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# CENTRALIZED VERSUS DECENTRALIZED PROVISION OF LOCAL PUBLIC GOODS: A POLITICAL ECONOMY ANALYSIS

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## ABSTRACT

This paper takes a fresh look at the trade-off between centralized and decentralized provision of local public goods. The point of departure is to model a centralized system as one in which public spending is financed by general taxation, but districts can receive *different* levels of local public goods. In a world of benevolent governments, the disadvantages of centralization stressed in the existing literature disappear, suggesting that the case for decentralization must be driven by political economy considerations. Our political economy analysis assumes that under decentralization public goods are selected by locally elected representatives, while under a centralized system policy choices are determined by a legislature consisting of elected representatives from each district. We then study the role of taste heterogeneity, spillovers and legislative behavior in determining the case for centralization.

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# Introduction

Which tier of government should be responsible for particular taxing and spending decisions? From Philadelphia to Maastricht, this question has vexed constitution designers. Yet still the issues are unresolved. Witness the recent debate in the U.S. over whether the States or Federal government should take responsibility for welfare policy. footnote  In Europe, the principal of subsidiarity dictates that functions should be decentralized where possible, without any clearly defined criteria for centralization to be desirable.

This paper takes a fresh look at the trade off between centralized and decentralized provision of local public goods. Our analysis differs from much of the existing literature in emphasizing the importance of the *politics* of decision making for the decision to centralize. Centralization requires a system of governance that balances regional interests. Accordingly, the decision making unit typically incorporates a legislature consisting of representatives from each member district. The behavior of such legislatures will be a key determinant of the performance of centralized systems. Legislatures that produce minimum winning coalitions expose members of federations to the risk of expropriation. More universalistic legislatures offer insurance against this. However, they are open to manipulation as citizens use the political process to exploit the budgetary externality that centralization creates. While influential commentators, such as Inman and Rubinfeld (1997a) and (1997b), have stressed their importance for the performance of federal systems, such issues have yet to be formally incorporated into models of fiscal federalism. Here we develop an analysis which integrates them with the more traditional concerns, emphasized in the seminal work of Oates (1972), of achieving the right balance between respecting heterogeneous tastes and internalizing externalities.

The existing literature has typically modelled a centralized system as one in which public spending is financed by general taxation and all districts receive a uniform level of the local public good. By contrast, a decentralized system is one in which local public goods are financed by local taxation and each district is free to choose its own level. The drawback with a decentralized system is that it produces public good levels which reflect only local benefits and hence results in underprovision when such goods provide significant benefits to the larger community. Centralized decision making, on the other hand, produces a “one size fits all” outcome, which is insufficiently sensitive to local needs. Such logic underpins Oates’ (1972) celebrated *Decentralization Theorem* stating that, in the absence of spillovers, decentralization is preferable. When spillovers are present, the appropriate level of government depends on a weighing of the benefits of internalizing externalities with the costs of uniformity.

The point of departure for this paper is to model a centralized system as one in which public spending is financed by general taxation, but districts can receive *different* levels of local public goods. The usual assumption of uniform provision of local public goods like roads, parks and airports, seems very hard to justify empirically. Using this model, decision making by benevolent governments would make centralization an attractive option — a planner charged with choosing public good levels will respect the preferences of citizens at the district level, while optimally accounting for cross-border externalities. If there is a case for decentralization, therefore, it stems from political economy considerations.

Our analysis assumes that, in a decentralized system, local public goods are selected by locally elected representatives, while in a centralized system policy choices are made by a central legislature consisting of elected representatives from each district. In specifying the behavior of the legislature, we draw on the political science literature on distributive policy-making. footnote We adopt a simple parametric specification which allows us to go between the two polar cases studied in this literature. These are the "minimum winning coalition" view that distributive policies will be obtained by a coalition of representatives of the smallest size (for example, Buchanan and Tullock (1962), Riker (1962), Ferejohn, Fiorina and McKelvey (1987) and Baron and Ferejohn (1989)) and the "universalism" view that representatives will develop a norm of reciprocity so that every district will receive its share (for example, Weingast (1979) and Weingast, Shepsle and Johnsen (1981)). In modelling the election of representatives we draw on recent work on the citizen-candidate model of representative democracy (Osborne and Slivinsky (1996), Besley and Coate (1997)), particularly the extension to legislative elections developed in Coate (1997).

There is a significant body of work on the economics of federal systems. The older literature, particularly that stemming from Tiebout (1956), is well reviewed in Rubinfeld (1987). This focused on whether a set of competing jurisdictions would provide an efficient allocation of pure local public goods. This approach paid little attention to the potential role for more centralized forms of government. This omission was remedied in Oates (1972) who developed an analysis of impure local public goods (i.e. those with spillovers) under the assumption that centralization would imply uniform expenditures.

More recently a number of papers have studied political economy questions concerning centralized and decentralized systems. Bolton and Roland (1997) ask the positive question of when would we expect a federation to break up. Again, they work with the assumption that provision is uniform under centralization. footnote They assume exogenously given efficiency gains from centralization so that their trade-off is then principally between reaping the benefits of these against the ability to tailor policies to individual districts' tastes. Alesina and Spoalare (1997) consider the optimal and equilibrium number of districts in a model that trades off scale economies against preference diversity. Persson and Tabellini (1994) ask whether a more centralized system of government will tend to lead to a larger government sector. Like us, they model policy choices under centralization as emerging from a legislature consisting of representatives from each district. However, they do not consider elections to such a legislature. Ellingsen (1998) considers the positive and normative economics of centralization of a pure public good. footnote The median voter of the larger community created by centralization selects policy. If the districts are identical, then centralization is attractive since there is scope for cost sharing and cross-district free-rider problems are eliminated. However, heterogeneity undermines the case for centralization.

Our exercise is also in the spirit of Persson and Tabellini (1996a,b) which contrasts risk sharing by centralized and decentralized governments. They focus on the trade-off between improved risk sharing under centralization with the increased moral hazard due to regions taking on more risk. They also consider how different federal constitutions shape regional transfers in political equilibrium and which type of constitutional arrangement performs better.

Finally, there is the independent work of Lockwood (1998). Like us, he is critical of the assumption that centralization implies uniformity in public spending across districts and develops a political economy analysis of decentralization versus centralization of public good provision. He also assumes that a centralized system forms policy in a legislature comprising of elected representatives from each district. Unlike us, he specifies an extensive form bargaining game for the legislature which predicts that spillovers affect the nature of the legislative outcome. However, in contrast to this paper, he assumes that the local public good in each district is discrete and that citizens are homogeneous. The latter assumption makes legislative elections straightforward. Overall, Lockwood's focus is complementary with ours, paying greater attention to legislative processes and less attention to election outcomes.

The remainder of the paper is as organized as follows. Section 2 outlines the framework for our analysis. Section 3 provides a brief review of Oates' analysis and demonstrates a version of his Decentralization Theorem. Section 4 presents our political economy analysis, while sections 5 and 6 develop its implications for the choice between decentralization and centralization in identical and non-identical districts. Section 7 discusses possible alternative modeling assumptions and concluding remarks appear in section 8.

# The Model

## The Economic Environment

The economy is divided into 2 geographically distinct districts indexed by $i \in \{1,2\}$. Each district has a continuum of citizens with a mass of unity. There are 3 goods in the economy, a single private good, *x*, and two local public goods, $g_1$ and $g_2$, each one associated with a particular district. Each citizen is endowed with an "income" of *y* units of the private good. To produce one unit of either of the public goods, requires *p* units of the private good.

Each citizen in district *i* is characterized by a public good preference parameter $\lambda$. The preferences of a type $\lambda$ citizen in district *i* are

$$(1-\lambda)\ln x + \lambda[(1-\kappa)\ln g_i + \kappa \ln g_{-i}].$$

The term $\kappa \in [0,1/2]$ indexes the degree of *spillovers*; when $\kappa = 0$ citizens care only about the public good in their own district, while when $\kappa = 1/2$ they care equally about the public goods in both districts. While spillovers are the same for all citizens, those with higher $\lambda$'s value public goods more highly.

The range of possible preference types is $[0,\overline{\lambda}]$ where $\overline{\lambda} < 1$. The distribution of citizens across types in district *i* is described by the cumulative distribution function $G_i(\lambda)$. The *median type* in each district *i* is denoted by $m_i$ and the *mean type* by $\overline{m}_i$. We assume throughout that $G_i(\lambda)$ is increasing on $[0,\overline{\lambda}]$, and that $2m_i < \overline{\lambda}$. The latter condition is needed for interior solutions to exist below.

## Decentralized and Centralized Systems

Under a *decentralized system*, the level of public good in each district is chosen by the government of that district and public expenditures are financed by a uniform head tax on local residents. Thus, if district *i* chooses a public good level $g_i$, each citizen in district *i* pays a tax of $pg_i$. Under a *centralized system*, the levels of both public goods are determined by a government that represents both districts, with spending being financed by a uniform head tax on all citizens. Thus, public goods levels $(g_1, g_2)$, result in a head tax of $\frac{p}{2}(g_1+g_2)$. footnote

# Oates' Analysis

To provide some background for our analysis, it will be useful to briefly review Oates' (1972) famous treatment of the relative merits of decentralization and centralization in our framework. Oates supposes that, in a decentralized system, each district's policy is chosen independently by a planner whose objective is to maximize average utility in the district.

Accordingly, we look for a pair of expenditure levels $(g_1^d, g_2^d)$ which satisfy

$$g_i^d = \arg\max_{g_i}\{(1-\overline{m}_i)\ln(y-pg_i)+\overline{m}_i[(1-\kappa)\ln g_i+\kappa\ln g_{-i}^d]\},\ i\in\{1,2\}.$$

Taking the first order conditions and solving, yields:

$$(g_1^d, g_2^d) = \left(\frac{\overline{m}_1(1-\kappa)y}{(1-\overline{m}_1\kappa)p}, \frac{\overline{m}_2(1-\kappa)y}{(1-\overline{m}_2\kappa)p}\right).$$

Thus, each district $i$ devotes a proportion $\overline{m}_i(1-\kappa)/(1-\overline{m}_i\kappa)$ of its income to the provision of local public goods, a proportion which is decreasing in the degree of spillovers.

In a centralized system, Oates supposes that policy is chosen by a planner whose objective is to maximize average utility but assumes that a common level of public goods is chosen for both districts. The latter, denoted $g^c$, satisfies

$$g^c = \arg\max_{g}\{(2-(\overline{m}_1+\overline{m}_2))\ln(y-pg)+[\overline{m}_1+\overline{m}_2]\ln g\}.$$

Taking the first order condition and solving, yields

$$g^c = \frac{(\overline{m}_1+\overline{m}_2)y}{2p}.$$

In this instance, the proportion of the economy's income spent on public goods depends on the average valuation of public goods in the society as a whole — $(\overline{m}_1+\overline{m}_2)/2$. This proportion is independent of the extent of spillovers.

To compare the public goods levels in the two regimes, observe that with spillovers, the average level of public goods in a decentralized system is less than under a centralized system. This can be seen by noting that the average levels are equal at $\kappa = 0$ and remembering that public goods expenditure is decreasing in $\kappa$ under decentralization. Thus decentralization induces free-riding and lower levels of public spending.

The above analysis implies that aggregate welfare under decentralization is given by:

$$W^d = \sum_{i=1}^{2}\{(1-\overline{m}_i)\ln\frac{y(1-\overline{m}_i)}{1-\overline{m}_i\kappa}+\overline{m}_i[(1-\kappa)\ln\frac{\overline{m}_i(1-\kappa)y}{(1-\overline{m}_i\kappa)p}+\kappa\ln\frac{\overline{m}_{-i}(1-\kappa)y}{(1-\overline{m}_{-i}\kappa)p}]\},$$

#

while aggregate welfare under centralization is given by:

$$W^c = (2-(\overline{m}_1+\overline{m}_2))\ln y(1-\frac{(\overline{m}_1+\overline{m}_2)}{2})+[\overline{m}_1+\overline{m}_2]\ln\frac{(\overline{m}_1+\overline{m}_2)y}{2p}.$$ #

Two facts are readily established. First, aggregate welfare under centralization is independent of $\kappa$, while welfare under decentralization is decreasing in $\kappa$. Second, when $\kappa = 0$ aggregate welfare under decentralization exceeds that under centralization except when $\overline{m}_1 = \overline{m}_2$, in which case, the two welfare levels are the same. We thus have:

**Proposition** ($i$) *If the average valuation of public goods is the same in the two districts* ($\overline{m}_1 = \overline{m}_2$) *and spillovers are present* ($\kappa > 0$), *a centralized system is welfare superior to a decentralized system. If there are no spillovers* ($\kappa = 0$), *the two systems generate the same level of welfare*.

($ii$) *If* $\overline{m}_1 \neq \overline{m}_2$, *either a decentralized system is welfare superior for all values of* $\kappa$ *or there exists a critical value of* $\kappa$, *strictly greater than* 0, *such that a centralized system is* *welfare superior if and only if $\kappa$ exceeds this critical level.*

This result suggests that the choice between a decentralized and a centralized system should reflect a trade-off between spillovers and taste heterogeneity. With no spillovers and identical districts, a decentralized system is superior - a result often referred to as *Oates' Decentralization Theorem*. With spillovers and identical districts, a centralized system is preferred. With spillovers and non-identical districts, the issue can only be resolved by comparing the magnitude of the two effects.

The trade-off identified by Oates relies on the assumption that provision under centralization is uniform. If the two districts are not identical, aggregate welfare could be enhanced by providing each district with a different level of local public goods. Thus, if the planner could offer the districts different levels of the public good, centralization would produce a policy outcome that was responsive to taste heterogeneity. Thus centralization could deliver the benefits of internalizing spillovers without generating any of the costs identified by Oates. footnote  Hence, since the reason for imposing the uniformity constraint seems obscure, Oates' analysis becomes suspect as a basis for issuing policy recommendations.

# A Political Economy Analysis

## Policy Determination Under Decentralization

Under decentralization, a single representative is elected to choose policy in each district. Following the citizen candidate approach, this representative is a citizen from the district in question. Accordingly, representatives are characterized by their public good preferences $\lambda$. There is no commitment, so that these preferences determine their policy choices if they win office.

The policy determination process has two stages. First, elections determine which citizens are selected to represent the two districts. Second, policies are chosen simultaneously by the elected representative in each district. Working backwards, let the types of the representatives in districts 1 and 2 be $\lambda_1$ and $\lambda_2$. Then the policy outcome $(g_1^*(\lambda_1), g_2^*(\lambda_2))$ satisfies

$$g_i^*(\lambda_i) = \arg\max_{g_i} \{(1-\lambda_i)\ln(y - pg_i) + \lambda_i[(1-\kappa)\ln g_i + \kappa \ln g_{-i}^*(\lambda_{-i})]\} \text{ for } i \in \{1,2\}.$$

Solving this yields

$$(g_1^*(\lambda_1), g_2^*(\lambda_2)) = \left(\frac{\lambda_1(1-\kappa)y}{(1-\lambda_1\kappa)p}, \frac{\lambda_2(1-\kappa)y}{(1-\lambda_2\kappa)p}\right). \qquad \#$$

Thus each district spends a fraction of its income on public goods which is decreasing in the level of spillovers and increasing in $\lambda_i$.

If the representatives in districts 1 and 2 are $\lambda_1$ and $\lambda_2$, a citizen of type $\lambda$ in district $i$ will enjoy a payoff

$$(1-\lambda)\ln[y - pg_i^*(\lambda_i)] + \lambda[(1-\kappa)\ln g_i^*(\lambda_i) + \kappa \ln g_{-i}^*(\lambda_{-i})] \qquad \#$$

These preferences over types determine voting decisions.  A pair of representative types $(\lambda_1^*, \lambda_2^*)$ is *majority preferred under decentralization* if, in each district $i$, a majority of citizens prefer the type of their representative to any other type $\lambda \in [0, \overline{\lambda}]$, given the other district's representative type $\lambda_{-i}^*$.

We assume that if a majority preferred pair of representative types exists, the elected representatives in the two districts will be of these types. There are two possible justifications. First, there is an equilibrium of the citizen-candidate model in which a candidate of the majority preferred type from each district runs and is elected unopposed. footnote Second, if in each district, two Downsian parties compete for office by selecting candidates, equilibrium will involve both parties in each district selecting candidates of the majority preferred type. Given our assumption, a pair of public goods levels $(g_1, g_2)$ is a *policy outcome under decentralization* if there exists a pair of representative types $(\lambda_1^*, \lambda_2^*)$ which is majority preferred under decentralization such that $(g_1, g_2) = (g_1^*(\lambda_1^*), g_2^*(\lambda_2^*))$.

The optimal type of representative for a citizen of type $\lambda$ in district $i$ maximizes $(1-\lambda)\ln[y - pg_i^*(\lambda_i)] + \lambda(1-\kappa)\ln g_i^*(\lambda_i)$. Since a type $\lambda$ candidate chooses the public good level which solves this problem, each citizen prefers a candidate of his own type. Citizens' preferences over types are *single-peaked* footnote implying that a pair of representative types is majority preferred under decentralization if and only if it is a median pair i.e., $(\lambda_1^*, \lambda_2^*) = (m_1, m_2)$. We have therefore established:

**Proposition** *There is a unique policy outcome under decentralization given by*

$$(g_1, g_2) = \left(\frac{m_1(1-\kappa)y}{(1-m_1\kappa)p}, \frac{m_2(1-\kappa)y}{(1-m_2\kappa)p}\right).$$

This has a conventional flavor since local public good provision respects the median taste within a district. This is well known for pure local public goods ($\kappa = 0$), and has served as the workhorse predictive model for local public finance — see Rubinfeld (1987). It also holds in our model when $\kappa > 0$. footnote

# Policy Determination under Centralization

The policy determination process under centralization also has an election and a policy selection stage. Citizens from each district are elected to serve in a legislature. We use a reduced form model of how this legislature makes policy that captures (parametrically) the main competing theories of distributive policy making. The latter fall into two main categories. Minimum winning coalition theories predict that distributive policies will be obtained by a coalition of representatives of the smallest size necessary to pass the legislation. Thus, a coalition of 51% of the representatives will form to propose a bill distributing benefits to their districts which will pass by the slimmest of margins. The theories are much less clear in predicting who will form the winning coalition and suggest that there could be a stochastic element in this.

The uncertainty about coalition membership motivates the universalism model. Representatives may prefer a less random outcome to the "feast or famine" implied by the minimum winning coalition theory. Accordingly, members of a decisive coalition may allocate benefits to those outside on the understanding that non-members would behave similarly if political power were allocated differently. Thus, a "norm of reciprocity" will emerge under which representatives behave more cooperatively.

Our reduced form model supposes that power in the legislature is randomly allocated to one of the representatives, with each district's representative equally likely to be selected. This captures the randomness inherent in the identity of the decisive coalition. The representative with power chooses public good levels in accordance with a "norm" represented by a parameter $\mu \in [1/2, 1]$. If district $i$'s representative holds power, he chooses a pair of public goods that maximize $\mu$ times his own utility plus $(1-\mu)$ times the other district's representative's utility. Thus, higher values of $\mu$ correspond to a less cooperative legislature, i.e., one in which the norm of reciprocity is weaker.

This formulation implies that, if the legislature is of type $\mu$ and the representatives are of types $\lambda_1$ and $\lambda_2$, the policy outcome will be $(g_1^1(\lambda_1,\lambda_2;\mu), g_2^1(\lambda_1,\lambda_2;\mu))$ with probability 1/2 and $(g_1^2(\lambda_1,\lambda_2;\mu), g_2^2(\lambda_1,\lambda_2;\mu))$ with probability 1/2 where $(g_1^i, g_2^i)$ solves

$$\max_{(g_i,g_{-i})} \mu \bullet \{(1-\lambda_i)\ln(y - \frac{p}{2}(g_i+g_{-i})) + \lambda_i[(1-\kappa)\ln g_i + \kappa \ln g_{-i}]\} +$$

$$(1-\mu)\bullet\{(1-\lambda_{-i})\ln(y-\frac{p}{2}(g_i+g_{-i})) + \lambda_{-i}[(1-\kappa)\ln g_{-i} + \kappa \ln g_i]\}.$$

It is easily checked that

$$(g_1^1, g_2^1) = (\frac{2[\mu\lambda_1(1-\kappa)+(1-\mu)\lambda_2\kappa]y}{p}, \frac{2[\mu\lambda_1\kappa+(1-\mu)\lambda_2(1-\kappa)]y}{p}), \qquad \#$$

and,

$$(g_1^2, g_2^2) = (\frac{2[(1-\mu)\lambda_1(1-\kappa)+\mu\lambda_2\kappa]y}{p}, \frac{2[(1-\mu)\lambda_1\kappa+\mu\lambda_2(1-\kappa)]y}{p}). \qquad \#$$

The case of $\mu = 1$, is akin to the prediction of the minimum winning coalition literature; the policy package maximizes the joint utility of a coalition of the smallest possible size and the composition of the minimum winning coalition is unknown ex ante. With $\mu = 1/2$, the policy outcome maximizes the joint utility of the representatives, which is more in line with the literature on universalistic legislative norms. footnote Moving between the two extremes yields hybrid outcomes where benefits are concentrated inside a minimum winning coalition with some partial reflection of preferences outside.

Turning to the election stage, if the representative types are $\lambda_1$ and $\lambda_2$, a citizen of type $\lambda$ in district *i* obtains an expected utility level of

$$\frac{1}{2}\sum_{k\in\{1,2\}}\{(1-\lambda)\ln(y-\frac{p(g_1^k+g_2^k)}{2}) + \lambda[(1-\kappa)\ln g_i^k + \kappa \ln g_{-i}^k]\}. \qquad \#$$

A pair of representative types $(\lambda_1^*, \lambda_2^*)$ is *majority preferred under centralization* if, in each district *i*, a majority of citizens prefer the type of their representative to any other type $\lambda \in [0,\overline{\lambda}]$, given the other district's representative type $\lambda_{-i}^*$. Again, we assume that when a majority preferred pair of representative types exists, the elected representatives in the two districts will be of these types. footnote Thus, a *policy outcome under centralization* consists of a pair of public goods pairs $\{(g_1^i, g_2^i)\}_{i\in\{1,2\}}$ that would be generated by a pair of majority preferred representative types $(\lambda_1^*, \lambda_2^*)$; i.e., such that $(g_1^i, g_2^i) = (g_1^i(\lambda_1^*,\lambda_2^*;\mu), g_2^i(\lambda_1^*,\lambda_2^*;\mu))$ for $i \in \{1,2\}$.

Under decentralization, public goods levels are only a function of the type elected in the district providing that good. As ( ref: a ) and ( ref: b ) show, under centralization, the public goods level now depends on the type of the legislator in *both* districts. This can generate an incentive for citizens in each district to delegate policy making strategically to a representative with different tastes in order to affect the policy outcome when their district does not hold power. This reasoning is key to understanding voters' preferences over representative types.

It is straightforward to show that a pair of representative types $(\lambda_1^*, \lambda_2^*)$ is majority preferred under centralization if and only if in each district $i$ the median type prefers $\lambda_i^*$ to any other type $\lambda \in [0, \overline{\lambda}]$, given the other district's representative type $\lambda_{-i}^*$. footnote We can exploit this observation in the following way. First, for $i \in \{1,2\}$ define $U_i(\lambda_1, \lambda_2, m_i)$ to be the expected payoff received by the median voter in district $i$, when the representative types are $(\lambda_1, \lambda_2)$; that is,

$$U_i(\lambda_1, \lambda_2, m_i) = \frac{1}{2} \sum_{k \in \{1,2\}} \{(1 - m_i)\ln(y - \frac{p(g_1^k + g_2^k)}{2}) + m_i[(1 - \kappa)\ln g_i^k + \kappa \ln g_{-i}^k]\}.$$

Then, $(\lambda_1^*, \lambda_2^*)$ is majority preferred under centralization if and only if $(\lambda_1^*, \lambda_2^*)$ is a Nash equilibrium of the two player game in which each player has strategy set $[0, \overline{\lambda}]$ and player $i \in \{1,2\}$ has payoff function $U_i(\lambda_1, \lambda_2, m_i)$. Thus, even though the outcome is supported by a game in which candidates run for office in each district, it is *as if* the two median voters get to choose the type of policy maker who is elected.

The district $i$ median citizen will try to manipulate $\lambda_i$ so that he obtains something close to his preferred policy outcome anticipating the subsequent working of the legislature. However, since he only has one degree of freedom, $\lambda_i$, and two objectives $(g_1, g_2)$, this instrument is rather blunt. Raising $\lambda_i$ leads to an increase in both districts' public goods, irrespective of which district's representative holds political power. However, it also raises taxes. This trade-off can be illustrated by differentiating $U_i$ with respect to $\lambda_i$. Using ( ref: a ) and ( ref: b ), we obtain:

$$\begin{aligned}\frac{\partial U_1}{\partial \lambda_1} &= \frac{m_1 y}{p}[\frac{\mu(1-\kappa)^2}{g_1^1} + \frac{\mu\kappa^2}{g_2^1} + \frac{(1-\mu)(1-\kappa)^2}{g_1^2} + \frac{(1-\mu)\kappa^2}{g_2^2}] \\ &\quad - \frac{(1-m_1)y}{2}[\frac{\mu}{y - T_1} + \frac{1-\mu}{y - T_2}],\end{aligned}$$

and

$$\begin{aligned}\frac{\partial U_2}{\partial \lambda_2} &= \frac{m_2 y}{p}[\frac{\mu(1-\kappa)^2}{g_2^2} + \frac{\mu\kappa^2}{g_1^2} + \frac{(1-\mu)(1-\kappa)^2}{g_2^1} + \frac{(1-\mu)\kappa^2}{g_1^1}] \\ &\quad - \frac{(1-m_2)y}{2}[\frac{\mu}{y - T_2} + \frac{1-\mu}{y - T_1}],\end{aligned}$$

where $T_i = p[g_1^i + g_2^i]/2$ for $i \in \{1,2\}$. The first term in these equations (which is positive) represents the marginal benefits of raising the representative's type, while the second (which is negative) reflects the marginal costs.

It is readily demonstrated that $\frac{\partial^2 U_1}{\partial \lambda_1^2} < 0$ and that $\frac{\partial^2 U_2}{\partial \lambda_2^2} < 0$, so that each player's payoff is a strictly concave function of his strategy. Since each player's strategy set is compact and convex, a pure strategy equilibrium of the game exists. Equilibria can either be interior where $\lambda_i^* \in (0, \overline{\lambda})$, or extremal where for some district $j$, $\lambda_j^* = 0$ or $\overline{\lambda}$. Interior equilibria are characterized by the first order conditions $\frac{\partial U_1}{\partial \lambda_1} = \frac{\partial U_2}{\partial \lambda_2} = 0$. It is also easy to show that $\frac{\partial^2 U_1}{\partial \lambda_1 \partial \lambda_2} < 0$ and that $\frac{\partial^2 U_2}{\partial \lambda_2 \partial \lambda_1} < 0$, implying that types are strategic substitutes.

# Decentralization vs Centralization: Identical Districts

## Majority Preferred Types and Policy Outcomes

A pair of representative types $(\lambda_1^*, \lambda_2^*)$ is *symmetric* if $\lambda_1^* = \lambda_2^*$. With identical districts, there is a unique symmetric pair of majority preferred types under centralization. To simplify the statement of this, define:

$$\sigma(\kappa,\mu) = \begin{cases} \frac{\kappa(1-\kappa)[\mu^2+(1-\mu)^2]+[\kappa^2+(1-\kappa)^2]\mu(1-\mu)}{\kappa(1-\kappa)[\mu^2+(1-\mu)^2]+[\kappa^3+(1-\kappa)^3]2\mu(1-\mu)} & \text{if } (\kappa,\mu) \neq (0,1) \\ 1 & \text{if } (\kappa,\mu) = (0,1). \end{cases}$$

It is easy to verify that $\sigma \leq 1$ and that $\sigma$ is increasing in $\kappa$ with $\sigma(0,\mu) = 1/2$ for all $\mu < 1$ and $\sigma(1/2,\mu) = 1$ for all $\mu$. In addition, $\sigma$ is increasing in $\mu$ for $\kappa > 0$ and satisfies $\sigma(\kappa,1) = 1$ and $\sigma(\kappa,1/2) = 1/2[\kappa^2 + (1-\kappa)^2]$ for all $\kappa$. We now have:

**Lemma** *Suppose that $m_1 = m_2 = m$. Then, a symmetric pair of representative types $(\lambda_1^*, \lambda_2^*)$ is majority preferred under centralization if and only if*

$$\lambda_1^* = \lambda_2^* = \frac{m}{m + (1-m)\sigma(\kappa,\mu)}.$$

This result makes the incentive for strategic delegation transparent, with each district desiring a representative with preferences above the median, except in the special case of either maximal spillovers ($\kappa = 1/2$) or a non-cooperative legislature ($\mu = 1$). The divergence from the median preference is greater, the smaller are the spillovers and the more cooperative is the legislature. The maximal divergence from the median preference occurs when there are no spillovers ($\kappa = 0$) and the legislature is fully cooperative ($\mu = 1/2$).

To develop some intuition for this, consider first the case where $\mu = 1$. In this case, district $i$'s representative only affects the outcome when he holds political power, in which case he selects the policy outcome which maximizes his own utility. To maximize the median voter's utility then requires having a representative just like himself. Next consider the case where $\mu = 1/2$ and $\kappa = 0$. In this case, when district $i$'s representative holds political power, he is compelled by legislative norms to take into account the preferences of the other representative. Accordingly, he does not exploit the budgetary externality as much as the median voter would like. Thus, the median voter elects a higher type. footnote As spillovers increase, putting in a higher type leads to more public good for both districts, dampening the incentive to put in a higher type. The intermediate cases of $\mu \in (1/2,1)$ display the same pattern as the cooperative case. The median voters in each district desire candidates of higher types when spillovers are low but their incentive to overstate their preferences is dampened as spillovers increase.

The impact of changing $\mu$ on equilibrium types is more subtle. In the case of zero spillovers, the type of each district's representative has no effect on the level of the public good received by the other district. As $\mu$ increases, the amount of the public good each representative selects for his district when he holds political power increases, while the amount he selects for the other district decreases. Thus, when his district's representative holds political power, the median voter's incentive to have a higher type is dampened, but when the other district's representative holds power the impact is reversed. These two effects turn out to offset each other which explains why when $\kappa = 0$, the equilibrium type is the same for all $\mu < 1$. With positive spillovers, the same two effects are present. However, there is an additional consideration since putting in a higher type also increases the amount of the public good received by the other district. This occurs irrespective of which district's representative holds power. Increasing $\mu$ magnifies this effect to a greater extent when the median voter's own representative holds power and hence the net impact is to dampen the incentive to put in a higher type.

A policy outcome under centralization is *symmetric* if $g_1^1 = g_2^2$ and $g_2^1 = g_1^2$. Using Lemma 1, we have the following:

**Lemma** *Suppose that $m_1 = m_2 = m$. Then, the unique symmetric policy outcome under centralization is given by*

$$(g_i^i, g_{-i}^i) = \left(\frac{2m[\mu(1-\kappa)+(1-\mu)\kappa]y}{[m+(1-m)\sigma(\kappa,\mu)]p}, \frac{2m[\mu\kappa+(1-\mu)(1-\kappa)]y}{[m+(1-m)\sigma(\kappa,\mu)]p}\right) \quad i \in \{1,2\}.$$

It is useful to distinguish between the average level of public goods and the divergence between the level received by the district holding power and the other district. The average level of public goods is given by

$$\frac{g_i^i + g_{-i}^i}{2} = \frac{my}{[m+(1-m)\sigma(\kappa,\mu)]p}.$$

The smaller the level of spillovers, the greater the overspending relative to the median's desired level of $my/p$. Spending is also higher in more cooperative legislatures. In addition, average spending on public goods is at least as large under centralization as it is under decentralization — this inequality holds strictly except when $\kappa = 0$ or $\mu = 1$. The advantage gained by the district holding power in political equilibrium is given by

$$g_i^i - g_{-i}^i = \frac{(2\mu-1)(1-2\kappa)2my}{[m+(1-m)\sigma(\kappa,\mu)]p}.$$

This is increasing in $\mu$ and zero only when $\mu = 1/2$ or $\kappa = 1/2$.

# Welfare Comparisons

Proposition 2 implies that aggregate welfare in a decentralized regime is given by:

$$W^d(\kappa) = 2\{(1-\overline{m})\ln\frac{y(1-m)}{1-m\kappa} + \overline{m}\ln\frac{m(1-\kappa)y}{(1-m\kappa)p}\},$$

while under a centralized regime, if the policy outcome is symmetric, Lemma 2 implies that aggregate welfare is given by

$$W^c(\kappa,\mu) = 2(1-\overline{m})\ln y\left(1-\frac{m}{m+(1-m)\sigma}\right) + \overline{m}\left[\ln\frac{2m\alpha y}{[m+(1-m)\sigma]p} + \ln\frac{2m\beta y}{[m+(1-m)\sigma]p}\right],$$

where $\alpha = [\mu(1-\kappa)+(1-\mu)\kappa]$ and $\beta = [\mu\kappa+(1-\mu)(1-\kappa)]$. We begin the comparisons with the case in which there is no bias in the political system stemming from a difference between the preferences of the median citizen and the average citizen; i.e., $m = \overline{m}$. In this case, under decentralization, we get exactly the prediction of Oates' analysis. The following four facts about welfare in this case are readily established. First, as noted in the discussion of Oates' analysis, aggregate welfare under decentralization is decreasing in $\kappa$. Second, for all $\mu$, aggregate welfare under centralization is increasing in $\kappa$. Third, for all $\kappa < 1/2$, aggregate welfare under centralization is decreasing in $\mu$. Finally, $W^c(1/2,\mu) = W^d(0)$ for all $\mu$. Putting these together, we obtain:

**Proposition** *Suppose that in each district the average valuation of public goods equals the median valuation ($\overline{m}_i = m_i$), that $\overline{m}_1 = \overline{m}_2$, and that the policy outcome under centralization is* *symmetric. Then, whatever the degree of non-cooperativeness of the legislature, there exists a critical value of* $\kappa$, *strictly greater than* 0 *and strictly smaller than* 1/2, *such that a centralized system is welfare superior if and only if* $\kappa$ *exceeds this critical level. Moreover, this critical level depends positively on the degree of non-cooperativeness of the legislature as measured by* $\mu$.

This result implies that decentralization is welfare superior for small spillovers, while centralization is better when spillovers are large. The result differs sharply from the conclusions of Oates' analysis, which suggests that centralization is always desirable when districts are identical.

Decentralization is favored with small spillovers due to the budgetary externality created by common financing of expenditures. Without spillovers, citizens prefer a high level of the local public good in their own district and none in the other. In a cooperative legislature ($\mu = 1/2$), the representative with power (more generally, the decisive coalition) does not exploit this externality. However, the voters respond by delegating decision making strategically to representatives that favor public spending, resulting in excessive public spending from a social viewpoint. In a less than fully cooperative legislature ($\mu > 1/2$), the representative who holds power exploits this by overproviding public goods to his own district and underproviding them to the other. The random allocation of decision power creates political risk which further lowers aggregate welfare.

The preference for centralization with high spillovers is due to two things. First, the free rider problem between the districts under decentralization is worsened. Second, high spillovers lead citizens to prefer more equitable spending on public goods and hence lessen the conflict of interest between districts. For a more cooperative legislature, this is manifested in representatives with tastes for public spending closer to those of the average citizen which generates lower levels of public spending. When the legislature behaves non-cooperatively, the decisive representative chooses to provide local public goods for both districts. In the extreme case, when $\kappa = 1/2$ the interests of citizens from different districts of the same taste are perfectly aligned and the degree of cooperativeness of the legislature does not affect the performance of centralization.

The Proposition assumes that the median and mean preference for public spending are the same. footnote It is straightforward to show that the case for centralization when spillovers are high holds provided the median valuation is no larger than the mean; i.e., $W^c(1/2, \mu) > W^d(0)$ for all $\mu$ when $m \leq \overline{m}$. In addition, the case for decentralization when spillovers are low holds irrespective of the relationship between the median and the mean when the legislature behaves non-cooperatively (i.e., $\mu = 1$). However, if the legislature is cooperative ($\mu = 1/2$) and the mean preference is sufficiently above the median, then it is possible for centralization to dominate even if there are no spillovers. This is because, absent spillovers, the political process under decentralization would tend to under-provide public goods relative to the welfare optimum. In this case, the budgetary externality under centralization becomes a virtue and allows an increased level of public goods. Thus centralization becomes preferred for reasons other than regulating spillovers. footnote

The effects at work in this section stem purely from the political consequences of centralization. In the background is the common financing of spending under centralization which generates a budgetary externality in spending decisions. With small spillovers, there is a conflict of interest between citizens resident in different districts which undermines the performance of the centralized system. Larger spillovers soften the conflict of interest between citizens and centralization performs better irrespective of legislative norms.

# Decentralization vs Centralization: Non-identical Districts

We now consider non-identical districts, assuming throughout that $m_1 < m_2$. Characterizing strategic delegation incentives and the consequences for political equilibrium is more complicated in this instance. The two extreme cases ($\mu = 1$ and $\mu = 1/2$) can be solved analytically. The intermediate cases will be investigated via simulations.

## Majority Preferred Types and Policy Outcomes

### The Non-cooperative Legislature

The case in which the legislature is non-cooperative ($\mu = 1$) is straightforward. There is no incentive for strategic delegation — district $i$'s representative only has an impact on the policy outcome when he has the power to decide and hence the median voter is content to choose someone with his own tastes. Thus, we have

**Proposition** *Suppose that $\mu = 1$. Then, a pair of representative types $(\lambda_1^*, \lambda_2^*)$ is majority preferred under centralization if and only if $(\lambda_1^*, \lambda_2^*) = (m_1, m_2)$. Accordingly, there is a unique policy outcome under centralization given by*

$$(g_1^i, g_2^i) = \left(\frac{2m_i(1-\kappa)y}{p}, \frac{2m_i\kappa y}{p}\right), \;\; i \in \{1, 2\}.$$

### The Cooperative Legislature

To state the solution in this case, define

$$\alpha_1 = \frac{(1-\kappa)^2}{(1-\kappa) + \kappa/\xi} + \frac{\kappa^2}{\kappa + (1-\kappa)/\xi},$$

and,

$$\alpha_2 = \frac{(1-\kappa)^2}{(1-\kappa) + \kappa\xi} + \frac{\kappa^2}{\kappa + (1-\kappa)\xi}.$$

where $\xi = \frac{(1-m_2)m_1[\kappa^3+(1-\kappa)^3]-(1-m_1)m_2\kappa(1-\kappa)}{(1-m_1)m_2[\kappa^3+(1-\kappa)^3]-(1-m_2)m_1\kappa(1-\kappa)}$. In addition, define $\widehat{\kappa}$ as the solution to

$$\frac{[\widehat{\kappa}^3 + (1-\widehat{\kappa})^3]}{\widehat{\kappa}(1-\widehat{\kappa})} = \frac{(1-m_1)m_2}{(1-m_2)m_1}.$$

It is straightforward to show that $\widehat{\kappa} \in (0, 1/2)$. Observe that $\alpha_i = 1$ when $\kappa = 0$ and $\alpha_i < 1$ when $\kappa \in (0, \widehat{\kappa})$ for $i = 1, 2$. In addition, $\alpha_1 < \alpha_2$ when $\kappa \in (0, \widehat{\kappa})$. footnote We now have:

**Lemma** *Suppose that $\mu = 1/2$. Then, there exists a unique pair of representative types $(\lambda_1^*, \lambda_2^*)$ which is majority preferred under centralization. When $\kappa < \widehat{\kappa}$*

$$(\lambda_1^*, \lambda_2^*) = \left(\frac{m_1}{m_1(1+1/\xi)/2 + (1-m_1)/2\alpha_1}, \frac{m_2}{m_2(1+\xi)/2 + (1-m_2)/2\alpha_2}\right),$$

*and when $\kappa \in [\widehat{\kappa}, 1/2]$*

$$(\lambda_1^*, \lambda_2^*) = (0, 2m_2).$$

Thus there are two cases. For low spillovers ($\kappa < \widehat{\kappa}$), there is an interior solution. For high spillovers ($\kappa \in [\widehat{\kappa}, 1/2]$), we have a boundary solution in which the district with the low median elects a citizen who does not value public spending, while the district with the high median heads towards the other extreme, electing a citizen with twice the median preference.

To understand this intuitively, consider the two extremes of $\kappa = 0$ and $\kappa = 1/2$. In the first of these, $\alpha_1 = \alpha_2 = 1$ and $\xi = 1$. Lemma 3 then implies that the majority preferred pair of representatives is

$$(\lambda_1^*, \lambda_2^*) = (\frac{m_1}{1/2 + (1 - m_1)m_2/2(1 - m_2)}, \frac{m_2}{1/2 + (1 - m_2)m_1/2(1 - m_1)}).$$

Using the fact that $2m_i < \bar{\lambda} < 1$, it is easy to show that $\lambda_i^* > m_i$ for $i \in \{1,2\}$ and $\lambda_1^* < \lambda_2^*$. Thus, both districts select representative types above the median. As above, this is explained by the voters' desire to exploit the budgetary externality. An increase in the median voter's type in one district, reduces the representative's type in the other district. The former raises taxes and hence reduces the relative value of local public goods on the margin.

When $\kappa = 1/2$, the strategic delegation incentives are different. Each district's median voter desires the same level of spending on the two public goods. However, district 2's median voter prefers more of both public goods than does district 1. Thus district 1's median voter has an incentive to have a lower representative type to reduce public goods spending, while that in district 2 will wish to have a representative with a higher valuation. They pull in opposite directions until one or both districts has put in their most extreme type. Our assumption that $2m_2 < \bar{\lambda}$ implies that district 2 can obtain its preferred level of public goods when district 1 has put in its most extreme type so that district 2's median voter is getting its preferred outcome. footnote

The policy outcomes associated with Lemma 3 are described in our next result.

**Proposition** *Suppose that $\mu = 1/2$. Then, if $\kappa < \hat{\kappa}$, the pair of public goods pairs $(g_1^i, g_2^i)_{i \in \{1,2\}}$ where*

$$g_1^i = [\frac{(1-\kappa)m_1}{m_1(1 + 1/\xi)/2 + (1 - m_1)/2\alpha_1} + \frac{\kappa m_2}{m_2(1 + \xi)/2 + (1 - m_2)/2\alpha_2}]\frac{y}{p}$$

*and*

$$g_2^i = [\frac{(1-\kappa)m_2}{m_2(1 + \xi)/2 + (1 - m_2)/2\alpha_2} + \frac{\kappa m_1}{m_1(1 + 1/\xi)/2 + (1 - m_1)/2\alpha_1}]\frac{y}{p}$$

*is the unique policy outcome under centralization. If $\kappa \geq \hat{\kappa}$, the pair of public goods pairs*

$$(g_1^i, g_2^i) = (\frac{2m_2\kappa y}{p}, \frac{2m_2(1-\kappa)y}{p}), \;\; i \in \{1,2\}.$$

*is the unique policy outcome under centralization.*

It is interesting to compare these outcomes with those produced by a non-cooperative legislature. With low spillovers, the fully cooperative legislature eliminates the political risk associated with centralization, but at the expense of over-provision of public goods stemming from strategic delegation. With high spillovers ($\kappa = 1/2$), the fully cooperative legislature produces, with probability one, one of the public goods pairs which is selected with equal probability by the non-cooperative legislature.

## Partially Cooperative Legislature

In cases where $\mu \in (1/2, 1)$, the legislature exposes each district to some political risk since the identity of the politically decisive district is uncertain. It also gives incentives for strategic delegation to affect policy outcomes when the other district holds political power. Hence, it combines features of the previous sub-sections. While it is no longer possible to obtain closed form solutions for $(\lambda_1^*, \lambda_2^*)$, we are able to solve the model numerically.

We present numerical results for three scenarios depending on the degree of heterogeneity between the districts. The first has almost identical districts, the second has medium heterogeneity and the third has a large degree of heterogeneity. For comparative purposes, we provide results for the cases $\mu = 1/2$, $\mu = 3/4$ and $\mu = 1$. footnote The equilibrium choice of representatives are given in Figures 1, 2, and 3. As in the cooperative case, when $\mu = 3/4$, there are two possible cases — interior solutions where the first order conditions hold with equality and extremal solutions where district 1's representative has preferences at the lower extreme of the distribution of preferences. The extremal solution arises in the case of high heterogeneity. Note also that, in contrast to the identical districts case, the equilibrium representative type for district 1 is no longer always decreasing in $\mu$.

# Welfare Comparisons

Welfare comparisons in the case in which the legislature is non-cooperative ($\mu = 1$) are straightforward. Using Proposition 4, it can be shown that welfare under centralization is increasing in the degree of spillovers. Moreover, it is clear that decentralization out-performs centralization when spillovers are zero, since the latter generates no public goods for one district. However, in contrast to the case of identical districts, centralization does not necessarily out perform decentralization when spillovers are maximal. When $\kappa = 1/2$, centralization produces a uniform level of public goods $m_1 y/p$ with probability 1/2 and a level $m_2 y/p$ with probability 1/2. This may be worse than the decentralized outcome where district $i$ receives outcome $m_i y/(2 - m_i)p$. Accordingly, either a decentralized system is welfare superior for all values of $\kappa$ or there exists a critical value of $\kappa$, strictly greater than 0, such that a centralized system is welfare superior if and only if $\kappa$ exceeds this critical level.

Welfare comparisons for more cooperative legislatures are less straightforward analytically and we must turn to simulations for guidance. In general, we can show that when $m_i = \overline{m}_i$ decentralization continues to be welfare superior for small spillovers. footnote In addition, it is clear that centralization does not necessarily out perform decentralization when spillovers are maximal. For example, with $\mu = 1/2$, Proposition 5 applies and the uniform level of public goods when $\kappa = 1/2$ will be $m_2 y/p$, which is too high from a social viewpoint (assuming $m_i = \overline{m}_i$). Analytically, the problem lies in showing that if centralization is preferred at some level of spillovers, it is preferred at all higher levels. This is difficult even in the case when $\mu = 1/2$ for which we have explicit solutions.

Figures 4, 5 and 6 provide the welfare comparisons corresponding to the scenarios described above. In all cases, there exists some critical level of spillovers such that centralization is welfare superior if and only if $\kappa$ exceeds this level. footnote Two further points are apparent from the Figures. First, the critical level of spillovers is increasing in the degree of heterogeneity for all values of $\mu$ we consider. Thus, the simulations support the idea that heterogeneity across the districts weakens the case for centralization. Second, the critical level is not necessarily decreasing in $\mu$, suggesting that there is no presumption that more cooperative legislative norms imply a stronger case for centralization when districts differ. This differs from the findings for identical districts reported in Proposition 3.

To summarize, the simulations suggest a picture that is broadly consistent with the conclusions of Oates' analysis of non-identical districts. If the median and mean tastes for public spending are close, a decentralized system is welfare superior for small spillovers. However, there exists a critical value of spillovers, such that a centralized system is welfare superior if and only if $\kappa$ exceeds this critical level. This level of spillovers is higher the greater the degree of heterogeneity. This similarity not-withstanding, a very different logic underlies our results. In Oates' analysis the poor performance of centralization when districts differ follows directly from the assumption of uniform provision. In our analysis, it is due the political consequences of setting up a centralized system with non-identical districts. Heterogeneity worsens the conflict of interest between jurisdictions in selecting policy choices. footnote

# Discussion

The paper has developed a particular modeling approach in order to make comparisons between centralized and decentralized systems of government. In this section, we discuss the implications of relaxing two key assumptions – uniform taxation and the absence of mobility between jurisdictions.

## Non-uniform Taxing and Spending

Throughout, we have assumed that financing decisions are uniform, while expenditure decisions are not. This can be justified on empirical grounds since most centralized systems of government appear to operate (approximately) according to such rules. footnote These assumptions can, however, be relaxed and, in doing so, we can gain some insight into why such conventions are the norm.

Suppose then that taxes can be different in each district, so that the representative holding political power chooses a pair of local public goods levels $(g_1, g_2)$ and a pair of district-specific taxes $(T_1, T_2)$. Such taxes can now serve two roles — they raise revenue for public spending and are a direct instrument for effecting cross district redistribution. While the former rids us of the budgetary externality that common pool financing generates, the latter increases political risk under centralization significantly.

Consider the extreme case where $\mu = 1$. Then, the representative holding political power can set a tax on the other district equal to $y$ and thereby extract all their resources. With our logarithmic preferences, zero private consumption is very bad indeed and decentralization is preferred no matter what the degree of spillovers. While this is the most extreme case, it is clear that the welfare cost of non-cooperative legislative behavior is likely to be high when cash based mechanisms for cross-district redistribution are feasible. Uniform tax codes eliminate an important source of such redistributions. We suspect that this finding explains the widely observed preference for uniform tax codes in federal systems.

It is also worth noting that even when the legislature behaves perfectly cooperatively $(\mu = 1/2)$, non-uniform taxation does not resolve the problems of centralization arising from strategic delegation. Suppose that in each district $i$, $m_i = \overline{m}_i$ so that if the majority preferred types are just the median types $(m_1, m_2)$, centralization produces the socially optimal allocation. This will be the outcome when there are no spillovers. However, when there are spillovers, the median voter in each district has an incentive to strategically delegate to a candidate with a lower preference for public spending. This is because district $i$'s tax depends only on the type of district $i$'s representative, while the public good levels depend upon both representatives' types. Suppose, for example, that $\kappa = 1/2$. Then, if the representatives are of types $(\lambda_1, \lambda_2)$, for each district $i$ it is the case that $g_i = y(\lambda_1 + \lambda_2)/2p$ and $T_i = \lambda_i y$. It is now straightforward to show that the majority preferred types are given by:

$$(\lambda_1^*, \lambda_2^*) = \left(\frac{m_1 - (1-m_1)m_2}{m_1 + (1-m_1)m_2}, \frac{m_2 - (1-m_2)m_1}{m_2 + (1-m_2)m_1}\right).$$

Local public goods are thus under-supplied in political equilibrium.

If uniform tax codes can be seen as a device to reduce the welfare costs associated with inter-district redistribution, one might also try to justify the traditional assumption that spending decisions are uniform with the same logic. The task would be to understand when welfare under centralization would be higher when representatives were constrained to set $g_1 = g_2$. In such circumstances, one might argue that we should expect to see a norm of uniform spending adhered to. In practice, however, we suspect that such a norm is much more difficult to establish than provisions of uniform taxation. Most notions of equal treatment would have little to do with equal expenditures. Deciding what constitutes equal provision of flood defences for a land-locked and coastal area has nothing to do with uniform provision. Similarly, defining equal access to roads in an urban and rural area may imply very different levels of spending. These inherent heterogeneities in spending needs make it hard indeed to imagine a satisfactory scheme of uniform provision.

# Mobility

We have assumed that the citizens continue to reside in the same location under either policy making arrangement that we have considered. This could be justified if there were other immovable local public goods such as language and culture that tie individuals to locations. It is interesting to consider the consequences of allowing mobility in the model. This could be incorporated by adding a prior stage to the model in which citizens chose which district to live in. If mobility is costless, equilibrium requires that no citizen wishes to relocate given the composition of the district in which they have chosen to live.

Free mobility has striking implications for the model with centralization; only two districts with identical median voters will satisfy the required equilibrium condition. Citizens make the same tax payment no matter where they choose to live and hence prefer to live in the district that has the greatest expected utility from public goods. Thus equilibrium requires that both districts must have the same expected value of public goods provision. This is true only if they elect identical representatives, which in our model requires that the median type be the same in both districts.

By contrast, decentralized policy making can lead to districts not being identical. The argument exploits the fact that, under decentralization, our preferences exhibit a single-crossing property in preferences familiar from Epple and Romer (1991) and Benabou (1996). Specifically, let $U_i(\lambda,(m_1,m_2))$ be the utility of a type $\lambda$ living in district $i$ when the median's in each district are $(m_1,m_2)$. Under the assumption that the equilibrium policy choices $(g_1^1,g_2^1)$ and $(g_1^2,g_2^2)$ are increasing in $m_i$, it is straightforward to show that $\partial^2 U_i(\lambda,(m_1,m_2))/\partial\lambda\partial m_i > 0$. This says that types with high preferences for public goods crave a district with a higher median and individuals sort. Thus, under this assumption, we would get an outcome in which all the high and low public good preference types reside together. Moreover, under reasonable definitions of stability (Benabou (1996)) the only possibility would be for a stratified outcome. That said, establishing that the (intuitively plausible) assumption that the equilibrium policy choices $(g_1^1,g_2^1)$ and $(g_1^2,g_2^2)$ are increasing in $m_i$ is generally true, appears difficult.

Thus our model seems to suggest that, with free mobility, decentralization gives maximal heterogeneity, while centralization promotes homogeneity. Where mobility costs are low, therefore, we would not expect our analysis (which held the degree of cross-district heterogeneity fixed) to be valid. It is somewhat more difficult to discern how this affects the welfare results. Our results have shown that homogeneity favors centralization and heterogeneity favors decentralization. Hence, migration will tend to bring the welfare in each case closer together, making it harder to say a priori which is better. This is an issue which could usefully be investigated in future work.

# Conclusion

The relative merits of decentralized and centralized systems of taxing and spending have long been of interest to public economists. This paper has modified the usual model of a centralized system by assuming that districts can receive different levels of local public goods. In this framework, a benevolent government can realize advantages of centralization without any of the costs (such as uniform provision) identified in the existing literature. The case for decentralization must, therefore, be driven by imperfections in the working of decision making institutions. Our political economy analysis develops models of policy making under the two regimes which capture features of decision making used in practice. We then identify the kinds of forces that determine which system will lead to a higher level of social welfare.

It is perhaps comforting that the conventional trade-off between cross-district heterogeneity and spillovers emerges intact from our analysis. However, it is important to refocus attention on the role of decision making institutions rather than the assumption of uniform provision in shaping that trade-off. Under the benevolent planner assumption, there seems little doubt that for many types of public spending, an unrestricted centralized system is, in principle, superior. The case for centralization in practice will then inevitably depend on how effectively the political process can harvest the benefits of centralization. We have emphasized how legislative norms and consequent incentives to delegate policy making strategically enter the fray. The focus on the political process also delivers new insights. Our observation that decentralization outperforms centralization with low spillovers even when districts are identical is an important example.

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# Appendix

**Proof of Lemma 1**: Suppose that $m_1 = m_2 = m$. We first check for symmetric interior equilibria in which $\lambda_1^* = \lambda_2^* = \lambda^* \in (0, \overline{\lambda})$. Any such equilibrium must satisfy the first order condition $\frac{\partial U_1(\lambda^*, \lambda^*, m)}{\partial \lambda_1} = 0$. Using the expression for $\frac{\partial U_1}{\partial \lambda_1}$ and the facts that

$$g_1^1 = g_2^2 = \frac{2\lambda^*[\mu(1-\kappa) + (1-\mu)\kappa]y}{p},$$

$$g_2^1 = g_1^2 = \frac{2\lambda^*[(1-\mu)(1-\kappa) + \mu\kappa]y}{p},$$

and $T_1 = T_2 = \lambda^* y$, the first order condition can be written as:

$$\frac{m}{\lambda^*}\left[\frac{\mu(1-\kappa)^2 + (1-\mu)\kappa^2}{\mu(1-\kappa) + (1-\mu)\kappa} + \frac{\mu\kappa^2 + (1-\mu)(1-\kappa)^2}{\mu\kappa + (1-\mu)(1-\kappa)}\right] = \frac{(1-m)}{(1-\lambda^*)}.$$

This is a linear equation for $\lambda^*$ and it is straightforward, if tedious, to show that

$$\lambda^* = \frac{m}{m + (1-m)\sigma(\kappa, \mu)}$$

where $\sigma(\kappa, \mu)$ is as defined in the text.

Since $0 < m < \lambda^* < 2m < \overline{\lambda}$, it must be the case that $\frac{\partial U_1(0,0,m)}{\partial \lambda_1} > 0 > \frac{\partial U_1(\overline{\lambda}, \overline{\lambda}, m)}{\partial \lambda_1}$. Thus, we cannot have boundary equilibria in which $\lambda_1^* = \lambda_2^* = 0$ or that $\lambda_1^* = \lambda_2^* = \overline{\lambda}$. We may therefore conclude that there is a unique symmetric equilibrium of the game in which both players select $\lambda^*$. [End Proof]

**Proof of Lemma 2**: It is clear from Lemma 1 that

$$(g_i^i, g_{-i}^i) = \left(\frac{2m[\mu(1-\kappa) + (1-\mu)\kappa]y}{[m + (1-m)\sigma(\kappa, \mu)]p}, \frac{2m[\mu\kappa + (1-\mu)(1-\kappa)]y}{[m + (1-m)\sigma(\kappa, \mu)]p}\right) \ i \in \{1, 2\},$$

is a symmetric policy outcome under centralization. Suppose there existed another such policy outcome $\{(\widehat{g}_i^i, \widehat{g}_{-i}^i) : i \in \{1, 2\}\}$ and let $(\lambda_1^*, \lambda_2^*)$ be the pair of majority preferred types generating it. Lemma 1 allows us to assume that $\lambda_1^* \neq \lambda_2^*$. But then $(\widehat{g}_1^1, \widehat{g}_2^1) = (\widehat{g}_1^2, \widehat{g}_2^2)$ if and only if $\mu(1-\kappa) = (1-\mu)\kappa$ or, equivalently, $\mu = \kappa = 1/2$. It follows that $\widehat{g}_1^1 = \widehat{g}_2^2 = \widehat{g}$ and, from the first order conditions,

$$\frac{my}{2p}\frac{1}{\widehat{g}} = \frac{(1-m)y}{2}\left[\frac{1}{y - 2\widehat{g}}\right],$$

which implies that

$$\widehat{g} = my/p.$$

But this is exactly the outcome generated by the symmetric solution described in Lemma 1 when $\mu = \kappa = 1/2$. [End Proof]

**Proof of Proposition 3**: We begin by establishing three of the claims made in the text.

**Claim 1**: For all $\mu$, $\partial W^c(\kappa,\mu)/\partial\kappa \geq 0$ with the inequality holding strictly if and only if $\kappa < 1/2$.

We know that

$$W^c(\kappa,\mu) = 2(1-\overline{m})\ln y(1-\frac{m}{m+(1-m)\sigma}) + \overline{m}[\ln\frac{2m\alpha y}{[m+(1-m)\sigma]p} + \ln\frac{2m\beta y}{[m+(1-m)\sigma]p}].$$

Differentiating with respect to $\kappa$ yields

$$\frac{\partial W^c(\kappa,\mu)}{\partial\kappa} = \overline{m}[\frac{\partial\alpha/\partial\kappa}{\alpha} + \frac{\partial\beta/\partial\kappa}{\beta}] - \frac{2[(1-\overline{m})m-\sigma(1-m)\overline{m}]\partial\sigma/\partial\kappa}{[m+(1-m)\sigma]\sigma}.$$

Evaluating the derivatives $\partial\alpha/\partial\kappa$ and $\partial\beta/\partial\kappa$, we have:

$$\frac{\partial W^c(\kappa,\mu)}{\partial\kappa} = \overline{m}\frac{(1-2\mu)^2(1-2\kappa)}{\alpha\beta} - \frac{2[(1-\overline{m})m-\sigma(1-m)\overline{m}]\partial\sigma/\partial\kappa}{[m+(1-m)\sigma]\sigma}.$$

The first term is always non-negative and positive unless $\kappa = 1/2$ or $\mu = 1/2$. When $m = \overline{m}$, the second term is always non-negative and positive unless $\kappa = 1/2$ or $\mu = 1$. The result follows.

**Claim 2**: For all $\mu$, $\partial W^c(\kappa,\mu)/\partial\mu \leq 0$ with the inequality holding strictly if and only if $\kappa < 1/2$.

Following the logic of the previous claim, we have that

$$\frac{\partial W^c(\kappa,\mu)}{\partial\mu} = \overline{m}\frac{(1-2\mu)(1-2\kappa)^2}{\alpha\beta} - \frac{2[(1-\overline{m})m-\sigma(1-m)\overline{m}]\partial\sigma/\partial\mu}{[m+(1-m)\sigma]\sigma}$$

The first term is always non-positive and negative unless $\kappa = 1/2$ or $\mu = 1/2$. When $m = \overline{m}$, the second term is always non-positive and negative unless $\kappa = 1/2$ or 0 or $\mu = 1$. The result follows.

**Claim 3**: For all $\mu$, $W^c(1/2,\mu) = W^d(0)$.

To see this, observe first that $\sigma(1/2,\mu) = 1$ for all $\mu$, which implies that the policy outcome under centralization is given by $g_i^i = g_{-i}^i = \frac{my}{p}$. This implies that

$$W^c(1/2,\mu) = 2(1-m)\ln y(1-m) + m[\ln\frac{my}{p} + \ln\frac{my}{p}] = W^d(0).$$

We can now prove the Proposition. Let $\mu$ be given. By Claim 3, $W^d(0) = W^c(1/2,\mu)$ and by Claim 1, $W^c(1/2,\mu) > W^c(0,\mu)$. It follows that $W^d(0) > W^c(0,\mu)$ and, since $W^d(\kappa)$ is decreasing in $\kappa$, that $W^d(1/2) < W^c(1/2,\mu)$. By the intermediate value theorem and Claim 1, there exists a unique value of $\kappa$, denoted $\widehat{\kappa}(\mu)$, such that $\widehat{\kappa}(\mu) \in (0,1/2)$ and $W^d(\widehat{\kappa}(\mu)) = W^c(\widehat{\kappa}(\mu),\mu)$. Claim 1 and the fact that $W^d(\kappa)$ is decreasing, implies that $W^d(\widehat{\kappa}) > (<) W^c(\widehat{\kappa}(\mu),\mu)$ for all $\kappa < (>) \widehat{\kappa}(\mu)$. That $\widehat{\kappa}(\mu)$ is decreasing in $\mu$ follows routinely from Claims 1 and 2 and the fact that $W^d(\kappa)$ is decreasing. [End Proof]

**Proof of Lemma 3**: For $i = 1,2$, let $r_i : [0,\overline{\lambda}] \to [0,\overline{\lambda}]$ denote the district $i$ median voter's *reaction function*. By definition, for all $\lambda_2 \in [0,\overline{\lambda}]$,

$$r_1(\lambda_2) = \arg\max\{U_1(r_1,\lambda_2,m_1) : r_1 \in [0,\overline{\lambda}]\},$$

and for all $\lambda_1 \in [0,\overline{\lambda}]$,

$$r_2(\lambda_1) = \arg\max\{U_2(\lambda_1, r_2, m_2) : r_2 \in [0, \overline{\lambda}]\}.$$

Since $(\lambda_1^*, \lambda_2^*)$ is majority preferred if and only if $(\lambda_1^*, \lambda_2^*) = (r_1(\lambda_2^*), r_2(\lambda_1^*))$, we may understand the set of majority preferred types by studying the players' reaction functions.

Some general features of the reaction functions follow from the properties of the payoff functions. Recall from the earlier discussion in the text that each player's payoff is a strictly concave and differentiable function of his strategy. This implies (i) that $r_1(\lambda_2) = 0$ if $\partial U_1(0, \lambda_2, m_1)/\partial\lambda_1 < 0$; (ii) that $r_1(\lambda_2) = \overline{\lambda}$ if $\partial U_1(\overline{\lambda}, \lambda_2, m_1)/\partial\lambda_1 > 0$; and (iii) that otherwise $r_1(\lambda_2)$ is implicitly defined by the first order condition $\partial U_1(r_1(\lambda_2), \lambda_2, m_1)/\partial\lambda_1 = 0$. In addition, the fact that types are strategic substitutes implies that $r_1(\lambda_2)$ is non-decreasing. Similar remarks apply to the district 2 median voter's reaction function

It remains therefore to determine the details of each player's reaction function. Let $\overline{\lambda}_1(\overline{\lambda}_2)$ denote the level of $\lambda_1(\lambda_2)$ beyond which district 2's median voter (district 1's median voter) would like a type 0 representative. These levels are implicitly defined by the equalities

$$\partial U_1(0, \overline{\lambda}_2, m_1)/\partial\lambda_1 = 0,$$

and

$$\partial U_2(\overline{\lambda}_1, 0, m_2)/\partial\lambda_2 = 0.$$

Using the facts that

$$\partial U_1/\partial\lambda_1 = \frac{p}{y}\{m_1[\frac{(1-\kappa)^2}{\lambda_1(1-\kappa)+\lambda_2\kappa} + \frac{\kappa^2}{\lambda_2(1-\kappa)+\lambda_1\kappa}] - [\frac{1-m_1}{2-(\lambda_1+\lambda_2)}]\},$$

and

$$\partial U_2/\partial\lambda_2 = \frac{p}{y}\{m_2[\frac{(1-\kappa)^2}{\lambda_2(1-\kappa)+\lambda_1\kappa} + \frac{\kappa^2}{\lambda_1(1-\kappa)+\lambda_2\kappa}] - [\frac{1-m_2}{2-(\lambda_1+\lambda_2)}]\},$$

we obtain

$$\overline{\lambda}_1 = \frac{2\phi(\kappa)m_2}{1-m_2+\phi(\kappa)m_2}$$

and

$$\overline{\lambda}_2 = \frac{2\phi(\kappa)m_1}{1-m_1+\phi(\kappa)m_1},$$

where

$$\phi(\kappa) = \frac{(1-\kappa)^3+\kappa^3}{\kappa(1-\kappa)}.$$

Observe that $\phi(\kappa)$ is decreasing in $\kappa$, that $\phi(1/2) = 1$ and that $\lim_{\kappa\to 0_+}\phi(\kappa) = \infty$. This implies that $\overline{\lambda}_1 \geq 2m_2$ and $\overline{\lambda}_2 \geq 2m_1$.

Next we characterize the highest type representative each district's median voter would want. It is straightforward to show that

$$\partial U_1(2m_1, 0, m_1)/\partial\lambda_1 = 0$$

and

$$\partial U_2(0, 2m_2, m_2)/\partial\lambda_2 = 0,$$

which implies that district $i$'s median voter desires a type $2m_i$ candidate when the other district selects a type 0 candidate. By assumption, $2m_i < \overline{\lambda}$, so that the upper bound constraint on type choice is not binding here. It follows that for both districts $i = 1, 2$, $r_i(0) = 2m_i$.

We may conclude from the above that for all $\lambda_2 \in [0, \min\{\overline{\lambda}_2, \overline{\lambda}\}]$, $r_1(\lambda_2)$ is implicitly defined by the first order condition

$$\partial U_1(r_1(\lambda_2), \lambda_2, m_1)/\partial\lambda_1 = 0$$

and for all $\lambda_2 \in (\min\{\overline{\lambda}_2, \overline{\lambda}\}, \overline{\lambda}]$,

$$r_1(\lambda_2) = 0.$$

Further, we know that $r_1(0) = 2m_1$ and that $r_1(\lambda_2)$ is downward sloping on $[0, \min\{\overline{\lambda}_2, \overline{\lambda}\}]$. Similarly, for all $\lambda_1 \in [0, \min\{\overline{\lambda}_1, \overline{\lambda}\}]$, $r_2(\lambda_1)$ is implicitly defined by the first order condition

$$\partial U_2(\lambda_1, r_2(\lambda_1), m_2)/\partial\lambda_2 = 0$$

and for all $\lambda_1 \in (\min\{\overline{\lambda}_1, \overline{\lambda}\}, \overline{\lambda}]$,

$$r_2(\lambda_1) = 0.$$

Further, we know that $r_2(0) = 2m_2$ and that $r_2(\lambda_1)$ is downward sloping on $[0, \min\{\overline{\lambda}_1, \overline{\lambda}\}]$.

We can now prove the lemma. Recall from above that $(\lambda_1^*, \lambda_2^*)$ is majority preferred if and only if $(\lambda_1^*, \lambda_2^*) = (r_1(\lambda_2^*), r_2(\lambda_1^*))$. Suppose first that $\kappa < \widehat{\kappa}$. Then, from the definition of $\widehat{\kappa}$ in the text, it follows that $\phi(\kappa) > \frac{(1-m_1)m_2}{(1-m_2)m_1}$, which in turn implies that

$$\overline{\lambda}_2 = \frac{2\phi(\kappa)m_1}{1 - m_1 + \phi(\kappa)m_1} > 2m_2.$$

This inequality implies that there exist no boundary equilibria in which $\lambda_i^* = 0$ for one or more districts. If $\lambda_1^* = 0$, then $\lambda_2^* = r_2(0) = 2m_2$, but since $2m_2 < \overline{\lambda}_2$ we know that $r_1(2m_2) > 0$ which contradicts the fact that $\lambda_1^* = 0$. If $\lambda_2^* = 0$, then $\lambda_1^* = r_1(0) = 2m_1$, but since $2m_1 < \overline{\lambda}_1$ we know that $r_2(2m_1) > 0$ which contradicts the fact that $\lambda_2^* = 0$. Since $\max r_i(\lambda_{-i}) < \overline{\lambda}$, it is apparent that there can be no boundary equilibria in which $\lambda_i^* = \overline{\lambda}$ for one or more districts.

It follows that there must exist an interior equilibrium. Any such equilibrium $(\lambda_1^*, \lambda_2^*)$ must satisfy the first order conditions $\frac{\partial U_i(\lambda_1^*, \lambda_2^*, m_i)}{\partial\lambda_i} = 0$ for $i \in \{1, 2\}$. Using the expressions for $\frac{\partial U_i}{\partial\lambda_i}$, $i \in \{1, 2\}$ from above, we may write these first order conditions as:

$$m_1\left[\frac{(1-\kappa)^2}{\lambda_1^*(1-\kappa) + \lambda_2^*\kappa} + \frac{\kappa^2}{\lambda_2^*(1-\kappa) + \lambda_1^*\kappa}\right] = \frac{1 - m_1}{2 - (\lambda_1^* + \lambda_2^*)},$$

and

$$m_2\left[\frac{(1-\kappa)^2}{\lambda_2^*(1-\kappa) + \lambda_1^*\kappa} + \frac{\kappa^2}{\lambda_1^*(1-\kappa) + \lambda_2^*\kappa}\right] = \frac{1 - m_2}{2 - (\lambda_1^* + \lambda_2^*)}.$$

Combining the two first order conditions, we obtain

$$\frac{m_1}{1-m_1}\left[\frac{(1-\kappa)^2}{\lambda_1^*(1-\kappa)+\lambda_2^*\kappa}+\frac{\kappa^2}{\lambda_2^*(1-\kappa)+\lambda_1^*\kappa}\right]$$
$$=\frac{m_2}{1-m_2}\left[\frac{(1-\kappa)^2}{\lambda_2^*(1-\kappa)+\lambda_1^*\kappa}+\frac{\kappa^2}{\lambda_1^*(1-\kappa)+\lambda_2^*\kappa}\right],$$

which implies that

$$\lambda_1^* = \xi\lambda_2^*,$$

where $\xi$ is as defined in the text. Notice that the assumption that $\kappa < \widehat{\kappa}$ implies that $\xi > 0$. Using this and the first order conditions for $\lambda_1^*$ and $\lambda_2^*$ respectively yields:

$$\lambda_1^* = \frac{m_2}{m_2(1+\xi)/2+(1-m_2)/2\alpha_2}$$

and

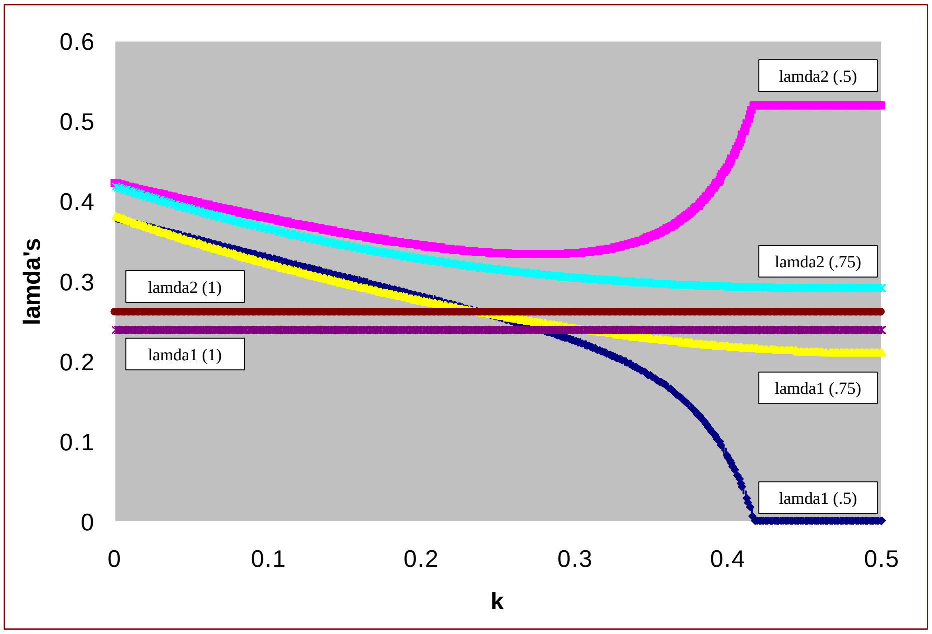
$$\lambda_2^* = \frac{m_1}{m_1(1+1/\xi)/2+(1-m_1)/2\alpha_1},$$

where $\alpha_1$ and $\alpha_2$ are as defined in the text.

Suppose now that $\kappa \geq \widehat{\kappa}$. Then, from the definition of $\widehat{\kappa}$ in the text, it follows that $\phi(\kappa) \leq \frac{(1-m_1)m_2}{(1-m_2)m_1}$, which in turn implies that

$$\overline{\lambda}_2 = \frac{2\phi(\kappa)m_1}{1-m_1+\phi(\kappa)m_1} \leq 2m_2.$$

This inequality implies that there exists a boundary equilibrium in which $(\lambda_1^*, \lambda_2^*) = (0, 2m_2)$. This is because $r_1(2m_2) = 0$ and $r_2(0) = 2m_2$. The same arguments from above imply that there exist no other boundary equilibria. To complete the proof, therefore, we need to show that there are no interior equilibria. We know that any such equilibrium $(\lambda_1^*, \lambda_2^*)$ must satisfy the first order conditions $\frac{\partial U_i(\lambda_1^*,\lambda_2^*,m_i)}{\partial\lambda_i} = 0$ for $i \in \{1,2\}$. Using the above logic, this means that $\lambda_1^* = \xi\lambda_2^*$. But the assumption that $\kappa < \widehat{\kappa}$ implies that $\xi \leq 0$ which, in turn, is inconsistent with the hypothesis that $(\lambda_1^*, \lambda_2^*) > (0,0)$. [End Proof]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lamda2 (.75)
lamda2 (1)
lamda1 (1)
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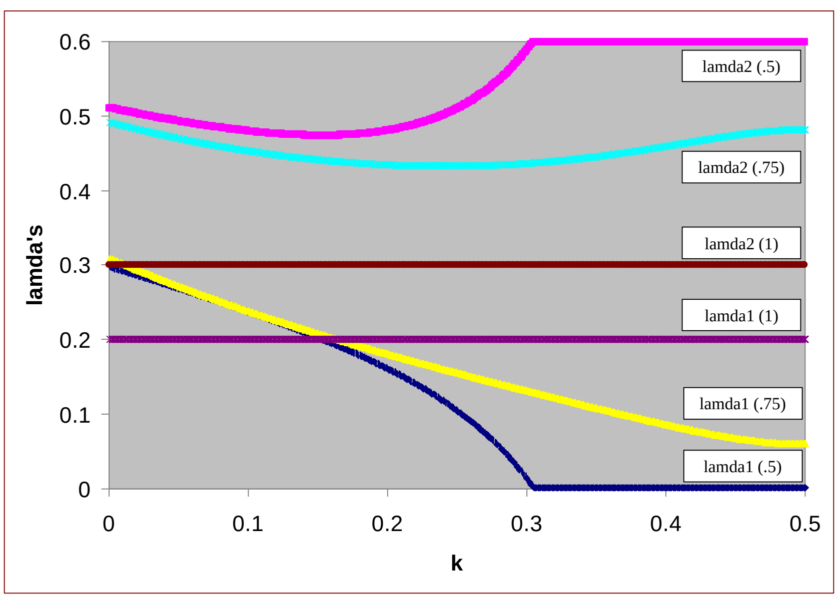
0.6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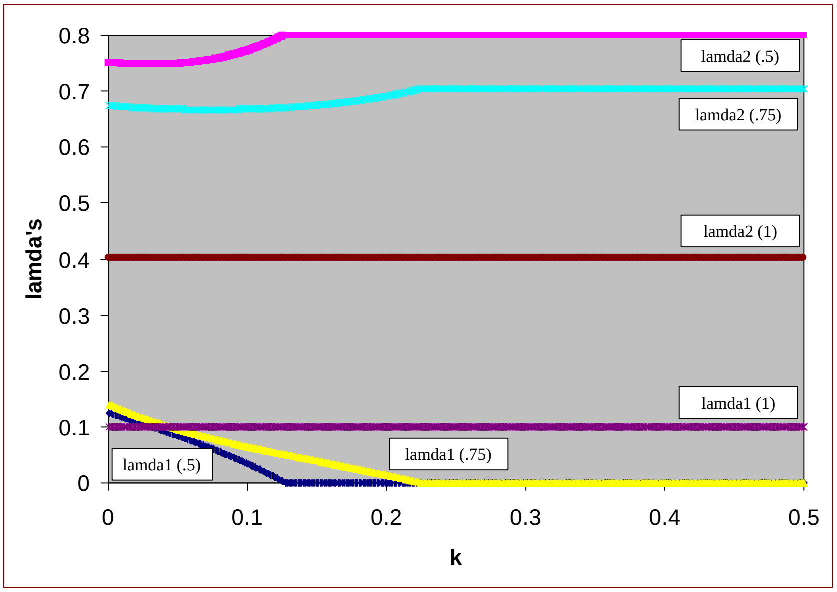

lamda's
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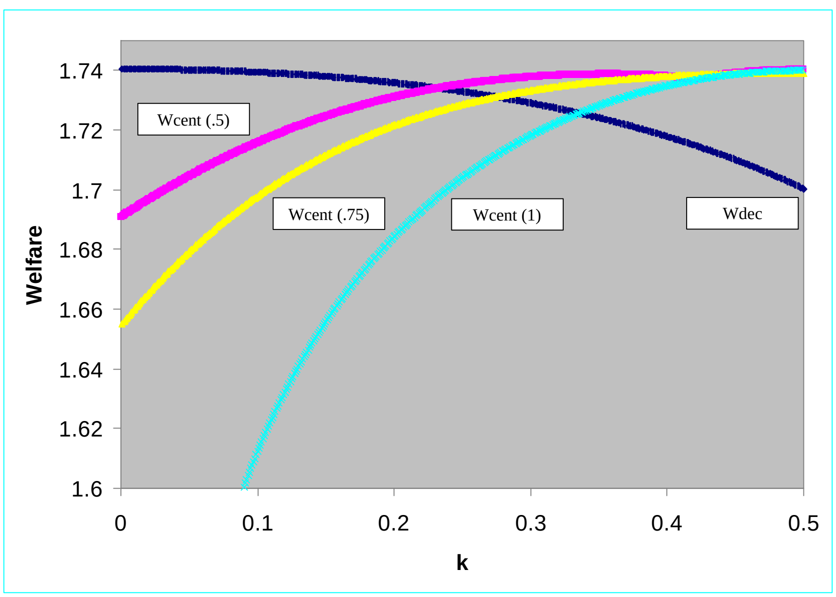

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1.64
1.66
1.68
1.7
1.72
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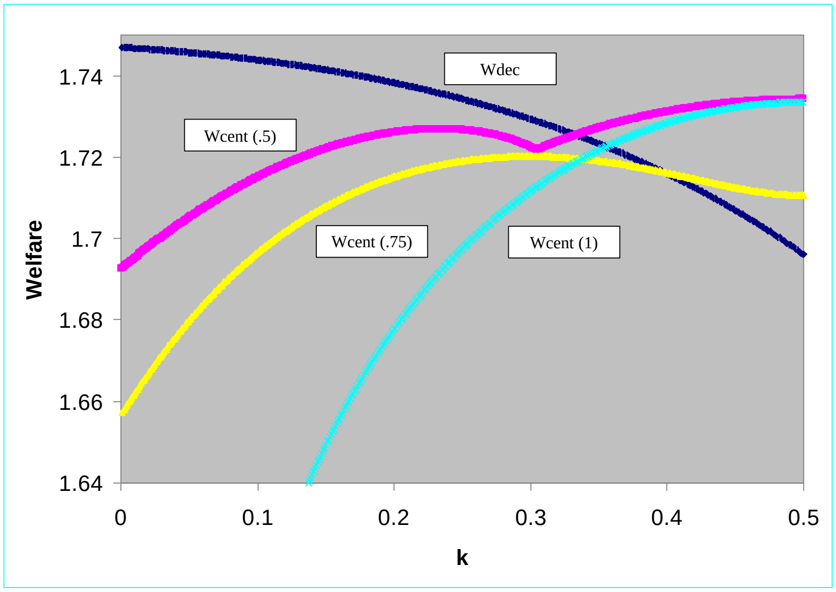
0.3
0.4
0.5
Wcent (.5)
Wcent (.75)
Wcent (1)
Wdec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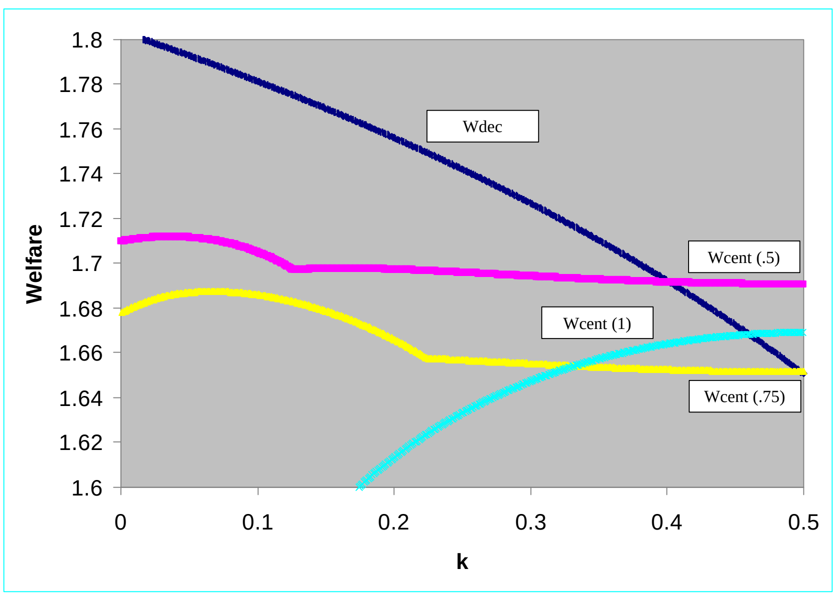
k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1.62
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k