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DISCRETE CHOICE WITH SOCIAL INTERACTIONS I: THEORY

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ABSTRACT

This paper provides an analysis of aggregate behavioral outcomes when individual utility exhibits social interaction effects. We study generalized logistic models of individual choice which incorporate terms reflecting the desire of individuals to conform to the behavior of others in an environment of noncooperative decisionmaking. Laws of large numbers are generated in such environments. Multiplicity of equilibria in these models, which are equivalent to the existence of multiple self-consistent means for average choice behavior, will exist when the social interactions exceed a particular threshold. Local stability of these multiple equilibria is also studied. The properties of the noncooperative economy are contrasted with the properties of an economy in which a social planner determines the set of individual choices. The model is additionally shown to be well suited to explaining a number of empirical phenomena, such as threshold effects in individual behavior, ethnic group fixed effects of income equations, and large cross-group differences in binary choice behavior.

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1. Introduction

A large body of recent research has begun to consider the role of social interactions in economic behavior. By social interactions, we refer to the idea that the utility or payoff an individual receives from a given action depends directly on the choices of others in that individual's reference group, as opposed to the sort of dependence which occurs through the intermediation of markets. When these spillovers are positive in the sense that the payoff for a particular action is higher for one agent when others behave similarly, the presence of social interactions will induce a tendency for conformity in behavior across members of a reference group. Further, as described by Bernheim (1994), even when the underlying intrinsic utility from the actions differs widely across individuals due to heterogeneity of individual characteristics, the presence of this desire to conform may create either a tendency either towards common behavior or towards a few polarized types of behavior within a reference group. In addition, social interactions can also represent an explanation for large cross-group variations in behavior if different groups conform to alternative types of self-reinforcing behavior. When social interactions act as strategic complementarities between agents, multiple equilibria may occur in absence of any coordination mechanisms, as described by Cooper and John (1988).

The intuition that individuals seek to conform to the behavior of reference groups has found successful application in a number of circumstances.[1] One obvious example follows from the consequences of social interactions within a neighborhood for individual behavior. Bénabou (1993) shows that when the cost of individual education investment is a decreasing function of the investment decisions of one's neighbors, neighborhoods can exhibit large discrepancies in the

[1] A very early study of the role of social interactions in binary choice is Schelling (1973), who provides a wealth of charming examples, ranging from driving patterns to styles of athletic play.

level of human capital formation. One interpretation of this spillover is that deviation from a neighborhood's mean education level is costly. This type of spillover can have powerful consequences for income distribution; in fact as shown by Durlauf (1994a, b), such effects, when intergenerational, can lead to permanent inequality between family dynasties. Alternatively, Schelling (1971) shows how preferences over neighborhood racial composition can lead to pronounced residential segregation, even when these preferences are relatively weak. Similarly, Glaeser, Sacerdote, and Scheinkman (1994) argue that social interactions can explain large differences in community crime rates.

The emphasis on social interactions as a determinant of behavior, while relatively recent in the context of economic theorizing, is of course not new from the perspective of sociology. One early analysis of the role of social interactions is found in the literature on the "culture of poverty" (see Lewis (1966) and Liebow (1967) for classic formulations and Montgomery (1994) for an interesting formalization), which argues that isolated poor groups exhibit different values towards work and childrearing from the population as a whole. More recent treatments of ghetto poverty, such as Wilson (1987), even while rejecting a strict cultural explanation for phenomena such as labor force withdrawal and out-of-wedlock births, nonetheless do emphasize the social multiplier which converts changes in private utility to changes in community-wide behavior. In fact, this interdependence between private incentives and imitative behavior will drive our theoretical framework.

Recent empirical evidence which is consistent with the presence of social interaction effects has been developed in a number of contexts. Case and Katz (1991) provide evidence that the probability of social ills in one neighborhood is increasing in the prevalence of these same ills in adjacent neighborhoods. Crane (1991) finds a relationship between both school dropout and teenage childbearing rates and the occupational composition of a community. Haveman and Wolfe (1994) present similar findings, concluding in terms of high school dropout rates, for example, that "If those who grew up in a "bad" neighborhood were to grow up in a "good" neighborhood, (the) probability of dropping out falls by 52%" (pg. 250). Finally, Anderson (1990) provides a fascinating portrait of the power of social interactions on individual behavior in the inner city of Philadelphia based on direct field observation.[2]

The potential role of social interactions has also been demonstrated in economic situations far removed from neighborhoods. Brock (1993) shows how these types of effects, when embedded in the expectations formation process, can help explain asset market volatility. Brock and Hommes (1995) further show how these effects, when embedded in a model of costly learning, can produce complex aggregate dynamics. Finally, we observe that models such as Frank (1985) which include relative status in individual utility, introduce social interaction effects as well in ways similar to our own analysis.

This paper is designed to provide an analytical framework for studying economies in which social interactions are embedded in individual decisions.[3] Our analysis follows Brock (1993) in exploiting the relationship between models of discrete choice with interaction effects and a class of stochastic processes known as random fields. Random fields modelling has proven useful in studying the potential for multiple equilibria and complex cross-section dynamics in large heterogeneous economies (see Föllmer (1974) for an early contribution and Durlauf (1993) and Bell (1994) for recent examples) in which all agent interactions are local, *i.e.* when individuals have incentives to conform to the behavior of a small number of appropriately defined neighbors. Our current

[2]At the same time, two caveats should be stressed in interpreting these types of studies. First, as well described in Jencks and Mayer (1990), the evidence of these effects is often not robust across studies and regression specifications. Second, as emphasized in Manski (1993a,b) the empirical literature has not successfully distinguished between evidence of social interactions and the presence of correlated (within-group) individual effects, especially when these effects are themselves among the determinants of group formation.

[3]See also Arthur (1987,1989), Ioannides (1990), Weidlich (1992), and Krugman (1995) for applications of stochastic process models to social phenomena which are very much in the spirit of the current analysis.

analysis shows how to derive complementary conclusions when the interactions are global, *i.e.* where individuals face incentives to conform their behavior to the mean of a common reference group.

One interesting feature of our analysis is the use of parameterizations suggested by the discrete choice literature to embody social interactions. This strategy leads us to consider binary decision problems for individual agents. Such a framework naturally fits a wide array of social phenomena, such as teenage pregnancy, participation in the above ground economy versus becoming a criminal, location in a city or suburb, entry or withdrawal from the labor force, staying in or dropping out of school, etc. Under the discrete choice parameterization, the model proves to have a number of interesting theoretical properties. A major goal of this paper is to demonstrate that our framework can be theoretically useful without any sacrifice of econometric tractability and therefore may prove valuable in a variety of applications. In the sequel to this paper, Brock and Durlauf (1995b), we develop the associated econometrics which will allow for empirical implementation of our framework.

Section 2 provides a baseline model of social interaction effects. Section 3 develops a probabilistic equilibrium characterization of individual choices under the assumption that these choices are made noncooperatively. Dynamic behavior, with a focus on the stability of various steady state average choice levels, is studied as well. Welfare analysis is conducted which shows how to rank the multiple steady states, when they exist. Section 4 develops a probabilistic equilibrium characterization of individual choices in the presence of a social planner. Section 5 considers the implications of alternative formulations of social utility. Section 6 discusses some empirical implications of the model. Section 7 provides summary and conclusions.

2. Utility maximization in the presence of social interactions

i. **Modelling individual choice with private social utility**

We consider the problem of individual choice in the presence of social interactions. Formally, each individual in a population of I agents must choose a binary action at some common time. Each of these binary actions is coded into ω_i, a realization with support $\{-1,1\}$. The space of all possible sets of actions by the population is the I-tuple $\underset{\sim}{\omega} = (\omega_1, \ldots, \omega_I)$. Finally, $\underset{\sim}{\omega}_{-i}$ denotes $(\omega_1, \ldots, \omega_{i-1}, \omega_{i+1}, \ldots, \omega_I)$, the choices of all agents other than i.

Individual utility, $V(\omega_i)$, consists of three components.

$$V(\omega_i) = u(\omega_i) + S(\omega_i, \mu^e(\underset{\sim}{\omega}_{-i})) + \epsilon(\omega_i) \tag{1}$$

The term $\mu^e(\underset{\sim}{\omega}_{-i})$ denotes the conditional probability measure agent i places on the choices of others at the time of making his own decision. The components of total utility are threefold: $u(\omega_i)$ is the private utility associated with a choice, $S(\omega_i, \mu^e(\underset{\sim}{\omega}_{-i}))$ is the social utility associated with the choice, and $\epsilon(\omega_i)$ is a random utility term, independently and identically distributed across agents. Agent i knows $\epsilon(\omega_i)$ at the time of his decision.

This formulation is closely related to a number of types of social interactions which have appeared in the literature. When $S(1, \mu^e(\underset{\sim}{\omega}_{-i})) - S(-1, \mu^e(\underset{\sim}{\omega}_{-i}))$ is increasing in a rightward shift of $\mu^e(\underset{\sim}{\omega}_{-i})$ (where a rightward shift in a probability measure is interpreted in a stochastic dominance sense), our social utility component will exhibit the expectational analogue to increasing differences in the sense of Milgrom and Roberts (1990), pg. 1261, and will represent a version of the binary choice models with externalities studied in Schelling (1973). When $S(\omega_i, \mu^e(\underset{\sim}{\omega}_{-i}))$ both exhibits increasing differences and takes the form , with

$$\bar{m}_i^e = (I-1)^{-1} \mathrm{E}(\sum_{j \neq i} \omega_j \mid \mu^e(\underset{\sim}{\omega}_{-i})), \tag{2}$$

social utility will exhibit the "totalistic" (*i.e.* dependent only on the average actions of others) form of strategic complementarities studied by Cooper and John (1988).

ii. Parametric assumptions

We restrict our analysis to parametric representations of both the social utility term and the probability density of the random utility term. These assumptions will render our model both theoretically and econometrically tractable.

First, we consider forms of social utility which exhibit a strategic complementarity that is both totalistic and constant. This means that we are interested in forms of $S(\omega_i, \overline{m}_i^e)$ which have the property that

$$\frac{\partial^2 S(\omega_i, \overline{m}_i^e)}{\partial \omega_i \partial \overline{m}_i^e} = J > 0 \tag{3}$$

A constant cross-partial allows one to measure the degree of dependence across agents with a single parameter. This assumption leads us to two functional forms for social utility. The first embodies a multiplicative interaction between individual and expected average choices,

$$S(\omega_i, \overline{m}_i^e) = J\omega_i \overline{m}_i^e. \tag{4}$$

We will designate this as the proportional spillovers case, since the percentage change in individual utility from a change in the mean decision level is constant, given the individual's choice.

The second parameterization captures a pure conformity effect of the type studied by Bernheim (1994),

$$S(\omega_i, \overline{m}_i^e) = -\frac{J}{2}(\omega_i - \overline{m}_i^e)^2. \tag{5}$$

This specification penalizes deviations far from the mean more strongly than the proportional spillovers case.

To see the relationship between the two forms of social utility, we rewrite (5) as

$$-\frac{J}{2}(\omega_i - \bar{m}_i^e)^2 = J\omega_i\bar{m}_i^e - \frac{J}{2}(1 + (\bar{m}_i^e)^2), \tag{6}$$

making use of the fact that $\omega_i^2 = 1$. This form of (5) when contrasted with (4) shows that while the two social utility specifications differ in levels, they coincide on those terms which contain the individual choice variable.

Finally, we assume that the errors $\epsilon(-1)$ and $\epsilon(1)$ are independent and extreme-value distributed, so that

$$Prob(\epsilon(-1) - \epsilon(1) \leq x) = \frac{1}{1 + exp(-\beta x)}. \tag{7}$$

As the probability that ω_i takes on the value -1 rather than 1 will equal the probability that $V(-1) > V(1)$, parameterization of the probability density of $\epsilon(-1) - \epsilon(1)$ will allow explicit calculation of $Prob(\underset{\sim}{\omega})$.

3. Equilibrium properties under noncooperative decisionmaking

i. Large economy behavior with proportional spillovers

We first study the behavior of the model with the proportional spillovers specification (4) under the assumption that agents act noncooperatively. Operationally, this means that each agent makes his choice given an expectation of the mean choice level which is independent of the realizations of $\epsilon(\omega_i)\ \forall\ i$. In other words, agents do not communicate or coordinate their decisions. It is standard[4] under the extreme-value assumption for $\epsilon(\omega_i)$ that each individual choice will obey the probability

$$Prob(\omega_i) = \frac{exp(\beta(u(\omega_i) + J\omega_i \bar{m}_i^e))}{\sum_{\nu_i \in \{-1,1\}} exp(\beta(u(\nu_i) + J\nu_i \bar{m}_i^e))}. \tag{8}$$

In this probability, β parametrizes the extent to which the deterministic component of utility determines actual choice. When $\beta \Rightarrow \infty$, the effect of $\epsilon(\omega_i)$ on the realized choice will vanish whereas as $\beta \Rightarrow 0$, the probability that $\omega_i = -1$ (or 1) will converge to $\frac{1}{2}$ regardless of the values of the private and social utility terms under each choice.

Since the $\epsilon(\omega_i)$ terms are independent across agents, the joint probability measure over all choices will equal

$$Prob(\underset{\sim}{\omega}) = \frac{exp(\beta(\sum_{i=1}^{I} (u(\omega_i) + J\omega_i \bar{m}_i^e)))}{\sum_{\nu_1 \in \{-1,1\}} \cdots \sum_{\nu_I \in \{-1,1\}} exp(\beta(\sum_{i=1}^{I} (u(\nu_i) + J\nu_i \bar{m}_i^e)))}. \tag{9}$$

When $J = 0$, this expression is proportional to the logistic density; the standard logistic form follows directly when one performs the change of variables $\kappa_i = \frac{\omega_i + 1}{2}$ in order to shift the support of the individual decisions from $\{-1,1\}$ to $\{0,1\}$. In this case, the applicability of laws of large numbers to the sample average of the individual decisions, $I^{-1}\sum_{i=1}^{I} \omega_i = \bar{\omega}_I$, is immediate. When $J \neq 0$, this standard form is augmented by social interactions. This case possesses a probability structure equivalent to the so-called mean field version of the Curie-Weiss model of statistical mechanics, (comprehensively developed in Ellis

[4]See Anderson, de Palma, and Thisse (1992) for a valuable discussion of the various motivations for the logistic model as well as derivations of many standard results.

(1985)). The properties of this joint probability measure, for large I, may be developed as follows.

First, we compute an equilibrium probability measure for the model under the assumption that

$$\bar{m}_i^e = \bar{m} \;\; fixed \; \forall \; i \tag{10}$$

In other words, each individual's expectation of the mean is replaced with a common value. Second, we convert eq. (9) so that the exponent in the expression only depends on ω_i linearly. This may be done as follows. Since we are dealing with binary choices, private utility $u(\omega_i)$ can be replaced with $h\omega_i + k$ where h and k are chosen so that $h + k = u(1)$ and $-h + k = u(-1)$. Notice that this implies that $h = \frac{1}{2}(u(1) - u(-1))$ and so this parameter is proportional to the deterministic private utility difference between the two choices.

This linear representation implies that the joint probability equals

$$Prob(\underset{\sim}{\omega}) = \prod_{i=1}^{I} \frac{exp(\beta(h\omega_i + J\bar{m}\,\omega_i))}{exp(\beta h + \beta J\bar{m}) + exp(-\beta h - \beta J\bar{m})} \tag{11}$$

since $exp(k)$ cancels out in numerator and denominator.

The probability density thus factors into the product of I independent and identically distributed Bernoulli random variables. Hence a law of large numbers will apply to the sample mean for any $\bar{m}$, as described for example in Ellis (1985) pg. 299. The expected value of each of these random variables will equal

$$\mathrm{E}(\omega_i) = \frac{exp(\beta h + \beta J\bar{m}) - exp(-\beta h - \beta J\bar{m})}{exp(\beta h + \beta J\bar{m}) + exp(-\beta h - \beta J\bar{m})} = tanh(\beta h + \beta J\bar{m}). \tag{12}$$

Finally, we require that this joint probability measure is self-consistent (or equivalently, that expectations are rational in a steady state), so that

$E(\omega_i) = \bar{m}$.[5] Together with eqs. (10) and (11), we have Proposition 1, which characterizes the behavior of the large economy limit under noncooperative decisionmaking.

Proposition 1. Law of large numbers for discrete choices with noncooperative decisionmaking

i. When agents choose actions noncooperatively given social utility specification eq. (4), and given commonly held expectations $\bar{m}$, then as $I \Rightarrow \infty$,[6,7]

$$\bar{\omega}_I \Rightarrow_s tanh(\beta h + \beta J \bar{m}). \tag{13}$$

ii. There exists at least one value m^* such that

$$m^* = tanh(\beta h + \beta J m^*). \tag{14}$$

Any such m^* is a self-consistent expectation of the mean of choices across all agents in the large economy limit.

This model will exhibit an analogue to multiple equilibria in deterministic models of strategic complementarities, such as those described in Cooper and John (1988), when there exist multiple solutions to eq. (14). These multiple solutions imply the existence of distinct average choice levels which are each compatible with individually optimal decisions. Conditions for the existence of multiple solutions may be immediately obtained from the properties of the

[5]This self-consistency requirement is the "social equilibrium" condition discussed in Manski (1993a, b).

[6]Throughout, "$\Rightarrow_s$" and "$\Rightarrow_w$" refer to strong and weak convergence respectively.

[7]See Ellis (1985) pg. 181 for a formal justification for using strong convergence in the proposition.

$tanh(\cdot)$ function and are summarized in Proposition 2.

Proposition 2. Existence of multiple average choice levels in equilibrium

i. If $\beta J > 1$ and $h = 0$, there exist three roots to eq. (14). One of these roots is positive, one root is zero, and one root is negative.

ii. If $\beta J > 1$ and $h \neq 0$, there exists a threshold H, (which depends on βJ) such that

a. for $|\beta h| < H$, there exist three roots to eq. (14), one of which has the same sign as h, and the others possessing opposite sign.

b. for $|\beta h| > H$, there exists a unique root to eq. (14) with the same sign as h.

Proposition 2 allows us to designate without ambiguity m^*_- as the mean choice level in which the largest percentage of agents choose -1, m^*_+ as the mean choice level in which the largest percentage of agents choose 1, and m^*_m as the root associated with a mean between these two values, when there are multiple roots.

One interesting feature of the proposition is that the potential for multiple average equilibrium choice levels depends both on the strength of the social utility as well as the magnitude of the bias towards one choice induced by private utility. In other words, for each β and J, there will exist a level for h which ensures that the equilibrium is unique. This implies one is most likely to observe multiplicity in those social environments in which private utility renders individuals relatively close to indifferent between choices.

One can establish the limiting properties of the percentage of positive choices in the population, $\overline{\kappa}_I = \frac{\overline{\omega}_I + 1}{2}$, when agents possess self-consistent expectations in the sense of eq. (14). Given Proposition 1, $\overline{\kappa}_I$ will converge strongly (according to Proposition 1) to a limit k^* which depends on h, β, J and the selection of a particular solution to (14) when multiple solutions exist. The following properties for k^* are straightforward to verify.

Proposition 3. Relationship between limiting percentage of positive choices and model parameters

i. If $h = 0$ and $\beta J < 1$, $k^* = \frac{1}{2}$.

ii. $lim_{h \Rightarrow \infty}\ k^* = 1$.

iii. $lim_{h \Rightarrow -\infty}\ k^* = 0$.

iv. If $h = 0$, $lim_{J \Rightarrow \infty}\ k^* = 1$, $\frac{1}{2}$, or 0 depending on whether m^*_+, m^*_m, or m^*_- is the root of eq. (14) which characterizes the equilibrium of the economy.

v. If $h \neq 0$, then $lim_{J \Rightarrow \infty}\ k^* = 1$ or 0, depending on which root of eq. (14) characterizes the equilibrium of the economy.

***ii*. Dynamic stability**

We consider the dynamic stability of the steady equilibrium choice levels m^*_-, m^*_+, and m^*_m. We do this by considering the dynamics of the mean choice levels under the assumption that the expectations term $\overline{m}^e_i$ obeys the relationship

$$\overline{m}^e_i = m_{t-1}\ \forall\ i \tag{15}$$

where

$$m_{t-1} = plim_{I \Rightarrow \infty} \overline{\omega}_I. \tag{16}$$

In other words, we consider the dynamics of a sequence of large economies in which expectations are myopic. While this analysis certainly does not exhaust the analysis of learning mechanisms in the model, it does illustrate how dynamic analogues of the model will evolve. Notice as well that if the spillover effects are intertemporal, as in Durlauf (1993), so that aggregate behavior last period affects current individual payoffs, our analysis will also apply.[8]

Eq. (11) immediately implies the existence of a unique m_t conditional on any m_{t-1}. Therefore, local stability of a particular steady state identified in Proposition 1 will require that it represents a limiting solution to

$$m_t = tanh(\beta h + \beta J m_{t-1}) \tag{17}$$

where m_0 is taken anywhere in some neighborhood of that steady state.

We sketch an argument on stability as follows, assuming $\beta J > 1$ and $h = 0$. In this case, the derivative of $m_t - m_{t-1}$ with respect to m_{t-1} will equal

$$\frac{\partial(m_t - m_{t-1})}{\partial m_{t-1}} = \beta J(1 - (tanh^2(\beta J m_{t-1})) - 1. \tag{18}$$

Suppose we start with the $m_0 = 0$. This would imply that $\frac{\partial(m_1(m_0) - m_0)}{\partial m_0} > 0$ since $tanh(0) = 0$. Hence $m_m^* = 0$ is not locally stable, since by continuity and symmetry of $tanh(\cdot)$, one could find a neighborhood around $m_0 = 0$ such that $m_1 > m_0$ if $m_0 > 0$ and $m_1 < m_0$ if $m_0 < 0$. Now suppose that we start with $m_0 = 0^+$. By eq. (17), $m_t > m_{t-1}$ $\forall\ t > 1$, so the sequence is monotonically increasing. Since m_t is bounded, the sequence must converge to some limit which, by Proposition 2.i, must be m_+^* since there are no other steady state

[8]For example, the payoff to labor force participation of generation t might depend on the labor force participation decisions of generation $t-1$ due to role model or labor market connection effects. Loury's (1977) notion of intergenerational social capital may also be given this interpretation.

solutions with positive value. Therefore m^*_+ is locally stable from below. Now suppose that $m_0 = 1$. In this case, eq. (18) implies $m_1 < m_0$, eq. (17) implies the sequence is monotonically decreasing, which will again require that its limit is m^*_+ since there are no other steady states with positive value; hence m^*_+ is stable from above. This verifies local stability of m^*_+. By symmetry, m^*_- must be locally stable as well.

Analysis of the case with three roots and $|h| \neq 0$ (but small) is parallel to the $h = 0$ case. Further, analogous reasoning can be used to show the stability of the unique steady states which occur when either $\beta J < 1$ or $|h|$ is large enough. Together, this leads to Proposition 4.

Proposition 4. Stability of steady state mean choice levels

Under the assumption of noncooperative decisionmaking and the expectations formation process eq. (15),

i. If eq. (14) exhibits a unique root, that root must be locally stable.

ii. If eq. (14) exhibits three roots, then the steady state mean choice levels m^*_- and m^*_+ are locally stable whereas the steady state mean choice level m^*_m is locally unstable.

This result provides an interesting complement to analyses in Miyao (1978a) and Bénabou (1993) on the instability of integrated neighborhood equilibria in which there are intra-group externalities. While those analyses show how agents will segregate themselves by type in the presence of externalities, thereby inducing within-group homogeneity and cross-group heterogeneity, our analysis illustrates how agents will choose to act relatively homogeneously when their types (defined in terms of realizations of $\epsilon(\omega_i)$ across i) are heterogeneous and they are required to form a common group.

In subsequent analysis, we shall focus only on the two stable equilibria.

***iii.* Welfare analysis**

Unlike the frameworks studied in Cooper and John (1988) and Milgrom and Roberts (1990), there will not exist a Pareto ranking across the two equilibrium mean choice levels given a realization of the individual utility errors $\epsilon_i(\omega_i)$. The reason for this is simple. Extreme realizations of these errors will cause some agents to choose -1 and others to choose 1 regardless of the social utility induced by the choices of others. Hence these agents will disagree on the relative desirability of the m^*_- and m^*_+ equilibrium means, and therefore no Pareto ranking will exist. However, one can exploit the preference symmetry across agents to calculate the expected utility of a typical agent (*i.e.* expected prior to the realization of his random utility terms) under the two equilibria, and use this to evaluate social welfare under the two mean choice levels. This calculation compares

$$\mathrm{E}(max_{\omega_i}V(\omega_i)\,|\,m^*_+) = \mathrm{E}max_{\omega_i}(h\omega_i + k + J\omega_i m^*_+ + \epsilon(\omega_i)) \tag{19}$$

to

$$\mathrm{E}(max_{\omega_i}V(\omega_i)\,|\,m^*_-) = \mathrm{E}max_{\omega_i}(h\omega_i + k + J\omega_i m^*_- + \epsilon(\omega_i)). \tag{20}$$

Anderson, de Palma and Thisse (1992) (see pg. 60-61 for a proof) show that for any root m^*, the expected utility can be written as

$$\begin{aligned}&\mathrm{E}(max_{\omega_i}V(\omega_i)\,|\,m^*) = \\ &\beta^{-1}(ln(exp(\beta h + \beta k + \beta J m^*) + exp(-\beta h + \beta k - \beta J m^*))).\end{aligned} \tag{21}$$

When $h = 0$, it is easy to show that $|\,m^*_+\,| = |\,m^*_-\,|$, so eqs. (19) and (20) must be equal. Thus in the absence of any private utility, each root provides equal expected utility. On the other hand, when $h > 0$ (< 0), $|m^*_+| > |m^*_-|$ ($|m^*_-| > |m^*_+|$), so that the expected utility under m^*_+ (m^*_-) must exceed the expected utility under m^*_- (m^*_+). Intuitively, the root whose sign is the same as the mean that private utility alone would induce is that which maximizes expected utility since the private and social utility effects work in the same direction. This verifies Proposition 5.

Proposition 5. Welfare rankings

i. When $h > 0$ (< 0), then the equilibrium associated with m^*_+ (m^*_-) provides a higher level of expected utility for each agent than the equilibrium associated with m^*_- (m^*_+).

ii. When $h = 0$, then the equilibrium associated with m^*_+ and the equilibrium associated m^*_- provide equal levels of expected utility for each agent.

***iv.* Large economy behavior with conformity effects**

Finally, we consider the properties of a noncooperative equilibrium when social utility embodies conformity effects of the form eq. (5). In this case, the joint probability for choices will obey

$$Prob(\underset{\sim}{\omega}) = \frac{exp(\beta(\sum_{i=1}^{I}(h\omega_i + J\omega_i\bar{m} - \frac{J}{2}(1+\bar{m}^2)))}{\sum_{\nu_1 \in \{-1,1\}} \cdots \sum_{\nu_I \in \{-1,1\}} exp(\beta(\sum_{i=1}^{I}(h\nu_i + J\nu_i\bar{m} - \frac{J}{2}(1+\bar{m}^2)))}. \quad (22)$$

Notice however, that $exp(-\frac{\beta J}{2}(1+\bar{m}^2))$ cancels out of the numerator and denominator of this expression. Hence, eqs. (22) and (11) are equivalent, which means that all features we have developed for the proportional spillovers specification apply to the conformity specification as well. Further, it is straightforward to replicate the analysis and conclusions in Section 2.*iii* under conformity effects, which leads to Proposition 6.

Proposition 6. Equivalence of noncooperative equilibrium under positive spillovers and conformity effects

Propositions 1-5 will still hold if social utility takes the form eq. (5) rather than eq. (4).

4. Large economy behavior under a social planner

We now consider the equilibrium probability measure which arises when individual choices are assigned by a social planner whose possesses a utility function over the set of choices, $P(\underset{\sim}{\omega})$, which consists of deterministic and random components,

$$P(\underset{\sim}{\omega}) = U(\underset{\sim}{\omega}) + \epsilon(\underset{\sim}{\omega}). \tag{23}$$

We will constrain the deterministic component of the social planner's utility to equal the sum of the deterministic components of the individual utilities in the population,

$$U(\underset{\sim}{\omega}) = \sum_{i=1}^{I} (u(\omega_i) + S(\omega_i, \bar{\omega}_I)). \tag{24}$$

Notice that by placing $\bar{\omega}_I$ in the individual social utility functions, the planner internalizes the individual-level spillover effects induced by the mean choice level.

Finally, we assume that the error $\epsilon(\underset{\sim}{\omega})$ is independent and extreme-value distributed across all 2^I possible configurations of $\underset{\sim}{\omega}$. This assumption will ensure that the joint probability measure characterizing individual choices under a social planner has the same logistic form as the noncooperative case as discussed in Anderson, de Palma and Thisse (1992 chapter 2). Notice that under this interpretation, the random utility of the social planner, rather than that for individuals, is germane to the determination of $\underset{\sim}{\omega}$.[9]

This specification of a social planner determining the vector of individual choices is of interest both in terms of its contrast with the noncooperative equilibrium as well as in terms of its possible empirical relevance. As described by Coleman (1988, 1990 chapter 12), the evolution of social capital, defined to include aspects of social structure which facilitate coordination across individuals and which may be embedded either in personal mores or organizations such as churches or schools, implies that in many types of social situations, coordinated behavior can emerge. This idea underlies a growing body of work in economic history. For example, Greif (1994) provides a discussion of the empirical importance of social capital in the evolution of trading relationships. See also Bowles and Gintis (1994) for additional empirical examples of social capital as well as a theoretical characterization of those conditions under which coordinated equilibria are most likely to prevail. From this perspective, our social planner represents a reduced form representation of the consequences of social capital which, at a community-wide level, act to coordinate individual behavior.

i. Social planner equilibrium with proportional spillovers

As before, we examine the case of proportional spillovers first. The equilibrium for our model may be derived by replacing $\bar{m}$ with $I^{-1}\sum_{i=1}^{I} \omega_i$ in eq.

[9] Since the random utility increments to individuals were assumed extreme-value distributed, they cannot be summed to produce an extreme-value distributed term for the social planner.

(9). The probability measure characterizing the joint choice of $\underset{\sim}{\omega}$ follows the same logistic form as the noncooperative case in the sense that

$$Prob(\underset{\sim}{\omega}) = \frac{exp(\beta(\sum_{i=1}^{I} h\omega_i + \frac{J}{I}(\sum_{i=1}^{I}\omega_i)^2))}{\sum_{\nu_1 \in \{-1,1\}} \cdots \sum_{\nu_I \in \{-1,1\}} exp(\beta(\sum_{i=1}^{I} h\nu_i + \frac{J}{I}(\sum_{i=1}^{I}\nu_i)^2))}. \tag{25}$$

However, unlike the noncooperative case, the likelihood of each $\underset{\sim}{\omega}$ will account for the spillover effects induced through by the impact of individual choices on mean behavior, as one would expect from the eq. (24).

In order to analyze this probability measure, which is known in the statistical mechanics literature as the Curie-Weiss model, it is necessary to eliminate the $(\sum_{i=1}^{I}\omega_i)^2$ terms in (25). This can be done by using the identity, whose usefulness was exploited by Kac (1968) and subsequently by Brock (1993) for economic models,

$$exp(a^2) = (2\pi)^{-1/2}\int_{-\infty}^{\infty} exp(-\frac{x^2}{2} + 2^{1/2}xa)dx. \tag{26}$$

This identity can be verified immediately by dividing both sides of the expression by $exp(a^2)$ and observing that eq. (26) is equivalent to the statement that the integral of the probability density of a normal $(2^{1/2}a, 1)$ random variable over its support is 1. When this identity is employed, substituting a with $(\frac{\beta J}{I})^{1/2}\sum_{i=1}^{I}\omega_i$, we have

$$exp((\frac{\beta J}{I})^{1/2}\sum_{i=1}^{I}\omega_i)^2 = (2\pi)^{-1/2}\int_{-\infty}^{\infty} exp(-\frac{x^2}{2} + x(\frac{2\beta J}{I})^{1/2}\sum_{i=1}^{I}\omega_i)dx. \tag{27}$$

Using the change of variable $y = (2\beta J/I)^{1/2}x$, it is therefore the case that

$$exp(\beta(\sum_{i=1}^{I} h\omega_i + \frac{J}{I}(\sum_{i=1}^{I}\omega_i)^2)) =$$

$$exp(\beta(\sum_{i=1}^{I} h\omega_i) \cdot (\frac{I}{4\pi\beta J})^{1/2}\int_{-\infty}^{\infty} exp(-\frac{Iy^2}{4\beta J} + y\sum_{i=1}^{I}\omega_i)dy)) =$$

$$\prod_{i=1}^{I} exp(\beta h\omega_i) \cdot (\frac{I}{4\pi\beta J})^{1/2}\int_{-\infty}^{\infty} exp(-\frac{Iy^2}{4\beta J})\prod_{i=1}^{I} exp(y\omega_i)dy =$$

$$(\frac{I}{4\pi\beta J})^{1/2}\int_{-\infty}^{\infty} exp(-\frac{Iy^2}{4\beta J})\prod_{i=1}^{I} exp((\beta h + y)\omega_i)dy. \tag{28}$$

Summing this expression over all possible realizations of $\underset{\sim}{\omega}$ yields

$$\sum_{\nu_1 \in \{-1,1\}} \cdots \sum_{\nu_I \in \{-1,1\}} exp(\beta(\sum_{i=1}^{I} h\nu_i + \frac{J}{I}(\sum_{i=1}^{I}\nu_i)^2)) =$$

$$\sum_{\nu_1 \in \{-1,1\}} \cdots \sum_{\nu_I \in \{-1,1\}} (\frac{I}{4\pi\beta J})^{1/2}\int_{-\infty}^{\infty} exp(-\frac{Iy^2}{4\beta J})\prod_{i=1}^{I} exp((\beta h + y)\nu_i)dy =$$

$$(\frac{I}{4\pi\beta J})^{1/2}\int_{-\infty}^{\infty} exp(-\frac{Iy^2}{4\beta J}) \cdot$$

$$\Big(\sum_{\nu_1 \in \{-1,1\}} exp((\beta h + y)\nu_1)\Big) \cdot \ldots \cdot \Big(\sum_{\nu_I \in \{-1,1\}} exp((\beta h + y)\nu_I)\Big)dy. \tag{29}$$

However, since

$$\sum_{\nu_i \in \{-1,1\}} exp((\beta h + y)\nu_i) = exp(\beta h + y) + exp(-\beta h - y) \;\forall\; i, \tag{30}$$

$Prob(\underset{\sim}{\omega})$ will equal

$$\frac{\int_{-\infty}^{\infty} exp(-\frac{Iy^2}{4\beta J})\prod_{i=1}^{I} exp((\beta h + y)\omega_i)dy}{\int_{-\infty}^{\infty}(exp(-\frac{y^2}{4\beta J})(exp(\beta h + y) + exp(-\beta h - y)))^{I} dy} \tag{31}$$

which can be rewritten as

$$Prob(\underset{\sim}{\omega}) = \int_{-\infty}^{\infty} K(I,y) \frac{exp((\beta h + y)(\sum_{i=1}^{I} \omega_i))}{(exp(\beta h + y) + exp(-\beta h - y))^I} dy \tag{32}$$

where

$$K(I,y) = \frac{(exp(-\frac{y^2}{4\beta J})(exp(\beta h + y) + exp(-\beta h - y)))^I}{\int_{-\infty}^{\infty} (exp(-\frac{y'^2}{4\beta J})(exp(\beta h + y') + exp(-\beta h - y')))^I dy'}. \tag{33}$$

Consider the function $K(I,y)$. Clearly, $\int_{-\infty}^{\infty} K(I,y)dy$ will equal 1 for all I. Further, the shape of the function with respect to y for fixed I will be determined by

$$(exp(-\frac{y^2}{4\beta J})(exp(\beta h + y) + exp(-\beta h - y)))^I \tag{34}$$

since the denominator of (33) is independent of y. As I increases, the ratio of the value of $K(I,y)$ evaluated at y^*, defined by

$$y^* = max_y(exp(-\frac{y^2}{4\beta J})(exp(\beta h + y) + exp(-\beta h - y))), \tag{35}$$

to the value of $K(I,y)$ at any other y must become arbitrarily large, so long as y^* is unique. Differentiating and rearranging terms implies that y^* must be a root of

$$y^* = 2\beta J tanh(\beta h + y^*). \tag{36}$$

Since y^* is a maximum, the root of eq. (36) which also solves eq. (35) must be the one that has the same sign as h, so that uniqueness is assured so long as $h \neq 0$.[10] We will assume that h is nonzero for the subsequent analysis, and return to the case $h = 0$ at the end.

Intuitively, this discussion leads one to expect $K(I,y)$ to asymptotically behave as a Dirac function, *i.e.* a function which equals zero everywhere except at y^*. Further, given the term

$$\frac{exp((\beta h+y)(\sum_{i=1}^{I}\omega_i))}{((exp(\beta h+y)+exp(-\beta h-y)))^I}=\prod_{i=1}^{I}\frac{exp((\beta h+y)\omega_i)}{exp(\beta h+y)+exp(-\beta h-y)} \quad (37)$$

in (32), one would expect that $K(I,y)$ acts on this term in such a way that the probability measure for $\underset{\sim}{\omega}$ will possess the property that

$$\bar{\omega}_I \Rightarrow_w \frac{exp(\beta h+y^*)-exp(-\beta h-y^*)}{exp(\beta h+y^*)+exp(-\beta h-y^*)}=tanh(\beta h+y^*). \quad (38)$$

This intuition can be formalized using LaPlace's method, (see Erdélyi (1956), section 2.4 for a general exposition and Kac (1968) for the development of the method in the context of the Curie-Weiss model). The actual application of the method in the current context is in fact quite subtle. Brock (1993), pg. 22, provides a formal demonstration of its validity in the current analysis. Combining eqs. (36) and (38), and rewriting y^* as $2\beta Jm^*$ leads to Proposition 7.

Proposition 7. Law of large numbers for individual choices in social planner equilibrium with proportional spillovers

Let m^* denote the root of $m^*=tanh(\beta h+2\beta Jm^*)$ with the same sign as h. If eq. (25) characterizes the joint distribution of discrete choices, then

$$\bar{\omega}_I \Rightarrow_w m^*. \quad (39)$$

[10]This uniqueness shows up in a wide range of stochastic processes of this type. For example, in Ising models, mean behavior conforms to the sign of the external field, which is equivalent to h in our framework. See Spitzer (1971), chapter 7, for further discussion of conditions for uniqueness versus multiplicity of invariant measures for models with nearest neighbor interactions.

A comparison of Proposition 1 and Proposition 7 reveals two differences between the noncooperative and social planner equilibria under proportional spillovers. First, multiple equilibrium levels of average choice can exist without cooperation when $\beta J > 1$, whereas the average choice level is unique under the social planner. This is unsurprising given the Pareto rankability of the multiple steady states in terms of individual expected utility in the noncooperative case, when contrasted with the formulation of the social planner's utility. Second, the average choice level chosen under the social planner equilibrium will be the same as would be chosen under the noncooperative solution if the signs of the means are preserved and the value of J is doubled, assuming one eliminates the multiplicity by always choosing the root of eq. (14) whose sign is the same as h. This means that even if the mean choice level in the noncooperative equilibrium has the same sign as the social planner equilibrium, the level will still be socially inefficient. Intuitively, while agents in the noncooperative equilibrium account for the effects of others on themselves, they do not account for their effect on others. By symmetry of the spillovers across agents, as given by eq. (4), the equilibrium probability measure under noncooperative decisions ignores half of the total spillovers induced by an individual decisions in the sense that while the spillovers onto individual i affect his behavior, he does not take account of the spillovers induced by his behavior. All such spillovers are accounted for by our social planner, however. This failure to internalize spillover effects can, however, be offset by doubling the social interaction parameter, leading to Proposition 8.

Proposition 8. Sustainability of social planner equilibrium under decentralized decisionmaking with proportional spillovers

The social planner's choice of $\underset{\sim}{\omega}_I$ in the large economy limit can be supported under decentralized decisionmaking by doubling the social utility payoff to each individual, in the sense of doubling J.

Of course, it is also possible in this case that a doubling of the social utility payoff could be counterproductive, if the equilibrium chosen noncooperatively were that which had an opposite sign to h. This is easily controlled for, in the context of a government subsidy, by ensuring that the doubling of J occurs only for choices whose sign is the same as h.

***ii.* Social planner equilibrium with conformity effects**

The social planner equilibrium with conformity effects will be characterized by the joint probability measure

$$Prob(\underset{\sim}{\omega}) = \frac{exp(\beta(\sum_{i=1}^{I} h\omega_i - \frac{J}{2}\sum_{i=1}^{I}(\omega_i - \bar{\omega}_I)^2))}{\sum_{\nu_1 \in \{-1,1\}} \cdots \sum_{\nu_I \in \{-1,1\}} exp(\beta(\sum_{i=1}^{I} h\nu_i - \frac{J}{2}(\sum_{i=1}^{I}(\nu_i - \bar{\nu}_I)^2)))}. \quad (40)$$

Since $-\frac{J}{2}\sum_{i=1}^{I}(\omega_i - \bar{\omega}_I)^2 = \frac{J}{2I}(\sum_{i=1}^{I}\omega_i)^2 - \frac{J}{2}I$, we can reexpress this probability as

$$Prob(\underset{\sim}{\omega}) = \frac{exp(\beta(\sum_{i=1}^{I} h\omega_i + \frac{J}{2I}(\sum_{i=1}^{I}\omega_i)^2)}{\sum_{\nu_1 \in \{-1,1\}} \cdots \sum_{\nu_I \in \{-1,1\}} exp(\beta(\sum_{i=1}^{I} h\nu_i + \frac{J}{2I}(\sum_{i=1}^{I}\nu_i)^2))} \quad (41)$$

which has the same form as (25) when J in that equation is replaced with $\frac{J}{2}$. Therefore, we can use the limiting behavior of (25) to conclude Proposition 9.

Proposition 9. Law of large numbers for discrete choices with social interactions

in social planner equilibrium with conformity effects

Let m^* denote the root of $m^* = tanh(\beta h + \beta J m^*)$ with the same sign as h. If eq. (41) characterizes the joint distribution of discrete choices, then

$$\overline{\omega}_I \Rightarrow_w m^*. \tag{42}$$

A comparison of Proposition 9 with Proposition 1 reveals an important difference between the proportional spillovers and conformity effects specifications. In the presence of conformity effects, the mean choice level in the social planner equilibrium is one of the steady state solutions under decentralized decisionmaking. This immediately implies Proposition 10.

Proposition 10. Sustainability of social planner equilibrium under decentralized decisionmaking with conformity effects

The social planner's choice of $\underset{\sim}{\omega}_I$ in the large economy limit can be supported under decentralized decisionmaking when social utility exhibits conformity effects of the form (5).

The intuition for the discrepancy between the relationship between the noncooperative and social planner equilibria for the two social utility parameterizations may be seen by computing the expected utility of a representative agent under noncooperation, as a function of the equilibrium mean choice level. Replacing $Jm^*\omega_i$ with an arbitrary $S(\omega_i, m^*)$ in eq. (21) and differentiating with respect to m^* reveals that

$$\frac{\partial \mathrm{E}(max_{\omega_i} V(\omega_i) \mid m^*)}{\partial m^*} = \frac{\partial \mathrm{E}(S(\omega_i, m^*) \mid m^*)}{\partial m^*}. \tag{43}$$

For the proportional spillovers model, the expected utility of a representative agent with respect to his own utility innovations will, in the large economy limit of a noncooperative equilibrium, have the feature that

$$\frac{\partial \mathrm{E}(h\omega_i + k + J\omega_i m^* \mid m^*)}{\partial m^*} = Jm^* \tag{44}$$

since by self-consistency, $\mathrm{E}\omega_i = m^*$. This means that when $m^* > 0$, a marginal increase in the average choice level raises the expected utility of the typical agent whereas when $m^* < 0$, a decrease in the average choice level raises expected utility. This means, from the perspective of the social interaction component on individual utility, that there is an externality in the mean choice level which is not accounted for by individuals, as expected utility could be increased by a coordinated change in the mean.

Under conformity effects, on the other hand, the associated derivative will follow

$$\frac{\partial \mathrm{E}(h\omega_i - \frac{J}{2}(\omega_i - m^*)^2 \mid m^*)}{\partial m^*} = \mathrm{E}(J(\omega_i - m^*) \mid m^*) = 0. \tag{45}$$

Hence there is no external effect which fails to be internalized by individuals, at least locally.[11] Since (45) holds for any self-consistent m^*, it must hold at the mean choice level as determined by the social planner, whose deterministic utility component is the sum of the private deterministic utility components, so that the social planner equilibrium is sustainable under decentralized decisionmaking. Intuitively, since the conformity specification more strongly punishes large deviations from the mean than the proportional spillovers specification, the total average utility benefit from a marginal change in the mean in the direction of the majority for those in the majority is exactly offset

[11]In fact, it is easy to see that the proportional spillovers specification is consistent with the condition for inefficiency of a noncooperative equilibrium in Cooper-John (1988), Proposition 2, whereas the conformity specification is not. See Bryant (1983) for a similar case where efficiency can be sustained in a noncooperative environment.

by the utility loss to those who choose differently from the majority. Such an exact offset does not hold under proportional spillovers.

iii. Social planner equilibrium in absence of deterministic private utility

Finally, we consider the case $h = 0$ for proportional spillovers. (The reasoning for the conformity specification is identical.) In this case, eq. (36) will have three solutions if $\beta J > 1$. Designate the two nonzero roots as y^*_+ and y^*_- and define $m^*_+ = \frac{y^*_+}{2\beta J}$ and $m^*_- = \frac{y^*_-}{2\beta J}$. Ellis (1985), pg. 100, shows that the limiting probability measure over choices will have the property that

$$\overline{\omega}_I \Rightarrow_w m^*_+ \text{ with probability } \frac{1}{2}$$

$$\overline{\omega}_I \Rightarrow_w m^*_- \text{ with probability } \frac{1}{2}. \tag{46}$$

In other words, the limiting measure for $\underset{\sim}{\omega}$ will be a mixture whose limiting behavior may differ across sample path realizations. This mixture has two interesting features. First, the root corresponding to $y^* = 0$ does not appear in the limiting expression. This parallels the instability of this root under noncooperative decisionmaking. Intuitively, the utility from bunching means that even when spillover effects are internalized in the sense of eq. (23), the system cannot rest at $m^* = 0$. Second, the probability weights on the two conditional (given y^* values) limiting means are equal. What this means is that under each sample path realization of the economy, there is an equal probability of producing the m^*_+ and m^*_- mean choice levels. Intuitively, social utility is embedded in eq. (23) in such a way that all spillovers from each individual choice are accounted for. When $h = 0$, the tendency of the mean choice level is irrelevant; what matters is that agents achieve high utility by tending to act similarly.

5. Extensions

We illustrate three extensions of the basic modelling framework, focusing on the noncooperative environment.

i. Dependence of social utility on past society behavior

It is natural in some social contexts to expect social utility to depend on the past level of the mean choice level. Examples of this feature would include intergenerational models of social norms in which offspring attitudes depend on the behavior of adult role models. A general formulation of this idea may be done using the analysis of section 3.*ii*, after incorporating the additional feature that the social utility parameter J depends on the lagged mean choice level. In an equilibrium, the mean choice level must solve

$$m_t = tanh(\beta h + \beta J(m_{t-1})m_{t-1}) \tag{47}$$

under either social utility specification. Fixed points of this equation will represent self-consistent steady states.

This equation, will, depending on the specification of $J(m_{t-1})$, be capable of exhibiting much more complicated behavior than the baseline model. For example, if $J(0) < \beta^{-1}$, whereas $J(m) > \beta^{-1}$ for $|m| > K$, then the model can (depending on K and h) exhibit a stable steady state at a mean level near 0 as well as at stable equilibria at mean levels near -1 and 1, unlike the analysis in section 3.*ii*. Additional unstable steady states can emerge as well.

ii. Asymmetric social utility

An alternative generalization of the social utility term would allow for an asymmetry in the consequence of a choice above the mean level of others versus a choice below this level. This would mean replacing J in (4) and (5) with J_+ if $\omega_i = 1$, J_- otherwise. Self-consistency would, under the noncooperative equilibrium with proportional spillovers, require that the mean choice level equals a root of

$$m^* = \frac{exp(\beta h + \beta J_+ m^*) - exp(-\beta h - \beta J_- m^*)}{exp(\beta h + \beta J_+ m^*) + exp(-\beta h - \beta J_- m^*)}. \tag{48}$$

Under a conformity effect, the self-consistent mean is a root of

$$m^* = \frac{exp(\beta h - \beta \frac{J_+}{2}(1 - m^*)^2) - exp(-\beta h - \beta \frac{J_-}{2}(-1 - m^*)^2)}{exp(\beta h - \beta \frac{J_+}{2}(1 - m^*)^2) + exp(-\beta h - \beta \frac{J_-}{2}(-1 - m^*)^2)}. \tag{49}$$

Hence the two solutions no longer coincide.

One interesting feature of these equations is that they illustrate how the relationship between large social utility effects in one direction and multiplicity of mean choice levels will depend critically on overall social utility specification. Suppose that $J_- = 0$, and consider the limiting behavior of (48) and (49) as $J_+ \Rightarrow \infty$. In the case of proportional spillovers, -1 and 1 are roots in the limit, whereas under conformity, -1 is still a root whereas 1 is not. Intuitively, while a large J_+ makes the choice of 1 under proportional spillovers extremely desirable for any positive mean, no such effect occurs under the conformity specification.

iii. Heterogeneity in deterministic private utility

A final extension would allow the $u(\cdot)$ term to vary across individuals. From the perspective of the development of the noncooperative equilibrium, this is equivalent to replacing the common h with different h_i across individuals. We associate the probability measure $dF_{h,I}(\cdot)$ with these individual characteristics. Reworking eqs. (8) to (12), it is straightforward to verify that a self-consistent mean for the noncooperative equilibrium must solve

$$m^* = \int tanh(\beta h + \beta J m^*) dF_h(h) \tag{50}$$

so long as $dF_{h,I}(\cdot)$ converges weakly to some probability measure $dF_h(\cdot)$. Using techniques developed in Amaro de Matos and Perez (1991), Propositions 1 to 5 can be generalized accordingly.

6. Empirical implications

The social interactions framework we have developed can provide some insights in interpreting a number of different existing empirical results.[12] This section identifies a few prominent recent studies of social interactions and their relationship to the model.

i. Threshold effects in individual behavior

Crane (1991) studies the influence of neighborhood characteristics in determining high school dropout rates and teenage pregnancy rates. In this analysis, Crane estimates logistic regressions explaining both high school dropout rates and teenage childbearing rates as a function of a host of neighborhood and family variables. One important finding is that there appears to be a threshold effect in both of these regressions. Specifically, the probabilities of both

[12] In this discussion, we focus exclusively on the ability of our model to replicate the empirical findings of these papers, rather than on the success with which each paper is able to map these findings into the particular structural interpretation each contains. Analysis of this latter question leads to the identification issues studied in Manski (1993a,b).

dropping out and childbearing increase dramatically as the percentage of workers in a teenager's neighborhood whose occupations are either managerial or professional falls below 5%.

Such a threshold effect is easily produced by the framework we have been studying. In the context of decentralized environments, one way in which a threshold effect will be produced is if there exist multiple equilibrium average choice levels due to strong social utility effects ($\beta J > 1$). Suppose now that a dearth of managerial and professional workers in a community is a consequence of some socially disrupting shock, such as the outflow of jobs from the community, as documented by Wilson (1987) in the context of inner cities across the US. If such disruptions lead to a rise in either childbearing or dropout rates, the stability analysis of section 3 can imply the existence of such a threshold in the data, if the implicit shock to h associated with the disruption is large enough.

Further, even if the equilibrium were unique due to the presence of a social planner, in the sense discussed in section 4, the model can exhibit threshold effects in the presence of cross-neighborhood variations in h. To see this, observe that as one moves from $h > 0$ to $h < 0$ in the model with a social planner, the equilibrium will jump discontinuously from a positive mean of large magnitude to a negative mean of large magnitude, when βJ is large. Thus small changes in the private utility of a choice can be magnified into large aggregate changes.

In our view, this sensitivity of aggregate outcomes to the interaction of h with βJ provides a natural reconciliation of the "culture of poverty" versus "deteriorating economic fundamentals" arguments which divide sociological explanations of persistent inner city poverty. Taking the noncooperative equilibrium as a baseline, a large component to social utility means that very disparate types of equilibria can emerge, so long as the private utility component is not too large, as illustrated in Proposition 2. Even in the presence of a social planner, large social utility effects can exacerbate small shifts in economic fundamentals into large equilibrium effects. From this perspective, the "culture of poverty" and "deteriorating economic fundamentals" arguments are complementary and mutually reinforcing explanations, rather the alternative explanations as which they are frequently portrayed.

***ii.* Ethnic differences in behavior**

A number of studies have documented substantial ethnic differences in social behavior. For example, Borjas (1992) shows that the fit of a regression of the log of offspring income $y_{i,t}$ on parental income $y_{i,t-1}$ is substantially improved when a fixed effect dummy variable is included which accounts for a family's ethnic group.[13] Borjas estimates

$$y_{i,t} = \alpha + \beta y_{i,t-1} + \gamma_j \delta_{i,j} + \epsilon_{i,t}, \tag{51}$$

where $\delta_{i,j} = 1$ if family i is a member of group j, 0 otherwise.

Such an empirical specification is quite compatible with a model of social interactions in which the interactions occur within ethnic groups.[14] Suppose that the level of income in generation t depends on parental investment out of $t-1$ income and a binary effort level,

$$Y_{i,t} = f(Y_{i,t-1}) \cdot g(\omega_{i,t}), \tag{52}$$

where the individual effort decision depends on the choices of fellow ethnic group

[13]Table III in Borjas (1992) indicates that the fixed effect method of accounting for ethnic differences fits the data slightly better than one which uses the mean income of all parents in the same ethnic group as a variable. Our discussion can be applied to the latter regression as well.

[14]This would perhaps be most likely to occur with respect to race, given persistence in patterns of racial residential segregation. See Massey and Denton (1993) for an extensive empirical documentation of the extent of contemporaneous segregation.

members. So long as β, h, and J are allowed to vary across ethnic groups, and following Borjas in treating $f(\cdot)$ as linear, any pattern of cross-ethnic group differences of the form eq. (51) can occur. To see this, vary β and J across the nonnegative numbers and h across the reals, which will produce roots for all values between -1 and 1 for eq. (14). This wide range of possible ethnic differences is in fact unsurprising, when cross-group differences in h are allowed to be large. Perhaps more interesting, small differences in h between blacks and whites for example can, when combined with a common large J value, produce very different equilibrium income behavior. At the same time, notice that if β, h, and J are constrained to be the same across ethnic groups, our model will allow bimodal ethnic fixed effects, but not the range of fixed effects found by Borjas.

***iii*. Cross-city differences in crime**

As described in Glaeser, Sacerdote, and Scheinkman (1994), a major puzzle in the sociology literature is the large cross-city variation in crime rates. More specifically, if one regresses city-wide crime rates against a host of city-wide characteristics such as the unemployment rate (which seem likely to explain crime rates in the absence of social interactions), the residuals produced in such a regression are far too different across cities to be explained by a model in which individuals decide whether or not to commit crimes independent of one another. Glaeser, Sacerdote, and Scheinkman illustrate how these extremely large cross-city differences may be produced by a model in which individuals are arrayed on a 1-dimensional lattice and choose to engage in crime at time t depending on whether neither, one, or both of their immediate neighbors engaged in criminal behavior at $t-1$. Specifically, these authors develop an interesting variation of the type of local interaction model known as a voting model in which cross-city differences are explained by the interaction of one set of agents whose choices exhibit social interactions with a second set of agents in the population whose preferences are unaffected by others.

It is straightforward to see that our model can produce similar results. As in the ethnic capital example, cross-city variation in β, h, and J can replicate any cross-section variation in crime rates found in the data. As indicated above, Glaeser, Sacerdote, and Scheinkman attempt to eliminate the influence of cross-city variation in (the equivalent of) h through a use of set of control variables. When J is constant across cities, and if their control set exhausts cross-city variation in h, then their model is distinguishable from ours in the sense that all residual cross-city variation in our model must be (approximately) bimodal, so long as multiple equilibria exist. This can be seen from observing that eq. (14) implies that the data within a city under group-wide social interactions will, in the large economy limit, appear to be conditionally independent across agents, with all dependence embedded in the m^* term which controls the expected value of each $\omega_{i,t}$. A bimodal support for these residuals will exist when there are multiple equilibrium mean choice levels across cities.

On the other hand, when the set of control variables is incomplete, and the βJ parameter is less than, but near one, then our model can again produce a cross-section distribution of crime rates. To see this, let $h_{1,k}$ denote the part of private utility controlled for by an econometrician looking at city k, and $h_{2,k}$ the part which is not controlled. The mean choice level in city k, under a social planner and proportional spillovers, will equal

$$m_k = tanh(\beta(h_{1,k} + h_{2,k}) + 2\beta J m_k) \quad (53)$$

When $h_{1,k}$ is near zero, as one would expect for "at risk" individuals, small changes in $h_{2,k}$ which in turn change the sign of $h_{1,k} + h_{2,k}$ can produce extremely large changes in m_k.

An interesting exercise suggested by the Glaeser, Sacerdote, and Scheinkman model is the determination of what different empirical implications exist between their framework and ours when agents who are unaffected by the behavior of others are introduced into our model.

7. Summary and conclusions

This paper has developed a simple framework for characterizing discrete decisions when individuals experience private as well as social utility from their choices. The model is shown to produce a number of interesting features. First, multiple, locally stable equilibrium levels of average behavior are shown to exist when social utility effects are large enough and decisionmaking is noncooperative. Second, a large social multiplier can exist in terms of relating small changes in private utility to large equilibrium changes in average behavior. Third, while a social planner eliminates the multiplicity of average outcomes, other features of the noncooperative equilibrium, such as the presence of a large social multiplier, are preserved. Fourth, the model provides some insights into a number of empirical phenomena.

In terms of future research, we would identify three areas of investigation suggested by the current analysis. First, the analysis in this paper does not deal with issues of econometric implementation, although the logistic functional forms we employ naturally lend themselves to econometric work. See Brock and Durlauf, (in progress) for an extensive analysis of the econometrics of this class of models. Second, it would be valuable to integrate the social utility analysis of the current model with a framework such as Bénabou (1993, 1994) or Durlauf (1994a, b), which allows for endogenous selection of one's reference group.[15] This integration would enhance the ability of the current framework to explain phenomena such as the emergence and perpetuation of ghettos.

[15]See Miyao (1978b) for an application of discrete choice methods to neighborhood location decisions and Arthur (1987) for an analysis of urban dynamics using complementary methods to ours.

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