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NOISE TRADING, DELEGATED PORTFOLIO MANAGEMENT, AND ECONOMIC WELFARE

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ABSTRACT

We consider a model of the stock market with delegated portfolio management. All agents are rational: some trade for hedging reasons, some investors optimally contract with portfolio managers who may have stock-picking abilities, and portfolio managers trade optimally given the incentives provided by this contract. Managers try, but sometimes fail, to discover profitable trading opportunities. Although it is best not to trade in this case, their clients cannot distinguish "actively doing nothing," in this sense, from "simply doing nothing." Because of this problem: (i) some portfolio managers trade even though they have no reason to prefer one asset to another (noise trade). We also show that, (ii), the amount of such noise trade can be large compared to the amount of hedging volume. Perhaps surprisingly, (iii), noise trade may be Pareto-improving. Noise trade may be viewed as a public good. Results (i) and (ii) are compatible with observed high levels of turnover in securities markets. Result (iii) illustrates some of the possible subtleties of the welfare economics of financial markets.

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I. Introduction

We show the possibility of rational trade in securities markets which is not motivated by an informational advantage nor by "genuine" motives such as hedging, portfolio rebalancing, or liquidity needs. It is pure noise trade: it does not maximize the utility of the owner of the portfolio. We also provide an example in which this noise trade makes all agents better off. We explain this apparent paradox in what follows.

We view these results in the context of two sets of issues. First, there is a long-standing debate concerning whether prices and trading volumes in securities markets reflect fundamentals or "animal spirits". To date there is no conclusive empirical test which has resolved this debate. For this reason the two points of view are essentially dogmas. On the one hand, there is the belief that participants in markets are "rational," and the few who are not are quickly eliminated by the natural selection effects of arbitrage. On the other hand, there is the belief that many individuals may not be rational and that there is no necessary tendency for irrationality to disappear in the aggregate. In other words, a market may display aggregate "irrationality," "fads," or "herding."

In the finance literature, this debate appears in the form of differing interpretations of "noise trade." Noise trade is a random error term introduced into net asset demand to allow agents with private information to profit by trading on their information (see Grossman and Stiglitz (1976, 1980) and Kyle (1985)). Noise traders lose money (on average) to privately informed traders. The rationale for noise trade is theoretical: noise is necessary for profitable informed trading, and profitable informed trading is necessary for market efficiency. Since the origin of noise trade was not explicitly modelled, different interpretations have arisen. It may be viewed as rational agents trading for liquidity or hedging motives, consistent with the rational or efficient-markets perspective (e.g. Diamond and Verecchia (1981), Ausubel (1990a, b), Biais and Hillion (1992), Dow and Gorton (1994a, b)). Alternatively, noise trade may represent the actions of irrational agents (e.g. Black (1986), De Long et al. (1990)). In this paper, we suggest a third possibility.

The second set of issues concerns not the existence, but the level, of trading volume: trading volume seems high. There appears to be a consensus that trading volume or turnover (trading volume as a fraction of total market value) is inexplicably high. For example, Ross: "...it is difficult to imagine that the volume of trade in securities markets has very much at all to do with the modest amount of trading required to accomplish the continual and gradual portfolio rebalancing inherent in our current intertemporal models" (1989, p. 94). Hard evidence for this is difficult to find because economic models have not been developed to predict trading volume.[1] Dynamic hedging strategies without transactions costs would result in infinitely high volume. In the presence of transactions costs, optimal dynamic hedging would presumably lead to much reduced volume but the literature has not developed to the point of providing a benchmark prediction (this seems like an interesting area for research). Below we briefly examine the evidence of "high" turnover in two markets: foreign exchange and the New York Stock Exchange. The evidence is not conclusive, but is suggestive. There also appears to be an empirical link between turnover and agency problems, as proxied by the fraction of the market controlled by institutions and intermediaries.[2]

The daily trading volume of foreign exchange transactions in all currencies (including forwards, swaps, and spot transactions) in 1992 was US$ 880 billion, according to the comprehensive survey for April 1992 carried out by the Bank for International Settlements (BIS) (1993). To put this number in perspective compare this *daily* volume to the total value of *annual* world trade in 1992. The total value of world trade in 1992 was US$ 3,646 billion and foreign direct investment was US$ 220 billion. Thus, roughly one quarter of the annual trade and investment flow is traded each day in the foreign exchange market.

Furthermore, of the US$ 880 billion traded daily in the foreign exchange market, US$ 648 billion was between financial intermediaries and dealers (see BIS (1993)). In other words, most of the trading volume is made up of interbank transactions. Note that in the BIS survey, interbank transactions specifically exclude all transactions made on behalf of bank customers. Figure 1 shows the total and interbank trading volume in major currencies for spot transactions, forwards, and swaps. In each case, interbank trading volume makes up the bulk of transactions. For each of the three types of contract, the correlation across currencies between total and interbank volume exceeds 0.99.

On the New York Stock Exchange, turnover in 1992 was 48%. While there is no convincing theoretical prediction for assessing these numbers, many observers have the view that turnover is very high. For example, the Presidential Task Force on Market Mechanisms (1988) presents this viewpoint. As in the foreign exchange market, on the NYSE the increase in turnover has been accompanied by a rise in institutional ownership. This casual observation of a positive correlation between turnover and institutional ownership is confirmed when we take account of the decline in real trading costs over the post-WWII period. A regression of turnover on institutional ownership and real commissions per share shows that institutional ownership is still highly significant in explaining turnover.[3] As with foreign exchange, the available evidence is at least suggestive of a causal link between turnover and institutional control.

It seems difficult to explain the level of trading activity purely on the basis of "rational" motives for trade. Hedging and liquidity seem likely to explain only a small fraction of this trade, and it seems unreasonable to suppose that a small amount of such uninformed trade can support a large amount of informed trade. Hence the appeal of the "irrational" point of view. In contrast, we consider another motive for rational, uninformed agents to trade. We argue that it is capable of explaining a significant amount of trade. It is also consistent with the observed correlation between volume and institutional presence.

The motive stems from a contracting problem between professional traders and their clients or employers. We have in mind two settings in which the problem may arise. In one setting, an investor hires a fund management firm. The other setting is one in which a firm hires an employee to trade securities on its behalf. In both settings, there is likely to be a difficulty in writing incentive compatible, efficient compensation contracts. In this context money managers may engage in ex ante unprofitable trades which have some chance of being profitable ex post. One goal of this paper is to investigate this possibility and to analyze the conditions under which such trade can be sizeable.

In our setting portfolio managers who engage in producing information do not always uncover profitable trading opportunities. It can happen that inactivity (i.e. not trading) is the (first-best) optimal decision because the portfolio manager's effort at finding mispriced securities did not uncover any. The contracting problem that arises in our model is whether the delegated portfolio manager can convince the client/employer that inactivity was his best strategy.[4] The difficulty is that the employer cannot distinguish "actively doing nothing" in this sense from "simply doing nothing." If the contract allows a reward for not trading, portfolio managers may simply do nothing; the contract may either attract incompetent managers or lead competent managers to shirk. If this makes it impossible to reward inactivity, and limited liability prevents punishing ex post incorrect decisions, then the optimal contract may induce trading by the portfolio manager which is simply a gamble to produce a satisfactory outcome by chance. We call this noise trade or churning.[5] In the first part of the paper we show that noise trade will occur in equilibrium.

The contracting environment we study is a simple one in which the portfolio manager is unable to convince his employer that any inactivity is optimal. In our setting inactivity is not rewarded because that would induce shirking by talented portfolio managers and it would attract incompetent portfolio managers. In many simple contracting environments this problem could be solved by a two-part contract that specifies: (i) a large bonus for taking correct trading positions; and (ii), a smaller, lump-sum, payment for inactivity. Talented portfolio managers would be attracted by the chance of the bonus, but ex post, if they happened not to uncover trading opportunities, the lump-sum would be chosen in preference to trading randomly as a gamble to earn the bonus. Incompetent portfolio managers would not sign the contract if the lump-sum is not as large as their opportunity cost. Our environment is chosen with care so that this contract, and others like it, cannot eliminate the agency problem. Of course, if the contracting problem does arise in reality, as we suggest, it is presumably in the context of a more complex, repeated, environment that would not be as analytically tractable as the one specified here.

In the latter part of the paper, we consider the implications of noise trading for agents' welfare. Noise trading would appear to be costly for the employer since it lowers the expected rate of return on the portfolio. On the other hand, it will benefit hedgers: if managed portfolios earn lower rates of return, then uninformed hedgers earn higher returns. However, there is another effect. The higher return earned by the hedgers effectively reduces the cost of hedging; as a result they will trade larger amounts. In turn, this increase in volume can support a larger amount of investment by an informed fund manager. If the manager earns a smaller (percentage) return on a sufficiently increased investment, then he will be better off. We provide an example in which noise trading or churning by portfolio managers is Pareto-improving. In the example we provide, the security market would actually fail to exist without noise trade. This may not be as extreme as it may appear. It is often held that "illiquidity" can prevent the opening of a new market. As we discuss in the final section of the paper, our analysis suggests that noise trade, by increasing liquidity, can be a public good.

Our model is a general equilibrium model of portfolio management in a security market in which: (i) there is a labour market for potential portfolio managers; (ii) these agents optimally contract with investors to manage their portfolios; (iii) the security is traded in a market where prices are formed by a competitive market-making process; (iv) the portfolio managers trade against hedgers who optimize when choosing their demands. Our elementary welfare analysis is possible because we explicitly model the motives for trade of all the agents and, in particular, the hedging demand. Models which introduce "noise" or "liquidity" trade as an exogenous random variable cannot be used for this purpose. Endogenizing this component of trade, as we have done here, is neither original nor difficult. However, anything less than this has nothing to say about welfare. The most that can be done is to measure the transfer from uninformed agents to privately informed agents.

In Section II, we present the model. The equilibrium is derived in Section III. We show that in equilibrium, the optimal contract induces noise trade. A small amount of uninformed hedging trade may support a large amount of noise trade. The welfare implications of this are considered in Section IV. An example shows that the noise trade caused by contracting problems can be Pareto-improving. The results and the related research are discussed in Section V.

II. The Model

In this section we first describe the assumptions of the model, in Subsections A, B and C. In Subsection D we discuss these assumptions.

There are two dates, 0 and 1. There are two securities: cash and a risky security. The securities are traded at date 0, and at date 1 the security pays a liquidating dividend of either H or L (with equal probability). We assume H = 1 and L = 0, but sometimes for clarity we maintain the H and L notation.

A. Delegated Portfolio Management

A single principal has an opportunity to employ an agent to manage his portfolio. There are two types of agent who may potentially be employed as professional portfolio managers. The first type (talented managers) may receive private information about the value of the security, which would allow them to earn an above average return for the principal. The second type (incompetent managers) have no chance of receiving private information. The population of agents consists almost entirely of incompetents, though there is a large number of talented managers as well.[6] An agent's type is private information. We assume agents have no resources of their own, have the protection of limited liability, and are risk-neutral.

Any agent has the choice of working for the principal (if offered a contract) or engaging in another activity (receiving remuneration k). However, this other activity is compatible with agreeing to manage the principal's portfolio so long as no trading actually occurs. The principal cannot observe whether the agent is engaging in the other activity. If a talented agent chooses to forego this other activity and work for the principal, then with probability α he discovers the liquidation value of the security. In the discussion below we will refer to an agent who foregoes the other activity as "actively working" for the principal. In contrast, an agent who accepts a contract to manage the portfolio, but who simultaneously engages in the alternative activity (so cannot trade for the principal), is "shirking." Agents might want to do this if the contract specifies a payment for doing nothing, i.e. a payment in the event of no trade.

If a talented agent actively works for the principal, he may or may not receive private information. If he receives private information, he can manage the portfolio to generate superior returns. Even if he does not receive private information, he can still choose to trade and by chance he may earn a superior return anyway. Of course, if he does trade without information he is equally likely to earn an inferior return, but if this happens the principal cannot penalize him because the agent has no private resources.

An incompetent agent faces the choice between shirking and actively working. If he shirks he will earn k, together with any payment specified in the contract for not trading. Alternatively, if he actively works he can trade at random in the hope of earning a superior return (of course he could instead collect the payment for doing nothing, but then it would be better to shirk and collect k in addition). In other words, if he is not going to trade he might as well get paid twice!

If the principal decides to hire an agent as his portfolio manager, he must design a contract to induce the agent to forego the alternative activity (i.e. to actively work for him), and to maximize the return on the portfolio net of management expenses. The contract cannot condition directly on whether the agent undertakes the alternative activity, nor on whether the agent receives the private information since both of these are unobservable to the principal. However, the contract can condition on the realized value of the security and on the position the agent took.

Note that any contract that attracts incompetents as well as talented managers will result in hiring an incompetent almost surely, since they predominate in the population. Clearly, such a contract will not arise in equilibrium since it entails a positive payment in return for nothing. By the same argument, talented managers under the optimal contract will choose to actively work rather than shirk (if they shirk, they are no better than the incompetents).

Since there is a large supply of talented agents, a portfolio manager will be paid just enough to induce him to forego the alternative activity.

Since the contract cannot condition directly on the agent's type, shirking decision, or on information arrival, the portfolio manager's incentives may be distorted. This may happen even though the contract is optimally designed and attracts only the talented managers. This agency cost may be reflected in the manager trading when he has received no information. We describe this as "noise trade" or "churning."

B. The Security Market

The security is traded in a centralized market where a market maker sets prices and clears the market, and other agents trade for hedging motives. Hedgers should be viewed as a continuum of small traders. However, for convenience we will simply refer to "a hedger" in what follows.

The probability that an agent with a hedging need arrives is δ. With probability $(1 - \delta)$ there is no hedger present.

Hedgers want to insure against an income shock. The income shock may be positively or negatively correlated with the security's liquidation value. With 50 percent probability it will be positively correlated and a hedger will be perfectly hedged by selling one share short. Specifically, the hedger's wealth is W when the asset is worth L and his wealth is worth W + 1 when the asset is worth H. With 50 percent probability a hedger will be perfectly insured by buying one share, that is, his wealth is worth W when the asset is worth H and W + 1 when the asset is worth L. No other agents know whether there is a hedger present, and, if so, whether his hedging need is positive or negative. In other words, in our market the amount of hedgeable risk is either -1, 0, or $+1$ with probabilities $\frac{1}{2}\delta$, $1-\delta$, and $\frac{1}{2}\delta$ respectively.

Prices are set by a risk neutral market maker who faces Bertrand competition as in Kyle (1985). The portfolio manager and the hedger submit their orders to the market maker, who sets the price equal to the expected value conditional on this order flow and meets the net order from his inventory.

C. The Benchmark Model: Direct Investment

Subsequently, we will want to compare the economy described in our main model (equilibrium with delegated portfolio management) to another economy where there is no agency problem, i.e. the owners of portfolios (the principals) may themselves become informed about the value of the asset. In this benchmark model, the principal manages his own portfolio (and foregoes other activities at cost k). With probability α he learns the true value of the asset. Since this investor is trading with his own money he will only trade if he receives information, and this will be understood by the market maker and the hedgers. We refer to this case as "direct investment" (DI). We solve this model in Appendix 2 below.

We stress that our comparison is between a world of agency problems and a world without such problems. We do not address the question of whether a portfolio owner with investment management ability would deliberately introduce an agency problem by hiring an agent as his portfolio manager.

D. Discussion of the Assumptions

1) What is the alternative activity for portfolio managers?

We have two alternative situations in mind. First, suppose the portfolio manager is an employee and the principal is his employer. Since the agent is an employee he can, presumably, be prevented from working at another firm. His alternative activity is to engage in leisure activities on the firm's time, e.g., watching the winter olympics on the Reuters screen rather than market prices, taking his friends out to lunch. It is not possible to engage in these activities to the same extent if the agent must actually trade.

A second situation is where the agent is a portfolio management firm which has been hired by an investor, for example a pension fund. The portfolio management firm is able to choose how much effort to devote to this client's account. If they do not carry out any trades for the client, they will be able to spend more time recruiting other clients and working for them instead. Here the alternative activity is managing the portfolios of other clients.

2) Is the assumption about "actively working" economically equivalent to simply modelling a choice of effort level?

Not exactly. We considered modelling the agent's decision as a discrete effort choice with cost k. The problem with that formulation is that the first-best could be achieved with a contract that rewards an agent who admits that he was unable to uncover information (the payment would simply be the expected reward to the agent from churning). Under the assumptions presented in this paper, the agency problem is substantive: first-best contracting is impossible because offering a contract which provides a payment for doing nothing will attract agents who intend to engage in this alternative activity. The agents will simply do nothing. In our examples, the employees will watch sports and the portfolio management firms will devote their time to other clients, in each case claiming that doing nothing was the ex post optimal strategy.

3) Why is there only one principal?

This assumption is made for analytical simplicity.

4) Why are there hedgers in the model?

Since the portfolio managers (averaging over both informed and uninformed) earn excess returns, other agents must be losing money to them. The standard device in the literature (as discussed above in the Introduction) is to model these money-losing agents as "noise" or "liquidity" traders who trade an exogenous random quantity. Rather than introduce such exogenous behaviour, we explicitly model the utility functions of all agents, for two reasons.

First, one goal of our paper is to explain the existence of "noise" trade that is not motivated by informational advantage, risk aversion or liquidity needs. Therefore, we cannot assume the result by introducing exogenous noise trade. We model the trade by agents who are willing to lose money as resulting from an explicit hedging need. When these agents trade they lose money on average, but they are better off because they are partially hedged.

Second, the amount the hedgers trade depends on the equilibrium prices, which in turn depend upon the optimal portfolio management contract and hence the amount of churning. Welfare analysis would be impossible without explicitly modelling the utility functions of all agents. Moreover, our example will depend crucially on the response of hedging demand to the equilibrium prices.

5) Is the discreteness of the hedging demand important?

No. Our hedgers have an income shock of ± 1. One could imagine, as an alternative, an income shock x, where x is a continuously distributed random variable. For example, x could have a Gaussian distribution; this would lead to a continuous (though obviously not Gaussian) distribution of hedging demands. This example illustrates that the closed-form solution of the model would become impossible in general. We chose the simplest distribution of the hedging need for tractability.

6) What is the point of the benchmark model?

Later, we will compare welfare in the model of delegated portfolio management to the benchmark model of direct investment. Our view is that the agency problem is unavoidable. Nevertheless, it is useful to understand the welfare implications of this agency problem by making the comparison with a suitable benchmark.

7) Is the price formation process a critical assumption?

No. We need a price formation process that allows informed traders to earn an excess return at the expense of the uninformed. The market maker institution we use is a common modelling device in the Finance literature.

III. Equilibrium with Delegated Portfolio Management

An equilibrium of the model of delegated portfolio management (DPM) specifies:

1) a decision for the principal on whether to employ an agent to manage his portfolio, and if so, a contract describing the agent's remuneration;

2) a decision for the portfolio manager on whether to shirk or actively work for the principal, and if the latter, a trading strategy (conditional on information arrival);

3) a decision for the hedger on how large a position to take;

such that everybody is maximizing utility, given the market maker's beliefs and the behavior of the others.

A. The Optimal Portfolio Management Contract

In this section we derive the optimal portfolio management contract. The analysis in this section is conditional on the assumption that the principal does decide to employ an agent. Later, we will have to verify that the principal does not pay too much money to cancel the benefits of having the agent work. In other words, hiring the portfolio manager to produce superior returns should be more profitable than saving the management fee and picking stocks at random. This will not always be true in equilibrium, in particular if the opportunity cost k is large relative to the amount of hedging demand, as will be seen below.

The hedgers will trade a quantity $\pm$ x, derived below. As we verify below, a portfolio manager will also either buy or sell x in order to pool with the hedgers (any other quantity would reveal his identity to the market maker).

As explained above, the optimal contract will only attract talented agents. It will induce them not to shirk, and they may or may not become informed. It is clear that (under an optimal contract) a portfolio manager who receives information will buy x on good news and sell x on bad news. An uninformed manager may or may not trade x depending on the incentives provided by the contract. We will show that the optimal contract will indeed induce an uninformed manager to trade.

The possible outcomes observed by the principal at $t = 1$ are:

1) Security value = H and portfolio manager bought x.

2) Security value = H and portfolio manager sold x.

3) Security value = L and portfolio manager bought x.

4) Security value = L and portfolio manager sold x.

5) The portfolio manager did not trade.

In cases (1) and (4) the portfolio manager was either informed or was an uninformed noise trader who was lucky (i.e., had no information but traded x anyway in the right direction). In cases (2), (3), and (5) it is clear that the portfolio manager was uninformed.

The payment in each of these cases could in principle be positive and we denote these payments by m_1, m_2, m_3, m_4 and m_5 respectively.[7] Note however, that because the agent has no private resources he cannot be penalized with a negative payment, notably in cases (2) and (3).[8]

Proposition 1 (Churning): *Under an optimal contract only talented agents will be employed, they will actively work for the principal, and they will churn in the event that they receive no information.*

Proof: If a contract attracts incompetent agents, it will almost surely result in employing an incompetent. Principals will not be willing to pay for this. Among other things, this implies $m_5 = 0$, since any contract which offers a positive payment for not trading will attract a flood of incompetents. A talented agent who does not actively work is no better than an incompetent. Again, principals will not pay for this. Thus the principal offers a contract that only attracts talented agents, and induces them to work. We now show that they will churn if they receive no information.

An agent who buys without information is equally likely to receive either m_1 (in case he correctly bought when the security turns out to high valued) or m_3 (the incorrect position). Similarly if he sells he will receive either m_2 or m_4. Thus the payoff from churning is:

$\max\ \{\frac{1}{2}m_1 + \frac{1}{2}m_3, \frac{1}{2}m_2 + \frac{1}{2}m_4\}$.

This must be strictly positive since at least one of the m_i ($i = 1, \ldots 5$) must be non-zero, all of them are non-negative and $m_5 = 0$. Therefore if the talented agent receives no information, churning is a strictly dominant strategy. ∎

An optimal contract must satisfy the following two conditions:

$$\max\ \{\tfrac{1}{2}m_1 + \tfrac{1}{2}m_3, \tfrac{1}{2}m_2 + \tfrac{1}{2}m_4\} < k \tag{2}$$

and

$$\alpha\ (\tfrac{1}{2}m_1 + \tfrac{1}{2}m_4) + (1 - \alpha)\ \max\ \{\tfrac{1}{2}m_1 + \tfrac{1}{2}m_3, \tfrac{1}{2}m_2 + \tfrac{1}{2}m_4\} = k \tag{3}$$

Condition (2) says that the expected payment must be small enough not to attract incompetent managers to actively work at managing portfolios. Since they never receive information, they would then churn. Note that when

$$\tfrac{1}{2}m_1 + \tfrac{1}{2}m_3 = \tfrac{1}{2}m_2 + \tfrac{1}{2}m_4 \tag{4}$$

the uninformed agent will be willing to randomize between buying and selling.

Condition (3) states that the expected payment to the talented manager must be just high enough to attract him away from alternative employment. With probability α he will get information and take the correct investment decision (given that the contract will induce the correct decision in this event). With probability $(1 - \alpha)$ he will not become informed and will churn (recall that the optimal contract cannot specify a strictly positive payment for doing nothing). Since there are many talented managers (even though almost all potential managers are incompetent), the equilibrium expected reward to the manager must be just enough to attract him to actively work.

We can now compute the payments for an optimal contract. We consider symmetric contracts where the payment is the same for both correct outcomes (i.e. $m_1 = m_4$) and also for both incorrect outcomes (i.e. $m_2 = m_3$). Define $m = m_1 = m_4$, and we set $m_2 = m_3 = 0$.

Substituting $m = m_1 = m_4$ and $m_2 = m_3 = 0$ into equation (3) gives:

$$m = 2k/(1+\alpha). \tag{5}$$

This solution satisfies inequality (2) since $\frac{1}{2}m < k$.

It is clear that there other contracts that are equivalent in terms of their incentives and their expected costs. Any contract where $m_1 = m_4$, $m_2 = m_3$, $m_1 + m_3 = 2k/(1+\alpha)$ and $m_1 > m_3$ is equivalent so long as m_3 is small enough to satisfy inequality (2). For our purposes this distinction is immaterial.[9]

B. Order Flow in Equilibrium

We now consider the order flow in equilibrium with portfolio management. The hedgers will trade $\pm$ x (the quantity x will be derived below). In order to pool with the hedgers, an informed portfolio manager will also either buy or sell x (any other quantity would reveal his information to the market maker).

Consider the decision problem of an uninformed portfolio manager. If he does not trade, he will be revealed as uninformed. As we showed above, the contract will not reward him in this event, because otherwise it would attract a flood of incompetents. On the other hand if he churns and trades x there are two possible outcomes: first, by luck he may mimic the actions of an informed portfolio manager, and be rewarded accordingly. Secondly, he may take the wrong action and reveal himself to be uninformed, but in this case he cannot be penalized because of limited liability. Therefore he will trade $\pm$ x at random. This was shown in Proposition 1.

The market maker can therefore observe five possible different order flows:

Sell 2x: If there is a hedger who sells, together with either an informed portfolio manager with bad news or an uninformed portfolio manager who sold at random.

Sell x: If there is no hedger, and there is either an informed portfolio manager with bad news or an uninformed portfolio manager who sold at random.

No trade: If there is a hedger who sells, together with either an informed portfolio manager with good news or an uninformed portfolio manager who buys at random, or vice versa.

Buy x: This is symmetric to selling x.

Buy 2x: This is symmetric to selling 2x.

C. Prices

It is straightforward to compute the market maker's updated beliefs (and prices) in these five cases. Since the asset takes the value of either 1 or 0, the price is equal to the market maker's belief that the asset value is high.

Sell 2x: The probability of this event is:

$$\tfrac{1}{2}\delta\,[\tfrac{1}{2}\alpha + \tfrac{1}{2}(1 - \alpha)] = \tfrac{1}{4}\delta,$$

and the probability of the joint event of sell 2x and the asset is valuable is:

$$\tfrac{1}{2}\delta\;\tfrac{1}{4}(1 - \alpha).$$

Therefore the price in this case is $\tfrac{1}{2}(1 - \alpha)$.

Sell x: The probability of this event is:

$$(1 - \delta)\,[\tfrac{1}{2}\alpha + \tfrac{1}{2}(1 - \alpha)] = \tfrac{1}{2}(1 - \delta),$$

and the probability of the joint event of sell x and the asset is valuable is:

$$(1 - \delta)\;\tfrac{1}{4}(1 - \alpha),$$

so the price is again $\tfrac{1}{2}(1 - \alpha)$. Note that this is the same as for sell 2x because the only difference between the two cases is the presence of the hedger, which conveys no information.

No trade: This conveys no information (by symmetry) so the price is the unconditional expectation, $\tfrac{1}{2}$. The probability of this case is $\tfrac{1}{2}\delta$.

Buy x: Since the average value of the updated belief must be $\tfrac{1}{2}$, the price is $\tfrac{1}{2}(1 + \alpha)$.

Buy 2x: Again the price is $\tfrac{1}{2}(1 + \alpha)$.

We will denote the prices p_S in the event of sell x or 2x, p_0 if no trade, and p_B in the event of buy x or 2x.

D. The Hedger's Decision

Consider the case of a hedger whose income shock is negatively correlated with the security value. When he *buys* x he will pay one of two prices: p_B or p_0. The possibilities are as follows:

1) there is another buyer and the security is worth L. This can only occur if the other buyer is an uninformed portfolio manager who randomly happens to buy (which occurs with probability $\frac{1}{2}\cdot\frac{1}{2}(1-\alpha)) = \frac{1}{4}(1-\alpha)$. Since there is another buyer the price is p_B.

2) there is another buyer and the security is worth H. This can occur if the other buyer, a portfolio manager, is informed (which occurs with probability $\frac{1}{2}\alpha$) or if the other buyer is uninformed and randomly happens to buy (which occurs with probability $\frac{1}{2}\cdot\frac{1}{2}(1-\alpha)$. The probability is, therefore, $\frac{1}{4}(1+\alpha)$. Since there is another buyer the price is p_B.

3) there is another order which is a sell order and the security is worth L. This can occur if the other order was submitted by an informed portfolio manager with bad news (which occurs with probability $\frac{1}{2}\alpha$) or the other order was randomly submitted by an uninformed portfolio manager (which occurs with probability $\frac{1}{2}\cdot\frac{1}{2}(1-\alpha)$). As before the total probability is $\frac{1}{4}(1+\alpha)$. In this case the price is p_0.

4) there is another order which is a sell order and the security is worth H. This occurs if an uninformed portfolio manager randomly submits a sell order (which occurs with probability $\frac{1}{4}(1-\alpha)$). Again, the price is p_0.

The hedger chooses to buy an amount x of the security to maximize:

$$\tfrac{1}{4}(1-\alpha)\,U(W - p_Bx + 1) + \tfrac{1}{4}(1+\alpha)\,U(W - p_Bx + x) +$$

$$\tfrac{1}{4}(1+\alpha)\,U(W - p_0x + 1) + \tfrac{1}{4}(1-\alpha)\,U(W - p_0x + x).$$

Appendix 1 describes the other case of a hedger who sells. Because of the symmetry in the hedger's decision problem, it is optimal to hedge equal but opposite amounts depending on whether the hedger has income shocks which are positively or negatively correlated with the asset value.

E. Out-of-Equilibrium Beliefs and Contract Payments

The contents of this subsection are purely technical but are included for the sake of completeness.

To this point we have only considered the possibility that all agents trade $\pm$x. To complete the construction of the equilibrium, it remains to verify that no agent has an incentive to deviate by trading other quantities. Recall that the market maker's belief is a function of the total quantity traded. The simplest specification of beliefs for the market maker at out-of-equilibrium quantities is that he believes the asset to be worth 1 for any positive (buy) order flow other than x or 2x, and worth 0 for any negative (sell) order flow other than x or 2x.

There are two possible deviations an agent can make. He can trade an amount different from x, but in the right direction (e.g. buy on good news). In this case the price will immediately become fully revealing. Clearly the contract payment will be designed not to induce this trading behaviour (e.g. a payment of zero). Alternatively he can trade in the wrong direction as well a different amount (e.g. sell on good news). In this case the price will be wrong, but so will his position (e.g. he will be short an undervalued asset). Again, the contract will clearly be designed to avoid inducing this.

Finally, the hedgers have no incentive to deviate by definition of x. Since the hedgers are a continuum of infinitesimal agents, an individual cannot affect the aggregate trading volume, and so cannot change the market maker's beliefs (see Section II.B). The quantity x is derived under precisely this assumption (Appendix 1).

F. The Return on the Delegated Portfolio

It remains to verify that the principal will indeed employ an agent as portfolio manager. If he does employ the agent as portfolio manager, he will earn a higher return than if he bought and held the market. On the other hand, he will have to pay a management fee on the portfolio. We now compute the expected return on the delegated portfolio.

When the portfolio manager buys, the price will be either p_0 (if the hedger arrives and sells) or p_B otherwise. The expected price is then:

$$\tfrac{1}{2}\delta p_0 + (1 - \tfrac{1}{2}\delta)p_B.$$

When the portfolio manager sells the expected price is:

$$\tfrac{1}{2}\delta p_0 + (1 - \tfrac{1}{2}\delta)p_S.$$

If the portfolio manager is informed, he buys when the security is worth H and sells when it is worth L, earning:

$$\tfrac{1}{2}[H - \tfrac{1}{2}\delta p_0 - (1 - \tfrac{1}{2}\delta)p_B] + \tfrac{1}{2}[\tfrac{1}{2}\delta p_0 + (1 - \tfrac{1}{2}\delta)p_S - L]$$

per share traded. If the portfolio manager is uninformed, he will pay the same expected prices but his orders will be uncorrelated with the value of the security:

$$[(\tfrac{1}{2}L + \tfrac{1}{2}H) - \tfrac{1}{2}\delta p_0 - (1 - \tfrac{1}{2}\delta)p_B] + [\tfrac{1}{2}\delta p_0 + (1 - \tfrac{1}{2}\delta)p_S - (\tfrac{1}{2}L + \tfrac{1}{2}H)]$$

per share traded. Thus, the expected earnings of the principal, net of management fees (expected value k), are:

$$x\,[\alpha\,\{\tfrac{1}{2}[H - \tfrac{1}{2}\delta p_0 - (1 - \tfrac{1}{2}\delta)p_B] + \tfrac{1}{2}[\tfrac{1}{2}\delta p_0 + (1 - \tfrac{1}{2}\delta)p_S - L]\}$$

$$+ (1 - \alpha)\,\{\tfrac{1}{2}[(\tfrac{1}{2}L + \tfrac{1}{2}H) - \tfrac{1}{2}\delta p_0 - (1 - \tfrac{1}{2}\delta)p_B] +$$

$$\tfrac{1}{2}[\tfrac{1}{2}\delta p_0 + (1 - \tfrac{1}{2}\delta)p_S - (\tfrac{1}{2}L + \tfrac{1}{2}H)]\}\,] - k$$

$$= x\,[\tfrac{1}{2}(1 - \tfrac{1}{2}\delta)(p_S - p_B) + \tfrac{1}{2}\alpha(H - L)] - k.$$

Substituting for the prices, we obtain:

$$\tfrac{1}{4}\delta\alpha(H - L)x - k. \tag{6}$$

If this expression is positive, the delegated portfolio management arises in equilibrium: it will be worthwhile for the principal to hire the agent. Clearly, this is more likely to hold if: (1) hedging demand is large (δx large); (2) the chance of a portfolio manager obtaining information is high (α large); (3) the variance of the security value is high ($H - L$ large); (4) the portfolio manager's opportunity cost is low (k small).

If the hedging demand is too small, then it is impossible to cover the fixed cost k of delegated portfolio management. This is because the superior return on a managed portfolio comes at the expense of the hedgers.

G. How Much Noise Trade Can the Market Support?

Substituting for H and L in (6) and defining $\delta^* = 4k/\alpha x$, we have that the market for portfolio money management exists as long as $\delta \geq \delta^*$. Since the expected amount of hedging trade is δx and the amount of expected noise trade is $(1 - \alpha)x$, the ratio of expected noise trade to expected hedging trade is

$$(1 - \alpha)/\delta.$$

Figure 2 illustrates this. As the amount of hedging trade, δ, falls, the ratio of noise trade to hedging trade increases. Furthermore it does so at an increasing rate, $(1 - \alpha)/\delta^2$. In this sense, a "small" amount of hedging can support a "large" amount of noise trade.

IV. Can Noise Trade Make Everybody Better Off?

Noise trading by portfolio managers reduces the profitability of an actively managed portfolio, relative to the benchmark case of direct investment. By the same token, hedgers are effectively able to insure their endowment risk at lower cost. Thus at first glance, it would appear that the noise trade resulting from the agency problem inherent in delegated portfolio management makes hedgers better off and investors worse off.

This conclusion would be too simplistic, however. Because noise trading lowers the effective cost of insurance, hedgers will respond by purchasing more. If the increase in hedging demand (x) is large enough, the investor-principal may actually earn a larger total amount. This is analogous to the standard result in consumer demand theory that, if the price elasticity exceeds 1, a price fall will cause an increase in expenditure.

If the hedgers do respond to delegated portfolio management by increasing their demand sufficiently that the profits on a managed portfolio improve (relative to the benchmark case of direct investment), then the agency problem that generates noise trade will have created a Pareto improvement: hedgers can hedge more cheaply, portfolio managers are indifferent (they are employed at a wage equal to their opportunity cost) and portfolio owners earn higher returns.

In this section, we formalize this argument. We compare an economy with delegated portfolio management to one where there are no agency problems because principals (the owners of the portfolios) have investment management talent, i.e. may become informed with probability α. We provide an example showing a Pareto improvement. Note that this conclusion could not be reached using the standard paradigm of inelastic liquidity demand.

Let x' be hedging demand in the benchmark case of direct investment (DI), and x in the delegated portfolio management case (DPM). We can derive the increase in hedging demand needed to increase the principal's profits:

Lemma 1: *Net profits for the principal are larger with delegated portfolio management than with direct investment if:*

$$x > x' (2 - \alpha - \delta)/(\alpha + \delta - 2\alpha\delta)$$

Proof: In Appendix 2 we solve for the equilibrium in the benchmark case of direct investment by the principal. There, we show that the expected profits of the trader, net of his opportunity cost **k**, are:

$$\tfrac{1}{4}\alpha x'\delta(2 - \alpha - \delta)/(\alpha + \delta - 2\alpha\delta) - \mathbf{k}. \tag{7}$$

The result follows from comparing (6) to (7). ▌

It may be verified that the coefficient $(2 - \alpha - \delta)/(\alpha + \delta - 2\alpha\delta)$ is greater than 1. Therefore a necessary condition for DPM to Pareto-dominate DI is for $x > x'$ by a sufficient margin.

We now provide an example in which hedging increases by a sufficient amount that delegated portfolio management is Pareto improving. To motivate the construction of the utility functions in the example, consider the wealth levels of hedgers in DPM and DI. We consider buying hedgers only; selling is symmetric (see the Appendices). A buying hedger in DPM will realize one of four possible wealth levels, in each of the four cases listed in section III.C:

1) probability $\frac{1}{4}(1 + \alpha)$; wealth $W + x(1 - p_B)$

2) probability $\frac{1}{4}(1 - \alpha)$; wealth $W + 1 - xp_B$

3) probability $\frac{1}{4}(1 - \alpha)$; wealth $W + x(1 - p_0)$

4) probability $\frac{1}{4}(1 + \alpha)$; wealth $W + 1 - xp_0$

In DI, a buying hedger will also realize one of four wealth levels, as given in Appendix 2.4.:

1) probability $\frac{1}{2}\alpha$; wealth $W + x'(1 - p_{+2})$

2) probability $\frac{1}{2}(1 - \alpha)$; wealth $W + 1 - x'p_{+1}$

3) probability $\frac{1}{2}(1 - \alpha)$; wealth $W + x'(1 - p_{+1})$

4) probability $\frac{1}{2}\alpha$; wealth $W + 1 - x'p_0$

In order for the investor-principal to be better off under DPM it must be that hedgers hedge sufficiently more than in DI (Lemma 1). We will construct an example where this property holds in the extreme, i.e., $x = 1$ and $x' = 0$. In other words, in DI the cost of hedging is so high that hedgers choose not to hedge at all, while in DPM they choose to hedge fully. In that case, the possible wealth levels in DPM are as follows: Cases (1) and (2), above, have wealth: $W + 1 - p_B = W + \frac{1}{2} - \frac{1}{2}\alpha$; Cases (3) and (4) reduce to: $W + 1 - p_0 = W + \frac{1}{2}$. Note that, since they hedge fully under DPM, the hedgers care about the price they are paying, but not about the value of the asset. Under DI, with $x' = 0$, the situation is reversed: the hedgers are exposed to asset risk, but not to price risk since they choose not to hedge: Cases (1) and (3) have wealth W; Cases (2) and (4) have wealth $W + 1$.

For hedgers to make these choices we need them to be very risk averse in the region from $W + \frac{1}{2} - \frac{1}{2}\alpha$ to $W + \frac{1}{2}$, and less risk averse on either side of this region. A likely candidate for such a preference is, therefore, a concave piecewise-linear utility function with kinks at $W + \frac{1}{2} - \frac{1}{2}\alpha$ and $W + \frac{1}{2}$. We will show that with such a utility function DPM is indeed Pareto-improving.

Proposition 2: *If hedgers have the following utility function:*

$u(\lambda) = \lambda$ for $\lambda < W + \frac{1}{2}(1-\alpha)$

$u(\lambda) = [W + \frac{1}{2}(1-\alpha)](1-a) + a\lambda$ for $W + \frac{1}{2}(1-\alpha) < \lambda < W + \frac{1}{2}$

$u(\lambda) = (W + \frac{1}{2})(1-b) - \frac{1}{2}\alpha(1-a) + b\lambda$ for $\lambda > W + \frac{1}{2}$

then Delegated Portfolio Management Pareto-dominates Direct Investment.

Proof: We begin by showing in DI, $x' = 0$ is optimal. Expected utility in DI is:

$$\tfrac{1}{2}\alpha W + \tfrac{1}{2}(1-\alpha)[W+x'(1-p_{+1})] +$$
$$\tfrac{1}{2}(1-\alpha)[(W+\tfrac{1}{2})(1-b) - \tfrac{1}{2}\alpha(1-a) + b(W+1-p_{+1}x')] +$$
$$\tfrac{1}{2}\alpha[(W+\tfrac{1}{2})(1-b) - \tfrac{1}{2}\alpha(1-a) + b(W+\tfrac{1}{2})]$$
$$= \tfrac{1}{2}W + \tfrac{1}{2}(1-\alpha)x'(1-p_{+1}) + \tfrac{1}{2}(W+\tfrac{1}{2})(1-b) - \tfrac{1}{4}\alpha(1-a) +$$
$$\tfrac{1}{2}bW + \tfrac{1}{2}b(1-\alpha)(1-p_{+1}x') + \tfrac{1}{4}\alpha b$$

We require that the derivative with respect to x' (from the right hand side at $x' = 0$) be negative, so:

$$\tfrac{1}{2}(1-\alpha)(1-p_{-1}) - \tfrac{1}{2}b(1-\alpha)p_{+1} < 0,$$

i.e.,

$$b > (1-p_{+1})/p_{+1} < 1.$$

Next we compute the expected utility under DPM:

$$\tfrac{1}{4}(1+\alpha)[W+x(1-p_B)] +$$
$$\tfrac{1}{4}(1-\alpha)\{[W + \tfrac{1}{2}(1-\alpha)](1-a) + a(W+1-p_Bx)\} +$$
$$\tfrac{1}{4}(1-\alpha)\{[W + \tfrac{1}{2}(1-\alpha)](1-a) + a(W+\tfrac{1}{2}x)\} +$$
$$\tfrac{1}{4}(1+\alpha)[(W+\tfrac{1}{2})(1-b) - \tfrac{1}{2}\alpha(1-a) + b(W+1-\tfrac{1}{2}x)]$$
$$= \tfrac{1}{4}(1+\alpha)W + \tfrac{1}{2}(1-\alpha)[W+\tfrac{1}{2}(1-\alpha)(1-a)] +$$
$$\tfrac{1}{4}(1+\alpha)[(W+\tfrac{1}{2})(1-b) - \tfrac{1}{2}\alpha(1-a)] +$$
$$\tfrac{1}{4}(1-\alpha)[a(W+1) + aW] + \tfrac{1}{4}(1+\alpha)b(W+1) +$$
$$x\{\tfrac{1}{4}(1+\alpha)(1-p_B) - \tfrac{1}{4}(1-\alpha)ap_B + \tfrac{1}{4}(1-\alpha)\tfrac{1}{2}a - \tfrac{1}{4}(1+\alpha)\tfrac{1}{2}b\}$$

We require that the derivative with respect to x (at $x=1$ from the left-hand side) be positive, i.e.,

$$\tfrac{1}{4}(1-\alpha)a(\tfrac{1}{2}-p_B) + \tfrac{1}{4}(1+\alpha)[1-p_B-\tfrac{1}{2}b] > 0.$$

Now, substituting for $p_B = \tfrac{1}{2}(1+\alpha)$,

$$(1-\alpha)a(-\tfrac{1}{2}\alpha) + (1+\alpha)[\tfrac{1}{2}(1-\alpha)-\tfrac{1}{2}b] > 0,$$

or,

$$b < (1-\alpha) - a\alpha(1-\alpha)/(1+\alpha).$$

It remains to verify that a and b can simultaneously satisfy this equation together with the previously derived condition under DI, as well as $0 < b < a < 1$ (see Figure 3). We now show that this is possible by taking as an example $a = b = (1-p_{+1})/p_{+1}$ (this point is circled in Figure 3). It remains to check algebraically that this point lies below the line $b = (1-\alpha) - a\alpha(1-\alpha)/(1+\alpha)$, i.e. that:

$$(1-p_{+1})/p_{+1} < (1-\alpha) - \alpha(1-\alpha)/(1+\alpha)(1-p_{+1})/p_{+1}.$$

Substituting $p_{+1} = (2\alpha + \delta - 3\alpha\delta)/2(\alpha + \delta - 2\alpha\delta)$ gives:

$$[\delta(1-\alpha)/(2\alpha+\delta-3\alpha\delta)][(1-\alpha)/(1+\alpha)] < 1.$$

Since each of the two terms in square brackets on the left hand side are less than one, the inequality is, indeed, satisfied. Notice that, by revealed preference, the hedgers are better off under delegated portfolio management, while the investor-principal is also better off since he makes zero profits under direct investment as there is no trade. This completes our example. ▌

Intuitively, this analysis may be interpreted as follows. Because of noise trading under DPM, the (percentage) return earned by the principal is lower than the return under DI. Since the principal's return comes at the expense of the hedgers, they are effectively able to purchase lower-cost insurance against their wage-income shock.

Since the "cost of insurance" is lower under DPM, the hedgers will choose larger quantities. If they amount they choose is larger by a sufficient margin, then, by Lemma 1, delegated portfolio management is a Pareto improvement. Intuitively, if the "elasticity of hedging demand" is large enough, then the lower "cost of hedging" under DPM can induce the hedgers to increase their hedging demand by so much that the total revenue of the investor is actually larger than in DI.

The reason for the decline in the cost of insurance is the presence of noise trade in DI. Of course, in a world without agency problems (DI) a principal who manages his own portfolio is free to engage in noise trade if he wants to. What lowers the "price of insurance" in a world of delegated portfolio management (DPM), however, is the credible precommitment to engage in noise trade by hiring an agent. A principal investing for himself (DI) would always prefer not to engage in noise trade, *ex post*, and thus the hedgers and the market maker would not anticipate any noise trade.

V. Discussion

In this section we discuss our results and relate them to previous work.

A. Why compare the model to the first-best world of no agency problems, if that is not a feasible alternative?

In the above analysis, we stress the interpretation that agency problems are unavoidable. In other words, the principal does not have the option of actively managing the portfolio himself (as a potentially informed investor). Thus, our welfare comparison between the world with agency problems to the unattainable world of direct investment is not a comparison of two feasible alternatives. We make the comparison simply to help understand the possible implications of the portfolio management contracting problem for economic welfare.

A possible alternative interpretation is that the portfolio owner does have the same ability to manage the portfolio as a talented manager. In this case, in our model he might (as in our example) choose to employ another manager as agent. In other words, hiring an agent might be the only way to precommit to churning. This interpretation is surely stretching the model too far. One reason why it seems implausible is that in a model with multiple principals, churning would be a public good (we discuss this further below).

B. What additional welfare implications would arise if the informativeness of asset prices mattered?

An obvious omission from our analysis is any benefit from more informative prices. It is generally accepted (although it has rarely been explicitly modelled) that more informative prices in secondary securities markets are better because they lead to more efficient resource allocation. Our model ignores this effect. At first glance, more noise makes prices less informative. If the price is used to guide resource allocation, this effect would therefore counteract the effect in our welfare example. However, as the original insight of Grossman and Stiglitz (1976, 1980) showed, more noise can allow more (costly) information production to become profitable. The overall effect is therefore ambiguous.

C. Is it surprising that an agency problem can be beneficial to the principal?

In our model the agency problem for the portfolio owner can end up (as in the example) making him better off. While this may seem paradoxical it is analogous to the fact that less information or more constraints in a decision problem can sometimes help an agent in a game-theoretic environment. This is a standard result; see for example, the discussion of Stackelberg and Cournot duopoly in Gibbons (1992).

D. Can noise trade be a public good?

In the extreme, the lemons problems caused by the presence of informed traders may cause a market to fail to exist. Indeed, our example illustrates this possibility.[10] Churning, then, may cause the market to open (as in our example). A similar problem was shown in Pagano (1989). His model may have more than one equilibrium, each with a different amount of shares in existence. In a "thin" equilibrium (i.e. a small number of shares in existence), risk-averse agents are unwilling to trade because of the risk that there will be few buyers when they need to sell. In a "thick" equilibrium this problem disappears.

Opening new markets, or keeping an existing market open, often requires subsidizing trade. This can occur in at least two ways. First, an agent may be paid to trade, or be willing to absorb a trading loss. Second, an agent making a market in a security may be willing to post prices with an unprofitably narrow bid-ask spread in order to induce uninformed trade. While in the second case there is no actual noise trade, there is a similar effect of lowering the price impact of a trade. In both cases, the smaller price response induces a larger volume of uninformed trade. In fact, subsidization of market making, rather than of noise trading, is simply a more direct method of creating liquidity.

To illustrate we give two examples. On the New York Stock Exchange, specialists are often assigned smaller, less liquid stocks in addition to the stock of a large-capitalization firm. The implicit understanding is that some monopoly profits from making a market in the large stock will be used to subsidize a lower bid-ask spread in the small-capitalization stock. Another example was Drexel Burnham Lambert's support of the junk-bond market. Apparently Drexel was willing to create liquidity in the secondary junk bond market, perhaps at a loss, in order to profit from underwriting in the primary market.

E. How does portfolio management differ from direct sale of information?

Instead of managing the portfolio, the agent could simply make trading recommendations. The principal could then manage the portfolio on the basis of this recommendation, making a payment to the agent depending on the accuracy of the recommendation. We have not directly addressed the question of how this differs from the agent combining the roles of information production and portfolio management. As discussed in subsection H, below, there is a literature which treats these two situations as economically equivalent.

Our view is that with direct sale of information it would be easier for the agent to undertake the alternative activity as well as making a false recommendation. In other words, it would be difficult to induce the agent to truly produce information because it would be too easy for him to deliver a random recommendation and engage in the alternative activity in the meantime.

Clearly there are instances of information sale as well as many cases of managed portfolios. Examples of information sale include ratings agencies, analysts' reports, financial newsletters, newspapers, Reuters and Bloomberg screens, etc. One could argue that the type of information that is most successfully sold by these media is information which cannot be generated at random. On the other hand, simple buy and sell recommendations can be generated at random with a nontrivial chance of success. Lists of buy and sell recommendations, while sometimes sold, are more likely generated internally by the organizations that act on them.

F. How do you reconcile the fact that typical portfolio management contracts pay a fixed percentage of funds under management with the premise of the model that payment for doing nothing cannot arise in equilibrium?

Fund managers who do nothing are likely to be fired. The same is true for employees. This is exactly analogous to the effect in our model, but it manifests itself slightly differently because of our one period setting.

G. How would the analysis change in a repeated setting?

In a repeated context, possibly with reputation, the agency problem would manifest itself somewhat differently, notably in the details of the contract. There is no reason to believe that the basic results would change, but the optimal contracting problem would become complex. For example, one attractive model might be a stopping problem where the payment to the agent is a fixed wage every period, but the employer can fire employees for poor performance over some interval. However, it is clear that the fixed-wage contract would be suboptimal except in a narrow class of environments. In general the optimal payment every period would depend on many variables, including in particular the agent's intertemporal consumption preferences (note that it would not be possible to improve the incentives by making the entire payment at the end of the sequence of periods because the manager would want to consume every period and has no personal resources). To avoid such inessential complications we restrict attention to the single-period setting.

H. Related Research

Our model is concerned with the effects of an agency problem on prices in a security market. The majority of the related literature has concentrated purely on the agency problem of eliciting information production and revelation. The main point of our paper is to integrate the following five features:

(i) an environment for information producers that makes it difficult for them to credibly "actively do nothing;"

(ii) an approach to the contracting problem that specifies an optimal contract given the available informational constraints;

(iii) a model of delegated portfolio management as distinct from a model of information sale;

(iv) equilibrium asset prices and trading volume that reflect this contracting problem;

(v) an explicit optimization problem for each agent so that welfare analysis is possible.

Two previous papers have derived churning as a result of the portfolio management contracting problem.

Allen and Gorton (1993) show the possibility of price bubbles and noise trade as a result of the agency problem. They study a setting where informed portfolio managers trade in one market and the uninformed managers (noise traders) trade in a market with a price bubble. The bubble is an exogenous price process whose role is to allow uninformed managers to carry out negative-present-value risky trades, as in this paper.

Trueman (1988) is a model of portfolio management where churning may occur. In his model, the relationship between manager and investor is not modelled. Instead, it is assumed that the objective function of the agent is (essentially) to maximize the posterior belief of an observer (e.g. the investor) that he will receive information. Churning occurs because if it did not, any trade would signal that the manager was informed. In other words, one cannot have a separating equilibrium where uninformed managers make no attempt to imitate the informed. This problem could be overcome by the incentives of a suitably designed contract. This issue is one of the starting point of our paper.

There are a number of other papers that consider the contracting problem of buying information from an agent. This issue is tangentially related because these models treat information sale as isomorphic to portfolio management. See the discussion in Allen (1989) and the references therein, e.g. Allen (1990), Bhattacharya and Pfleiderer (1985), Kihlstrom (1988). These models do not imply churning (i.e. reporting false signals) because the agents can effectively be punished sufficiently hard to deter churning/lying. In our model, this possibility is precluded by the (binding) constraints of limited liability and limited personal resources of the agent. These models of information sale also do not consider the effect of the agency problem on security market prices or trading volume.

In our model, unlike the above models, the direct sale of information by a potentially informed agent to an investor is different from an agreement under which the agent actually trades with the investor's portfolio. The reason for this is that, in our setting, agents can accept contracts to produce information without foregoing their reservation wage. In other words, an uninformed agent has nothing to lose by accepting such a contract. Any such contract would therefore attract a flood of incompetents.

VI. Conclusion

A portfolio manager will frequently find that the best investment policy is simply to hold the existing portfolio. In other words, to do nothing. The question is whether, in this situation, he will be able to credibly convince his client or employer that he is "actively" doing nothing. The client may instead believe that he is simply doing nothing. He may think that the portfolio manager has not spent any effort on producing information or he has no talent. Our paper describes a contractual relationship, and its economic consequences, where actively doing nothing is indistinguishable from simply doing nothing. Ultimately it is an empirical question as to when these are indistinguishable. Designing a contractual relationship for portfolio management is to a large extent a matter of maximizing this distinction.

Noise trade is a manifestation of this agency problem. Because all our agents' objectives are specified we can examine the welfare implications of this agency problem. Our example shows that noise trade, by making the market more liquid, can benefit everyone. This illustrates that welfare effects can be more subtle and more complex than is allowed by standard models with exogenous noise traders.

Appendix 1: The Hedgers' Decisions under Delegated Portfolio Management

If the hedger buys x there are four possible cases. First, note that when the asset is worth 0 his wealth, before hedging, is W + 1 and when the asset is worth 1 his wealth is W.

Case 1: There is another buy order and the asset is worth 1. Then the price is p_B. This can happen either as a result of churning, with probability $\frac{1}{4}(1 - \alpha)$, or as a result of informed buying, with probability $\frac{1}{2}\alpha$. Thus, the probability of this case is $\frac{1}{4}(1 + \alpha)$. The agent's wealth is $W + x(1 - p_B)$.

Case 2: There is another sell order and the asset is worth 0. Again the price is p_B. This can only happen if the other buy order comes from churning and the probability of this is $\frac{1}{4}(1 - \alpha)$. The wealth of the agent is $W + 1 - xp_B$.

Case 3: The other order is a sell order but the asset is worth 1. The price is p_0. The probability of this happening is $\frac{1}{4}(1 - \alpha)$ and the agent's wealth is $W + x(1 - p_0)$.

Case 4: The other order is a sell and the asset is worth 0. The price is p_0. The probability of this happening is $\frac{1}{4}(1 + \alpha)$ and the agent's wealth is $W + 1 - xp_0$.

Thus, the agent's expected utility from buying the quantity x is:

$$\tfrac{1}{4}(1 + \alpha)U(W + x(1 - p_B)) + \tfrac{1}{4}(1 - \alpha)U(W + 1 - xp_B) +$$

$$\tfrac{1}{4}(1 - \alpha)U(W + x(1 - p_0)) + \tfrac{1}{4}(1 + \alpha)U(W + 1 - xp_0).$$

Note that $p_S = 1 - p_B$ and $p_0 = \frac{1}{2}$. Substituting these into the above expression for the objective function gives the identical objective function for hedgers who want to sell shares. Therefore, hedgers who buy and sell will both choose the same amount x. Also, recall that $p_B = \frac{1}{2}(1 + \alpha)$ so that the hedgers' objective function and their choice of x are independent of δ. The reason is that the prices do not depend on δ. If a hedger arrives he loses the same amount of money (on average) independent of δ, but as δ increases the chance of a hedger arriving increases so that the amount of money lost by hedgers to informed traders increases.

The first order condition for utility maximization is:

$$\tfrac{1}{4}(1 + \alpha)U'(W + x(1 - p_B))(1 - p_B) +$$

$$\tfrac{1}{4}(1 - \alpha)U'(W + 1 - xp_B)(- p_B) +$$

$$\tfrac{1}{4}(1 - \alpha)U'(W + \tfrac{1}{2}x)\tfrac{1}{2} +$$

$$\tfrac{1}{4}(1 + \alpha)U'(W + 1 - \tfrac{1}{2}x)(-\tfrac{1}{2}).$$

Evaluating at $x = 0$ and requiring the derivative to be positive gives

$$U'(W)/U'(W+1) > (2 - \alpha)(1 + \alpha)/(2 + \alpha)(1 - \alpha),$$

which is the condition that hedgers are sufficiently risk averse for hedging to be non-zero.

Appendix 2: Equilibrium with Direct Investment

A2.1 Order Flow under Direct Investment

As before, orders must be multiples of x' since that is the amount the hedger will trade if he arrives. The market maker will observe five possible order flows:

Sell $2x'$: This occurs if there is both an informed agent who sells and a hedger who sells.

Sell x': This occurs if there is either an informed agent who sells (and no hedger) or a hedger who sells (and the agent does not receive information).

No trade: If no hedger arrives and the agent does not learn any information; or if a hedger arrives to sell and the informed agent learns that the security is of high value and buys or vice versa.

Buy x': This is symmetric to selling x'.

Buy $2x'$: This is symmetric to selling $2x'$.

A2.2 Prices and Beliefs

The market maker's beliefs (and the prices) in these five cases are:

Sell $2x'$: The true value of the security is revealed so the market maker's belief that the asset is worth 1 is zero and the price is also zero. We denote this price by p_{-2}. The probability of this event is $\frac{1}{4}\alpha\delta$.

Sell x': The probability of this event is:

$$\tfrac{1}{2}\alpha(1 - \delta) + \tfrac{1}{2}(1 - \alpha)\delta,$$

and the probability of the joint event of sell x' and the asset is valuable is:

$$\tfrac{1}{4}(1 - \alpha)\delta.$$

Therefore, the market maker's belief about the value of the asset when he observes a single sell order is:

$$\delta(1 - \alpha)/2(\alpha + \delta - 2\alpha\delta)$$

which is the price. We denote this price by p_{-1}.

No trade: This conveys no information (by symmetry) so the price, p_0, is the unconditional expectation, $\frac{1}{2}$. The probability of this event is $1 - \alpha - \delta + (3/2)\alpha\delta$.

Buy x': The probability of buying x' is

$$\tfrac{1}{2}\alpha(1 - \delta) + \tfrac{1}{2}(1 - \alpha)\delta.$$

The probability of buying x' and the asset is highly valued is:

$$\tfrac{1}{2}\alpha(1 - \delta) + \tfrac{1}{4}(1 - \alpha)\delta.$$

Therefore, the updated belief is:

$$(2\alpha + \delta - 3\alpha\delta)/2(\alpha + \delta - 2\alpha\delta)$$

which is the price p_{+1}.

Buy 2x': This is symmetric with sell 2x', i.e., the price, p_{+2}, is H (=1).

A2.3 The Investor's Expected Returns

From the point of view of an informed investor with good news, the probabilities of these possible events (and corresponding prices) are:

2 buy orders: probability $\frac{1}{2}\delta$. This happens when there is a hedger present and he buys.

1 buy order: probability $1 - \delta$. This occurs when there is no hedger.

No (net) trade: probability $\frac{1}{2}\delta$. This occurs when there is a hedger but he is selling.

If the informed investor has good news it is impossible to have net asset sales. From the point of view of an informed investor with bad news, the probabilities of two sell orders, one sell order, and no net trade are, respectively, the same.

The expected profits of the investor, therefore, are:

$$\alpha x'\{\tfrac{1}{2}[1 - (\tfrac{1}{2}\delta\cdot 1 + (1 - \delta)(2\alpha + \delta - 3\alpha\delta)/2(\alpha + \delta - 2\alpha\delta) + \tfrac{1}{2}\delta\cdot\tfrac{1}{2})]$$

$$+ \tfrac{1}{2}[(\tfrac{1}{2}\delta\cdot 0 + (1 - \delta)\delta(1 - \alpha)/2(\alpha + \delta - 2\alpha\delta) + \tfrac{1}{2}\delta\cdot\tfrac{1}{2}) - 0]\} - k$$

$$= \tfrac{1}{2}\alpha x'[1 - \tfrac{1}{2}\delta - \alpha(1 - \delta)^2/(\alpha + \delta - 2\alpha\delta)] - k.$$

$$= \tfrac{1}{4}\alpha x'\delta(2 - \alpha - \delta)/(\alpha + \delta - 2\alpha\delta) - k$$

This must be positive for the agent to forego his alternative activity, i.e.,

$$\tfrac{1}{4}\delta(2 - \alpha - \delta)/(\alpha + \delta - 2\alpha\delta) \geq k/\alpha x'.$$

A2.4 The Hedgers' Decisions

Consider a hedger whose wealth is negatively correlated with the asset value, i.e., his wealth is W + 1 when the asset is worth 0, and his wealth is W when the asset is worth 1. From the point of view of this hedger there are four possible cases when he buys.

Case 1: There is another buy order and the asset is worth 1. This can only happen if the other trader is an informed trader who knows that the asset value is 1. The price is $p_{+2} = 1$. The wealth of the hedger in this case is $W + x' - p_{+2}x'$ and the probability of this event is $\frac{1}{2}\alpha$.

Case 2: There is no other order and the asset is worth 0. Again, there will be no other order only if there is no informed trader (in which case the asset is equally likely to be worth 1 or 0). The wealth of the hedger is $W - p_{+1}x' + 1$ and the probability of this event is $\frac{1}{2}(1 - \alpha)$.

Case 3: There is no other order and the asset is worth 1. There will be no other order only if there is no informed trader in which case the asset is equally likely to be worth 1 or 0. The wealth of the hedger is $W - p_{+1}x' + x'$ and the probability of this event is $\frac{1}{2}(1 - \alpha)$.

Case 4: There is a sell order and the asset is worth 0. This occurs if the informed trader sells the asset in which case the price is p_0 and the wealth of the hedger is $W - p_0x' + 1$. This event occurs with probability $\frac{1}{2}\alpha$.

The utility of the hedger is given by:

$$\tfrac{1}{2}\alpha U[W] + \tfrac{1}{2}(1 - \alpha)U[W + 1 - x'p_{+1}]$$

$$+ \tfrac{1}{2}(1 - \alpha)U[W + x'(1 - p_{+1})] + \tfrac{1}{2}\alpha U[W + 1 - \tfrac{1}{2}x']$$

The utility of a hedger whose wealth is positively correlated with the asset value can be similarly derived. It turns out to be the same function as above so that the choice of x' for buying and selling hedgers is the same.

The derivative of expected utility with respect to x' is:

$$+ \tfrac{1}{2}(1 - \alpha)U'[W + 1 - x'p_{+1}](-p_{+1})$$

$$+ \tfrac{1}{2}(1 - \alpha)U'[W + x'(1 - p_{+1})](1 - p_{+1}) + \tfrac{1}{2}\alpha U'[W + 1 - \tfrac{1}{2}x'](-\tfrac{1}{2}).$$

Evaluating at $x' = 0$ and setting the derivative to be positive gives:

$$U'(W)/U'(W+1) > [\tfrac{1}{4}\alpha + (1 - \alpha)p_{+1}]/[(1 - \alpha)(1 - p_{+1})]$$

which is the condition for non-zero hedging in the case of direct investment. Note that $p_{+1} = (2\alpha + \delta - 3\alpha\delta)/2(\alpha + \delta - 2\alpha\delta)$; it may be verified that this condition is more stringent than the corresponding condition under delegated portfolio management (see Appendix 1).

Footnotes

1. Tobin (1988) makes this point: "... the accounts of asset markets standard in both economics and finance ... do not explain the volume of *transactions*." See also Tobin (1978).

2. Higher trading volume by intermediaries is also consistent with the explanation that they have lower transactions costs.

3. Annual data on turnover and institutional ownership were taken from the NYSE (1993) and the data on real commission costs per share were kindly provided by Harold Mulherin (see Mulherin (1990)). For the period 1955-1988 the regression results were:

$$\text{Turnover} = \underset{(0.16)}{-\ 0.40} + \underset{(0.48)}{2.58}\ \text{Institutional Ownership \%} + \underset{(9.04)}{20.00}\ \text{Real Trading Cost}$$

Standard errors in parentheses. $R^2 = 0.82$.

4. In practice, a similar effect would often arise when the manager identifies an asset as overvalued, but short sale costs prevent holding a short position.

5. The word "churning" is sometimes used in the different sense of trading to generate brokerage commissions.

6. We omit the measure theoretic representation of this statement since it is obvious.

7. In principle, there could be infinitely many other outc mes, e.g. the agent bought 25,000,000 shares rather than x, etc. It is easy to show that a zero payment in all of these outcomes is an optimal level. See Section II.E.

8. By convention, we assume that principals do not offer trivial contracts that pay zero in exchange for no work.

9. Note also that this contract is not renegotiation-proof, since a (talented) portfolio manager who (actively works and) receives no information can offer not to churn (which is costly to the principal) in exchange for a payment of at least ½m. However, principals who leave open the possibility of such renegotiation would be vulnerable to a flood of incompetent applicants for the management contract. We therefore assume that they are able to credibly precommit not to renegotiate. One could imagine a number of different ways to achieve this, e.g. the principal is a legal entity with a governing charter that precludes paying agents for doing nothing.

10. Our model was chosen to analyze the effects of delegated portfolio management with a single principal. The details cannot readily be modified to the case of many principals, particularly because of the discreteness of the hedging demand. However, the intuition that noise trade could be a public good is straightforward from our model

References

Allen, Franklin (1989), "Information Contracting in Financial Markets," in Sudipto Bhattacharya and George M. Constantinides, eds., *Financial Markets and Incomplete Information* (Rowman and Littlefield publishers; Totowa, New Jersey), p. 188-198.

Allen, Franklin (1990), "The Market for Information and the Origin of Financial Intermediation," *Journal of Financial Intermediation* 1: 3-30.

Allen, Franklin and Gary Gorton (1993), "Churning Bubbles," *Review of Economic Studies* 60(4): 813-836.

Ausubel, L. (1990a), "Partially-Revealing Rational Expectations Equilibrium in a Competitive Economy," *Journal of Economic Theory* 50: 93-126.

Ausubel, L. (1990b), "Insider Trading in a Rational Expectations Economy," *American Economic Review* 80: 1022-1041.

Bank for International Settlements (1993), Central Bank Survey of Foreign Exchange Market Activity in April 1993 (March; Basle).

Bhattacharya, S. and P. Pfleiderer (1985), "Delegated Portfolio Management," *Journal of Economic Theory* 36: 1-25.

Biais, Bruno and Pierre Hillion (1992), "Insider Trading and Stock Options Markets," Groupe HEC working paper.

Black, Fischer (1986), "Noise," *Journal of Finance* 41(3): 529-543.

De Long, Bradford, Andrei Shleifer, Lawrence Summers and Robert Waldman (1990), "Noise Trader Risk in Financial Markets," *Journal of Political Economy* 98: 703-738.

Diamond, D. and Verrechia, R. (1981), "Information Aggregation in a Noisy Rational Expectations Economy," *Journal of Financial Economics* 9: 221-35.

Dow, James and Gary Gorton (1994a), "Arbitrage Chains," *Journal of Finance* (July), forthcoming.

Dow, James and Gary Gorton (1994b), "Profitable Informed Trading in a Simple General Equilibrium Model of Asset Pricing," working paper.

Gibbons, Robert (1992), *A Primer on Game Theory* (Harvester Wheatsheaf).

Grossman, S. G. and J. E. Stiglitz (1976), "Information and Competitive Price Systems," *American Economic Review* 66: 246-253.

Grossman, Sanford and Joseph Stiglitz (1980), "On the Impossibility of Informationally Efficient Markets," *American Economic Review* 70: 393-408.

Holmstrom, Bengt (1982), "Managerial Incentive Problems: a Dynamic Perspective," in *Essays in Economics and Management in Honor of Lars Wahlbeck.*

Kihlstrom, Richard (1988), "Optimal Contracts for Security Analysts and Portfolio Managers," *Studies in Banking and Finance* 5: 291-325.

Kyle, Albert (1985), "Continuous Auctions and Insider Trading," *Econometrica* 53: 1315-1335.

Mulherin, Harold J. (1990), "Regulation, Trading Volume and Stock Market Volatility," Revue Economique 41(5) (September), 923-937.

New York Stock Exchange (1993), Fact Book for the Year 1992 (New York Stock Exchange; New York).

Pagano, Marco (1989), "Endogenous Market Thinness and Stock Price Volatility," *Review of Economic Studies* 56(2): 269-287.

Presidential Task Force on Market Mechanisms, January 1988.

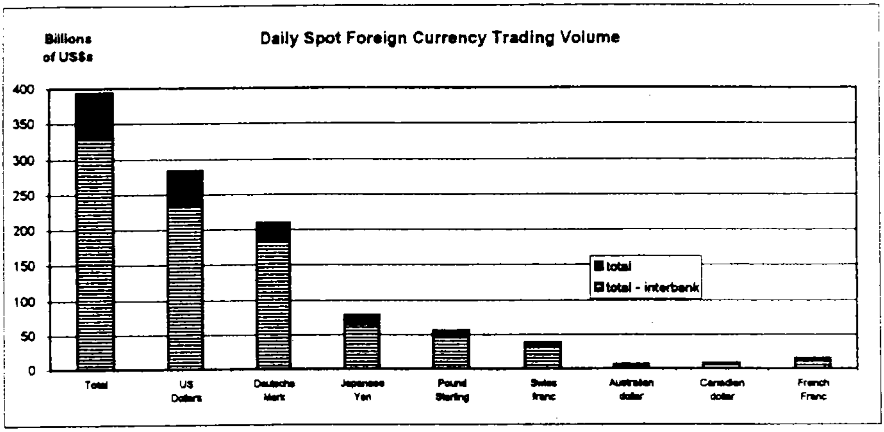
Ross, Stephen A. (1989), "Discussion: Intertemporal Asset Pricing," in *Theory of Valuation*, edited by Sudipto Bhattacharya and George M. Constantinides (Rowman and Littlefield publishers; Totowa, New Jersey), pp. 85-96.

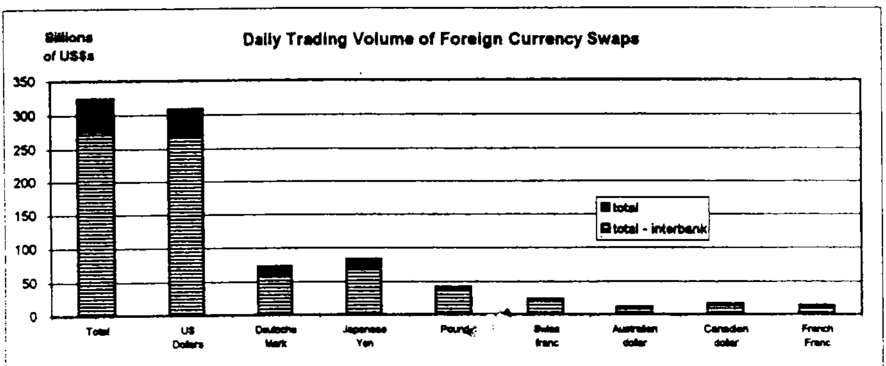
Tobin, James (1988), "Overview," Financial Market Volatility (Federal Reserve Bank of Kansas City).

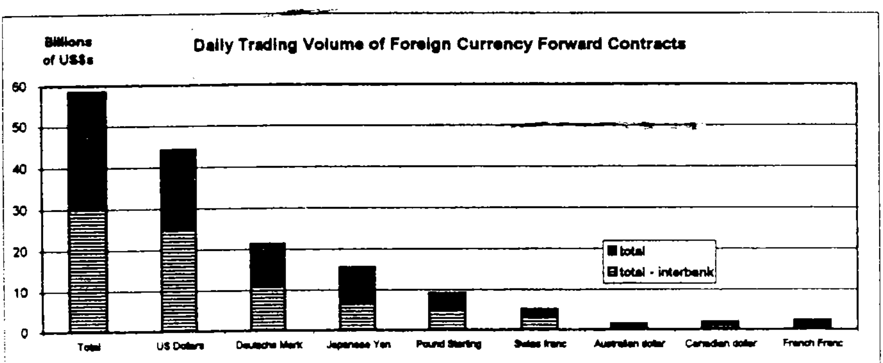
Tobin, James (1978), "A Proposal for International Monetary Reform," Eastern Economic Journal (July).

Trueman, Brett (1888), "A Theory of Noise Trading in Securities Markets," *Journal of Finance* 43(1): 83-95.

Figure 1: Foreign Exchange Trading Volume







Source: Bank for International Settlements (1993).

Figure 2

Ratio of Noise Trade to Hedging Trade

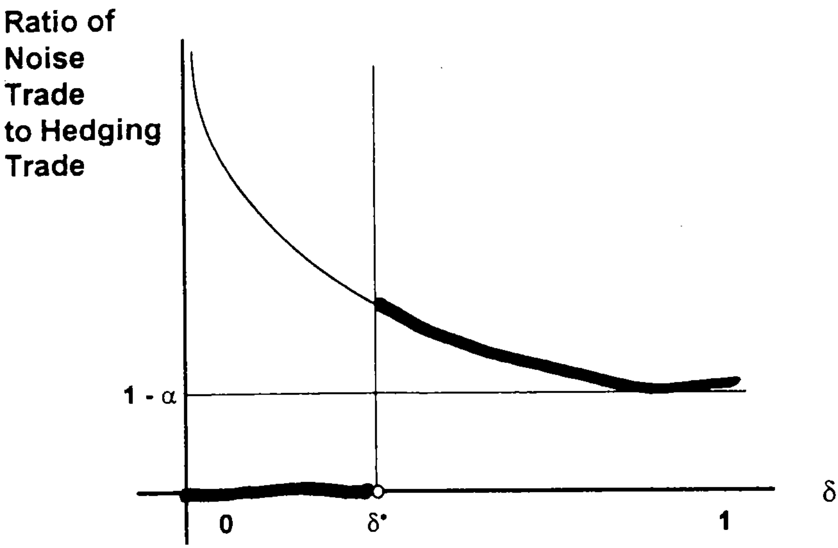


Figure 3

Parameter Region Where Delegated Portfolio Management Dominates Direct Investment