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QUALITY CHANGE IN CAPITAL GOODS AND ITS IMPACT ON ECONOMIC GROWTH

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ABSTRACT

This paper argues that productivity puzzles like the Solow Paradox arise, in part, from the omission of an important dimension of the debate: the resource cost of achieving a given rate of technical change. A remedy is proposed in which a new parameter, defined as the cost elasticity of producing capital with respect to the rate of technical change, is introduced. This parameter is shown to be latent in the Hall-Jorgenson user-cost of capital, as well as in the Solow residual. It is also shown that an increase in the rate of embodied technical change may actually cause a *decrease* in the Solow residual, in the short run, if the parameter is greater than the ratio of the user cost to the corresponding asset price.

Different values of the new parameter also correspond to different theories of technological innovation: the Solow-Swan and Cass-Koopmans assumption of costless technical change is consistent with a zero value of the cost elasticity parameter, while the model of endogenous growth with R&D externalities implies a larger value. Finally, the appropriate investment-good price deflator is shown to be a function of the cost-elasticity. When the parameter equals zero, no quality adjustment should be undertaken, but values greater than zero lead to a partial adjustment for quality change, and a value of one leads to a full correction.

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QUALITY CHANGE IN CAPITAL GOODS AND ITS IMPACT ON ECONOMIC GROWTH

I. Introduction

Persistent improvements in the quality of new machinery and equipment are a salient feature of modern technical progress. Studies of computer prices suggest large annual improvements in quality (e.g., Cole et. al. (1986)), and Robert Gordon (1990) finds significant quality changes in a wide variety of other producers' durable equipment. But just how important they are for the measurement of economic growth remains a matter of debate. Denison (1989) argues that quality adjustments should not be applied to official prices indexes on the grounds that this merely reclassifies growth from one source to another, and Greenwood et. al. (1992) argue that the quality adjustment of capital goods may obscure the welfare gains obtained from innovation. There is also the unanswered "Solow Paradox": if computers are so important and their quality has improved so much, why hasn't total factor productivity growth been stronger?

This paper argues that these productivity puzzles arise, in part, from the omission of an important dimension of the debate: the resource cost of achieving a given rate of technical change. A remedy is proposed, in which a new parameter, defined as the <u>cost elasticity of producing capital with respect to the rate of technical change</u>, is introduced. This parameter is shown to be latent in the Hall-Jorgenson user-cost of capital, as well as in the price of capital goods, and to generalize the Hall-Jorgenson model in the following way: when the cost elasticity equals zero, the reformulated user-cost model reduces to the Hall (1967) form of this variable; on the other hand, the user cost reduces to the original Hall-Jorgenson formulation when the elasticity is one.

The theoretical foundations of the "Solow Paradox" are examined in this framework, and it is shown that when the cost-elasticity parameter takes on a value of one, the Solow residual disappears along the optimal growth path even if the rate of embodied technical change is positive. It is also shown that an increase in the rate of embodied technical change may actually cause a *decrease* in the Solow residual, in the short run. This arises when the cost elasticity is greater than the ratio of the user cost to the asset price or, put differently, when the capitalized cost of innovation exceeds the corresponding user cost. This paradox thus gives one answer to the Solow question: quality change costs resources to obtain.

The utility of the new parameter also arises from the fact that different values correspond to different theories of technological innovation: the "Manna from Heaven" assumption of costless technical change, implicit in the Solow-Swan and Cass-Koopmans models of economic growth, implies a cost elasticity of zero; on the other hand, the model of endogenous growth with R&D externalities implies a value between zero and one. The new parameter thus links asset prices and user costs to alternative paradigms of technological innovation, and might therefore be used to discriminate among the competing models.

The proposed reformulation of the user cost also sheds new light on the debate of the appropriate treatment of quality change in capital goods prices. This debate, described in Triplett (1983), is usually seen as a conflict between those who want to correct the price of capital for the full extent of quality change (the "user-value" view), and those, like Denison, who advocate a "resource-cost" standard in which there is no correction at all. This paper comes out in favor of the Triplett resolution of this debate, in which the resource-cost approach is correct for deflating the price of investment goods, while the user-value approach is correct for capital input prices. There is, however, an important qualification: the magnitude of the resource-cost deflator is shown to be a function of the cost-elasticity parameter. This leads to the conclusion that the resource-cost approach might possibly imply a full correction for quality change, and not necessarily a zero correction as is often believed. Indeed, only when the parameter equals zero does the resource-cost approach imply that no quality adjustment at all should be undertaken. Values of the cost elasticity greater than zero will lead to a partial adjustment for quality change, and a value of one leads to full correction.

The final sections discuss how the cost elasticity of technical change might be estimated using price-hedonic techniques. The main result is that, under certain conditions, the standard price-hedonic regression can be shown to be homogenous of degree μ, the magnitude of the cost elasticity parameter. This implies a strategy for measuring this parameter, and a brief review of the price-hedonic literature suggests values around one. A final section sums up and sketches implications for the quality-deflation of consumption.

II. Capital Prices and Technical Change

A. The Neoclassical User Cost of Capital

Two prices are implicit in the standard growth and production theory. In optimal growth models of a one sector economy, the shadow price of capital goods is the costate variable of the Hamiltonian function, while the rental price (or user cost) is equal to the marginal product of capital along the optimal growth path.[1] These two capital prices also constitute the core of the neoclassical investment theory developed by Jorgenson (1963) and extended in Hall and Jorgenson (1967), and the user cost is a central part of the neoclassical theory of cost and production functions.

The user cost of capital and the price of capital goods are connected by the equilibrium relationship that requires the price of the latter to equal the present value of the future flow of user costs. Assuming, for simplicity of exposition, that the discount rate, r, is constant over time, this present value expression takes the form

$$(1) \qquad P^{I}_{t,s} = \sum_{\tau=0}^{\infty} \frac{P^{K}_{t+\tau,s+\tau}}{(1+r)^{\tau+1}} \quad ,$$

where $P^{I}_{t,s}$ is the price of an asset of age s in year t, and $P^{K}_{t,s}$ is the expected flow of user costs (or value of marginal product) over the remainder of its useful life. The price $P^{I}_{t,s}$ has two time subscripts, the first referring to the passage of calendar time and the other to asset age. Each subscript advances from year to year, tracing the price history, or *vintage-price profile*, of a given capital good.

In the special case in which the physical productivity of a capital good decays at a constant rate with age, δ, the user cost of old assets is proportional to that of new assets: $P^K_{t,s} = (1-\delta)^s P^K_{t,0}$. This can be inserted into equation (1) to express the present value in terms of the new user-cost equivalents. The result can then be inverted to yield an expression for the user cost in terms of the capital-goods price. For the case of new assets (s=0),

$$(2) \qquad P^K_{t,0} = rP^I_{t,0} + \delta P^I_{t+1,0} - [P^I_{t+1,0} - P^I_{t,0}] \quad .$$

This user cost is the sum of three terms: the opportunity cost of the investment, $P^I_{t,s}$, the economic depreciation on this amount, and a revaluation term in brackets that allows for changes in the price level over time. This is the conventional Hall-Jorgenson user cost minus taxes, for s=0.

An extension of the capital pricing model was adapted by Hall (1967) to the case in which technical change is embodied in the design of new capital goods. In the model of embodied technical change, successive vintages of capital differ in (marginal) productivity by a fixed parameter $1+\theta$ (Fisher (1965)). If embodied technical change proceeds at a constant rate, θ, then the relation between the user cost of an old capital good and the cost of its new equivalent is $P^K_{t,s} = [(1-\delta)/(1+\theta)]^s \ P^K_{t,0}$. This leads directly to an expanded version of (2):

$$(3) \qquad P^K_{t,0} = rP^I_{t,0} + \delta \frac{P^I_{t+1,0}}{1+\theta} - [\frac{P^I_{t+1,0}}{1+\theta} - P^I_{t,0}] \quad .$$

In this case, a persistent rate of embodied technical change drives up the user cost of capital in order to compensate the owner-user for obsolescence.

B. The Supply Side of the Capital Price

The conventional user-cost model is essentially demand driven. Demand is determined by comparing the present value, (1), with the cost of acquiring another unit of capital, which is taken as given. The supply side of the pricing problem is suppressed by assuming that the supply curve is perfectly elastic at the prevailing price (see the comment by Hall (1994)). This supply curve may shift over time, as general inflation or changes in relative prices increase the marginal cost of producing capital goods, but the supply price of capital is assumed to be fixed in any time period and the shift factors are not explicitly represented in the user-cost model. Instead, they are implicit in the revaluation terms of equations (2) or (3).

This paper introduces supply-side parameters into the user-cost formula. This might be accomplished in several ways, but for economy of exposition, we will retain the assumption of perfect elasticity at each point in time but allow for shift parameters. Specifically, marginal cost, MC, is assumed to be constant at each point in time ($\partial MC/\partial Q=0$), but period-to-period shifts in marginal cost follow (in discrete time):

$$MC_{t+1} = (1+\mu\theta)(1+\rho)MC_t. \tag{4}$$

The second shift factor allows for an increase in marginal cost due to changes in factor cost, e.g., if factor prices rise at a common rate ρ. The parameter μ is defined as the elasticity of the marginal cost of producing a unit of capital good with respect to the rate of technical change, i.e., $\mu = \partial MC_t/\partial\theta$. The parameter μ thus allows for the possibility that process of technical change may increase the cost of producing capital. When $\mu = 0$, innovation occurs without affecting the cost of new capital; otherwise, the parameter μ affects the price of capital goods.

The cost elasticity parameter affects the price of capital goods through the marginal cost expression (4). When the price of capital goods equals the marginal cost of production, and marginal cost is constant with respect to the quantity of investment goods produced, pricing evolves over time as a mark-up for inflation and for the cost to embodied technical change; thus the condition $P^I_{t+1,0} = (1+\mu\theta)(1+\rho)P^I_{t,0}$. A time-invariant mark-up over marginal cost due to other causes may also be included without affecting this result.

C. Interpretation of the Cost Elasticity

The parameter μ has a straight-forward macroeconomic interpretation. In a one-good macro model in which consumption and investment are perfect substitutes, $C+I = F(K,L)$, and in a world without quality change, a one unit increase in investment requires a one unit decrease in consumption. The same is true of a world in which costless quality improvements occur. But when quality change costs resources of obtain (i.e., is "consumption-using"), the tradeoff between C and I (measured, as before, in transactions units rather than <u>effective</u> units) is no longer one-to-one. If the new model of capital good is $(1+\theta)$ times as productive as the preceding year's model, $(1+\mu\theta)$ units of consumption must be given up to get an additional unit of investment.[2] When consumption is taken as the numeraire good, the marginal rate of transformation between C and I is thus $(1+\mu\theta)$, and since this is the ratio of marginal products, (4) follows immediately.[3]

In a macro framework, $\mu\theta$ can be thought of as the annual cost in consumption goods of sustaining the path of embodied technical change defined by the rate θ. However, μ is not just a macro concept, but rests on strong microeconomic foundations. Indeed, two sets of micro factors determine the magnitude of μ. First, machines embodying new technology may be more or less costly to build, either because they are made of more costly materials, or use more costly types of labor and capital, or a new design may simply require a greater quantity of inputs. Examples abound: the new halogen light bulb technology is approximately four times more efficient but also costs four times as much as conventional light bulb technology; the limits of the silicon-based computer technology are at hand and future innovations will probably require the more expensive gallenium arsenide; the robotics and numerical control revolutions in machine tools caused the price of individual machines to rise significantly. In each case, the new capital goods are produced at a higher marginal cost than those they supplant, with the result that $\mu > 0$.

Research and development expenditures are a second, and perhaps more important, micro mechanism through which technical change may drive up the cost of new machines. Most advances in knowledge are the result of systematic investments in research and development. Expenditures for acquiring new knowledge are inputs to the production of new goods and contribute to the cost of production, because some fraction of the R&D expenditures must be capitalized in the price of the new good in order to make the research investment profitable. The pattern of amortization may be the result of a complex economic calculation in which R&D costs are distributed unevenly across future years, but it is clear that the amortization process implies $\mu > 0$, and a shift in the supply curve of new capital goods over some span of time.

It is also clear that there are other situations in which innovation does not affect the cost of producing capital. It is often the case that the benefits of innovation cannot be fully appropriated by the innovator. Indeed, there is a large body of evidence in the R&D literature to suggest that the wedge between the social and private marginal returns to R&D is important. In this case, the recipients of the spillover may receive the technology at little or no cost, resulting in a value of μ equal to zero for these beneficiaries. This must be averaged against the actual developmental cost of the innovators to arrive at an average value of μ for the economy.[4] Alternatively, innovation may arise costlessly from pure inspiration or serendipity. In neither case does the marginal cost function shift upward by the full cost of the innovation.

The process of innovation and diffusion is extremely complex, and thus the link between R&D expenditures and the μ formulation in equation (4) is hardly straightforward. For any given innovation and resulting "model year" of capital good, the pattern of R&D amortization may be the result of a complex economic calculation in which R&D costs are distributed unevenly across future years. We will largely ignore this complexity, since the goal of this paper is to point out the utility of the μ formulation even, in its most rudimentary form.

III. μ and the Price of Capital Goods

A. The μ-Modified User Cost

Since the magnitude of μ is an empirical issue, any complete analysis of capital goods prices should allow for the possibility that it can assume any feasible value. This can be accomplished by introducing the parameter μ into the user cost by substitution of $(1+\mu\theta)(1+\rho)P^I_{t,0}$ for $P^I_{t+1,0}$ in (3). This yields an alternative form of the Hall-Jorgenson user-cost formula for the case of constant parameter values:

<u>Proposition 1</u>: Under the assumption that the relevant parameters (the rate of discount, r, the rate of inflation, ρ, the rate of asset decay, δ, the rate of embodied technical change, θ, and μ) are constant over time, the user cost of capital, $P^K_{t,0}$, is the following constant proportion of the new investment good price, $P^I_{t,0}$:

$$(5) \qquad P^K_{t,0} = [r - \frac{1+\mu\theta}{1+\theta}\rho + \frac{1+\mu\theta}{1+\theta}\delta + \frac{(1-\mu)\theta}{1+\theta}]\, P^I_{t,0} \quad .$$

When the derivation is done in discrete time, second order effects (e.g., $\rho\mu\theta$) are set to zero. It is important to note that this modified user-cost refers to transaction unit of capital, unadjusted for changes in quality.

When the value of the parameter μ takes on a value of zero, the user cost reduces to the Hall form shown in equation (3) above, which includes the rate of embodied technical change, $P^K_{t,0} = [(r-\rho+\delta+\theta)/(1+\theta)]P^I_{t,0}$. On the other hand, when μ equals one, the original Jorgenson form of the user cost holds, (2), and $P^K_{t,0} = [r-\rho+\delta]P^I_{t,0}$; i.e., the user cost is invariant to θ and is equivalent to the user cost prevailing in the absence of embodied technical change.

The extension of the user-cost model proposed in Proposition 1 is thus a generalization of the existing framework. However, it is important to recognize that the parameter μ has always been *latent*, or implicit, in the user-cost formulation, and has not been added *de novo* in this paper. In the conventional treatment, μ is suppressed into the revaluation term $[(P^I_{t+1,0}/(1+\theta))-P^I_{t,0}]$ and, in this suppressed form, differences in assumptions about μ appear as different paths for the price of new capital goods over time.

B. From User Cost to Asset Price

Since the parameter μ affects the user cost, the present value equation (1) implies that it should also affect the capital goods price, $P^I_{t,0}$. Indeed, the new user-cost formulation can be inserted into the present value expression to yield:

> Proposition 2: Under the assumption that the relevant parameters are constant over time (see Proposition 1), the vintage-price profile can be expressed as

$$(6) \qquad P^I_{t+s,s} = \frac{(1+\mu\theta)^s}{(1+\theta)^s}(1-\delta)^s(1+\rho)^s P^I_{t,0} \quad .$$

As with the user cost, conventional treatments of the capital-goods price have recognized the factors δ, ρ, and θ, but have not allowed for μ. And, again, it should be noted that this vintage-price profile refers to transaction unit of capital, unadjusted for changes in quality.

According to Proposition 2, a value of $\mu = 1$ causes the technical change parameter θ to disappear from the vintage price path, implying that the decline in the value of a capital good over its life is invariant to the rate of technical change in new capital. This means that the arrival of newer and technologically superior substitutes does not drive down the price of old capital, because the technical advantage of the new assets is just balanced against the increased cost of acquiring those assets. The welfare gains from innovation are exactly the same as the gains from investing in any other form of capital good.μ However, when $\mu < 1$, this balance no longer holds and the net advantage of the new assets is capitalized in the value of their older counterparts. This effect is termed "obsolescence." When $\mu = 0$, the technological disadvantage of old assets is fully capitalized into their price and obsolescence is at its maximum. This effect is familiar to owners of older models of personal computers.

B. The Separate Effects of Age and Time

The vintage-price path of a given asset can be expressed as the sum of two factors: a pure aging effect (analogous to the partial derivative of the price, $P^I_{t,s}$, with respect to age, s), and a pure time effect (analogous to the partial derivative of $P^I_{t,s}$ with respect to time, t).[5] The age effect is caused by the decay in asset productivity and is represented by the *age-price profile*, defined as the set of asset prices $[P^I_{t,s}]$ for t held fixed. This effect is also called *economic depreciation*, and is of central importance in the measurement of income and wealth.

The time effect, on the other hand, is associated with changes in the general price level, changes in relative prices, and changes in the quality of capital goods, and is called *revaluation*. It is represented by the *time-price profile* defined as the set of asset prices $[P^I_{t,s}]$ with s held fixed. This effect captures changes in the general price level and other factors that cause the price of new assets to increase over time, as well as the price of assets of all ages. In light of our discussion of μ above, it is not surprising that the vintage price profile of Proposition 2 can be "disaggregated" to obtain results about its component parts:

Proposition 3: Under the constant parameter assumptions of Proposition 1, the vintage profile can be expressed as the product of the *age-price profile*

$$P^I_{t,s} = \frac{(1-\delta)^s}{(1+\theta)^s} P^I_{t,0} \quad , \tag{7}$$

and the *time-price profile*

$$P^I_{t+s,0} = (1+\mu\theta)^s (1+\rho)^s P^I_{t,0} \quad . \tag{8}$$

Note that the age-price profile does not depend on the cost parameter μ. This leads to the conclusion that economic depreciation depends on the pure decay effect, δ, and the obsolescence effect, θ, but not on the cost of technical change. This, in turn, leads to the conclusion that economic depreciation can be estimated from age-price profiles (e.g., following Hulten and Wykoff) independently of the value of μ. But, unlike economic depreciation, revaluation does depend on μ, as well as the rate of technical change and the change in prices. Thus, μ enters the determination of capital goods prices only through the time (revaluation) side of the problem.

The cost elasticity again plays a latent role in the determination of asset price, $P^I_{t,s}$. This price can therefore be measured and analyzed without explicit reference to μ. However, the price of capital depends implicitly on the value of this parameter, a fact which is illustrated graphically in Figures 1 and 2 of the appendix. These figures are derived from different arrays of asset prices calculated using equation (6): Table A1 examines the case in which $\mu = .5$, $\rho = 0$, $\delta = 0$, and $\theta = .1$; the implications of changing the cost parameter μ are shown in Tables A2 (μ=0) and A3 (μ=1).[6] The impact of the μ parameter on asset price is readily apparent in these examples.

IV. Taxonomies of Technical Change and the Parameter μ

Although the cost elasticity is a latent parameter of the user cost-asset price model, it has several practical applications. First, it will be shown in this section that the cost elasticity has interesting taxonomic properties in classifying alternative theories of technical change. Then, in the following section, we will see that μ plays an important role in the theory and measurement of capital-good price deflators. Finally, we show that μ has implications for the Solow Paradox.

The first of these applications arises because different values of μ are implicit in different growth theories, and thus the user cost and asset price differ systematically from one paradigm to the next. On the one hand, the Solow-Swan and Cass-Koopmans models of economic growth are silent about the mechanisms through which technical change is achieved, but the implicit assumption is that improvements in technology occur through inspiration, serendipity, or imitation, but without a resource cost. The Johansen-Salter-Solow model of embodied technical change also assumed that technical advances could be achieved without explicit resource cost (with dissents by Domar (1963) and Jorgenson (1966)). In view of the definition of μ, this is tantamount to assuming that $\mu = 0$.

On the other hand, the growth model of Romer (1986) offers a different paradigm of technical change in which innovation is endogenized through systematic investment in knowledge capital and technical change becomes an endogenous function of the rate of investment along the growth path of the economy. There are several mechanisms through which this can occur, but all involve a commitment of resources to produce and use new knowledge and therefore imply a value of μ greater than zero.[7] The same can be said for the class of endogenous technology models, which is broader than the class of endogenous growth models.[8]

This taxonomic property of μ has a potentially important empirical implication: since the alternative models of economic growth predict different values of the latent parameter μ, measurement of this parameter could in principle be used to discriminate among the competing theories. In other words, the different price paths evident in Figures 1 and 2 are associated with different innovation processes and therefore might be used to test hypotheses about the endogeneity of technical change. We will return to this possibility in Section VII below, after discussing two other applications of the cost elasticity.

V. The Implications of μ for Investment Price Deflators

A. Background of the Deflation Issue

Total expenditure on investment goods is, by definition, the price of capital goods times the quantity purchased, $V=P^I I$. The units of measurement are specified by the market: I denotes the number of transaction units exchanged in the market and P^I the transaction price. This is the usual convention of economic accounting, and it poses no problem when changes in quality are negligible. This convention is ambiguous, however, when there are quality improvements from one year to the next. In this case, the physical unit of transaction represents more productive capacity in one year than another. An adjustment for the productivity (or "quality") improvement would reflect the increase, and replace I_t with quality-adjusted measure, H_t. The price of investment goods must be adjusted in the opposite direction, since total expenditure on the investment is invariant to the quality adjustment, implying $V_t = P^I{}_t I_t = P^H{}_t H_t$. In this formulation, $P^H{}_t$ is the price of investment measured in efficiency units.

As noted above, there has been an active debate over the use of quality-adjusted measures of capital in production and growth theory, and three distinct positions are discernible. The first is the original neoclassical formulation in which no adjustment is made to the output of investment goods or to the capital stock. This position has been defended by Denison (1989) in his critique of the decision by the Bureau of Economic Analysis to adjust computers for quality change. In the Denison view, the aggregate production function should defined as $Q_t = C_t+I_t = F(L_t,K_t,t)$, where capital stock is defined by the accumulation process

$$(9) \qquad K_t = I_t + (1-\delta)I_{t-1} + \ldots + (1-\delta)^t I_0 \quad .$$

In this concept of capital, physical depreciation proceeds at a constant rate, implying that $(1-\delta)$ units of productive capacity remains after the first year of asset life, etc. The pattern of productivity decay is not limited to the geometric case. Technical change is represented as a shift in the aggregate production function, and is absent from the concepts of investment and capital stock.

In the alternative Johansen-Salter-Solow view, technical innovations are embodied in the design of new capital. When embodied technical change proceeds at the constant rate θ, the stock measured in quality-adjusted units is given by

$$(10) \qquad J_t = (1+\theta)^t I_t + (1+\theta)^{t-1}(1-\delta)I_{t-1} + \ldots + (1-\delta)^t I_0 \quad .$$

This formulation differs from the preceding accumulation equation by the quality factor $(1+\theta)^t$ associated with annual investment.[9] This suggests a quality-adjusted concept of investment based on θ.[10] In this case, the aggregate production function has the form $Q_t = F(L_t, J_t)$.

The allocation of output Q_t between consumption and investment is the subject of another controversy within the embodied technical change literature. In the original embodiment model, the output of capital goods was not adjusted for quality, yielding $C_t+I_t= F(L_t,J_t)$. An alternative view was proposed by Domar (1963) and Jorgenson (1966), who opt for quality adjustment on both sides of the production function: $C_t+H_t = F(L_t,J_t)$.

This debate over quality adjustments to the quantity of investment and capital has a parallel in the literature of quality adjustments on the price side of the problem. The identity $V_t = P^I{}_t I_t = P^H{}_t H_t$ indicates that quality adjustment in prices is not independent of the adjustment for quantities, and that they must arise from the same source. However, the debate on the price side has used a different vocabulary, emphasizing the distinction between the "resource-cost" approach to quality deflation and the "user-value" approach (Triplett (1983)). As a resolution to this debate, Triplett advocates the use of a resource-cost standard for the output of investment goods, and a user-value approach for capital input. In other words, the Triplett resolution implies separate deflators for capital input and output. In terms of the preceding discussion, this amounts to the advocacy of the $C_t+I_t= F(L_t,J_t)$ version of the production function *if* it is assumed that investment measured in transaction units, I_t, corresponds to the resource cost of capital formation and that $H_t = (1+\theta)^t I_t$ corresponds to the value of capital goods to the user. And, we will see in the following section that this is precisely what is implied by the logic of the resource-cost rationale for quality adjustment of output.

B. Price Deflators for Investment Goods

The resolution of the price deflator debate rests on the Koopmans' argument that measurement should be linked to underlying economic theory, and that measurement issues of growth and production should therefore be consistent with the production theory. The aggregate production possibility frontier (PPF) of an economy determines the maximum quantity of output that may be obtained from a given amount of capital and labor and given technological possibilities. The output side of the technology indicates the attainable combinations of consumption and investment, and the slope of the PPF (the marginal rate of transformation) indicates the additional quantity of investment goods that can be obtained from a unit of foregone consumption -- i.e., the cost of investment in terms of consumption. On the input side, the PPF indicates the combinations of capital and labor required to produce a given combination of output, and the slope of the PPF (the marginal rate of technical substitution) indicates the tradeoffs between the various inputs, including the various vintages of capital goods.

If a quality correction procedure is to be consistent with the PPF, the relative output prices implied by the convention should equal the corresponding slopes of the PPF. Thus, relative output prices should equal the MRT, and relative inputs should equal the MRTS. This elementary proposition leads directly to the resource-cost and user-value deflation conventions: when relative output prices are equated to the MRT, a resource-cost standard is implied for investment goods (i.e., the consumption foregone in producing a marginal unit of capital); when relative capital input prices are based on the MRTS, the ratio of the user cost is implied and this equals (via the Leontief Aggregation condition) the relative marginal product of vintage capital goods *measured in units of productive efficiency.*

The parameter μ enters this discussion by observing that it virtually has been defined with respect to the MRT. With consumption as the numeraire good, the consumption-investment price ratio is just the real marginal cost of investment, and the real marginal cost is defined by equation (4) as $MC_{t+1} = (1+\mu\theta)MC_t$. This leads to the following production-theoretic approach to output price deflation: in some initial time period, normalize quantity units so that the investment-consumption marginal-cost ratio equals one, and thereafter, adjust the price of investment so that the ratio in year t equals $(1+\mu\theta)^t$. This can be expressed formally as

> <u>Proposition 4</u>: Let investment measured in physical units, I_0, be equal to the investment measured in efficiency units, H_0, in a base year t=0 (i.e., let the efficiency index of investment goods be normalized to one in this year). Let θ be the rate of embodied technical change. The marginal rate of transformation between consumption and investment is $(1+\mu\theta)$ and the implied quality adjustment for the *transaction* price of new investment goods (on the output side) is
>
> $$P^H_{t,0} = P^I_{t,0}/(1+\mu\theta)^t. \tag{11}$$
>
> The corresponding quality adjustment for new investment on the output side is thus
>
> $$H_t = (1+\mu\theta)^t I_t \quad . \tag{12}$$

This parameterizes the "resource-cost" versus "user-value" debate by invoking the link between μ and the relative marginal cost of consumption and investment. If no consumption is foregone in producing investment goods of higher quality, $\mu = 0$, and the uncorrected amount of investment, I_t, defines the opportunity cost irrespective of the rate of technical change, θ. On the other hand, when $\mu = 1$, consumption must be given up to obtain θ in the same proportion as the improvement in the efficiency of capital.[11]

The logic of this argument can be extended to the input side to show that the parameter μ does not enter the quality deflation of inputs. The input correction corresponding to the slope of the production possibility frontier is based on the marginal rate of technical substitution between capital of different vintages. The relevant deflator for the input side is given in the following proposition:

Proposition 5: Under the conditions of Proposition 4, the quality adjustment for the price on new capital goods added to the stock (on the input side) that corresponds to the marginal rate of technical substitution between vintages of capital is

$$(13) \qquad P^{H*}_{t,0} = P^{I}_{t,0}/(1+\theta)^{t}.$$

$P^{H*}_{t,0}$ is the efficiency price of new investment goods, and corresponds to the user-value of a unit of new capital, while the deflator for the quantity of new additions to the capital stock is

$$(14) \qquad H^{*}_{t} = (1+\theta)^{t}I_{t}.$$

The MRTS does not generally equal the MRT, leading to a different correction procedure on the input side than on the output side of the PPF. However, the wedge between the output and input corrections, H_t and H^{*}_{t}, is not a free-floating "error term," but instead represents a critical aspect of the innovation process: the extent of costless (welfare-inducing) productivity change.

A remark are in order, here. When investment good prices are deflated by $(1+\mu\theta)$ as per Proposition 4, the preceding propositions are no longer valid. Those propositions were derived under the assumption that investment goods are measured in transaction units, and when the transaction price is deflated by the factor $(1+\mu\theta)$, this term is removed from the vintage-price profile (6), yielding

$$(6') \qquad P^H_{t+s,s} = \frac{P^I_{t+s,s}}{(1+\mu\theta)^s} = (1+\theta)^s (1-\delta)^s (1+\rho)^s P^I_{t,0} \quad .$$

and thus from the time-price profile (8), giving

$$(8') \qquad P^H_{t+s,0} = \frac{P^I_{t+s,0}}{(1+\mu\theta)^s} = (1+\rho)^s P^I_{t,0} \quad .$$

This, in turn, removes the μ terms from the modified user-cost equation (5), leaving the Hall formulation shown in equation (3). This sheds additional light on the role of the parameter μ: when investment is measured in observed transaction units rather than conceptual resource-cost units, a parameter μ must be introduced into the pricing equations in order to compensate for the omitted resource-cost effect. When the units of measurement are appropriately changed, the μ parameter is no longer necessary.

VI. The Solow Residual

A. The Solow Residual with the Parameter μ

The choice between the "resource cost" versus "user value" approach to adjusting the price and quantity of capital goods is, like many issues of economic measurement, a matter of measurement convention (c.f., Jorgenson (1966) and Denison (1989)). A convention based on the PPF has great intuitive appeal, but alternative conventions are neither inherently right or wrong so long as internal consistency is maintained, i.e., $P^H_{1,0}H_1 = P^I_{1,0}I_1$. However, as Jorgenson (1966) points out, any change in the units of measurement of capital simply changes the magnitude of the Solow (1957) productivity residual. The Solow residual is defined as the growth rate of output (uncorrected for quality change) minus the share-weighted growth rates of labor and capital (also uncorrected). This measures changes in output per unit of total input, or "total factor productivity." In continuous time, the Solow residual has the following form

$$(15) \qquad T_t = [(1-\sigma_t)\dot{C}_t + \sigma_t\dot{I}_t] - [\pi_t\dot{K}_t + (1-\pi_t)\dot{L}_t] \quad .$$

where π denotes capital's income share, σ is the share of investment in the value of output, and dots over variables indicate rates of growth. If the underlying production function exhibits constant returns to scale and technical change is both costless and disembodied, i.e., $C_t+I_t = F(L_t,K_t,t)$, π is equivalent to the output elasticity in competitive equilibrium, and the Solow residual measures the shift in the aggregate production function (Solow (1957)).

The significance of the Solow residual in the context of Propositions 4 and 5 arises from the mismeasurement that occurs in the absence of the quality adjustments implied by these propositions. Suppose that the technology exhibits quality change at a constant rate θ with cost elasticity μ, and that the true form is $C_t+H_t = F(L_t,J_t)$. This can be expressed, with Triplett deflation, as $C_t+\lambda_t I_t = F(L_t,\psi_t K_t)$, where $\lambda_t = e^{h\theta t}$ is the continuous time counterpart of $(1+\mu\theta)^t$ and $\psi_t = J_t/K_t$ is the average embodied technical efficiency, whose arguments can be shown to be an investment-weighted average of the θ (see Hulten (1992)). In this case, the Solow residual depends on the relative size of the efficiency parameters:

Proposition 6: When the true form of the technology is $C_t+\lambda_t I_t = F(L_t, \psi_t K_t)$, with constant returns to scale, the Solow residual, computed as per equation (15), is equivalent to

$$(16) \qquad T_t = \pi_t \psi_t - \sigma_t \mu_t \theta_t \; .$$

This extends a similar formulation in Hulten (1992), which did not include the parameter μ (see equation (11) of that paper). Equation (16 indicates that the failure to use the PPF-based "resource cost-user value" deflation convention when the quality change is present merely suppresses the parameters of embodied technical change parameters θ and ψ into the Solow residual.

Proposition 6 introduces additional structure to the total factor productivity model. When θ is constant, $\psi = \theta$ and the Solow residual can be expressed as $(\pi-\mu\sigma)\theta$. In this case, when quality change is costless, $\mu = 0$ and the Solow residual retrieves the rate of embodied technical change, $\pi\theta$. This is the rate at which the production function, defined with respect to the K-concept of capital, shifts over time. This interpretation is similar to the original total factor productivity interpretation of (15), which was based (following Solow) on the rate of disembodied technical change. On the other hand, when $\mu = 1$, the residual disappears when the Golden Rule condition $\pi = \sigma$ holds (see Jorgenson (1966)). This is an intuitively appealing result, since technical change is fully "costed-out" and is like an investment in tangible capital. There is thus no shift in the production function in this situation, and technical change is characterized as a movement along the function in the dimension of knowledge capital. There are thus no direct welfare gains from innovation.[12] Instead, welfare is improved by the intertemporal reallocation of consumption (less consumption today gives more tomorrow, etc.). This is an essential implication of treating technical change as an endogenous process operating through investments in knowledge.

A variant of this point was raised by Jorgenson and Griliches (1967), who argued that the residual should disappear if all inputs were measured correctly. Proposition 6 gives conditions under which the Jorgenson-Griliches dictum holds.

Proposition 6 also sheds light on the argument by Greenwood et. al. (1992) that investment should not be deflated by the factor $(1+\theta)$ because this would result in a stationary equilibrium in which consumption per capita does not grow at all. Their results might be interpreted, in the framework of this paper, as arguing that μ is falling over time. In that case, the results above indicate that their view of the deflation issue is certainly true when μ approaches 0. However, it is also true that if μ=1 and the production function also exhibits constant returns to scale, an improvement in the quality of capital is exactly like an increase in the quantity and a steady state is attained in which consumption per capita is constant (in the absence of other forms of technical change). Indeed, the case μ=1 requires increasing returns to scale to generate increases in consumption per capita.

B. The Solow Paradox

A third application of Proposition 6 sheds light on the Solow question about the apparent lack of "payoff" to the measured rate of quality change in computers. While there are many potential explanations of this phenomenon, the analysis of the preceding section suggests that there is no *a priori* reason to expect total factor productivity growth to rise with the rate of embodied technical change. The rate of growth of total factor productivity is just the Solow residual, $T = (\pi\psi - \mu\sigma\theta)$ under the conditions of Proposition 6. We have already seen that this expression can be zero under optimal steady-state growth with $\mu = 1$, implying that $\partial T/\partial\theta = 0$. And, this is not the only possibility: a sudden increase in θ may actually cause a short-run <u>decrease</u> in the rate of total factor productivity growth.

This paradox arises because the rate of technical change for new capital enters as a negative element of the Solow residual, and the sign of the derivative of the residual with respect to θ is, therefore, ambiguous. In the special case where the capital share and the rate of saving are constant, this derivative has the form $\partial T/\partial\theta = [\pi(\partial\psi/\partial\theta) - \mu\sigma]$, implying that the rate of growth of total factor productivity falls when $\mu > (\pi/\sigma)(\partial\psi/\partial\theta)$. Since $\pi = \theta$ in the long run, this condition reduces to $\mu > (\pi/\sigma)$. In the short run, it can be shown that $(\partial\psi/\partial\theta) = H^*/J$, resulting in the condition $\mu > P^J/P^{H^*}$. This, in turn, can be written as $\mu P^{H^*} > P^J$.[13]

These results arise because the Solow residual is measured using I-K concepts of capital even when technical change is actually capital-embodied and should be represented by the H-J convention. An increase in the rate of embodied technical change will reduce the Solow residual if the measured "error" associated with the I-K convention is greater for quality-corrected investment, H^*, than for the quality-corrected stock, J. The former is the cost of acquiring additional units of efficiency, μP^{H^*}, the latter is the value of the marginal product of those units, P^J μP^J. In other words, if the cost of additional efficiency outweighs the benefit, technical change reduces the measured Solow residual.

C. Extension to Disembodied Technical Change

Improvements in the management and organization of production are another source of productivity growth. They appear in the conventional Solow residual as costless disembodied technical change, but there is no logical reason that the latter should be seen as entirely costless - indeed, the adoption of some organizational changes may involve a large cost in foregone output. An alternative approach is to treat the level organizational efficiency as an intangible asset, and regard disembodied technical change as the return to "investment" in this stock. The portion of this investment that involves a resource cost should be added to output while the stock of intangibles should enter on the input side in user-value units. By symmetry with the embodied technical change case, the Solow residual would then be composed of positive and negative terms with a parallel interpretation to Proposition 6.

VII. Measurement Issues and Price Hedonics

A. Hedonic Interpretations of μ

It is clear from Propositions 2 and 3 that competing paradigms of economic growth implicitly make different predictions about the path of asset prices over time. The costless Manna case corresponds to a zero value of the parameter μ and while the endogenous technology case corresponds to a value greater than zero. Since the prices of new and used capital goods are often observable in data from market transactions, estimation of the vintage-price profile can serve as an indirect test of alternative models of technical change. The empirical problem is to identify the parameters of the vintage-price profile from data in the price array $P^I_{t,s}$.

As shown in the Appendix, this price array can be visualized as a two dimensional matrix in which age (s) increases along each row in any year (i.e., for fixed t), and time increases along each column for any age (i.e., for a fixed s). The diagonal of this array (ν = t-s) tracks the price history of any individual piece of capital from the time it is new, $P^I_{t,0}$, to its level in the following year, $P^I_{t+1,1}$, and so on. The resulting path is simply the vintage-price profile. Specific examples are given in the Appendix, each corresponding to different values of the parameters of equation (6) of Proposition 2.

The simplest procedure for using this price array data is to regress the price $P^I_{t,s}$ (or its natural logarithm) on age and time (or on age and time dummy variables). However, Hall (1967) shows that this procedure runs afoul of an identification problem: there are two dimensions in the price array,

$P^I_{t,s}$, with which to identify the three parameters θ, ρ, and δ. This is immediately apparent from the price profiles of Proposition 3. The age-price profile involves two parameters associated with age, θ and δ, and the time-price involves the parameters θ and ρ. The addition of the parameter μ in this paper only adds to the identification problem.

The Hall Identification Theorem refers to the case in which growth occurs at a constant (geometric) rate. Accelerations and decelerations in the rate of inflation, the rate of embodied technical change, or the rate of physical depreciation lead to changes in the growth rate of $P^I_{t,s}$, and the variation can be used for identification. Alternatively, identification can be achieved using exogenous estimates of some parameters (e.g., an estimate of rate of inflation), or by restricting some parameters to be zero. In the case of economic depreciation, Hulten and Wykoff (1981) estimate the composite age effect of δ and θ, since economic depreciation is defined as the rate of change of $P^I_{t,s}$ with respect to s.

Price hedonics is a variant of this solution to the identification problem. Under the hedonic approach, goods are viewed as bundles of characteristics, or as Griliches puts it:

> "The hedonic ... approach to the construction of price indexes is based on the empirical hypothesis (or research strategy) which asserts that the multitude of models and varieties of a particular commodity can be comprehended in terms of a much smaller number of characteristics or basic attributes of a commodity such as "size," "power," "trim," and "accessories," and that viewing a commodity in this way will reduce greatly the magnitude of the pure new commodity or "technical change" problem, since most (though not all) new "models" of commodities may be viewed as new combinations of "old" characteristics (1971, page 4).

In the hedonic view of the matter, improvements in the quality of goods or commodities are equivalent to increases in the quantity of the relevant characteristics. Indeed, it is the growth of these characteristics that is the source of productivity change.

On the dual side of the problem, the price of any good is just the total value of the characteristics embodied in the good. The price-hedonic regression estimates the contribution of each characteristic by regressing commodity price on the characteristic set $[X_1, .. , X_n]$, as well as on time in order to account for inflation in time series analyses. In the log-log specification of the hedonic regression, for example, the logarithm of the commodity price is on the left-hand side of the regression and the logarithm of the characteristics on the other side:

$$(17) \qquad \mathcal{L}n\, P^{I}_{t,0} = \boldsymbol{\beta}_0 + \boldsymbol{\beta}_1 \mathcal{L}n\, \mathbf{X}_1 + \ldots + \boldsymbol{\beta}_n \mathcal{L}n\, \mathbf{X}_n + \boldsymbol{\beta}_t t + \boldsymbol{\varepsilon}_t$$

β_0 can be interpreted as the price prevailing in the base year and β_t as the coefficient of pure asset revaluation. The β_i can be interpreted, in this formulation, as price elasticities, but it is well known that they are determined both by supply (cost) and demand-side (productivity) conditions, and that further structure must be given in order to put restrictions on the magnitudes of these coefficients.

There is at least one set of restrictions that links the hedonic regression (17) to the vintage-price profile (6) via the parameter μ. This link is based on the assumption that the level of embodied technical efficiency is determined by a set of underlying characteristics according to a Cobb-Douglas function $\Pi X_i^{\alpha i}$, where α_i is the elasticity of embodied efficiency with respect to the ith characteristic. In this case, the rate of quality change can be expressed as $\theta = \Sigma\ \alpha_i \chi_i$, where $[\chi_1, \ldots, \chi_n]$ are the growth rates of corresponding characteristics. Estimates of the α_i provide estimates of θ, given the measures of $[X_1, \ldots, X_n]$.

The α_i are not identical to the β_i, but are closely related if $P^I{}_{t,0}$ follows the time-price profile of Proposition 3. In this case, the left-hand side of the hedonic regression grows at a rate determined by $\mu\theta$ and ρ while the right-hand side grows at the rates θ and ρ. In view of $\theta = \Sigma\ \alpha_i \chi_i$, a regression of the logarithmic time-difference of the $P^I{}_{t,0}$ on the $[\chi_1, \ldots, \chi_n]$ will (in principle) yield estimates of β_i that equal $\mu\alpha_i$, not α_i, though collinearity may cause a variant of the Hall identification problem in practice. This, in turn, leads (in principle) to estimates of μ under the further restriction $\Sigma\ \alpha_i = 1$, since $\Sigma\ \beta_i = \Sigma\ \mu\alpha_i = \mu$ in this case (also yielding estimates of the individual $\alpha_i = \beta_i / \Sigma\ \beta_i$).[14]

This analysis shows that cost elasticity μ is an implicit parameter of the price-hedonic framework, even if identification problems are present in the implementation of (17). This conclusion is not surprising, in light of existing formulations of hedonic theory. Figure 5.3 of Triplett (1983) presents a "hedonic" production function (i.e., a production function for characteristics) in order to illuminate the debate over the proper treatment of costless quality change. This exposition could also be expressed in terms of the cost-elasticity parameter, since the case $\mu = 0$ corresponds to a shift in the hedonic production function in Triplett's Figure 5.3 while $\mu = 1$ corresponds to a movement along this function.

A variant of the new price-hedonic regression is of interest in light of the age-price profile which, according to Proposition 3, does not depend on μ. A cross-section hedonic regression using the logarithm of the *used* capital goods price, $P^I_{t,s}$, for a given year t, can be carried out on the χ_i and age time (to capture the effects of productivity decay). If $P^I_{t,s}$ follows the age-price profile of Proposition 3, the left-hand side of the hedonic regression declines at a rate determined by θ and ρ. Since the right-hand side grows at the same rates and since $\theta = \Sigma\ \alpha_i\chi_i$, the resulting regression should yield estimates of β_i which are identical to the α_i. A pooled cross-section and times series analysis might thus provide estimates of the parameters of the vintage-price profile without the restriction $\Sigma\ \alpha_i = 1$.

Finally, it should be noted that price hedonics is also carried out on samples that include different models of capital that do not necessarily come from different technological vintages (housing is an example). The equilibrium outcome in the simple model of this paper suggests that different models with varying "packages" of characteristics will coexist when $\mu = 1$, i.e., when the productivity advantage of a "better" model is just matched by a proportionate increase in cost, and may also coexist with models embodying newer technology or simply more characteristics. The price array $P^I_{t,0}$ may thus contain assets of different technological vintages and models with varying packages of characteristics, complicating the interpretation of results. This may, under certain circumstances, lead to an estimate of μ that is biased toward one, if estimates of the relevant parameters are obtained from a sample dominated by different "repackaged" models of the technological vintage and applied to situations in which new technology appears.[15]

B. Implied Estimates of μ from Existing Studies

There is more to price-hedonic theory than has been described thus far, and the complexity of the problem is far greater than suggested by the simple formulation of the preceding section. Nevertheless, the apparent utility of the cost elasticity parameter, and the possibility that $\Sigma \beta_i = \mu$, invites the question about the size of $\Sigma \beta_i$ in existing price-hedonic studies. The table shows the implied magnitude of this sum for a sample of price-hedonic studies. The overall picture is mixed, but there is a tendency for the sum to be near one. Moreover, values greater than one may reflect a high mark-up over marginal cost, due to the need to amortize the cost of R&D over the early years of product life. Indeed, this effect may be growing as product cycles shorten.

In any case, there is little evidence that the sum is greatly less than one as predicted by the Manna-from-Heaven model of technical change. However, even these relatively weak conclusions need to be couched in the warning that these studies were not designed to measure μ, that different functional forms of (13) are used, that there are dummy variables whose coefficients are not reported in Table 1, and that dummy variables in some studies may be interpreted as deviations from a rate of growth.

TABLE 1

Results from Selected Hedonic Price Studies

Study	Good	Source Table	Form	Years	Sum of Characteristic Estimates
Griliches (1971)	Autos	Table 3.3	SL	Annual 1957,59,60	0.55-0.65
Dhrymes (1971)	Refrigerators:				
	Fridgidaire	Table A.27	DL	Pooled 1950-65	2.32
	GE	Table A.28	DL	Pooled 1950-65	1.18
Cole et. al. (1986)	Mainframe:				
	Computer Processors	Table 2	DL	Pooled 1972-84	0.90-1.00
	Disk Drives	Table 2	DL	Pooled 1972-84	0.87-1.15
	Printers	Table 2	DL	Pooled 1972-84	1.16
	Displays	Table 2	DL	Pooled 1972-84	0.95
Oliner (1993)	Mainframe Computers	Table 1.2 Table 1.3	DL	Pooled 1970-86	0.93-1.04
Berndt, Griliches, & Rapport (1995)	Personal Computers	Table 5	Mixed	Pooled 1989-92	3.42

VIII. Conclusion

This paper has introduced a new parameter of technical change with surprisingly widespread applications. It is latent in the existing debates about the appropriate treatment of quality change, and when made explicit, not only helps inform those debates, but also provides a parametric connective tissue. Moreover, it links the quality change debate to different paradigms of economic growth, as well as to the model of price hedonics, thus suggesting an empirical strategy for testing the competing models and shedding new light on the debates themselves. Several substantive results have also been obtained, including the paradox that an increase in the rate of embodied technical change can actually lower the total factor productivity residual.

The paper has dealt only with quality change in investment goods and, to a minor extent, with disembodied technical change, but there are a parallel set of issues on the consumption side. If quality change in consumption requires resources to obtain, then the rationale used to adjust investment goods for quality change should be extended to the production of consumption goods. In other words, a measurement convention based on the marginal rate of transformation between consumption and investment should reflect quality improvements in <u>both</u> goods to the extent of their cost. Thus, if the consumption counterpart of μ is b, and the rate of quality improvement in consumption goods is γ, the price of consumption should be deflated by $(1+b\gamma)$ and the quality-corrected quantity written up by this factor. However, it must be recognized that this approach reflects the treatment of quality change from the standpoint of production. From the consumers' standpoint, the utility *of the good* has increased by the full extent of the quality improvement $(1+\gamma)$. This leaves a wedge between the welfare (utility) measure of quality change and the supply-side measure, that is, between $(1+\mu\gamma)$ and $(1+b\gamma)$. This wedge is the resource-free improvement in utility that is obtained by the innovation. This points to the need to expand the usual supply-side perspective of technical change to include a utility side in the growth accounting exercise. This is well beyond the scope of this paper, but under the Golden Rule condition $\pi = \sigma$ and constant returns to scale, a full consumption-production residual looks like an expanded version of the conventional residual: $(1-\mu)\sigma\theta+(1-b)(1-\sigma)\gamma$.

1. The rental price of capital is the one-period cost of hiring a capital good for use in production. When the capital good is owner-utilized, there is no explicit rental price and the shadow price is typically termed the "user cost" to signify an implicit transaction involving the owner-user. This one-period cost is also terms a "quasi-rent" to signify that owner-utilized capital is a residual claimant on revenue, and short-term fluctuations in demand may cause the ex post user cost to vary around its long-run value. These distinctions are not essential to this paper, and we will generally use the term "user cost" to denote the one-period cost of using capital goods.

2. Suppose, for example, that the rate of embodied technical change is 10 percent between one year and the next. A physical unit of new capital is now 10 percent more productive than the same model in the preceding year. If the latter cost one unit of consumption to produce and $\mu = 0.5$, the new model costs 1.05 units of consumption.

3. We will examine, in a subsequent section, the case in which the price and quantity of investment goods are adjusted by the factor $(1+\mu\theta)$. This is the "resource-cost" approach to quality adjustment. It leads to a specification of technology in which the relevant tradeoff is $C+(1+\mu\theta)I$ rather than $C+I$ on the left-hand side of the production function.

4. According to Dowrick and Nguyen (1989), spillovers at the international level are a particularly important source of aggregate technical change. Their findings suggest that diffusion of technology is a major component of the Solow residual in developing countries that are in the process of "TFP catch-up." This, in turn, implies a lower value of the parameter μ for developing countries in the process of "catch-up" (perhaps one close to zero) than for developed countries at or near the technological frontier.

5. See Hall (1967, 1971) and Hulten and Wykoff (1981) for an elaboration of this distinction, or the review by Hulten (1990).

6. The appendix tables should be read in the following way. Table A1 portrays the case in which a new asset in the base year 1990 has a value of $100; in the following year, it is a one-year old asset and has a value of $92; in the next year, the asset's value falls to $86, and so on down the principal diagonal. This diagonal, denoted by solid arrows, is the vintage price profile of an asset put into service in 1990. The vintage price profile of an asset placed in service in 1991 lies on a diagonal directly below, etc.

The age-price profile of the new 1990 asset (equation (7) in Proposition 3) is found along the first row of the array. The second entry ($86) is the price of a one-year old asset in 1990, the next price ($75) is the price of a two-year old asset in that year, and so on. The time-price profile of the new 1990 asset (equation (8) in Proposition 3) is read along the first column. A new asset put in service in 1991 has a price of $107, and a new asset in 1992 costs $115, and so on.

Tables A2 and A3 repeat this exercise for different values of μ. Table A2 shows the price dynamics of the "Manna-from-Heaven" model, that is, when $\mu = 0$, but the other parameters remain the same. In this case, the time-price profiles are flat (recall that there is no price inflation in this example), and the age-price profiles are identical. This situation is portrayed in Figure 1. It is also the model analyzed by Solow (1970) to describe the pure effects of technological obsolescence. A different situation arises when $\mu = 1$, shown in Table A3 and Figure 2. Here, there is no loss of asset value with age, and the vintage-price profile is constant over time. Technologically superior assets do arrive in the market, but their additional cost just equals their productivity advantage, so the ownership of older assets is not penalized. Since decay and inflation are zero, there is no change in the price of assets as they age.

7. In the "AK" model, output per worker is assumed to be a linear function of capital per worker, where the concept of capital is broadened to include intangible as well as tangible capital. In this version of the endogenous growth model, knowledge investment displaces other investments (and consumption) on a dollar-for-dollar basis, implying that the implicit rate of technical change is fully paid for. In the framework of this paper, this is equivalent to assuming that $\mu = 1$. In contrast, the spillover variant of the endogenous growth model developed by Romer is based on externalities that drive a wedge between the marginal social return to knowledge investment and the corresponding marginal private return. As noted above, this leads to a value of μ between zero and one.

8. The former encompass any situation in which technical change is resource using, while the latter explicitly relies on increasing returns to scale. This distinction is important for the discussion of the welfare effects of technical change in a subsequent section, since increasing returns to scale is needed to insure sustained growth in consumption per capita over time, while a constant returns to scale version of the endogenous growth model would tend to lead to a steady-state path along which consumption per capita is constant.

9. A more general formulation of the accumulation equations (9) and (10) would replace the geometric parameters δ and θ with a single parameter $\phi_{t,s}$ that declines with age, s, and rises with time, t. To fix the value of the efficiency sequence relative to the base year, the normalization $\phi_{t-s,0} = 1$ is used.

10. Note that the term $H_t = (1+\theta)^t I_t$ can be inserted into (10) to yield

$$J_t = H_t + (1-\delta)H_{t-1} + \ldots + (1-\delta)^t H_0 \quad .$$

This is the form in which the quality-adjusted capital stock is most often presented.

11. It is worth noting, here, that the both Denison (1989) and Greenwood et. al. (1992) base their case against quality deflation on arguments involving the consumption-investment tradeoff. They seem to imply that the resource-cost approach implies no correction for quality change in investment goods, but Proposition 4 suggests that this depends on the magnitude of the parameter μ.

12. For this reason, some may be reluctant to use the term "technical change" to describe an innovation of this sort. However, a clear distinction should be made between cause and effect. The consequences of a technological advance that results in, say, the doubling of the productivity of a particular type of capital are the same regardless of whether this advance was caused by a costly program of research or whether it was achieved costlessly by luck or imitation. Moreover, if a costly innovation is not deemed to be a "technical change" in the country in which it originated, how should it be characterized in those countries into which the innovation is diffused at little or not cost?

13. The average embodied technical efficiency, ψ, is a weighted average of the individual vintage θ's. Therefore, the former change more slowly than the latter, the derivative $\partial\psi/\partial\theta$ is thus less than one in the short-run. Equation (4) of Hulten (1992)) gives the exact form of the relation between ψ and θ, and can be used to show that $\partial\psi/\partial\theta$ is equal to H^*/J. It then follows that $(\pi/\sigma)(\partial\psi/\partial\theta) = P^{H*}/P^J$, since capital's income share is $\pi = P^J J/P^Q Q = P^K K/P^Q Q$ and the rate of investment is $\sigma = P^{H*} H^*/P^Q Q = P^I I/P^Q Q$ (the invariance result in Jorgenson (1966)). In the long run, ψ converges to θ, so $(\partial\psi/\partial\theta) = 1$.

14. The restriction $\Sigma\ \alpha_i = 1$ is plausible, since, in combination with the assumption that $\theta = \Sigma\ \alpha_i \chi_i$, it implies that two machines with half of the quantity of characteristics $[X_1, \ldots, X_n]$ are the equivalent of one machine with the full quantity. However, the degree of homogeneity of $\Sigma\alpha_i$ is an empirical issue.

15. Problems may also arise because it is not always easy to agree on the list of relevant characteristics, much less insure that this list is exhaustive. By seizing on one or two of the most prominent characteristics, like CPU speed and memory in computers, the hedonic regression may overstate the true increase in computing power, particularly if other, unmeasured characteristics, limit the significance of those that are measured. For example, an increase in motor vehicle engine power to, say, 10,000 horsepower, is irrelevant given the other design features and limitations of autos and trucks, as well as the roads on which they travel. Other problems arise when each characteristic has its own cost parameter, μ_i.

REFERENCES

Berndt, Ernst R., Zvi Griliches, and Neal R. Rappaport, "Econometric Estimates of Price Indexes for Personal Computers in the 1990's," Journal of Econometrics, 68, 1995, 243-268.

Cass, David, "Optimum Growth in an Aggregative Model of Capital Accumulation," Review of Economic Studies, 32, July 1965, 233-240.

Cole, Rosanne, Chen, Y.C., Barquin-Stolleman, J.A., Dullberger, E., Helvacian, N., and Hodge, J.H., "Quality-Adjusted Price Indexes for Computer Processors and Selected Peripheral Equipment," Survey of Current Business, 66, January 1986, 41-50.

Denison, Edward F., Estimates of Productivity Change by Industry, Washington, D.C.: The Brookings Institution, 1989.

Dhrymes, Phoebus J., "Price and Quality Changes in Consumer Capital Goods: An Empirical Study," in Zvi Griliches, ed., Price Indexes and Quality Change, Cambridge, Massachusetts: Harvard, 1971, 83-150.

Domar, Evsey D., "Total Factor Productivity and the Quality of Capital," Journal of Political Economy, 71, December 1963, 586-588.

Dowrick, Steve and Duc-Tho Nguyen (1989). "OECD Comparative Economic Growth 1950-85: Catch-up and Convergence," The American Review, 79, 1010-1030.

Fisher, Franklin M., "Embodied Technical Change and the Existence of an Aggregate Capital Stock," Review of Economic Studies, 32, 1965, 263-88.

Gordon, Robert J., The Measurement of Durable Goods Prices, Chicago: University of Chicago Press for the National Bureau of Economic Research, 1990.

Greenwood, Jeremy, Zvi Hercowitz, and Per Krusell, "Macroeconomic Implications of Investment-Specific Technical Change," unpublished working paper, 1992.

Griliches, Zvi, "Hedonic Price Indexes for Automobiles: An Econometric Analysis of Quality Change," in, The Price Statistics of the Federal Government, New York: National Bureau of Economic Research, General Series, 73, 1961, 137-196.

Griliches, Zvi, "Introduction: Hedonic Price Indexes Revisited," in Zvi Griliches, ed., Price Indexes and Quality Change, Cambridge, Massachusetts: Harvard University Press, 1971, 3-16.

Griliches, Zvi, "Research Expenditures and Growth Accounting," in B.R. Williams, ed., Science and Technology in Economic Growth, The Price Statistics of the Federal Government, MacMillan Press, 1973.

Griliches, Zvi, "The Search for R&D Spillovers," Scandinavian Journal of Economics, 94, 1992, 29-47.

Hall, Robert E., "Technical Change and Capital From the Point of View of the Dual," Review of Economic Studies, 35, 1968, 34-46.

Hall, Robert E., "The Measurement of Quality Change From Vintage Price Data," in Zvi Griliches, ed. Price Indexes and Quality Change, Cambridge, MA: Harvard University Press, 1971.

Hall, Robert E., "Comment on 'A Reconsideration of Investment Behavior Using Tax Reforms as Natural Experiments'," in Brookings Papers of Economic Activity, 2, 1994, 60-62.

Hall, Robert E. and Dale W. Jorgenson, "Tax Policy and Investment Behavior," American Economic Review, 57, 1967, 391-414.

Hulten, Charles R., "The Measurement of Capital," in Ernst R. Berndt and Jack E. Triplett, eds., Fifty Years of Economic Measurement, Studies in Income and Wealth, Volume 54, Chicago: Chicago University Press for the National Bureau of Economic Research, 1990, 119-152.

Hulten, Charles R., "Growth Accounting when Technical Change is Embodied in Capital," American Economic Review, 82, No. 4, September 1992, 964-980.

Hulten, Charles R. and Frank C. Wykoff, "The Measurement of Economic Depreciation," in Charles R. Hulten, ed., Depreciation, Inflation, and the Taxation of Income From Capital, Washington: The Urban Institute Press, 1981, 81-125.

Johansen, Leif, "Substitution versus Fixed Production Coefficients in the Theory of Economic Growth: A Synthesis," Econometrica, 27, April 1959, 157-176.

Jorgenson, Dale W., "Capital Theory and Investment Behavior," American Economic Review, 53, May 1963, 247-259

Jorgenson, Dale W., "The Embodiment Hypothesis," Journal of Political Economy, 74, February 1966, 1-17.

Jorgenson, Dale W., "The Economic Theory of Replacement and Depreciation," in W. Sellykaerts, ed., Econometrics and Economic Theory, New York: MacMillan 1973.

Jorgenson, Dale W. and Zvi Griliches, "The Explanation of Productivity Change," Review of Economic Studies, 34 (July 1967), 349-83.

Koopmans, T.C., "On the Concept of Optimal Economic Growth," Pacifica Academia Scientiarus, Rome, 1965, 276-279.

Leontief, Wassily W., "Introduction to a Theory of the Internal Structure of Functional Relationships," Econometrica, 15, October 1947, 361-373.

Lucas, Robert E. Jr., "On the Mechanics of Economic Development," Journal of Monetary Economics, 22, 1988, 3-42.

Oliner (1993) in Murray F. Foss, Marilyn E. Manser, and Allan H. Young, eds., Price Measurements and Their Uses, Studies in Income and Wealth, Volume 57, Chicago: Chicago University Press for the National Bureau of Economic Research, 1993, 19-61.

Rebelo, Sergio T, "Long-Run Policy Analysis and Long-Run Growth," Journal of Political Economy, 99, No. 3, 1991, 500-521.

Romer, Paul M., "Increasing Returns and Long-Run Growth," Journal of Political Economy, 94, No. 5, 1986, 1002-1037.

Romer, Paul M., "Endogenous Technical Change," Journal of Political Economy, 98, No. 5, Pt. 2, 1990, S71-S102.

Salter, W.E.G., Productivity and Technical Change, Cambridge: Cambridge University Press, 1960.

Solow, Robert M.,"A Contribution to the Theory of Economic Growth," Quarterly Journal of Economics, 70, February 1956, 65-94.

Solow, Robert M., "Technical Change and the Aggregate Production Function," Review of Economics and Statistics, 39, August 1957, 312-320.

Solow, Robert, "Investment and Technical Progress," in K.Arrow, S. Karlin and P. Suppes, eds., Mathematical Methods in the Social Sciences, 1959, Stanford: Stanford University Press, 1960.

Solow, Robert M., Growth Theory: An Exposition, Oxford University Press, New York and Oxford, 1970.

Swan, T.W. "Economic Growth and Capital Accumulation," The Economic Record, XXXII, 1956, 334-361.

Triplett, Jack E., "Concepts of Quality in Input and Output Price Measures: A Resolution of the User Value-Resource Cost Debate," in Murray F. Foss, ed., The U.S. National Income and Product Accounts: Selected Topics, Studies in Income and Wealth, Volume 47, Chicago: University of Chicago Press for the National Bureau of Economic Research, 1983, 296-311.

Triplett, Jack E., "Hedonic Functions and Hedonic Indexes," in John Eatwell, Murray Milgate and Peter Newman, eds., The New Palgrave Dictionary of Economics, Vol. 2, New York: The Macmillan Press Limited, 1987, 630-634.

INVESTMENT PRICE ARRAY ($P^{I}_{t,s}$)

age

	0	1	2	3	4	5
1990	100	86	75	64	56	48
1991	107	92	80	69	60	51
1992	115	99	86	74	64	55
1993	123	106	92	79	68	59
1994	132	114	98	85	73	63
1995	141	122	105	91	78	68

$\delta = .05$

$\rho = .02$

$\theta = .10$

$h = .50$

Table 1

INVESTMENT PRICE ARRAY ($P^{I}_{t,s}$)

age

	0	1	2	3	4	5
1990	100	91	83	75	68	62
1991	100	91	83	75	68	62
1992	100	91	83	75	68	62
1993	100	91	83	75	68	62
1994	100	91	83	75	68	62
1995	100	91	83	75	68	62

$\delta = 0$

$\rho = 0$

$\theta = .10$

$h = 0$

Table 2

INVESTMENT PRICE ARRAY ($P^{I}_{t,s}$)

age

	0	1	2	3	4	5
1990	100	91	83	75	68	62
1991	110	100	91	83	75	68
1992	121	110	100	91	83	75
1993	133	121	110	100	91	83
1994	146	133	121	110	100	91
1995	161	146	133	121	110	100

$\delta = 0$

$\rho = 0$

$\theta = .10$

$h = 1$

Table 3

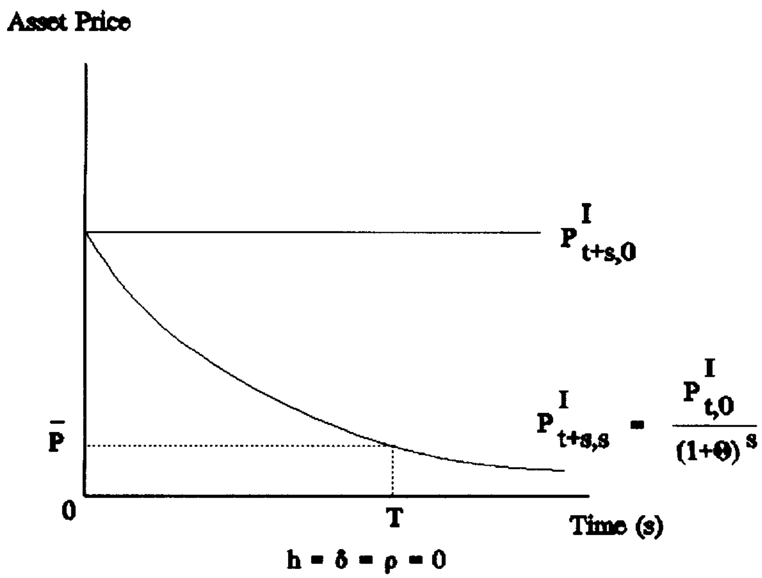

FIGURE 1

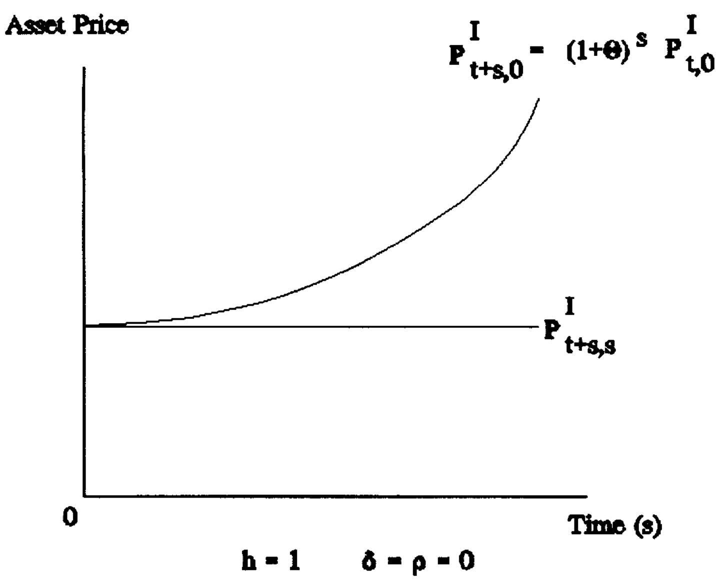

FIGURE 2