

NBER WORKING PAPER SERIES

INTERGENERATIONAL EARNINGS
MOBILITY, INEQUALITY, AND GROWTH

Ann L. Owen
David N. Weil

Working Paper 6070

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
June 1997

We are grateful to Rachel Friedberg, Oded Galor, Harl Ryder, Michael Spagat, Dani Tsiddon, and especially to Fernando Alvarez for helpful comments. The opinions expressed in this paper are those of the authors and do not necessarily reflect those of the Federal Reserve Board of Governors, or the National Bureau of Economic Research. This paper is part of NBER's research program in Economic Fluctuations and Growth.

© 1997 by Ann L. Owen and David N. Weil. All rights reserved. Short sections of text, not to exceed two paragraphs, may be quoted without explicit permission provided that full credit, including © notice, is given to the source.

Intergenerational Earnings Mobility, Inequality,
and Growth

Ann L. Owen and David N. Weil

NBER Working Paper No. 6070

June 1997

JEL Nos. O40, J62, J31

Economic Fluctuations and Growth

ABSTRACT

We examine a model in which per capita income, inequality, intergenerational mobility, and returns to education are all determined endogenously. Individuals earn wages depending on their ability, which is a random variable. They purchase an education with transfers received from their parents, and are subject to liquidity constraints. In the model, multiple steady state equilibria are possible: countries with identical tastes and technologies can reach differing rates of mobility, inequality, and per capita income. Equilibria with higher levels of output also have lower inequality, higher mobility, and more efficient distribution of education.

Ann L. Owen
Federal Reserve Board
Mail Stop 73
20th and C Streets, NW
Washington, DC 20551
aowen@frb.gov

David N. Weil
Department of Economics
Box B
Brown University
Providence, RI 02912
and NBER
david_weil@brown.edu

1 Introduction

A common measure of individuals' economic satisfaction is their success in achieving a higher income than their parents. Families can advance economically as a result of three different processes. First, all families in a country can become richer by an equal amount, increasing the mean of the country's income distribution without changing the variance. Second, a family can move up to a higher rank within a country's unchanged distribution of income. In such a case, of course, one family's economic advance is balanced by another family's decline. Finally, the variance of the country's income distribution can change in a way that makes the family better off without changing the family's rank in the distribution -- for example a decrease in income inequality will make families in the bottom part of the income distribution better off.

All three of these methods of advancing are interpretable as aspects of welfare and as legitimate targets for economic policy-making. The average level of income per capita is the focus of conventional studies of economic growth. The cross-sectional variance of income, or income inequality, is also viewed as a topic of interest by itself, as well as a determinant of growth. And intergenerational mobility, the ease with which families move between different parts of the income distribution, has been used as a measure of equality of opportunity within an economy.

In this paper we examine a model of economic growth in which all three of these phenomena are present. The model combines two existing aspects of the literature: first, the joint determination of economic mobility and inequality, which has previously been pursued in the context of an unchanging macroeconomic environment, and second, the relation between inequality and economic growth, which has previously not allowed for economic

mobility. Our approach highlights the interaction between economic growth and intergenerational mobility and allows us to examine the degree of mobility at various levels of development. In particular, we are able to show that mobility increases when rising incomes relax constraints on the purchase of education. In addition, increased mobility allows resources to be allocated in a manner that leads to higher per capita income. Another important result of this analysis is that, when individuals are constrained by income in their purchase of education, multiple steady state equilibria are possible. Each of these equilibria is associated with different levels of mobility, inequality, and per capita income.

A natural question to ask of our model is how different policies which affect the ability of individuals to acquire education affect the three outcome variables in which we are interested: income distribution, mobility, and the average level of income. Recent literature has pointed to the intergenerational transmission of human capital as a key factor determining the persistence of poverty (See for example, Borjas (1992)). We use our model to evaluate two policies: the public provision of education on a meritocratic basis, and the elimination of credit constraints in the acquisition of education.

Becker and Tomes (1986) and (1979) have contributed notably to the theory of intergenerational earnings mobility, developing a model in which children's incomes are related to their parents' both via the inheritability of abilities and via spending on training by altruistic parents. In their model, children's native abilities are correlated with those of their parents, and these native abilities are enhanced by parents' investments in their children's human capital. A constant wage per unit of human capital is paid, creating higher wages to people who own more human capital. Both inherited abilities and earnings regress toward the

mean. Because of the effect of parental investment in children, however, earnings regresses more slowly than does ability. Loury (1981) develops a similar model where endowments of innate ability are random. He shows that when some parents' investments in their children's human capital are constrained by income, an inefficient allocation of resources results. A key feature of these models is that families act in isolation from each other, unaffected by the macroeconomic implications of their investments in human capital.

While the work mentioned above studies the incomes of individuals within a stagnant economy, related work in growth theory has shown how the economy's aggregate investment in human capital affects the level of per-capita income. In these models, human capital can be a factor of production or the impetus driving technological change. A strand of this literature specifically focuses on income distribution, human capital, and economic growth. In Galor and Zeira (1993), unequal distributions of income combined with credit market imperfections are a constraint to growth. In this model, since the poor cannot invest in human capital, they must use an inferior production technology. Tamura (1993) and Galor and Tsiddon (1997) analyze economies in which increases in the average level of human capital in the economy make all workers more productive.¹

This paper builds on the works mentioned above, merging the two approaches to examine intergenerational earnings mobility in a growing economy. It places families subject

¹A related set of papers (Benabou, 1993; Durlauf, 1994) examine a different channel through which inequality and growth are related. In these papers, there are local complementarities in the production of human capital. Higher inequality, accompanied by stratification of neighborhoods by income, reduces the efficiency with which human capital is supplied to the young, thus lowering output. Benabou (1992) presents a general framework for analyzing how the integration of heterogenous neighborhoods affects growth when there are both local and global externalities in the accumulation of human capital.

to stochastic ability shocks in a macroeconomic environment where wage rates are influenced by the actions of individuals as well as the aggregate production technology. In the model described below, educated and uneducated labor are complements in production. Because individuals interact with each other in production, individuals' investments in human capital affect both the level of income per capita and relative wages.²

We consider an economy where ability is randomly distributed in the population independently of parents' ability or wealth. Ability is enhanced by education, but, because of borrowing constraints, the affordability of an education depends on parental wealth. In the model we present, increases in the educated work force cause economic growth and a change in relative wages. Due to the complementarity of educated and uneducated workers, a developed economy with high levels of human capital will have high relative wages for uneducated workers, making it more likely that the children of uneducated workers will be able to afford an education. Downward mobility is also more likely in the developed economy since the smaller wage gap decreases the incentive for children of educated workers to become educated. Conversely, in a less developed economy with lower levels of education, the wage gap between educated and uneducated workers will be larger, decreasing the number of children of uneducated workers who can afford an education and increasing the incentive for children of educated workers to remain in the educated class. Thus, mobility between the two classes and per capita income will both be low. Further, differences in

²The association between changing relative wages and economic growth is empirically supported by Williamson (1985) who documents such a phenomenon in 19th century industrializing England. It is also supported by cross-country evidence on returns to education collected by Psacharopoulos (1985 and 1993).

mobility will translate into differences in the efficiency with which education is provided: in an economy with high mobility, a higher proportion of educational resources will be devoted to individuals with high ability than in an economy with low mobility.

These results are developed in the next four sections. Section 2 describes the economic environment in which individuals live. Section 3 discusses the existence and properties of steady states of the economy and evaluates policy alternatives. Section 4 analyzes the dynamic evolution of earnings mobility as an economy develops. Section 5 concludes.

2 The Economic Environment

2.1 Production

Aggregate production is given by a CRS function of physical capital, K_t , and an aggregate labor input, L_t :

$$Y_t = F(K_t, L_t) \tag{1}$$

where the subscript t denotes time. The aggregate labor input is in turn a function of the number of educated and uneducated efficiency units:

$$L_t = L(U_t, E_t) \tag{2}$$

where $L(U, E)$ is homogenous of degree one, strictly quasiconcave and satisfies the Inada conditions. U_t is the number of efficiency units supplied by workers who do not have an education, and E_t is the number of efficiency units supplied by educated workers. In addition,

$L_{1,2} > 0$ -- educated and uneducated efficiency units are complements in the production of aggregate labor input.³

It is important to note that our use of the term efficiency unit applies to both educated and uneducated labor. Efficiency units are *not* the output of the educational sector. In other words, education does not increase the number of efficiency units that a worker supplies. Rather, receiving an education means that a worker supplies his efficiency units as educated labor, while a worker without an education supplies his efficiency units as uneducated labor. As will become clear in the next section, we have applied the concept of efficiency units to both educated and uneducated labor to capture the idea that productivity differs within each of these groups. We assume that the total number of efficiency units is constant over time, so that growth in this economy will occur when efficiency units are reallocated into a more productive combination of educated and uneducated labor. Although this reallocation generates transitional and not steady state growth, it may occur over several generations.

Factors are paid their marginal products. We assume that the economy is small and open to the world capital market. As a result, physical capital will flow into or out of the country so that the marginal product of physical capital will be equal to the world interest

³An example of a production function that satisfies these properties is

$$Y_t = K_t^\alpha U_t^{\beta(1-\alpha)} E_t^{(1-\beta)(1-\alpha)}$$

In this case, both $F(K,L)$ and $L(U,E)$ are Cobb-Douglas. See Owen (1996) for a discussion of a similar economy in a non-stochastic environment.

Note that we do not assume that educated workers have any intrinsic advantage over uneducated workers. In an economy with sufficiently few uneducated workers, wages for educated workers would be lower than for uneducated workers.

rate, \bar{r} . The constancy of the world interest rate implies that the economy's capital to labor ratio will be fixed through time, which in turn implies that the marginal product of a unit of aggregate labor will be constant at \bar{F}_L .⁴

Labor is not internationally mobile. Define w^u as the wage per efficiency unit to people with no education and w^e as the wage per efficiency unit to people with education:

$$w^u = \bar{F}_L L_U \quad w^e = \bar{F}_L L_E \quad (3)$$

Noting that $U = 1 - E$, wages per efficiency unit can be written as a function of E alone. The properties of $L(U,E)$ ensure that

⁴ Specifically, let

$$\frac{Y_t}{L_t} = f(k_t) ; \quad k_t \equiv \frac{K_t}{L_t}$$

Then, given the mobility of physical capital, the economy's capital to labor ratio, k , is fixed through time:

$$r_t = f'(k_t) \equiv \bar{r} ; \quad k_t = f'^{-1}(\bar{r}) \equiv \bar{k}$$

Total wages to a unit of aggregate labor can then be expressed as a constant:

$$f(\bar{k}) - f'(\bar{k})\bar{k} \equiv \bar{F}_L$$

and the wage bill for the economy as a whole is

$$\bar{F}_L L(U, E)$$

$$\frac{dw^u}{dE} > 0, \quad \frac{dw^e}{dE} < 0 \quad (4)$$

$$\lim_{E \rightarrow 0} w^e = \infty, \quad \lim_{E \rightarrow 1} w^e = 0, \quad \lim_{E \rightarrow 0} w^u = 0, \quad \lim_{E \rightarrow 1} w^u = \infty$$

Thus, given E , (3) determines wage rates for educated and uneducated efficiency units. We summarize this relationship

$$\{w_t^u, w_t^e\} = W(E_t) \quad (5)$$

2.2 Individuals

We model the economy as being made up of overlapping generations of individuals, each of whom lives for two periods. A continuum of individuals is born at each time t . In the first period of life, individuals receive a transfer from their parents, may purchase an education, and work as either educated or uneducated workers.⁵ For simplicity we assume that they do not consume during the first period of life. They invest their wages plus the value of their transfer net of spending on education at the world interest rate. In the second period of life, individuals consume and make transfers to their children. Each individual has a single parent and a single child. The child is in the first period of life when the parent is in

⁵Education must be purchased *before* entering the work force. Thus, the first period is essentially divided into two parts. In the first part, education may be purchased; and in the second part, individuals work.

the second. Thus, the economy is made up of a collection of dynasties, each of which is made up of one working child and one retired parent in each period of time.

We model individuals within a cohort as differing in two respects: first, individuals receive different transfers from their parents, and, second, individuals differ in their innate abilities. Let $q_{i,t}$ be the ability of the single member of dynasty i who is of working age in period t . We take ability to be equal to the number of efficiency units of labor, either educated or uneducated, that an individual can supply. Thus, an educated worker will have total earnings of $q_{i,t}w_t^e$, while the same worker would earn $q_{i,t}w_t^u$ if uneducated. q has a probability density function $Q(q)$, which is strictly positive over a support with upper boundary \bar{q} and lower boundary $\underline{q} > 0$.⁶ $Q(q)$ is invariant across time and individuals. Although q is a random variable from the perspective of the individual, we assume that the number of dynasties in the economy is large enough that the sum of efficiency units can be taken as a constant which we normalize to one. The cost of getting an education is assumed to be fixed at \bar{e} , resulting in a higher net rate of return to becoming educated for a high-ability individual. The minimum level of ability for which it is profitable for an individual to obtain an education at time t , q_t^* , is determined by the cost of an education relative to the wages of educated and uneducated efficiency units:

$$q_t^* = \frac{\bar{e}}{w_t^e - w_t^u} \quad (6)$$

Since wages are determined by the number of educated efficiency units, it will always be the

⁶Throughout, we use capital letters to denote density functions.

case (as long as education has positive cost) that in equilibrium $w^e > w^u$ and $q^* > 0$.⁷

Not all individuals for whom an education would be profitable are able to afford one, however. Let $x_{i,t}$ be the amount of financial support given to the working-age member of dynasty i by his parent in period t . We assume that educational spending within the first period of life takes place before individuals work, so that education cannot be funded out of one's own wages. Due to credit market imperfections, educational loans are not available and individuals can only receive an education if the transfer they receive is larger than \bar{e} , the cost of an education.⁸ Combining these two constraints, an individual will purchase education only if

$$x_{i,t} \geq \bar{e} \quad \text{and} \quad q_{i,t} > q^* \quad (7)$$

Thus, resources net of spending on education of member i of generation t are

$$\begin{aligned} x_{i,t} - \bar{e} + q_{i,t}w_t^e & \quad \text{if } x_{i,t} \geq \bar{e} \text{ and } q_{i,t} > q^* \\ x_{i,t} + q_{i,t}w_t^u & \quad \text{otherwise} \end{aligned} \quad (8)$$

Individuals get utility from consumption in the second period of life and from the support they give to their children. We assume that individuals have a log-linear utility function, with

⁷Note that $q^* \leq 0$ implies $w^u \geq w^e$. In such a case, no worker would choose to get an education, implying $E=0$. But as shown above,

$$\lim_{E \rightarrow 0} w^e = \infty \quad \text{and} \quad \lim_{E \rightarrow 0} w^u = 0$$

which contradicts $w^u \geq w^e$.

⁸Credit market imperfections exist in the educational loan market since it is not possible to offer human capital as collateral. This problem is not present in the market for physical capital. See Barro, Mankiw, and Sala-i-Martin (1995) for further discussion of this type of borrowing constraint.

weight γ , ($0 < \gamma < 1$), on their transfer to their child, and $(1-\gamma)$ on their own consumption.

$$U(c_{t+1}, x_{t+1}) = \gamma \ln(x_{t+1}) + (1-\gamma) \ln(c_{t+1}) \quad (9)$$

Thus, an individual will give a fraction γ of his second-period wealth as a transfer to his single child.⁹

2.3 The Evolution of Dynasties

We assume for simplicity that an individual's ability is unrelated to that of his parent. Thus, the amount that an individual receives as a transfer serves as a sufficient statistic to summarize the entire history of the dynasty. The transfer an individual gives is a function of that individual's transfer receipt and net labor income (income after paying for an education). The individual's net labor income is in turn a function of his ability and transfer receipt, as well as the wage structure. For the assumptions about preferences made above:

$$x_{i,t+1} = g(x_{i,t}, q_{i,t}, w_t^u, w_t^e) \equiv \begin{cases} \gamma(1+\bar{r})(x_{i,t} - \bar{e} + q_{i,t} w_t^e) & \text{if } q_{i,t} > q_t^* \text{ and } x_{i,t} \geq \bar{e} \\ \gamma(1+\bar{r})(x_{i,t} + q_{i,t} w_t^u) & \text{otherwise} \end{cases} \quad (10)$$

where $x_{i,0}$ is given. We also impose the condition that $\gamma(1+\bar{r}) < 1$ so that the size of transfers does not grow indefinitely. For a given set of wages, Equation (10) defines a Markov process

⁹This particular form of the utility function is used for simplicity. Alternative specifications that allow for consumption in the first period of life or an altruistic bequest motive (parents care about the *utility* of their children) leave the qualitative results unchanged. The essential element of this utility function is that, as a result of income-constrained utility maximization, some poor parents cannot afford to give a transfer large enough to educate their children. As long as some children do not receive enough funds to purchase education because their parents are too poor, the conditions listed in (7) are still relevant and the following analysis is qualitatively unchanged.

where the probability of inheriting a particular value, $x_{i,t+1}$, is conditioned on the value, $x_{i,t}$.

Note that since $g(x_{i,t}, q_{i,t}; w_t^u, w_t^e)$ is a strictly increasing and continuous function of $q_{i,t}$, we can also define a new function that, given $x_{i,t}$, w_t^u , and w_t^e , returns the value of q necessary to obtain a particular value of $x_{i,t+1}$.¹⁰ In particular,

$$q_{i,t} = \hat{g}(x_{i,t+1}, x_{i,t}; w_t^u, w_t^e) \equiv \begin{cases} \frac{x_{i,t+1} + \gamma(1+\bar{r})[\bar{e} - x_{i,t}]}{\gamma(1+\bar{r})w_t^e} & \text{if } \frac{x_{i,t+1} + \gamma(1+\bar{r})[\bar{e} - x_{i,t}]}{\gamma(1+\bar{r})w_t^e} > q^* \text{ and } x_{i,t} > \bar{e} \\ \frac{x_{i,t+1} - \gamma(1+\bar{r})x_{i,t}}{\gamma(1+\bar{r})w_t^u} & \text{otherwise} \end{cases} \quad (11)$$

Given $x_{i,t}$, w_t^e , and w_t^u , the probability density function of dynasty i 's transfer in period $t+1$ can be expressed as

$$X_{i,t+1}(x) = Q(\hat{g}(x, x_{i,t}; w_t^u, w_t^e)) \quad (12)$$

Because the distribution of q is bounded and $\gamma(1+\bar{r}) < 1$, it is easy to show that there exists a recurrent distribution of x that will also be bounded. An upper bound of the support of the recurrent distribution is given by

$$\bar{x} = \frac{\gamma(1+\bar{r})}{1-\gamma(1+\bar{r})} (\bar{q}w^e - \bar{e}) \quad (13)$$

Individuals who received a transfer larger than \bar{x} , even if they had the maximum possible level of ability, would pass on to their children a transfer smaller than the one they received.

A lower bound of the support of the recurrent distribution of transfers is given by

¹⁰ $g(x_{i,t}, q_{i,t}; w_t^e, w_t^u)$ is a continuous function of q because $q_t^* w_t^e - \bar{e} = q_t^* w_t^u$.

$$\underline{x} = \frac{\gamma(1+r)}{1-\gamma(1+r)} q w^u \quad (14)$$

Individuals who received a transfer of less than \underline{x} , even if they had the lowest possible ability, would pass on to their children a transfer larger than the one they received.¹¹

3 Steady States

We now turn to an examination of the existence and characteristics of steady states in the economy described above. We define an economy as being in a steady state if the values of w^u , w^e , and E are constant and the distribution of x_i across individuals is unchanging from generation to generation. Note that even when an economy is in a steady state, individual dynasties will experience movements within the income and wealth distributions due to shocks to ability.

3.1 Conditional Steady states

We begin by looking at how the distribution of transfers across individuals will evolve while holding the macroeconomic environment, in particular wage rates, constant. This part of our analysis is similar to previous microeconomic analyses of earnings mobility (e.g., Becker and Tomes, 1979 and 1986, and Loury, 1981). Since wages are held constant in a conditional steady state, we suppress the time subscripts on wages in this section.

¹¹In some special cases, the bounds of the recurrent distribution of transfers may be more restrictive than those derived here. Specifically, if nobody becomes educated (no dynasties are able to afford an education or if $q < q^*$) or if everybody becomes educated (all workers can afford an education and $q > q^*$), then the exact formulation of the upper and lower bounds of the recurrent distribution will differ from those stated in the text. However, we show below that neither of these special cases can occur if wages are set endogenously.

Holding the wages of educated and uneducated workers constant, the transfers received by young members of a dynasty in period t are functions only of their parents' abilities and transfer receipts. Equation (12) shows how the distribution of transfers within a given dynasty, $X_{i,t}(x)$ evolve over time. The probability density function of transfers for the economy as a whole in period $t+1$, $X_{t+1}(x)$, is given by

$$\begin{aligned} X_{t+1}(x) &= \int_0^{\infty} Q(\hat{g}(x,j;w^u,w^e)) X_t(j) dj \\ &\equiv G(X_t(\cdot), w^u, w^e) \end{aligned} \tag{15}$$

where $X_0(x)$ is given.¹²

A *conditional* steady state of the economy is defined as an economy-wide distribution of transfers having a density function, $X_{css}(x)$, that is unchanging from period to period, holding constant the wages to educated and uneducated workers. In other words, the invariance of $X_{css}(x)$ is conditional on an exogenously given wage structure. In particular, $X_{css}(x)$ satisfies

$$X_{css}(x) = G(X_{css}(\cdot), w^u, w^e) \tag{16}$$

Conditional steady states can be classified by the existence of mobility between classes. We define mobility as a change in educational attainment between generations. We

¹²For any j , there is a unique $q_{i,t}$ that gives a particular value of $x_{i,t+1}$. Thus, there is not a positive probability mass point in $X_{t+1}(x)$

will assume throughout that $\bar{q} > q^*$, that is, people with the highest ability find an education profitable.¹³ The condition for the existence of upward educational mobility is that a dynasty which is initially uneducated and in which each generation receives the highest draw from the ability distribution eventually gives large enough transfers to purchase an education:

$$\frac{\gamma(1+\bar{r})}{1-\gamma(1+\bar{r})}\bar{q}w^u > \bar{e} \quad (17)$$

The condition for downward educational mobility in the case where $q^* \leq \underline{q}$ (individuals with the lowest ability find it profitable to get educated), is that a dynasty of low-ability people eventually give transfers small enough that they do not cover the cost of an education:

$$\gamma(1+\bar{r})\underline{q}w^e < \bar{e} \quad (18)$$

In the case where $q^* > \underline{q}$, downward mobility will always exist since there is a positive probability that an educated parent will have a child with less than q^* efficiency units.

It may be the case that mobility is possible only upwards, only downwards, in both directions, or in neither. These four possibilities are illustrated in different panels of Figure 1. In these figures, the line marked $g(x_{it}; \bar{q}, w^u, w^e)$ shows the transfer given to a child as a function of the transfer received by a parent when the parent has maximum ability, and $g(x_{it}; \underline{q}, w^e, w^u)$ shows the same function for a parent of minimum ability. Panels A-D of Figure 1 are drawn assuming that $q^* < \underline{q}$, that is, that even the lowest ability person would profit from an education. Upward mobility occurs when a person who received a transfer of less than \bar{e} passes on to his or her child a transfer greater than \bar{e} ; downward mobility is when a

¹³Below we show that this assumption always holds true if wages are determined endogenously.

person who received a transfer of greater than \bar{e} passes on a transfer of less than \bar{e} . If both $g(x_{it}; \bar{q}, w^u, w^e)$ and $g(x_{it}; \underline{q}, w^e, w^u)$ cross the 45-degree line twice (Figure 1A), there is no mobility. If each crosses the 45-degree line only once (Figure 1D), then there is mobility in both directions. Finally, if either $g(x_{it}; \bar{q}, w^u, w^e)$ or $g(x_{it}; \underline{q}, w^e, w^u)$ crosses the 45-degree line twice, while the other crosses it only once, then there is mobility in only one direction. Thus in Figure 1B there is only upward mobility, while in Figure 1C there is only downward mobility.

Panels E-G of Figure 1 consider the case where $q^* > \underline{q}$ -- that is, people with the lowest level of ability will choose not to become educated even if they can afford it. In this case downward mobility is always possible. Panel E shows the case where there is only downward mobility. Panels F and G show the case where there is both upward and downward mobility. The two panels differ in the relative positions of \underline{x} , the lower bound of the recurrent distribution, and \bar{e} , the cost of an education. In panel F, $\underline{x} > \bar{e}$, so that children of even the poorest families can afford an education. By contrast, in panel G, $\underline{x} < \bar{e}$, so that not all children born into poor families can afford an education. In this case, there will be "gradual" upward mobility: an uneducated dynasty will have to have several good draws from the ability distribution before one generation can become educated. By contrast, in panel F there is "instant" upward mobility: a bright child of even the poorest dynasty will get an education.¹⁴

If there is mobility in only one direction, then eventually all dynasties will move to

¹⁴We do not describe the possibility that $\underline{x} < \bar{e}$ in the case where $q^* < \underline{q}$ because it is trivial: all dynasties will be educated all of the time. Below we show that such a case cannot arise in general equilibrium.

one state. Below we will show that such an outcome will never take place when wages are endogenously determined. There are two types of conditional steady states possible in which the distribution of education does not collapse to a single level: class mobility (Panels D, F, and G) or class immobility (Panel A). If both downward and upward mobility exist, then, as we prove in the Appendix, there exists a unique conditional steady state distribution of x , the transfer between generations. When there is no mobility between classes, there are an infinite number of steady state distributions, with the number of dynasties in each educational class depending simply on initial conditions. In the Appendix we show that in such a case the steady state distribution of transfers *within* each class will be invariant to initial conditions. Thus, the overall distribution of transfers in the economy will be a weighted average of the two invariant distributions, with weights E on the educated distribution and $(1-E)$ on the uneducated distribution.

3.2 Unconditional Steady States

We now consider the question of whether there exist *unconditional* steady states in the economy. An unconditional steady state is a conditional steady state that generates the wage structure on which it is conditioned. An unconditional steady state is described more specifically in the remainder of this section.

To define an unconditional steady state, we use three relationships in this economy. First, as noted in Equation 5, wages for educated and uneducated workers are uniquely determined by the fraction of total efficiency units of labor supplied by educated workers, E_t . The second relationship, $G(X_t(x), w^u, w^e)$ as defined in equation (15), gives the distribution of

transfers received in period t+1 as a function of wages and transfers received in period t.

Finally, we can consider the mapping of $X_t(x)$, the distribution of transfers received at time t, and $\{w^u, w^e\}_t$, the wage structure, onto the stocks of educated and uneducated labor. Define this relationship as $J(X_t(x), w_t^u, w_t^e)$:

$$E_t = \int_{\bar{e}}^{\bar{x}} \int_{q_t^*}^{\bar{q}} q Q(q) X_t(x) dq dx \quad (19)$$

$$\equiv J(X_t(x), w_t^u, w_t^e)$$

where q_t^* , defined in equation (4), is a function of the wages in period t.

An unconditional steady state is a fixed point $\{E_{ss}, \{w^u, w^e\}_{ss}, X_{ss}\}$ such that:

$$\{w^u, w^e\}_{ss} = W(E_{ss})$$

$$X_{ss}(x) = G(X_{ss}, \{w^u, w^e\}_{ss}) \quad (20)$$

$$E_{ss} = J(X_{ss}(x), \{w^u, w^e\}_{ss})$$

To study the existence and properties of unconditional steady states, we introduce two new functions that incorporate the relations described above. First, define G^* as the function that maps a set of wages, $\{w^u, w^e\}$, into a conditional steady state distribution of x:

$$G^*(\{w^u, w^e\}) = \{X_{css} | X_{css} = G(X_{css}, \{w^u, w^e\})\} \quad (21)$$

For values of w^u and w^e for which there is mobility in the economy, we show in the appendix that there is unique conditional steady state distribution of transfers, X_{css} . Thus for these values of $\{w^u, w^e\}$, $G^*(\{w^u, w^e\})$ will be a singleton. For values of wages for which there is no

mobility, there will be large set of conditional steady state distributions.

The second function we introduce to study the properties of unconditional steady states, $\pi(E)$, takes a value of E and, through the wage rates implied by this value of E and the conditional steady state distribution of transfers, $X_{css}(x)$, for these wage rates, gives a resulting value of E . Given the definition of $G^*({w^u, w^e})$, we can define $\pi(E)$ as

$$\pi(E) = J(G^*(W(E)), W(E)) \quad (22)$$

In other words, given a value of E , $W(E)$ can be used to compute wages which in turn can be used to determine the $X_{css}(x)$ through $G^*({w^u, w^e})$. $J(X_{css}(x), {w^u, w^e})$ then returns a resulting value of E .

An unconditional steady state for the economy can then be defined in terms of $\pi(E)$:

$$E_{ss} = \pi(E_{ss}) \quad (23)$$

3.3 Existence and Properties of Unconditional Steady States

In this section, we derive several results about the existence of unconditional steady states, and we discuss their implications. We discuss our results with reference to the $\pi(E)$ function. As shown above, an unconditional steady state will be an intersection of the graph of $\pi(E)$ and the 45 degree line. Figure two summarizes three different possible configurations of the $\pi(E)$ function, and shows three different types of steady states.¹⁵ Proofs for the following results can be found in the Appendix.

¹⁵Since we are interested in using $\pi(E)$ only to identify unconditional steady states, when drawing the $\pi(E)$ function for values of E which imply no mobility, we show only the values of $\pi(E)$ which lie along the 45-degree line.

Proposition 1: There cannot be an unconditional steady state in which there is mobility in only one direction.

If there were mobility in only one direction, then eventually the entire population would be either educated or uneducated. Before such a state could be reached, however, the wages of the two groups would adjust to shut off mobility or to allow for mobility in the other direction. This proposition rules out unconditional steady states such as those corresponding to the figures shown in 1B, 1C, and 1E.

This result leaves open the possibility of two types of steady states: one type with mobility in neither direction, or a second type with mobility in both directions. We first show that the former of these will exist for any set of parameters. We then show that the latter will exist for some values of the parameters.

Proposition 2: For any set of parameters there exists a non-trivial unconditional steady state with no mobility.

If the initial stock of educated workers is sufficiently small, there will be a steady state with no mobility. With few educated workers, the wages of educated workers will be sufficiently high that even the least able child of an educated parent will both receive a sufficient transfer and find it profitable to get educated. Similarly, the wages of the uneducated will be sufficiently low that no child of uneducated parents will be able to afford an education. Thus, for any value of the parameters, it is possible that a no-mobility

equilibrium of the type shown in Figure 1A will obtain.

This result implies that for some range of values greater than zero, the $\pi(E)$ function will follow the 45 degree line. Thus there is a continuum of steady state levels of E . Since there is no mobility between classes in this region, an economy which starts with a level of E anywhere in this region will not move to a different level of E .

Proposition 3: For any values of the other parameters, there is a value of \bar{e} low enough such that there exists an unconditional steady state with mobility.

We show in the Appendix that there is always a value of \bar{e} such that the $\pi(E)$ function has the graph shown in figure 2A: it follows the 45-degree line from zero for a time, then jumps up to one, then falls, ending up at zero for $E=1$. In such a case it must cross the 45 degree line at least once, producing a steady state. This steady state will be characterized by both upward and downward mobility between classes. The particular steady state we are able to prove exists is of the type shown in figure 1F. Since $\bar{e} < x$ in this case, wealth does not matter for educational attainment. In the next section, we show with a numerical example that a steady state with mobility where wealth does matter exists, corresponding to the situation pictured in figures 1D and 1G.

Proposition 4: For any values of the other parameters, there is a value of \bar{e} high enough such that there do not exist any unconditional steady states with mobility.

For sufficiently high values of \bar{e} , the $\pi(E)$ function will have the shape shown in figure 2B. That is, it will not cross the 45 degree line above the level of E^* . In such a case, the only steady states will be those with no mobility.

A final possibility is shown in figure 2C: the $\pi(E)$ function may initially fall to zero when $E > E^*$ but then may rise and cross the 45 degree line one or more times. In this case, there are multiple steady states with mobility. Although we do not derive the exact conditions under which such a case will exist, in the next section we consider an example where it is present. Thus, for some ability distributions, multiple steady states with mobility are possible.

Summary

The possible existence of an unconditional steady state with mobility, for some sets of parameters, is a key result of this paper. Since unconditional steady states without mobility exist for all sets of parameters, this result implies that the two different types of steady states can exist for a given set of parameters describing the economy. Therefore, the steady state characterizing an economy will be history dependent. Two economies, identical in tastes and technologies, may find themselves in steady states which differ in output, education, and mobility.

3.4 Numerical Analysis of Steady States

In this section we use a numerical approach to examine the properties of $\pi(E)$. To implement this approach, we choose a production function of the form

$$Y_t = K_t^\alpha U_t^{\beta(1-\alpha)} E_t^{(1-\beta)(1-\alpha)} \quad (24)$$

We discretize the continuous distribution of transfers, x , and use a discrete distribution of q , the measure of ability. Specifically, we assume that q has a normal distribution, truncated at 0, with a mean of one and a variance of 0.2. We are then able to calculate a transition matrix for the distribution of transfers conditional on a given level of E . This in turn allows calculation of the conditional steady state distribution, $X_{\text{css}}(x)$ and the resulting level of E in the conditional steady state. This allows us to draw $\pi(E)$. The details of our calculations are discussed further in the Appendix.

Figure 3 shows the $\pi(E)$ correspondence for a baseline set of parameters.¹⁶

Intersections of the $\pi(E)$ curve with the 45 degree line define steady state levels of E . The three panels of the figure are drawn holding other parameters constant, and varying only the cost of education. For sufficiently low costs of education, shown here as $\bar{e}=6.5$, there is a single steady state with mobility. For sufficiently high costs of education, shown here as $\bar{e}=12$, there are no steady states other than those with no mobility. Finally, for an intermediate cost of education ($\bar{e}=10$), the $\pi(E)$ function crosses the 45 degree line more than once, producing multiple steady states in which there is mobility.

It should be noted that this figure does not give any information about out-of-steady-state dynamics, because the $\pi(E)$ function is derived under the assumption that the economy is in steady state. We discuss the model's dynamics further in section 4.

¹⁶The values of the parameters are $\beta=0.5$, $\Theta=30$, $\gamma(1+\bar{r})=0.5$. See the appendix for definition of Θ .

In the case where $\bar{e}=10$, we examine the properties of three steady states. Point A is the steady state with no mobility in which education is highest. Points B and C are steady states with mobility. In the case where $\bar{e}=6.5$, we examine two steady states: Point D is the highest value of E for which there is a steady state with no mobility and point E is the unique steady state with mobility.

The five steady states shown here correspond to panels in Figure 1. Points A and D correspond to panel A in Figure 1 in which there is neither upward nor downward mobility. Point B is characterized by both upward and downward mobility, where downward mobility results when the child of a low-ability educated parent is not able to afford an education. This is panel D of figure 1. At points C and E, there is also both upward and downward mobility, but downward mobility is generated when low-ability children of educated parents choose not to get educated. At point E, any child, even if his parent is uneducated and of low ability, is able to afford an education -- this is panel F of Figure 1. At point C, by contrast, only some children of uneducated parents (those whose dynasty has been characterized by high ability in recent generations) can afford an education -- this is panel G of Figure 1.¹⁷

Each steady state in figure 3 corresponds to a steady state distribution of x , as well as a steady state transition matrix between levels of x . Figure 4 presents the smoothed distributions of wealth and earnings that correspond to points A, B, and C marked in figure 3 ($\bar{e}=10$). At the lower steady state ($E=.09$), there is no mobility between classes and only a

¹⁷The statements about mobility can be verified by looking at Tables 1A and 1B below and by examining the wage structure at each point.

small fraction of the population is able to earn the higher wages paid to educated workers. As a result, the distributions of wealth and earnings are bimodal, with an area of zero probability mass between the upper and lower classes. At the middle steady state ($E=.18$), although there is very little mobility between classes, the disparity between the earnings and wealth of the educated vs. uneducated has diminished. At the upper steady state ($E=.35$) mobility has increased significantly, almost eliminating the bimodality in the distribution of wealth.

Figure 5 presents the distributions of earnings and wealth for steady states corresponding to points D and E. These mirror the distributions at the high and low steady states of the previous example. At the lower steady state with no mobility the distributions of wealth and earnings are bimodal, with only a small percentage of the population in the upper hump. At the higher steady state, the disparity in the earnings distribution has decreased and the wealth distribution is unimodal.

At each steady state we can quantify the degree of mobility using two different measures of mobility. The first is the correlation between wealth and/or earnings of parents and children. The second measure is the relative odds of being uneducated for children of uneducated parents compared to children of educated parents. Specifically, we calculate the odds ratio

$$\frac{P(\text{child is uneducated} | \text{parent is uneducated})}{P(\text{child is uneducated} | \text{parent is educated})} \quad (25)$$

An odds ratio equal to one implies complete mobility between classes. As the odds ratio approaches infinity, mobility approaches zero. Correlation coefficients for wealth and

earnings and the odds ratio at each of the steady states marked on figure 3 are presented in Tables 1A and 1B.¹⁸

These tables reflect the fact that mobility is highest at the high education steady state for both values of \bar{e} . An interesting conclusion from Table 1 is that, when the cost of education is higher ($\bar{e}=10$), even at the high education steady state (corresponding to a more developed economy), there is not complete equality of opportunity. This is evidenced by the fact that the odds ratio is still greater than 1, indicating that there are some people who have ability levels greater than q^* but do not purchase education due to a wealth constraint. The economy is not efficiently allocating resources since it could achieve the same ratio of educated to uneducated efficiency units by educating some of the wealth-constrained high ability individuals.¹⁹ Such a redistribution of education could be achieved at a lower total cost since education costs are allocated per person and not per efficiency unit. At a lower cost to education ($\bar{e}=6.5$), however, the high education steady state generates complete equality of opportunity and the correlation between parents' and children's earnings is zero.

¹⁸We also experimented with using the rank correlation between parents' and children's places in the wealth or income distributions as measures of mobility. This measure indicates that at the low education steady state, when a large percentage of the population is uneducated, mobility is high. The reason for this is that when a large percentage of the population is uneducated, it is relatively easy to move from the lowest rank in the earnings distribution to a rank near the top even if the purchase of education is constrained by wealth. Thus, an economy with no mobility between classes but with most people in a single class will show a high degree of mobility.

¹⁹We use the term efficient here and in the next section to refer to an outcome in which output is maximized for a given level of inputs.

3.5 Policy Evaluation

The analysis above demonstrates that, even in an economy in which mobility exists, an inefficient allocation of education can result. In this section, we extend our numerical analysis of this economy to evaluate several different policies designed to address the inefficiency. The simplest of these policies is a program of student loans—all individuals are permitted to borrow \bar{e} and repay the loan out of their first period wages.²⁰ The student loan case serves as a benchmark to show the effect of liquidity constraints, the key imperfection in the model. Table 2 gives the mean and standard deviation of steady state consumption in the original, liquidity-constrained version of our model as well as for the model with a student loan program.

As can be seen by comparing the two high education steady states when $\bar{e}=6.5$, a student loan program has no effect on the distribution of consumption if no individuals are liquidity constrained. However, the student loan program does create an increase in the average consumption level in all other cases by allowing some previously constrained individuals to purchase education.²¹

²⁰Since education and work take place in the same period, the interest rate on educational loans is necessarily zero. While a positive interest rate on loans would affect the quantity of education, it would not affect our conclusions regarding the optimality of this policy.

²¹If individuals were able to sign contracts before ability was revealed, there would be the possibility of insuring the shock. Alternatively, in a model with altruistic preferences, parents could insure their children's ability. In a world with no borrowing constraints and complete ability insurance, the mean level of consumption would be the same as in such a world without insurance, but the cross-sectional variance of consumption would be zero. When there are borrowing constraints, analysis of the effects of ability insurance becomes quite complicated. For example, a poor individual might want to buy insurance that paid off if he had high ability (so that he could afford an education) or if he had very low ability (to provide extra consumption). We would guess that the moral hazard and adverse selection

The second policy that we consider is the public provision of education. Here two questions immediately arise: first, who receives education; and second, how much education is provided? To answer the first question, we assume that education is provided on a meritocratic basis: that is, it is given to the highest ability individuals, who will profit the most (in terms of increased wages) from it. The alternative assumption--that education is provided to children on the basis of their parent's wealth or their transfer receipt--would lead to a model much like the original one. In reality, both ability and parental wealth are determinants of education.

To answer the second question, we assume that education is provided to the point that maximizes the net-of-education level of output produced, assuming that the government is able to raise this much revenue.²² Assuming that the government's revenue constraint does not bind, we get the interesting result that the distribution of education when it is provided publicly exactly matches the distribution of education when non-liquidity constrained people purchase it for themselves.²³

Of course, it might not be possible for the government to provide this much education. In an economy with a low level of output, total wage income could easily be less than what

²²An alternative assumption might be that education was provided to everyone who wanted it -- that is, until such a point that the wages of educated and uneducated workers were equalized. In this case, however, individuals would not consider the full costs of becoming educated in their decision and education would be over-provided.

²³To see this point, note that output net of education costs is maximized when the marginal product of an educated efficiency unit less the cost of educating one efficiency unit is equal to the marginal product of an uneducated efficiency unit. Specifically, when $w^e - e/q^* = w^u$, output net of education expenses is maximized. This is exactly the criteria faced by the non-constrained individual.

would be required to educate all of the population for which education would be efficient. This raises the question of whether, assuming the parameters are such that there exist steady states both with and without mobility, government policy would always be able to *eventually* move the economy out of a steady state with no mobility and up to the level of output that would be achieved in the *non-liquidity constrained* economy.

The answer to this question depends on the form of the $\pi(E)$ function, which in turn depends on the cost of education. If the cost of education is sufficiently low, so that $\pi(E)$ has the shape shown in figure 2A, then it will always be possible for the government to move the economy to the high education steady state. Specifically, consider a policy in which all income is taxed away and spent on education for the most able. Consider a value of E associated with a steady state with no mobility. Since at the wages implied by this value of E , all educated dynasties paid for their children's education, regardless of the children's ability, seizing all income and spending it on education of the most able students will certainly raise the value of E in the next period. If the resulting value of E is such that the $\pi(E)$ function lies above the 45 degree line, E will continue to rise, since for this level of wages even some uneducated dynasties would have been able to afford an education. If the resulting value of E is such that $\pi(E)$ lies on the 45 degree line, a repeated application of the policy will, by the argument above, increase E as well. But if $\pi(E)$ has the form shown in figure 2C, this conclusion does not hold. In this case, it may happen that even a draconian policy of income confiscation and meritocratic public education cannot generate enough resources to move the economy to the level of education seen in the steady state with *mobility*.

4 Dynamic Analysis

We now turn to analysis of the dynamics of our model. The analysis of steady states presented above gives little information about the stability of the different steady states. We wish to ask under what conditions the economy will be attracted to one or another of the different steady states.

Analysis of the dynamics of the model is complicated by the fact that the state of the economy cannot be summarized by one or two variables. Rather, the state of the economy is represented by the entire distribution of transfers, $X_t(x)$. Two economies with similar values for aggregate variables such as the fraction educated or average wealth, but different distributions of wealth, will evolve differently over time.

We confine ourselves to numerical analysis of the model. Analysis of dynamics uses the same basic machinery described above. The dynamic model can be described with the following set of equations:

$$\begin{aligned} E_t &= J(X_t(\cdot), \{w^u, w^e\}_t) \\ \{w^u, w^e\}_t &= W(E_t) \\ X_{t+1}(x) &= G(\{w^u, w^e\}_t, X_t(\cdot)) \end{aligned} \tag{26}$$

where $X_0(x)$ is given. Note that the levels of human capital and wages in period t are determined simultaneously: the decision to purchase an education depends on the equilibrium level of wages. Solving W and J simultaneously yields human capital and wages in period t as a function only of the distribution of transfers received in period t . We can define H as the solution of these two equations, which gives wages in period t as a function of transfers

received in period t:

$$\{w^u, w^e\}_t = H(X_t(x)) \quad (27)$$

The distribution of transfers received in period t+1 is, in turn, a function of transfers received and wages in period t. Thus, transfers received in period t+1 can be written as a function only of transfers received in period t:

$$X_{t+1}(x) = G(H(X_t(\cdot)), X_t(\cdot)) \equiv \Phi(X_t(\cdot)) \quad (28)$$

where $X_0(x)$ is given. Using this function, we can trace the evolution of an economy through time.

4.1 Stability of Steady States

In order to examine the local stability of steady states, we consider small perturbations of the wealth distributions that correspond to the steady states depicted in Figure 3.

Figure 6 shows the effect of shocking the steady state wealth distributions in the case of $\bar{e}=6.5$, that is, where there is a unique steady state with mobility. The lower steady state wealth distribution (corresponding to point D in figure 3) was perturbed in two ways: once by moving a small percentage of the population from the point below \bar{e} to the point above \bar{e} , and once by moving a small percentage of the population in the other direction. Since at the upper steady state (corresponding to figure 3, point E), all individuals already have wealth greater than \bar{e} , our experiment at this point was limited to examining the effects of a negative shock to the steady state wealth distribution. The figure shows the path of E over time and

demonstrates that the steady state with mobility is locally stable. In the case of the steady state with no mobility, we get an interesting result. A negative shock to wealth moves the economy to a different steady state without mobility. A positive shock to wealth, however, puts the economy on a path to the steady state with mobility.

Figure 7 looks at the case where $\bar{e}=10$, where there are two steady states with mobility. We conduct similar experiments at each of these steady states with somewhat different results. In this case, a small positive shock to the lower steady state does not allow this economy to escape this poverty trap. In addition, the middle steady state is unstable. A positive shock to the middle steady state results in the economy converging to the upper steady state while a negative shock to the middle steady state results in the economy moving towards the lower steady state. As in the previous case, the upper steady state is locally stable²⁴.

One can also see from this graph that the transition from the middle steady state to the upper steady state occurs more rapidly than the transition from the middle steady state to the lower steady state. The reason for the difference in the speed of convergence towards the upper and lower steady states is that the steady state distribution of wealth at the mid-level steady state is one in which the wealthy people have accumulated a great deal of wealth.

²⁴The comparison between the dynamics of economies with two different costs of education suggests that it would be interesting to study the dynamics of an economy in which the cost of education changed with per capita income. The cost of education would be a decreasing function of E if there were fixed costs in the education sector or if the cost of education was an increasing function of w^e (i.e., if educated workers provided education). Since a decreased cost of education and a higher wage to uneducated workers both increase mobility, our results relating growth and increased mobility would be strengthened under such an assumption.

This accumulation of wealth creates inertia in this system since mobility from the educated to uneducated class will happen only after a dynasty has suffered a large number of low draws from the ability distribution. In addition, since wages to the educated increase as E decreases, the movement of E decelerates as it approaches the low steady state. When an economy is growing towards the upper steady state, the opposite phenomenon occurs. As E increases, wages to the uneducated increase, removing the wealth constraint for more children and causing the growth of E to accelerate.

5 Conclusion

In this paper we have examined a model in which the levels of output and human capital accumulation, the degree of income inequality, and the rate of intergenerational mobility are all endogenously determined. We find that there is the possibility of multiple equilibria: two economies with identical taste and technology parameters but with different initial wealth distributions can end up in different steady states, one with high income, education, and mobility; the other with lower levels of these measures and with a higher level of income inequality. An interesting feature of the steady states with mobility is that, due to the existence of both upward and downward mobility, individual families within an economy can advance economically even though the aggregate level of per capita income remains constant.

In addition, we find that the causation between intergenerational mobility and growth in per capita income runs both ways. Mobility increases as a result of changes in the wage structure that accompany economic growth. In particular, increases in the fraction of the labor force that is educated reduce the wage gap between educated and uneducated workers,

thus raising the probability that the children of uneducated workers will be able to afford an education. Conversely, economic growth occurs as intergenerational mobility allows a more efficient allocation of resources. Mobility-induced growth may initially occur at increasing rates as the economy moves towards a higher level of per capita income.

Using our model, we analyzed the effect of different education finance policies on mobility, income distribution, and the mean level of output. In the absence of policy, the most able individuals, who would most profit from education, may not be able to afford it. Removing barriers to the acquisition of education, either by removing liquidity constraints or by publicly funding education, increases per capita output and the mean level of consumption. An important result from this analysis is that the benefit derived from such policy can depend on the equilibrium in which the economy finds itself prior to the imposition of policy.

The analysis of policy that we present only scratches the surface, however. While a policy of allowing freer access to education improves the lot of the majority of families, in the absence of a redistribution scheme, there are dynasties that will lose by the policy. In particular, in an equilibrium with no (or low) mobility, members of wealthy dynasties earn high wages even if they have low ability because they are able to afford an education. We have not examined the positive question of when such policies are likely to be implemented, but we consider this to be a profitable area for future research.

The model presented here is not an endogenous growth model. Although this type of transitional growth may be sustained for several generations, once a steady state has been reached, output per person is constant. Steady state growth could occur with the introduction of technological progress. A natural way to endogenize the technological progress would be

to consider an aggregate production externality that increases with the average level of education or human capital in the economy. In such a case, a more mobile, more highly educated economy would also have higher steady state income growth.

Appendix

A.1 Proofs of Propositions Stated in Text

Proof of A.1.1 (A) relies on the following result presented in Hopenhayn and Prescott (1992):

H&P Theorem 2: Suppose P is increasing, S contains a lower bound (which we will denote by a) and an upper bound (which we will denote by b), and the following condition is satisfied:

Monotone Mixing Condition: There exists a point $s^* \in S$ and an integer m such that $P^m(b, [a, s^*]) > 0$ and $P^m(a, [s^*, b]) > 0$.

Then there is a unique stationary distribution λ^* for process P and for any initial measure μ , $T^n \mu = \int P^n(s, \cdot) \mu(ds)$ converges to λ^* .

A.1.1 Proofs of Propositions About Conditional Steady States

A) There is a unique conditional steady state when there is mobility between classes.

Let $P^N(x, \cdot)$ give the transition probabilities from wealth x after N generations. Noting that

- (i) P is increasing. The monotonicity of P can be shown by examining equation 10 and noting that increasing x_t results in an increase in x_{t+1} for every realization of q_t ,
- (ii) The upper bound of the distribution of wealth is $\max(\bar{x}, x^*)$ and the lower bound of the distribution of wealth is $\min(x, x^{**})$ where x^* is the largest inheritance received by any individual at $t=0$ and x^{**} is the smallest inheritance received by any individual at $t=0$. \bar{x} and x are the upper and lower bounds of the recurrent distribution (see equations 13 and 14).
- (iii) The Monotone Mixing Condition applies when there is mobility between classes.

This can be seen by first noting that within a class, wealth mobility is determined by the realization of ability. Since ability is i.i.d. among individuals, there is complete wealth mobility within a class. When there is class mobility, the Monotone Mixing Condition follows directly from the definition of class mobility.

we can then apply H&P Theorem 2 to show that the distribution of wealth converges to a unique distribution.

B) There is an infinite number of conditional steady states when there is no mobility between classes.

Proof: Let A be the set of all possible transfers from educated parents and B the set of all possible transfers from uneducated parents. When there is no mobility between classes, $P^N(a_1, [b_1, b_2]) = 0$ and $P^N(b_1, [a_1, a_2]) = 0$ for all $b_1, b_2 \in B$ and $a_1, a_2 \in A$ and $N \geq 1$. As shown in Part (A), $P^N([a_1, a_2], [a_2, a_3]) > 0$ for all $a_1, a_2, a_3 \in A$ and $P^N([b_1, b_2], [b_2, b_3]) > 0$ for all $b_1, b_2, b_3 \in B$. Thus, when there is no mobility between classes, the state space can be divided into two ergodic sets, A and B , which in turn correspond to two invariant measures. An invariant measure over the entire state space will be a convex combination of these two invariant measures. Since there are an infinite number of convex combinations, there are an infinite number of invariant measures and thus of distributions of wealth.

A.1.2 Proofs of Propositions About Unconditional Steady States

Proposition 1: There does not exist an unconditional steady state in which there is mobility in only one direction.

Proof: Suppose not. Suppose mobility in the downward direction exists but there is no mobility in the upward direction. Then as $t \rightarrow \infty$, $E \rightarrow 0$. Since

$$\lim_{E \rightarrow 0} q^* = 0 < \underline{q} \quad \text{and} \quad \lim_{E \rightarrow 0} w^e = \infty$$

and given (18) (condition for downward mobility) this leads to a contradiction. A similar argument holds for mobility in the upward direction only.

Proposition 2: *For any set of parameters there exists an unconditional steady state with no mobility.*

Proof: By (17) and (18) (conditions for upward and downward mobility), there is no mobility between classes when

$$\frac{\gamma(1 + \bar{r})\bar{q}w^u}{1 - \gamma(1 + \bar{r})} \leq \bar{e} < \gamma\underline{q}(1 + \bar{r})w^e \quad (\text{NM})$$

By (4)

$$\lim_{E \rightarrow 0} w^u = 0 \quad \text{and} \quad \lim_{E \rightarrow 0} w^e = \infty$$

Therefore, by choosing E within an ε -neighborhood of 0, condition NM is satisfied

Proposition 3: *Given any values of the other parameters that satisfy the assumptions stated in the text, there exists a value of $\bar{e} > 0$ low enough such that there exists an unconditional steady state with mobility.*

Remark: In particular, we show there is a steady state with "perfect mobility" in which education does not depend on parental wealth, such as that found on the second line of Table 1B. We do not prove here the existence of a steady state with imperfect mobility in which education does depend on parental wealth, although our numerical exercise identifies such a steady state (Table 1A, line 3).

We prove this result in the following three steps:

Step 1: We show for $\bar{e} = 0$, there exists an unconditional steady state with mobility.

Step 2: We show the proposition to be true for $\bar{e} > 0$ but ignoring the restriction that $x_{i,t} \geq \bar{e}$ in order for an individual to obtain education.

Step 3: Finally, we show that we can choose an \bar{e} that is small enough so that the restriction $x_{i,t} \geq \bar{e}$ is not binding.

Step 1: Let $\bar{e} = 0$. Then, it must be the case that $w^e = w^u$. (If not, then $E=1$ or $E=0$. But $E=1$ contradicts $\lim_{E \rightarrow 1} w^u = \infty$ and $E=0$ contradicts $\lim_{E \rightarrow 0} w^e = \infty$) Denote the number of educated efficiency units consistent with $w^e = w^u$ as E_0 . Since, in this case, all individuals are indifferent between working as educated or uneducated workers, we can choose a rule that assigns a set of agents to the educated class by ability level. In particular we can choose a q^* such that all individuals with $q \geq q^*$ become educated. Because there will be intergenerational mobility under such a regime, we can invoke result A.1.1 (A) to claim that there will be an invariant distribution. In addition, we can choose q^* such that, given the distribution of abilities $Q(q)$, $E=E_0$. Thus, an unconditional steady state with mobility exists when $\bar{e} = 0$. The invariant distribution of transfers has support $[x, \bar{x}]$ and, noting equation (14), $x > 0$.

Step 2: Let $\bar{e} > 0$ but $x_{i,t} \geq \bar{e}$ not be a binding restriction for any individual i to obtain education. Define $E_1(\bar{e}) = E : (w^e - w^u) = \bar{e}/q$ and $E_2(\bar{e}) = E : (w^e - w^u) = \bar{e}/q$. $E_1(\bar{e})$ is the value of E at which the lowest ability person is indifferent between getting an education or not getting an education and $E_2(\bar{e})$ is the value of E at which the highest ability person is indifferent about education. When $x_{i,t} \geq \bar{e}$ is not binding, intergenerational mobility will occur and by result A.1.1 (A), we know that for each $E \in [E_1(\bar{e}), E_2(\bar{e})]$ there is a unique invariant distribution.

This unique invariant distribution defines uniquely the value of $\pi(E)$.

We know that: (1) w^u and w^e are continuous functions of E , (2) q^* is a continuous function of w^e and w^u , and (3) when liquidity constraints do not bind, E is a continuous function of q^* . Thus, when liquidity constraints do not bind, $\pi(E)$ is continuous for $E \in [E_1(\bar{e}), E_2(\bar{e})]$. Clearly, $\pi(E_1(\bar{e})) = 1$ and $\pi(E_2(\bar{e})) = 0$. Thus, given the continuity of $\pi(E)$, a fixed point can be found in $[E_1(\bar{e}), E_2(\bar{e})]$.

Step 3: We wish to prove that there exists an $\bar{e} > 0$ such that $x_{i,t} > \bar{e}$ is not binding. By equation (14), the lower bound of the support of X , x , is a continuous function of E . In Step 1, we showed that for $E(0) = E_0$, $x > 0$. Thus for a small enough \bar{e} , we must have $x > \bar{e}$.

Proposition 4: *Given any values of the other parameters that satisfy the assumptions stated in the text, there exists a value of $\bar{e} > 0$ high enough such that there does not exist an unconditional steady state with mobility.*

Proof: Following Proposition 2, define E^* as a level of E such that there is no mobility.

Define λ as the upper bound of the steady state distribution of transfers among the educated

$$\lambda = \frac{\gamma(1 + \bar{r})(\bar{q}w^e - \bar{e})}{1 - \gamma(1 + \bar{r})}$$

Since $\partial\lambda/\partial\bar{e} < 0$, if we increase \bar{e} to equal $\lambda + \epsilon$, for any $\epsilon > 0$, the upper bound decreases and $x_i < \bar{e}$ for all i . Thus, $\pi(E^*) = 0$. Since $\partial\lambda/\partial E < 0$, given our choice of \bar{e} , $\pi(E^* + \delta) = 0$ for all $\delta > 0$. Therefore, there are no fixed points of $\pi(E)$ that generate mobility. This implies no unconditional steady state with mobility exists.

A.2 Numerical Analysis

Under the assumptions outlined in Section 2, wages can be expressed as

$$w_t^u = \beta \theta \left(\frac{E_t}{1-E_t} \right)^{1-\beta} \quad \text{and} \quad w_t^e = (1-\beta) \theta \left(\frac{E_t}{1-E_t} \right)^{-\beta}$$

where

$$\theta \equiv (1-\alpha) \left(\frac{\alpha}{r} \right)^{\frac{\alpha}{1-\alpha}}$$

As noted in section 2, the evolution of transfers within a dynasty is governed by a Markov process. Given values of $x_{i,t}$ and $q_{i,t}$, and holding constant the wages of educated and uneducated workers, equation (10) gives the exact value of $x_{i,t+1}$. Alternatively, if one does not know the actual value of the parent's ability, $x_{i,t+1}$ is a function of the random variable q with a distribution as specified in equation (12). Our approach will be to approximate (12) by calculating a transition matrix, T . Since the value of x_{t+1} depends not only on x_t but also on $\{w^u, w^e\}_t$, and since $\{w^u, w^e\}_t$ depends on E_t , the transition matrix between periods t and $t+1$ will depend on the level of education in period t . We call this matrix $T(E_t)$. For the population as a whole, the distribution of x in period $t+1$ can be written as a function of the distribution of x in period t :

$$X_{t+1} = T(E_t)X_t$$

We begin by dividing the continuous distribution of x into a large number (200) of discrete values. It is also necessary to choose a discrete distribution for q , the random variable that measures ability. Specifically, we assume that q takes the form where η has a normal distribution with a mean of zero and a variance of 0.2. We use 65

$$q_{i,t} = 1 + \eta_{i,t}$$

discrete values of η to approximate this distribution, and truncate the distribution to ensure that $q_{i,t} > 0$. Results for different levels of the variance of η were not materially different from those presented here.

Given the distribution of q and values of the two wage rates, it is then straightforward to calculate the transition matrix $T(E)$.²⁵ Finally, the conditional steady state distribution, X_{css} , is a convex combination of the rows of the limit matrix, P , where

$$P = \lim_{n \rightarrow \infty} T^n$$

Since, as shown in Appendix A1, steady states with mobility have a unique invariant distribution, when there is mobility, a row of P is the conditional steady state distribution. We also show in Appendix A1 that, for steady states with no mobility, P converges to a block diagonal matrix with a unique invariant distribution of transfers among the educated and a unique invariant distribution of transfers among the uneducated. In this case, the conditional steady state distribution is a weighted average of the two invariant distributions with the weight E on the educated distribution and weight $1-E$ on the uneducated distribution.

For each value of E , we can calculate the implied wages and the transition matrix produced by these implied wages. We can then use this transition matrix to calculate the fraction of workers who would be educated in a conditional steady state. Thus we can trace

²⁵Specifically, to calculate each row of T , we start with the value of x_t and, for each possible value of q_t , consider the resulting value of x_{t+1} . We then find the discrete value of x that is closest to this value of x_{t+1} , and add $1/65$ (where 65 comes from the number of discrete values of η) to this cell of the transition matrix.

out the $\pi(E)$ function.

References

- Barro, R. J., N. G. Mankiw, and X. Sala-i-Martin, 1995, "Capital Mobility in Neoclassical Models of Growth," *American Economic Review*, 85, 103-115.
- Becker, G. S. and N. Tomes, 1986, "Human Capital and the Rise and Fall of Families," *Journal of Labor Economics*, 4, S1-S39.
- _____ and _____, 1979, "An Equilibrium Theory of the Distribution of Income and Intergenerational Mobility," *Journal of Political Economy*, 87(6), 1153-1189.
- Benabou, R., 1992, "Heterogeneity, Stratification, and Growth," mimeo, M.I.T..
- _____, 1993, "Workings of a City: Location, Education, and Production," *Quarterly Journal of Economics*, 108, 619-652.
- Borjas, George J., 1992, "Ethnic Capital and Intergenerational Mobility," *Quarterly Journal of Economics*, 107(1), 123-150.
- Durlauf, Steven N., 1994, "Spillovers, Stratification, and Inequality," *European Economic Review* 38, 836-845.
- Galor, Oded and Daniel Tsiddon, 1997, "The Distribution of Human Capital and Economic Growth," *Journal of Economic Growth*, 2, 96-124.
- Galor, Oded and Joseph Zeira, 1993, "Income Distribution and Macroeconomics," *Review of Economic Studies*, 60, 35-52.
- Hopenhayn, Hugo A. and Edward C. Prescott, 1992, "Stochastic Monotonicity and Stationary Distributions for Dynamic Economies," *Econometrica*, 60, 1387-1406.

- Loury, Glenn C., 1981, "Intergenerational Transfers and the Distribution of Earnings," *Econometrica*, 49, 843-867.
- Lucas, Robert E., Jr., 1988, "On the Mechanics of Economic Development," *Journal of Monetary Economics*, 22, 3-42.
- Owen, Ann L., 1996, "The Distribution of Education and Long-Run Growth," mimeo, Federal Reserve Board of Governors.
- Psacharopoulos, George, 1993, "Returns to Investment in Education: A Global Update," World Bank Policy Research Working Paper 1067.
- _____, "Returns to Education: A Further International Update and Implications," 1985, *The Journal of Human Resources*, 583-600.
- Romer, Paul M., 1986, "Increasing Returns and Long-Run Growth," *Journal of Political Economy*, 94, 1002-1037.
- Stokey, Nancy L. and Robert E. Lucas Jr. with Edward C. Prescott, 1989, *Recursive Methods in Economic Dynamics*, Cambridge, (Harvard University Press, Massachusetts).
- Tamura, Robert, 1993, "From Decay to Growth: A Dynamic Equilibrium Model of Income Distribution," mimeo.
- Williamson, J.G., 1985, *Did British Capitalism Breed Inequality?* (Allen & Unwin, Boston).

TABLE 1A: Mobility In Steady States: $\bar{e}=10$

Steady State	Wealth Mobility	Earnings Mobility	Occupational Mobility
	Correlation Coefficients		Odds Ratio
E = .09	.96	.86	∞
E = .18	.89	.70	280.02
E = .35	.56	.16	2.46

TABLE 1B: Mobility In Steady States: $\bar{e}=6.5$

Steady State	Wealth Mobility	Earnings Mobility	Occupational Mobility
	Correlation Coefficients		Odds Ratio
E = .06	.97	.86	∞
E = .41	.46	0	1

TABLE 2: Policy Evaluation--Student Loans

		$\bar{e}=10$		$\bar{e}=6.5$	
	Steady State	Consumption Mean	Consumption SD	Consumption Mean	Consumption SD
Liquidity Constrained Case	high education steady state	11.72	2.41	12.79	2.41
	mid education steady state	7.11	4.43	--	--
	low education steady state	6.69	9.87	2.86	5.97
Student Loan Case	high education steady state	11.81	2.44	12.79	2.41

Figure 1

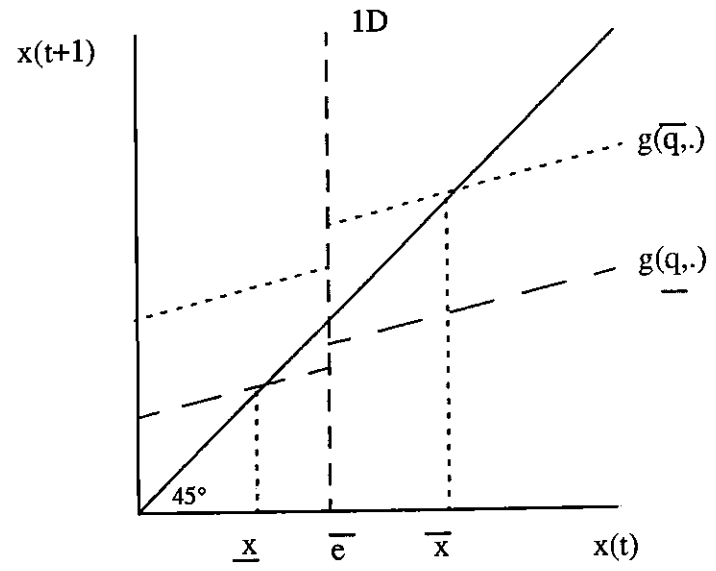
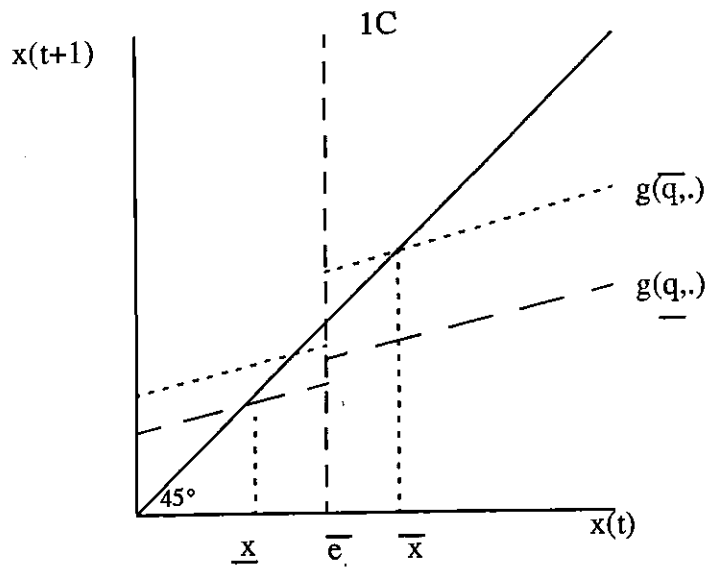
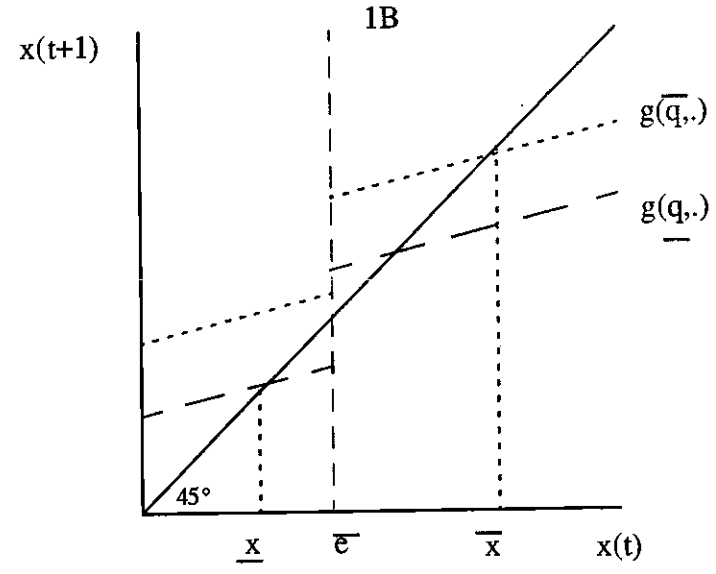
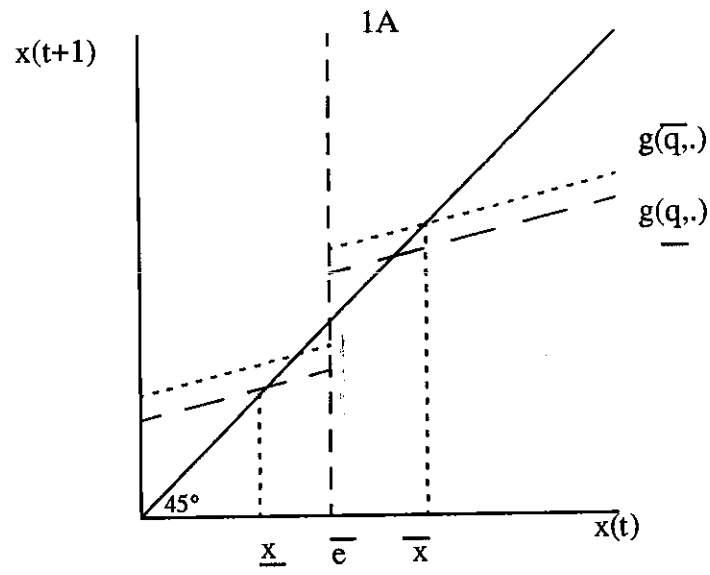


Figure 1

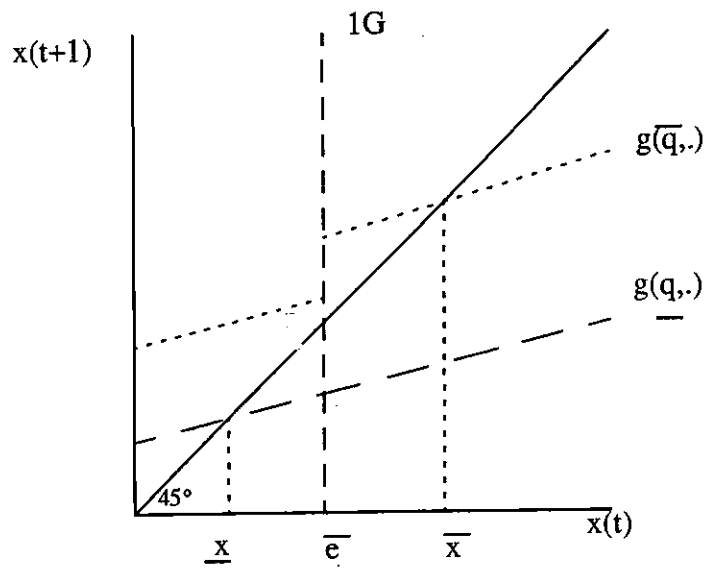
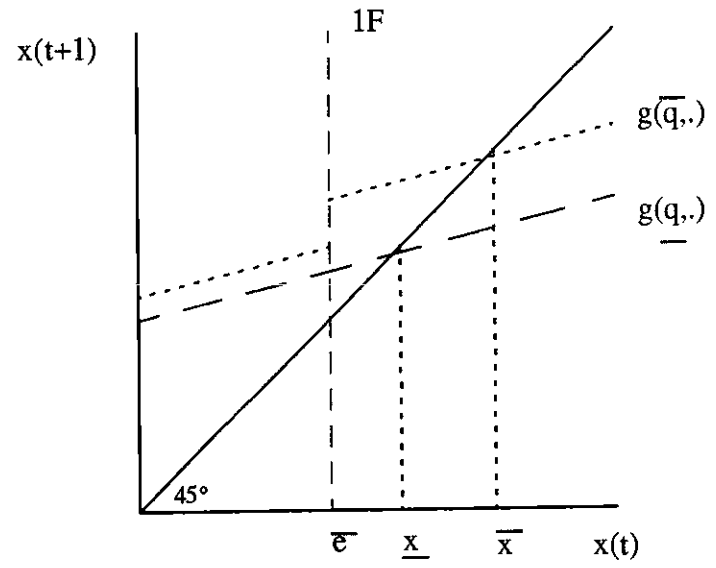
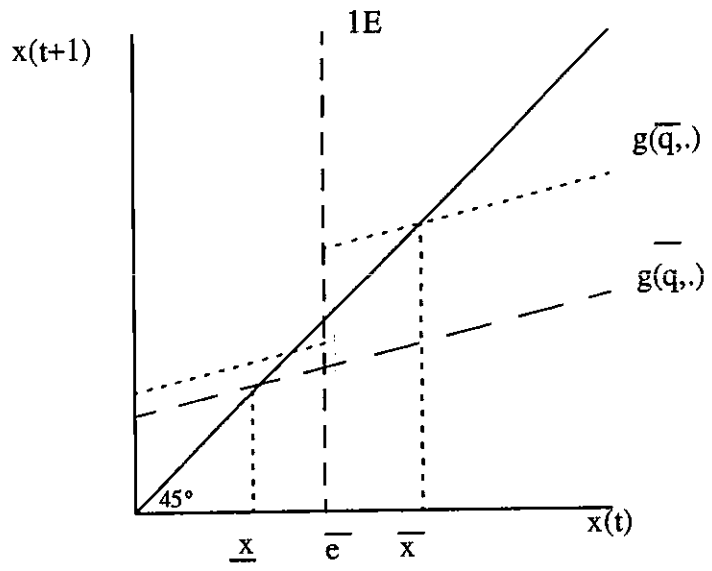


Figure 2

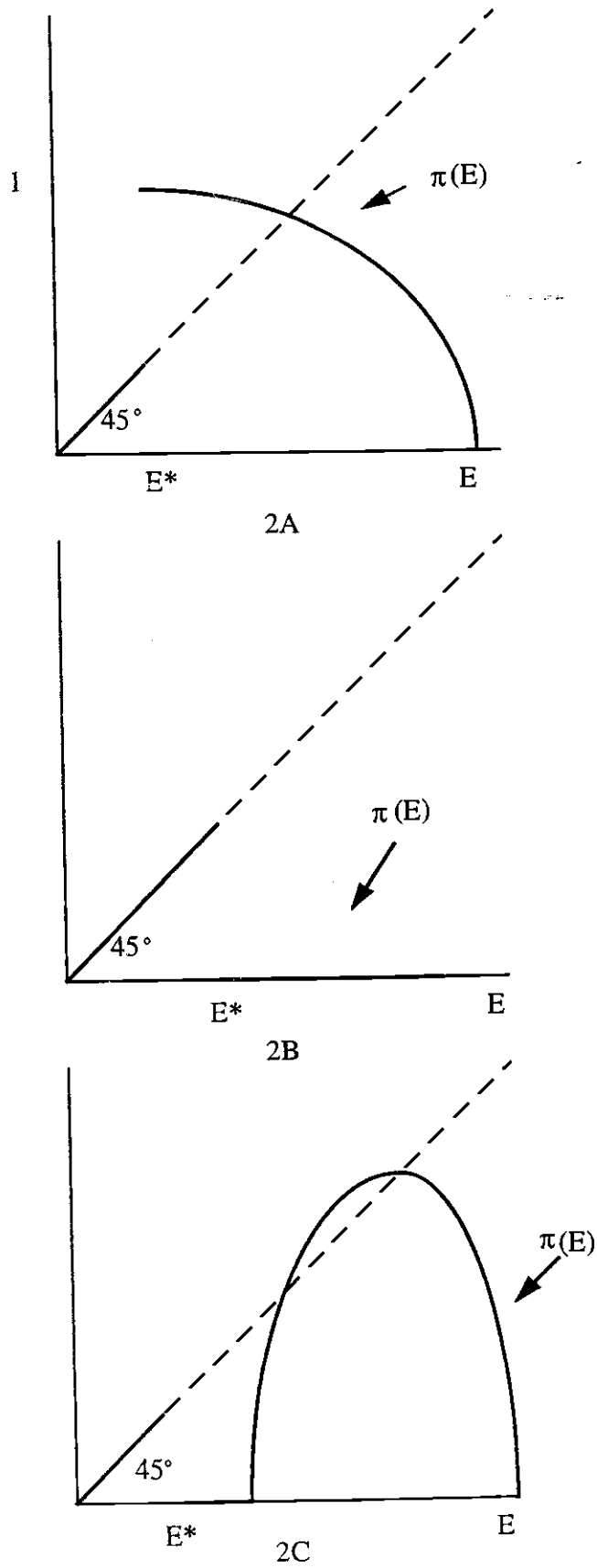


Figure 3
 $\pi(E)$

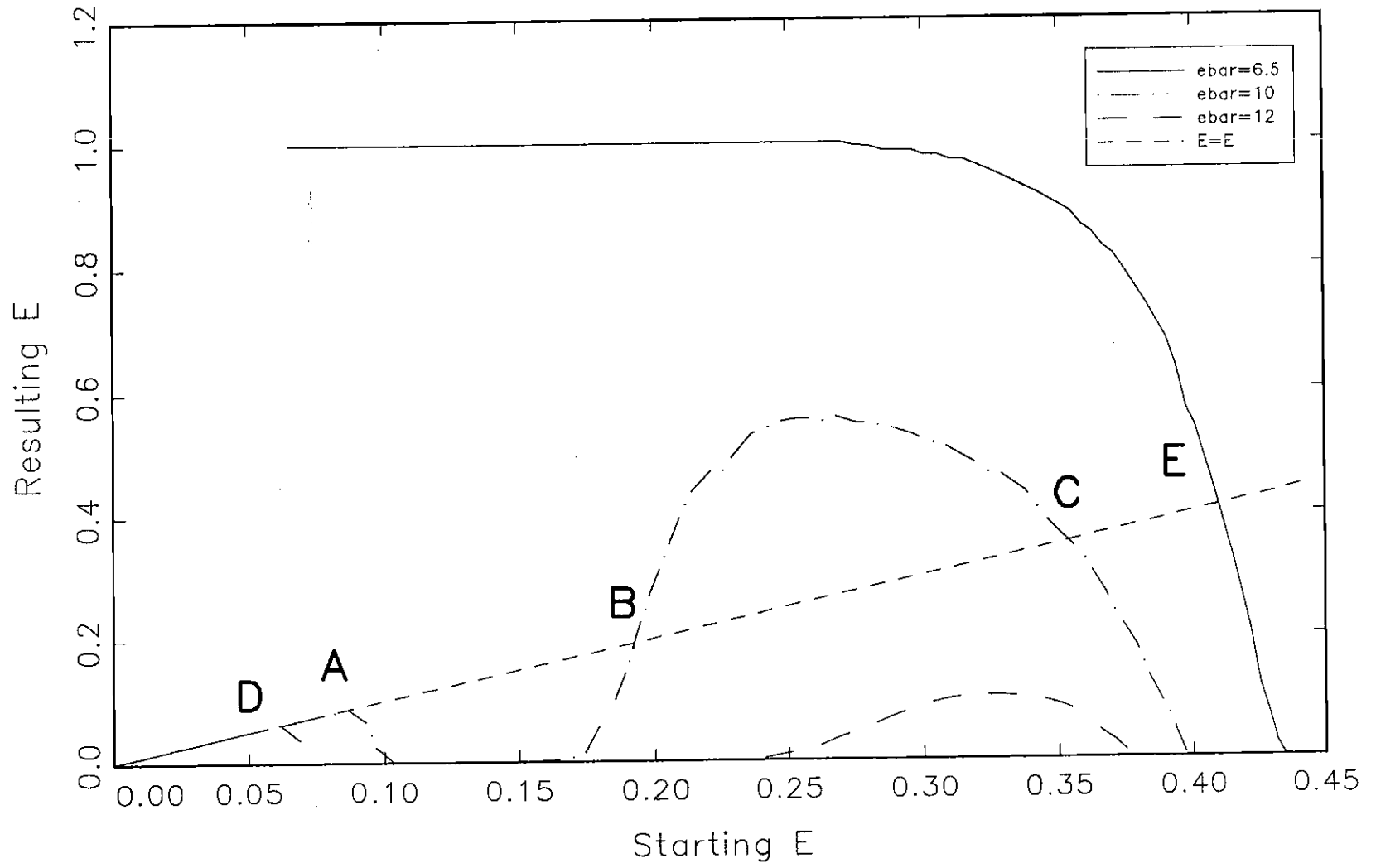


FIGURE 4

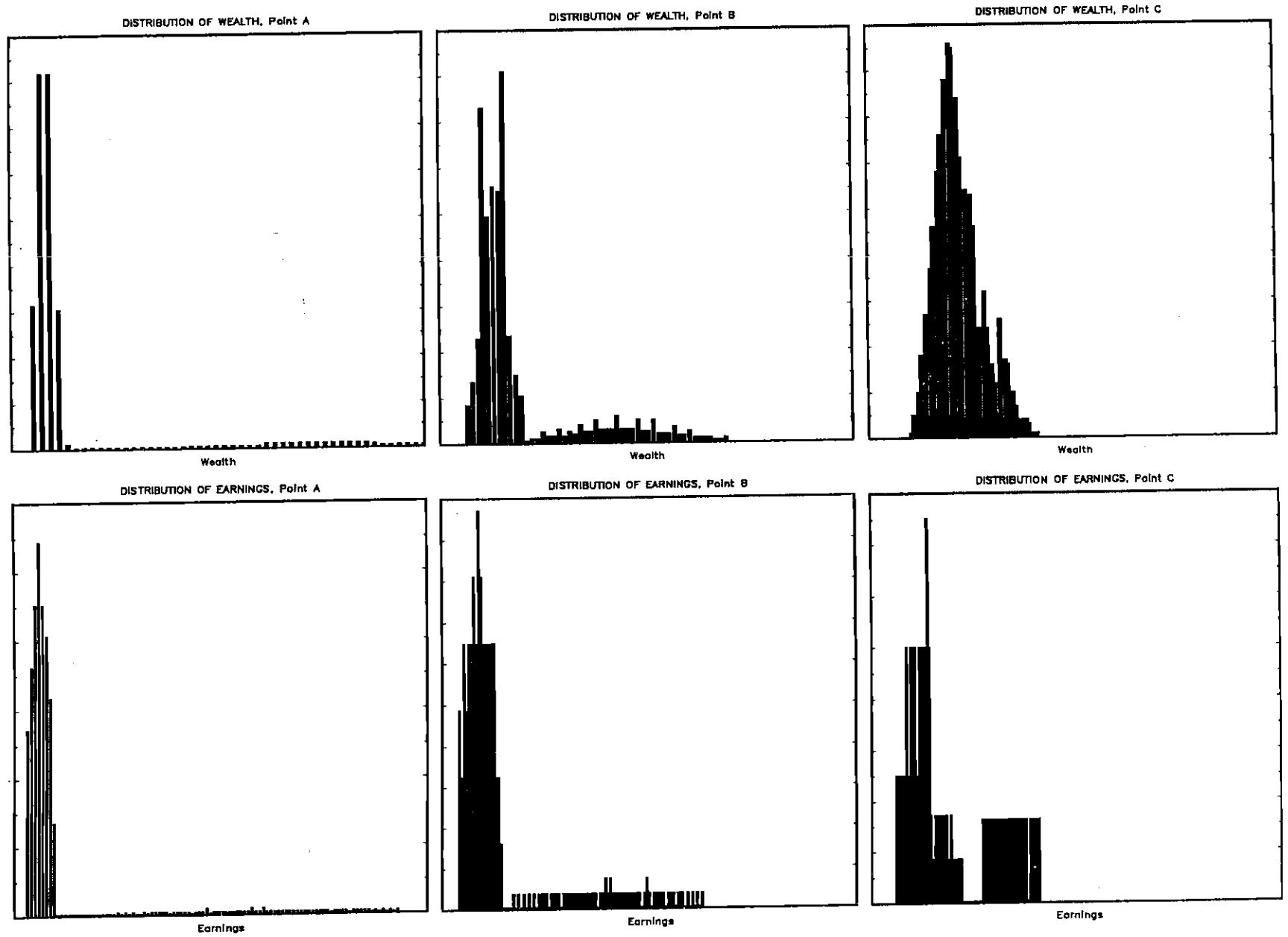
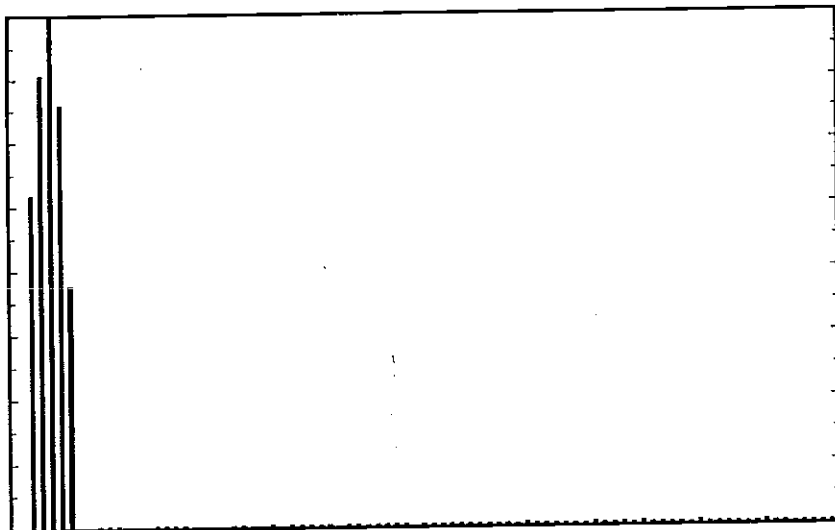


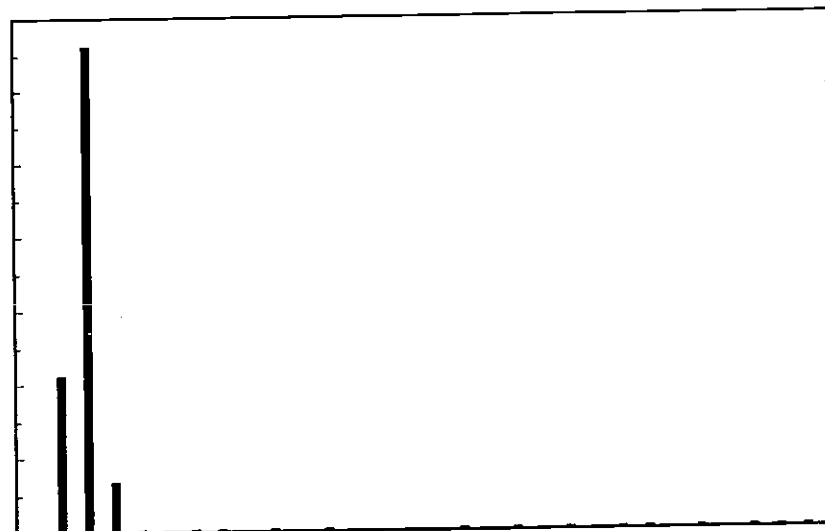
FIGURE 5

DISTRIBUTION OF EARNINGS, Point D



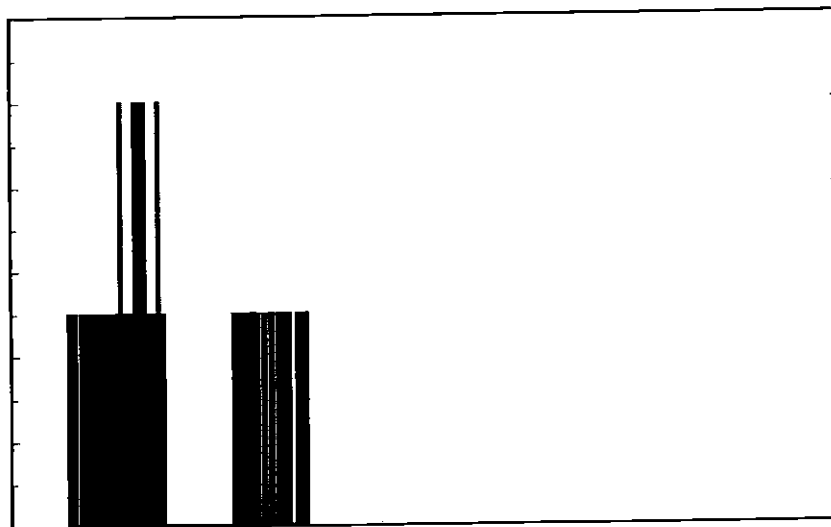
Earnings

DISTRIBUTION OF WEALTH, Point D



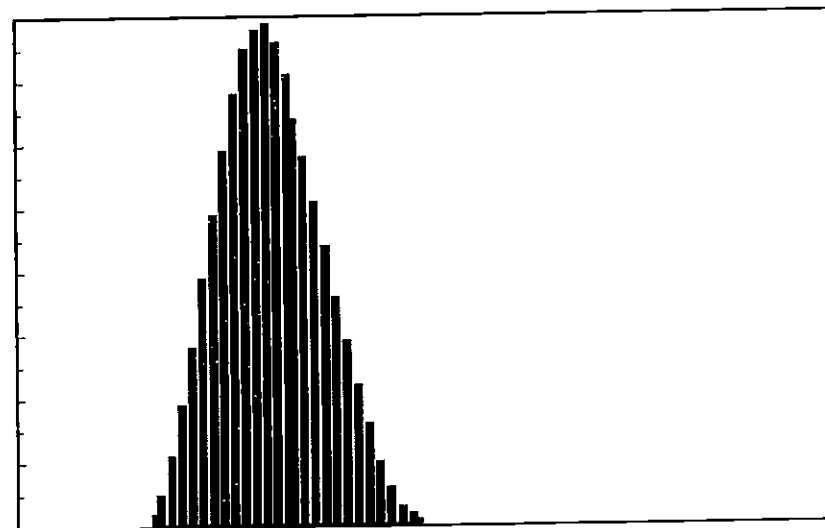
Wealth

DISTRIBUTION OF EARNINGS, Point E



Earnings

DISTRIBUTION OF WEALTH, Point E



Wealth

Figure 6
Shocks to Steady States, $\bar{e}=6.5$

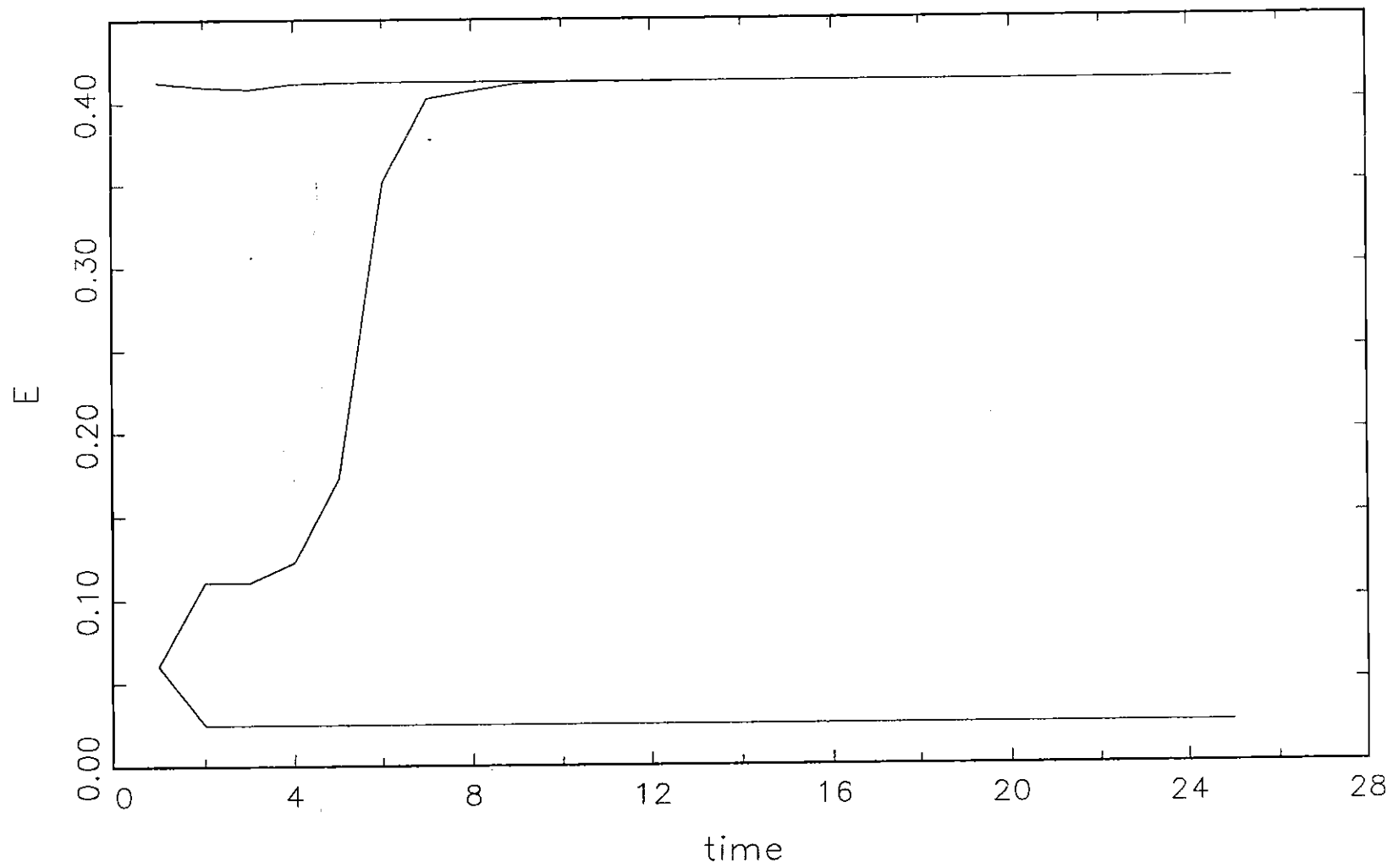


Figure 7
Shocks to Steady States, $\bar{e}=10$

