NBER WORKING PAPER SERIES

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Working Paper 5729

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
August 1996

This paper is part of NBER's research program in Labor Studies. Any opinions expressed are those of the authors and not those of the National Bureau of Economic Research.

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ABSTRACT

We present a two-period model of the assignment market with uncertainty in the first period regarding productive characteristics of participants. This model is used to understand incentives toward early contracts or unraveling in labor markets for entry-level professionals. We study two contractual situations, one where firms are bound by *ex post* unsuccessful early contracts, and the other where they can buy out of unsuccessful early contracts. The economic benefit of unraveling is to provide insurance in the absence of complete markets, but it can come at the cost of inefficient assignments. Without reentry, unraveling need not occur. It is more likely, the smaller the applicant pool or the proportion of more promising applicants in the pool, and the greater the degree of heterogeneity in the pool. A ban on early contracts always hurts firms and benefits less promising applicants, but the welfare effects on more promising applicants depends on how the gains from early contracts are shared. With buyouts, inefficiencies in assignments are eliminated, and unraveling always occurs between firms and the more promising applicants. The efficiency gains of buyouts can be distributed unevenly and sometimes firms benefit from a ban on buyouts.

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1. Introduction

The timing of transactions is important in markets where buyers and sellers need to be matched to each other. There are many examples. Elite colleges have both regular and early admission programs. Most graduate and undergraduate admission procedures agree not to inform applicants prior to a common date. New entrants into many professional sports competitions choose when to expose themselves to the draft. Much attention lately has been focused on the trend toward recruiting younger and less experienced players in recent National Basketball Association drafts. This year the NBA draft set a record in that the first 7 picks and 17 among 29 first-round selections were not college seniors. Relative to previous years, it is astonishing that the 13th-ranked, 17-year old Kobe Bryant skipped college altogether. In spite of the posturing by NBA executives urging players to stay in school and finish their education, this year's outcome is likely to signal a long term trend toward earlier entrance into the NBA.

Difficulties in controlling the timing of recruiting new entrants of highly trained professionals have been pointed out in a set of interesting papers by Roth [1984,1991], and Mongell and Roth [1991]. Roth and Xing [1994] observe that these difficulties appear in many other markets, including the medieval weekly markets for ordinary commodities, medical interns and residents, and postseason college football bowl games.

Two related aspects of timing of market transactions must be distinguished. In one, labeled "jumping the gun" by Roth and Xing, participants compete for a limited supply of the best-qualified candidates or best positions in timing their bids and acceptances. For instance, firms sometimes make exploding offers which expire within a very short time, and job candidates often try to delay making a choice from available offers in the hope of receiving better ones. The second aspect of timing is unraveling of the appointment date. In entry-level professional labor markets, employment begins only after graduation and attainment of professional qualifications, but the appointment date sometimes unravels to a few years before that. Unraveling of the appointment date may take the form of signing employment contracts long before employment is to begin, as in the placement of medical interns and residents before 1947, or it may be in the form of summer job programs of law students prior to offers of longer term positions upon graduation as is happening in law firms today. Since in many situations there are natural dates when the potential of candidates can be assessed and appointments can be made, unraveling of the appointment date is not so much about competing through the timing of proposing and accepting offers, as about the inefficiencies of early contracts due to incomplete information of all participants concerning the productive characteristics of the candidates.

These two aspects of market timing problems are clearly related. Sometimes jumping the gun can be the principal reason for unraveling of the appointment date. Still, isolating the unraveling aspect from the jumping-the-gun aspect helps us understand what features of the market imperfections are responsible for the timing problems. It also enables a more accurate assessment of the social cost and benefit of early contracting, as well as the welfare effects on different groups of participants.

We analyze the determinants of market unraveling in what follows. The model is built upon incomplete information about agents' future productive characteristics. This creates both individual uncertainty about one's traits as well as aggregate uncertainty about the market value of those traits. Aggregate uncertainty arises in these market because of indivisible (pair-wise) assignments. If all appointments are made after individual uncertainty is resolved, efficiency of assignments is guaranteed, but all agents are exposed to payoff risks that they can be on the long or short side of the market. Complete Arrow-Debreu securities markets would eliminate payoff risks to agents, as well as guarantee efficient job assignments. However, in the absence of such markets, unraveling or early contracting can bring about limited insurance gains at the expense of inefficient assignments. Unraveling is a manifestation of risk aversion under incomplete insurance markets which relieve some of the anxiety about the availability of job positions and qualified applications.

We analyze markets where individual uncertainty applies only to job applicants, not to firms, and consider two contractual situations, one where firms are bound by *ex post* unsuccessful early contracts, and another where buyouts by firms are allowed.

Without buyouts, unraveling need not occur. It is more likely the smaller the proportion of more promising candidates in the applicant pool and the smaller the total applicant pool relative to the number of positions. The absence of individual uncertainty for firms implies that payoff risks to firms are the main source of insurance gains from early contracts. These gains are large when the applicant pool or proportion of promising applicants are small, because firms are less likely to fill their positions in the spot market. The degree of heterogeneity in the applicant pool can have ambiguous effects on the likelihood of unraveling, because it affects both the average quality of the applicant pool as well as the size of insurance gains. A "compensated" increase in the degree of heterogeneity increases the likelihood of unraveling for the more promising applicants. In this model, unraveling always reduces the probability that productive applicants will be short and firms will be long in the spot market. Therefore a ban on early contracts always decreases the *ex ante* welfare of firms. The welfare effects on applicants depends on how the gains from early contracting are shared. Since less promising applicants never receive any rents from early contracts, the ban unambiguously improves their welfare. Whether the ban increases or decreases the welfare of more promising applicants depends on the size of the gains they receive.

When buyouts of *ex post* unsuccessful early contracts are allowed, unraveling of firms and more promising applicants always occurs. Some less promising applicants also sign early contracts if the number of more promising applicants is smaller than the number of positions. Buyouts are Pareto optimal for all market participants as a whole because they eliminate inefficient assignments and increase total gains from trade. Buyouts unambiguously increase the welfare of less promising candidates. In many situations they benefit all parties, but examples can be constructed where buyouts benefit applicants at the expense of firms. Sometimes prohibitions on buyouts may be in the interests of firms.

This paper is organized as follows. In the next section, we introduce a two-period assignment model and use a simple example to define individual and aggregate uncertainties that are central to analysis of unraveling. We also identify the market imperfections that cause unraveling. Section 3 analyzes how unraveling occurs as a competitive equilibrium without buyouts, and section 4 considers comparative statics issues regarding the relative size and composition of the applicant pool and the degree of heterogeneity in the pool. In section 5 we consider unraveling when buyouts are allowed. The last section is a brief discussion on how to extend the model to more general setups. Proofs of pure technical nature can be found in the appendix.

2. The Assignment Market: Background

There are two models of markets where matching considerations are crucial: the assignment market of Koopmans and Beckmann [1953] and Shapley and Shubik [1972], and the marriage market of Gale and Shapley [1962]. These two markets differ from most others in that the objects of trade are indivisible. Each participant has only one unit to buy or sell. [1] The difference between the two markets is that in the assignment market money is transferable among all participants, while in the marriage market only rank orderings of preferences matter. [2] Since we emphasize the role of market and market prices, we choose the assignment market as the framework. In this section we review the setup of the assignment market, illustrate the role of uncertainty, and present a simple example of how unraveling can occur when markets are incomplete.

2.1. Competitive equilibrium in the assignment market

Formally, the assignment market can be described as follows. There are two groups of agents, M and W. There is a production function for each pair of agents such that the joint output of an assignment of an M agent and a W agent is non-negative, and the joint output of assigning either two M agents or two W agents to each other is zero. The output of an unassigned agent is normalized to zero. The largest total output of any number of agents is the maximal sum of outputs that can be produced by pair-wise assignments of these agents.

A pair-wise assignment is feasible if each agent is matched to at most one other agent. Associated with a feasible assignment is a feasible outcome, a vector of non-negative payoffs to the agents such that the sum of the payoffs for each matched pair is no greater than the joint output of the pair and the payoff for each unmatched agent is zero. An assignment is efficient if among all feasible assignments it yields the largest sum of joint outputs, or equivalently, the largest sum of payoffs. A feasible outcome is stable if the sum of the

[1] More generally, the matching or assignment can be one-to-many, in which case buyers have a fixed, integer-valued demand.

[2] For a discussion of the modeling issues in the two markets, see Crawford [1991].

payoffs to each pair of agents is greater than or equal to the joint output they can produce. Thus, in a stable outcome, the sum of payoffs equals the joint output for each matched pair. For each unmatched pair, the sum of payoffs for the two agents is greater than the joint output they can produce, so that the pair does not form a "blocking" coalition that improves the payoffs for both of them.

The main result in the theory of assignment markets is that if a feasible outcome is stable then the associated assignment is efficient, and if a feasible assignment is efficient then there exists a feasible outcome associated with it that is stable. This is analogous to the first and second welfare theorem in a decentralized economy. Koopmans and Beckmann [1953] formulate the efficient assignment as a solution to a linear programming problem and give a competitive market equilibrium interpretation to the associated outcome. Shapley and Shubik [1972] formulate stable outcomes as equilibria of a cooperative game and show that side-payments do not occur in equilibrium. In this paper, we refer to each stable outcome as a competitive equilibrium.

2.2. Payoff risks

The customary level of abstraction of the assignment theory conceals its connections with more familiar market analysis. The parallels are best revealed by analyzing a situation where there are two distinct types of identical agents in each group. Let M_1 and M_2 denote the two types of M agents of sizes m_1 and m_2. Similarly, W_1 and W_2 are the two types of W agents of sizes w_1 and w_2. Assume that M_1 agents are more productive than M_2 agents, and similarly for the two types in W. To simplify further, assume that assignment of an M_1 agent to a W_1 agent produces unit output and all other kinds of assignments produce nothing. [3]

It is useful to think of one group of agents as residual income recipients, say W, to put the assignment market in a competitive framework. In this example W_2 agents are not productive and cannot bid positive prices for either type of M agents. They receive a payoff of zero in the market. Similarly, M_2 agents cannot ask positive prices of

[3] In the last section, we consider assignment markets with more general specifications of joint outputs.

any type of W agents and receive zero payoff. Only M_1 and W_1 agents potentially have positive payoffs. Their actual payoffs in a competitive market depend on m_1 and w_1. For example, if $m_1 = w_1$, it is possible to match each M_1 agent with a W_1 agent. The unit output produced by such a match must be divided between the two agents. Any division that gives productive agents in each group the same non-negative payoff is a competitive equilibrium when $m_1 = w_1$. Suppose, however, that $m_1 > w_1$. Now the supply of M_1 agents exceeds demand and their payoff is driven down to zero. W_1 agents get all the return because they are the scarce factor. If $m_1 < w_1$ then M_1 agents are the scarce factor and receive 1 in the competitive equilibrium.

The discontinuities in equilibrium prices or payoffs with respect to supply and demand are inherent in the indivisibility of assignments (pairwise here) and in the discreteness in the types. Indivisibility implies that payoffs are sensitive only to the sign of excess demand or supply, not to its magnitude. Whether m_1 exceeds w_1 by a few or by many does not change the equilibrium payoffs of zero for M_1 agents and 1 for W_1 agents. The discontinuity does not disappear as the size of the market gets large.

The discontinuity is more sensitive to the discreteness of types. For example, suppose the joint output between less and more productive agents of each group is $\nu \in (0, 1/2)$, instead of 0. Then the consequences of being on the long or short side of the market are less severe. If $m_1 > w_1$ but $m_1 < w_1 + w_2$ so that all M_1 agents are matched to the two types of W agents, their equilibrium payoff is ν instead of zero because they add value ν to assignments with W_2 agents. W_1 agents receive $1 - \nu$ instead of 1. In the limit of a continuum of types, the discontinuity disappears. In our simple model of two types of each group, the uncertainty about the relative size of the two productive types translates into payoff risks before they enter the market. Avoiding this kind of payoff risks is a main motivation for individual agents to contract early and "unravel" the market. [4]

[4] Roth and Xing [1994] identify two other causes of unraveling in a two-period model of the assignment market. One is instability: If the assignment in the second period is unstable, then there may be incentives for agents to reach agreements early. However, this cannot happen in a competitive assignment market. They identify market competition as another cause of unraveling and give an example where despite the stability of assignment in the second period, some agents have incentives to deviate and make offers in the first period to their next best candidates by committing to not competing for these agents in case they reject the offers.

2.3. An example of unraveling

The essential aspects of market unraveling are illustrated by a two period model in which there is both individual uncertainty and aggregate uncertainty. Individual uncertainty is incomplete information in the first period by all agents of how productive a particular agent will be in the second period. Individual uncertainty creates randomness in supplies of different types of agents, and results in aggregate uncertainty in the form of discontinuous equilibrium payoffs. Both types of uncertainty are necessary to capture the essence of unraveling. Unraveling (first period contracting) eliminates aggregate uncertainty but at the costs of inefficient assignments when there is individual uncertainty. Contracting after individual uncertainty is resolved ensures efficient assignments, but exposes agents to payoff risks due to aggregate uncertainty.

These points can be illustrated by extending the above example. In the second period there are productive and unproductive agents in M and W as above. In the first period the productivities of the agents within each group M and W are unknown. Instead, each agent has probability λ of becoming productive in the second period. We use superscripts for period and subscripts for type. For example, M_1^2 denotes the productive M agents in the second period, with size m_1^2. All agents are risk-averse with utility function u. For simplicity, we assume that there are equal numbers of M and W agents.

The second period competitive equilibrium gives productive agents a payoff of 1 or 0 depending on excess demand or supply. It is indeterminate if there are equal numbers of productive agents. We assume that when $m_1^2 = w_1^2$, the equilibrium payoff to all M_1^2 agents is 0 or 1 with probability 1/2. This assumption is non-consequential for what follows. When n is sufficiently large, the probability that $m_1^2 = w_1^2$ in the second period becomes arbitrarily small. Since we are interested in assignment markets with large numbers of participants, the event $m_1^2 = w_1^2$ is irrelevant.

The uncertainty of second period payoffs may motivate agents to contract with each other in the first period, before their productivity is known. Imagine the following game. In the first period, there are random pair-wise meetings between M^1 agents and W^1 agents. Each pair of agents chooses whether to sign a contract in which the M^1 agent receives payoff r and the W^1 agent receives $1-r$ in the second period if both turn out to be productive, and

0 otherwise. The second period market is said to unravel if there exists some $r \in [0,1]$ such that some pairs of agents choose to sign the first period contract rather than to wait. From the social point of view, the contracts create assignment inefficiencies because information necessary to achieve efficient assignments is available only in the second period. From the individual point of view, signing the first period contract eliminates payoff risks that the two agents may be on the long or short side of the market, should they be productive, but it also increases payoff risks due to individual uncertainty, because the payoff from the contract depends on the joint probability that both are productive.

When λ is sufficiently large the insurance gains from contracting early and eliminating aggregate uncertainty outweigh the additional individual uncertainty it creates. The market unravels. The argument runs as follows. In the second period, the payoffs to productive agents are 1 or 0 depending on excess demand. The expected utility to each agent from waiting is $\lambda[u(1)/2 + u(0)/2] + (1-\lambda)u(0)$. If two agents sign the first period contract the expected utility to the M^1 agent is $\lambda^2 u(r) + (1-\lambda^2)u(0)$ and the expected utility to the W^1 agent is $\lambda^2 u(1-r) + (1-\lambda^2)u(0)$. Let r^m be the minimum payoff required for M^1 agents to sign the contract and r^w be the maximum payoff that W^1 agents are willing to pay M^1 agents to sign the contract. Equating the expressions above we have that the ask price r^m and the bid price r^w are implicitly defined by the condition $u(r^m) = u(1-r^w) = u(0) + [u(1) - u(0)]/(2\lambda)$. If λ is large enough then the ask price r^m falls below the bid price r^w and unraveling occurs. There is not enough structure in this example to determine the market equilibrium price r uniquely, but it must be between the bid and ask. When λ is sufficiently small, $r^w < r^m$ and the market does not unravel: The value of insurance from a first period contract is small because the probability of exposure to aggregate uncertainty in the second period is small. The additional individual uncertainty dominates the insurance gains.

2.4. Unraveling and market incompleteness

Complete analysis of richer assignment problems is presented the following sections. Before we go on, it is important to understand that the existence and inefficiency of unraveling is caused by market incompleteness. In the above example, the optimal insurance arrangement is for all agents to sign a first period contract in which they all receive the same share of maximum total joint output after waiting till the second period to achieve efficient assignments. An Arrow-Debreu securities market together with a second period spot market is the decentralized mechanism that achieves this.

Imagine that in the first period each agent sells 100 perfectly divisible shares of the claim to the agent's payoff from the second period spot market. For example, holding 50 shares of one claim entitles the buyer to 1/2 of the seller's equilibrium payoff from the spot market. A competitive equilibrium in the securities market is the set of share prices that equates the supply and demand for shares of all claims. Given these prices, each agent purchases the numbers of shares that maximize expected utility subject to the budget constraint, and the market clears for shares of each claim. Since all agents are indistinguishable in the first period the competitive equilibrium has equal share prices for all claims and each agent holds an equal number of shares of each claim, including the agent's own claim. This equilibrium is optimal *ex post*, because the second period spot market produces the efficient assignment with maximal total output. It is also optimal *ex ante*, because each agent spreads risks as much as possible.

If markets were complete and agents could legally sell claims to their future payoffs, the empirical phenomena associated with unraveling would not exist. Unraveling and early, inefficient contracting is a manifestation of market failure. Unraveling is never said to be observed in markets where futures contracts are common, such as agricultural commodities. Rather, it is restricted to those labor markets where indivisible assignments are important and where complete markets of the kind described above are legally prohibited as too restrictive on persons for other reasons. In what follows we take it for granted that complete markets do not exist, so that unraveling may occur.

3. Unraveling without Reentry

In this section, we consider a richer version of the example above. There agents are indistinguishable in the first period and unraveling affects all market participants in the same way. Here we allow for two kinds of heterogeneity among participants. First, the two groups of participants in the assignment market are not symmetric in terms of their individual uncertainty. In labor market for entry-level professionals, individual uncertainty about future productivity is a substantial issue only for the potential candidates. Firms have operated in the market for a long time and have established reputations. Second, there is heterogeneity within each group of agents. Some are more likely to become productive later than others in the same group. Unraveling in this situation can affect different types of agents in one group differently. The simplest model that captures these kinds of heterogeneity in assignment markets is described next, and the competitive equilibrium in the first period market is characterized. In this section we assume that agents who sign first period contracts do not enter the spot market in the second period. The case of unraveling with buyouts and reentry is considered in the next section.

3.1. The model

As before, in the second period there are two types of agents in each group, productive and unproductive, denoted as M_1^2 and M_2^2, and W_1^2 and W_2^2. The joint product is 1 between M_1^2 and W_1^2 agents and 0 otherwise. Now, there are two different types of agents in each group in the first period, type 1 and type 2, who face different individual uncertainty. For each $i = 1, 2$. M_i^1 agents, of size m_i^1, have probability λ_i of becoming productive in the second period, and probability $1 - \lambda_i$ of becoming unproductive. We assume that $\lambda_1 > \lambda_2$ so that M_1^1 agents are the more promising job candidates. To capture the idea that firms face little uncertainty about their own types, we assume that W_1^1 agents, of size w_1^1, have probability 1 of becoming productive, and that type 2 W agents have probability 1 of becoming unproductive. Both types of M agents have the same concave utility function u over payoffs, and both types of W agents have the same utility function v.

We look for an equilibrium where both the terms of first period contracts and the number of each type of agents who sign them are endogenous. For agents who choose to wait, the equilibrium in the second period spot market is described as before: Productive agents receive a payoff of 1 if they are on the short side of the market and all other agents receive zero. Since there is no reentry, the first period contract between an M_i^1 agent

($i = 1, 2$) and a type 1 W agent takes the following form: Any M_i^1 agent who turns out to be productive receives r_i and the W_1^1 agent receives $1 - r_i$; otherwise both receive zero. We refer to r_i as the price of the first period contract with M_i^1 agents.

Since type 2 W agents are known to be unproductive, first period contracts are only between the two types of M agents and type 1 W agents. Again, we proceed by defining bid and ask prices for the M agents. There are two complications: Different types may have different bid and ask prices, and these prices depend on the number y_i of M_i^1 agents who sign first period contracts. Ask and bid prices depend on y_1 and y_2 because aggregate uncertainty, the discontinuous payoffs in the second period spot market conditional on excess demand and supply, depends on y_1 and y_2. For any y_1 and y_2, let $\pi(y_1, y_2)$ be the probability of the event $m_1^2 \leq w_1^2$. That is, $\pi(y_1, y_2)$ is the probability that *ex post* productive M agents receive 1 in the second period spot market. [5]

It represents the aggregate uncertainty. To shorten notation, we write π instead of $\pi(y_1, y_2)$ whenever the meaning is clear.

3.2. Bid and ask prices

For each $\pi \in [0, 1]$, in order for M_1^1 agents to be indifferent between signing a first period contract at price r_i and waiting until the second period spot market, we must have

$$(1 - \lambda_i)u(0) + \lambda_i u(r_i) = (1 - \lambda_i)u(0) + \lambda_i[\pi u(1) + (1 - \pi)u(0)].$$

This simplifies to

$$\pi u(1) + (1 - \pi)u(0) = u(r_i).$$

Therefore, the ask price of M_1^1 agents is the same as the ask price of M_2^1 agents. Let $r^m(\pi)$ denote the common ask price as a function of π.

In order for W_1^1 agents to be indifferent between signing a first period contract with M_i^1 agents at price r_i and waiting for the second period spot market, we must have

$$(1 - \pi)v(1) + \pi v(0) = (1 - \lambda_i)v(0) + \lambda_i v(1 - r_i).$$

[5] For simplicity, indeterminacy of the spot market equilibrium is resolved by our assumption that when $m_1^2 = w_1^2$, the equilibrium is the same as when $m_1^2 < w_1^2$.

Let $r_i^w(\pi)$ denote the bid price for M_i^1 agents. It is straightforward to show that since $\lambda_1 > \lambda_2$, $r_1^w(\pi) \geq r_2^w(\pi)$ for all π, and the equality holds if and only if $\pi = 1$.

Taking derivatives with respect to π,

$$\frac{dr^m(\pi)}{d\pi} = \frac{u(1) - u(0)}{u'(r^m(\pi))}, \quad \frac{dr_i^w(\pi)}{d\pi} = \frac{v(1) - v(0)}{\lambda_i v'(1 - r_i^w(\pi))}.$$

Thus, the ask price $r^m(\pi)$ is increasing and convex, and the bid prices $r_1^w(\pi)$ and $r_2^w(\pi)$ are increasing and concave. It is straightforward to check that $r^m(0) = 0$, $r_i^w(1 - \lambda_i) = 0$, and $r^m(1) = r_i^w(1) = 1$ for $i = 1, 2$.

In Figure 1, r^m intersects r_1^w at $\pi_1 \in (0,1)$ and it intersects r_2^w at $\pi_2 \in (\pi_1, 1)$. $r^m(\pi) > r_1^w(\pi) > r_2^w(\pi)$ for $\pi \in [0, \pi_1)$, $r_1^w(\pi) > r^m(\pi) > r_2^w(\pi)$ for $\pi \in (\pi_1, \pi_2)$, and $r_1^w(\pi) > r_2^w(\pi) > r^m(\pi)$ for $\pi \in (\pi_2, 1)$. Therefore, above π_2 insurance gains exist from first period contracts with W_1^1 agents for both type 1 and type 2 M agents. Between π_1 and π_2 insurance gains exist for type 1 M agents but not for type 2 M agents. Below π_1 the ask price exceeds both bid prices so there is no gain from insurance. That insurance gains exist only when π is high is due to the absence of individual uncertainty for W_1^1 agents. When π is very low, the ask price of both types of M agents is close to zero, but because of the individual uncertainty of the M agents, the bid price for each type of M agent is negative: W_1^1 agents prefer waiting for the spot market to signing first period contracts with either type of M agents at any price. An interpretation of this result is that payoff risks to firms are the main source of insurance gains in our model.

Two other configurations of bid and ask functions are possible. It can happen that the ask price only intersects the bid price function for M_1^1 agents and lies entirely above the bid price for M_2^1 agents for all $\pi < 1$. Then M_2^1 agents never participate in unraveling. It can also happen that the three curves intersect only at $\pi = 1$. Here the ask price function lies above both bid price functions for any $\pi < 1$ and there is no possible gain from insurance. Which case occurs depends on the curvatures of r^m and r_i^w, and on the probabilities λ_1 and λ_2 if u and v are concave. The curvatures of r^m and r^w in turn depend on risk aversion. If both u and v are linear (risk neutral), $dr^m/d\pi = 1$ and $dr_i^w/d\pi = \lambda_i$: There are no gains from first period contracts regardless of λ_1 and λ_2. The more risk averse the agents, the more likely are insurance gains. If u and v are concave, larger λ_i causes the bid price function for M_i^1 agents to shift up, and insurance gains are more likely to exist. In what follows, we incorporate all cases by considering the case shown in Figure 1, with π_2 and π_1 set to 1 if they do not exist.

3.3. Ordering property and monotonicity property

To define the market equilibrium, it is useful to think of the first period market as an assignment market. If an M_i^1 agent signs a first period contract with a W_1^1 agent, they are said to be assigned to each other. If an agent chooses to wait until the second period market, the agent is said to be unassigned. In a "pure" assignment problem the payoff from not being unassigned is exogenous. Here it depends on y_1 and y_2 because these numbers affect the aggregate uncertainty in the second period spot market. But these are endogenous variables, determined by the first period market equilibrium.

Unraveling is identified with signing a first period contract. Different kinds of unraveling are possible because there can be many corner solutions in this problem. Unraveling of M_i^1 agents occurs if the bid price exceeds the ask price at some $y_i > 0$. The market equilibrium price r_i lies between the bid and ask. Unraveling of M_i^1 agents is complete if all M_i^1 agents sign a first period contract, so that $y_i = m_i^1$. Unraveling is incomplete if $y_i < m_i^1$. In this case all M_i^1 agents must be indifferent between signing and waiting for the second period spot market. For unraveling of W_1^1 agents to occur, the bid price for at least one type of M agents must exceed the ask price at the corresponding y_1 and y_2 such that $y_1 + y_2 > 0$. Again, the equilibrium price r_i must fall in between. If $y_1 + y_2 = w_1^1$, there is complete unraveling for W_1^1 agents. In this case, if both y_1 and y_2 are positive, W_1^1 agents must be indifferent between signing first period contracts with M_1^1 agents and with M_2^1 agents. If $y_1 + y_2 < w_1^1$ there is incomplete unraveling for W_1^1 agents, and $y_i > 0$ implies that W_1^1 agents are indifferent between signing first period contracts with M_i^1 agents and waiting for the second period spot market.

Two properties of the model greatly simplify the characterization of market equilibrium. First, unraveling is ordered: Unraveling of M_1^1 agents precedes that of M_1^2 agents. This ordering property limits the number of corner solutions that must be considered.

Second, the probability $\pi(y_1, y_2)$ is monotonically decreasing in its arguments. This monotonicity property simplifies the calculation of equilibrium, given the initial conditions of the problem.

The greater probability of becoming productive for type 1 M agents than for type 2 M agents implies that the bid price for M_1^1 agents is greater than the bid price for M_2^1 agents for any $\pi < 1$. Since the ask price for the two types is the same, any unraveling of M_2^1 agents implies complete unraveling of M_1^1 agents. This result can be proven as follows. Assuming unraveling for M_2^1 agents, we have $r^m(\pi) \leq r_2$. If there is incomplete unraveling for M_1^1 agents, we have $r^m(\pi) = r_1$. It follows that $r_1 \leq r_2$. Since $\lambda_1 > \lambda_2$,

$$(1-\lambda_1)v(0) + \lambda_1 v(1-r_1) > (1-\lambda_2)v(0) + \lambda_2 v(1-r_2).$$

Thus, W_1^1 agents strictly prefer a first period contract with M_1^1 agents at price r_1 to a first period contract with M_2^1 agents at price r_2, which is a contradiction because W_1^1 agents must be indifferent between contracting with M_1^1 and M_2^1 agents if $y_1, y_2 > 0$.

We now establish the monotonicity property of $\pi(y_1, y_2)$. By definition, $\pi(y_1, y_2)$ is given by

$$\sum_{i=0}^{w_1^1-y_1-y_2} \sum_{j=0}^{i} \left[\sum_{k=0}^{j} C^j_{m_1^1-y_1} \lambda_1^k (1-\lambda_1)^{j-k}\right] \left[\sum_{k=0}^{i-j} C^{i-j}_{m_2^1-y_2} \lambda_2^k (1-\lambda_2)^{i-j-k}\right],$$

where $C_i^j = j!/(i!(j-i)!)$ is the binomial coefficient. Since $y_2 > 0$ implies $y_1 = m_1^1$ we only need to know how $\pi(y_1, 0)$ changes with y_1 for $y_1 \leq m_1^1$ and how $\pi(m_1^1, y_2)$ changes with y_2. The absence of individual uncertainty for W_1^1 agents implies that $\pi(y_1, 0)$ decreases with y_1 for all $y_1 \leq m_1^1$ and $\pi(m_1^1, y_2)$ decreases with y_2. In other words, the probability of excess demand for productive M agents in the second period spot market decreases as more agents sign first period contracts.

The reason can be understood as follows. If an additional pair of M_1^1 and W_1^1 agents signs a first period contract, the number of W_1^2 agents remaining in the second period spot market falls by one. Individual uncertainty for M_1^1 agents means that the expected number of M_1^2 agents participating in the second period market falls by less than one. Therefore the probability that productive M agents will be short in the second period spot market decreases with y_1. More precisely, consider $\pi(y_1-1,0)$ and $\pi(y_1,0)$ for any $y_1 \leq m_1^1$. Applying the definition above,

$$\begin{aligned}
&\pi(y_1-1,0)\\
=&\sum_{i=0}^{w_1^1-y_1+1}\sum_{j=0}^{i}\left[\sum_{k=0}^{j}C^{j}_{m_1^1-y_1+1}\lambda_1^k(1-\lambda_1)^{j-k}\right]\left[\sum_{k=0}^{i-j}C^{i-j}_{m_2^1}\lambda_2^k(1-\lambda_2)^{i-j-k}\right]\\
=&\lambda_1\sum_{i=0}^{w_1^1-y_1}\sum_{j=0}^{i}\left[\sum_{k=0}^{j}C^{j}_{m_1^1-y_1}\lambda_1^k(1-\lambda_1)^{j-k}\right]\left[\sum_{k=0}^{i-j}C^{i-j}_{m_2^1}\lambda_2^k(1-\lambda_2)^{i-j-k}\right]\\
&\quad+(1-\lambda_1)\sum_{i=0}^{w_1^1-y_1+1}\sum_{j=0}^{i}\left[\sum_{k=0}^{j}C^{j}_{m_1^1-y_1}\lambda_1^k(1-\lambda_1)^{j-k}\right]\left[\sum_{k=0}^{i-j}C^{i-j}_{m_2^1}\lambda_2^k(1-\lambda_2)^{i-j-k}\right]\\
>&\sum_{i=0}^{w_1^1-y_1}\sum_{j=0}^{i}\left[\sum_{k=0}^{j}C^{j}_{m_1^1-y_1+1}\lambda_1^k(1-\lambda_1)^{j-k}\right]\left[\sum_{k=0}^{i-j}C^{i-j}_{m_2^1}\lambda_2^k(1-\lambda_2)^{i-j-k}\right]\\
=&\pi(y_1,0).
\end{aligned}$$

A similar argument shows that $\pi(m_1^1, y_2)$ decreases with y_2.

3.4. Market equilibria

Different kinds of equilibria are possible, depending on the relative number of each type. Assume for now that $m_1^1 < w_1^1 < m_1^1 + m_2^1$. $\pi(m_1^1,0)$ is the probability that productive M agents will be on the short side of the second period spot market given that all M_1^1 agents sign first period contracts and no M_2^1 agents sign them. The ordering property implies that the relation between this probability and π_1 and π_2 in Figure 1 is critical, because there is no unraveling for M_2^1 agents unless unraveling for M_1^1 is complete. Depending on the location of $\pi(m_1^1,0)$ relative to π_1 and π_2, there are three possible kinds of unraveling equilibria.

(i) Complete unraveling of the more promising type. Suppose $\pi_2 > \pi(m_1^1,0) > \pi_1$. See Figure 1. By the ordering property, unraveling of M_2^1 agents only occurs after complete unraveling for M_1^1 agents, at $\pi(m_1^1,0)$. But there is no gain from insurance for M_2^1 agents at or below that point because $\pi(m_1^1,0) < \pi_2$, and by the monotonicity property, any unraveling of M_2^1 agents causes π to decrease further. Thus there is no unraveling for M_1^2 agents. Since $\pi(0,0) > \pi(m_1^1,0) > \pi_1$, unraveling for M_1^1 agents continues until all M_1^1

agents sign first period contracts with W_1^1 agents. The equilibrium π is $\pi(m_1^1, 0) > \pi_1$. There is incomplete unraveling for W_1^1 agents, who are indifferent between signing first period contracts with M_1^1 agents and waiting for the second period spot market. The equilibrium price of the first period contract is $r_1 = r_1^w(\pi(m_1^1, 0))$, on the bid function for M_1^1 agents. They receive all the rent from first period contracts.

(ii) Incomplete or no unraveling of the promising type. Suppose $\pi(m_1^1, 0) < \pi_1$. Again, since $\pi_1 < \pi_2$, the ordering property and the monotonicity property imply that there is no unraveling for M_2^1 agents. If $\pi(0,0) < \pi_1$ there is no unraveling for M_1^1 agents either. All agents wait for the second period spot market because there is no gain from insurance at or below $\pi(0,0)$ and unraveling always reduces π. If $\pi(0,0) > \pi_1$ there is incomplete unraveling for type 1 M agents. Unraveling continues until the equilibrium π is driven down to π_1. [6] There is also incomplete unraveling for W_1^1 agents. Both W_1^1 and M_1^1 agents are indifferent between signing first period contracts with each other and waiting for the second period spot market. The price of first period contracts is $r_1 = r^m(\pi_1) = r_1^w(\pi_1)$ and all insurance gains are exhausted.

(iii) Unraveling of both types. Suppose $\pi(m_1^1, 0) > \pi_2$. Here there are insurance gains for M_2^1 agents after complete unraveling of M_1^1 agents. There is incomplete unraveling of M_2^1 agents. The equilibrium π occurs at π_2, where all insurance gains for M_2^1 agents are exhausted. M_2^1 agents are indifferent between signing first period contracts and waiting for the second period market, and the equilibrium price for first period contracts with M_2^1 agents is $r_2 = r^m(\pi_2) = r_2^w(\pi_2)$. The equilibrium price of first period contracts with M_1^1 agents is $r_1 = r_1^w(\pi_2) > r^m(\pi_2)$. M_1^1 agents extract all the rents from first period contracts with W_1^1 agents, and they strictly prefer signing first period contracts to waiting for the second period spot market. Again there is incomplete unraveling for W_1^1 agents, and they are indifferent between first period contracts with M_1^1 and M_2^1 agents and waiting for the second period spot market.

For completeness, we discuss what happens when $m_1^1 > w_1^1$. Then $\pi(m_1^1, 0)$ is not defined. If $\pi(0,0) < \pi_1$ unraveling does not occur. If $\pi(0,0) > \pi_1$, there is incomplete

[6] We ignore the integer problem and assume that y_1 can take real values. More rigorously we can assume agents use mixed strategies in the sense that the probability of signing a first period contract can be any number between 0 and 1.

unraveling of type 1 M agents. The equilibrium price of first period contracts with type 1 M agents is on the ask price function. Unraveling continues until either the insurance gains of first period contracts between M_1^1 agents and W_1^1 agents are exhausted at π_1, or until all W_1^1 agents sign first period contracts. The second case cannot occur if the total number of M agents is sufficiently large, because any single W_1^1 agent could almost certainly obtain the maximum utility $v(1)$ by waiting for the second period spot market when all other W_1^1 agents sign first period contracts. The equilibrium therefore has incomplete unraveling of both type 1 M agents and type 1 W agents, and the equilibrium price is $r_1 = r^m(\pi_1) = r^w(\pi_1)$. The less interesting case of $m_1^1 + m_2^1 < w_1^1$ can be similarly addressed.

4. Unraveling with No Reentry: Comparative Statics

The characterization of equilibria in the last section allows us to address how changes in relative numbers of different kinds of agents and in the degree of heterogeneity in the first period market affect the likelihood of different forms of unraveling, equilibrium prices and welfare in terms of first period expected utility.

It follows from the ordering property and the monotonicity property that the necessary and sufficient condition for unraveling of M_1^1 agents is $\pi(0,0) > \pi_1$, and the necessary and sufficient condition for unraveling f M_2^1 agents is $\pi(m_1^1, 0) > \pi_2$. As mentioned previously, the first period market is not a pure assignment market because the payoffs from being unassigned are endogenous. In a pure assignment market, equilibrium payoffs are determined by relative numbers of different types of market participants up to a boundary condition that the unassigned receive an exogenous payoff. Here, the "effective" numbers of market participants, $\pi(0,0)$ and $\pi(m_1^1, 0)$ (relative to π_1 and π_2) replace the relative numbers m_1^1, m_2^1 and w_2^1 as the determinants of equilibrium.

4.1. Relative numbers and unraveling

π_1 and π_2 do not depend on relative numbers of market participants, but $\pi(0,0)$ and $\pi(m_1^1, 0)$ decrease as the number of either M_1^1 or M_2^1 agents increases or as the number of W_1^1 agents decreases. Therefore, unraveling of M_1^1 and M_2^1 agents becomes less likely as the relative supply of the two type of M agents increases. Furthermore, given the total number of M_1^1 and M_2^1 agents, unraveling for both types is less likely the greater the proportion of M_1^1 agents, because a greater proportion of M_1^1 agents decreases both $\pi(0,0)$ and $\pi(m_1^1,0)$. An interpretation of these results is that early contracting is less likely in assignment markets where firms expect to fill their positions in the spot market and therefore face small payoff risks.

Whether the numbers of market participants affects the equilibrium prices depends on the kind of equilibrium that occurs. Relative numbers do not affect the bid and ask functions in Figure 1, so if the equilibrium involves incomplete unraveling of either M_1^1 or M_2^1 agents, changes in the numbers of different types of market participants on the margin have no effect on prices and hence no effects on the expected utility of any type of agents.

However, if there is complete unraveling of M_1^1 agents, changes in the numbers of different types of market participants affect the equilibrium price r_1 through $\pi(m_1^1,0)$. An increase in m_1^1, an increase in m_2^1, or a decrease in w_1^1 all have the same effect of reducing r_1 because they reduce $\pi(m_1^1,0)$. Also, given the total number of M_1^1 and M_2^1 agents, a greater proportion of M_1^1 agents decreases the equilibrium price r_1 of first period contracts for M_1^1 agents. In either case, the welfare of type 1 M agents decreases, and the welfare of W_1^1 agents increases. The welfare of type 2 M agents falls because r^m, the shadow price of first period contracts for type 2 M agents, decreases as the equilibrium π falls.

4.2. Heterogeneity and unraveling

Consider an increase in λ_2 that reduces heterogeneity among M agents. The bid function r_2^w shifts up, and the function r_1^w remains unchanged. See Figure 2. π_2 decreases and insurance gains increase for M_2^1 agents. This tends to increase the likelihood of unraveling for M_2^1 agents. But a greater λ_2 also decreases $\pi(0,0)$ and $\pi(m_1^1,0)$ because it increases the average quality of M agents. This tends to decrease the likelihood of unraveling for both types of agents. Putting all these together, unraveling for type 1 M agents becomes less likely, but we cannot say whether the likelihood of unraveling for type 2 M agents increases or decreases without additional structure on preferences and other parameters.

If there is incomplete unraveling or no unraveling of type 1 M agents, the equilibrium price r_1 is unaffected, and the expected utilities of all market participants are unaffected by a decrease in λ_2. If there is complete unraveling of type 1 M agents, the equilibrium price r_1 falls, either due to a smaller $\pi(m_1^1, 0)$ if there is no unraveling for type 2 M agents, or due to a smaller π_2 if incomplete unraveling of type 2 M agents occurs. In either case, the welfare of type 1 M agents decreases, and the welfare of W_1^1 agents increases. The effect on the shadow price r^m if there is no unraveling of M_2^1 agents, and on the equilibrium price r_2 if there is unraveling of M_2^1 agents, is ambiguous because λ_2 has increased but equilibrium π has decreased. The effect on the welfare of type 2 M agents is therefore ambiguous.

Less heterogeneity caused by a decrease in λ_1 has different effects. The bid function r_1^w shifts down, π_1 increases, and π_2 remains unchanged. See Figure 3. $\pi(0,0)$ increases, but $\pi(m_1^1, 0)$ does not change because it does not depend on λ_1. There is no effect on the likelihood of unraveling for type 2 M agents, and the effects for type 1 are ambiguous.

If there is incomplete unraveling of type 1 M agents, the equilibrium price r_1 increases, but the effect on their welfare is ambiguous because λ_1 has decreased. The shadow price r^m of type 2 M agents increases, making them better off. W_1^1 agents are worse off. If there is complete unraveling of type 1 M agents, the equilibrium price r_1 decreases due to a downward shift of the bid price function $r_1^w(\pi)$, whether or not there is unraveling of type 2 M agents. M_1^1 agents are worse off because both r_1 and λ_1 are lower. There is no change in either the shadow price r^m if there is no unraveling of M_2^1 agents, or the equilibrium price r_2 if there is unraveling of M_2^1 agents, and their welfare remains the same. The welfare of W_1^1 agents does not change either because the better terms of first period contracts with type 1 M agents is offset by their decreased probability of being productive.

Changes in λ_1 and λ_2 considered above affect the average quality of M agents differently, and change both the initial conditions $\pi(0,0)$ and $\pi(m_1^1, 0)$, and the critical values π_1 and π_2. A "compensated" decrease in the heterogeneity of M agents is a change in both λ_1 and λ_2 that isolates the two effects. Consider decreasing λ_1 and increasing λ_2 so that the initial condition $\pi(0,0)$ remains unchanged. Since π_1 is increased by a smaller λ_1, unraveling for M_1^1 agents becomes less likely.

4.3. Prohibitions on early contracts

We conclude this section by discussing the welfare effects of banning first period contracting. Clearly, a ban has no effects if $\pi(0,0) \leq \pi_1$. If the ban is effective, we use the shadow price of each type of agents evaluated at $\pi(0,0)$ as an index of welfare with the ban and compare it to the price or shadow price in the relevant competitive equilibrium without the ban. Since in any unraveling equilibrium type 2 M agents are indifferent between signing a first-period contract and waiting for the second period spot market, and since unraveling reduces the equilibrium π and hence r^m, a ban on first period contracting unambiguously benefits them. Similarly, since in any equilibrium W_1^1 agents are indifferent between signing a first-period contract with type 1 M agents and waiting for the second period spot market, and since unraveling reduces the equilibrium π and hence the equilibrium price r_1, a ban on first period contracting unambiguously hurts them.

The welfare effects on type 1 M agents depend on what kind of equilibrium would otherwise obtain without the ban. If $\pi(0,0) > \pi_1 > \pi(m_1^1, 0)$, the equilibrium value of π is $\pi_1 \leq \pi(0,0)$, and unraveling is incomplete for type 1 M agents. In this case, a ban increases the shadow price of M_1^1 agents and makes them better off. If $\pi(m_1^1, 0) > \pi_1$, the market equilibrium involves complete unraveling for type 1 M agents. The welfare effect of banning first period contracts on M_1^1 agents depends on the initial condition $\pi(0,0)$. If this is large enough, the shadow price $r^m(\pi(0,0))$ for M_1^1 agents with the ban is greater than the competitive equilibrium price $r_1^w(\pi(m_1^1, 0))$ without the ban. But if $\pi(0,0)$ is small so that $r^m(\pi(0,0)) < r_1^w(\pi(m_1^1, 0))$, the ban makes them worse off.

5. Unraveling with Reentry

First and second period inefficiencies are associated with the contracts analyzed above. Pairwise first period contracts cannot replicate the full risk sharing implied by complete contingent contracts. In addition, second period inefficiencies are caused by the restriction on reentry for those *ex post* productive agents whose first period contracts prove unsuccessful because their partners turn out to be unproductive. In this section we allow these *ex post* productive agents to enter the second period spot market. This amounts to allowing

W_1^1 agents to "buy out" of their first period contracts *ex post*, should the contracts prove unsuccessful. [7] First period inefficiencies remain, but all second period inefficiencies are eliminated because all *ex post* unsuccessful first period assignments are dissolved.

5.1. Buyout provisions

Buyouts require analyzing the terms on which they are negotiated. If the M_i^1 agent ($i = 1, 2$) in a first period contract with a W_1^1 agent turns out to be productive, the terms set by the initial contract cannot be advantageously renegotiated. For instance, if the spot market turns out to be short on productive W agents and they receive a spot payoff 1, the W_1^1 agent must pay the first period price of r_i to the M_i^1 agent in order to enter the spot market, which leaves them with exactly the same net payoffs from not buying out. Similarly, there are no gains for the W_1^1 agent to buy out of an unsuccessful first period contract if the spot price for productive M agents is 1. Therefore, buyout occurs only when the contract is unsuccessful and the spot price of productive M agents is 0.

Because agents participating in a first period contract anticipate the possibility of buyout, renegotiation amounts to an up front contingency clause in the contract which specifies the terms and the market circumstances of a buyout: The W agent pays the *ex post* unproductive M_i^1 agent r' to buy out of the contract when there is excess supply of productive M agents in the spot market. Since the joint gain from buyout is 1, the two agents in the first period contract are in the same position as they would have been had the M_i^1 agent been productive. It then follows that an optimal first period contract should share risks in the same way: $r' = r_i$. [8]

[7] The reason why W_1^1 agents buy out unsuccessful contracts here is that they have no individual uncertainty: *ex post* productive M agents who sign a first period contract are always efficiently assigned to productive W agents. The buyout would be two-sided if there was individual uncertainty for W_1^1 agents as well as for M agents.

[8] When the cost of reentry is a number $k \in (0, 1)$, instead of 0 in this section and 1 in the previous section, equilibrium first period contracts specify the division of the output 1 when the M_i^1 agent turns out to be productive and of the output $1 - \nu$ when he is not productive but $m_1^2 > w_1^2$ in the spot market. The payoff risks in these two contingencies must be optimally shared between the two agents in a first period contract. Lemma A.3 in the appendix characterizes the form of first period contracts, with $1 - k$ replacing ν_2. The analysis of unraveling in this case of costly reentry is very similar to that in this section. The only change is that insurance gains may not exist when π is small because reentry is more likely and the cost of reentry reduces the value of the first period insurance contract.

5.2. Market equilibria

The analysis of equilibrium unraveling with reentry proceeds as in the previous section. In fact, the analysis is easier here because with buyout all *ex post* inefficient assignments are eliminated and the key probability of the event $m_1^2 \leq w_1^2$ is independent of the numbers of M_1^1 and M_2^1 agents who sign first period contracts with W_1^1 agents. Thus π is no longer endogenous and this greatly simplifies the characterization of equilibrium unraveling. To simplify notation, let $\pi_0 = \pi(0,0)$ be the probability of the event that $m_1^2 \leq w_1^2$ with reentry.

As in the previous section, we first characterize the bid and ask functions r^m and r^w. For any fixed $\pi \in [0,1]$ and $i = 1,2$, let $r_i^m(\pi)$ be the solution to the equation

$$(1-\lambda_i)\pi u(0) + [1-(1-\lambda_i)\pi]u(r) = \lambda_i \pi u(1) + (1-\lambda_i\pi)u(0),$$

and let $r_i^w(\pi)$ be the solution to the equation

$$(1-\lambda_i)\pi v(0) + [1-(1-\lambda_i)\pi]v(1-r) = \pi v(0) + (1-\pi)v(1).$$

Thus with reentry there are two ask price functions as well as two bid price functions, one for each type of agents. The properties of these functions are established in the appendix (Lemma A.1). The key aspects are: For either type of M agents, (i) the ask and bid price functions are increasing in π, (ii) ask and bid prices equal zero at $\pi = 0$ and equal 1 at $\pi = 1$, (iii) bid price is strictly greater than ask price for all π between 0 and 1. Therefore, unlike the previous section, insurance gains exist for all values of π for both types of M agents. Figure 4 depicts the ask and bid price functions for type i M agents ($i = 1,2$).

Recall that without reentry, ask prices are the same for both types of M agents and the bid price is higher for type 1 M agents, so the value of insurance is always greater for type 1, and unraveling of M agents is ordered by type. Here, one can easily check that because $\lambda_1 > \lambda_2$, both the bid and ask prices are larger for type 1 M agents than for type 2 at each value of π. It is proven in the appendix (Lemma A.2) that although type 2 M agents are willing to accept a lower price for first period contracts, the fact that $\lambda_1 > \lambda_2$ still implies that W_1^1 agents prefer to sign first period contracts with type 1 M agents at their ask price $r_1^m(\pi)$ rather than with type 2 at $r_2^m(\pi)$ at any π. With this result, we can show that any unraveling of type 2 M agents implies complete unraveling of type 1, so that the order is preserved when buyout and reentry are permitted.

The argument is as follows. Unraveling of M_2^1 agents implies that there is a price $r_2 \geq r_2^m(\pi_0)$ such that W_1^1 agents prefer a first period contract with a type 2 M agent at r_2 to waiting for the second period spot market. But since W_1^1 agents prefer to sign first period contracts with type 1 M agents at $r_1^m(\pi_0)$ rather than with type 2 at $r_2^m(\pi_0)$, there exists a price $r_1 > r_1^m(\pi_0)$ such that W_1^1 agents prefer a first period contract with a type 1 M agent rather than with a type 2 M agent at r_2. But type 1 M agents also prefer a first period contract at such r_1 to waiting for the second period spot market. Thus, there must be unraveling of type 1 M agents if there is unraveling of type 2 M agents. Moreover, the unraveling of type 1 M agents must be complete, because incomplete unraveling would imply that $r_1 = r_1^m(\pi_0)$ and W_1^1 agents are indifferent between signing with type 1 M agents at r_1 and signing with type 2 M agents at some $r_2 \geq r_2^m(\pi_0)$, which we know is impossible.

Since π is no longer endogenous, the first period contract equilibrium can be thought of as a pure assignment problem. The following characterization of equilibrium with reentry follows immediately. There are two possible cases. [9] If $w_1^1 < m_1^1$ so that W_1^1 agents are in short supply in the first period, the unique competitive equilibrium has complete unraveling for W_1^1 agents and incomplete unraveling for type 1 M agents at their ask price $r_1^m(\pi_0)$. There is no unraveling for type 2 M agents. If $m_1^1 < w_1^1 < m_1^1 + m_2^1$ so that type 1 M agents are short in the first period, the unique competitive equilibrium has complete unraveling for type 1 M agents and for W_1^1 agents, and incomplete unraveling for type 2 M agents. The equilibrium price for type 2 M agents is their ask price $r_2^m(\pi_0)$. The equilibrium price r_1 for type 1 M agents is given by the following indifference condition by W_1^1 agents:

$$(1-\lambda_1)\pi_0 v(0) + [1-(1-\lambda_1)\pi_0]v(1-r_1) = (1-\lambda_2)\pi_0 v(0) + [1-(1-\lambda_2)\pi_0]v(1-r_2^m(\pi_0)).$$

[9] The less interesting case of $w_1^1 > m_1^1 + m_2^1$ is ignored.

5.3. Welfare comparisons

Unraveling without buyouts and reentry is both *ex ante* and *ex post* inefficient. Unraveling with buyouts and reentry provide limited insurance in the first period without affecting the efficiency of assignments. Therefore compared to a situation where first period contracting is prohibited, unraveling with free reentry generally improves welfare of market participants. In particular, if W_1^1 agents are short in the first period market ($w_1^1 < m_1^1$), banning first period contracts decreases their welfare without changing the welfare of either type of M agents. If type 1 M agents are short ($m_1^1 < w_1^1$), banning first period contracts decreases their welfare as well as the welfare of W_1^1 agents without affecting the welfare of type 2 M agents.

It is also of interest to compare welfare with and without reentry. In general reentry must improve welfare, but there is a question of the distributions of the gains among various types of agents. It is possible that some agents become worse off with reentry. Reentry causes both bid price functions to shift up (because W_1^1 agents are willing to pay more for first period contracts with either type of M agents at any π), and the ask price function to shift down (because M agents of either type are willing to accept less to sign first period contracts at any π) and become two ask functions.

It is straightforward to show that reentry generally benefits M_2^1 agents. With or without reentry, in an unraveling equilibrium M_2^1 agents are indifferent between signing and not signing first period contracts with W_1^1 agents. In either case the welfare of M_2^1 agents is negatively related to the equilibrium π. Since without reentry the equilibrium π is smaller than $\pi(0,0)$, and since with reentry the equilibrium π always equals $\pi(0,0)$, M_2^1 are better off with reentry. Of course, if in the equilibrium without reentry there is no unraveling of any type, reentry does not change their welfare.

Welfare comparisons for M_1^1 and W_1^1 agents are more complicated. If there is no unraveling of any type without reentry, then allowing reentry makes W_1^1 agents better off and leaves M_1^1 either indifferent or better off. With reentry, the equilibrium either involves incomplete unraveling of M_1^1 agents when W_1^1 are in short supply in the first period, or complete unraveling of M_1^1 agents when they are in short supply. In the first case ($w_1^1 < m_1^1$), reentry does not change the welfare of type 1 M agents, because it does not change the equilibrium π and type 1 M agents are indifferent between signing first period contracts with W_1^1 agents and waiting, with and without reentry. The equilibrium price with reentry $r_1^m(\pi_0)$ is lower than the shadow price $r^m(\pi(0,0))$ without reentry, and reentry allows W_1^1 agents to reap all the efficiency gains of better insurance, because In the second case $(m_1^1 < w_1^1)$ reentry makes both M_1^1 agents and W_1^1 agents better off: In the equilibrium with reentry M_1^1 agents prefer a first period contract at the equilibrium r_1 to waiting and obtaining a shadow payoff $r_1^m(\pi_0)$, and W_1^1 agents get an equilibrium price $r_2^m(\pi_0)$ from first period contracts with type 2 M agents, lower than the shadow price $r_2^w(\pi(0,0))$ without reentry.

If there is incomplete unraveling of type 1 M agents without reentry, allowing reentry improves the welfare of M_1^1 agents. This is because in the equilibrium without reentry type 1 M agents are indifferent between signing a first period contract and waiting for the spot market at π_1, but in the equilibrium with reentry they weakly prefer the equilibrium contract with W_1^1 agents at $\pi_0 > \pi_1$. But the welfare effect on W_1^1 agents is ambiguous in this case: W_1^1 can be either worse off or better off with reentry. If π_1 is sufficiently small relative to π_0, W_1^1 agents are able to sign first period contracts with M_1^1 agents at a very low price $r^m(\pi_1)$ in the unraveling equilibrium without reentry, and allowing reentry makes them worse off. This can happen when the risk-aversion of M_1^1 agents is great for low values of π so that the ask price function without reentry is very convex. In this case, π_1 can be very small because M_1^1 agents are willing to accept very unfavorable terms in first period contracts with W_1^1 agents when π is low.

6. Extension

In concluding this paper, we briefly describe how the model can be extended to more general assignment markets. Let us consider the case of a general symmetric 2×2 matrix of joint outputs. Suppose that in the second period an assignment of two unproductive agents of opposite groups produces ν_1 and an assignment of a productive agent and an unproductive agent of the opposite group produces ν_2. The output of two productive agents of opposite groups is still 1. We assume that $\nu_1 \leq \nu_2$ and $\nu_2 \leq 1$ so that one type of either group (M_1^2 and W_1^2 agents) is more productive that type 2 of the same group (M_2^2 and W_2^2 agents), and that $2\nu_2 < 1 + \nu_1$ so that productive agents of opposite groups are optimally assigned to each other whenever possible. Assume that reentry is not allowed, and the individual uncertainty for each type of agents is the same as before.

Now the equilibrium payoffs of participants in the second period market are not simply characterized as 1 or 0 depending on relative numbers of the *ex post* productive agents. Recall that the payoff from being unassigned in the second period is assumed to be 0. This boundary payoff is not sufficient to pin down the equilibrium payoffs if the two groups have exactly the same size because it is possible to assign all agents to each other and rents can be distributed differently. [10] In order to solve the problem of indeterminacy, we distinguish two cases: over-supply of W agents ($m_1^1 + m_2^1 < w_1^1 + w_2^1$), and over-supply of M agents ($m_1^1 + m_2^1 > w_1^1 + w_2^1$). We consider the first case only, the second case can be dealt with similarly. If $m_1^1 + m_2^1 < w_1^1 + w_2^1$, then regardless of the numbers of different types of agents who sign first period contracts, W agents are over-supplied in the second period spot market, so that the unproductive W agents (W_2^2 agents) obtain an equilibrium payoff of zero. Since some of M_2^2 agents are matched with W_2^2 agents, they receive ν_1 regardless of the market supply and demand condition. The payoffs to productive M and W agents depend their relative numbers: If the productive W agents are over-supplied in the spot market ($m_1^2 < w_1^2$), their equilibrium payoff is $\nu_2 - \nu_1$, and the equilibrium payoffs for productive M agents are $1 + \nu_1 - \nu_2$; if the productive W agents are under-supplied ($m_1^2 < w_1^2$), their equilibrium payoff is $1 - \nu_2$, and the equilibrium payoffs for M_1^2 agents are ν_2.

As in previous sections, type 2 W agents are not involved in first period contracting. The best terms that a type 2 W agent can offer an M_i^1 agent in a first period contract is ν_2 when the M_i^1 agent turns out to be productive and ν_1 when he is unproductive. However, any M_i^1 agent obtains either $1 + \nu_1 - \nu_2 > \nu_2$ or ν_2 from the second period spot market if he turns out to be productive, and ν_1 with certainty if he turns out to be unproductive.

[10] Note that this kind of payoff indeterminacy is not present with the previous specification of the joint output matrix.

The expected utility from waiting is greater than the maximum utility from signing a first period contract with a type 2 W agent.

The general specification of the joint output matrix brings in some new considerations to the analysis. Any first period contract between an M_i^1 agent and a W_1^1 agent specifies the division of the joint output in two contingencies, when the M_i^1 agent turns out to be productive and when he is unproductive. Since risk-sharing between the M_i^1 agent and the W_1^1 agent must be optimal in the market equilibrium, the terms of the contract in the two contingencies are not independent. Suppose the first period contract specifies r and s as the payoffs to the M_i^1 agent in the first period contract when he is productive and when he is unproductive respectively. Payoffs to the W_1^1 agent in the two contingencies are $1 - r$ and $\nu_2 - s$ respectively. Since payoffs to the W_1^1 agent must be non-negative, $r \leq 1$ and $s \leq \nu$. In the appendix (Lemma A.3) we establish that s can be written as a (weakly) increasing, piecewise differential function of r.

The ask and bid price functions can be defined as before: For any fixed $\pi \in [0,1]$ and $i = 1, 2$, $r_i^m(\pi)$ is the solution to the equation

$$\lambda_i u(r) + (1-\lambda_i)u(s(r)) = \lambda_i[\pi u(1+\nu_1-\nu_2) + (1-\pi)u(\nu_2)] + (1-\lambda_i)u(\nu_1),$$

and $r_i^w(\pi)$ is the solution to the equation

$$\lambda_i v(1-r) + (1-\lambda_i)v(\nu_2 - s(r)) = \pi v(\nu_2 - \nu_1) + (1-\pi)v(1-\nu_2).$$

Note that $s(r)$ represents the payoff to the M_i^1 agent when he is unproductive as a function of his payoff when he turns out to be productive.

A comparison of the two ask price functions reveals that the ask price function of type 1 M agents may be either below or above that of type 2. For a given π, a greater λ tends to increase the ask price because the agent has a higher probability of being productive and hence a higher expected utility from waiting), but at the same time it tends to decrease the ask price because the agent has greater probability of obtaining the better terms r_i rather than $s(r_i)$. One can show that if ν_1 is close to ν_2, the second tendency dominates and the ask price function of type 1 M agent lies below the type 2 ask price function. Since the bid price function for type 1 M agent lies above the bid price function for type 2 (because $\lambda_1 > \lambda_2$), the order of unraveling is preserved when ν_1 is close to ν_2: There must be complete unraveling of type 1 M agents before any unraveling of type 2 M agents. The monotonicity property of $\pi(y_1, y_2)$ established in section 3 clearly remains valid in this more general assignment market. With these two properties, different types of unraveling equilibria can be established in a similar way as in section 3.

Appendix

Lemma A.1. *For each* $i = 1, 2$, $r_i^m(\pi)$ *and* $r_i^w(\pi)$ *are increasing in* π. *For any* $\pi \in (0,1)$, $r_i^m(\pi) < r_i^w(\pi)$. $r_i^m(0) = r_i^w(0) = 0$ *and* $r_i^m(1) = r_i^w(1) = 1$.

Proof. By the definitions of r_i^m and r_i^w, we have

$$(1 - \pi + \lambda_i \pi)u(r_i^m(\pi)) = (1-\pi)u(0) + \lambda_i \pi u(1),$$

$$(1 - \pi + \lambda_i \pi)v(1 - r_i^w(\pi)) = \lambda_i \pi v(0) + (1-\pi)v(1),$$

and thus $r_i^m(\pi)$ and $r_i^w(\pi)$ are increasing functions of π. Direct inspection of the above two equations reveals that $r_i^m(0) = r_i^w(0) = 0$ and $r_i^m(1) = r_i^w(1) = 1$. Taking derivatives with respect to π, we have

$$u'(r_i^m(\pi))\frac{dr_i^m(\pi)}{d\pi} = \frac{\lambda_i[u(1) - u(0)]}{(1 - \pi + \lambda_i \pi)^2},$$

$$v'(1 - r_i^w(\pi))\frac{dr_i^w(\pi)}{d\pi} = \frac{\lambda_i[v(1) - v(0)]}{(1 - \pi + \lambda_i \pi)^2}.$$

Therefore,

$$\frac{dr_i^m(0)}{d\pi} = \frac{\lambda_i[u(1) - u(0)]}{u'(0)}, \quad \frac{dr_i^m(1)}{d\pi} = \frac{u(1) - u(0)}{\lambda_i u'(1)},$$

$$\frac{dr_i^w(0)}{d\pi} = \frac{\lambda_i[v(1) - v(0)]}{v'(1)}, \quad \frac{dr_i^w(1)}{d\pi} = \frac{v(1) - v(0)}{\lambda_i v'(0)}.$$

Since u and v are concave, $(u(1) - u(0))/u'(0) < 1$ and $(u(1) - u(0))/u'(1) > 1$, $(v(1) - v(0))/v'(0) < 1$ and $(v(1) - v(0))/v'(1) > 1$. Therefore, we have $dr_i^m(0)/d\pi < dr_i^w(0)/d\pi$ and $dr_i^m(1)/d\pi > dr_i^w(1)/d\pi$. Since u and v are continuously differentiable, there is π_i' such that $dr_i^m(\pi)/d\pi < dr_i^w(\pi)/d\pi$ for all $\pi < \pi_i'$ and $dr_i^m(\pi_i)/d\pi = dr_i^w(\pi_i)/d\pi$. Then, for all $\pi > \pi_i'$,

$$\begin{aligned}\frac{dr_i^m(\pi)}{d\pi} &= \frac{\lambda_i[u(1) - u(0)]}{u'(r_i^m(\pi))(1 - \pi + \lambda_i \pi)^2} > \frac{\lambda_i[u(1) - u(0)]}{u'(r_i^m(\pi_i))(1 - \pi + \lambda_i \pi)^2} \\ &= \frac{\lambda_i[v(1) - v(0)]}{v'(r_i^w(1 - \pi_i))(1 - \pi + \lambda_i \pi)^2} > \frac{\lambda_i[v(1) - v(0)]}{v'(r_i^w(1 - \pi))(1 - \pi + \lambda_i \pi)^2} \\ &= \frac{dr_i^w(\pi)}{d\pi}.\end{aligned}$$

Since $r_i^m(0) = r_i^w(0) = 0$ and $r_i^m(1) = r_i^w(1) = 1$, $r_i^m(\pi)$ does not intersect $r_i^w(\pi)$ for all $\pi \in (0,1)$, and $r_i^m(\pi) < r_i^w(\pi)$.

Q.E.D.

Lemma A.2. *For all* $\pi \in (0,1)$,

$$(1 - \lambda_1)\pi v(0) + [1 - (1 - \lambda_1)\pi]v(1 - r_1^m(\pi)) > (1 - \lambda_2)\pi v(0) + [1 - (1 - \lambda_2)\pi]v(1 - r_2^m(\pi)).$$

Proof. Given any $\pi \in (0,1)$, let

$$V(\lambda_i) = (1-\lambda_i)\pi v(0) + [1-(1-\lambda_i)\pi]v(1-r_i^m(\pi))$$

be the expected utility for W_1^1 agents from first period contracts with M_i^1 agents at price $r_i^m(\pi)$. Given π, we can think of $V(\lambda)$ as a function of λ given by

$$V(\lambda) = (1-\lambda)\pi v(0) + [1-(1-\lambda)\pi]v(1-r^m(\lambda)),$$

where $r^m(\lambda)$ is a function of λ defined by

$$(1-\pi+\lambda\pi)u(r^m(\lambda)) = (1-\pi)u(0) + \lambda\pi u(1).$$

The statement of the lemma follows if we can show that $V(\lambda)$ is increasing in λ. By the definition of $V(\lambda)$,

$$dV(\lambda)/d\lambda = \pi[v(1-r^m(\lambda)) - v(0)] - (1-\pi+\lambda\pi)v'(1-r^m(\lambda))dr^m(\lambda)/d\lambda.$$

By the definition of $r^m(\lambda)$,

$$dr^m(\lambda)/d\lambda = \frac{[u(1)-u(0)]\pi(1-\pi)}{u'(r^m(\lambda))(1-\pi+\lambda\pi)^2}.$$

Combining the above expressions, we find that $dV(\lambda)/d\lambda > 0$ if and only if

$$\frac{v(1-r^m(\lambda)) - v(0)}{v'(1-r^m(\lambda))} > \frac{u(1-r^m(\lambda)) - u(0)}{u'(r^m(\lambda))}\frac{1-\pi}{1-\pi+\lambda\pi}.$$

By the definition of $r^m(\lambda)$, we have

$$\frac{1-\pi}{1-\pi+\lambda\pi}[u(1-r^m(\lambda)) - u(0)] = u(1) - u(r^m(\lambda)).$$

Thus, $dV(\lambda)/d\lambda > 0$ if and only if

$$\frac{v(1-r^m(\lambda)) - v(0)}{v'(1-r^m(\lambda))} > \frac{u(1) - u(r^m(\lambda))}{u'(r^m(\lambda))}.$$

Since u and v are concave, the left-hand side of the above inequality is greater than $1-r^m(\lambda)$ while the right-hand side is less than $1-r^m(\lambda)$. This completes the proof of the lemma.

Q.E.D.

Lemma A.3. *There exists $\underline{r} \in (0, 1-\nu_2)$ and $\bar{r} \in (\nu_2, 1)$ with $\underline{r} < \bar{r}$ such that in any optimal risk-sharing arrangement, $s = 0$ if $r \in [0, \underline{r}]$, $s = \nu_2$ if $r \in [\bar{r}, 1]$, and*

$$\frac{u'(r)}{u'(s)} = \frac{v'(1-r)}{v'(\nu_2 - s)}$$

if $r \in (\underline{r}, \bar{r})$.

Proof. Let $\underline{r}$ be the solution to the equation

$$\frac{u'(r)}{u'(0)} = \frac{v'(1-r)}{v'(\nu_2)}.$$

Since the left-hand-side of the above equation is greater than the right-hand-side when $r = 0$ and the opposite is true when $r = 1-\nu_2$, and since the left-hand-side is decreasing in r and the right-hand-side is increasing in r, the above equation defines a unique $\underline{r} \in (0, 1-\nu_2)$. Similarly, the equation $u'(r)/u'(\nu_2) = v'(1-r)/v'(0)$ defines a unique $\bar{r} \in (\nu_2, 1)$. Note that by definitions $\underline{r} < \bar{r}$. Otherwise,

$$\frac{u'(0)}{v'(\nu_2)} = \frac{u'(\underline{r})}{v'(1-\underline{r})} \leq \frac{u'(\bar{r})}{v'(1-\bar{r})} = \frac{u'(\nu_2)}{v'(0)},$$

resulting in a contradiction.

Suppose $r \in [0, \underline{r}]$. If $s > 0$, we would have

$$\frac{u'(r)}{u'(s)} > \frac{u'(\underline{r})}{u'(0)} = \frac{v'(1-\underline{r})}{v'(\nu_2)} > \frac{v'(1-r)}{v'(\nu_2 - s)}.$$

This implies that the two agents in the contract can be made better off if r is increased slightly and s decreased slightly, a contradiction. Thus, $s = 0$ if $r \in [0, \underline{r}]$. By a symmetric reasoning, $s = \nu_2$ if $r \in [\bar{r}, 1]$. If $r \in (\underline{r}, \bar{r})$, the first-order condition that an optimal risk-sharing arrangement must satisfy leads the relation stated in the lemma.

Q.E.D.

References

Vincent Crawford [1991]:"Comparative Statics in Matching Markets," *Journal of Economic Theory*, 54(2), 389-400.

David Gale and Lloyd Shapley [1962]: "College Admissions and the Stability of Marriage," *American Mathematical Monthly*, 69(1), 9-15.

Tjalling Koopmans and Martin Beckmann [1953]:"Assignment Problems and the Location of Economic Activities," *Econometrica* 25(1), 53-76.

Susan Mongell and Alvin Roth [1991]: "Sorority Rush as a Two-sided Matching Mechanism," *American Economic Review*, 81(3), 441-464.

Alvin Roth [1984]:"The Evolution of the Labor Market for Medical Interns and Residents: A Case Study in Game Theory," *Journal of Political Economy*, 92(6), 991-1016.

Alvin Roth [1991]:"A Natural Experiment in the Organization of Entry-Level Labor Markets: Regional Markets for New Physicians and Surgeons in the United Kingdom," *American Economic Review*, 81(3), 415-440.

Alvin Roth and Xiaolin Xing [1994]:"Jumping the Gun: Imperfections and Institutions Related to the Timing of Market Transactions," *American Economic Review* 84(4), 992-1044.

Lloyd Shapley and Martin Shubik [1972]:"The Assignment Game I: The Core," *International Journal of Game Theory* 1(1), 111-130.

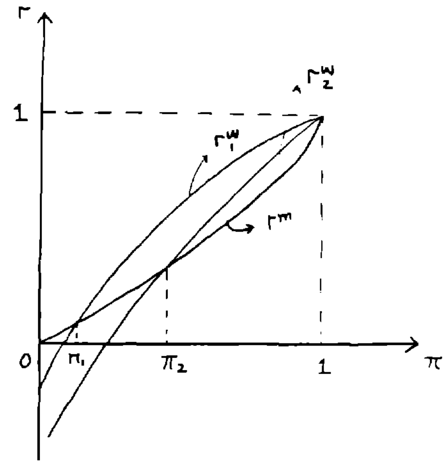

Figure 1. Ask and bid price functions without reentry.

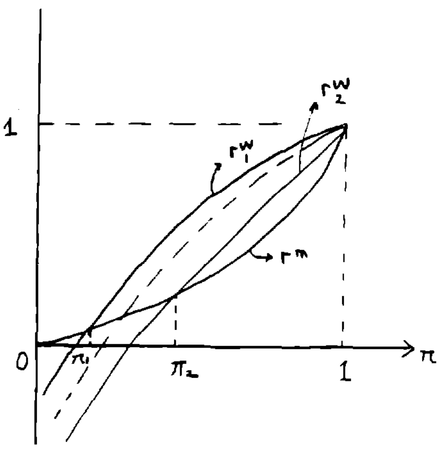

Figure 2. Ask and bid price functions when λ_2 increases.

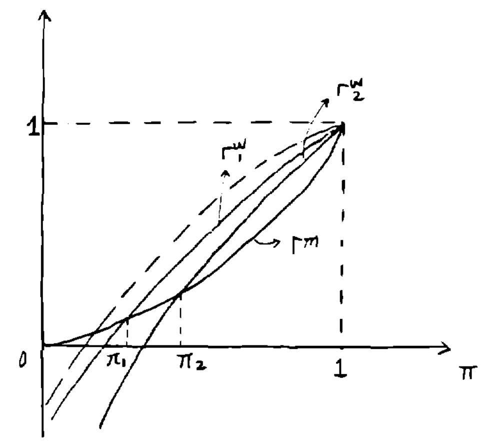

Figure 3. Ask and bid price functions when λ_1 decreases.

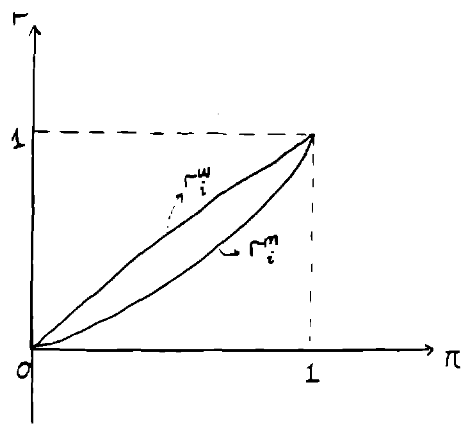

Figure 4. Ask and bid price functions with reentry.