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# FORWARD AND BACKWARD INTERGENERATIONAL GOODS: A THEORY OF INTERGENERATIONAL EXCHANGE

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## ABSTRACT

This paper develops a theory of intergenerational exchange for generations that are either selfish or have non-dynastic altruism. The main building blocks of the theory are forward and backward intergenerational goods (FIGs and BIGs) and the relationship between them. A FIG is a transfer from present to future generations, like parental investments in education and the preservation of the environment. A BIG is a transfer from future to present generations, like pay-as -you-go social security or taking care of elderly parents. We show that there is a fundamental difference between BIGs and FIGs. BIGs generating a positive surplus are self-sustainable, but FIGs never are. However, even with selfish generations, optimal investment in future generations can take place if the equilibrium social norm links BIGs and FIGs. The tools developed here can be used to understand a wide class of intergenerational problems, from the political economy of environmental treaties to the economics of seniority institutions. Two applications are developed in the paper: (1) the political economy of intergenerational public expenditures, and (2) investment in children within the family.

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"Why should I care about future generations? What have they done for me?"
*Addison*

"Be nice to your children, they will pick your nursing home."
*Anonymous bumper sticker*

"As others planted before me, so do I plant for my children."
*Talmud*

"The improvements made by the dead form a charge against the living who take the benefit of them. ... There seems then to be a foundation in the nature of things, in the relation which one generation bears to another, for the descent of obligations from one to another. Equity requires it. Mutual good is promoted by it."
*James Madison*

"...expenditures on the elderly are part of a "social compact" between generations. Taxes on adults help finance efficient investments in children. In return, adults receive public pensions and medical payments when old."
*Gary Becker and Kevin Murphy (1988)*

# 1 Introduction

Intergenerational exchange is ubiquitous in economic life. It takes place at the level of the family, organization, community, nation, and planet. Traditional examples at the family level are parental investments in the education of their children and the care that middle-aged adults provide for their elderly parents. The seniority system in political bodies like the U.S. Congress is an example of intergenerational exchange in long-lived organizations.[1] In the United States, the government performs intergenerational exchange at the local level, through investments in infrastructure and public education, and at the national level, through investment in R&D, the financing of wars, and programs like pay-as-you-go social security and Medicare. Finally, environmental problems such as Global Warming and the depletion of the ozone layer are examples of intergenerational exchange at the international level.

The main contribution of this paper is to show that this diverse class of problems can be understood using a common set of principles, that are captured by a simple theory of intergenerational exchange. In each of the previous examples the welfare of a generation depends on the actions taken by other generations and, in turn, its actions affect the well-being of other generations. This pattern of intergenerational spillovers is at the core of the problem: Do present generations have an incentive spend optimally in other generations? And if so, why?

Of course, the answer depends crucially on the degree and type of intergenerational altruism. In a model with infinitely lived dynasties the problem of intergenerational exchange is not particularly interesting. After all, for an infinitely lived agent investing in other generations is not an intergenerational transfer, but an investment in himself. Although the dynastic model has yielded important insights, it is invalid in many cases of interest.

First of all, in some cases a model with selfish generations is a better approximation of reality. Take for example the case of the seniority system in the U.S. Congress. It is hard to believe that politicians fully internalize the preferences of future generations of statesmen. Furthermore, although more work remains to be done, the studies of Altonji, Hayashi and Kotlikoff (1992,1997) suggests that the selfish model may even be a good approximation for intergenerational exchange within the family.

[1] See Shepsle (1999) and Shepsle and Nalebuff (1990).

Another problem with the dynastic model is that agents might be altruistic, but have non-dynastic preferences.[2] With paternalistic altruism agents care about how their actions affect future generations, but not about the *utility* of future generations. For example, they might be willing to pay for the college education of their children, but unwilling to finance other consumption goods.

This paper presents a theory of intergenerational exchange with non-dynastic preferences, including the case of selfish generations. The problem of optimal intergenerational exchange is particularly interesting with selfish generations because they fail to internalize the effect of their actions on other generations. Intuitively, one might expect selfish parents to under-invest in the education of their children and selfish generations to deplete the planet of its natural resources too quickly. As it turns out, these problems can be alleviated with an appropriate trading relationship between the different generations.

The first part of the paper develops a basic model of intergenerational exchange. The model abstracts from institutional details and emphasizes the key forces at work in all of the examples described above. The analysis provides two main insights. First, it is useful to distinguish between *Forward Intergenerational Goods* (FIGs) and *Backward Intergenerational Goods* (BIGs). In broad terms, FIGs are transfers from present to future generations. Examples of FIGs are investments in children, investments in long-lived infrastructure, and the preservations of environmental quality, all of which generate a cost for present generations and a benefit for future generations. By contrast, BIGs are transfers from future to present generations, or from young to old. Examples of BIGs are taking care of elderly parents and pay-as-you-go social security, which amount to transfers from the young to the old. Thus, in FIGs the transfers go forward in time, from the present to the future; in BIGs they go backwards, from the present to the past.

BIGs and FIGs generate rather different incentive problems. The difference is due to the timing of exchange. Consider, as an extreme but illustrative example, a selfish family and the difference between investing in children, a FIG, and taking care of elderly parents, a BIG. Look at the BIG first. Every period a middle age adult needs to decide whether or not to take care of his parents. Since he is selfish, he would rather not do it. However, he also wants his children to take care of him in the future. If the value of being taken care of by one's children outweighs the cost

[2] Non-dynastic altruism has also been called paternalistic altruism. See Ray (1987).

of taking care of one's parents, there is an equilibrium in which the BIG is always provided. Although there are many equilibria that support this outcome, all of them have the following feature: (along the equilibrium path) a generation believes that their children will take care of them only if they take care of their own parents. Because of the timing of exchange (pay first and receive later) the provision of the BIG for a generation can be conditioned on whether or not it provided a BIG for the previous generation.

By contrast, now look at the education FIG. Again, for expositional purposes suppose that parents are completely selfish. By the time they have to decide to invest in their children they have already received their own education. As a result, since they are selfish, they have no incentive to provide the FIG. Notice that the timing of exchange is exactly the opposite from the one in BIGs: receive the benefit first and decide whether or not to pay later. This difference in the timing of exchange implies that backward intergenerational exchange can be self-sustained, but forward intergenerational exchange cannot.

The second key insight is that optimal investment in future generations can take place even with selfish agents. Selfish generations can be given an incentive to invest in FIGs by linking them with BIGs. To see the intuition, consider again the selfish family and suppose that the middle-aged adult believes that tomorrow his children will take care of him only if: (1) he takes care of his own parents *and* (2) he invests optimally in their education. In other words, the cost of buying a BIG for himself is paying for his children's education *and* taking care of his parents. If being taken care off in old age is valuable enough, the middle-aged parent provides both the BIG and the FIG. Of course, this works only if the cost of providing a FIG is low enough and the value of the BIG is high enough. But in some cases the link is able to generate optimal investment in future generations.

The basic theory of BIGs and FIGs can be applied to a wide class of problems. The second part of the paper explores two natural applications: (1) the political economy of intergenerational public expenditures, and (2) investment in children within the family. Many other applications are possible.

## 2 Previous Literature

There exists a large literature on intergenerational problems, which for the most part has focused on concrete problems like bequests, the family, the environment, or the effect of the government debt. By contrast, the goal of this paper is to provide a common framework for intergenerational problems. Several other papers have also attempted to do this. Sandler and Smith (1976) and Sandler (1978,1982) are the first references known by the author to develop the concept of "intergenerational goods." These papers focus on characterizing the optimality conditions for the provision of intergenerational goods, but ignore the incentive problems that are at the heart of the theory of BIGs and FIGs.[3] Doeleman and Sandler (1998) study two and three period overlapping generation models in which earlier generations can make investments that benefit later generations. Afterwards, later generations voluntarily choose whether to refund previous generations for their efforts. They show that under-investment takes place because later generations do not have an incentive to pay for the investment once they have received it. In this sense, they lay out the fundamental market and political failure – which they refer to as "missing markets and missing voters"– which is at the core of this paper, but they do not explore how intergenerational exchange can overcome these political and market failures.

The theory of intergenerational exchange developed in this paper is an application of the theory of repeated games with overlapping generations of players (Salant (1991), Kandori (1992), Smith (1992)). From a game theoretical point of view, the only innovation is to point out and exploit the distinction between backward and forward intergenerational exchange. The paper also uses of the well-known fact that linkages across games can help in sustaining cooperation. Bernheim and Whinston (1990) show that cooperation among (infinitely-lived) oligopolists is more likely to arise when they compete simultaneously in several industries (see also Bendor and Mookherjee (1990)). Folmer et al. (1993) apply similar ideas to international cooperation in environmental problems. All of these papers study cooperation among infinitely-lived agents who play several games simultaneously. However, this paper applies these ideas to intergenerational exchange and emphasizes the distinction between FIGs and BIGs: forward intergenerational exchange can be sustained *only* by linking it to a game with the opposite timing of exchange.

A significant number of papers have studied the sustainability of backward intergenerational exchange in different contexts. See, among others, Bohn (1988), Cooley and Soares (1999), Hammond (1975), Hansson and Stuart (1989), Kocher-

[3] See also Silvestre (1994,1995).

lakota (1998), Kotlikoff, Persson, and Svensson (1988) and Kreps (1990). However, only a small number of papers have studied backward and forward intergenerational exchange together. Of particular relevance are Becker and Murphy (1988) and Boldrin and Montes (1988) who, as in the first application developed below, also study the political economy of intergenerational public expenditures. Becker and Murphy suggested that one could think of old age social security programs and educational investments as a large intergenerational trade in which every new generation receives education from its parents and in exchange pays their retirement benefits. However, they did not study the dynamic consistency of these arrangements. Boldrin and Montes study a three period OLG model in which expenditures in public education and social security are decided by majority rule. They show that the optimal allocation can be sustained using trigger strategies similar to the ones studied in this paper.[4]

## 3 Basic Model of Intergenerational Exchange

Consider a simple overlapping generations model. Every period $t = ..., -1, 0, 1, ...$ a new generation is born and lives for three periods: $t$, $t+1$, and $t+2$. Agents are infants in the first period, middle-aged in the second, and old in the third. They receive an endowment or wage in the second period, and can save (or borrow) for retirement using a storage technology with a constant rate of return $1+r$. A generation consists of a single individual.

Every period $t$ the middle-aged generation $t-1$ can invest in a *Forward Intergenerational Good* (FIG) that only benefits that next generation. Let $f_t$ denote the level of investment in period $t$. The FIG can be produced at only two levels: $f_t \in \{0, 1\}$. Every period, the middle-aged can also invest in a *Backward intergenerational Good* (BIG) that only benefits the previous generation. Let $b_t \in \{0, 1\}$ denote the level of investment in the BIG. The cost of investing in the FIG is $F > 0$, the cost of investing in the BIG is $B > 0$.[5] The timing of decision making is illustrated in Figure 1.

To simplify the exposition we start with the case of selfish generations and add

[4]The political economy model of intergenerational exchange presented here and the one in Boldrin and Montes (1998) were developed independently. The political economy model presented in Section 7 is a revised version of Rangel (1997).

[5]The theory remains unchanged if these costs are modelled as "utility" costs, which may be more appropriate for some applications like intergenerational exchange within the family.

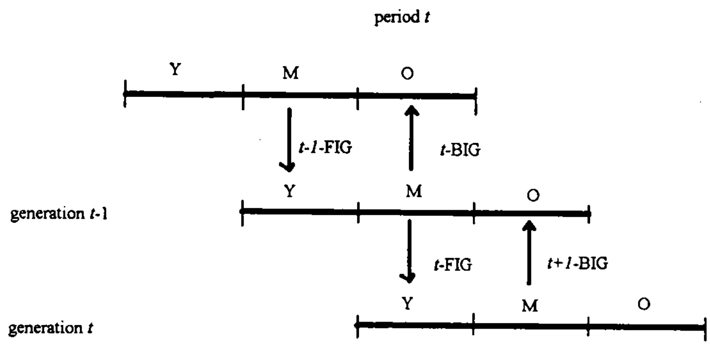


Figure 1: Timing of exchange

non-dynastic altruism afterwards. The preferences of generation $t$ are given by

$$U_t(c^m, c^o, f_t, b_{t+2}) = u(c^m) + \delta u(c^o) + V^F(f_t) + V^B(b_{t+2});$$

where $V^F(1) > V^F(0)$ and $V^B(1) > V^B(0)$. Agents only care about their own consumption, and about receiving a FIG when they are young and a BIG when they are old. Children do not derive utility from consumption, but they care about the FIG that they get from the previous generation.[6] Finally, $u(0) = -\infty$, which implies that agents have a very strong preference for consuming a positive amount in both periods.

The following notation will simplify the exposition below. Let

$$\sigma^*_{(w^m,w^o)} = \underset{-\frac{w^o}{(1+r)} \leq \sigma \leq w^m}{\arg\max} \quad u(w^m - \sigma) + \delta u((1+r)\sigma + w^o)$$

denote the optimal level of savings for an agent that has income $w^m$ in middle age and $w^o$ in old age. (The endowment of the old is zero, but in equilibrium they may receive transfers of the consumption good). Also, let

$$v^*(w^m, w^o) = u(w^m - \sigma^*_{(w^m,w^o)}) + \delta u((1+r)\sigma^*_{(w^m,w^o)} + w^o)$$

denote the indirect utility of consumption. Finally, to minimize notation time subscripts always refer to the period in which the relevant event is taking place. For example, $f_t$ is the level of FIG investment in period $t$ and $w_t$ are the wages in that period (which are paid to the middle-aged of generation $t-1$).

[6] As we will see below, the results easily extend to the case of non-dynastic altruism and to the case of impure FIGs, where the generation producing the FIG gets some of the benefits.

Under these assumptions the model of intergenerational exchange reduces to an infinitely repeated game that the generations play with each other. Every period $t$, as illustrated in Figure 1, generation $t-1$ picks an action $(f_t, b_t) \in \{0,1\} \times \{0,1\}$. The payoff of a generation depends only on his actions, the actions of the previous generation, and the actions of the next generation. The history $h_t$ of previous decisions is known, where $h_t = ((f_{t-1}, b_{t-1}), (f_{t-2}, b_{t-2}), ...)$ denotes every action taken prior to period $t$. A strategy for generation $t$-1 is a function $s_t(h_t)$ that specifies an action contingent on the entire history of play. An equilibrium is simply a subgame perfect equilibrium of this game.

# 4 BIGS vs FIGs: the crucial role of the timing of exchange

In this stylized model of intergenerational exchange the only difference between FIGs and BIGs is the timing of exchange. In both cases the *structure* of exchange is similar: buy something for generation $x$ and receive something from generation $y$. The *timing* of exchange, however, is rather different. In the case of BIGs, agents pay first, by providing a BIG for the previous generation, and benefit later, by receiving a BIG from the next generation. By contrast, agents receive their own FIG first and pay only later by providing a FIG for the next generation.

The goal of this section is to show that this simple difference in the timing of exchange has important implications for the type of intergenerational exchange that can take place. Two concepts are useful in achieving this goal. First, is the *surplus* generated by a BIG, which is equal to the value of "cooperating" in the BIG. For generation $t-1$ the surplus is given by

$$[v^*(w_t - B, 0) + V^B(1)] - [v^*(w_t, 0) + V^B(0)].$$

The first term is the payoff if the BIG is produced every period. The second term is the payoff if the BIG is never produced.

Another useful notion is self-sustainability. To understand this notion consider an even simpler exchange game in which there are only BIGs. Here, the only decision that agents make is whether or not to produce a BIG for the previous generation. A BIG is *self-sustainable* if there is an equilibrium in the simpler game in which only BIGs are produced every period. Intuitively, a BIG is self-sustainable if it generates, on its own, enough incentives to sustain backward intergenerational exchange. The definition of self-sustainability for FIGs is analogous.

**PROPOSITION 1:** *There is a fundamental asymmetry between forward and backward intergenerational exchange: (1) BIGs that generate a positive surplus for every generation are self-sustainable; (2) FIGs are not self-sustainable even when they generate an arbitrarily large surplus for every generation.*

The proof of this proposition is a straightforward application of the theory of repeated games. Consider first the case in which there is only a BIG and suppose that it generates a surplus for every generation. We need to show that there is an equilibrium in which the BIG is always produced. To do so, consider the following trigger-strategies:

$$s_t(h_t) = \begin{cases} b_t = 1 & \text{if } b_{t-1} = b_{t-2} = ... = 1 \\ b_t = 0 & \text{otherwise} \end{cases} \quad . \tag{1}$$

Under these strategies a generation produces the BIG if, and only if, every previous generation has done the same.

To establish that the strategies in (1) are an equilibrium we need to show that every generation has an incentive to follow it given that every other generation does so. Consider the incentives of generation $t-1$ who, in period $t$, can find itself in two types of histories. First, there are histories in which a previous generation has failed to produced the BIG. In this case, as (1) specifies, the best response is not to produce the BIG since the actions of the next generation are fixed at $b_{t+1} = 0$. Second, there are histories in which every previous generation has provided the BIG. In this case generation $t-1$ gets

$$u(w_t - B - \sigma^*_{(w_t - B, 0)}) + \delta u((1+r)\sigma^*_{(w_t - B, 0)}) + V^B(1)$$

if it provides the BIG, and

$$u(w_t - \sigma^*_{(w_{t-1}, 0)}) + \delta u((1+r)\sigma^*_{(w_{t-1}, 0)}) + V^B(0)$$

if it does not. Clearly, generation $t-1$ provides the BIG as long as

$$V^B(1) - V^B(0) \geq v^*(w_t, 0) - v^*(w_t - B, 0); \tag{2}$$

i.e., as long as the BIG generates a surplus. We can conclude that if (2) is satisfied for every generation, the trigger-strategies are an equilibrium in which the BIG is always produced.

For the second part of the proof consider the case in which there are only FIGs. It is easy to see that there is a unique equilibrium in which no FIGs are produced. Since agents receive their own FIG before they have to make a decision, providing a FIG for the next generation is a pure transfer. Thus, regardless of the surplus generated by the FIG, it is never produced. This concludes the proof.

The timing of exchange is essential to understand why BIGs that generate a small but positive surplus are self-sustainable but FIGs never are. Providing a BIG or a FIG for someone else is a costly action that generations do not want to undertake. They do so only if this is the only way of getting a good that they care about. In the case of BIGs, cooperation is possible because agents must choose first to buy a BIG for someone else, and only later receive the BIG that they care about. This allows the social norm to condition the production of the BIG that a generation cares about on their behavior towards previous generations. By contrast, in the case of FIGs, a generation receives its own FIG at birth and it does not care about how its actions affect future behavior.

The trigger-strategies described in (1) are somewhat draconian and unrealistic. In particular, under this social norm intergenerational exchange collapses if a single generation fails to produce the BIG. However, if intergenerational cooperation is feasible with trigger-strategies, then typically it is also feasible with non-trigger strategies. For example, a variation of the previous argument shows that the strategies

$$s_t(h_t) = \begin{cases} b_t = 1 & \text{if } b_{t-1} = b_{t-2} = ... = 1 \\ b_t = 1 & \text{if } b_{t-k} = b_{t-(k+1)} = 0 \text{ and } b_{t-j} = 1 \text{ for all } j \neq k, k+1 \\ b_t = 0 & \text{otherwise} \end{cases} \tag{3}$$

are also an equilibrium that support the production of the BIGs. But in this case a single "deviation" does not collapse all intergenerational exchange. The first generation that fails to produce the BIG is punished by not receiving a BIG in old age, but afterwards cooperation resumes. More complicated strategies are possible.[7]

The rest of the paper makes extensive used of trigger strategies. This is justified by the following well known result. In this class of games, if intergenerational cooperation can be sustained using non-trigger-strategies, then it can also be sustained

[7] Bhaskar (1998) shows that, in this type of overlapping generation games, the existence of cooperative equilibria depends crucially on the observability of the entire history of play. In particular, no-cooperation is the unique equilibrium when generations can observe *at most* the actions of the last $n$ predecessors.

with trigger-strategies. Thus, one can use trigger-strategies to establish *necessary* conditions for intergenerational cooperation. However, as the previous paragraph shows, this does not mean that trigger-strategies are the only way of supporting cooperation. They are a simply convenient analytic device, not the most realistic description of the equilibrium social norm. A straightforward implication of this result is that Proposition 1 can be strengthen: BIGs are self-sustainable *if and only if* they generate a non-negative surplus for every generation. This follows directly from (2) in the proof of Proposition 1.

The multiple equilibria problem is severe and unavoidable in this class of games. In particular there are always equilibria in which the BIG is never produced. After all, since producing a BIG is costly, a generation does so only if it believes that its actions will affect the behavior of the next generation. As a result, $s_t(h_t) = 0$ for all $h_t$ is always an equilibrium. This "bad" equilibrium exists even if the BIG generates an arbitrarily large surplus.

# 5 Linking FIGs and BIGs

Although FIGs are not self-sustainable, in this section we show that there are equilibria in which they are provided. Furthermore, the results show that there is an important relationship between backward and forward intergenerational exchange: FIGs are provided *only* when they are "linked" to BIGs.

**PROPOSITION 2:** *FIGs are provided in every period if and only if (1) for every generation, the surplus generated by the BIG is larger than the cost of providing the FIG; and (2) the equilibrium strategies link BIGs and FIGs by making cooperation on FIGs a necessary condition for cooperation on BIGs.*

To prove this result note that two types of strategies are possible when generations pick BIGs and FIGs. In the first type there is no link between BIGs and FIGs, the history of BIGs influences only the choice of BIGs and the history of FIGs influences only the choice of FIGs. These strategies can be written as

$$s_t(h_t) = (b_t(b_{t-1}, b_{t-2}, \ldots), f_t(f_{t-1}, f_{t-2}, \ldots)).$$

It is easy to see that the set of no-link equilibria is given simply by the juxtaposition of the equilibria described in the previous section. Thus,

$$s_t(h_t) = (0, 0)$$

and

$$s_t(h_t) = \begin{cases} b_t = 1,\ f_t = 0 & \text{if } b_{t-1} = b_{t-2} = ... = 1 \\ b_t = f_t = 0 & \text{otherwise} \end{cases}$$

are two possible no-link equilibria. In the first no intergenerational exchange takes place, in the second only BIGs are produced.

By contrast, in the second type of strategies given by

$$s_t(h_t) = (b_t(h_t), f_t(h_t))$$

there is a link between BIGs and FIGs. Here the choice of BIGs depends on the history of *both* BIGs and FIGs. This link has important consequences for the production of FIGs. In particular, consider the trigger-strategies

$$s_t(h_t) = \begin{cases} b_t = f_t = 1 & \text{if } f_{t-1} = b_{t-1} = f_{t-2} = b_{t-2} = ... = 1 \\ b_t = f_t = 0 & \text{otherwise} \end{cases}, \tag{4}$$

where failure to produce either a BIG or a FIG collapses both types of intergenerational trade.

As before, to show that the strategies in (4) are an equilibrium we need to show that every generation has an incentive to follow them at every possible history, given that every other generation does so. Once more, start by looking at histories in which there was a previous breakdown in intergenerational cooperation. Here generation $t-1$ knows that, regardless of his actions, the next generation will not provide the BIG. The best response, as (4) specifies, is to produce neither the BIG nor the FIG.

Now consider the history in which every previous generation has provided both goods. If generation $t-1$ cooperates it gets a payoff

$$u(w_t - B - F - \sigma^*_{(w_t - B - F, 0)}) + \delta u((1+r)\sigma^*_{(w_t - B - F, 0)}) + V^F(1) + V^B(1).$$

On the other hand, the payoff at the best possible deviation is given by

$$u(w_t - \sigma^*_{(w_t, 0)}) + \delta u((1+r)\sigma^*_{(w_t, 0)}) + V^F(1) + V^B(0)$$

Clearly, generation $t-1$ provides the BIG and the FIG as long as

$$V^B(1) - V^B(0) \geq v^*(w_{t-1}, 0) - v^*(w_{t-1} - (B+F), 0). \tag{5}$$

In other words, cooperation is possible as long as the surplus generated by the BIG outweighs the cost of providing both the BIG and the FIG. This concludes the proof.

The key insight behind this result is that a self-sustainable BIG plus a low cost FIG looks like a self-sustainable BIG. If everyone behaves according to (4), the only way that a generation can purchase its own BIG is to provide both the BIG and the FIG. But then the choice that every generation faces is to give up $B + F$ when middle-aged to receive $V^B(1)$ when old. This looks exactly like a BIG that costs $B + F$.

Can the link between BIGs and FIGs generate optimal investment in future generations? Yes, as long as the surplus generated by the BIGs outweighs the cost of providing the BIGs. However, over-investment in future generations is also possible. A generation invests in the FIG only as way of maintaining cooperation in the BIG. It does not care about the benefits generated by the FIG, only about its costs. Thus, it is indifferent between investing in efficient and inefficient FIGs.

All of the equilibria in this model can be divided into three categories. First, there are Markovian or history independent equilibria in which no intergenerational trade, backward or forward, takes place. Second, there are history dependent non-linkage equilibria in which only BIGs are produced. Finally, there are history dependent equilibria with linkage in which both BIGs are FIGs are produced. A comparison of these categories yields the following insights:

1. Investment in future generations is possible even with selfish generations. For some parameters, the linkage strategy may even yield optimal intergenerational exchange.

2. Optimal investment in future generations can take place *only if*: (1) BIGs and FIGs are linked in the equilibrium social norm, and (2) the cost of providing the FIG is not too large and the surplus generated by the BIG is large enough.

3. Since FIGs are produced only when BIGs are also produced, there is a bias in favor of backwards intergenerational exchange.

4. The benefits generated by the FIG play no role on its sustainability, only the production costs matter. Thus, a FIG that generates arbitrarily large benefits, but is costly, will not be produced. Whereas a FIG that generates lower net benefits, but is relatively cheap, can be sustained.

These results are at work in a wide range of problems, from the political economy of the environment to the economics of the family.

A natural extension of the model is to consider the case in which there are multiple FIGs and BIGs. Suppose, as before, that all of these goods can be produced at two levels (0 and 1) and that preferences are additively separable in each good. Some of the BIGs might be self-sustainable, others may not.

**PROPOSITION 3:** *(1) The provision of an expensive FIG can be sustained by linking it to several BIGs. This is feasible as long as, for every generation, the sum of the surpluses generated by the BIGs is larger than the cost of providing the FIG. (2) A non-self-sustainable BIG is provided in equilibrium if and only if it is linked to a self-sustainable BIG that generates a large enough surplus.*

The proof of this result is a straightforward repetition of previous arguments and is omitted. The key to the first part of the result is that two BIGs that generate a surplus look like a BIG that generates an even larger surplus. As a result, several BIGs can be linked together to provide incentives to support a FIG. If BIGs 1 to $n$ are linked together then the incentive constraint in (5) becomes

$$\sum_{i=1}^{n}(V^{B_i}(1) - V^{B_i}(0)) \geq v^*(w_t, 0) - v^*(w_t - (\sum_{i=1}^{n} B_i + F), 0),$$

which gives more incentive leverage to support forward intergenerational exchange. It is also easy to see that a non-sustainable BIG can be supported by linking it with a sustainable one. In some sense, producing a non-sustainable BIG is analogous to producing a FIG.

The possibility of a bad outcome in BIGs is essential to provide incentives for investment in future generations. However, an increase in the size of the surplus generated by the BIG can be a mixed blessing. Consider the case in which $V^B(0) << 0$; i.e., a generation that doesn't get its BIG gets a very low payoff. Here the BIG generates a very large surplus. However, to avoid the bad equilibrium in which the BIG in not produced, agents might be tempted to look for an exogenous commitment device. This would be valuable because it eliminates the possibility of a very bad outcome. But it also eliminates the good equilibria in which FIGs are produced. Furthermore, since $V^B(1) - V^B(0)$ is very large, the BIG provides a large amount of incentive leverage that could be used to support the production of a large class of FIGs.

In some applications we might want to understand the conditions under which intergenerational cooperation begins. Unfortunately, given the multiplicity of equilibria, a precise answer for BIGs cannot be given. If BIGs generate a surplus in every period, for every natural number $k$ there is an equilibrium in which the first $k-1$ generations do not produce intergenerational goods, and every generation born afterwards plays one of the cooperative strategies. However, given the need for a link between BIGs and FIGs, something can be said about the timing at which cooperation in FIGs must begin. We know that a generation will finance a FIG only if it is necessary for getting a BIG in old age. Thus, cooperation on BIGs must arise, *at the latest*, one period after FIGs are first produced. This occurs, for example, in the following equilibrium:

$$s_t(h_t) = (b_t = 0, f_t = 0) \text{ for } t < k \ ,$$

$$s_k(h_k) = (b_1 = 0, f_k = 1) \ ,$$

and, for $t > k$,

$$s_t(h_t) = \begin{cases} b_t = f_t = 1 & \text{if } f_{t-1} = b_{t-1} = ... = f_{k+1} = b_{k+1} = 1 \text{ and } f_k = 1 \\ b_t = f_t = 0 & \text{otherwise} \end{cases} .$$

There are also equilibria in which for a while generations condition cooperation on BIGs on the history of BIGs, and the link with FIGs is introduced only later on.

## 6 Extensions

The basic model of intergenerational exchange is very stylized. Generations are completely selfish, they do not benefit at all from investing in FIGs or BIGs, and there is only a one period lag between the production and the consumption of the FIG. These assumptions are problematic in some applications. For example, a natural way of modelling families is to assume that parents have altruistic but paternalistic preferences. Also, FIGs like investment in public infrastructure benefit many generations, including the one that makes the investment. The goal of this section is to show that these complications can easily be incorporated without changing the insights of the basic theory.[8]

[8] The extension of the theory to "non-linear" economies in which interest rates and wages depend on the capital stock is not considered below. However, conceptually it is straightforward. The only difference is that in some cases agents take into account the effect of their actions on the capital stock. When this happens conditions like (5) need to be modified to include these effects, but the key insights remain unchanged. For example, see Cooley and Soares (99) and Boldrin and Montes (98) for a non-linear model of the pay-as-you-go social security BIG.

### 6.1 Multi-generational FIGs

Consider first a more general production structure for FIGs. As before, every generation has to decide whether or not to produce a FIG at a cost $F > 0$. However, the FIG now may benefit more than one generation, and there may be a long lag between the production and the consumption of the FIG. An example of such a FIG is investment in long-term R&D, where the benefits appear only several generations after the investment is made, and every generation born afterwards benefits from the invention.

General production structures can be modelled using the following utility function

$$U_t(.) = u(c^m) + \delta u(c^o) + \sum_{k=0}^{\infty} \rho_k V^F(f_{t-k}) + V^B(b_{t+2}),$$

where $\rho_k$ is a sequence of positive weights satisfying $\sum_{k=0}^{\infty} \rho_k < \infty$. For example, the basic model is a special case of this specification with $\rho_0 = 1$ and $\rho_k = 0$ for any $k > 0$. A FIG that benefits every generation born 10 periods after the investment is made, but that also exhibits depreciation, can be represented by $\rho_k = 0$ for $k = 0, ..., 9$ and $\rho_k = \beta^{k-10}$ $(0 < \beta < 1)$ for all $k > 9$.

It is easy to see that the previous results remain unchanged. The key is to notice that, although the new production function looks more complicated, the incentives of the different generations have not changed. In particular, the trigger-strategy in (4) generates production of FIGs and BIGs if, and only if, (5) holds. The intuition is simple. Since generation $t$ does not derive any value from the FIG, it is indifferent between producing a FIG that benefits generation $t+1$, one that benefits generation $t+1000$, or one that benefits every future generation. After all, the only reason that the generation produces the FIG is to keep cooperation going in the BIGs.

An interesting difference with the basic model has to do with the form of the punishments that support cooperation. In the basic model, generation $t$ knows that if it doesn't produce the FIG for generation $t + 1$ it will be punished by them. In this case the generation carrying out the punishment is the same generation that is hurt when the FIG is not produced. By contrast, consider a FIG that benefits every generation born 10 periods after it is produced. Why should generation $t + 1$ punish here? After all, it does not care about the period $t$ FIG. If the social norm is a trigger-strategy, generation $t + 1$ punishes by not producing the BIG because it knows that intergenerational exchange collapses after a single deviation. If the social norm is not a trigger strategy, generation $t+1$ punishes because it knows that it will not be rewarded for helping a deviator.

## 6.2 Non-dynastic Altruism and "Impure" FIGs

The basic theory is also robust to the introduction of non-dynastic altruism. The key difference is that now generations receive benefits from producing a FIG. A simple complication of the basic model allows us to incorporate this effect.

Suppose that the BIG decision is discrete, $b_t \in \{0,1\}$, but the FIG decision is continuous: $f_t \in [0,\infty)$. $f_t$ can be interpreted as the amount of the private good invested in FIGs. Also, suppose that preferences are given by

$$U_t(.) = u(c^m) + \delta u(c^o) + (1-\theta)V^F(f_t) + \theta V^F(f_{t+1}) + V^B(b_{t+2}),$$

where $\theta \in (0,1)$ is a parameter that reflects the degree to which the generation producing the FIG benefits from it. If $\theta = 0$ the model reduces the basic case. If $\theta = 1$ FIGs do not generate intergenerational spillovers.[9]

Let $f_t^*$ denote the efficient level of FIGs (at an interior allocation) and $\widehat{f}_t$ the level of FIGs that a generation chooses if it only takes into account its preferences; i.e., the level that it chooses in the absence of game theoretic considerations. Clearly, $f_t^* > \widehat{f}_t$. In contrast to the selfish case, with non-dynastic altruism every generation wants to produce *some* of the FIG. Thus, the conflict is not over producing a positive level of FIGs, but over producing more than the short-sighted amount. In some sense, agents produce two goods: (i) a non-intergenerational good that costs $\widehat{f}_t$, and (ii) a pure FIG given by expenditures above $\widehat{f}_t$.

This variation of the model also covers the case of impure FIGs. An *impure FIG* is a good that benefits the generation producing it and future generations. In other words, an impure FIG has short-run and long-run benefits. A good example of an impure FIG is saving the rain forests, which benefits current and future generations.

**PROPOSITION 4:** *In the case of impure FIGs or non-dynastic altruism, the optimal amount of FIGs is provided in every period if and only if: (1) for every generation, the surplus generated by the BIG is larger than the (utility) cost of providing the optimal amount of FIG; and (2) the equilibrium strategies*

[9]The basic theory can also easily be extended to the case of two-sided non-dynastic altruism. This variation of the model is used below in the application to investment in children within the family.

*link BIGs and FIGs by making cooperation on FIGs a necessary condition for cooperation on BIGs.*

Since the proof of this result is almost identical to the proof of Proposition 2, only a sketch is provided. First, to check the conditions under which optimal investment can take place we can use the trigger-strategies

$$s_t(h_t) = \begin{cases} b_t = 1, f_t = f_t^* & \text{if } f_{t-k} = f_t^* \text{ and } b_{t-k} = 1 \text{ for all } k \\ b_t = 0, f_t = \widehat{f}_t & \text{otherwise} \end{cases},$$

which link BIGs and FIGs by conditioning provision of the BIG on the *optimal* provision of the FIG. Second, one can show that the necessary and sufficient conditions for these strategies to be an equilibrium are given by

$$(V^B(1) - V^B(0)) + \theta(V^B(f_t^*) - V^B(\widehat{f}_t)) \geq v^*(w_t - \widehat{f}_t, 0) - v^*(w_t - (B + f_t^*), 0).$$

This condition, analogous to (5), states that the surplus generated by the BIG must be larger than the cost of investing in FIGs beyond the short-sighted amount. Finally, it is easy to see that in any no-link strategy every generation chooses $\widehat{f}_t$.

Thus, we can conclude that the introduction of non-dynastic altruism or "impure" FIGs does not change the main results of the theory. To induce a generation to invest above the short-sighted amount it is still necessary to link BIGs and FIGs. The intuition is simple. From the point of view of the generation making the investment, any expenditure in FIGs beyond what it internalizes looks like a FIG because the costs of additional provision outweigh the benefits.

### 6.3 Intergenerational Transfers and Endowment Effects

The previous results show that FIGs are provided, or provided optimally in the case of non-dynastic altruism, only if they are linked with a BIG. A natural question follows: Can the production of FIGs be supported using transfers of the consumption good? After all, generation $t+1$ could compensate generation $t$ for producing its FIG by giving away part of its endowment. The answer is yes and follows from the fact that intergenerational transfers are a BIG. However, as we will see below, this BIG is not always self-sustainable.

This question is particularly relevant when FIGs generate endowment effects; i.e., when a generation's endowment depends on whether previous generations invested in FIGs. Natural examples are investments in infrastructure, R&D and human capital expenditures.

A simple way of incorporating endowment effects is to return to the basic model, but make the endowments a function of the history of FIGs. In particular, suppose that

$$w_t(h_t) = \begin{cases} (1+\lambda)w_{t-1}(h_t) & \text{if } f_{t-1} = 1 \\ w_{t-1}(h_t) & \text{otherwise} \end{cases}, \tag{6}$$

where $\lambda > 0$ is the endowment effect.[10] Here, the wage of generation $t$ is equal to the endowment of the previous generation if there is no investment in the FIG, but grows by $(1+\lambda)$ as a result of the investment. We can think of $\lambda$ as the gross rate of return of investing in the FIG.

To keep things simple suppose that investing in the FIG costs a fraction $\phi \in (0,1)$ of the endowment of the generation making the investment, where $\phi$ can be interpreted as the fraction of labor income that is devoted to increasing the productive capacity of the economy. Assume also that FIGs have no utility effects. (An example of this type of FIG is a narrow interpretation of investments in human capital.)

To explore the role of intergenerational transfers consider an exchange game in which every generation picks an action $(T_t, f_t) \in \Re_+ \times \{0,1\}$. In this case the transfers, which amount to a BIG, are continuous and the FIG is discrete.

**PROPOSITION 5:** *Consider an equilibrium in which (1) present generations invest in increasing the endowment of the next generation and (2) the next generation compensates them with a fraction $\mu$ of their endowment. This equilibrium is feasible only if the rate of return of investing in FIGs exceeds the rate of return of the storage technology: $\lambda \geq r + (1+r)\frac{\phi}{\mu}$.*

Once more, since the proof of this result is a small variation of previous arguments, only the key steps are presented. First note that a sequence of transfers $\{T_t\}$ is a BIG: give a transfer to the previous generation when young and in exchange receive a transfer when old. Also, since agents can borrow and save at the rate $1+r$, they care about the rate of return implicit in the transfers, but not about their timing. As a result, this BIG is self-sustainable only if $T_{t+1} \geq (1+r)T_t$ for all $t$. Intuitively, the "transfer game"[11] amounts to a voluntary savings program that is attractive only if it pays a better rate of return than the storage technology.

[10] Recall that $w_t$ is the endowment or wage that generation $t-1$ gets in middle age, which takes place in period $t$. Therefore, $w_t$ can be a function of $h_t = (f_{t-1}, b_{t-1}, ...)$.

[11] This transfer game was first suggested by Samuelson (1958) and has also been studied by Hammond (1975).

Next, as before, the FIG is produced only if (1) the generations play history dependent strategies that link BIGs and FIGs, and (2) the surplus generated by the BIG is larger than the cost of providing the FIG. The BIG's surplus is given by $T_{t+1} - (1+r)T_t$. It is easy to check that the trigger-strategy linking FIGs and transfers is an equilibrium only if, for every $t$ and $h_t$,

$$T_{t+1} \geq (1+r)(T_t + \phi w_t(h_t)), \tag{7}$$

where $\phi w_t(h_t)$ is the cost of providing the FIG. Clearly, a sequence of transfers satisfies (7) only if it grows faster than the interest rate, which is possible only if $\lambda > r$. Now consider transfers of the form $T_t(h_t) = \mu w_t(h_t)$. These transfers satisfy the equilibrium condition only if

$$\mu(1+\lambda)w_t(h_t) \geq w_t(h_t)(\mu + \phi)(1+r) \text{ for all } t,$$

which is satisfied only if

$$\lambda \geq r + (1+r)\frac{\phi}{\mu}.$$

Consider the implications of this condition. In this model society has two investment technologies: the storage technology and the FIGs. Both of them transform units of the consumption good in one period into units of the consumption good in the following period. If $\lambda > r$, it is socially optimal to invest only on FIGs. Ideally, present generations would invest in FIGs and future generations would use the increase in their endowment to compensate them. However, Proposition 6 shows that the incentive conditions are more stringent than the conditions for social optimality. In particular, this arrangement is dynamically consistent only if the FIG's rate of return exceeds the one of the storage technology by at least $(1+r)\frac{\phi}{\mu}$.

# 7 Application 1: Political Economy of Intergenerational Expenditures

The basic model of intergenerational exchange can be applied to a wide class of problems. In the rest of the paper we explore some of these applications. The first application is to study the political economy of intergenerational public expenditures. In most nations, a significant amount of intergenerational exchange takes place through the government. Prominent examples are pay-as-you-go programs that benefit the elderly, like Social Security and Medicare (BIGs), and expenditures in public education, R&D, and infrastructure (FIGs). In the U.S. some of these programs take place at the federal level, others, like education, are carried out at the state level. In this section we present a dynamic model of the political economy of intergenerational expenditures at the federal or central level, and show that the underlying political forces are easily understood in terms of the theory of BIGs and FIGs.[12]

## 7.1 Model

Consider an economy with a somewhat realistic demographic structure in which every period a new generation of agents is born and lives for 80 periods. Agents are children for the first 20 periods, workers for the next 45 periods, and retire at age 65. Every generation has a continuum of agents with mass 1. Workers in period $t$ receive a wage $w_t(a)$, which might depend on the age of the worker. This could reflect, for example, differences in education levels across cohorts. For simplicity, suppose that wages do not depend on the size of the capital stock, and that they grow at a constant growth rate . The interest rate is fixed and equal to $1 + r$ and, as before, agents can borrow and lend at this rate.[13] Furthermore, assume that the interest rate exceeds the rate of growth of wages, so that the economy is dynamically efficient.

There are two government programs: (1) pay-as-you-go social security, and (2) a generic FIG that can be interpreted as environmental protection, long-term R&D, or any other public expenditure that disproportionately benefits the young. Decisions on these programs are made by majority rule. Agents can vote only after they become workers at age 21.

The pay-as-you-go social security system is characterized by a sequence of (lump-sum) payroll taxes $T_t$, which are always equal to a fraction $\pi$ of wages. The system is balanced period by period and thus each retiree receives benefits $B_t = 3T_t$.[14] A pay-

[12]For an analysis of the political economy of investment in future generations at the local level see Conley and Rangel (1999), Kotlikoff and Rosenthal (1993), and Soares (1999).

[13]These partial equilibrium assumptions simplify the analysis of the model, but none of the insights discussed below depend on them. Also, to avoid dealing with borrowing constraints, we assume that agents can borrow in international financial markets at the rate $1 + r$.

[14]We assume that the agents' retirement age is exogenously given and that social security benefits start at that date. Thus, in this simple model social security does not distort retirement decisions. An interesting question, however, is why so many retirement programs around the world introduce such distortions. Two papers addressing this issue are Diamond and Mirrlees (1978) and Mulligan and Sala-i-Martin (1999).

as-you go social security system amounts to a sequence of backward intergenerational transfers and thus is analogous to the transfer BIG discussed in Section 5.

The generic FIG can only be produced at two levels, $f_t \in \{0, 1\}$. The cost of producing the FIG is born equally by all taxpayers, an assumption that is justified if FIGs are financed out of general revenues. The (lump-sum) cost for each taxpayer of producing the $t$-FIG is $F_t$. To keep things simple we also assume that (1) only children and unborn generations benefit from producing a FIG and (2) $F_t$ is a constant fraction of aggregate wages. Under these assumptions, taxpayers benefit from the production of past FIGs, but not from the production of current or future FIGs. These two restrictions are made for analytical convenience, but are not necessary for the results or their interpretation.

As before, let $h_t = \{(T_{t-1}, f_{t-1}), (T_{t-2}, f_{t-2}), ...\}$ denote the history of government policy. The preferences of generation $t$ are given by

$$V_t(c(1), ..., c(80), h_{t+21}) = \sum_{a=21}^{80} \beta^{a-21} u(c(a), h_{t+21}). \tag{8}$$

Under these preferences agents do not consume before they become workers, care about FIGs produced before their birth or during their childhood, and are not affected by FIGs produced after they reach adulthood. (This follows from the fact that $V_t(.)$ only depends on $h_{t+21}$).

Every period voters choose social security policy and whether or not to finance the FIG. However, the analysis is simplified if we assume that the size of the benefits and transfers are exogenously given and that voters only decide whether or not to execute them. Let $\{\overline{T}_t\}$ denote the exogenously given system. In this case the policy space in period $t$ is given by the familiar

$$P_t = \{b_t = 1, b_t = 0\} \times \{f_t = 1, f_t = 0\}.$$

This assumption is not as restrictive as it seems. If a system $\{\overline{T}_t\}$ can be supported as an equilibrium of the unrestricted game, then it also arises as an equilibrium with the restricted policy space. Thus, the restricted analysis fully characterizes the set of transfers that can be supported in the general case.

Let $s_t(a, h_t) \in P_t$ denote the (history dependent) voting strategy for an agent of age $a$ in period $t$, and let $s$ denote a complete description of the behavior of every agent in every period. Given $s$ we can construct the functions

$$W^B(s, h_t) \text{ and } W^F(s, h_t) \tag{9}$$

which denote the outcome of the election, for BIGs and FIGS, if the economy reaches period $t$ under the history $h_t$. Another useful object is

$$h^c(s, (b_t, f_t), h_t),$$

which denotes the continuation outcomes of choosing $(b_t, f_t)$ after history $h_t$; i.e., it describes the outcome of the election in periods $t+1$, $t+2$, ... . $h^c$ is computed by iterating (9).

To characterize the equilibria of the voting game we need to derive agents' preferences over present and future public policy. Consider a voter of age $a$ in period $t$. Since the voter does not benefit from the production of new FIGs, and is able to borrow and lend at the rate $1+r$, he only cares about the effect of public policy on his budget constraint. Thus, his preferences over public policy are given by

$$U_t(a, \{(b_k, f_k)\}_{k \geq t}) = PV_t^B(a, \{(b_k, f_k)\}_{k \geq t}) + PV_t^F(a, \{(b_k, f_k)\}_{k \geq t}),$$

where $PV_t^B$ denotes the present value of all present and future social security contributions and $PV_t^F$ denotes the present value of the taxes that the agent will have to pay in this and subsequent periods if the sequence $\{f_k\}_{k \geq t}$ of FIGs is produced. (Benefits are positive and taxes are negative numbers). In other words, $PV_t^B$ and $PV_t^F$ denote the effect that BIGs and FIGs policy have on the agent's budget constraint.

**Political Equilibrium:** A vector $s^*$ of voting strategies is a political equilibrium if, for all $a, t$, and $h_t$, it satisfies

$$s_t^*(a, h_t) \in \arg\max_{b_t, f_t} U_t(a, (b_t, f_t), h^c(s^*, (b_t, f_t), h_t)).$$

Two assumptions are built into this notion of equilibrium. The first one is sincere voting: agents vote for their favorite policy even though they are never pivotal voters. This assumption is standard in the political economy literature and is a way of avoiding the paradox of voting: why do voters vote at all? It also eliminates a lot of "bad" equilibria. In fact, without sincere voting any policy could be supported as an equilibrium. The second assumption is that voters are not short-sighted, they take into account the effect that current electoral outcomes have on future elections.

### 7.2 Political Economy of Pay-as-you-go Social Security

As before, it is useful to analyze first the political equilibria of a simpler case in which there is only social security. Using the insights from the basic model the analysis is rather simple.

The key is to notice that the continuation value of social security improves with age. The present value of the system is positive for retirees, since they only receive benefits, and for those that are close enough for retirement. In fact, if the social security system is balanced, payroll tax rates and wage growth rates are constant, and the economy is dynamically efficient we have that

$$PV_t^B(65) > PV_t^B(64) > ... > PV_t^B(21). \tag{10}$$

Let $a_t^{\min}$ be the youngest age, in period $t$, at which the continuation value of social security is non-negative (i.e., $PV_t^B(a_t^{\min}) \geq 0$ and $PV_t^B(a_t^{\min} - 1) < 0$). Also, let $a_t^{med}$ denote the median age of the voting population.

**PROPOSITION 6:** *Social security can arise as a political equilibrium in every period as long as the median voter is always a worker with a positive continuation value for social security ($a_t^{med} \geq a_t^{\min}$ for all $t$).*

The proof of this result is analogous to the proof that BIGs are self-sustainable if they generate a positive surplus. In particular, it is sufficient to show that the following voting strategies are a political equilibrium when $a_t^{med} \geq a_t^{\min}$:

$$s_t(a, h_t) = \begin{cases} b_t = 1 & \text{if } a > 65 \\ b_t = 1 & \text{if } a_t^{\min} \leq a \leq 65 \text{ and } b_{t-k} = 1 \text{ for all } k \\ b_t = 0 & \text{otherwise} \end{cases}. \tag{11}$$

The incentives of retirees are straightforward: since they are net beneficiaries, they always vote for social security. The incentives of agents younger than $a_t^{\min}$ are also straightforward. Since the continuation value of social security is negative for them, they always vote against social security. These two elements are incorporated into (11).

Things are more complicated for middle-aged voters with ages between $a_t^{\min}$ and 65. If the only choice is to kill or save social security forever, the agents are in favor of keeping the system. However, since they have to pay payroll taxes until their retirement, they would prefer to suspend social security in the short-term and reinstate it when they retire.

Under (11), this group votes for social security as long, and only as long as, social security has always been chosen by the political process. In other words, middle-aged voters play a trigger-strategy. To see that (11) is an equilibrium it is enough to notice that from the point of view of the middle-aged, voting for social security is analogous to providing a BIG in the basic theory. This follows from the definition of equilibrium: agents vote sincerely and take the response of future voters as given. Thus, since the elderly vote for social security, the middle-aged group always determines policy and is playing an intergenerational exchange game with future generations of middle-aged voters, the ones that will determine the social security benefits of current workers. But then, as we showed before, the trigger-strategies defined in (11) are an equilibrium as long as the social security generates a surplus for at least half of the population; that is, as long as $PV_t^B(a) \geq 0$ for all $a \geq a_t^{med}$. This condition is satisfied when $a_t^{\min} \leq a_t^{med}$.

At first sight this political economy model does not fit the framework of the basic theory. There is a continuum of voters, agents do not determine policy by themselves, and voters make decisions in every period of their adult lives. However, the previous argument shows that with sincere voting the political economy model can be reduced to a BIG game among the "pivotal" middle aged generations.

The contribution of this section of the paper is not to provide a new model of the politics of social security. Bohn (1998), Boldrin and Montes (1998), Cooley and Soares (1999), and Hanson and Stuart (1989) have studied similar models. The goal here is to show that complicated models of intergenerational exchange can be understood in simple terms using the ideas of BIGs and FIGs. Incidentally, Bohn also studies a model with a realistic demographic structure and shows that pay-as-you-go social security is supportable as long as the median voter is old enough. He shows that historically the condition $a_t^{\min} < a_t^{med}$ has been satisfied by the U.S. population and that this will continue to be true in the future. This provides at least part of the explanation for why the American Social Security system remains politically viable even though it is not actuarially fair.

### 7.3 Political Economy of Investment in Future Generations

Now let's add the generic FIG. Will a selfish electorate finance investments that do not benefit anyone who votes? The basic theory of BIGs and FIGs provides a simple answer. It depends on (1) whether voting strategies link social security and FIGs, and (2) on whether the continuation value of social security for the pivotal middle aged voters outweighs the cost of purchasing the FIGs.

As before, if $a_t^{med} \geq a_t^{\min}$ for all $t$ there are equilibria with social security but not FIGs. But equilibria with public investment in future generations are also possible. Let $PV_t^{B+F}(a)$ denote the effect for an agent of age $a$ of having social security and FIGs in every period from $t$ onwards. Clearly,

$$PV_t^{B+F}(a) < PV_t^{B}(a) \text{ for all } a \geq 21,$$

since agents now pay for social security *and* FIGs. Furthermore, as long as the cost of the FIGs doesn't fluctuate too much we have that

$$PV_t^{B+F}(65) > PV_t^{B+F}(64) > ... > PV_t^{B+F}(21),$$

which is guaranteed by the assumption that the cost of FIGs is a constant fraction of aggregate wages. If the FIG is very expensive then $PV_t^{B+F}(a)$ could be negative even for retirees. However, if the FIG is small relative to social security, or if social security benefits are very generous, there is an age $\widehat{a}_t^{\min} < 80$ such that the continuation value of the joint BIG and FIG policy is positive.

**PROPOSITION 7:** *If $a_t^{med} \geq \widehat{a}_t^{\min}$ for all $t$ there are political equilibria in which social security and the generic FIG are provided in every period. These equilibria are supported by voting strategies that link social security and the FIG.*

To prove this result it is enough to show that the following voting strategy is an equilibrium when $a_t^{med} \geq \widehat{a}_t^{\min}$:

$$s_t(a, h_t) = \begin{cases} b_t = f_t = 1 & \text{if } \widehat{a}_t^{\min} \leq a \text{ and } b_{t-k} = f_{t-k} = 1 \text{ for all } k > 0 \\ b_t = f_t = 0 & \text{otherwise} \end{cases} . \quad (12)$$

The argument is analogous to the proof that FIGs can be supported by linking them with BIGs. Consider any voter older than $\widehat{a}_t^{\min}$. This voter prefers not to finance the FIG and to have a social security system only during his retirement. However, if the choice is between social security *and* FIGs, or none of the two programs, the voter will favor keeping both policies. Furthermore, since agents vote sincerely and take future voting strategies as given, this agent is playing an intergenerational game with future voters in the age range $\widehat{a}_t^{\min}$ to 80. Thus, his incentives look exactly like the ones of the middle-aged agent in the basic model. From Proposition 2 it follows that the trigger-strategies in (12) are an equilibrium as long as $PV_t^{B+F}(a)$ is non-negative for a majority of the electorate, which is satisfied if $a_t^{med} \geq \widehat{a}_t^{\min}$.

Note that now the only voters with a dominant strategy are those between 21 and $a_t^{\min}$ years old, who always vote against social security and against the generic FIG. Agents between $a_t^{\min}$ and $\widehat{a}_t^{\min}$ would like to have social security, but not if it is linked by the FIG. Agents older than $\widehat{a}_t^{\min}$ would prefer to have social security without FIGs, but are willing to finance FIGs if this is the only way of making sure that future voters will finance their social security benefits.

Proposition 8 shows that investment in future generations can take place even if present generations are selfish and do not benefit from the expenditures. However, as in the basic theory, this can happen only if voting strategies link social security with FIGs such as investments in the environment and public education. Of course, with impure FIGs or altruistic generations there is some investment in FIGs even if the link is not present. However, in the absence of dynastic altruism, agents typically do not invest the optimal amount without a link to the BIG.

A surprising comparative static is that, contingent on being in an equilibrium that links social security and FIGs, the aging of the population can help investment in future generations. The reason is that for workers the continuation value of social security increases with age. As a result, older workers have more surplus that can be used to support the production of FIGs.

Other predictions of the basic model are also at work here. First, pure investments in future generations take place only when BIGs, like social security, are also produced. In this sense there is a bias in favor of backwards intergenerational exchange: public expenditures on the elderly are more likely to arise than public expenditures in future generations. Second, the benefits generated by FIGs play no role on the political calculations of selfish voters, only the costs of producing them matter. As a result, FIGs that generate arbitrarily large benefits, but are very costly, will not be produced. By contrast, FIGs that generate lower net benefits, but are relatively cheap can be sustained. Thus, the model predicts that costly FIGs are less likely to be produced and that the net benefits produced by a FIG should not affect its level of funding. In this sense, expensive but potentially very valuable FIGs like avoiding global warming or space exploration may be under-funded, while cheaper but less attractive FIGs, like minor environmental improvements, may be funded.

Social Security is not the only BIG that can be used to support investment in future generations. Any government program that disproportionately benefits or hurts the old can also be used. Examples of such programs are Medicare, capital taxation, or a combination of the two.

In the capital taxation BIG, voters choose in every period the capital tax rate for that period. Young voters would like to expropriate the savings of the elderly, but do not want to be expropriated when they are old. The bad equilibrium in this BIG is rather grim: no one saves because they know that their savings will be taxed away. To avoid this outcome societies might want to make use of an exogenous constitutional constraint against capital taxation. This would prevent a very bad outcome but would also eliminate a lot incentive leverage that could be used to support investment in future generations.

One might be tempted to think that using debt to finance FIGs would get rid of these problems. After all, by using debt one could invest in future generations and pass them the bill. This intuition is incorrect. The problem is twofold. First, if the government has the authority to issue debt to finance FIGs, it also has the authority to issue debt to finance consumption. Hence, present voters always internalize the cost of investing in FIGs and the problem remains. A possible solution would be to introduce a capital budget constraint under which the government can issue debt only to invest in future generations. The problem with this solution is that it is difficult to define precisely what is an investment and what is not.[15] Furthermore, even if this problem could be solved, future generations would still have to vote to service the debt.

# 8 Application 2: A "Selfish" Theory of the Altruistic Family

Another natural application of BIGs and FIGs are the economics of the family, where a significant amount of intergenerational exchange takes place. Parents involvement and expenditures affect their children's human capital and emotional well-being[16] (FIGs). Middle-aged parents also provide tangible and intangible benefits to elderly parents (BIGs).

[15] A recent presidential commission on capital budgeting arrived at this conclusion. See President's Commission to Study Capital Budgeting (1999).

[16] If you do not think that this is an economic good, consider the amount of resources that are devoted every year to psychological counseling and therapy.

Even casual observation suggests that altruism plays an important role in the economics of the family. However, it is important to differentiate between paternalistic and dynastic altruism. To the knowledge of the author, there is no strong evidence in favor of the dynastic model, and some evidence against it.[17] This motivates the study of models of the family based on two-sided paternalistic altruism, where agents care about their elderly parents and about their children.[18] This model has implications that are different from the standard dynastic model and that can be used to compare the two models empirically. Furthermore, it also provides new insights into the relationship between government policy and family behavior.

Consider a simple model of an imperfectly altruistic family. Agents live for three periods. They are children in the first period, parents in the second, and grandparents in the third. At any time there are three coexisting generations.

Each period the parent chooses how much to invest in his child and how much to help his elderly parents. Let $f_t$ and $b_t$ denote, respectively, the amount of resources devoted to the children and the elderly in period $t$. This notation emphasizes that investing in a child is a FIG and helping an elderly parent is a BIG.

The preferences of generation $t$ are given by

$$U_t(.) = u(c^m) + \delta u(c^o) + V^F(f_t) + \theta^F V^F(f_{t+1}) + \theta^B V^B(b_{t+1}) + V^B(b_{t+2}),$$

where $\theta^B, \theta^F \in [0,1)$ denote the degree to which it internalizes the intergenerational spillover on the young and old. In the selfish model $\theta^c = \theta^g = 0$. Thus, the model allows for selfish and paternalistic generations. Finally, let $\widehat{f}_t$ denote the amount of investment in children than parents make in the absence of game theoretic effects and $f_t^*$ the optimal amount of investment. Define $\widehat{b}_t$ and $b_t^*$ similarly. It is natural to assume that $f_t^* > \widehat{f}_t$ and $b_t^* > \widehat{b}_t$.

From the basic theory, it follows that without government intervention there are equilibria in which parents choose short-sightedly and under-invest in the children and the elderly; equilibria in which the middle-aged invest optimally in the elderly, but not in the children; and equilibria with optimal and even over-investment in

[17] See Altonji, L., F. Hayashi, and L. Kotlikoff (1992,1997).

[18] For an excellent review of the literature of the economics of the family see Laitner (1997). As far as the author knows, this is the first paper to study a truly *dynamic* model of the family with two-sided altruism. Laitner (1988) studies a two period model in which parents make decisions before their children. However, in that framework multi-generational incentives of the type "I take care of my parents so that my children will take care of me" cannot arise. These type of incentives are at the core of the theory of BIGs and FIGs.

both. However, there is no equilibria in which parents invest optimally in the children but not in the elderly.

Since parents are altruistic, they always spend some resources on the old and on their children. However, optimal investment requires additional incentives. The incentive to take optimal care of the elderly stems from desire be taken care in old age. This BIG is self-sustainable as long as the value of the optimal care provided by one's children outweighs the cost of taking optimal care of one's parents. By contrast, the FIG "invest an additional $f_t^* - \widehat{f}_t$ units in children" is not self-sustainable. Parents invest the extra resources in children if, and only if, they belief that this is the only way of purchasing optimal care in old age. As in the basic theory, optimal investment in FIGs is supported with a family norm that links BIGs and FIGs. In this norm, agents receive optimal care in old age only if: (1) they took optimal care of their own elderly parents, and (2) they invested optimally in their children.

Government policy has interesting and surprising implications in this model of the family. In particular, government interventions on BIGs crowd out BIG and FIG provision within the family. To see this, suppose that the government introduces a law that forces the middle-aged to provide optimally for their parents. An example is old age public housing that is financed with taxes on the middle-aged. Then parents know that their children will *have* to provide them with optimal care when they are in old age – even if the care is provided indirectly through government services. But this eliminates the possibility of a family norm that, by linking FIGs and BIGs, provides incentives to invest optimally in children. As a result, the introduction of a government BIG crowds out investment in children within the family.

Of course, this assumes that the government intervenes only in BIGs. One could argue that the problem disappears if the government intervenes in both BIG and FIGs. But, at best, this can only be a partial solution. First of all, government programs require revenue that must be raised with costly distortionary taxes. Second, these programs are not likely to be tailored optimally to the specific needs of different families. Finally, and perhaps most importantly, the nature of some BIGs and FIGs is such that they can only be provided in the context of the family. For example, there does not seem to a substitute for the impact that parents' care and love have on the emotional and character development of children.

## 9 Conclusions

Many problems of intergenerational exchange share a common structure which is captured by the difference between BIGs and FIGs and the relationship between them. Broadly speaking, FIGs are forward transfers, from present to future generations, and BIGs are backward transfers, from young to old. This difference in the time of exchange has important implications for the type of intergenerational trade that can be sustained. BIGs that generate a positive surplus are self-sustainable, but FIGs never are. Furthermore, transfers to future generations take place in equilibrium only if generations link BIGs and FIGs. As we saw in the applications, these forces are at work in a wide range of problems, from the political economy of public intergenerational expenditures to investment by private firms.

The theory of BIGs and FIGs has positive and normative applications. On the positive side, it can applied to understand the economic problems in which intergenerational exchange is one of the key forces at work. Besides the three problems studied above, other possible applications are intergenerational risk-sharing;[19] "seniority" institutions like the US. Congress,[20] academia, and the governance structure of many companies; the political economy of debt and taxation; optimal taxation in the presence of political constraints; and the political economy of international intergenerational goods like the global climate.

On the normative side, the theory can also be used to try to design better intergenerational institutions. Suppose, for example, that future empirical analysis shows that voters do not link BIGs and FIGs, and that there is under-investment in future generations. One could then look for plausible modifications to the electoral and congressional systems that create the necessary link.

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