous section, the test of HJ-distance equal zero always produces similar results to those of the J test from optimal GMM. Hence, that is implicitly a criterion. In total we have 12 models satisfying both conditions. In addition we provide information on the monthly FF3 model with non-scaled factors for comparison. This section first discusses monthly models, then quarterly models.

Table 5 reports parameter estimates from minimizing the HJ-distance measure for all interesting models. Each panel has two parts. The first part presents estimates for b as in equation (3). If b_1 for one factor is significantly different from zero, then the factor is an important determinant of the pricing kernel. The second part of each panel presents estimates for Λ as in equation (7). It provides information on whether the factors significantly influence the expected returns.

Monthly Models. The first model is the monthly CAPM with factors scaled by IP. The model marginally passes the test of HJ-distance equal zero with a p-value of 0.0255. Both R_{VW} and IP are important for the correct pricing kernel. Thus the business cycle incorporated in IP cannot be omitted from the pricing kernel. The same two factors are significantly priced for the basic twenty-six portfolios with the positive signs. Thus IP helps to explain the size effect and the B/M effect. In the framework of Jagannathan and Wang (1996), IP could be a proxy for beta-sensitivity. Panel A of Figure 4 reports the model's pricing errors, with its non-scaled counterpart. With two more factors, IP and $R_{VW} \cdot IP$, most of the improvements are for low B/M portfolios, and the biggest one happens for the smallest growth firms. With size increasing, the improvement becomes smaller. However, the scaled factors model cannot eliminate either the size effect or B/M effect. The monthly CAPM with factors scaled by IP also does not pass the supLM test at the 5% level, so the estimates may be unstable.

The second monthly model is the CCAPM with factors scaled by IP. Parameter estimates are reported in Panel B of Table 5. The test of HJ-distance equal zero is passed with a p-value of 0.0408. From the estimates for b, we find Δc, IP and $\Delta c \cdot IP$ are all significantly priced for the SDF. The estimates for Λ indicate that both Δc and IP are priced for the underlying twenty-six portfolios with the correct signs. As in the monthly CAPM with factors scaled by IP, the business cycle element IP is important for both the pricing kernel and the pricing of individual portfolios. The monthly CCAPM with factors scaled by JAN also satisfies both conditions for being "interesting". For comparison, its parameter estimates are provided in Panel C of Table 5. Now only the interaction between Δc and JAN is significant for both the pricing kernel and prices of risk. While this result literally implies that the consumption growth rate is important only in January, an alternative interpretation is that the return characteristics of the underlying twenty-six portfolios are most evident in January. The pricing errors for the two scaled factor versions of the CCAPM together with the non-scaled factor benchmark are given in Panel B of Figure 4. One finds that when the factors are scaled by IP, the improvements mostly happen for the high B/M portfolios by 0.1% to 0.2% per month, thus the pricing errors when factors are scaled by IP are flatter than the original non-scaled CCAPM. When the factors are scaled by JAN, both the size effect and the B/M effect are much smaller. As a result, the line connecting the pricing errors is somewhat flatter.

Panel D of Table 5 reports the parameter estimates for the monthly JW model with factors scaled by IP. The p-value for the test of HJ-distance equal zero is 0.0574. First, both R_{VW} and $R_{PREM} \cdot IP$ are important factors for the correct pricing kernel. The same two factors with $R_{LBR} \cdot IP$ significantly affect risk premiums. Panel E of Table 5 presents the parameter estimates for the monthly JW model with factors scaled by JAN. From the estimation of b, both R_{PREM} and $R_{PREM} \cdot JAN$ are significant determinants of the model's pricing kernel. It is interesting to find the default premium is priced differently in January(-0.28+0.13=-0.15) and outside January(-0.28). Jagannathan and Wang (1996) find a positive price of risk for R_{PREM} which appears to be driven primarily by a January effect. The pricing errors of the above two models together with their non-scaled factors benchmark are presented in Panel C of Figure 4. When the factors are scaled by IP, the pricing errors are smaller for both the small firms and high B/M firms. Thus IP helps dampen both the size effect and the B/M effect. When the factors are scaled by JAN, the pricing errors are even smaller, as in the CCAPM above. The p-value of the test of HJ-distance equal zero is 0.6497. However, neither of the models passes the supLM test.

Campbell's model with non-scaled factors is reported in Panel F of Table 5. The model passes the test of HJ-distance equal zero with a p-value 0.3471. Both DIV and TRM are important for the correct pricing kernel from the estimates of b. In the lower part of the panel, we present the estimate for the prices of risks Λ. R_{VW}, DIV and TRM are all significantly priced. Neither labor income nor the relative bill rate is important. Panel D of Figure 4 reports the model's pricing errors along with the errors from the FF3 model as the benchmark. No size effect is apparent and Campbell's model prices the small growth firms better than FF3. While a B/M effect is present, its magnitude is not large. Overall, the pricing errors for Campbell's model are not bigger than those of FF3's, while the latter model is constructed to price the size effect and B/M effect. However, Campbell's model fails the supLM test. Thus the parameter estimates are not stable and should be used cautiously.

The last monthly models we report are FF3 with non-scaled factors and FF3 with factors scaled by JAN. FF3 is reported because we want to examine whether it can price the size and B/M effects which it is constructed to do. It does not pass the test of HJ-distance equal zero. Parameter estimates for FF3 are presented in Panel G of Table 5. It is somewhat surprising to find that only R_{VW} and HML are important for the pricing kernel, and they are also significantly priced risk factors. Panel E of Figure 4 provides the pricing errors for FF3. The problem portfolios are the lowest B/M with smallest and second smallest sizes, which are overpriced by the model. Thus, the factor SMB cannot adequately capture the size effect in the portfolios, and SMB is not significantly priced in the unconditional version when risk prices are held constant.

The monthly FF3 with factors scaled by JAN is reported in Panel H of Table 5. It passes the test of HJ-distance equal zero with a p-value of 0.1012. From the estimates of b, R_{VW}, SMB and $SMB \cdot JAN$ are important factors for the pricing kernel. For the prices of risks, R_{VW}, HML and $SMB \cdot JAN$ are significant. This is consistent with the view that the size effect is primarily a January effect.

As mentioned in the previous section, if the B/M effect mainly occurs in January, and HML explains the B/M effect, HML will not be priced outside January. Thus, the results tell us either there is still a significant B/M effect outside of January or there are some other risks which can be priced by HML. We also examine the pricing errors to see whether scaling by JAN really improves on the performance of the FF3 model in an interesting way. In the Panel E of Figure 4, we find that adding JAN actually reduces the pricing errors by 0.2% for the smallest growth stocks. Since the FF3 model already captures the B/M effect reasonably well, JAN does not improve this dimension. Both models pass the supLM test.

Quarterly Models. The first quarterly model is the JW model. It marginally passes the test of HJ-distance equal zero with a p-value 0.0370. The parameter estimates are presented in Panel I of Table 5. Only R_{PREM} is important in the pricing kernel. For the prices of risks, R_{PREM} is also significant, but with a negative sign in contrast to Jagannathan and Wang (1996). In addition, the price of market risk is marginally significant. The pricing errors of the JW model are reported in Panel F of Figure 4 together with the quarterly FF3 with non-scaled factors as benchmark. Both the size effect and the B/M effect are evident in the Figure, and most of the errors range from 0.5% per quarter to 2% per quarter. These pricing errors are quite large compared to those of FF3. Thus the quarterly JW model passes the HJ-distance test not because it has small pricing errors but because it has larger standard errors. Hence, the JW model with non-scaled factors is not an economically interesting model. It also fails the supLM test indicating that the parameter estimates are not stable.

The second quarterly model is Campbell's model with non-scaled factors. The test of HJ-distance equal zero has a p-value 0.0159. Panel J of Table 5 provides the parameter estimates. As in the monthly models, the term premium is important in the pricing kernel. Both market risk and term premium risk are priced factors for the risk premiums. The pricing errors are reported in Panel G of Figure 4 together with the benchmark FF3. The pattern of the errors is very similar to the monthly models we provide in Panel D. Campbell's model improves on the smallest growth portfolio, but it has an evident B/M effect. It also fails the supLM test.

The third quarterly model is Cochrane's model with factors scaled by the cyclical element in lag GNP. The parameter estimates are given in Panel K of Table 5. For the pricing kernel, both $RINV$ and $RINV \cdot GNP$ are important. This is consistent with Cochrane (1996) who demonstrates the importance of residential investment. For the twenty-six portfolios we are considering, only the latter factor is significantly priced with a correct sign. The HJ-distance measure drops from 0.6255 for Cochrane's non-scaled factors model to 0.5585 for its scaled factors model. In all of the above models, scaled-factor models perform better than non-scaled models, and we confirm the scaling factors are economically interesting by looking at the pricing errors and parameter estimates. However, for Cochrane's model, the improvement in HJ-distance does not actually come from the improvements on pricing errors. This can be seen in Panel H of Figure 4. The pricing errors of the non-scaled model show a distinct pattern of size and B/M effects. The scaled factors model shifts most of the pricing error upward by 0.5-1%. There is improvement only for the first portfolio. The smaller HJ-distance for the scaled factor model arises because the additional free parameters make it easier for Cochrane's model to solve the minimization problem with the particular weighting matrix. This is significant statistically, but it is not interesting economically. Panel L of Table 5 reports the quarterly Cochrane model with factors scaled by JAN. Both JAN and $NRINV \cdot JAN$ are important for the pricing kernel. Thus the January effect itself as a constant is important for the pricing kernel. The same two factors are also priced significantly for the size effect, the B/M effect and the riskfree rate. By looking at Panel H of Figure 4, we find after controlling for the January effect, the pricing errors are shifted downward by 1-1.5%, which is a big improvement for value firms. The B/M effect is mitigated but still present. Thus we conclude that the improvement on HJ-distance measure is from the improvement of pricing errors. Both Cochrane's scaled factors models are stable, and they both pass the supLM test.

The quarterly FF5 model with non-scaled factors is provided in Panel M of Table 5. It passes the test of HJ-distance equal zero with p-value 0.0180. From the estimates of b, we find that R_{VW} and HML are priced, as in FF3, and the two macro factors, $TERM$ and R_{PREM} are both important for the correct pricing kernel. However, the latter two factors are not significantly priced risk factors for the twenty-six portfolios. The pricing errors from FF5 in Panel H of Figure 4 are almost the same as those in FF3. There are only small improvements on the smallest growth portfolios. Unfortunately, the two additional macro factors bring instability into the model as it fails the supLM test.

There is one last issue to note. All of the models do well in pricing the gross return of the T-bill. This implies that although the minimization problem does not put a particularly large weight on the T-bill return, it does not ignore it either. Others, such as Lettau and Ludvigson (1999) and Jagannathan and Wang (1996), only include stock portfolios and have big estimates for the zero-beta rate. We estimate the zero-beta rate for each model. For monthly models, the rate is around 0.4% per month; for quarterly models, it is around 1.8% per quarter. We believe these estimates are more reasonable.

4.4 Robustness

In the above results, we obtain parameter estimates and conduct tests using non-scaled returns. To examine whether these models are robust, we change the underlying assets from non-scaled returns to scaled returns, and we investigate whether the parameter estimates obtained from non-scaled return models (the first stage estimates) can price the scaled returns. We scale returns with the term premium, the difference in yields between a thirty-year government bond and a one-year government bond. If a model is able to price the basic assets (non scaled-returns), and it is specified correctly, it should be able to price the managed portfolios (scaled-returns).

Table 6 provides the information on these experiments. We use the estimates obtained from the first stage by optimal GMM, to calculate test of the HJ-distance equal zero and the J-statistic for optimal GMM for the new orthogonality conditions, as in equations (41), (42) and (43). These p-values are denoted p_1 and p_2. We also use the first-stage estimates of HJ-distance to calculate second-stage HJ-distance tests, and the p-value is denoted p_3. None of the monthly models successfully prices the new assets.

5 Conclusion

The purpose of this paper is to evaluate a number of asset pricing models that have been advanced in light of the anomalies that have been uncovered in testing the CAPM. The models are compared on a common set of returns: twenty-five size and book-to-market portfolios constructed as proposed in Fama and French (1993) for a sample period from 1952 to 1997. Average excess returns across these portfolios are as low as 0.36 percent per month and as high as 1.13 percent per month. Within a size quintile, higher book-to-market portfolios have higher average returns. Within all but the lowest book-to-market quintiles, average returns are generally decreasing in size. The unconditional CAPM cannot explain these returns.

We consider only linearized versions of the models, and we evaluate the models with both non-scaled factors and scaled factors, where the scaling reflects either business-cycle movements or a January dummy. The models are compared using the methodology of Hansen and Jagannathan (1997) who recognize that the estimated distance between a model's pricing kernel and the true pricing kernel also is an estimate of the maximal mis-pricing of a portfolio of the assets with norm of the portfolio return equal to one. We also evaluate the models using the optimal GMM test of Hansen (1982). In general, we find little disagreement between the two tests. Finally, we evaluate the temporal stability of the parameters using the supLM test of Andrews (1993).

For monthly models with non-scaled factors, Campbell's (1996) model is the only model that passes the test of HJ-distance equals zero, and its estimated HJ-distance is also smaller than that of the Fama-French (1993) three-factor model. Only three of the five factors in the model appear to be important: the return on the market portfolio, the dividend yield, and the term premium. Unfortunately, the Campbell model fails to pass the stability test. While the simulation study of Ahn and Gadarowski (1999) provides some support that the small-sample distributions of the HJ-distance test are reliable for our sample size, no comparable study of the small-sample distributions of the stability test has been conducted. Thus, additional study of the Campbell model appears to be desirable. In particular, we evaluate only the linearized version of the model.

Scaling the risk factors of the models with the cyclical element in industrial production as measured by the Hodrick-Prescott (1997) filter improves the performance of several of the models. The CAPM, CCAPM, and Jagannathan and Wang (1996) models all have significant coefficients on the scaled factors. There is also evidence that pricing in January is significantly different than pricing outside of January. For example, when the three factors of the Fama-French (1993) model are entered without scaling, only the market return and the HML portfolio are significant risk factors. When the factors are also scaled with a January dummy, the market return and the HML portfolio retain their significance and the SMB portfolio is significant in January. This latter model also passes the stability test.

With quarterly data, none of the models with non-scaled factors passes the test of HJ-distance equal to zero. Nevertheless, the simulation results of Ahn and Gadarowski (1999) suggest that these results should be interpreted with care as the sizes of the tests appear to deteriorate in this sample size. Neither scaling with the cyclical component of GNP as measured by the Hodrick-Prescott (1997) filter nor scaling with the consumption-wealth series of Lettau and Ludvigson (1999) has much of an influence on the results.

Additionally, none of the models, either monthly or quarterly appears to be robust in the following sense. When we estimate the parameters of the models using the basic returns and ask the models to price the set of assets constructed by scaling returns with the term premium, all of the models fail.

There are several directions in which this study could be extended. First, we construct our estimates as if there are no transactions costs in asset markets. Hanna and Ready (1999) find that transaction costs reduce but do not eliminate the CAPM anomalies.. Luttmer (1996) notes that a small transaction costs can have large implications for the variability of implied stochastic discount factors. Future research should be directed to determine how transaction costs affect the estimates of HJ-distance. Liquidity and market impact of trading individual assets may also be important. The study by Brennan, Chordia, and Subrahmanyam (1998) suggests that average returns on individual equities are affected by trading volume, which is consistent with differences in liquidity premiums across assets. Understanding how liquidity is priced and the role it plays in portfolio returns is an open issue. The presence of these market frictions implies that it may be difficult if not impossible to realize the returns that certain trading strategies imply. It is only truly available returns that require adjustment for risk.

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Table 1: summary statistics for Fama-French 25 portfolios

Panel A: means

portfolios	BM1	BM2	BM3	BM4	BM5
SIZE1	0.36	0.77	0.83	1.03	1.13
SIZE2	0.49	0.78	0.96	1.00	1.15
SIZE3	0.59	0.76	0.80	0.97	1.04
SIZE4	0.60	0.60	0.82	0.87	1.02
SIZE5	0.57	0.63	0.68	0.67	0.85

Panel B: standard errors

portfolios	BM1	BM2	BM3	BM4	BM5
SIZE1	7.17	6.25	5.56	5.26	5.53
SIZE2	6.49	5.62	5.11	4.85	5.39
SIZE3	5.94	5.04	4.66	4.50	5.14
SIZE4	5.32	4.80	4.61	4.52	5.22
SIZE5	4.54	4.39	4.09	4.24	4.91

Panel C: t-statistics

portfolios	BM1	BM2	BM3	BM4	BM5
SIZE1	1.18	2.91	3.52	4.58	4.82
SIZE2	1.76	3.25	4.41	4.85	5.03
SIZE3	2.33	3.55	4.05	5.04	4.76
SIZE4	2.64	2.93	4.17	4.50	4.60
SIZE5	2.97	3.36	3.89	3.74	4.07

The data are monthly returns on Fama-French 25 portfolios from 1952:01 to 1997:12 in excess of 1-month T-bill rate. Increasing portfolio numbers indicate increases in either size or book-to-market ratio.

Table 2: Predictive power of conditioning variables used to scale factors

Panel A: monthly cycle = IP

	constant	cycle	R^2
b	0.01	-0.13	0.01
s.e.(b)	0.00	0.06	

Panel B: quarterly cycle = GNP

	constant	cycle	R^2
b	0.01	-0.77	0.03
s.e.(b)	0.01	0.34	

Panel C: quarterly cycle = CAY

	constant	cycle	R^2
b	0.00	2.52	0.11
s.e.(b)	0.01	0.56	

The estimated OLS regression is $R_{vw}(t) = b_0*\text{constant} + b_1*\text{cycle}(t-1)+\varepsilon(t)$. R_{vw} is the value-weighted return from CRSP. For the monthly regression, the sample period is 1952:01 to 1997:12. For the quarterly regression, the sample period is 1953:01 to 1997:04. The series IP and GNP are the Hodrick-Prescott (1997) filtered industrial production and real income respectively. The series CAY is the consumption-wealth ratio calculated by Lettau and Ludivigson (1999).

Table 3: summary of models using excess returns with T-bill (26 portfolios)

Panel A: monthly non-scaled returns with non-scaled factors

MODEL	NULL	CAPM	CCAPM	JW	CAMP	FF(3)	FF(5)
HJ-dist(δ)	0.4198	0.3900	0.4293	0.3861	0.2961	0.3230	0.3164
p(δ=0)	0.0000	0.0000	0.0000	0.0000	0.3471	0.0000	0.0007
se(δ)	0.0510	0.0503	0.0633	0.0519	0.0648	0.0524	0.0547
p-Wald(err)	0.0000	0.0000	0.0000	0.0000	0.3059	0.0010	0.0053
p(J)	0.0000	0.0000	0.0000	0.0000	0.1944	0.0011	0.0045
supLM stat	216.5006	3.5479	4.2343	38.2902	193.9762	9.9709	58.8892
supLM test	Fail	pass	pass	fail	fail	pass	fail
No. of para	1	2	2	4	6	4	6

Panel B: monthly non-scaled returns with scaled factors by cycle(IP)

MODEL	NULL	CAPM	CCAPM	JW	CAMP	FF(3)	FF(5)
HJ-dist(δ)	0.4101	0.3515	0.3890	0.3138	0.2556	0.3021	0.2728
p(δ=0)	0.0000	0.0255	0.0408	0.0574	0.5804	0.0096	0.1431
se(δ)	0.0543	0.0639	0.0838	0.0502	0.0789	0.0616	0.0620
p-Wald(err)	0.0000	0.0640	0.0072	0.0491	0.6149	0.0239	0.2039
p-Wald(b)	0.0639	0.0123	0.0233	0.0136	0.6157	0.3502	0.3717
p(J)	0.0004	0.2694	0.0015	0.0624	0.5336	0.0265	0.2180
p-Wald(b*)	0.0063	0.0028	0.0205	0.0156	0.4859	0.3287	0.3981
supLM stat	10.0277	15.9634	9.8311	28.2542	73.9089	16.6455	40.2039
supLM test	Pass	fail	pass	fail	fail	pass	fail
No. of para	2	4	4	8	12	8	12

Panel C: monthly non-scaled returns with scaled factors by JAN

MODEL	NULL	CAPM	CCAPM	JW	CAMP	FF(3)	FF(5)
HJ-dist(δ)	0.3963	0.3657	0.3665	0.2738	0.2842	0.2866	0.2682
p(δ=0)	0.0000	0.0000	0.0574	0.6497	0.1260	0.1012	0.3351
se(δ)	0.0598	0.0668	0.0892	0.0863	0.0640	0.0493	0.0667
p-Wald(err)	0.0000	0.0003	0.0427	0.7780	0.1023	0.0167	0.2576
p-Wald(b)	0.0001	0.0419	0.0479	0.0213	0.9521	0.1535	0.5431
p(J)	0.0000	0.0002	0.0223	0.8086	0.0652	0.0253	0.0976
p-Wald(b*)	0.0000	0.1651	0.0257	0.0180	0.9616	0.2378	0.5935
supLM stat	5.6920	6.2444	10.3446	52.6631	180.9788	13.4695	39.2249
supLM test	pass	pass	pass	fail	fail	pass	fail
No. of para	2	4	4	8	12	8	12

Panel D: quarterly non-scaled returns with non-scaled factors

MODEL	NULL	CAPM	CCAPM	JW	CAMP	COCH	FF(3)	FF(5)
HJ-dist(δ)	0.6490	0.6206	0.6186	0.5784	0.5501	0.6255	0.5368	0.5156
p(δ=0)	0.0000	0.0000	0.0007	0.0370	0.0159	0.0000	0.0010	0.0180
se(δ)	0.1034	0.0973	0.1076	0.1246	0.1074	0.1126	0.1157	0.1046
p-Wald(err)	0.0005	0.0005	0.0026	0.0545	0.0187	0.0003	0.0087	0.0684
p(J)	0.0005	0.0005	0.0048	0.0832	0.0499	0.0002	0.0102	0.1250
SupLM stat	55.0231	3.6710	10.0706	31.0784	55.9571	10.0257	8.7462	52.1701
SupLM test	fail	pass	pass	fail	fail	pass	pass	fail
No. of para	1	2	2	4	6	3	4	6

Panel E: quarterly non-scaled returns with scaled factors by cycle(lag GNP)

MODEL	NULL	CAPM	CCAPM	JW	CAMP	COCH	FF(3)	FF(5)
HJ-dist(δ)	0.6418	0.6004	0.6129	0.5432	0.5038	0.5585	0.4522	0.4291
p(δ=0)	0.0000	0.0013	0.0000	0.0878	0.1473	0.1080	0.4881	0.3624
se(δ)	0.0990	0.0820	0.1061	0.1106	0.1039	0.1285	0.1078	0.0990
p-Wald(err)	0.0002	0.0043	0.0004	0.1414	0.1162	0.1084	0.2346	0.1949
p-Wald(b)	0.1501	0.2030	0.5238	0.4105	0.7939	0.1426	0.2384	0.3347
p(J)	0.0002	0.0105	0.0014	0.0558	0.1007	0.0859	0.4233	0.2537
p-Wald(b*)	0.2194	0.0507	0.7989	0.0129	0.5747	0.0084	0.1108	0.2422
SupLM stat	10.8365	11.0756	11.5782	37.0059	44.6401	9.8478	11.2852	34.0714
SupLM test	pass	pass	pass	fail	fail	pass	Pass	fail
No. of para	2	4	4	8	12	6	8	12

Panel F: quarterly non-scaled returns with scaled factors by CAY

MODEL	NULL	CAPM	CCAPM	JW	CAMP	COCH	FF(3)	FF(5)
HJ-dist(δ)	0.6342	0.6134	0.6080	0.5443	0.5152	0.6234	0.5278	0.4975
p(δ=0)	0.0000	0.0000	0.0001	0.2691	0.0985	0.0000	0.0010	0.0105
se(δ)	0.0993	0.1102	0.1049	0.1543	0.1246	0.1143	0.1054	0.0895
p-Wald(err)	0.0007	0.0003	0.0015	0.0778	0.0198	0.0001	0.0046	0.0383
p-Wald(b)	0.0767	0.4116	0.3338	0.8286	0.7796	0.9450	0.8092	0.8587
p(J)	0.0011	0.0002	0.0010	0.4282	0.0967	0.0005	0.0030	0.0324
p-Wald(b*)	0.0122	0.5416	0.2529	0.4044	0.8340	0.6092	0.9312	0.9304
SupLM stat	14.0275	14.3103	7.1698	39.1712	40.3727	16.7572	20.1487	30.9369
SupLM test	fail	pass	pass	fail	fail	pass	pass	fail
No. of para	2	4	4	8	12	6	8	12

Panel G: quarterly non-scaled returns with scaled factors by JAN

MODEL	NULL	CAPM	CCAPM	JW	CAMP	COCH	FF(3)	FF(5)
HJ-dist(δ)	0.5903	0.5638	0.5818	0.3908	0.3791	0.5099	0.5085	0.3941
p(δ=0)	0.0006	0.0014	0.0003	0.9970	0.9745	0.4290	0.0053	0.8703
se(δ)	0.1349	0.1274	0.1312	0.2389	0.1952	0.1329	0.1294	0.1488
p-Wald(err)	0.0091	0.0038	0.0078	0.9939	0.9708	0.4013	0.0038	0.3941
p-Wald(b)	0.0000	0.0001	0.0101	0.1530	0.4625	0.0175	0.3532	0.4322
p(J)	0.0106	0.0032	0.0096	0.9973	0.9844	0.5999	0.0037	0.9097
p-Wald(b*)	0.0000	0.0001	0.0055	0.2059	0.4346	0.0011	0.6764	0.4995
SupLM stat	8.5860	9.1810	9.1330	32.2225	28.3110	11.7944	20.1440	52.1233
SupLM test	pass	pass	pass	fail	pass	pass	pass	fail
No. of para	2	4	4	8	12	6	8	12

The data are returns on Fama-French 25 portfolios in excess of the T-bill rate and the return on T-bill. Monthly data are from 1952:01 to 1997:12; quarterly data are from 1953:01 to 1997:04. Cycle (IP) is the cyclical element in industrial production index; cycle (GNP) is the cyclical element in real GNP; CAY is from Lettau and Ludvigson (1999). All conditioning variables are lagged at least one period; JAN is a dummy variable which has value 1 for January (monthly models) or first quarter (quarterly models).
HJ-dist(δ) is Hansen-Jagannathan distance. p(δ=0) is the p-value for the test δ=0 under the null δ=0. se(δ) is standard error for HJ-distance under the hypothesis that $\delta \neq 0$. p-Wald(err) is a Wald test for all model errors=0 using estimates of b from minimizing HJ-distance. p-wald(b) is a Wald test on that all conditional elements of b are 0. The p-value of the optimal GMM test is p(J). p-Wald(b*) is a Wald test on that all conditional elements of b* are 0. supLM stat is the value for the supLM test statistics. The supLM test is based on a 5% significance level. No. of para is the number of parameters.

Table 4: correlation of adjustments $y-\tilde{m}=\tilde{\lambda}'R$ in pricing proxies

Panel A: monthly models

	NULL	CAPM	CCAPM	JW	LCM	FF3	FF5	NULL (IP)	CAPM (IP)	CCAPM (IP)	JW (IP)	LCM (IP)	FF3 (IP)	FF5 (IP)	NULL (JAN)	CAPM (JAN)	CCAPM (JAN)	JW (JAN)	LCM (JAN)	FF3 (JAN)	FF5 (JAN)
NULL	1	0.93	0.93	0.92	0.71	0.77	0.75	0.98	0.84	0.82	0.75	0.61	0.72	0.65	0.94	0.87	0.77	0.65	0.68	0.68	0.64
CAPM	0.93	1.00	0.92	0.99	0.76	0.83	0.81	0.87	0.90	0.78	0.80	0.66	0.77	0.70	0.88	0.94	0.75	0.70	0.73	0.73	0.69
CCAPM	0.93	0.92	1.00	0.90	0.73	0.81	0.78	0.90	0.84	0.91	0.73	0.62	0.79	0.68	0.92	0.90	0.86	0.68	0.72	0.71	0.63
JW	0.92	0.99	0.90	1.00	0.76	0.79	0.77	0.86	0.90	0.75	0.81	0.66	0.74	0.68	0.85	0.91	0.71	0.71	0.74	0.72	0.67
LCM	0.71	0.76	0.73	0.76	1.00	0.76	0.72	0.69	0.78	0.57	0.68	0.86	0.77	0.69	0.72	0.76	0.56	0.69	0.96	0.72	0.57
FF3	0.77	0.83	0.81	0.79	0.76	1.00	0.98	0.72	0.78	0.69	0.75	0.73	0.93	0.84	0.84	0.90	0.82	0.66	0.76	0.89	0.83
FF5	0.75	0.81	0.78	0.77	0.72	0.98	1.00	0.72	0.79	0.67	0.73	0.66	0.93	0.86	0.83	0.88	0.80	0.65	0.72	0.86	0.85
NULL(IP)	0.98	0.87	0.90	0.86	0.69	0.72	0.72	1.00	0.86	0.82	0.77	0.62	0.74	0.67	0.91	0.80	0.77	0.65	0.66	0.66	0.60
CAPM(IP)	0.84	0.90	0.84	0.90	0.78	0.78	0.79	0.86	1.00	0.76	0.89	0.73	0.86	0.78	0.78	0.82	0.76	0.74	0.74	0.75	0.66
CCAPM(IP)	0.82	0.78	0.91	0.75	0.57	0.69	0.67	0.82	0.76	1.00	0.62	0.43	0.73	0.66	0.82	0.75	0.84	0.52	0.58	0.65	0.57
JW(IP)	0.75	0.80	0.73	0.81	0.68	0.75	0.73	0.77	0.89	0.62	1.00	0.67	0.79	0.79	0.65	0.72	0.67	0.63	0.64	0.75	0.67
LCM(IP)	0.61	0.66	0.62	0.66	0.86	0.73	0.66	0.62	0.73	0.43	0.67	1.00	0.76	0.59	0.61	0.65	0.51	0.57	0.82	0.65	0.45
FF3(IP)	0.72	0.77	0.79	0.74	0.77	0.93	0.93	0.74	0.86	0.73	0.79	0.76	1.00	0.90	0.79	0.82	0.80	0.70	0.77	0.86	0.77
FF5(IP)	0.65	0.70	0.68	0.68	0.69	0.84	0.86	0.67	0.78	0.66	0.79	0.59	0.90	1.00	0.67	0.68	0.66	0.56	0.68	0.80	0.76
NULL(JAN)	0.94	0.88	0.92	0.85	0.72	0.84	0.83	0.91	0.78	0.82	0.65	0.61	0.79	0.67	1.00	0.92	0.83	0.69	0.72	0.72	0.68
CAPM(JAN)	0.87	0.94	0.90	0.91	0.76	0.90	0.88	0.80	0.82	0.75	0.72	0.65	0.82	0.68	0.92	1.00	0.82	0.75	0.78	0.78	0.73
CCAPM(JAN)	0.77	0.75	0.86	0.71	0.56	0.82	0.80	0.77	0.76	0.84	0.67	0.51	0.80	0.66	0.83	0.82	1.00	0.53	0.58	0.80	0.72
JW(JAN)	0.65	0.70	0.68	0.71	0.69	0.66	0.65	0.65	0.74	0.52	0.63	0.57	0.70	0.56	0.69	0.75	0.53	1.00	0.73	0.56	0.50
LCM(JAN)	0.68	0.73	0.72	0.74	0.96	0.76	0.72	0.66	0.74	0.58	0.64	0.82	0.77	0.68	0.72	0.78	0.58	0.73	1.00	0.70	0.56
FF3(JAN)	0.68	0.73	0.71	0.72	0.72	0.89	0.86	0.66	0.75	0.65	0.75	0.65	0.86	0.80	0.72	0.78	0.80	0.56	0.70	1.00	0.94
FF5(JAN)	0.64	0.69	0.63	0.67	0.57	0.83	0.85	0.60	0.66	0.57	0.67	0.45	0.77	0.76	0.68	0.73	0.72	0.50	0.56	0.94	1.00

The HJ-distance is calculated as *min* $\|y - m\|$, and $y-\tilde{m}=\tilde{\lambda}'R$ is the adjustment for *y* to make it a true pricing kernel. LCM stands for linear version of Campbell's (1996) asset pricing model. The data are returns on Fama-French 25 portfolios in excess of T-bill rate and the return on T-bill rate. Monthly data are from 1952:01 to 1997:12, 552 observations.

Panel B: quarterly models

	NULL	CAPM	CCAPM	JW	LCM	COCH	FF3	FF5	NULL(GNP)	CAPM(GNP)	CCAPM (GNP)	JW(GNP)	LCM(GNP)	COCH (GNP)	FF3(GNP)	FF5(GNP)	NULL(CAY)	CAPM(CAY)	CCAPM (CAY)	JW(CAY)	LCM(CAY)	COCH (CAY)	FF3(CAY)	FF6(CAY)	NULL(JAN)	CAPM(JAN)	CCAPM (JAN)	JW(JAN)	LCM(JAN)	COCO (JAN)	FF3(JAN)	FF5(JAN)
NULL	1.00	0.96	0.95	0.89	0.85	0.96	0.83	0.79	0.99	0.93	0.94	0.84	0.78	0.96	0.70	0.66	0.98	0.95	0.94	0.84	0.79	0.96	0.81	0.77	0.91	0.87	0.90	0.60	0.58	0.96	0.78	0.61
CAPM	0.96	1.00	0.94	0.93	0.89	0.97	0.87	0.83	0.97	0.97	0.93	0.88	0.81	0.97	0.73	0.69	0.98	0.99	0.94	0.88	0.83	0.97	0.85	0.80	0.86	0.91	0.87	0.63	0.62	0.97	0.82	0.64
CCAPM	0.95	0.94	1.00	0.81	0.84	0.94	0.88	0.87	0.96	0.87	0.99	0.76	0.76	0.94	0.71	0.68	0.95	0.93	0.98	0.77	0.78	0.94	0.86	0.84	0.91	0.91	0.94	0.62	0.59	0.94	0.83	0.64
JW	0.89	0.93	0.81	1.00	0.85	0.87	0.75	0.70	0.89	0.92	0.79	0.94	0.77	0.87	0.64	0.59	0.90	0.92	0.81	0.94	0.78	0.87	0.72	0.65	0.72	0.75	0.70	0.68	0.56	0.87	0.69	0.49
LCM	0.85	0.89	0.84	0.85	1.00	0.87	0.87	0.84	0.85	0.89	0.84	0.79	0.92	0.87	0.85	0.80	0.87	0.87	0.85	0.78	0.94	0.87	0.84	0.80	0.79	0.82	0.77	0.66	0.69	0.87	0.83	0.71
COCH	0.96	0.97	0.94	0.87	0.87	1.00	0.88	0.86	0.97	0.94	0.94	0.83	0.83	1.00	0.72	0.69	0.98	0.97	0.96	0.84	0.84	1.00	0.86	0.82	0.91	0.91	0.90	0.61	0.62	1.00	0.82	0.69
FF3	0.83	0.87	0.88	0.75	0.87	0.88	1.00	0.96	0.86	0.80	0.90	0.72	0.79	0.88	0.84	0.80	0.85	0.88	0.89	0.69	0.87	0.88	0.98	0.93	0.84	0.88	0.86	0.68	0.69	0.88	0.95	0.73
FF5	0.79	0.83	0.87	0.70	0.84	0.86	0.96	1.00	0.82	0.78	0.87	0.70	0.77	0.86	0.83	0.83	0.81	0.83	0.87	0.63	0.83	0.86	0.95	0.96	0.80	0.85	0.83	0.63	0.58	0.86	0.87	0.76
NULL(GNP)	0.99	0.97	0.96	0.89	0.85	0.97	0.86	0.82	1.00	0.94	0.95	0.85	0.78	0.97	0.70	0.67	0.98	0.96	0.95	0.84	0.80	0.97	0.84	0.79	0.91	0.90	0.91	0.61	0.59	0.97	0.81	0.65
CAPM(GNP)	0.93	0.97	0.87	0.92	0.89	0.94	0.80	0.78	0.94	1.00	0.86	0.90	0.83	0.94	0.75	0.71	0.94	0.95	0.88	0.83	0.83	0.94	0.80	0.76	0.81	0.84	0.80	0.57	0.59	0.94	0.74	0.62
CCAPM(GNP)	0.94	0.93	0.99	0.79	0.84	0.94	0.90	0.87	0.95	0.86	1.00	0.75	0.78	0.94	0.73	0.70	0.95	0.94	0.99	0.78	0.79	0.94	0.87	0.85	0.92	0.92	0.94	0.62	0.61	0.94	0.86	0.66
JW(GNP)	0.84	0.88	0.76	0.94	0.79	0.83	0.72	0.70	0.85	0.90	0.75	1.00	0.73	0.83	0.64	0.63	0.84	0.86	0.77	0.87	0.75	0.83	0.71	0.67	0.63	0.67	0.63	0.57	0.52	0.83	0.64	0.48
LCM(GNP)	0.78	0.81	0.76	0.77	0.92	0.83	0.79	0.77	0.78	0.83	0.78	0.73	1.00	0.83	0.81	0.75	0.81	0.80	0.79	0.74	0.85	0.83	0.74	0.71	0.73	0.76	0.72	0.64	0.68	0.83	0.75	0.73
COCH(GNP)	0.96	0.97	0.94	0.87	0.87	1.00	0.88	0.86	0.97	0.94	0.94	0.83	0.83	1.00	0.72	0.69	0.98	0.97	0.96	0.84	0.84	1.00	0.86	0.82	0.91	0.91	0.90	0.61	0.62	1.00	0.82	0.69
FF3(GNP)	0.70	0.73	0.71	0.64	0.85	0.72	0.84	0.83	0.70	0.75	0.73	0.64	0.81	0.72	1.00	0.95	0.69	0.71	0.70	0.51	0.81	0.72	0.85	0.82	0.69	0.72	0.69	0.58	0.61	0.72	0.80	0.68
FF5(GNP)	0.66	0.69	0.68	0.59	0.80	0.69	0.80	0.83	0.67	0.71	0.70	0.63	0.75	0.69	0.95	1.00	0.67	0.66	0.67	0.44	0.76	0.69	0.82	0.81	0.63	0.67	0.65	0.49	0.56	0.69	0.72	0.67
NULL(CAY)	0.98	0.98	0.95	0.90	0.87	0.98	0.85	0.81	0.98	0.94	0.95	0.84	0.81	0.98	0.69	0.67	1.00	0.97	0.96	0.86	0.81	0.98	0.83	0.78	0.89	0.89	0.88	0.60	0.63	0.98	0.80	0.64
CAPM(CAY)	0.95	0.99	0.93	0.92	0.87	0.97	0.88	0.83	0.96	0.95	0.94	0.86	0.80	0.97	0.71	0.66	0.97	1.00	0.95	0.89	0.85	0.97	0.86	0.81	0.87	0.92	0.87	0.64	0.64	0.97	0.85	0.64
CCAPM(CAY)	0.94	0.94	0.98	0.81	0.85	0.96	0.89	0.87	0.95	0.88	0.99	0.77	0.79	0.96	0.70	0.67	0.96	0.95	1.00	0.81	0.79	0.96	0.87	0.85	0.89	0.91	0.92	0.60	0.61	0.96	0.84	0.65
JW(CAY)	0.84	0.88	0.77	0.94	0.78	0.84	0.69	0.63	0.84	0.83	0.78	0.87	0.74	0.84	0.51	0.44	0.86	0.89	0.81	1.00	0.73	0.84	0.64	0.60	0.70	0.74	0.68	0.68	0.54	0.84	0.66	0.42
LCM(CAY)	0.79	0.83	0.78	0.78	0.94	0.84	0.87	0.83	0.80	0.83	0.79	0.75	0.85	0.84	0.81	0.76	0.81	0.85	0.79	0.73	1.00	0.84	0.85	0.80	0.75	0.79	0.73	0.61	0.74	0.84	0.83	0.77
COCH(CAY)	0.96	0.97	0.94	0.87	0.87	1.00	0.88	0.86	0.97	0.94	0.94	0.83	0.83	1.00	0.72	0.69	0.98	0.97	0.96	0.84	0.84	1.00	0.86	0.82	0.91	0.91	0.90	0.61	0.62	1.00	0.82	0.69
FF3(CAY)	0.81	0.85	0.86	0.72	0.84	0.86	0.98	0.95	0.84	0.80	0.87	0.71	0.74	0.86	0.85	0.82	0.83	0.86	0.87	0.64	0.85	0.86	1.00	0.94	0.82	0.87	0.84	0.62	0.69	0.86	0.94	0.73
FF5(CAY)	0.77	0.80	0.84	0.65	0.80	0.82	0.93	0.96	0.79	0.76	0.85	0.67	0.71	0.82	0.82	0.81	0.78	0.81	0.85	0.60	0.80	0.82	0.94	1.00	0.79	0.83	0.81	0.57	0.54	0.82	0.85	0.72
NULL(JAN)	0.91	0.86	0.91	0.72	0.79	0.91	0.84	0.80	0.91	0.81	0.92	0.63	0.73	0.91	0.69	0.63	0.89	0.87	0.89	0.70	0.75	0.91	0.82	0.79	1.00	0.96	0.99	0.66	0.64	0.91	0.86	0.67
CAPM(JAN)	0.87	0.91	0.91	0.75	0.82	0.91	0.88	0.85	0.90	0.84	0.92	0.67	0.76	0.91	0.72	0.67	0.89	0.92	0.91	0.74	0.79	0.91	0.87	0.83	0.96	1.00	0.96	0.70	0.68	0.91	0.90	0.70
CCAPM(JAN)	0.90	0.87	0.94	0.70	0.77	0.90	0.86	0.83	0.91	0.80	0.94	0.63	0.72	0.90	0.69	0.65	0.88	0.87	0.92	0.68	0.73	0.90	0.84	0.81	0.99	0.96	1.00	0.67	0.63	0.90	0.87	0.66
JW(JAN)	0.60	0.63	0.62	0.68	0.66	0.61	0.68	0.63	0.61	0.57	0.62	0.57	0.64	0.61	0.58	0.49	0.60	0.64	0.60	0.68	0.61	0.61	0.62	0.57	0.66	0.70	0.67	1.00	0.61	0.61	0.68	0.44
LCM(JAN)	0.58	0.62	0.59	0.56	0.69	0.62	0.69	0.58	0.59	0.59	0.61	0.52	0.68	0.62	0.61	0.56	0.63	0.64	0.61	0.54	0.74	0.62	0.69	0.54	0.64	0.68	0.63	0.61	1.00	0.62	0.76	0.67
COCH(JAN)	0.96	0.97	0.94	0.87	0.87	1.00	0.88	0.86	0.97	0.94	0.94	0.83	0.83	1.00	0.72	0.69	0.98	0.97	0.96	0.84	0.84	1.00	0.86	0.82	0.91	0.91	0.90	0.61	0.62	1.00	0.82	0.69
FF3(JAN)	0.78	0.82	0.83	0.69	0.83	0.82	0.95	0.87	0.81	0.74	0.86	0.64	0.75	0.82	0.80	0.72	0.80	0.85	0.84	0.66	0.83	0.82	0.94	0.85	0.86	0.90	0.87	0.68	0.76	0.82	1.00	0.78
FF5(JAN)	0.61	0.64	0.64	0.49	0.71	0.69	0.73	0.76	0.65	0.62	0.66	0.48	0.73	0.69	0.68	0.67	0.64	0.64	0.65	0.42	0.77	0.69	0.73	0.72	0.67	0.70	0.66	0.44	0.67	0.69	0.78	1.00

The HJ-distance is calculated as $min \, || \, y - m \, ||$, and $y - \tilde{m} = \tilde{\lambda}' R$ is the adjustment for y to make it a true pricing kernel. LCM stands for linear version of Campbell's (1996) asset pricing model. COCH is a modified model for Cochrane(1996) model, with the factors as investment growth rates. . The data are returns on Fama-French 25 portfolios in excess of T-bill rate and the return on T-bill rate. Quarterly data are from 1953:01 to 1997:04, 180 observations.

Table 5: Parameters estimates of interesting models from HJ-distance

Panel A: CAPM monthly non-scaled returns with scaled factors by IP

	Constant	R_{VW}	IP	R_{VW}*IP
Parameters of the pricing kernel				
$\hat{b}$	1.03	-0.04	-0.34	0.02
s.e.	0.05	0.02	0.12	0.03
Factor risk prices				
Λ		0.66	2.16	0.58
s.e.		0.27	0.74	2.75

Panel B: CCAPM monthly non-scaled returns with scaled factors by IP

	Constant	Δc	IP	Δc *IP
Parameters of the pricing kernel				
$\hat{b}$	1.14	-0.75	-0.28	0.22
s.e.	0.10	0.36	0.11	0.12
Factor risk prices				
Λ		0.43	1.38	-0.49
s.e.		0.21	0.65	0.55

Panel C: CCAPM monthly non-scaled returns with scaled factors by JAN

	Constant	Δc	JAN	Δc *JAN
Parameters of the pricing kernel				
$\hat{b}$	1.05	-0.12	0.58	-3.93
s.e.	0.06	0.37	0.90	1.62
Factor risk prices				
Λ		0.26	0.02	0.20
s.e.		0.22	0.06	0.08

Panel D: JW monthly non-scaled returns with scaled factors by IP

	Constant	R_{VW}	R_{LBR}	R_{PREM}	IP	R_{VW}*IP	R_{LBR}*IP	R_{PREM}*IP
Parameters of the pricing kernel								
$\hat{b}$	1.38	-0.04	-0.66	0.68	0.38	0.00	-0.40	-0.40
s.e.	0.68	0.02	0.64	0.71	0.38	0.03	0.31	0.22
Factor risk prices								
Λ		0.65	0.05	-0.05	1.01	0.80	1.72	1.09
s.e.		0.28	0.12	0.13	0.98	2.68	1.02	0.41

Panel E: JW monthly non-scaled returns with scaled factors by JAN

	Constant	R_{VW}	R_{LBR}	R_{PREM}	JAN	R_{VW}*JAN	R_{LBR}*JAN	R_{PREM}*JAN
Parameters of the pricing kernel								
$\hat{b}$	-0.68	0.02	0.53	2.54	4.33	-0.45	0.35	-8.36
s.e.	0.90	0.05	0.78	1.13	3.34	0.40	3.65	3.26
Factor risk prices								
Λ		0.59	-0.14	-0.28	0.07	0.70	0.07	0.13
s.e.		0.34	0.15	0.18	0.05	0.62	0.07	0.06

Panel F: Campbell monthly non-scaled returns with non-scaled factors

	constant	R_{VW}	LBR	DIV	RTB	TRM
Parameters of the pricing kernel						
$\hat{b}$	-1.07	0.01	0.10	0.67	0.90	-0.72
s.e.	1.30	0.03	0.41	0.34	4.33	0.28
Factor risk prices						
Λ		0.66	0.02	-0.69	-0.05	1.11
s.e.		0.31	0.27	0.33	0.04	0.35

Panel G: FF3 monthly non-scaled returns with non-scaled factors

	Constant	R_{VW}	SMB	HML
Parameters of the pricing kernel				
$\hat{b}$	1.07	-0.05	-0.01	-0.10
s.e.	0.02	0.01	0.02	0.02
Factor risk prices				
Λ		0.65	0.14	0.39
s.e.		0.21	0.12	0.10

Panel H: FF3 monthly non-scaled returns with scaled factors by JAN

	Constant	R_{VW}	SMB	HML	JAN	R_{VW}*JAN	SMB*JAN	HML*JAN
Parameters of the pricing kernel								
$\hat{b}$	1.07	-0.08	0.12	-0.06	1.38	0.21	-0.98	0.15
s.e.	0.05	0.03	0.06	0.05	1.22	0.26	0.43	0.42
Factor risk prices								
Λ		0.63	0.16	0.39	0.01	-0.07	0.74	0.19
s.e.		0.27	0.21	0.16	0.06	0.47	0.32	0.23

Panel I: JW quarterly non-scaled returns with non-scaled factors

	Constant	R_{VW}	R_{LBR}	R_{PREM}
Parameters of the pricing kernel				
$\hat{b}$	-0.35	0.00	-0.20	1.01
s.e.	0.85	0.02	0.64	0.48
Factor risk prices				
Λ		1.29	-0.02	-0.74
s.e.		0.84	0.12	0.33

Panel J: Campbell quarterly non-scaled returns with non-scaled factors

	constant	R_{VW}	LBR	DIV	RTB	TRM
Parameters of the pricing kernel						
$\hat{b}$	0.22	0.00	0.10	0.28	-0.20	-0.56
s.e.	1.00	0.02	0.16	0.27	2.64	0.22
Factor risk prices						
Λ		1.52	-0.13	-0.28	-0.03	0.85
s.e.		0.79	0.37	0.24	0.02	0.34

Panel K: Cochrane quarterly non-scaled returns with scaled factors by lag GNP

	constant	NRINV	RINV	GNP	NRINV*GNP	RINV*GNP
Parameters of the pricing kernel						
$\hat{b}$	0.92	-0.01	-0.16	0.12	-0.04	-0.09
s.e.	0.27	0.16	0.07	0.22	0.07	0.04
Factor risk prices						
Λ		0.33	1.76	0.03	0.86	5.33
s.e.		0.85	1.31	0.58	1.21	3.24

Panel L: Cochrane quarterly non-scaled returns with scaled factors by JAN

	constant	NRINV	RINV	JAN	NRINV*JAN	RINV*JAN
Parameters of the pricing kernel						
$\hat{b}$	1.41	-0.24	0.09	-1.44	0.90	-0.19
s.e.	0.21	0.17	0.07	0.53	0.37	0.15
Factor risk prices						
Λ		-0.63	-1.38	0.15	-1.25	-0.03
s.e.		0.75	1.44	0.08	0.59	0.61

Panel M: FF5 quarterly non-scaled returns with non-scaled factors

	constant	R_{VW}	SMB	HML	TERM	R_{PREM}
Parameters of the pricing kernel						
$\hat{b}$	1.23	-0.05	0.00	-0.06	-0.21	1.25
s.e.	0.52	0.02	0.02	0.02	0.11	0.78
Factor risk prices						
Λ		1.51	0.58	1.12	0.23	-0.06
s.e.		0.79	0.42	0.41	0.51	0.10

All parameters are calculated by both optimal GMM and minimizing HJ-dist. The risk prices for factors, Λ, are defined in equation (7). The data are returns on Fama-French 25 portfolios in excess of T-bill rate and the return on T-bill rate. Monthly data are from 1952:01 to 1997:12; quarterly data are from 1953:01 to 1997:04.

Table 6: Robustness test for non-scaled returns models

Panel A: monthly scaled returns by TERM with non-scaled factors

	NULL	CAPM	CCAPM	JW	CAMPBELL	FF3	FF5
p1	0	0	0	0	0	0	0
p2	0	0	0.0001	0.0001	0	0.0005	0.0033
p3	0	0	0	0	0	0	0

Panel B: monthly scaled returns by TERM with scaled factors by IP

	NULL	CAPM	CCAPM	JW	CAMPBELL	FF3	FF5
p1	0	0	0	0.0022	0	0	0
p2	0	0.0035	0	0.0044	0	0.0173	0.0004
p3	0	0	0.0001	0	0	0.0001	0

Panel C: monthly scaled returns by TERM with scaled factors by JAN

	NULL	CAPM	CCAPM	JW	CAMPBELL	FF3	FF5
p1	0	0.0001	0.0013	0	0	0	0
p2	0	0.0006	0.0361	0.0023	0	0.0074	0.0855
p3	0	0	0.0751	0.0038	0	0.0006	0.0008

Panel D: quarterly scaled returns by TERM with non-scaled factors

	NULL	CAPM	CCAPM	JW	CAMPBELL	COCH	FF3	FF5
p1	0.0001	0.0001	0.0135	0.002	0	0	0.0015	0.0028
p2	0.0028	0.0054	0.0122	0.0063	0	0.0003	0.0403	0.0494
p3	0	0.0001	0.0064	0.0005	0	0	0.0012	0.0017

Panel E: quarterly scaled returns by TERM with scaled factors by lag GNP

	NULL	CAPM	CCAPM	JW	CAMPBELL	COCH	FF3	FF5
p1	0	0.0021	0.0096	0.001	0	0	0.0387	0.0176
p2	0.0011	0.0173	0.0149	0.0077	0.0004	0.0005	0.3613	0.4949
p3	0	0.0016	0.0035	0	0	0	0.0690	0.0156

Panel F: quarterly scaled returns by TERM with scaled factors by CAY

	NULL	CAPM	CCAPM	JW	CAMPBELL	COCH	FF3	FF5
p1	0	0.0014	0.0001	0.0052	0	0	0.0005	0.0012
p2	0.0005	0.0058	0.0022	0.012	0	0.0009	0.0528	0.1095
p3	0	0.0002	0.0004	0.0025	0	0	0.0022	0.0061

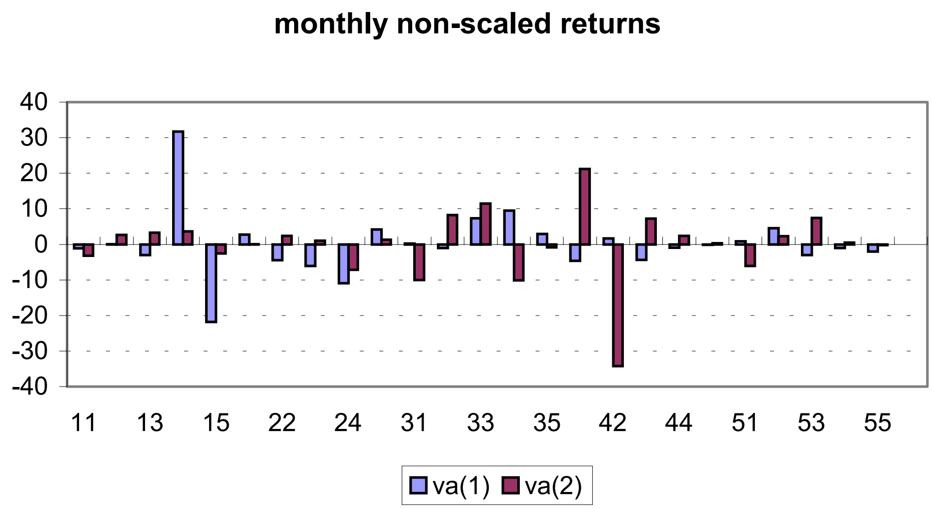
Panel G: quarterly scaled returns by TERM with scaled factors by JAN

	NULL	CAPM	CCAPM	JW	CAMPBELL	COCH	FF3	FF5
p1	0.0003	0.0022	0.0009	0.0204	0	0.0641	0.0004	0.0060
p2	0.0154	0.0199	0.0111	0.103	0.0166	0.2361	0.0315	0.2157
p3	0.0007	0.0017	0.0003	0.0159	0	0.0385	0.0007	0.0035

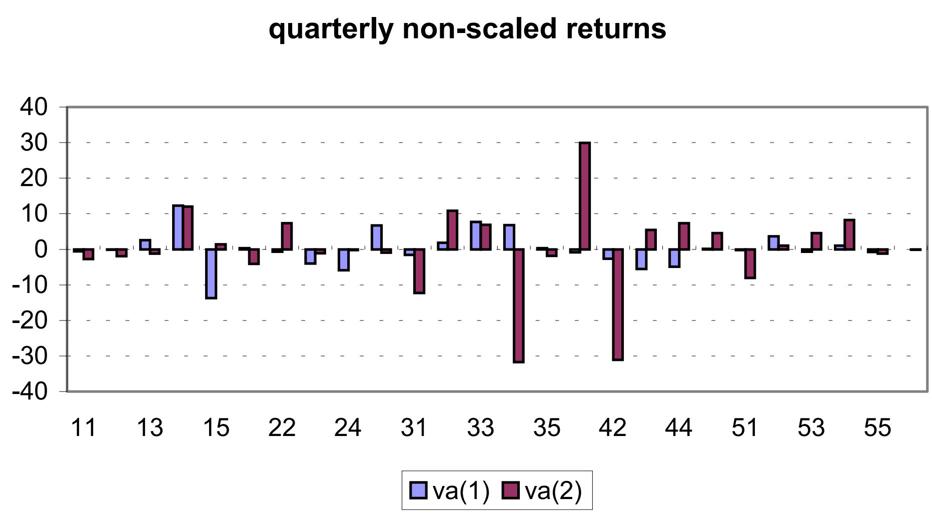
The p-values are:
p1: test of HJ-distance =0 using parameter estimates from optimal GMM for corresponding non-scaled return models; p2: test of optimal GMM over-identification using parameter estimates from optimal GMM for corresponding non-scaled return models; p3: test of HJ-distance =0 using parameter estimates from minimizing HJ-distance for corresponding non-scaled return models.
The tests are based on returns on the FF25 portfolios in excess of T-bill rate and the return of T-bill, conditioned on the term premium, the difference in yields between a thirty-year government bond and a one-year bond. Monthly data are from 1952:01 to 1997:12; quarterly data are from 1953:01 to 1997:04.

Figure 1. Diagnostic of Weighting matrix = $E[R'R]^{-1}$. Standardized eigenvector corresponding to the smallest eigenvalue of E[R'R]

Panel A: monthly non-scaled returns, condition number=13548, 1st va(E[R'R]) = 0.00007, 2nd va(E[R'R])=0.00009.



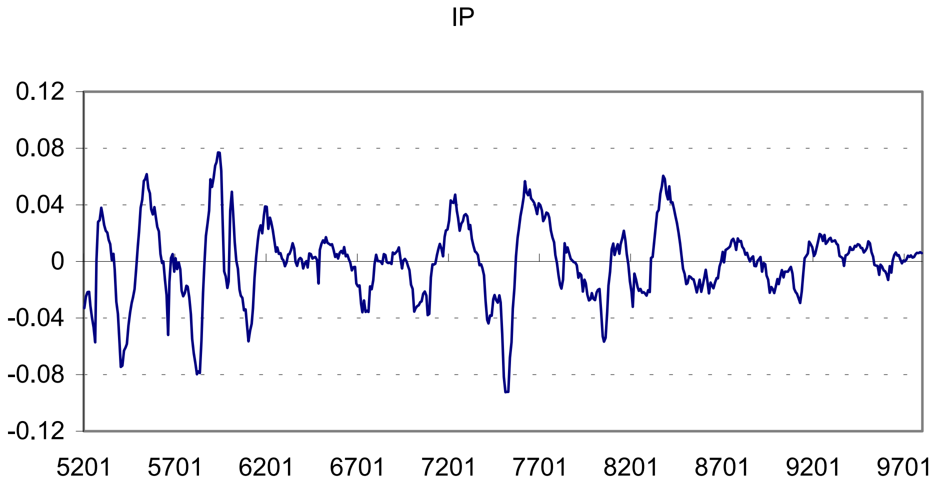
Panel B: quarterly non-scaled returns, condition number=7851, 1st va(E[R'R]) = 0.00013, 2nd va(E[R'R])=0.00018.



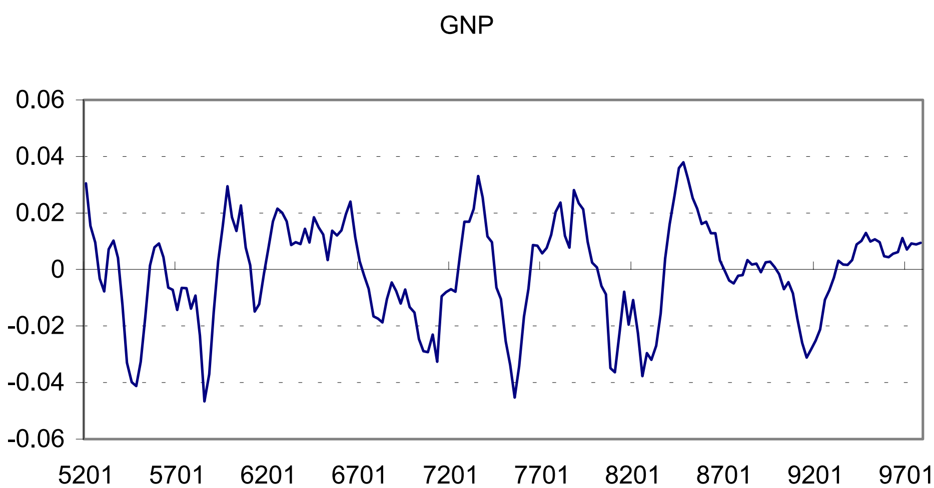
The data are monthly and quarterly returns of Fama-French 25 portfolios in excess of T-bill rate and the return on the T-bill. Monthly data start at 1952:01, end at 1997:12, 552 observations. Quarterly data start at 1953:01, end at 1997:04, 180 observations. The smallest eigenvalue of E[R'R] is the biggest eigenvalue for the weighting matrix = inv E[R'R], thus the corresponding eigenvector is the most important weight on the model errors.

Figure 2. Time series of three conditioning variables

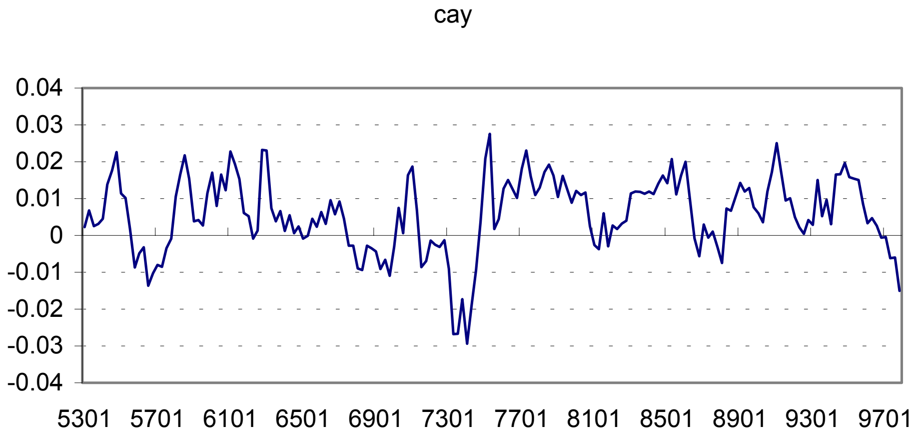
Panel A: monthly conditioning variable cycle(IP)



Panel B: quarterly conditioning variable cycle (GNP)



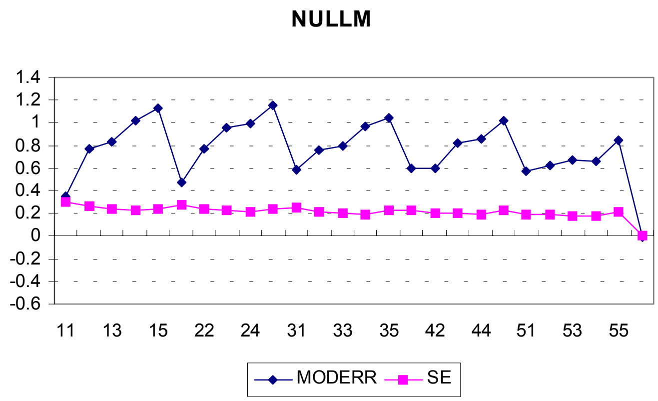
Panel C: quarterly conditioning variable cycle(CAY)



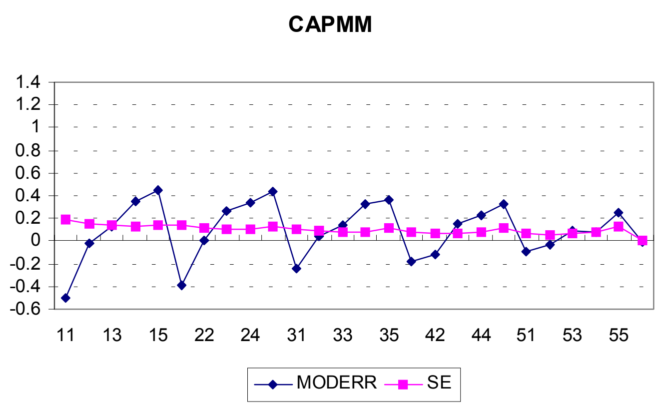
Cycle (IP) is the cyclical element in industrial production. Monthly data begin at 1952:01 and end at 1997:12. Cycle (GNP) is the cycle element of GNP. Cycle (CAY) is constructed as the change in aggregate consumption-wealth ratio, derived in Lettau and Ludvigson (1999a). Cycle (GNP) starts at 1952:01, and ends at 1997:04. Cycle (CAY) starts at 1953:01 and ends at 1997:04.

Figure 3: model errors for monthly models with non-scaled factors

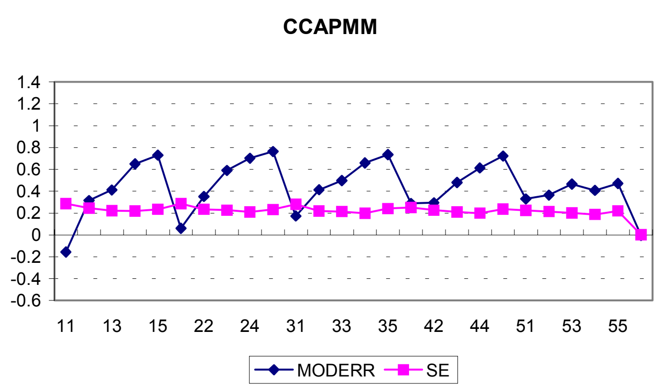
Panel A: Null



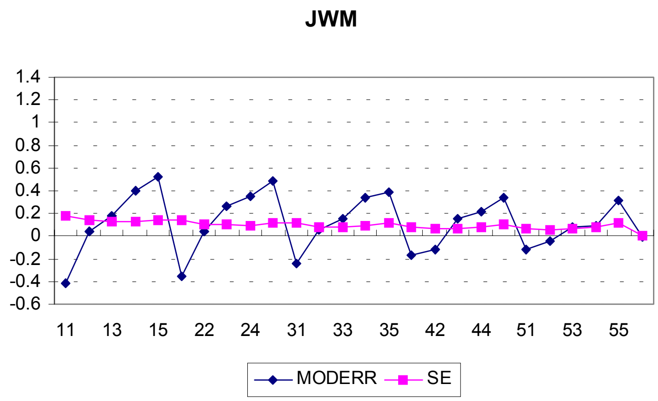
Panel B: CAPM



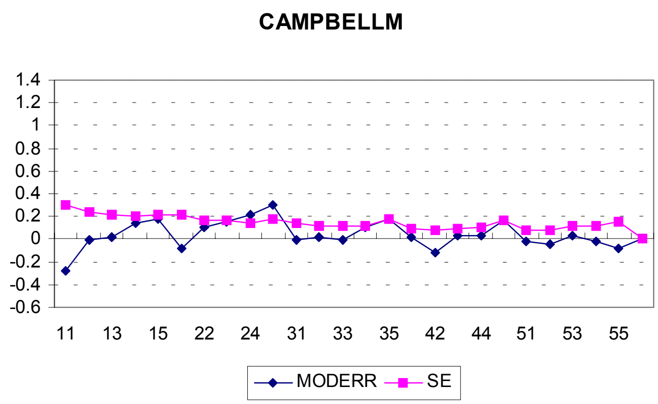
Panel C: CCAPM



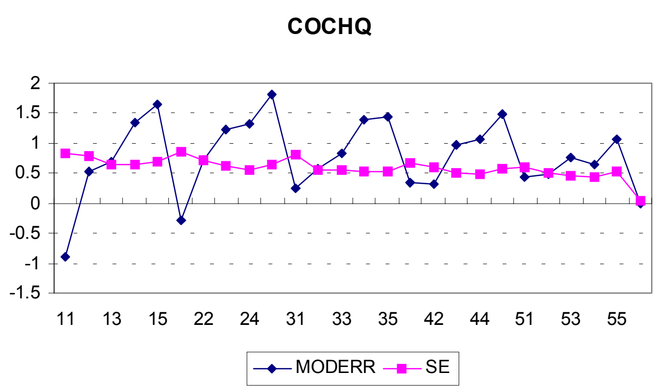
Panel D: JW



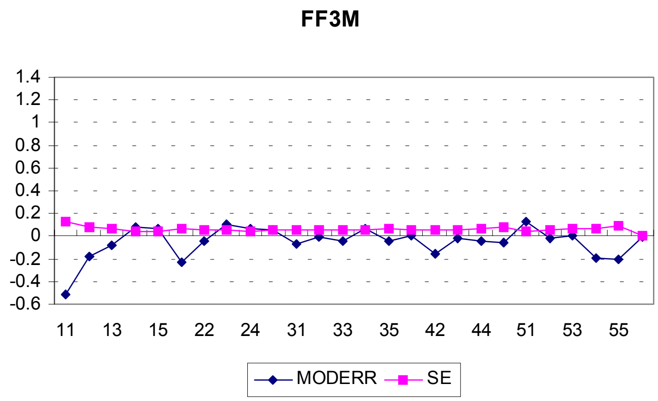
Panel E: CAMPBELL



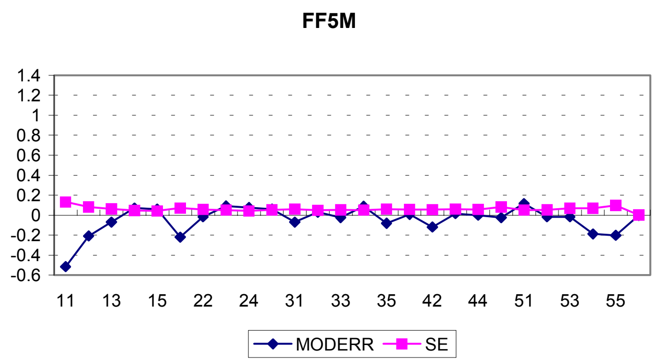
Panel F: COCHRANE



Panel G: FF3



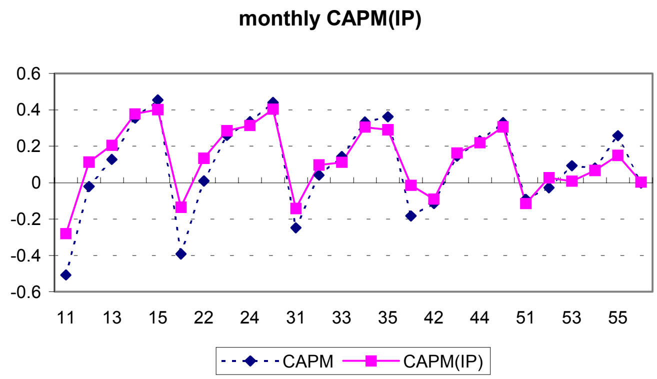
Panel H: FF5



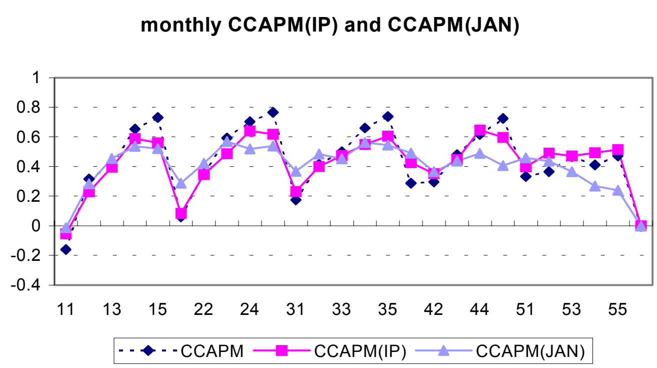
The data are monthly and quarterly returns of Fama-French 25 portfolios in excess of T-bill rate and the return on the T-bill. Monthly data start at 1952:01, end at 1997:12, 552 observations. Quarterly data start at 1953:01, end at 1997:04, 180 observations. Model errors are defined in equation (20).

Figure 4. Pricing Errors for Interesting Models

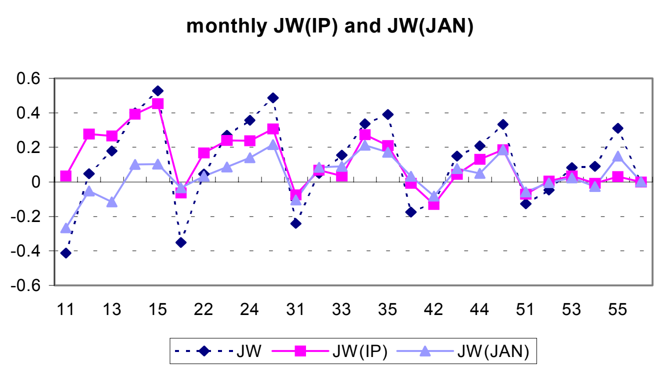
Panel A: monthly CAPM with scaled factors by IP



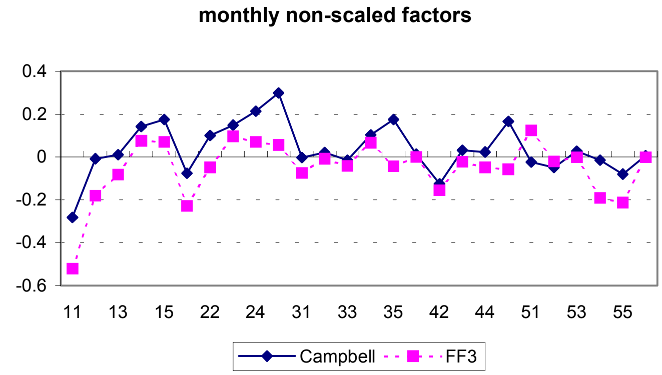
Panel B: monthly CCAPM with scaled factors by IP and JAN



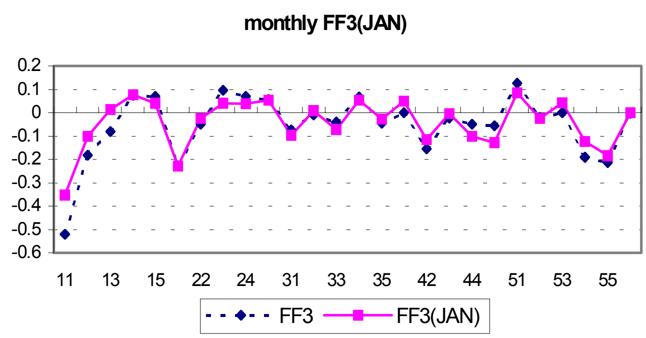
Panel C: monthly JW with scaled factors by IP and JAN



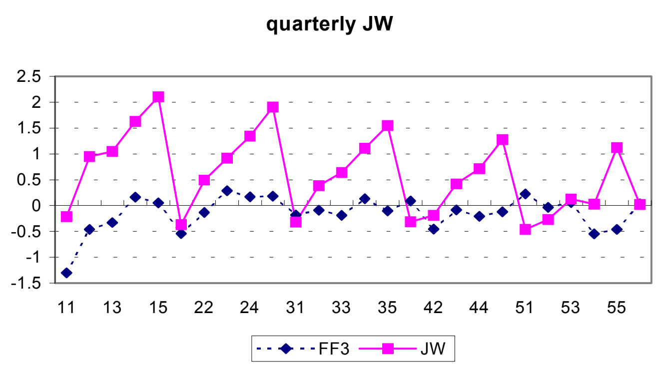
Panel D: monthly Campbell's model with non-scaled factors (with benchmark FF3)



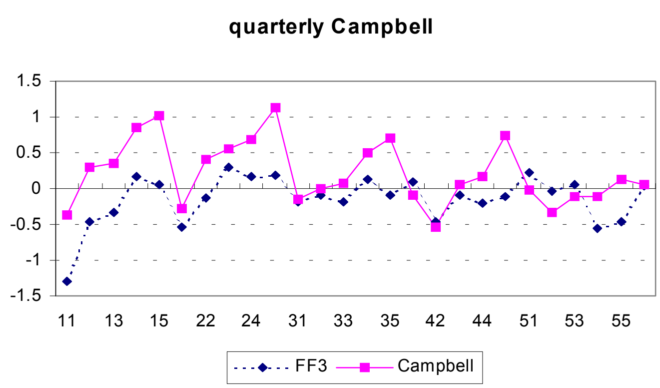
Panel E: monthly FF3 with scaled factors by JAN



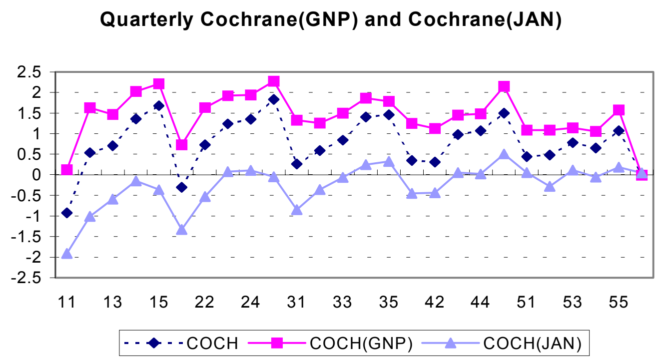
Panel F: quarterly JW with non-scaled factors (with benchmark FF3)



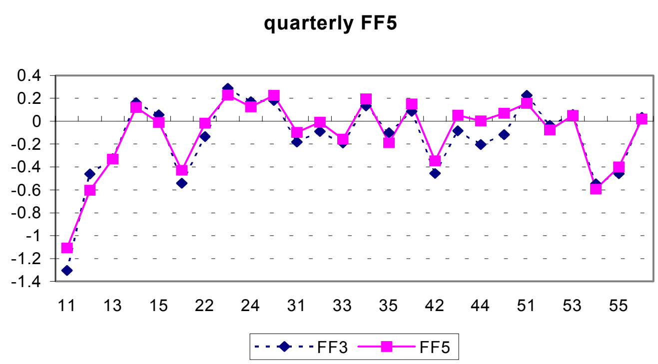
Panel G: quarterly Campbell's model with non-scaled factors (with benchmark FF3)



Panel H: quarterly Cochrane's model with scaled factors by GNP and JAN



Panel I: quarterly FF5 with non-scaled factors (with benchmark FF3)



The data are monthly and quarterly returns of Fama-French 25 portfolios in excess of T-bill rate and the return on the T-bill. Monthly data start at 1952:01, end at 1997:12, 552 observations. Quarterly data start at 1953:01, end at 1997:04, 180 observations. Pricing errors are defined in equation (33).