# GLOBAL DIVERSIFICATION, GROWTH AND WELFARE WITH IMPERFECTLY INTEGRATED MARKETS FOR GOODS

Bernard Dumas
Raman Uppal

Working Paper 6994
http://www.nber.org/papers/w6994

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
March 1999

We are grateful for comments to Pierluiggi Balduzzi, Geert Rouwenhorst, Angel Serrat, Jiang Wang, and Tan Wang. We would also like to acknowledge suggestions from seminar participants at Carnegie Mellon University, Catholic University (Leuven), Dartmouth College, MIT, Northwestern University, the NBER Finance Lunch Seminar Series, Stanford University, and the 1998 meetings of the European Finance Association and the Western Finance Association. The views expressed in this paper are those of the authors and do not reflect those of the National Bureau of Economic Research.

Global Diversification, Growth and Welfare
with Imperfectly Integrated Markets for Goods
Bernard Dumas and Raman Uppal
NBER Working Paper No. 6994
March 1999
JEL No. F36, F41, G15

## ABSTRACT

In this article, we examine the effect of the imperfect mobility of goods on international risk sharing and, through that, on the investment in risky projects, welfare and growth. We find that the welfare gain of financial market openness is not monotonic with respect to investors' risk aversion and the aggregate volatility of output growth. Our main result is that the welfare gain from integration is not drastically reduced by the presence of goods market imperfections, modeled as a cost of transferring goods from one country to the other. Hence, financial market integration may be a worthwhile goal to pursue even at a time when full goods mobility has not been achieved.

Bernard Dumas
HEC School of Management
78351 Jouy-en-Josas Cedex
France
dumas@gwsmtp.hec.fr

Raman Uppal
Sloan School of Management
MIT
50 Memorial Drive
Cambridge, MA 02142
and NBER
uppal@mit.edu

# Global Diversification, Growth and Welfare with Imperfectly Integrated Markets for Goods

The benefits, along with the impact on growth, of free trade in goods are frequently contrasted with those of the free movement of capital around the world. While, on the one hand, the benefits of freer trade, provided competition is pure and perfect, are viewed as substantial and are opposed only by the need of some governments to raise revenues in the form of tariffs, unfettered capital flows on the other are regarded with more suspicion. The reasons for this suspicion may be valid ones (deviations from rational expectations, herding behavior, bubbles, crises etc.) but they ought to be brought into play only after the impact of free capital flows under ideal conditions is well understood. It is undeniable, however, that the benefits of free capital flows under ideal conditions, and their merits relative to free trade, have not been the subject of as much research as free trade itself.

What seems to be needed is a study that would help decide to what extent, and in what sense, the degree of openness of goods markets and the degree of openness of asset markets are substitutes or complements to each other. In the limit of full market segmentation, there is no question that they have to be regarded as complements: in the absence of any physical transfers between countries, the financial markets would be closed *de facto* because the payment (physical delivery) of dividends across countries would become impossible. So, in effect, a closure of the goods market acts as a closure of the asset market.[1] The unwillingness to trade in the asset market is inherently linked to the inability to transfer goods across borders because of the role of the financial market as a reflection of the future of the physical economy.

But, in other ways, the two forms of openness are substitutes. The increased scope of competition which accompanies free trade in goods is often viewed as an impetus to growth. But, it is also true that the ability to finance investment projects by means of foreign capital must, in most case situations, foster growth.[2] Hence, the two forms of openness have comparable

[1] Conversely, in a one-good model, a closing of the financial market amounts to complete autarky. Cole and Obstfeld (1991) have studied the opening of asset markets in a two-good, pure-exchange, frictionless economy with a logarithmic utility function and found that there are cases where the trading of financial securities is unnecessary.

[2] Obstfeld (1994) has proposed a model of international asset trade linking an economy's long-run output growth rate to its financial openness. In his model, an economy that opens its asset market to trade may

effects. It is this link between asset trade and commodity trade that we wish to analyze.

In our view, developed economies today are characterized by financial markets that may or may not be integrated but by commodity markets in which goods are not perfectly mobile. Evidence to that effect is presented below in Section 1. In the current paper, our objective is to measure the benefits of asset market integration when there exist costs to transferring (shipping) physical resources from one country to the other. Our focus is on the reduction of the welfare gain from integration, brought about by the imperfections of the real side of the world economy, while financial markets are complete, integrated and frictionless.[3,4]

When we study only the extreme cases of zero transfer cost and infinite transfer cost, as we do in Section 3, the analysis is identical to a comparison between complete autarky and complete integration of both markets, as was done by Obstfeld (1994). But, in Section 4, we intend to gradually increase transfer costs; as we move away from mobility in the market for goods, we maintain the assumption of full integration in the financial market. Investors will remain free to modify costlessly their portfolio of domestic and foreign assets. In so doing, however, they take into account which country they live in; the physical resources currently trapped in their country act as a (temporarily) non-traded asset.

Financial flows between nations arise from saving and investment decisions, which are made on the basis of intertemporal considerations. To account for the intertemporal nature of these decisions, it would not be realistic to examine a pure-exchange economy. Thus, we explicitly model the production decision.[5] Also, for internal consistency, and because we are studying financial flows between large economies, our model is a general equilibrium one. That is, we do not make the small country assumption which is typically made in models analyzing capital flows.[6] Thus, our model allows for the inter-temporal general equilibrium analysis of

---

experience a substantial increase in growth (and welfare) because the ability to diversify risk allows a more intensive use of the riskier, higher-expected return technologies. The assumption made by Obstfeld when opening the asset market to trade, is that the market for goods is frictionless. There are no impediments whatsoever to transferring physical resources from one country to the other.

[3] Basak (1995), Errunza and Losq (1989) and Subrahmanyam (1975) examine the welfare loss from capital controls. They study how the expansion in the set of financial claims, which one can invest in, affects welfare.

[4] Brennan and Solnik (1989) also provide an empirical estimate of the welfare gains from international financial flows. Obstfeld (1992) suggests that the gains from capital flows reported in Brennan and Solnik (1989) may have been over-estimated.

[5] See Obstfeld (1982) for a discussion of why it is necessary to account for intertemporal considerations that arise in this context. Cole and Obstfeld (1991) underscore the need to model the production decision explicitly.

[6] In particular, the interest rate in our model is determined endogenously and the equilibrium that we analyze is, in contrast to Lucas (1982), not a perfectly pooled one.

the linkages between the real and financial sectors of an open economy.[7]

The ability to invest physically confers to goods asset characteristics, similar to those of storable goods. For this reason, our expectation, as we start this research, is that physical frictions reduce the benefits of asset market integration comparatively little: the *optimal timing* of physical transfers mitigates the effect of transfer costs. At times when it is not optimal to transfer goods, foreigners do not collect their payoffs. The financial market makes it possible for them to simply accumulate claims which are invested in the home production process and which will be redeemed later. Thanks to a well functioning financial market, a good deal of shipping expenditures is avoided.

The balance of the paper is organized as follows. In Section 1, we present the evidence from earlier studies showing that the movements of goods around the world is far from free. In Section 2, we describe the model and indicate how the equilibrium can be obtained as a welfare maximization problem. In Section 3, we derive the effect of financial integration on welfare and growth. In Section 4, we obtain the equilibrium as one gradually increases transfer costs and we study a calibration based on stock market data, in order to gain some insight into the likely effect of integration in the real world. In Section 5, we examine the case with transfer costs as one changes the various parameter values. The appendix contains a description of the numerical procedure used to solve the problem.

# 1 Empirical evidence on the international mobility of goods

In our view, developed economies today are characterized by financial markets that may or may not be integrated but by commodity markets in which goods are not perfectly mobile. For example, investors in the United States can trade in Japanese financial claims quite easily but incur significant costs for trading goods with another country, say Japan.[8] Similarly, it is much easier for investors within Western European countries to trade in foreign financial claims, but barriers to trade in goods still exist. Thus, in our model we will assume that financial markets are perfectly integrated. This assumption is supported by the following observations documented in Frankel (1986): (i) the removal of capital controls following the

[7] See Helpman and Razin (1978) for early work on the relation between the financial and real sectors in international economies. Arndt and Richardson (1987) discuss why it is important to study the issues that arise in this context. See, also, Allen and Stein (1990).

[8] See Cooper (1986) for the empirical evidence on the openness of the U.S. economy.

end of the Bretton Woods agreement,[9] (ii) the development of Euro-currency markets, and (iii) the absence of arbitrage opportunities in international financial markets.[10]

However, as Gagnon (1989) shows, and as the importance attached to achieving world trade agreements attests, significant barriers to commodity trade still exist. Even though tariff levels have fallen in the last twenty years, this decrease has been more than offset by the increase in the magnitude of non-tariff barriers. A comparison of the distortionary costs of quantitative restrictions in the three sectors [textiles, automobiles and steel] with the distortionary costs of the remaining tariff protection reveals that the costs of quantitative restrictions are ten times larger than the costs of tariffs (Melo and Tarr, 1992, p. 12).[11]

Deardorff and Stern (1990, Table A7) report that the average tariff in industrialized countries is about 6.6%. Deardorff (1989, Tables 4 and B5) reports, on the basis of several studies, the low and high estimates of *ad valorem* equivalents of non-tariff barriers for the various traded goods in the major industrialized countries. For example, in the US, the low (high) estimates range from 0% (0%) for machinery to 14.5% (20.5%) for food. For Japan, the low (high) estimates are 0% (0%) for machinery and 27.1% (58.1%) for food. Melo and Tarr (1992, pp. 200), based on their study of the textiles, automobile and steel industries in the US, estimate that the average tariffs that would generate the same welfare costs as those resulting from quotas in the three sectors is 49%.[12]

As has been documented by Marston (1990), an additional imperfection of the goods market, made possible by the existence of protections and transfer costs, arises from the fact that firms are able to price discriminate between markets.

As a result of these various imperfections, wide and persistent deviations from parity in the prices of goods between countries are pervasive. Engel and Rogers (1996) and many others have studied the deviations from the Law of One Price and the deviations from Purchasing

[9] Halliday (1989) reports on the absence of capital controls in the financial markets of Austria, Belgium, Denmark, Germany, Ireland, Italy, Japan, Netherlands, U.K. and U.S.

[10] Gultekin, Gultekin and Penati (1989) and Wheatley (1988) also provide evidence in support of integrated financial markets. See Frankel (1986), and the references therein, for why, even with integrated capital markets, savings and investment in a country may be highly correlated.

[11] For details on the type of barriers to international trade that exist, for example, restrictive licenses and differences in technical regulations across countries, see van Nunen (1990).

[12] Erzan and Kuwahara report that the effective tariff rate is as high as 66% in Central America and 51% in South America, while the effective tariff rate in Africa is about 37%, in West Asia 5% and in the rest of Asia 25%. A significant proportion of this effective tariff arises from non-tariff barriers. For example, the percentage of the effective tariff that is a consequence of non-tariff barriers is 100% in Central America, 85% in Africa, and 21% for Asia (other than W. Asia).

Power Parity (PPP) arising from it. When attempts have been made to fit an Autoregressive model to PPP deviations, it has frequently been found that they follow a martingale behavior that would allow them randomly to drift away from zero indefinitely.

In the present paper, as in Dumas (1992), we model the imperfection of commodities markets by introducing a proportional cost (tariff) for exporting the physical good from one country to another. This specification implies the existence of two regimes prevailing at different times: one in which transfers of goods take place and cause price deviations quickly to revert to some definite level imposed by transfer costs, and one in which transfers of goods do not occur at the present time so that prices drift away from parity more freely. The recent introduction into the empirical literature on PPP-deviations of Threshold AutoRegressive statistical models (TAR)[13] and of Smooth Transition AutoRegressive models (STAR)[14] provides support for this specification.

## 2 Social choice in an economy with frictions

The economy is made up of two countries[15] with perfectly integrated financial markets. The frictions in the goods market are modeled by introducing a proportional shipping cost for exporting/importing the physical good from one country to another.[16] One good exists in two versions, differentiated only by geographic location. The physical good that is produced in each country can be consumed, exported to the other country or reinvested in the production process.

The two countries are *ex ante* symmetric in all respects. Each country is populated by a large number of identical firms and consumers; the domestic and foreign representative consumers have identical preferences; and, the production technologies in both countries are of the same form. The only factor that distinguishes one country from another is that production in each country is subject to a country-specific shock, and it is costly to export the physical good from one country to another. That is, each country is characterized by a unique innovation in

[13]See Obstfeld and Taylor (1997) and O'Connell and Wei (1997).

[14]See Michael, Nobay and Peel (1997).

[15]The two countries are identified as the home and foreign country, and an asterisk (*) is used to denote all variables for the foreign country.

[16]For a model where the transfer cost is a fixed amount, rather than a proportional one, see Shrikhande (1992). Benninga and Protopapadakis (1988) and Coleman (1995) model the shipping cost as one that arises because trade takes time. Dumas (1988, Appendix I) shows how the shipping cost in our model can be re-interpreted as one that arises because shipping takes time.

its output process. Each representative investor is constrained to consume only from the stock of the good located in her own country: before one can consume from the foreign good, the good must be imported to the home country and the shipping cost must be paid.

## 2.1 The production set

Firms use as input the same good that they produce and this good is also the consumption good. It is assumed that all firms in a country have access to the same two production technologies: one is riskless with a rate of return $r$; the other is a constant-returns-to-scale, risky production technology which is characterized by its instantaneous expected rate of return, $\mu$, and instantaneous standard deviation, $\sigma$. We assume that $\mu > r$. The fraction of physical capital allocated to the risky technology is denoted $w$ and the fraction allocated to the riskless one is $1 - w$. Adjustments in the allocation of capital to the two technologies *within* a country are instantaneous and costless. Non-negative allocations are not feasible: $0 \leq w \leq 1$. There are no barriers to entry in an industry. With these assumptions, perfect competition obtains and the value of the domestic [foreign] industry is equal to the value of the existing stock of the domestic [foreign] good, $K(t)$ $[K^*(t)]$. The instantaneous return on an investment in the domestic industry is:

$$\begin{aligned} dY(t) &= [r + w(\mu - r)]\, K(t)dt + w\sigma K(t)dz(t), \\ K(0) &= K_0. \end{aligned}$$

The flow $dY(t)$ exceeds $dK(t)$, the formation of capital in the domestic industry, by the amount of domestic consumption and net exports. Similarly, the instantaneous return on an investment in the foreign industry is:

$$\begin{aligned} dY^*(t) &= [r + w^*(\mu - r)]\, K^*(t)dt + w^*\sigma K^*(t)dz^*(t), \\ K^*(0) &= K_0^*. \end{aligned}$$

The two fundamental sources of uncertainty in this model are the production shocks in the home and foreign country. For symmetry, the technology shocks in the two countries have the same volatility, $\sigma$. In contrast to Dumas (1992), we allow the shocks to output in the two countries to be correlated, and we denote this correlation between the two shocks by $\eta$.

Exporting the good from one country to another entails a proportional cost. Let $\tau$, where $\tau > 0$, denote the proportional (effective) cost for moving goods from one location to another.

Thus, for every unit of good that is exported from one country, only $(1-\tau)$ arrives in the other country. To account for the export of the good from one country to another, we introduce two non-decreasing processes, $X(t)$ and $X^*(t)$, with $X(0) = X^*(0) = 0$ that represent the cumulative amounts of the goods exported, since time 0, from the home to the foreign location, and from the foreign to the home location, respectively. These processes regulate the joint process for the stock of the domestic and foreign goods, $K(t)$ and $K^*(t)$.[17] The regulated processes for $K(t)$ and $K^*(t)$, adjusted for consumption $c(t)$ and exports/imports between the two countries, are described by the following stochastic differential equations:

$$\begin{aligned} dK(t) &= [r + w(\mu - r)]\, K(t)dt + w\sigma K(t)dz(t) \\ &\quad - c(t)dt - dX(t) + (1-\tau)dX^*(t), \\ K(0) &= K_0;\ 0 \leq w \leq 1;\ \mu > r; \end{aligned} \tag{1}$$

$$\begin{aligned} dK^*(t) &= [r + w^*(\mu - r)]\, K^*(t)dt + w^*\sigma K^*(t)dz^*(t) \\ &\quad - c^*(t)dt - dX^*(t) + (1-\tau)dX(t); \\ K^*(0) &= K_0^*;\ 0 \leq w^* \leq 1;\ \mu > r. \end{aligned} \tag{2}$$

Given that the home and foreign investors are identical and that the production shocks have the same mean and variance, in an economy with frictionless commodity markets it would be optimal to diversify production risk by always maintaining equal stocks of the good at home and abroad. That is, it would be optimal to regulate the processes for $K(t)$ and $K^*(t)$ continuously so that $K(t) = K^*(t)$. However, in the presence of the shipping cost it is not optimal to correct an imbalance in the relative quantity of the good at home and abroad at each instant. This is because the benefits obtained by exporting or importing the good may be less than the cost incurred in doing so. Dumas (1988) and Davis and Norman (1990) have shown that, in the presence of transfer costs, there will exist a region of the state space within which it is optimal not to export or import the good. With proportional transfer costs,[18] this no-trade region is a cone in $(K, K^*)$ space, delineated by two rays from the origin. Further, given that

[17]The theory of the instantaneous control of a Brownian motion, first examined in Benes *et al.* (1980), is developed in Harrison and Taksar (1983) and Harrison (1985). For a rigorous treatment, see Davis and Norman (1990). For a simple exposition of the mathematical theory, see Dixit (1991) and Dumas (1991). Dixit (1992) and Pindyck (1991) provide a review of the applications of this theory to various economic problems.

[18]And with isoelastic utility; see below the description of the consumption set and the homogeneity properties of the optimization program.

the transfer costs are proportional, the optimal shipping policy is to trade the infinitesimal amount of physical goods just necessary to stay within the no-trade region.

## 2.2 Consumer preferences

Consumption of the domestic investor, at any instant $t$, is denoted $c(t)$ and that of the foreign investor by $c^*(t)$. Preferences of the representative consumer in the domestic country are given by an isoelastic utility function, with degree of relative risk aversion (RRA) equal to $1-\alpha$.[19] Separately, we specify the elasticity of intertemporal substitution (EIS) as being equal to $1/(1-\rho)$.[20] The distinction between these two aspects of preferences is made possible by the use of non-time additive, recursive utility proposed by Kreps and Porteus (1978), Epstein and Zin (1989) and Duffie and Epstein (1992).[21] The preferences of the foreign representative investor are similarly defined. We also assume that the representative consumers in the two countries start out with identical endowments and have the same rate of impatience, $\beta$.

Geoffard (1995) has shown, under certainty, that the objective function of an investor-consumer with the aforementioned preferences may be written in a pseudo time-additive form; he calls this Variational Utility. El Karoui, Peng and Quenez (1997) and Dumas, Uppal and Wang (1997) show that Geoffard's deterministic formulation can be extended to a stochastic setting and is then equivalent to stochastic recursive utility given in Duffie and Epstein (1992).[22] Using the results in Dumas, Uppal and Wang, we can express the objective of a single investor as follows:

$$\sup_{c} \inf_{\nu} E_t \left\{ \int_t^{\infty} x_t F(c_t, \nu_t) dt \right\}, \tag{3}$$

$$\text{s.t.} \quad : \quad dx = -\nu_t x_t dt,$$

$$\text{and} \quad : \quad \text{usual budget or feasibility constraints,}$$

[19] We impose: $\alpha < 1$.

[20] We impose: $\rho < 1$. The special case of time-additive expected lifetime utility function is obtained for $\alpha = \rho$. When utility is isolelastic, risk aversion and elasticity of intertemporal substitution are intertwined with another aspect of investor preferences: the preference for early or late resolution of uncertainty. It can be shown (see Kreps and Porteus (1978) and Skiadas (1997)) that $\rho < \alpha$ ($\rho > \alpha$) implies preference for late resolution when $\alpha < 0$ ($\alpha > 0$). In the case of preference for early resolution, the inf operator in equation (3) should be replaced by a sup operator.

[21] The use of recursive utility also allows us to compare our results with those of Obstfeld (1994).

[22] The reader must be warned that the assumptions of the theorems stated in Dumas, Uppal and Wang are not satisfied in the case of constant risk aversion and constant elasticity of intertemporal substitution which is hypothesized below as Equation (4). Hence, the existence of variational utility may be an issue. However, a recent paper by Schroder and Skiadas (1997) provides a foundation for stochastic differential utility that would be valid for the case at hand.

where $F(c,\nu)$, called the "felicity function", is in our case equal to[23]

$$F(c,\nu) = \beta\frac{c^\alpha}{\alpha}\left[-\frac{\rho-\alpha}{\alpha-\frac{\rho\nu}{\beta}}\right]^{\frac{\alpha}{\rho}-1}. \tag{4}$$

Here, $\alpha, \rho < 1, \neq 0$, and $x_t$ may be interpreted as a psychological discount factor applicable to utility. The discount factor is endogenous, however, since its dynamics are governed by the choice variable $\nu$.

## 2.3 Social choice

The equilibrium of the economy is solved for by means of the equivalent social planner's problem. The results in Dumas, Uppal and Wang (1997) show that the objective function of the central planner is equal to the sum of the individual objective functions (3). Using these results and the above formulation of the single-agent objective function allows a straightforward treatment of the welfare optimization problem by means of continuous-time, stochastic dynamic programming, which is a considerable simplification in comparison with extant methods.[24]

The central planner's optimization problem, for the times at which no shipment takes place, may be written as:

$$\begin{aligned} V(K,K^*,x,x^*) &\equiv \sup_{c,c*,w,w^*}\inf_{\nu,\nu*} E_t\left\{\int_t^\infty [xF(c,\nu)+x^*F(c^*,\nu^*)]\,dt\right\}, \quad (5)\\ \text{s.t.} &: \\ dK &= ([r+w(\mu-r)]K-c)\,dt + w\sigma K dz;\ 0\le w\le 1;\\ dK^* &= ([r+w^*(\mu-r)]K^*-c^*)\,dt + w^*\sigma K^* dz^*;\ 0\le w^*\le 1; \end{aligned}$$

[23]There exists a close correspondence between Geoffard's "felicity function" and the "aggregator function" of Duffie and Epstein (1992). Denoting the latter by $f(c,v)$, one can show that the two functions are Legendre transforms of each other:

$$F(c,\nu) = \sup_v\,[f(c,v)+\nu v]\,;$$

and:

$$f(c,v) = \inf_\nu\,[F(c,\nu)-v\nu]\,.$$

The felicity function (4) is the Legendre transform of the aggregator (with variance multiplier equal to zero) proposed by Duffie and Epstein (1992, page 367) for isoelastic, Kreps-Porteus preferences:

$$f(c,v) = \frac{\beta}{\rho}\frac{c^\rho-(\alpha v)^{\rho/\alpha}}{(\alpha v)^{\frac{\rho}{\alpha}-1}}.$$

[24]If one used the approach suggested in Duffie, Geoffard, and Skiadas (1994), one would need to solve a system of $2\times N$ differential equations, where $N$ is the number of agents. Thus, in the case of our economy, one would have to solve a system of four differential equations.

$$
\begin{aligned}
dx &= -\nu x dt, \\
dx^* &= -\nu^* x^* dt.
\end{aligned}
$$

with initial conditions $K(0) = K_0, K^*(0) = K_0^*, x(0) = x^*(0) = 1$.

The Hamilton-Jacobi-Bellman differential equation for the value function $V$ is:

$$
\begin{aligned}
0 = \sup_{c,c*,w,w^*} \inf_{\nu,\nu*} & \{xF(c,\nu) + x^*F(c^*,\nu^*) - V_x \nu x - V_{x^*}\nu^* x^* \\
& + V_K \left( [r + w(\mu - r)] K - c \right) + V_{K^*} \left( [r + w^*(\mu - r)] K^* - c^* \right) \\
& + \frac{1}{2}\sigma^2 V_{KK} w^2 K^2 + \frac{1}{2}\sigma^2 V_{K^*K^*} (w^*K^*)^2 + \sigma^2 \eta V_{KK^*} wKw^*K^* \Big\}.
\end{aligned} \tag{6}
$$

subject to boundary conditions at the points where shipment of goods takes place: $V_{K^*} = (1-\tau)V_K$ at one end, and: $(1-\tau)V_{K^*} = V_K$ at the other (smooth-pasting boundary conditions; see Dumas (1992)).

After substituting in the first-order conditions for the $\inf_{\nu,\nu^*}$ problem, we get:

$$
\begin{aligned}
0 = \sup_{c,c*,w,w^*} & \{xf(c, V_x) + x^* f(c^*, V_{x^*}) \\
& + V_K \left( [r + w(\mu - r)] K - c \right) + V_{K^*} \left( [r + w^*(\mu - r)] K^* - c^* \right) \\
& + \frac{1}{2}\sigma^2 V_{KK} w^2 K^2 + \frac{1}{2}\sigma^2 V_{K^*K^*} (w^*K^*)^2 + \sigma^2 \eta V_{KK^*} wKw^*K^* \Big\}.
\end{aligned} \tag{7}
$$

where $f$ is, as we recall, the aggregator function of the Epstein-Utility specification. It is to be noted how the approach adopted here provides an elegant and straightforward derivation of the Bellman equation in terms of the aggregator $f$.

After substituting in the definition of the aggregator function $f$, the first-order conditions for all decision variables except the portfolio choices,[25] the differential equation for the value function $V$ is:[26]

$$
\begin{aligned}
0 = \sup_{0 \le w,w^* \le 1} & \left\{ (\frac{1}{\rho} - 1) V_K \left[ \frac{V_K (\alpha V_x)^{\frac{\rho}{\alpha} - 1}}{x\beta} \right]^{\frac{1}{\rho - 1}} \right. \\
& + (\frac{1}{\rho} - 1) V_{K^*} \left[ \frac{V_{K^*} (\alpha V_{x^*})^{\frac{\rho}{\alpha} - 1}}{x^*\beta} \right]^{\frac{1}{\rho - 1}} - \frac{\beta}{\rho} \alpha V \\
& + V_K [r + w(\mu - r)] K + V_{K^*} [r + w^*(\mu - r)] K^* \\
& \left. + \frac{1}{2}\sigma^2 V_{KK} (wK)^2 + \frac{1}{2}\sigma^2 V_{K^*K^*} (w^*K^*)^2 + \sigma^2 \eta V_{KK^*} (wK)(w^*K^*) \right\}.
\end{aligned} \tag{8}
$$

---

[25] $c = \left[ \frac{V_K (\alpha V_x)^{\frac{\rho}{\alpha} - 1}}{x\beta} \right]^{\frac{1}{\rho - 1}}$; $c^* = \left[ \frac{V_{K^*} (\alpha V_{x^*})^{\frac{\rho}{\alpha} - 1}}{x^*\beta} \right]^{\frac{1}{\rho - 1}}$

[26] An interpretation of this equation is provided below in (13).

Our task is to solve, for the unknown function $V$, this four-dimensional partial differential equation which, furthermore, incorporates a constrained optimization problem. Examining (5), it is clear that $V$ has some homogeneity properties: doubling $K$ and $K^*$ will double $c$ and $c^*$ and multiply $V$ by $2^\alpha$ while doubling $x$ and $x^*$ will leave $\nu$ and $\nu^*$ unchanged and double $V$. Whenever $V(K, K^*, x, x^*)$ is a solution to the optimization problem, so are $\zeta^{-\alpha}V(\zeta K, \zeta K^*, x, x^*)$ and $\zeta^{-1}V(K, K^*, \zeta x, \zeta x^*)$ for any $\zeta$. We will take advantage of these properties in solving for the value function, $V$.

# 3 Benefits to financial market integration

The levels of welfare and growth achieved in the above economy can be calculated. As shown in Obstfeld (1994), the calculations can be done analytically for the extreme cases of perfect integration ($\tau = 0$) and total autarky ($\tau = 1$). The intermediate case of international commodity markets with frictions ($0 < \tau < 1$) is studied in the next section by solving numerically the welfare optimization problem (5).

## 3.1 Effect of integration on welfare and growth

Consider the two extreme cases in turn. First, *under autarky* ($\tau = 1$), the solution to problem (5) exists provided that:

$$\xi^{aut} > 0,$$

where:

$$\begin{aligned} \frac{c}{K} &= \xi^{aut} \equiv \frac{\rho}{1-\rho}\left[\frac{\beta}{\rho} - r - w^{aut}(\mu - r) + \frac{1}{2}(1-\alpha)\left(w^{aut}\right)^2\sigma^2\right], \\ w^{aut} &= \min\left[\frac{\mu - r}{(1-\alpha)\sigma^2}, 1\right]. \end{aligned} \tag{9}$$

It is noteworthy that risk aversion, $1 - \alpha$, only plays a role in the decision $w$ to invest in risky *vs.* riskless production whereas the elasticity of intertemporal substitution $1/(1-\rho)$ only affects the consumption decision $c/K$. After lengthy but simple calculations, the solution for the unknown function $V$ is found to be:

$$V^{aut}(K, K^*, x, x^*) = \kappa^{aut}\left[xK^\alpha + x^*(K^*)^\alpha\right], \tag{10}$$

where:

$$\kappa^{aut} = \frac{1}{\alpha}\left[\left(\frac{1}{\beta}\right)^{\frac{1}{1-\rho}}\xi^{aut}\right]^{\frac{\alpha(\rho-1)}{\rho}}.$$

*Under complete integration* ($\tau = 0$), the solution exists provided that:

$$\xi^{opt} > 0,$$

where:

$$\begin{aligned} \frac{c + c^*}{K + K^*} &= \xi^{opt} \equiv \frac{\rho}{1-\rho}\left[\frac{\beta}{\rho} - r - w^{opt}(\mu - r) + \frac{1}{4}(1-\alpha)\sigma^2 \left(w^{opt}\right)^2 (1+\eta)\right], \quad (11) \\ w^{opt} &= \min\left[\frac{2}{1+\eta}\frac{\mu - r}{(1-\alpha)\sigma^2}, 1\right]. \end{aligned}$$

The remark above, concerning the separate roles of risk aversion and elasticity of intertemporal substitution, also applies here. The solution for $V$ is found to be:[27]

$$V^{opt}(K, K^*, x, x^*) = \kappa^{opt}(x + x^*)(K + K^*)^{\alpha}, \quad (12)$$

where:

$$\kappa^{opt} = \frac{1}{\alpha}\left[\frac{1}{2}\left(\frac{2}{\beta}\right)^{\frac{1}{1-\rho}} \xi^{opt}\right]^{\frac{\alpha(\rho-1)}{\rho}}.$$

The levels of welfare and the rates of growth achieved under autarky and integration can now be compared. Our measure of the welfare cost, i.e. compensating variation in initial endowments, is the same as that used in Obstfeld (1994). The compensating variation is the increase, $\pi$, in initial endowments ($K$ and $K^*$) that is required in the economy with completely segmented real sectors so that the lifetime expected utility in this economy is increased to the same level as that in the economy with integrated markets.[28] That is, the fraction $\pi$ is one that makes $V^{opt}[K, K^*, x, x^*] = V^{aut}[(1+\pi)K, (1+\pi)K^*, x, x^*]$. At the point $K = K^*$ and $x = x^*$, this compensating difference is given by:

$$\pi = \left[\frac{\xi^{opt}}{\xi^{aut}}\right]^{-\frac{1-\rho}{\rho}} - 1.$$

Since in the two extreme cases ($\tau = 0$ and $\tau = 1$), the ratio of consumption over wealth is a constant, growth rates of consumption and wealth are equal and their values corresponding to the two extreme situations are given by:

$$g^{aut} = w^{aut}(\mu - r) + r - \xi^{aut}$$

$$g^{opt} = w^{opt}(\mu - r) + r - \xi^{opt}.$$

[27] Under integration, perfect physical balance is maintained: $K = K^*$, at all times.

[28] The calculation is done for the case in which the countries under autarky have equal capital stocks: $K = K^*$.

## 3.2 The comparative statics of savings behavior

Before we proceed, we digress briefly to recall some basic principles concerning the comparative statics of savings behavior. These are useful in interpreting the comparison of the two extreme situations of autarky and integration: integration permits diversification of risk and is, therefore, equivalent to a fall in the level of risk of an investor's portfolio.

Consider a simple two-period, deterministic consumption problem and consider a shift in the rate of interest. The usual Slutzky analysis of income and substitution effects indicates that a rise in the rate of interest unambiguously raises period-two consumption. It reduces period-one consumption (or increases savings) if and only if the substitution effect dominates the income effect, which happens when the intertemporal elasticity of substitution is greater than 1. The increase in period 2 consumption from a rise in the rate of interest is large enough so that, irrespective of the effect on period 1 consumption, the growth rate of consumption between periods 1 and 2 is raised unambiguously.

The extension of these basic results to the case of uncertainty is straightforward when the assumption of recursive, isoelastic utility function is made, for it allows, in many cases, a simple separation of the consumer's attitudes towards risk and towards time. A consumer endowed with this form of utility treats a risky return exactly as he would a riskless one equal to the risky return's certainty equivalent value.[29] Under that assumption, the certainty equivalent of a random portfolio return $\widetilde{R}$ is equal to: $E\left[\widetilde{R}\right] - \frac{1}{2}(1-\alpha)\text{var}\left(\widetilde{R}\right)$.[30] As far as savings behavior is concerned, a reduction in portfolio risk is then equivalent to a rise in expected return; only the certainty equivalent matters. Hence, we can repeat what has been said for the case of certainty: a rise in the expected rate of return on a person's portfolio (or a drop in its variance) unambiguously raises the *certainty equivalent of* period-two consumption. It reduces period-one consumption (or increases savings) if and only if the substitution effect dominates the income effect, which happens when the elasticity of intertemporal substitution is greater than 1 ($\rho > 0$).

---

[29]See Selden (1979). The Kreps-Porteus recursive utility function, which we use here, is a special case of the Ordinal-Certainty-Equivalent (OCE) utility proposed by Selden. Epstein and Zin (1989) have criticized OCE utility functions for generally being time inconsistent, the special case of recursive utilities being the only one leading to time consistency.

[30]Under autarky the certainty equivalent rate of return in both countries is: $r + w(\mu - r) - \frac{1}{2}(1-\alpha)\left(w^{aut}\right)^2\sigma^2$. See (9). Under integration, given that capital is allocated equally to the two countries, the combined certainty equivalent rate of return is: $r + w^{opt}(\mu - r) - \frac{1}{4}(1-\alpha)\sigma^2\left(w^{opt}\right)^2(1+\eta)$. See (11).

As for the case of certainty, the *certainty equivalent of* the growth rate of consumption between periods 1 and 2, defined as $(1/dt)\{E[dc/c] - \frac{1}{2}(1-\alpha)[dc/c]^2\} = (1/dt)\{E[dc/c]\} - \frac{1}{2}(1-\alpha)\text{var}(\widetilde{R})$, is raised unambiguously by a rise in the expected rate of return (or a drop in risk) of an investor's portfolio.[31]

However, the *expected rate of growth of consumption* between the two periods, which rises unambiguously with a rise in the expected rate of return, may rise or fall with a drop in risk, depending on whether or not the fall in the term $\frac{1}{2}(1-\alpha)\text{var}\left(\widetilde{R}\right)$, which must be added back, overrides the rise in the certainty equivalent rate of return. When the investor combines optimally a risky and a riskless asset and, in the absence of binding constraints, it is found that a drop in risk induces a portfolio shift, then the expected rate of growth of consumption is unambiguously increased. In a corner situation, such as $w = 1$, there is no portfolio composition effect and it turns out that the expected rate of growth is increased if and only if the intertemporal elasticity of substitution is greater than 1 ($\rho > 0$).[32]

Given the complicating effect of price fluctuations, these same basic principles cannot be invoked in the general case of partial goods mobility. To illustrate this point, consider the following equation:

$$\frac{V_K c + V_{K^*} c^*}{V_K K + V_{K^*} K^*} = \frac{1}{1-\rho}\left[\beta - \rho\frac{\text{drift of } V \text{ at } c = c^* = 0 \text{ and at } \nu = \nu^* = 0}{V_K K + V_{K^*} K^*}\right] \tag{13}$$

which may be derived from (8) by a division by $\alpha V = V_K K + V_{K^*} K^*$. The left-hand side of this equation is the ratio of consumption to wealth where consumption and capital are valued at marginal utility. The coefficient $\frac{1}{1-\rho}$ is, of course, the elasticity of intertemporal substitution (EIS). The terms within the square bracket on the right-hand side represent the excess of the rate of impatience $\beta$ over $\rho$ times the "certainty equivalent rate of return" from the production side defined as: $\frac{\text{drift of } V \text{ at } c=c^*=0 \text{ and } \nu=\nu^*=0}{V_K K + V_{K^*} K^*}$.

Such an equation relating the rate of consumption out of wealth and the anticipated returns is a standard feature of constant-EIS utility functions. But we see that, generally, the definition of the certainty equivalent rate of return, involves the value function $V$ and is not simply

[31] That is,

$$CEg^{aut} \equiv w^{aut}(\mu - r) + r - \frac{1}{2}(1-\alpha)\left(w^{aut}\right)^2\sigma^2 - \xi^{aut}.$$
$$CEg^{opt} \equiv w^{opt}(\mu - r) + r - \frac{1}{4}(1-\alpha)\sigma^2\left(w^{opt}\right)^2(1+\eta) - \xi^{opt}.$$

[32] See Devereux and Smith (1994).

based on the degree of risk aversion, as it would be if the certainty equivalent of a risky return $\widetilde{R}$ were given, for instance, by: $\mathrm{E}\left(\widetilde{R}\right) - \frac{1}{2}(1-\alpha)\mathrm{var}\left(\widetilde{R}\right)$. Only in special cases will the general definition simplify to that form.[33] The complexity of the general case arises from the endogenous, state-dependent fluctuations of the price of capital located in one country relative to the price of capital located in the other: $p(K, K^*, x, x^*) = \frac{V_{K^*}}{V_K}$. Thus, in Section 4, we shall use numerical methods to analyze the case with shipping costs $0 < \tau < 1$.

### 3.3 A calibration with imperfect mobility of goods

Obstfeld (1994) provides several example situations to illustrate the role played by the increased willingness to bear risk under integration. His examples are meant only to highlight a number of analytical properties of the economy and not to provide a realistic assessment of the welfare gains brought about by integration, perfect or imperfect. He also provides two numerical calibrations representing the quantitative situation of several regions of the world. His first calibration is based on observed consumption behavior, the second on observed stock returns.

We unify our interpretation of the effects at play by considering a single calibration accompanied by some comparative statics exercises. We report the effect of shipping costs for Obstfeld's calibration based on stock market data.[34] Given that our model has been constructed for a symmetric situation, we amend his numbers slightly to estimate welfare gains as they would be approximately when one integrates Western Europe and Northern America: $r = 0.02, \mu = 0.08, \sigma = 0.20$, and $\eta = 0.554$.

The quantities corresponding to this exercise are reported in Table 1. As explained by Obstfeld, the welfare gain does not simply arise from improved risk bearing; it also arises from the fact that investors' willingness to invest in risky projects rises from 0.375 under autarky to 0.4826 under integration. In total, the welfare impact, $\pi$, of integration is quite large: 13%.[35] Thanks to the increased willingness to bear risk with integration, the expected growth rate of consumption rises from 1.69% to 2.17%.

[33] Autarky and full integration are cases in which the relative price of capital in the two countries is either irrelevant or constant.

[34] Because of some numerical difficulties that we have encountered, however, risk aversion is kept at 4, somewhat below Obstfeld's number which was 6.

[35] In some cases, Obstfeld found much higher welfare gains. The reason for the difference is that Obstfeld is able to accumulate the benefits of integrating many countries whereas we are limited to two countries only.

# 4 The general case with frictions

We now solve the optimization problem (5) for $0 < \tau < 1$. The only parameter that we need to specify, in addition to the ones specified in the previous section, is the transfer cost ($\tau$). On the basis of the evidence of Section 1 above, we aim to be conservative by considering values for $\tau$ between 0% and 25%. The appendix explains how one exploits the homogeneity, and then solves the differential equation numerically by successive interpolations over two variables: $\omega = \frac{K}{K+K^*}$ and $\lambda = \frac{x}{x+x^*}$.

The goal of our undertaking is to determine how rapidly welfare and growth decrease as one gradually introduces sand in the gear, i.e. lets transfer costs impede the flow of goods between countries. Is it or is it not the case that moderate transfer costs would drastically reduce the welfare and growth gains of integration? We answer the question at the point of equal weights for the two countries, both in terms of welfare weights: $\lambda = 0.5$ and in terms of physical balance: $\omega = 0.5$.

Table 2 gives the drop in the welfare gain in the numerical calibration, as one varies the proportional shipping costs. The drop is not negligible. Nonetheless, for seemingly large transfer costs of 25%, the welfare gain over and above autarky is still as large as 10.77%, to be compared with the 13% gain from full integration. Evidently, as the flow of international physical transfers has been optimized, a compromise has been found between reaping the benefits of international diversification (arising both from risk sharing and from optimal investment) and saving on shipping costs. The flow of physical transfers has been sufficiently slowed down to economize on transfer costs and preserve the larger part of the welfare gains, and this has been made possible because of the presence of financial markets. The comparative dynamics exercise conducted in the next section will verify that this conclusion is robust to changes in parameter values.

In Table 2, we report also the effect of shipping costs on the expected growth rate of consumption. Even with a 25% transfer cost, the expected growth rate of consumption is 1.99%, which represents a reduction by only a third of the gain arising from perfect integration.

Shipping costs act on growth in two interrelated ways. First, they represent, of course, a deadweight loss to society. For this reason alone, a reduction of shipping costs enhances growth. But, the size of this impact depends on shipping activity which is dampened by the costs. Second, as shipping activity is reduced, the ability to diversify is hindered and, with it, the willingness to undertake risky projects. We saw that an effective reduction of the willingness to undertake risky projects produces a drop in expected growth. We illustrate these two effects by means of a comparative dynamic exercise.

## 5 Comparative dynamics

In this section, we conduct a comparative analysis of the equilibrium under varying parameters values. For the base case, we consider the calibration described above in which the portfolio weights on the risky technology are less than unity. These results are reported in Table 3.

First, consider in Panel A of the table, a variation in the level of output risk, $\sigma$ (the size of productivity shocks) from 0.1 to 0.2 (the base case). Row #A1 of Table 3 shows that an increase in output risk raises the welfare gain of imperfect integration relative to autarky, for as long as the investment in the risky industry (the portfolio weight, row #A10) is constrained at 1, but decreases it when the weight becomes free to move. This is exactly as under perfect integration (row #A2).

The expected growth rate is, of course, reduced by the presence of shipments costs (compare rows #A3, A4 and A5). This is true for two reasons. First, shipment costs drain resources and their mere existence would reduce growth. The second reason is better understood if we, first, examine the effect of the level of risk on the opening of the cone (row #A12): a higher level of risk causes the cone of no shipment to widen. This is because a higher level of risk, for an unchanged position of the cone, would imply more frequent shipments and excessive costs.[36] But as the cone is widened, the benefits of diversification are reduced which induces agents to invest less in the risky technologies than they would under perfect integration (compare rows #A10 and A11).[37] Because of this physical investment choice, the expected growth rate is further reduced relative to perfect integration.

Returning to the effect of risk on expected growth (row #A4), we find here again a non-monotonic variation, as under perfect integration (row #A5). The maximum would generally occur at a different point than under full integration because the portfolio weights are lower, so that the $w \leq 1$ constraint becomes operative at a different level of risk. Finally, the rate of

[36]This effect overcomes the desire to tighten the cone arising from increased risk which enhances the benefits of good diversification.

[37]That is, whenever the weight is not bumping against the 100% constraint.

consumption out of wealth drops monotonically as risk increases, because of the precautionary motive (row #A7). This is true equally with or without shipping costs (rows #A6, A7 and #A8).

Consider next a variation (Panel B of the table) in the degree of risk aversion, $1-\alpha$, from 0.5 to 4 (the base case). Checking back on the equations corresponding to the perfect integration and autarky situations (in Section 3.2), it is easy to verify that in these two extreme cases risk and risk aversion played a role only via the compound parameter $(1-\alpha) \times \sigma^2$. Under imperfect integration, however, an increase in risk aversion does not produce the same effect as an increase in risk. In fact, it causes the cone of no shipment to tighten (row #B12) for as long as the investment into risky technology is constrained. This is because increased risk aversion enhances the benefits of good diversification. Admittedly, an increase in risk has this effect as well but an increase in risk, as we saw, also has the effect in increasing the frequency of shipments, which risk aversion does not. However, whenever the investment into risky technology is not constrained (as is the case for sufficiently high risk aversion), an increase in risk aversion widens the cone (row #B12 again). In short, because of the portfolio-shift effect, the effect of risk aversion on the cone is not monotonic. Nonetheless, the effects of risk aversion on the welfare gain, the expected growth rate and the consumption rate (rows #B1 to B8) are qualitatively similar to those of risk and the direction of these effects depends entirely on whether the portfolio weight assigned to risky projects is currently constrained or not (row #B9 to B11)

Finally, in Panels C and D of the table, we turn to the elasticity of intertemporal substitution (EIS), varying from 0.1 to 1.2 (0.5 being the base case), for two values of risk aversion: one low and one high. The lower risk aversion of 1.5 is introduced in order to examine a situation in which the $w \leq 1$ constraint is binding.[38] All the effects of EIS are monotonic and they are under imperfect integration as under perfect integration and under autarky: welfare and growth increase with EIS and the rate of consumption declines. As one increases EIS, society is more willing to displace consumption between different points in time. As a result, the cone of no shipment widens slightly (rows #C12 and D12). The EIS has very little effect on the share invested into the risky technologies (row #C10). This is to be contrasted with the perfect integration and autarky cases in which it has *no* impact whatsoever on it, this being

[38] This allows us to verify whether the effect of EIS differs depending on whether the constraint is binding or not. We find that it does not.

a manifestation of the separation between EIS which affects intertemporal choices and risk aversion which affects attitude to risk. In the presence of frictions, the separation is somewhat lost: the widening of the cone which is induced by an increase in EIS, is accompanied by a small increase in the willingness to invest in risky projects.

## 6 Conclusion

In this article, we have studied, in a general equilibrium setting, the impact on welfare and growth of the imperfect mobility of goods. We have modeled a two-country, one-good economy in which financial markets are complete and frictionless but the markets for real goods are partially segmented so that the single good exists really in two versions, differentiated by location. The imperfection of the goods market is modeled by introducing a proportional cost for exporting goods from one country to another. In each country, two technologies are present: one risky and the other riskless.

We have shown that the welfare gain from financial market integration is not drastically reduced by the presence of imperfect goods mobility, modeled as a cost of transferring goods from one country to the other. Similarly, the quantitative impact of goods market imperfection on growth is not very large. While the welfare gain is not monotonic with investors' risk aversion and the aggregate volatility of output growth, qualitatively speaking, most of the effects obtained under perfect goods mobility are preserved under imperfect mobility. This is because the *optimal timing* of physical transfers mitigates the effect of transfer costs. At times when it is not optimal to transfer goods, foreigners do not collect their payoffs. The financial market makes it possible for them to simply accumulate claims which will be redeemed later.

Thus, financial market integration may be a worthwhile goal to pursue even at a time when goods market integration has not been achieved. That is, even though trade agreements such as GATT and NAFTA have not removed all barriers to trade in goods and services, the flow of financial capital could still raise economic welfare. Furthermore, if financial markets behave in accordance with our model and mitigate the imperfections in the goods market, the welfare gains to be reaped by bringing down tariffs from their current level, which is about 6% among developed countries, seem trivial.

Admittedly, the setting that we have chosen, which featured a risky constant-return-to-scale production economy with a single good which is differentiated only by location and which is investible, was meant to give the central role to the asset market. If all goods were perishable, it is clear that financial markets would not provide the same benefits that we have identified. Nonetheless,, even in a pure-exchange economy with only perishable goods, endowments shocks with sufficient persistence would still give a *raison d'être* to inter-country state-contingent consumption loans. In future research, other settings ought to be investigated in an attempt to measure the relative importance of welfare benefits arising from goods market openness and asset market openness, depending on the nature and number of the goods, on the nature of the stochastic processes for endowments and/or productivity shocks and on the number and degree of asymmetry of the countries making up the world.

# A Characterizing the planner's problem

In this appendix, we modify the differential equation that characterizes the problem in (5), and show how one can use the homogeneity of the problem to reduce the state space. Then, we describe the boundary conditions for this differential equation. The numerical procedure used to solve this problem is described in Appendix B.

## A.1 Deriving the general differential equation for $H(\omega, \lambda; \tau)$

After substituting in the first-order conditions for all six decision variables, the differential equation for the value function $V$ is:

$$\begin{aligned} 0 \; = \; & (\frac{1}{\rho} - 1) V_K \left[ \frac{V_K (\alpha V_x)^{\frac{\rho}{\alpha} - 1}}{x\beta} \right]^{\frac{1}{\rho - 1}} - x \frac{\beta}{\rho} \alpha V_x \\ & + (\frac{1}{\rho} - 1) V_{K^*} \left[ \frac{V_{K^*} (\alpha V_{x^*})^{\frac{\rho}{\alpha} - 1}}{x^* \beta} \right]^{\frac{1}{\rho - 1}} - x^* \frac{\beta}{\rho} \alpha V_{x^*} \\ & + V_K \left[ r + w \left( \mu - r \right) \right] K + V_{K^*} \left[ r + w^* \left( \mu - r \right) \right] K^* \\ & + \frac{1}{2} \sigma^2 V_{KK} \left( wK \right)^2 + \frac{1}{2} \sigma^2 V_{K^*K^*} \left( w^* K^* \right)^2 + \sigma^2 \eta V_{KK^*} \left( wK \right) \left( w^* K^* \right) . \end{aligned} \tag{A1}$$

where the portfolio weights are optimized under the constraint that they must lie between 0 and 1:

$$\{w, w^*\} \equiv \begin{cases} \{0,1\} & \text{if } w_1 < 0 & \text{and } w_0^* > 1 \\ \{w_1, 1\} & \text{if } 0 < w_1 < 1 & \text{and } w_\cap^* > 1 \\ \{1,1\} & \text{if } w_1 > 1 & \text{and } w_1^* > 1 \\ \{0, w_0^*\} & \text{if } w_\cap < 0 & \text{and } 0 < w_0^* < 1 \\ \{w_\cap, w_\cap^*\} & \text{if } 0 < w_\cap < 1 & \text{and } 0 < w_\cap^* < 1 \\ \{1, w_1^*\} & \text{if } w_\cap > 1 & \text{and } 0 < w_1^* < 1 \\ \{0,0\} & \text{if } w_0 < 0 & \text{and } w_0^* < 0 \\ \{w_0, 0\} & \text{if } 0 < w_0 < 1 & \text{and } w_\cap^* < 0 \\ \{1,0\} & \text{if } w_0 > 1 & \text{and } w_1^* < 0 \end{cases} \tag{A2}$$

and where:

$$\begin{aligned} w_\cap (K, K^*, x, x^*) \; & \equiv \; -\frac{\mu - r}{K\sigma^2} \frac{V_K V_{K^*K^*} - V_{K^*} V_{KK^*} \eta}{V_{KK} V_{K^*K^*} - (V_{KK^*} \eta)^2} \\ w_1 (K, K^*, x, x^*) \; & \equiv \; -\frac{V_K (\mu - r) + \sigma^2 \eta V_{KK^*} K^*}{\sigma^2 V_{KK} K} \\ w_0 (K, K^*, x, x^*) \; & \equiv \; -\frac{V_K (\mu - r)}{\sigma^2 V_{KK} K} \end{aligned}$$

while $w_\cap^*, w_1^*$ and $w_0^*$ are defined analogously.

Taking advantage of the homogeneity properties of $V$, we define the function $H$:

$$V(K, K^*, x, x^*) \equiv (K+K^*)^{\alpha}(x+x^*)\,H(\omega,\lambda), \qquad \text{(A3)}$$
$$\textit{where} \; : \; \omega \equiv \frac{K}{K+K^*},\; \lambda \equiv \frac{x}{x+x^*}.$$

Substituting this expression into (A1), we get a differential equation for $H$:

$$\begin{aligned} 0 &= (\frac{1}{\rho}-1)\,[\alpha H + (1-\omega)H_\omega]^{\frac{\rho}{\rho-1}} \left[\frac{(\alpha\,(H + H_\lambda(1-\lambda)))^{\frac{\rho}{\alpha}-1}}{\lambda\beta}\right]^{\frac{1}{\rho-1}} \\ &+ (\frac{1}{\rho}-1)\,[\alpha H - \omega H_\omega]^{\frac{\rho}{\rho-1}} \left[\frac{(\alpha\,(H - \lambda H_\lambda))^{\frac{\rho}{\alpha}-1}}{(1-\lambda)\beta}\right]^{\frac{1}{\rho-1}} \\ &- \frac{\beta}{\rho}\alpha H + \alpha H\left[r + \mathbf{w}'\mathbf{r} + \frac{1}{2}\mathbf{w}'\mathbf{\Sigma}\mathbf{w}\right], \end{aligned}$$

where:

$$\begin{aligned} \mathbf{r} &\equiv \begin{bmatrix} (1+(1-\omega)H_\omega/(\alpha H)) \times (\mu - r) \\ (1-\omega H_\omega/(\alpha H)) \times (\mu - r) \end{bmatrix}. \\ \mathbf{w} &\equiv \begin{bmatrix} w\omega \\ w^*(1-\omega) \end{bmatrix} \end{aligned} \qquad \text{(A4)}$$

Here, $\{w, w^*\}$ is as defined in (A2), but with

$$\begin{aligned} \begin{bmatrix} w_\cap \omega \\ w_\cap^*(1-\omega) \end{bmatrix} &= -\mathbf{\Sigma}^{-1}\mathbf{r}, \\ \begin{bmatrix} w_1 \omega \\ w_1^*(1-\omega) \end{bmatrix} &= \begin{bmatrix} -\frac{\mu - r + \Sigma_{12}(1-\omega)}{\Sigma_{11}} \\ -\frac{\mu - r + \Sigma_{21}\omega}{\Sigma_{22}} \end{bmatrix}, \\ \begin{bmatrix} w_0 \omega \\ w_0^*(1-\omega) \end{bmatrix} &= \begin{bmatrix} -\frac{\mu - r}{\Sigma_{11}} \\ -\frac{\mu - r}{\Sigma_{22}} \end{bmatrix}, \end{aligned}$$

where

$$\begin{aligned} \mathbf{\Sigma} &= \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{12} & \Sigma_{22} \end{bmatrix}, \\ \Sigma_{11} &= \sigma^2(\alpha-1)\left[1 + 2\,(1-\omega)\,H_\omega/\,(\alpha H) + (1-\omega)^2\,H_{\omega\omega}/\,(\alpha(\alpha-1)H)\right] \\ \Sigma_{12} &= \sigma^2\eta(\alpha-1)\left[1 + (1-2\omega)H_\omega/\,(\alpha H) - \omega(1-\omega)H_{\omega\omega}/\,(\alpha(\alpha-1)H)\right] \\ \Sigma_{22} &= \sigma^2(\alpha-1)\left[1 - 2\omega H_\omega/\,(\alpha H) + \omega^2 H_{\omega\omega}/\,(\alpha(\alpha-1)H)\right]. \end{aligned}$$

Finally, dividing through by $\alpha H$:

$$0 = (\frac{1}{\rho}-1)\left[1 + (1-\omega)\frac{H_\omega}{\alpha H}\right]^{\frac{\rho}{\rho-1}} (\alpha H)^{\frac{\rho/\alpha}{\rho-1}} \left[\frac{\left(1 + (1-\lambda)\frac{H_\lambda}{H}\right)^{\frac{\rho}{\alpha}-1}}{\lambda\beta}\right]^{\frac{1}{\rho-1}}$$

$$+ (\frac{1}{\rho} - 1)\left[1 - \omega\frac{H_\omega}{\alpha H}\right]^{\frac{\rho}{\rho-1}} (\alpha H)^{\frac{\rho/\alpha}{\rho-1}} \left[\frac{(1-\lambda\frac{H_\lambda}{H})^{\frac{\rho}{\alpha}-1}}{(1-\lambda)\beta}\right]^{\frac{1}{\rho-1}}$$

$$- \frac{\beta}{\rho} + r + \mathbf{w}'\mathbf{r} + \frac{1}{2}\mathbf{w}'\mathbf{\Sigma}\mathbf{w}, \tag{A5}$$

where $\mathbf{w}$ is given by (A4).

## A.2 Deriving the boundary conditions for the PDE for $H(\omega, \lambda; \tau)$

We now take care of shipments which occur at the lower edge, $\omega = \underline{\omega}$, and the upper edge, $\omega = \overline{\omega}$, of the cone of no action.

We start by obtaining the boundary conditions for $H(\omega, \lambda)$ at $\underline{\omega}$. That is, we wish to determine the level of the value function at the lower edge, $\omega = \underline{\omega}$, where: $V_{K^*} = (1-\tau)V_K$ (smooth-pasting boundary condition; see Dumas (1992)). At that point, the most convenient way of imposing the boundary condition is to write the value function locally as:

$$H(\underline{\omega}, \lambda) = \Gamma(\underline{\omega}, \lambda)[\underline{\omega} + (1-\tau)(1-\underline{\omega})]^\alpha, \tag{A6}$$

implying that,

$$H_\omega(\underline{\omega}, \lambda) = \alpha H \frac{\tau}{[\underline{\omega} + (1-\tau)(1-\underline{\omega})]}, \tag{A7}$$

$$H_{\omega\omega}(\underline{\omega}, \lambda) = \alpha(\alpha-1) H \frac{\tau^2}{[\underline{\omega} + (1-\tau)(1-\underline{\omega})]^2}. \tag{A8}$$

To use (A6) and (A7) as boundary conditions, we need to determine $\Gamma(\underline{\omega}, \lambda)$. This can be done by substituting the expressions for $H_\omega$ and $H_{\omega\omega}$ into (A5), the PDE for $H(\omega, \lambda)$. Doing this gives,

$$\begin{aligned}
0 = {} & \left(\frac{1-\rho}{\rho}\right)\left[1 + \frac{\tau(1-\omega)}{[\underline{\omega} + (1-\tau)(1-\underline{\omega})]}\right]^{\frac{\rho}{\rho-1}} \left(\alpha\Gamma(\underline{\omega}, \lambda)\left[\underline{\omega} + (1-\tau)(1-\underline{\omega})\right]^\alpha\right)^{\frac{\rho/\alpha}{\rho-1}} \\
& \times (\lambda\beta)^{\frac{1}{1-\rho}} \left(\left[1 + \frac{(1-\lambda)H_\lambda}{\Gamma(\underline{\omega}, \lambda)[\underline{\omega} + (1-\tau)(1-\underline{\omega})]^\alpha}\right]^{\frac{\rho}{\alpha}-1}\right)^{\frac{1}{\rho-1}} \\
& + \left(\frac{1-\rho}{\rho}\right)\left[1 - \frac{\tau\omega}{[\underline{\omega} + (1-\tau)(1-\underline{\omega})]}\right]^{\frac{\rho}{\rho-1}} \left(\alpha\Gamma(\underline{\omega}, \lambda)\left[\underline{\omega} + (1-\tau)(1-\underline{\omega})\right]^\alpha\right)^{\frac{\rho/\alpha}{\rho-1}} \\
& \times ((1-\lambda)\beta)^{\frac{1}{1-\rho}} \left(\left[1 - \frac{\lambda H_\lambda}{\Gamma(\underline{\omega}, \lambda)[\underline{\omega} + (1-\tau)(1-\underline{\omega})]^\alpha}\right]^{\frac{\rho}{\alpha}-1}\right)^{\frac{1}{\rho-1}} \\
& - \frac{\beta}{\rho} + r + \mathbf{w}'\mathbf{r} + \frac{1}{2}\mathbf{w}'\mathbf{\Sigma}\mathbf{w}.
\end{aligned}$$

Noting the following two simplifications,

$$\left[1 + (1-\omega)\frac{\tau}{[\underline{\omega} + (1-\tau)(1-\underline{\omega})]}\right] = \frac{1}{[\underline{\omega} + (1-\tau)(1-\underline{\omega})]}, \tag{A9}$$

$$\left[1 - \omega\frac{\tau}{[\underline{\omega} + (1-\tau)(1-\underline{\omega})]}\right] = \frac{1-\tau}{[\underline{\omega} + (1-\tau)(1-\underline{\omega})]}, \tag{A10}$$

and collecting terms, allows us to write the differential equation as:

$$\begin{aligned} 0 = & \left(\frac{1-\rho}{\rho}\right)[\alpha\Gamma(\underline{\omega},\lambda)]^{\frac{\rho}{\alpha(\rho-1)}}\beta^{\frac{1}{1-\rho}} \\ & \times \left\{ \lambda^{\frac{1}{1-\rho}}\left(\left[1 + \frac{(1-\lambda)H_\lambda}{\Gamma(\underline{\omega},\lambda)[\underline{\omega} + (1-\tau)(1-\underline{\omega})]^\alpha}\right]^{\frac{\rho}{\alpha}-1}\right)^{\frac{1}{\rho-1}} \right. \\ & \left. + (1-\tau)^{\frac{\rho}{\rho-1}}(1-\lambda)^{\frac{1}{1-\rho}}\left(\left[1 - \frac{\lambda H_\lambda}{\Gamma(\underline{\omega},\lambda)[\underline{\omega} + (1-\tau)(1-\underline{\omega})]^\alpha}\right]^{\frac{\rho}{\alpha}-1}\right)^{\frac{1}{\rho-1}} \right\} \\ & -\frac{\beta}{\rho} + r + \mathbf{w}'\mathbf{r} + \frac{1}{2}\mathbf{w}'\Sigma\mathbf{w}. \end{aligned} \tag{A11}$$

where $\mathbf{w}$ is as defined before, and is evaluated at $\underline{\omega}$. Using the results in (A6), (A7) and (A8), we also have that at $\underline{\omega}$,

$$\begin{aligned} \Sigma_{11} &= \sigma^2(\alpha-1)\left[1 + 2(1-\underline{\omega})\frac{\tau}{[\underline{\omega} + (1-\tau)(1-\underline{\omega})]} + (1-\underline{\omega})^2\frac{\tau^2}{[\underline{\omega} + (1-\tau)(1-\underline{\omega})]^2}\right], \\ \Sigma_{12} &= \eta\sigma^2(\alpha-1)\left[1 + (1-2\underline{\omega})\frac{\tau}{[\underline{\omega} + (1-\tau)(1-\underline{\omega})]} - \underline{\omega}(1-\underline{\omega})\frac{\tau^2}{[\underline{\omega} + (1-\tau)(1-\underline{\omega})]^2}\right], \\ \Sigma_{22} &= \sigma^2(\alpha-1)\left[1 - 2\underline{\omega}\frac{\tau}{[\underline{\omega} + (1-\tau)(1-\underline{\omega})]} + \underline{\omega}^2\frac{\tau^2}{[\underline{\omega} + (1-\tau)(1-\underline{\omega})]^2}\right]. \end{aligned}$$

Equation (A11) defines $\Gamma(\underline{\omega},\lambda)$ implicitly; thus, $\Gamma(\underline{\omega},\lambda)$ can be obtained by solving this equation.

To obtain the BC for $H(\omega,\lambda)$ at $\bar{\omega}$, we repeat the above analysis at the point where $\omega = \bar{\omega}$.

# B Details of the numerical procedure

We wish to obtain the solution, $H(\omega, \lambda = 0.5)$, for the partial differential equation (PDE) given in equation (A5). Note that this differential equation includes $H_\lambda(\omega,\lambda)$, a term for the first derivative with respect to $\lambda$, but no terms for the second derivative, $H_{\lambda\lambda}$, or the cross-derivative, $H_{\lambda\omega}$. We will take advantage of this by treating the partial differential equation for

$H(\omega, \lambda)$ as an ordinary differential equation, where we approximate the term $H_\lambda(\omega, \lambda)$ with the finite difference:

$$H_\lambda(\omega, \lambda = 0.5) = \frac{H(\omega, \lambda = 0.5 + \Delta) - H(\omega, \lambda = 0.5 - \Delta)}{2\Delta}.$$

We will then reduce the size of $\Delta$ until we have a good approximation of the derivative. The steps in our numerical procedure are as follows:

1. Start by choosing $\Delta = 0.5$. Thus, to approximate $H_\lambda(\omega, \lambda = 0.5)$, we need to solve the differential equations for $H(\omega, \lambda = 1)$ and $H(\omega, \lambda = 0)$. The differential equation for $H(\omega, \lambda = 1)$ is given below, in equation (B1).
   - Determine the two boundary conditions for the differential equation for $H(\omega, \lambda = 1)$ at $\underline{\omega}$ and at $\bar{\omega}$. The ones for $\underline{\omega}$ are given in equations (A6-A7), and the ones at $\bar{\omega}$ are analogous.
   - To solve the differential equation for $H(\omega, \lambda = 1)$, use the "shooting technique:" start with a guess for the initial value for $\underline{\omega}$, and use the boundary conditions in (A6-A7) at $\underline{\omega}$ to solve this differential equation. To determine whether the initial guess for $\underline{\omega}$ is optimal, use the boundary conditions at $\bar{\omega}$ to check if these are satisfied by the solution. Revise the guess for $\underline{\omega}$ until boundary conditions at both extreme points are satisfied.
2. From the solution for $H(\omega, \lambda = 1)$, infer the solution for $H(\omega, \lambda = 0)$. That is, $H(1 - \omega, \lambda = 0) = H(\omega, \lambda = 1)$. Interpolate from $H(\omega, \lambda = 1)$ and $H(\omega, \lambda = 0)$ the value of $H_\lambda(\omega, \lambda = 0.5)$; use this to solve the original differential equation, $H(\omega, \lambda = 0.5)$
3. Now decrease $\Delta$ to 0.25. Solve for $H(\omega, \lambda = 0.75)$ with the derivative $H_\lambda(\omega, \lambda = 0.75)$ approximated from the solution of the differential equations for $H(\omega, \lambda = 1)$ and $H(\omega, \lambda = 0.5)$. Continue decreasing the size of $\Delta$ until convergence.

To obtain the differential equation $H(\omega, \lambda = 1; \tau)$, start with the differential equation that we already have for $H(\omega, \lambda)$ in (A5). Setting $\lambda = 1$ yields,

$$0 = (\frac{1}{\rho} - 1)\left[1 + (1 - \omega)\frac{H_\omega}{\alpha H}\right]^{\frac{\rho}{\rho-1}} (\alpha H)^{\frac{\rho/\alpha}{\rho-1}} \left[\frac{1}{\beta}\right]^{\frac{1}{\rho-1}} - \frac{\beta}{\rho} + r + \mathbf{w}'\mathbf{r} + \frac{1}{2}\mathbf{w}'\mathbf{\Sigma}\mathbf{w}. \quad \text{(B1)}$$

To obtain the boundary conditions for $H(\omega, \lambda = 1)$ at $\underline{\omega}$ we take advantage of equations (A6-A8). To use (A6) and (A7) as boundary conditions, we need to determine the constant $\Gamma(\underline{\omega}, \lambda = 1)$. This can be done by substituting the expressions for $H_\omega$ and $H_{\omega\omega}$ into (B1), the differential equations for $H(\omega, \lambda = 1)$. Doing this gives,

$$\begin{aligned} 0 &= \left(\frac{1-\rho}{\rho}\right)\left[1 + \frac{\tau(1-\omega)}{[\underline{\omega} + (1-\tau)(1-\underline{\omega})]}\right]^{\frac{\rho}{\rho-1}} \left(\alpha\Gamma(\underline{\omega})\left[\underline{\omega} + (1-\tau)(1-\underline{\omega})\right]^{\alpha}\right)^{\frac{\rho/\alpha}{\rho-1}} \beta^{\frac{1}{1-\rho}} \\ &\quad - \frac{\beta}{\rho} + r + \mathbf{w}'\mathbf{r} + \frac{1}{2}\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}. \end{aligned}$$

Using the result in (A9), we can write the above differential equation as,

$$0 = \left(\frac{1-\rho}{\rho}\right)\beta^{\frac{1}{1-\rho}}\left(\alpha\Gamma(\underline{\omega})\right)^{\frac{\rho/\alpha}{\rho-1}} - \frac{\beta}{\rho} + r + \mathbf{w}'\mathbf{r} + \frac{1}{2}\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}.$$

Simplifying, we get

$$\Gamma(\underline{\omega}, \lambda = 1) = \frac{1}{\alpha}\left\{\beta^{\frac{1}{\rho-1}}\left(\frac{\rho}{1-\rho}\right)\left[\frac{\beta}{\rho} - r - \mathbf{w}'\mathbf{r} - \frac{1}{2}\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}\right]\right\}^{\frac{\alpha(\rho-1)}{\rho}}, \qquad \text{(B2)}$$

where $\mathbf{w}$ is evaluated at $\underline{\omega}$ as before.

To obtain the boundary conditions for $H(\omega, \lambda = 1)$ at $\bar{\omega}$, we repeat the above analysis at the point where $\omega = \bar{\omega}$.

| Parameter value | Outcome |
|---|---|
| $r = 0.02$ | $w^{aut} = 0.3750$ |
| $\mu = 0.08$ | $\xi^{aut} = 0.0266$ |
| $\sigma = 0.2$ | $g^{aut} = 0.0169$ |
| $\beta = 0.02$ | |
| $\frac{1}{1-\rho} = 0.5$ | $w^{opt} = 0.4826$ |
| $1 - \alpha = 4$ | $\xi^{opt} = 0.0272$ |
| $\eta = 0.554$ | $g^{opt} = 0.0217$ |
| | welf. gain, $\pi = 0.13$ |

Table 1: **Results and parameter values for base case calibration.** The first column of this table gives the parameter values for the calibration, while the second column reports the portfolio holdings, consumption rate, and the growth rate under autarky and perfect integration. The last number in the second column gives the welfare gain from perfect integration of commodity (and financial) markets.

| **Shipping cost** | **Welfare gain** | **Expected growth rate** |
|---|---|---|
| 0% | 0.1300 | 0.02172 |
| 1% | 0.1290 | 0.02163 |
| 2% | 0.1281 | 0.02155 |
| 3% | 0.1272 | 0.02146 |
| 4% | 0.1263 | 0.02138 |
| 5% | 0.1254 | 0.02130 |
| 10% | 0.1209 | 0.02092 |
| 15% | 0.1164 | 0.02058 |
| 20% | 0.1121 | 0.02027 |
| 25% | 0.1077 | 0.01999 |
| Autarky | 0 | 0.01690 |

Table 2: **The effect of shipping costs on welfare and growth.** Panel A of this table presents the percentage increase, $\pi$, in initial capital stocks that is required to make the welfare level under autarky equal to that under financial integration with different levels of shipping costs. The table also presents the expected rate of growth of consumption with different levels of shipping costs. The case with zero shipping costs corresponds to that of perfect integration in both financial and commodity markets. The parameter values are as in Table 1: $r = 0.02; \mu = 0.08; \eta = 0.554; \beta = 0.02; 1 - \alpha = 4; 1/(1 - \rho) = 0.5; \sigma = 0.20$.

| | **Panel A: Varying $\sigma$** | $\sigma = .100$ | $\sigma = .125$ | $\sigma = .150$ | $\sigma = .175$ | *$\sigma = .200$* |
|---|---|---|---|---|---|---|
| A1 | $\pi$, Welfare gain: 10% shipping cost | 0.1050 | 0.1948 | 0.1854 | 0.1494 | *0.1209* |
| A2 | zero shipping cost | 0.1146 | 0.2112 | 0.2005 | 0.1602 | *0.1300* |
| A3 | Expected growth rate: autarky | 0.0400 | 0.0432 | 0.0300 | 0.0220 | *0.0169* |
| A4 | 10% shipping cost | 0.0372 | 0.0414 | 0.0371 | 0.0274 | *0.0209* |
| A5 | zero shipping cost | 0.0378 | 0.0421 | 0.0386 | 0.0284 | *0.0217* |
| A6 | Consumption rate: autarky | 0.0200 | 0.0172 | 0.0150 | 0.0137 | *0.0128* |
| A7 | 10% shipping cost | 0.0210 | 0.0188 | 0.0163 | 0.0146 | *0.0135* |
| A8 | zero shipping cost | 0.0211 | 0.0189 | 0.0164 | 0.0147 | *0.0136* |
| A9 | Portfolio weights: autarky | 1.0 | 0.9600 | 0.6667 | 0.4898 | *0.3750* |
| A10 | 10% shipping cost | 1.0 | 1.0 | 0.8444 | 0.6212 | *0.4753* |
| A11 | zero shipping cost | 1.0 | 1.0 | 0.8580 | 0.6304 | *0.4826* |
| A12 | Cone opening: 10% shipping cost | 0.3721 | 0.3648 | 0.3280 | 0.2658 | *0.2128* |
| A13 | zero shipping cost | 0.5 | 0.5 | 0.5 | 0.5 | *0.5* |
| | **Panel B: Varying RRA** | $1-\alpha = .5$ | $1-\alpha = 1.5$ | $1-\alpha = 2$ | $1-\alpha = 3$ | *$1-\alpha = 4$* |
| B1 | $\pi$, Welfare gain: 10% shipping cost | 0.0324 | 0.1710 | 0.1883 | 0.1495 | *0.1209* |
| B2 | zero shipping cost | 0.0502 | 0.2003 | 0.2173 | 0.1627 | *0.1300* |
| B3 | Expected growth rate: autarky | 0.0350 | 0.0450 | 0.0338 | 0.0225 | *0.0169* |
| B4 | 10% shipping cost | 0.0326 | 0.0401 | 0.0405 | 0.0277 | *0.0209* |
| B5 | zero shipping cost | 0.0339 | 0.0417 | 0.0434 | 0.0290 | *0.0217* |
| B6 | Consumption rate: autarky | 0.0225 | 0.0175 | 0.0156 | 0.0138 | *0.0128* |
| B7 | 10% shipping cost | 0.0229 | 0.0189 | 0.0170 | 0.0147 | *0.0135* |
| B8 | zero shipping cost | 0.0231 | 0.0192 | 0.0172 | 0.0148 | *0.0136* |
| B9 | Portfolio weights: autarky | 1.0 | 1.0 | 0.7500 | 0.5000 | *0.3750* |
| B10 | 10% shipping cost | 1.0 | 1.0 | 0.9379 | 0.6322 | *0.4753* |
| B11 | zero shipping cost | 1.0 | 1.0 | 0.9652 | 0.6435 | *0.4826* |
| B12 | Cone opening: 10% shipping cost | 0.2687 | 0.3167 | 0.3160 | 0.2575 | *0.2128* |
| B13 | zero shipping cost | 0.5 | 0.5 | 0.5 | 0.5 | *0.5* |
| | **Panel C: Varying EIS (RRA=4)** | $\frac{1}{1-\rho} = .10$ | $\frac{1}{1-\rho} = .30$ | *$\frac{1}{1-\rho} = .50$* | $\frac{1}{1-\rho} = 1.1$ | $\frac{1}{1-\rho} = 1.2$ |
| C1 | $\pi$, Welfare gain: 10% shipping cost | 0.0995 | 0.1092 | *0.1209* | 0.1767 | 0.1912 |
| C2 | zero shipping cost | 0.1077 | 0.1178 | *0.1300* | 0.1883 | 0.2036 |
| C3 | Expected growth rate: autarky | 0.0124 | 0.0146 | *0.0169* | 0.0236 | 0.0248 |
| C4 | 10% shipping cost | 0.0152 | 0.0181 | *0.0209* | 0.0295 | 0.0309 |
| C5 | zero shipping cost | 0.0159 | 0.0188 | *0.0217* | 0.0304 | 0.0318 |
| C6 | Consumption rate: autarky | 0.0151 | 0.0139 | *0.0128* | 0.0094 | 0.0089 |
| C7 | 10% shipping cost | 0.0164 | 0.0150 | *0.0135* | 0.00929 | 0.00857 |
| C8 | zero shipping cost | 0.0165 | 0.0151 | *0.0136* | 0.00928 | 0.00855 |
| C9 | Portfolio weights: autarky | 0.3750 | 0.3750 | *0.3750* | 0.3750 | 0.3750 |
| C10 | 10% shipping cost | 0.4744 | 0.4749 | *0.4753* | 0.4766 | 0.4768 |
| C11 | zero shipping cost | 0.4826 | 0.4826 | *0.4826* | 0.4826 | 0.4826 |
| C12 | Cone opening: 10% shipping cost | 0.2170 | 0.2150 | *0.2128* | 0.2041 | 0.2025 |
| C13 | zero shipping cost | 0.5 | 0.5 | *0.5* | 0.5 | 0.5 |
| | **Panel D: Varying EIS (RRA=1.5)** | $\frac{1}{1-\rho} = .10$ | $\frac{1}{1-\rho} = .30$ | $\frac{1}{1-\rho} = .50$ | $\frac{1}{1-\rho} = 1.1$ | $\frac{1}{1-\rho} = 1.2$ |
| D1 | $\pi$, Welfare gain: 10% shipping cost | 0.1218 | 0.1424 | 0.1710 | 0.4172 | 0.5455 |
| D2 | zero shipping cost | 0.1433 | 0.1670 | 0.2003 | 0.4940 | 0.6525 |
| D3 | Expected growth rate: autarky | 0.0330 | 0.0390 | 0.0450 | 0.0630 | 0.0660 |
| D4 | 10% shipping cost | 0.0261 | 0.0331 | 0.0401 | 0.0613 | 0.0648 |
| D5 | zero shipping cost | 0.0270 | 0.0343 | 0.0417 | 0.0637 | 0.0673 |
| D6 | Consumption rate: autarky | 0.0235 | 0.0205 | 0.0175 | 0.0085 | 0.0070 |
| D7 | 10% shipping cost | 0.0261 | 0.0225 | 0.0189 | 0.0082 | 0.0064 |
| D8 | zero shipping cost | 0.0265 | 0.0228 | 0.0192 | 0.0082 | 0.0063 |
| D9 | Portfolio weights: autarky | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| D10 | 10% shipping cost | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| D11 | zero shipping cost | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| D12 | Cone opening: 10% shipping cost | 0.3223 | 0.3195 | 0.3167 | 0.3086 | 0.3073 |
| D13 | zero shipping cost | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |

Table 3: **The comparative statics for the calibration.** Shipping costs are set at 10%. All quantities are measured at the center point where $x/(x+x^*) = 0.5$, $K/(K+K^*) = 0.5$. The "Expected growth rate" is the expected growth rate of consumption. The "Consumption rate" is the consumption of one country relative to the total capital stock in both countries $c/(K+K^*)$. The "Portfolio weights" are the share of each country's capital stock invested in the risky technology, $w$. The number for the "Cone opening" is compared to 0.5, which would represent perfect balance at all times and a zero cone opening. Base case numbers are displayed in italics.

## References

Allen, P. and J. Stein, 1990, "Capital Market Integration," *Journal of Banking and Finance* 14, 909-928.

Arndt, S. and J. Richardson, 1987, "Real-Financial Linkages among Open Economies: An Overview," in Arndt, S. and J. Richardson ed. *Real-Financial Linkages among Open Economies*, MIT Press, Cambridge, MA.

Basak, S., 1995, "An Intertemporal Model of International Capital Market Segmentation," *Journal of Financial and Quantitative Analysis*, 31, 161-188.

Benes, V., L. Shepp, and H. Witsenhausen, 1980, "Some Solvable Stochastic Problems," *Stochastics* 4, 134-160.

Benninga, S. and A. Protopapadakis, 1988, "The Equilibrium Pricing of Exchange Rates and Assets when Trade Takes Time," *Journal of International Money and Finance* 7, 129-149.

Black, F., 1973, "Transportation Costs, Tariffs, and Taxes in a One-sector Model of Trade," Working paper, M.I.T.

Brennan, M. and B. Solnik, 1989, "International Risk Sharing and Capital Mobility," *Journal of International Money and Finance* 8, 359-373.

Cole, H. and M. Obstfeld, 1991, "Commodity Trade and International Risk Sharing: How Much Do Financial Markets Matter?," *Journal of Monetary Economics* 28, 3-24.

Davis, M. and A. Norman ,1990, "Portfolio Selection with Transaction Costs," *Mathematics of Operations Research* 15, 676-713.

Devereux, M., and G. Smith, 1994, "International Risk Sharing and Economic Growth," *International Economic Review* 35, 535-550.

Dixit, A., 1991, "A Simplified Treatment of the Theory of Optimal Regulation of Brownian Motion," *Journal of Economic Dynamics and Control* 15, 657-673.

Duffie, D. and L. Epstein, 1992, "Stochastic Differential Utility," *Econometrica*, 60, 353-394.

Duffie, D., P.-Y. Geoffard and C. Skiadas (1994): "Efficient and Equilibrium Allocations with Stochastic Differential Utility," *Journal of Mathematical Economics*, 23, 133-146.

Dumas, B., 1988, "Pricing Physical Assets Internationally," Working Paper, University of Pennsylvania.

Dumas, B.,1991, "Super Contact and Related Optimality Conditions," *Journal of Economic Dynamics and Control* 15, 675-685.

Dumas, B., 1992, "Dynamic Equilibrium and the Real Exchange Rate in a Spatially Separated World," *Review of Financial Studies* 5, 153-180.

Dumas, B., R. Uppal and T. Wang, 1997, "Efficient Intertemporal Allocations with Recursive Utilities," working paper UBC and HEC.

El Karoui, N., S. Peng and M. Quenez (1997): "Backward Stochastic Differential Equations in Finance," *Mathematical Finance*, 7, 1-73.

Engel, C. and J. H. Rogers, 1996, "How Wide is the Border?," *American Economic Review* forthcoming.

Epstein, L., 1987, "The Global Stability of Efficient Intertemporal Allocations," *Econometrica*, 55, 329-358.

Epstein, L. and S. Zin, "Substitution, Risk Aversion and the Temporal Behavior of Consumption and Asset returns: A Theoretical Framework," *Econometrica*, 57, 937-969.

Errunza, V. and E. Losq,1989, "Capital Flow Controls, International Asset Pricing and Investors Welfare: A Multi-country Framework," *Journal of Finance* 44, 1025-1038.

Frankel, J., 1986, "International Capital Mobility and Crowding-out in the U.S. Economy: Imperfect Integration of Financial Markets or of Goods Markets," in R. Hafer ed. *How Open is the U.S. Economy ?*, Lexington Books, MA.

Gagnon, J., 1989, "Adjustment Costs and International Trade Dynamics," *Journal of International Economics* 26, 327-344.

Gultekin, M., B. Gultekin and A. Penati, 1989, "Capital Controls and International Capital Market Segmentation: The Evidence from the Japanese and American Stock Markets," *Journal of Finance* 44, 849-869.

Halliday, L., 1989, *The International Stock Exchange Directory*, Institutional Investor, March, 197-204.

Harrison, J., 1985, *Brownian Motion and Stochastic Flow Systems*, Wiley, NY.

Harrison, J. and M. Taksar, 1983, "Instantaneous Control of Brownian Motion," *Mathematics of Operations Research* 8, 439-453.

Helpman, E. and A. Razin, 1978, *A Theory of International Trade under Uncertainty*, Academic Press, NY.

Lucas, R., 1982, "Interest Rates and Currency Prices in a Two-Country World," *Journal of Monetary Economics* 10, 335-359.

Kreps, D. and E. Porteus, 1978, "Temporal Resolution of Uncertainty and Dynamic Choice Theory," *Econometrica* 46, 185-200.

Marston, R. C., 1990, "Pricing to Market in Japanese Manufacturing," *Journal of INternational Economics*, 29, 217-2356.

Melo, J. de and D. Tarr, 1992, *A General Equilibrium Analysis of US Foreign Trade Policy*, MIT Press, Cambridge MA.

Mendoza, E., "Capital Controls and the Gains from Trade in Business Cycle Model of a Small Open Economy," *International Monetary Fund Staff Papers* 38, 480-505.

Merton, R., 1971, "Optimum Consumption and Portfolio Rules in a Continuous Time Model," *Journal of Economic Theory* 3, 373-413.

Michael, P., A. R. Nobay and D. A. Peel, 1997, "Transactions Costs and Nonlinear Adjustment in Real Exchange Rates: an Empirical Investigation," *Journal of Political Economy*, 105, 862-879.

Obstfeld, M., 1986, "Capital Mobility in the World Economy: Theory and Measurement," *Carnegie-Rochester Conference Series on Public Policy* 24, 55-103.

Obstfeld, M., 1994, "Risk-Taking, Global Diversification, and Growth," *American Economic Review* 84.5, 1310-1329.

Obstfeld, M. and A. M. Taylor, 1997, "Non-linear Aspects of Goods Market Arbitrage and Adjustment: Heckscher's Commodity Points Revisited," CEPR working paper no. 1672.

O'Connell, P. G. J., 1997, "The Bigger They Are, the Harder They Fall: How Price Differences Across U.S. Cities Are Arbitraged," NBER working paper no. 6089.

Pindyck, R., 1991, "Irreversibility, Uncertainty and Investment," *Journal of Economic Literature* 29, 1110-1148.

Schroder, M. and C. Skiadas, 1997, "Optimal Consumption and Portfolio Selection with Stochastic Differential Utility," working paper No. 226, Kellogg School of Management, Northwestern University.

Selden, L., 1979, "An OCE Analysis of the Effect of Uncertainty on Saving under Risk Independence," *Review of Economic Studies*, 73-82.

Shrikhande, M., 1992, "The Cost of Operating Abroad in International Capital Market Equilibrium," Working Paper, Georgia Institute of Technology.

Skiadas, C., 1997, "Recursive Utility and Preferences for Information," *Economic Theory* forthcoming.

Stockman, A., 1988, "Sectoral and National Aggregate Disturbances to Industrial Output in Seven European Countries," *Journal of Monetary Economics* 21, 387-409.

Subrahmanyam, M., 1975, "On the Optimality of International Capital Market Integration," *Journal of Financial Economics* 2, 3-28.

van Nunen, A., 1990, "Europe 1992: Boundless Optimism Misplaced," in H. van Gemert ed. *Economic Interdependence and Financial Stability*, Tilburg University Press, Netherlands.

Wheatley, S., 1988, "Some Tests of International Equity Market Integration," *Journal of Financial Economics* 21, 177-213.