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A TIME TO SOW AND A TIME TO REAP: GROWTH BASED ON GENERAL PURPOSE TECHNOLOGIES

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ABSTRACT

We develop a model of growth driven by successive improvements in "General Purpose Technologies" (GPT's), such as the steam engine, electricity, or micro-electronics. Each new generation of GPT's prompts investments in complementary inputs, and impacts the economy after enough such compatible inputs become available. The long-run dynamics take the form of recurrent cycles: during the first phase of each cycle output and productivity grow slowly or even decline, and it is only in the second phase that growth starts in earnest. The historical record of productivity growth associated with electrification, and perhaps also of computerization lately, may offer supportive evidence for this pattern. In lieu of analytical comparative dynamics, we conduct simulations of the model over a wide range of parameters, and analyze the results statistically. We extend the model to allow for skilled and unskilled labor, and explore the implications for the behavior over time of their relative wages. We also explore diffusion in the context of a multi-sector economy.

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1 Introduction

In any given "era" there typically exist a handful of technologies that play a far-reaching role in fostering technical change in a wide range of user sectors, thereby bringing about sustained and pervasive productivity gains. The steam engine during the first industrial revolution, electricity in the early part of this century and micro-electronics in the past two decades are widely thought to have played such a role. Bresnahan and Trajtenberg (1994) refer to them as "General Purpose Technologies" (GPTs hereafter)

GPTs are characterized by the following features: (a) They are extremely pervasive; that is, they are used as inputs by a wide range of sectors in the economy. This stems from the fact that GPTs perform some generic function (such as *continuous rotary motion* for the steam engine, or *binary logic* for micro-electronics) that happens to have virtually universal applicability throughout the economy.[1] (b) The potential for continuous technical advances in the GPT itself, which manifests itself *ex post* as sustained improvements in performance. (c) The presence of complementarities with the user sectors, which can arise in manufacturing or in the R&D technology.

These features provide a mechanism by which the GPTs play a role as "engines of growth": as a better GPT becomes available it gets adopted by an increasing number of user sectors and it fosters complementary advances that raise the attractiveness of its adoption. For both reasons the demand for the GPT increases, inducing further technical progress in the GPTs, which prompts in turn a new round of advances downstream, and so forth. As the use of a GPT spreads throughout the economy its effects become significant at the aggregate level, thus impacting overall growth.

We study in this paper the economy-wide dynamics that a GPT may generate. For this purpose we embody the notion of GPTs, as developed in Bresnahan and Tra-

[1] The range of applicability of a given generic function is of course a function of the state of the art in the technologies of the user sectors, hence the time dependence of this feature (binary logic could have hardly had wide applicability before the advent of electricity).

jtenberg (1994), into a growth model á la Romer (1990) and Grossman and Helpman (1991, ch. 3). Since a full-fledged general equilibrium model of GPTs proves to be exceedingly complex and intractable, however, we analyze a scaled-back version in which advances in the GPTs are exogenous. As a result we ignore the feedback from user sectors to the GPTs.

We refer to each GPT as prompting the development of "compatible components", but of course these can be any sort of inputs or, more generally, complementary investments of any kind. It may help to visualize it by thinking of GPTs as being, say, computers, and the components as compatible software packages. Or, the GPTs being successive generations of integrated circuits, and the "components" the other parts of the appliances and instruments that incorporate those circuits.

We present the basic GPT-based growth model in the next section. In Section 3 we analyze the long-run dynamics, in the form of repetitive cycles, that result from the arrival of new GPTs within fixed time intervals. The consequent behavior over time of GDP, total factor productivity, real wages and factor shares, are described in Section 4. A number of important results emerge there. First, the immediate impact of the arrival of the new, more productive GPT, is to lower output. Second, a typical cycle contains two distinct phases. During the first phase output and productivity experience negative growth, the real wage rate stagnates, and the share of profits in GDP declines. The benefits from a more advanced GPT manifest themselves during the second phase, after enough complementary inputs have been developed for it. During this later phase there is a spell of growth, with rising output, real wages and profits. Over the entire cycle the economy grows at the rate determined by the rate of advance in the GPT itself.

The growth path thus obtained in the model seems to correspond to the historical record of productivity growth following the introduction of electricity a century ago, and may likewise resemble the economy-wide impact of computerization of the last few decades. In section 5 we present computer simulations of the model over a wide range of parameters. The statistical analysis of these simulation results provides estimates of the relationships between key endogenous variables (primarily the rate of growth of GDP over different phases and the length of the low growth phase) and the economy's basic parameters, such as the elasticity of substitution between complementary inputs. This type of analysis can be seen as fulfilling the role of "quasi comparative dynamics".

In Section 6 we present two modifications that shed light on additional issues. First we show how to extend the model to a multiple sector economy in order to examine the role of diffusion in the growth process. In this case the acceleration of growth in the second phase is driven by both the development of complementary inputs and by the gradual diffusion of the new GPT throughout the economy. Second, we examine the role of skilled and unskilled workers, where R&D is skill intensive relative to manufacturing. An economy with two types of labor may still follow the aggregate dynamics that were previously described, except that now output need not decline over time following the arrival of a new general purpose technology. Moreover, we show that during the first phase of slow growth real wages of unskilled workers stagnate while real wages of skilled workers increase. On the other hand, in the second phase of accelerated growth unskilled workers make real income gains while skilled workers may experience real gains in the early part of the phase and a loss at the end of the entire cycle. We provide concluding remarks in the closing section.

2 Building Blocks

Suppose that a final good is produced with the aid of a General Purpose Technology (GPT) i and an assembly of a continuum of components $x_i(j)$, $j \in [0, n_i]$, that have to be compatible with the particular GPT in use, where n_i denotes the number (measure) of available components. The production function is given by

$$Q_i = \lambda^i D_i, \quad \lambda > 1, \tag{1}$$

where λ^i stands for the productivity level of GPT i, and

$$D_i = \left[\int_0^{n_i} x_i(j)^\alpha dj\right]^{1/\alpha}, \quad 0 < \alpha < 1. \tag{2}$$

The elasticity of substitution between any two components is thus $1/(1-\alpha) > 1$. We assume that GPTs become available in an ordered fashion from $i = 1$ onward. Therefore with m available GPTs aggregate output of final goods equals $Q = \sum_{i=1}^m Q_i = \sum_{i=1}^m \lambda^i D_i$.

Each available component is supplied by a firm that owns the property right to the component's blueprint, and all blueprints are the results of past R&D efforts. Suppliers engage in monopolistic competition. All components can be manufactured with one unit of labor per unit output, independently of the GPTs with which they are compatible. Consequently marginal costs equal the wage rate w.

As is well known (see Grossman and Helpman [1991, ch. 3]), (2) implies constant elasticity demand functions,

$$x_i(j) = \frac{p_i(j)^{-1/(1-\alpha)} D_i}{\left[\int_0^{n_i} p_i(j)^{-\alpha/(1-\alpha)} dj\right]^{1/\alpha}}, \quad j \in [0, n_i],$$

where $p_i(j)$ is the price of component j for the i^{th} general purpose technology. Under these circumstances each component manufacturer equates marginal revenue to marginal costs, and hence all components will be equally priced according to

$$p_i(j) = p = \frac{1}{\alpha} w. \tag{3}$$

It follows that all components for the i^{th} GPT are employed in equal quantities x_i, and that in equilibrium

$$D_i = n_i^{(1-\alpha)/\alpha} X_i, \tag{4}$$

where $X_i \equiv n_i x_i$ represents aggregate employment of components by users of the i^{th} GPT. Given that a unit of labor is required for the manufacturing of a unit of components, however, X_i also represents labor employment in the manufacturing of components for the i^{th} general purpose technology.

It is clear from (1)-(4) that total labor input per unit of final output for the i^{th} GPT is $b_i \equiv X_i/Q_i = \lambda^{-i} n_i^{-(1-\alpha)/\alpha}$. This labor input is lower for GPT i the more components are available for i. Evidently, a general purpose technology is more valuable the more compatible components have been developed for its use.

Competitive suppliers of the final output minimize unit manufacturing costs of Q. Therefore they choose to manufacture with those general purpose technologies whose productivity level λ^i in combination with the number of available components n_i yield the lowest unit costs. Thus the price of final output equals

$$p_Q = \frac{1}{\alpha} wb, \quad \text{where } b = \min_{1 \leq i \leq m} b_i, \quad b_i \equiv \lambda^{-i} n_i^{-(1-\alpha)/\alpha}. \tag{5}$$

A technology whose unit costs exceed wb/α is not used, which implies that manufacturers of components that are compatible with this technology have no sales and make no profits.

Suppose that at first there is just one GPT, $m = 1$, and that suitable complementary inputs for it have been developed. Then a second generation GPT appears. However, in order for the second generation GPT to be used in production, appropriate complementary inputs have to be developed (the previous inputs are not compatible). The switch to the second generation GPT will occur only after "enough" such inputs have been developed; i.e., only after $n_2/n_1 > 1/\lambda^{\alpha/(1-\alpha)}$ (see (5)). In general, there will be a switch from the i^{th} GPT to the $(i+1)^{th}$ as soon as,

$$n_{i+1} > \eta\, n_i, \quad 0 < \eta \equiv 1/\lambda^{\alpha/(1-\alpha)} < 1. \tag{6}$$

Now, if Q_i units of final output are produced with the i^{th} general purpose technology, then its users employ $x_i = b_i Q_i / n_i$ units of each one of the n_i components. In view of the pricing equation (3) this implies that each component yields a profit stream

$$\pi_i = \frac{(1-\alpha) w b_i Q_i}{\alpha n_i}. \tag{7}$$

These profits equal zero whenever $b_i > b$ (see (5)); i.e., whenever GPT i requires labor input per unit output that is not the lowest.

Next suppose that each firm possesses indefinite monopoly power in the supply of its component. Then the value of a firm equals the present value of its profits,

$$v_i(t) = \int_t^\infty e^{-R(t,\tau)}\pi_i(\tau)d\tau, \tag{8}$$

where $R(t,\tau) = \int_t^\tau r(z)dz$ stands for the discount factor from time τ to t and r is the interest rate. Differentiation with respect to t yields the no-arbitrage condition

$$\frac{\pi_i}{v_i} + \frac{\dot{v}_i}{v_i} = r. \tag{9}$$

This condition is satisfied at each point in time for each existing component.

We assume that the R&D process rendering new components over a time interval of length dt takes the form $\dot{n}_i dt = (l_i/a)dt$, where l_i is the amount of labor devoted to the development of new components for the i^{th} GPT. Each new component is worth v_i in present value terms. Therefore the employment of l_i workers in product development over a time interval of length dt generates the value $v_i l_i/a$ per unit time at a cost of wl_i per unit time. Net value maximizing investors in R&D will thus abstain from product development whenever $v_i < wa$. On the other hand, $v_i > wa$ provides infinite profit opportunities, which cannot prevail in equilibrium. It follows that free entry into R&D implies the equilibrium condition

$$v_i \leq wa, \quad \text{with equality whenever } \dot{n}_i > 0. \tag{10}$$

Evidently, if new components are developed at all, they are developed for those GPTs that have the highest valued components. As a result, whenever it is profitable to develop components for the i^{th} GPT it is not profitable to develop components for older general purpose technologies.

In this type of economy labor demand arises from two sources: product development, which uses $a\dot{n}_i$ units of labor, and the manufacturing of components, which requires b_iQ_i. Therefore an economy with m general purpose technologies and L units of labor faces the resource constraint

$$a\sum_{i=1}^{m}\dot{n}_i + \sum_{i=1}^{m} b_iQ_i = L. \tag{11}$$

In this equation the flow of new products $\dot{n}_i$ has to be non-negative for all i.

To complete our model we need to specify intertemporal preferences and the appearance of new general purpose technologies. We will deal with the latter in the next section. As for preferences, we assume that at each point in time t consumers allocate consumption over time so as to maximize a logarithmic intertemporal utility function $\int_t^\infty \exp(-\rho\tau)\log C(\tau)d\tau$, where ρ is the subjective discount rate and $C(\tau)$ is consumption at time τ. Consumers of this type, who face an intertemporal budget constraint according to which the present value of consumption equals the present value of income plus the value of initial asset holdings, allocate consumption according to the rule $\dot{C}/C = r - \dot{p}_Q/p_Q - \rho$. And in equilibrium $C = \sum_{i=1}^{m} Q_i$. We normalize total consumer spending to equal a constant value E at each point in time, and we choose for simplicity $E = 1$. This normalization entails no loss of generality. In this event the value of output equals one at each point in time, which together with the differential equation for consumption implies that the nominal interest rate r equals the subjective discount rate. Namely,

$$p_Q(t)\sum_{i=1}^{m} Q_i(t) = 1 \quad \text{and} \quad r(t) = \rho \quad \text{for all } t. \tag{12}$$

For further details about this specification see Grossman and Helpman (1991, ch. 3).

3 Dynamics and the Impact of New GPTs

Our analysis of the system's dynamics will focus on the wage rate and on the number of available components. This section is in three parts. First we discuss the arrivals of new general purpose technologies and the phases of typical long-run cycles. Then we characterize the motion of the wage rate and the number of components within each phase separately. Finally, we specify the equilibrium links between phases and the resulting global dynamics.

3.1 Arrivals of new GPTs

Suppose that new GPT's arrive at predetermined time intervals of equal length $\Delta = T_i - T_{i-1}$, where T_i represents the time at which the i^{th} general purpose technology becomes available. We will refer to each such time interval as a "cycle", and to sub-periods within it as "phases" of the cycle. In what follows we focus on long-run equilibria in which each cycle looks the same, except for suitable factors of proportionality whose precise nature we explain below. We assume that the development of components for a particular GPT cannot start before the GPT is actually introduced.[2] There are two possible scenarios in this case, depending on how long Δ is: (i) that a moment before T_{i+1} components for GPT i are still being developed, and hence the appearance of the new GPT brings to an abrupt end those developments; and (ii) that the development of the components compatible with the i^{th} GPT has ceased before T_{i+1}, and hence after a hiatus when there is no product development, R&D for the new components starts at T_{i+1}.

In case (i) the sequence of events is as described in panel (a) of Figure 1. Recall from (6) that production with GPT $i+1$ becomes profitable as soon as $n_{i+1} \geq \eta\ n_i$. We denote with Δ_1 the length of time that elapses from the arrival of a new GPT up to the point in time at which it becomes profitable to switch to the new GPT in the production of components. During this time interval the *old* GPT is used to manufacture final output while R&D is used to develop components for the *new* GPT. We call this initial period "phase 1". In phase 2, which is of length $\Delta_2 \equiv \Delta - \Delta_1$,

[2]Clearly, that is not always the case in reality (e.g., some software for a new operating system is developed before the launching of the latter). In this case, however, the developers of components need to know enough about the upcoming GPT in order to start their own R&D early on. This raises important issues regarding the flows of information between the GPT and the user sectors, that may impinge on the growth process (see Bresnahan and Trajtenberg, 1994).

the old GPT is superseded by the new one in production while the development of components continues for the new GPT. Since we assume that Δ and λ (and hence η) are constant, so will be Δ_1 and Δ_2 in a stationary long-run equilibrium.

Panel (b) of Figure 2 describes case (ii). Unlike case (i), here the development of components for the i^{th} GPT ceases at $T_i + \Delta_n < T_{i+1}$ rather than at T_{i+1}, while other features remain the same as in case (i). It follows that case (ii) has in addition to phases 1 and 2 a third phase in the time interval $[T_i + \Delta_n, T_{i+1})$, during which production takes place with the new GPT and no R&D takes place whatsoever.

3.2 Phase one

Consider the time interval $[T_i, T_i + \Delta_1)$ during which producers of final output employ the $i - 1^{th}$ general purpose technology, but innovators invest resources in order to develop components for the i^{th} GPT. During this phase the number of components compatible with the old GPT remains constant at the level $n_{i-1}(T_i)$ and profits for the upstarts equal zero. In this event the no-arbitrage condition (9) together with the free entry condition (10) and the normalization (12) imply

$$\frac{\dot{w}}{w} = \rho \quad \text{for} \quad t \in [T_i, T_i + \Delta_1).$$

During this phase innovators make no profits, but engage in innovation nevertheless, because they expect profits in the future and they are indifferent as to when to innovate, since R&D costs are rising at the rate of interest rate. As a result the capital value of R&D costs is the same at each point in time. In addition, (5), (11) and (12) imply

$$\dot{n}_i = \frac{1}{a}\left(L - \frac{\alpha}{w}\right), \quad \text{for} \quad w \geq \frac{\alpha}{L},\ n_i(T_i) = 0 \quad \text{and} \quad t \in [T_i, T_i + \Delta_1).$$

The condition on the wage rate ensures non-negative employment in R&D. Given an initial wage rate $w(T_i)$, these differential equations yield the following solutions for the evolution of the wage rate and the number of products during phase 1:

$$w(t) = w(T_i)e^{\rho(t-T_i)} \quad \text{for} \quad t \in [T_i, T_i + \Delta_1), \tag{13}$$

$$n_i(t) = \frac{L}{a}(t - T_i) - \frac{\alpha}{\rho a w(T_i)}\left[1 - e^{-\rho(t-T_i)}\right] \quad \text{for} \quad t \in [T_i, T_i + \Delta_1). \tag{14}$$

Figure 2 describes two feasible trajectories of system (13)-(14) (the higher trajectory begins with a higher initial wage rate). As can be seen, both the wage rate and the number of products rise over time along these trajectories.

3.3 Phase two

Now suppose that we are in the time interval $[T_i + \Delta_1, T_i + \Delta_e)$, where $\Delta_e = \Delta$ if case (i) applies and $\Delta_e < \Delta$ if case (ii) applies. In either case during this phase GPT i has the lowest unit labor requirement coefficient b_i and all manufacturers of final output employ this technology. In addition, it is profitable to keep investing in the development of new components for this general purpose technology and no R&D takes place for components of other GPTs. In this event (10) is satisfied with equality, which together with (5), (7), (9) and (12) imply

$$\dot{w} = \rho w - \frac{1-\alpha}{a n_i}, \quad \text{for} \quad t \in [T_i + \Delta_1, T_i + \Delta_e). \tag{15}$$

This differential equation for the wage rate holds as long as the i^{th} GPT remains the best practice technology and new components can profitably be developed for its use. Next observe that as long as i remains the lowest cost GPT, (5), (11) and (12) imply as before,

$$\dot{n}_i = \frac{1}{a}\left(L - \frac{\alpha}{w}\right), \quad \text{for} \quad w \geq \frac{\alpha}{L} \quad \text{and} \quad t \in [T_i + \Delta_1, T_i + \Delta_e). \tag{16}$$

Equations (15) and (16) describe an autonomous system of differential equations whose motion we depict in Figure 3. The hyperbola $\dot{w} = 0$ describes the rest points of (15) while the horizontal line $\dot{n} = 0$ describes the rest points of (16). The shaded area below the horizontal line identifies a region in which investment in R&D is negative. Therefore this region is not feasible. Three arrowed curves describe dynamic trajectories. The intermediate trajectory is a saddle path that converges to the stationary point A. If no new GPTs are expected to become available, then this saddle path is the unique equilibrium trajectory that satisfies the valuation equation (8) and the intertemporal budget constraint (see Grossman and Helpman [1991, ch. 3]). If, on the other hand, new GPTs are expected to appear in the future, as described in section 3.1, then the equilibrium trajectory depends on the length of a cycle and on how much better a new technology is relative to the old.

3.4 Phase three

In phase 3 (applicable to case ii only) there is no R&D, all resources are employed in manufacturing, and all manufacturers of final output employ the newest general purpose technology. In this event the number of components is constant and so is the wage rate.

3.5 Global Dynamics

We proceed now to examine separately each of the two cases identified in section 3.1. The analysis throughout is predicated on the assumption of perfect foresight.[3]

3.5.1 Case (i)

Recall that in this case a cycle consists of two phases. In the first phase final output is manufactured with the old general purpose technology and innovators develop components for the new GPT. The evolution of the wage rate and the number of components of the new GPT follows the pattern that we discussed in section 3.2, namely,

[3]There also exists a degenerate equilibrium with no product development for new general purpose technologies. Suppose that when a new GPT i arrives, every potential innovator expects that no one will invest in R&D in order to develop components for its use; i.e., everyone expects $n_i(t) = 0$ for all time periods. As a result it does not pay to invest in the development of a single component, because the new GPT will never be used in manufacturing. In this event the pessimistic expectations are self fulfilling and no new GPTs are implemented. This is an expectations driven equilibrium that leads to stagnation, but it is a decentralized equilibrium nevertheless. We do not discuss these types of equilibria in what follows.

both rise over time. In the second phase, after sufficiently many complementary inputs have been developed for the new general purpose technology, manufacturers of final output switch to the new GPT while innovators continue to develop components for the new GPT. It follows that in the second phase, when the new GPT dominates both production and product development, the dynamics of the wage rate and the number of components (for the new GPT) follow the pattern described in section 3.3. It leaves open the question which of the many possible trajectories prevail in equilibrium. To answer this question we need to identify appropriate links between the two phases, to which we turn next.

Consider a typical cycle, one that starts at T_i and ends at T_{i+1}. At T_i the i^{th} general purpose technology becomes available. At that point in time the number of components that have been developed for the $i-1^{th}$ GPT has reached its peak $n_{i-1}(T_i)$, and innovators switch their efforts to develop complementary inputs for the new i^{th} GPT. The evolution of $n_i(t)$ is described in panel (a) of Figure 4. Starting from zero at T_i, the number of components for the new GPT grows over time until a still newer general purpose technology appears at T_{i+1}. From that point on the number of components n_i remains constant at the level $n_i(T_i)$, while innovators switch to develop components for the $(i+1)^{th}$ GPT.

Panel (b) of Figure 4 describes the evolution of profits. Profits π_i equal zero as long as manufacturers of final output employ the $(i-1)^{th}$ GPT. This changes at $T_i + \Delta_1$, at which point the number of components available for the i^{th} GPT, $n_i(T_i + \Delta_1)$, makes manufacturers of final output just indifferent between the old and the new general purpose technology. From (6) this implies

$$n_i(T_i + \Delta_1) = \eta n_{i-1}(T_i). \tag{17}$$

From this point on profits become positive for suppliers of components for the i^{th} GPT. Since profits are (see (5), (7) and (12))

$$\pi_i(t) = \frac{1-\alpha}{n_i(t)} \quad \text{for} \quad t \in [T_i + \Delta_1, T_{i+1} + \Delta_1), \tag{18}$$

these profits fall as the number of components compatible with the new GPT rises over time. The rise of n_i during phase 2 is brought to a halt with the appearance of a still newer GPT at T_{i+1}. From this point on π_i remains constant while innovators develop complimentary inputs for the $i+1^{th}$ GPT, and profits drop to zero when enough inputs have been developed to induce manufacturers of final output to abandon the i^{th} general purpose technology and to adopt the $i+1^{th}$. They remain zero thereafter.

Consider a manufacturer of a component for the $i-1^{th}$ GPT. She knows that her profits will remain constant at the level $(1-\alpha)/n_{i-1}(T_i)$ over the time interval $[T_i, T_i+\Delta_1)$ and drop to zero thereafter, as described above. In this event (8), (12) and (18) imply that the value of her firm will be

$$\begin{aligned} v_{i-1}(t) &\equiv \int_t^{T_i+\Delta_1} e^{-\rho(\tau-t)}\pi_{i-1}(\tau)d\tau \\ &= \tfrac{(1-\alpha)}{\rho n_{i-1}(T_i)}\left[1-e^{-\rho(T_i+\Delta_1-t)}\right] \quad \text{for} \quad t\in[T_i, T_i+\Delta_1). \end{aligned} \tag{19}$$

Now, in order for the development of the "last" component for GPT $i-1$ at time T_i^- to have taken place, it must have been that $v_{i-1}(T_i^-)=w(T_i^-)a$ (see (10)). This "free entry" condition together with (19), and the continuity of $v_{i-1}(t)$ that follows from (8), imply

$$w(T_i^-)=\frac{(1-\alpha)}{\rho a\; n_{i-1}(T_i)}(1-e^{-\rho\Delta_1}). \tag{20}$$

This equation then establishes the wage rate that prevails at the end of a cycle.

Just before and just after $T_i+\Delta_1$ (the point in time at which producers of final output switch to the i^{th} general purpose technology) innovators invest in R&D in order to develop components for the i^{th} GPT. In this event $v_i(t)=w(t)a$ just before and just after $T_i+\Delta_1$. Since by (8) the value of a firm is continuous in time, it follows that the wage rate is also continuous at $T_i+\Delta_1$. Equation (13) implies then

$$w(T_i+\Delta_1)=w(T_i)e^{\rho\Delta_1}. \tag{21}$$

In addition, (14) implies

$$n_i(T_i+\Delta_1)=\frac{L}{a}\Delta_1-\frac{\alpha}{\rho a w(T_i)}\left(1-e^{-\rho\Delta_1}\right). \tag{22}$$

Now we have a complete system. It consists of two sets of dynamic equations (13)–(14) and (15)–(16) for the wage rate and the number of components, the initial conditions $n_i(T_i) = 0$, and the connecting equations (17), (20), (21) and (22). The system has a long-run stationary equilibrium with cycles of constant length that are depicted in Figure 5, in which Δ_1 is constant, $w(T_i) = w(T_{i-1})$, $w(T_i^-) = w(T_{i-1}^-)$, and $n_i(T_{i+1}) = n_{i-1}(T_i)$.[4] In the first phase of each cycle the wage rate rises and it declines in the second phase. With the appearance of a new general purpose technology the wage rate jumps upwards, which makes it unprofitable to further invest in the development of components for the old GPT (the old GPT is at the margin of profitability before the appearance of the new one). If the appearance of a new GPT does not bid up wages then case (ii) rather than (i) applies.

In order to compute the stationary long-run equilibrium we seek a fixed point in (w, n) space. Begin by specifying initial values for the wage rate at the beginning of a cycle $w(T_i) = w_0$, and for the number of components at the end of a cycle $n_{i-1}(T_i) = n_{\max}$. Using these initial values calculate $w(T_i + \Delta_1)$, $n_i(T_i + \Delta_1)$, and Δ_1 from (17), (21) and (22). Then apply the differential equations (15) and (16) for a time interval of length $\Delta - \Delta_1$, starting with the initial values $w(T_i + \Delta_1)$ and $n_i(T_i + \Delta_1)$, in order to compute $w(T_{i+1}^-)$ and $n_i(T_{i+1})$. If $n_i(T_{i+1}) = n_{\max}$ and $w(T_{i+1}^-)$ satisfies (20); i.e.,

$$w(T_{i+1}^-) = \frac{(1-\alpha)}{\rho a\, n_{\max}}(1 - e^{-\rho\Delta_1}),$$

then we have found a fixed point. Otherwise adjust the initial values and repeat the previous steps until you find a fixed point. Once a fixed point has been found we can use the initial values $(w_0, n_{\max})$ to calculate a representative cycle of the long-run equilibrium trajectory.

[4]The stationarity conditions for the span of the first phase and the number of components are independent of normalization. On the other hand, the stationarity conditions on the wage rate result from our normalization (12). Alternatively, and independently of normalization, we could have expressed the stationarity conditions on the wage rate in terms of real rather than nominal wages.

3.5.2 Case (ii)

This case is similar to case (i), except that at the end of phase 2 the wage rate has to equal α/L and it remains at this level for the rest of the cycle (this wage rate clears the labor market in the absence of employment in R&D). In this event $w(T_{i+1}^-)$ is known to equal α/L. Therefore, instead of $w(T_{i+1}^-)$ we now calculate the length of phase 2, Δ_n, which is shorter than $\Delta - \Delta_1$. This is done by running the differential equations (15)–(16) from the initial values $w(T_i + \Delta_1)$ and $n_i(T_i + \Delta_1)$ until the wage rate drops to α/L.

4 Dynamics of Key Economic Aggregates

As mentioned in the introduction, our prime goal is to understand the role of GPTs and of complementary investments in economic growth. Thus we proceed now to analyze the dynamic behavior of output and of other economic aggregates.

4.1 GDP Growth

The real gross domestic product represents a standard measure of an economy's real output. GDP can be measured either on the output or on the input side (both lead of course to the same result). In our case GDP consists of wages plus profits on the input side. From (5) and (7) profits equal the fraction $1 - \alpha$ of the value of final output $p_Q Q$, and the latter equals one by our normalization (12). It follows that nominal GDP equals $wL + 1 - \alpha$. To calculate real GDP we divide nominal GDP by the price of final output p_Q as defined in (5), yielding the following measure:

$$G(t) = \left[L + \frac{1-\alpha}{w(t)}\right] \alpha\lambda^k n_k(t)^{(1-\alpha)/\alpha}, \tag{23}$$

where k is the index of the general purpose technology actually used in the production of final output.[5] Observe that the term outside the square brackets on the right hand side of (23) is continuous in time in view of the fact that each n_i is continuous in time and the GPT switching condition (6). Therefore real GDP is continuous as long as the wage rate is continuous. Recall, however, that the wage rate jumps upwards with the appearance of a new general purpose technology, when innovators abandon the development of components for the old GPT and redirect their innovative efforts to the development of complementary inputs for the new GPT. As a result, real GDP jumps downwards at the beginning of each new cycle. It is important to note that this initial fall in GDP (which includes the output of the R&D sector) stems from the (static) allocative inefficiency caused by monopolistic competition in the manufacturing of components. The departure from marginal cost pricing there implies that the manufacturing sector is too small to begin with relative to the R&D sector (where "output" is priced according to marginal costs; i.e., $v = wa$). When a new GPT appears the upward jump in the wage rate causes a sudden diversion of resources away from manufacturing towards R&D, further enhancing the initial distortion and bringing about the fall in GDP.[6]

Beyond the fall in output right at the start of a cycle, it is apparent from (23) that real GDP keeps declining throughout the first phase. This is due to the fact that the wage rate rises during the first phase of a cycle, while the number of components available for production of the final good (i.e., those associated with the previous GPT) remains constant. We conclude that (temporary) reductions in real output are

[5] In other words, $k = \begin{cases} i-1 & for \quad t \in [T_i, T_i + \Delta_1) \\ i & for \quad t \in [T_i + \Delta_1, T_{i+1}) \end{cases}$.

[6] Of course, without positive markups in the manufacturing of components new ones would never be developed, and likewise, without a redeployment of resources away from the old GPT-components the economy would never reap the benefits of the new technologies. This is, then, a further example of Schumpeter's "gales of creative destruction", and the consequent trade-offs between static and dynamic efficiency.

an integral feature of the long-run equilibrium.

Actual growth begins only in the second phase, when the economy has just developed "enough" components for the new GPT and hence production of final goods switches to the newest technology. Real GDP increases throughout the second phase as the number of components for the prevailing GPT keeps rising, and as the wage rate declines. Thus, it is only in phase 2 that the opportunities opened by the advent of a new GPT translate into a growth spell that continues until an even newer GPT appears. Figure 6 shows the simulated path of real GDP over two cycles, highlighting the sharp differences between the two phases in each cycle. Over an entire cycle, though, the average rate of growth is simply $g = \log \lambda / \Delta$.

The phenomenon sketched in Figure 6 underlies the critical role of complementary investments in the growth process: contrary to what happens in the context of the neoclassical growth model with exogenous technological progress, in our context technical progress in a key technology does not bring about growth in total factor productivity by itself, and certainly not right away (in our model total factor productivity growth is indistinguishable from GDP growth). Rather, the appearance of a new, more efficient GPT *induces* an initial deployment of resources into complementary investments (i.e., the development of compatible components), and it is only after there are enough such investments in place that the potential of the new technology begins to manifest itself in output and productivity gains. The economy suffers TFP (total factor productivity) and output *losses* during the first phase of a cycle, but output and total factor productivity gains re-appear with a vengeance in the second phase of each cycle. Observe, however, that the economy experiences average output and TFP growth over an entire cycle.

The history of technology offers supportive evidence to this highly stylized sequence. In fact, economists and other scholars have been repeatedly puzzled by the fact that new GPTs fail to deliver noticeable benefits for quite a long time after introduction, but then they "kick off" and ignite a spell of sustained growth. That was certainly the case with electricity, and that seems to be the case presently with computers and the associated "productivity puzzle". David's (1990) documentation of the introduction of electricity is of particular interest. As he has shown, substantial productivity gains appeared only about four decades after the introduction of dynamos. And his Figure 4a, which depicts the evolution of productivity during the diffusion of electricity in manufacturing, resembles our Figure 6.[7] Surely there are other forces at work behind the productivity path of GPTs (e.g., plain diffusion as in Griliches, 1957, or continuous improvements in the GPT itself as in Bresnahan and Trajtenberg, 1994), but it seems that complementary investments play a critical role, which has been largely overlooked.[8] In this simple model those investments take the form of compatible inputs, but of course in actuality there is a wide array of different types of complementary investments, including organizational and institutional changes, both within firms and across vertically-related firms.

Thus, for example, it is becoming quite clear that in order to reap the benefits from computerization firms have to redesign the organization of work (e.g., emphasize team-work rather than hierarchical links), decentralize decision making, and make flexibility a prime goal in planning production and product design. In contrast to the traditional view of organizations as exogenous and of organizational changes as costless, the history of technology suggests that changes in technology and changes in organization and institutions are intimately related (see Chandler, 1977), and that tangible investments in such changes in response to the opportunities offered by new GPTs may be crucial for growth.

In our model the distinction between the two phases of the cycle is very pointed, and involves sharp discontinuities. This is for the most part an artifact due to the fact that we do not allow new and old GPTs to coexist, neither in production nor in

[7]Figure 9, shown in Section 6, resembles David's Figure 4a even more closely.

[8]See, however, Milgrom and Roberts (1990) for the importance of complementarities in manufacturing.

R&D.[9] Such coexistence would smooth out the transition from phase 1 to phase 2, and may even reverse the negative growth of phase 1. Still, growth during the second phase would be significantly faster than in phase 1. Thus, the main inference from our analysis is that, even if substantially more efficient, new technologies may barely make a dent at first in actual growth, since they have to await for the development of a sufficiently large pool of complementary assets to make a significant and lasting impact. Moreover, these assets use up resources, and hence in the short run growth may be adversely affected.

4.2 Real Wages, Profits, and Factor Shares

In previous sections we discussed the behavior of the wage rate over time in units that reflect the particular normalization chosen there (i.e., equating the value of final output to one). We turn now to *real* wages and profits, which measure the return to factors in units of the final good; i.e., w/p_Q and π/p_Q. In view of (5), the real wage rate at time t equals $\alpha\lambda^k n_k(t)^{(1-\alpha)/\alpha}$, and real profits are $(1-\alpha)\alpha\lambda^k n_k(t)^{(1-\alpha)/\alpha}/w(t)$. Notice that the continuity of n_k and the switching condition (6) imply that the real wage rate is continuous. In fact, real wages remain constant during phase 1 of each cycle, and then rise over the course of phase 2, as more and more components of the new GPT are being developed (hence boosting labor productivity). Real profits, on the other hand, do drop at the beginning of each cycle, as a result of the upward jump in the (normalized) wage rate, and decrease further along phase 1 (recall that real GDP declines over that period, and that the real wage bill remains constant). During phase 2 though, real profits rise continuously.

As to factor shares, the labor share is $wL/(1-\alpha+wL)$, and the share of profits is

[9] We started our work on this project with a model that allowed for overlapping GPTs, but it soon became clear that it would be exceedingly difficult to work with it, because of the need to keep track in the dynamic calculations of the variables associated with all previous GPTs (primarily n_i). However, we can envision extending the model to allow for the coexistence of two contiguous GPTs at a time.

$(1-\alpha)/(1-\alpha+wL)$. Recalling the behavior of w over time, it is thus clear that the profit share is pro-cyclical (as is the case with real profits), whereas the labor share is counter-cyclical.

4.3 The Stock Market

The fact that the appearance of new GPTs eventually renders older ones obsolete, implies that the know-how for the manufacturing of at most two types of components can simultaneously command a positive value; those associated with the latest GPT and with the next to the last. When the economy is in the second phase of a typical cycle, only components of the best practice GPT are valuable, because at that time it is known that no component of older technologies will ever be used. On the other hand, when the economy is in phase 1, then components of both the best practice GPT and of the previous one have positive value. This is so because in phase 1 the older GPT is still used in manufacturing, thereby providing a profit stream to owners of components that go with it. At the same time, owners of components that go with the newer GPT do not collect profits yet, but they expect profits in the future. For these reasons the technological know-hows of both types of components are valuable.

It follows that the value of the stock market at time t can be expressed as $S(t) = n_{i-1}(t)v_{i-1}(t) + n_i(t)v_i(t)$, where i is the index of the best practice GPT at time t. In case (i), on which we focus, the value of a component for the best practice technology always equals R&D costs; i.e., $v_i(t) = w(t)a$. As for the $(i-1)^{th}$ GPT, during phase 1 the value of each component is given by (19), while in phase 2 their values equal zero. Therefore the value of the stock market is

$$S(t) = \begin{cases} \frac{1-\alpha}{\rho}\left[1 - e^{-(T_i+\Delta_1-t)}\right] + n_i(t)w(t)a & \text{for} \quad t \in [T_i, T_i+\Delta_1), \\ n_i(t)w(t)a & \text{for} \quad t \in [T_i+\Delta_1, T_{i+1}). \end{cases} \tag{24}$$

We first use this formula to simulate the stock market to GDP ratio $S/(1-\alpha+wL)$ (see Figure 7). Notice that the introduction of a new GPT brings not only to a sharp decline in real GDP, but to an even sharper decline in the real value of the stock market, so that the S/GDP ratio falls. This ratio continues to fall during a substantial part of phase 1, but it picks up towards the end of the phase. Thus, an upturn in S/GDP *precedes* the arrival of phase 2, and thereby predicts the forthcoming upturn in productivity and output growth. In the second phase the stock market out-performs output and productivity, and the ratio of the stock market to GDP rises.

Next we consider the price earning ratio. In phase 2, when only the best practice GPT is valued, all components with positive stock market values have the same PE ratio. But in phase 1, in which components of two GPTs are positively valued on the stock market, each type has a different PE ratio. In particular, since components of the best practice technology have no earnings but are positively valued nevertheless, their PE ratio is infinite. In either case, however, we can calculate the average PE ratio, as the value of the stock market divided by aggregate profits; i.e., $S(t)/(1-\alpha)$.

Figure 8 presents a plot of the price earning ratio over two consecutive cycles. In phase 1 the PE ratio declines initially and rises subsequently, while in phase 2 it rises initially and declines subsequently. The average value of this ratio is, however, lower during the first phase, when real output and productivity decline, and higher in the second phase, when real output and productivity rise. Two points are worth making about the plot in Figure 8. First, it shows that the price earning ratio is more volatile than output and also more volatile than the S/GDP ratio. Second, it shows that upward trends as well as downward trends in the PE ratio can take place both during periods of economic contraction (phase 1) and during periods of expansion (phase 2).

5 Simulations

As mentioned above, the model presented here cannot be solved analytically, and hence we had to resort to simulations in order to perform an analysis akin to "comparative dynamics." We do that for case (i) only, focusing on variations in λ, α and ρ, holding the other parameters fixed at the following values: $L = a = 1$ and

$\Delta = 20$. We run a large number of simulations for a wide range of parameter values. Those that converged and fell within the domain of case (i) are shown in the appendix, Table A1.[10] The range of parameter values used in those simulations were: $\lambda \in [1.3, 28]$, $\alpha \in [0.1, 0.6]$, and $\rho \in [0.025, 0.075]$. As of now, then, the "comparative dynamics" results can be regarded as valid only for those areas of the parameter space actually covered in the simulations.

Ultimately, the magnitude of interest in these simulations is the rate of growth of real GDP (as defined in (23)); that is, we seek to establish how variations in the parameters λ, α and ρ affect the growth rate. As mentioned in Section 4.1, however, the average rate of growth over an entire cycle is simply $g = \log \lambda / \Delta$.[11] For a fixed Δ, then, g depends just on λ in a transparent way, and does not require simulations. However, we know that the behavior of GDP is far from uniform over the cycle: it declines over the first phase, and it is only in the second phase that growth starts in earnest. Note that, by definition, $g = \delta_1 g_1 + (1 - \delta_1) g_2$, where $\delta_1 \equiv \Delta_1 / \Delta$ denotes the fraction of the cycle taken up by phase 1. Thus, the interesting issues that can be addressed with the simulations refer to the differential impact of the parameters on g_1, g_2 , δ_1, and also n, the total number of components developed for each GPT. We do that by running OLS regressions of each of the magnitudes of interest on the varying parameter values, using the simulation results of table A1 as "data." Table 1 shows simple statistics of the variables used. Thus, within the parameter combinations examined, GDP grows at an average rate of 10% per period, whereas the initial phase exhibits on average *negative* growth of 4% and lasts for about half of the overall cycle.

The most interesting results of the regressions, shown in Table 2, are those related

[10] Our simulations are precise up to one percentage point of the value of a variable. Parameter combinations that lead to case (ii) involved mostly very low α's, often in conjunction with high λ's.

[11] This is due to the stationary nature of the long-term equilibrium, which implies *inter alia* that the end values of w and n are the same over each cycle.

to the impact of α.[12] Notice that this parameter has two opposite effects: on the one hand it reduces g_1 and g_2, but on the other hand it shortens a great deal the length of the first phase, during which there is negative growth.[13] The reason for the negative impact of α on δ_1 stems from the condition for switching between the current and the new GPT, as shown in (6). A higher α implies a lower η, that is, a smaller (minimal) number of new components necessary to tip the balance in favor of the new GPT, hence ending the first phase. The negative impact of α on both g_1 and g_2 is due to the fact that α represents in this context an aspect of *appropriability*.[14] Indeed, higher α's lower profits and hence the expected value of the firms (since the various inputs become closer substitutes for each other), thus reducing the incentives to develop new components. In this model, then, one parameter controls two important but opposite forces: the easiness by which new, more productive technologies can displace older ones, and the appropriability of rents that would induce complementary investments. The two are undoubtedly related in reality, but of course they need not be as tightly paired as our model implies.

The results for λ are more difficult to interpret. We know that λ speeds up growth over the entire cycle, but it seems to do that primarily by shortening the first phase of negative growth. Once again, this is related to the switching condition: higher λ's mean that it takes fewer new components to achieve parity and surpass the previous GPT. As for the discount rate, it has a clear detrimental effect on the total number of components developed, but an ambiguous impact on the other variables of interest.

[12]We experimented with different functional forms and found strong indications of concavity, hence the semi-log form.

[13]The net effect of α on the growth rate over the entire cycle is of course zero, since as said, g depends only on λ.

[14]The fact that in this case appropriability comes in via a technology parameter (i.e., substitutability) just reflects the particular modeling strategy chosen here, but is not important in itself. Notice, however, that we are referring here to appropriability for the development of the complementary components, not for innovation in the GPT itself.

6 Technological Diffusion and Relative Wages

We present in this section two modifications of our "base case" model, that help to address a whole set of interesting, more micro-oriented issues.[15] The first extends the model to an economy with many sectors, with the GPT having a different productivity impact on each one of them. As a result, new GPTs spread gradually across them, and consequently growth is seen to depend not only on the development of complementary inputs as in the base case, but also on the rate of diffusion of the GPT. The second modification allows for the existence of skilled and unskilled labor, with R&D being relatively skill intensive. We investigate in this framework the evolution of *relative* and *real* wages over each of the phases of the cycle, and the growth of GDP. The analysis has a bearing on the recent debate concerning the observed decline of real wages of unskilled workers in industrial countries during the eighties.

6.1 Diffusion

Suppose that there exists a continuum of final goods, indexed by z, with $z \in [0,1]$. Each z uses for manufacturing the same GPT-compatible components, as described in Section 2, except that now we allow the productivity level of the GPT to differ across sectors. In particular, suppose that $\lambda = \lambda(z)$ is a declining function of z. That is, the productivity of a new GPT is highest for sector $z = 0$, and declines as we move to higher index sectors. Under these circumstances the price of good z equals $p_Q(z) = \alpha^{-1} w \min_{1 \le i \le m} \left[\lambda(z)^{-i} n_i^{-(1-\alpha)/\alpha}\right]$ (see (5)). Observe that the number of available components is the same for all goods. Thus, as the number of available components for a new GPT increases over time, a larger fraction of sectors switches to manufacture with it. In fact, given $\lambda(\cdot)$, we can derive a non-decreasing function $f(n_i/n_{i-1})$, that describes the fraction of sectors that manufacture with the newest general purpose technology i. This fraction encompasses all sectors $z \in [0, Z_i]$, where

[15] We refer to the model developed in previous sections as the "base case" model.

$Z_i = f(n_i/n_{i-1})$.

Next, consider the profits derived from a typical component, assuming that consumers allocate equal fractions of spending to all final goods. During the first phase of cycle i, profits are the same as in the base case; i.e., they are zero for components of the i^{th} GPT (the latest), and they equal $\pi_{i-1} = (1-\alpha)/n_{i-1}$ for components of the previous GPT. Unlike in the base case, however, at time $T_i + \Delta_1$ not everyone switches to the i^{th} GPT simultaneously. Rather, only sector $z = 0$ does so. But as more components for the i^{th} GPT are developed over time, more and more sectors adopt the latest GPT. Thus, during the second phase the profit streams are different from those that obtain in the base case. For those still manufacturing components for the older GPT, profits are $\pi_{i-1} = (1-\alpha)(1-Z_i)/n_{i-1}$, whereas in the base case nobody does so and hence $\pi_{i-1} = 0$. For those manufacturing components for the latest GPT, profits are $\pi_i = (1-\alpha)Z_i/n_i$, whereas in the base case they were just $\pi_i = (1-\alpha)/n_i$.

Our discussion assumed that by time T_{i+1} (i.e., by the end of the i^{th} cycle) all sectors have adopted the i^{th} GPT; i.e., $Z_i(T_{i+1}) = 1$. This ensures the existence of the two phases that were central in our base case. It might happen, however, that some sectors still have not adopted the i^{th} GPT as an even newer GPT appears. If the latter is true, then long-term equilibria may involve longer cycles, defined not by the time of appearance of new GPTs (as in the base case), but by the time when all sectors switch to the latest GPT. This may offer a richer (and perhaps more realistic) characterization of technologically driven long cycles.

Turning now to GDP growth, it can be shown that real GDP in units of aggregate consumption equals now

$$G = \left(L + \frac{1-\alpha}{w}\right)\alpha n_i^{\frac{1-\alpha}{\alpha}Z_i} n_{i-1}^{\frac{1-\alpha}{\alpha}(1-Z_i)} e^{\Lambda},$$

where $\Lambda = i\int_0^{Z_i} \log\lambda(z)dz + (i-1)\int_{Z_i}^1 \log\lambda(z)dz$. Comparing it to (23), we see that in the base case only additions of new components and the concomitant changes in the wage rate drive growth during phase 2. In the present case there is an additional force contributing to productivity growth, and that is the rate of diffusion of the latest GPT in the manufacturing sector; i.e., the rise in Z_i.

6.2 Relative Wages

Suppose now that the labor force is not uniform, but rather that there are skilled workers that are suitable primarily for R&D and unskilled workers suitable primarily for manufacturing (we revert to the single final good as in the base case). For simplicity, we focus on the special case whereby there is complete segmentation between the two sectors; namely, the manufacturing technology requires only unskilled workers and R&D only skilled workers.

Assume further that manufacturing of components requires one unit of unskilled labor per unit of output. In this event total production of components per unit time equals L. Components are priced according to (3) and final output according to (5), as in the base case, where w stands now for the wage rate of unskilled labor. This implies a constant wage rate for unskilled labor of $w = \alpha/L$. Denoting by H the supply of skilled labor, we choose labor units such that one unit of skilled labor develops one new product per unit time. Thus, $\dot{n}_i = H$; i.e., the flow of new components per unit time is constant and equals H. This implies that the number of components available at time t is $n_i(t) = (t - T_i)H$ for $t \in [T_i, T_{i+1})$, and that the length of the first phase is simply determined by $\Delta_1 = \eta\Delta$.

We restrict the discussion to parameter values that produce a two-phase cycle, such as in case (i) of Section 3. In this event the no-arbitrage condition (9) implies that the evolution of the wage rate of skilled workers, denoted by w_H, satisfies:

$$\dot{w}_H = \begin{cases} \rho w_H & \text{for} \quad t \in [T_i, T_i + \eta\Delta), \\ \rho w_H - \frac{1-\alpha}{(t-T_i)H} & \text{for} \quad t \in [T_i + \eta\Delta, T_{i+1}). \end{cases}$$

In addition, if it is profitable to develop components for the $i-1^{th}$ GPT at time T_i^-, then (see (20)):

$$w_H(T_{i+1}^-) = \frac{1-\alpha}{\rho H \Delta}\left(1 - e^{-\rho\eta\Delta}\right).$$

These provide us with a differential equation and an end condition. The solution to this system is:

$$w_H(t) = \begin{cases} e^{\rho(t-T_i)} w_H(0) & \text{for} \quad t \in [T_i, T_i + \eta\Delta), \\ e^{\rho(t-T_i)} \left\{ w_H(0) - \frac{1-\alpha}{H}\left[\Phi\left(\rho\eta\Delta\right) - \Phi\left(\rho(t-T_i)\right)\right] \right\} & \text{for} \quad t \in [T_i + \eta\Delta, T_{i+1}), \end{cases} \tag{25}$$

where $w_H(0) = \frac{1-\alpha}{H}\left[\frac{1}{\rho\Delta}e^{-\rho\Delta}\left(1 - e^{-\rho\eta\Delta}\right) - \Phi(\rho\Delta) + \Phi\left(\rho\eta\Delta\right)\right]$, and $\Phi(y) = \int_y^\infty \frac{e^{-x}}{x}dx$.

We can now describe the evolution of relative wages. Recalling that the wage rate of unskilled workers is constant here, the time trend of relative wages is fully determined by the wage rate of skilled workers, as described in (25). It follows that the relative wage rate of skilled workers rises during the first phase. However, during phase 2 the wage rate of skilled workers is always lower than its peak at the end of phase 1. Therefore the *relative* wage rate of skilled workers has to decline at least at the beginning of phase 2, and it may in fact decline all the way, until the arrival of a new GPT.[16]

Now let us turn to real wages. From (5) and the fact that the wage rate of unskilled workers is constant, it follows that the real wage rate of unskilled workers, w/p_Q, is constant in phase 1 and rising in phase 2.[17] On the other hand, the real wage rate of skilled workers, w_H/p_Q, rises during phase 1, because w_H rises and p_Q does not change. In phase 2 the real wage rate of skilled workers may decline.[18] We therefore

[16]For example, when $\alpha = 0.5$, $\lambda = 1.5$, $\rho = 0.025$, $\Delta = 20$, $L = 1$, and $H = 0.3$, the wage rate of skilled workers declines at each point in time during phase 2.

[17]An extension of the model can produce a declining real wage rate of unskilled workers during phase 1. For this we need to assume that both types of labor are used in manufacturing. Then, as innovators increase their demand for skilled labor with the appearance of a new GPT, skilled workers will be reallocated from manufacturing to R&D. As a result unskilled workers will collaborate with fewer skilled workers in manufacturing, and the marginal product of unskilled workers will decline, leading to lower wages of unskilled workers.

[18]For the parameter values presented in the previous footnote the real wage rate of skilled workers

see that in the early part of a long-run cycle, when productivity stagnates, so does the real wage of unskilled workers, while skilled workers gain higher real wages over time. Then, with the adoption of the newest general purpose technology, real income of unskilled workers turns around. In that phase both types of labor may experience rising real incomes, at least for some time. But further along the cycle skilled workers may suffer falling real wages.

As already mentioned, the changes in relative wages of skilled versus unskilled labor have received a great deal of attention in recent times, in view of the deterioration suffered by unskilled workers during the eighties, and the large increase in their unemployment rate in countries with rigid labor markets. Two hypothesis compete for the explanation of these trends: (a) that the pressure on relative wages emanates from foreign competition, and especially from labor intensive exports of less developed countries; and (b) that technological progress in the industrial countries has differentially affected the demand for labor with varying skills. Our analysis suggests that the observed changes in relative wages are consistent with economies that are in the first phase of a long cycle, driven perhaps by a computer-based GPT.

Next consider real GDP, now given by

$$G = \frac{1}{p_Q}(1 - \alpha + wL + w_H H) = L(1 + w_H H)\lambda^k (t_k H)^{\frac{1-\alpha}{\alpha}} \quad \text{for} \quad t \in [T_i, T_{i+1}),$$

where k is the index of the GPT actually used in production. We have $k = i - 1$ and $t_k = \Delta$ for $t \in [T_i, T_i + \eta\Delta)$, and $k = i$ and $t_k = t - T_i$ for $t \in [T_i + \eta\Delta, T_{i+1})$. Notice that real GDP is homogeneous of degree $1/\alpha > 1$ in L and H (recall that w is inversely proportional to H, and hence $w_H H$ is invariant to factor endowments). Thus, this economy exhibits aggregate economies of scale. As to growth, real GDP drops right at the beginning of a cycle (as in the base case), but then it grows over phase 1, contrary to the negative growth seen in the base case. The behavior of real GDP during the second phase is less clear: the simulations that we carried out

rises during the early part of phase 2 and declines subsequently.

for a particular parameter combination show that, as in the base case, growth is significantly faster in the second phase (see Figure 9).[19] However, we have not been able to establish thus far how general this result is.

7 Concluding Remarks

The point of departure of this paper is a view of the growth process in which the vague notions of technology-related "increasing returns" or "non-convexities" that underlie a great deal of the new growth theory acquire a very concrete meaning: that of general purpose technologies fostering complementary advances in user sectors. The building blocks of our analysis can therefore be related to specific technological and historical processes (e.g., the electrification of manufacturing in the early 20th century); similarly, the outcomes of the analysis can be judged against that same historical and context-specific backdrop.

The long-run equilibrium notion that emerges from our analysis is that of a recurrent cycle, associated with each new and ever improving generation of GPTs. From a purely historical viewpoint, that seems to be a more compelling representation of actual processes than either stationary growth rates or convergence to "saddle points". Within each cycle the analysis shows the centrality that complementary investments play in the growth process, and how the sequential and cumulative nature of such investments may induce different phases along each cycle, exhibiting very different features. Particularly striking is the initial phase of negative or slow growth, which results from the fact that there is a threshold level of complementary inputs that need to be developed before the latest GPT can outcompete and displace the previous one.

As to policy implications, the results point out to the fact that the effectiveness (and even desirability) of different policy measures may well depend upon where along

[19]Notice that the pattern shown in Figure 9 resembles the productivity path of electricity already alluded to in Section 4 (David (1991)) more closely than the one in Figure 6, which represents the base case.

the cycle one stands, and the time horizon of the policy maker. If for example one stands at the beginning of a new cycle and has a short horizon (or, equivalently, a high discount rate), then one would want to minimize the length of the first phase. That, in turn, could involve taking measures to foster competition in the components sector. If one where standing at the beginning of the second phase, however, then an acceleration of growth would involve increasing appropriability, which implies exactly the opposite. And just before a cycle is over a main concern may be the impending drop in the level of output (and means to cushion it).

Mirroring the ambiguity in the formulation of growth-enhancing policies, the computation of social rates of return to R&D also depends upon the stage along the cycle and discount factors. Thus, if one were to discount heavily the future while standing at the beginning of a cycle, investments in R&D might look quite unattractive. Similarly, ex post rates of return calculations done, say, at the end of the first phase, may lead us to question the wisdom of investments in new technologies that would seemed to have failed to deliver productivity gains (as questions were raised in the late eighties about the profitability of investments in information technologies).

Of course, increasing lambda is always desirable, and as far as that is related to advances in "basic science" then this is as good a rationale for supporting basic research as any other (there are plenty). What our analysis suggests though is that it takes much more than that for advances in key technologies to have a sustained impact on growth. Moreover, the growth process associated with the unfolding of GPTs appears to be time-dependent and non-uniform in a fundamental way (e.g., the initial investment in complements is bound to divert resources from current production without offering immediate tangible benefits), giving raise to phase-dependent assessments of costs and benefits and related policy trade-offs.

The extensions sketched in Section 6 suggest that this model may offer a suitable framework for the analysis of a series of important issues that arise in the interface of technology and economics. In particular, the model offers one possible way of addressing the troublesome decline in relative wages of unskilled workers during the eighties. In fact, the predicted dynamic behavior of wages in our extended model is consistent with the observed phenomenon, and carries also an optimistic message: these trends are supposed to turn around once the latest GPT (computers?) reaches beyond its gestational stage and starts having a real impact on productivity. Clearly, though, the model developed here suffers from a series of readily apparent limitations, which we see as an agenda for future research. Prominent among them is the fact that we have ignored the endogenous character of advances in the GPT itself, and the associated (positive) feedback going from the user sectors to the GPT. This is an important part of the mechanism by which GPTs are thought to play their role of "prime movers", and its (hopefully transient) omission is due only to the modeling difficulties that we have encountered.

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Appendix

Table A1 : Simulation Results

	λ	α	ρ	g	g_1	g_2	δ_1	n
1	1.3	0.6	0.025	0.0131	-0.0104	0.0565	0.6483	6.411
2	1.35	0.6	0.025	0.015	-0.0169	0.055	0.556	6.336
3	1.5	0.5	0.025	0.0203	-0.0125	0.0805	0.6477	8.316
4	1.75	0.5	0.025	0.028	-0.0312	0.082	0.4773	8.148
5	2	0.4	0.025	0.0347	-0.0183	0.1157	0.6048	10.376
6	2.5	0.3	0.025	0.0458	-0.014	0.1674	0.6702	12.498
7	2.5	0.4	0.025	0.0458	-0.0366	0.121	0.4773	10.246
8	2.5	0.4	0.05	0.0458	-0.0258	0.1264	0.5295	9.144
9	3	0.3	0.025	0.0549	-0.0201	0.1716	0.6085	12.553
10	3.5	0.3	0.025	0.0626	-0.0265	0.1755	0.5586	12.568
11	3.5	0.4	0.075	0.0626	-0.0567	0.1348	0.377	8.255
12	4	0.3	0.025	0.0693	-0.0333	0.1792	0.5172	12.534
13	4	0.3	0.05	0.0693	-0.0265	0.1854	0.5479	11.371
14	4.5	0.2	0.025	0.0752	-0.0141	0.2693	0.6849	14.804
15	4.5	0.3	0.025	0.0752	-0.0402	0.1828	0.4825	12.494
16	4.5	0.3	0.05	0.0752	-0.0328	0.1872	0.509	11.44
17	5	0.2	0.025	0.0805	-0.0159	0.2719	0.6652	14.829
18	5	0.3	0.025	0.0805	-0.0473	0.1863	0.4529	12.438
19	5	0.3	0.05	0.0805	-0.0394	0.1892	0.4757	11.464
20	5.5	0.2	0.025	0.0852	-0.0176	0.2743	0.6477	14.848
21	5.5	0.3	0.025	0.0852	-0.0545	0.1896	0.4275	12.381
22	5.5	0.3	0.05	0.0852	-0.0462	0.1914	0.447	11.471
23	6	0.2	0.025	0.0896	-0.0193	0.2766	0.632	14.863
24	6	0.3	0.025	0.0896	-0.0618	0.1927	0.4052	12.317
25	6	0.3	0.05	0.0896	-0.0531	0.1937	0.4219	11.446
26	6	0.3	0.075	0.0896	-0.0417	0.2	0.4566	10.464
27	6.5	0.2	0.025	0.0936	-0.0209	0.2787	0.6178	14.875
28	6.5	0.3	0.025	0.0936	-0.0691	0.1957	0.3857	12.252
29	6.5	0.3	0.05	0.0936	-0.0602	0.196	0.3998	11.412
30	6.5	0.3	0.075	0.0936	-0.0487	0.2006	0.4292	10.52
31	7	0.2	0.025	0.0973	-0.0225	0.2807	0.6048	14.878
32	7	0.3	0.025	0.0973	-0.0764	0.1986	0.3684	12.187
33	7	0.3	0.05	0.0973	-0.0676	0.1984	0.38	11.376
34	7	0.3	0.075	0.0973	-0.0559	0.2017	0.4053	10.528
35	7.5	0.2	0.025	0.1007	-0.0241	0.2826	0.5929	14.885
36	7.5	0.3	0.025	0.1007	-0.0838	0.2014	0.3529	12.122
37	7.5	0.3	0.05	0.1007	-0.0746	0.2007	0.3631	11.335
38	7.5	0.3	0.075	0.1007	-0.063	0.203	0.3844	10.567
39	8	0.2	0.025	0.104	-0.0257	0.2844	0.5819	14.89
40	9	0.2	0.025	0.1099	-0.0286	0.2878	0.5623	14.895
41	10	0.2	0.025	0.1151	-0.0315	0.2908	0.5451	14.897
42	11	0.2	0.025	0.1199	-0.0342	0.2937	0.53	14.895
43	11	0.2	0.05	0.1199	-0.0296	0.2997	0.546	13.894
44	12	0.2	0.025	0.1242	-0.0369	0.2963	0.5164	14.891
45	12	0.2	0.05	0.1242	-0.0321	0.3017	0.5315	13.923
46	13	0.2	0.025	0.1282	-0.0395	0.2988	0.5042	14.885
47	14	0.2	0.025	0.132	-0.042	0.3011	0.4931	14.878
48	14	0.2	0.05	0.132	-0.0369	0.3054	0.5067	13.958
49	16	0.2	0.025	0.1386	-0.0468	0.3054	0.4736	14.862
50	16	0.2	0.05	0.1386	-0.0415	0.3088	0.4859	13.984
51	18	0.2	0.025	0.1445	-0.0513	0.3093	0.4569	14.843
52	18	0.2	0.05	0.1445	-0.0458	0.312	0.4681	14
53	20	0.2	0.025	0.1498	-0.0556	0.3128	0.4424	14.818
54	20	0.2	0.05	0.1498	-0.05	0.315	0.4526	14.009
55	22	0.2	0.025	0.1546	-0.0598	0.316	0.4297	14.798
56	22	0.2	0.05	0.1546	-0.054	0.3178	0.439	14.013
57	22	0.2	0.075	0.1546	-0.047	0.3238	0.4564	13.132
58	24	0.2	0.025	0.1589	-0.0638	0.319	0.4183	14.773
59	24	0.2	0.05	0.1589	-0.0579	0.3204	0.4269	14.012
60	24	0.2	0.075	0.1589	-0.0508	0.3256	0.4429	13.171
61	26	0.1	0.025	0.1629	-0.0142	0.5684	0.6959	17.304
62	28	0.1	0.025	0.1666	-0.0148	0.5703	0.69	17.309

Table 1

Simple Statistics

Variable	Mean	Std. Dev.	Minimum	Maximum
λ	9.684	7.219	1.300	28.000
α	0.269	0.100	0.100	0.600
ρ	0.038	0.002	0.025	0.075
g	0.099	0.039	0.013	0.167
g_1	-0.040	0.018	-0.084	-0.010
g_2	0.242	0.096	0.055	0.570
δ_1	0.505	0.093	0.353	0.696
n	12.774	2.366	6.336	17.309

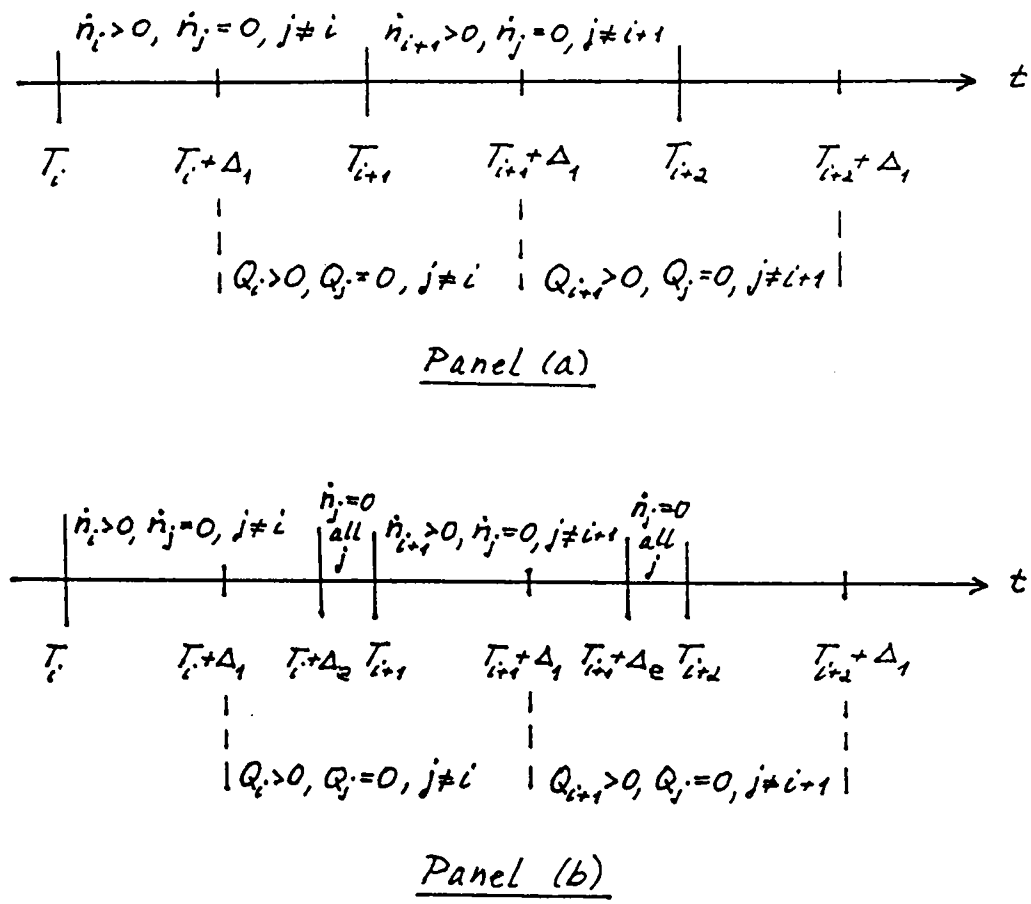
Table 2

Regression Results

	g_1	g_2	δ_1	n
c	-0.06	-0.12	0.25	-0.43
	(-4.7)	(-4.8)	(4.2)	(-0.8)
log λ	-0.04	0.02	-0.18	0.11
	(-9.1)	(2.8)	(-10.2)	(0.7)
log α	-0.07	-0.23	-0.40	-6.21
	(-8.2)	(-14.0)	(-10.3)	(-16.4)
log ρ	0.002	-0.001	-0.02	-1.32
	(0.4)	(-0.13)	(-1.0)	(-7.8)
R^2	0.64	0.95	0.73	0.96

t - statistics in parenthesis

Figure 1



$\dot{n}_i > 0,\ \dot{n}_j = 0,\ j \neq i$
$\dot{n}_{i+1} > 0,\ \dot{n}_j = 0,\ j \neq i+1$
t
T_i
$T_i + \Delta_1$
T_{i+1}
$T_{i+1} + \Delta_1$
T_{i+2}
$T_{i+2} + \Delta_1$
$Q_i > 0,\ Q_j = 0,\ j \neq i$
$Q_{i+1} > 0,\ Q_j = 0,\ j \neq i+1$
Panel (a)
$\dot{n}_i > 0,\ \dot{n}_j = 0,\ j \neq i$
$\dot{n}_j = 0$ all j
$\dot{n}_{i+1} > 0,\ \dot{n}_j = 0,\ j \neq i+1$
$\dot{n}_j = 0$ all j
t
T_i
$T_i + \Delta_1$
$T_i + \Delta_2$
T_{i+1}
$T_{i+1} + \Delta_1$
$T_{i+1} + \Delta_e$
T_{i+2}
$T_{i+2} + \Delta_1$
$Q_i > 0,\ Q_j = 0,\ j \neq i$
$Q_{i+1} > 0,\ Q_j = 0,\ j \neq i+1$
Panel (b)

Figure 2

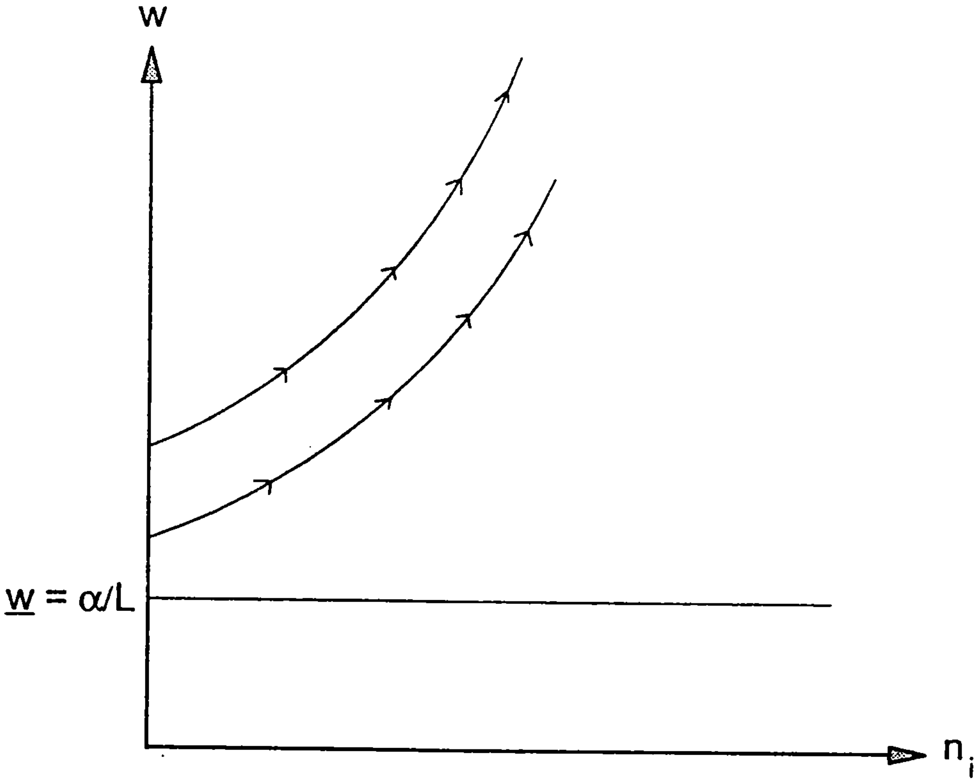


Figure 3

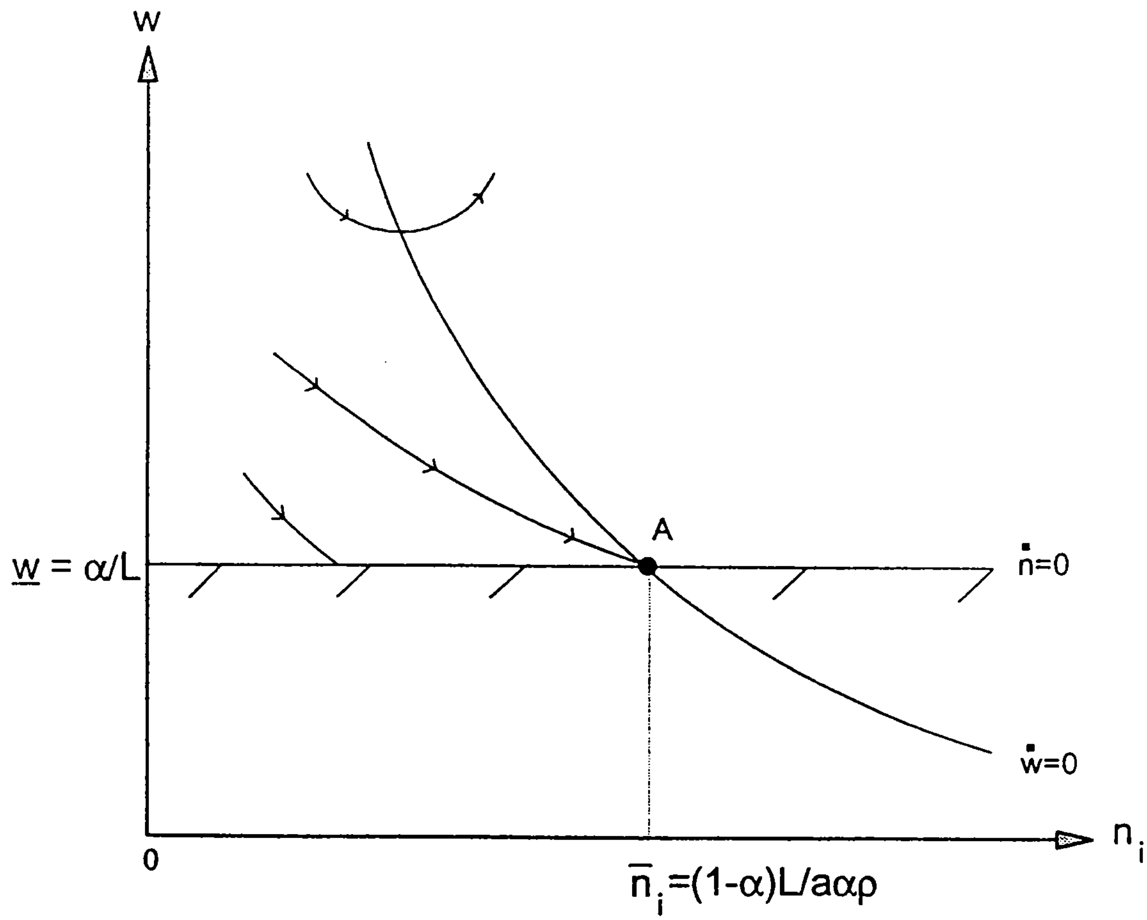
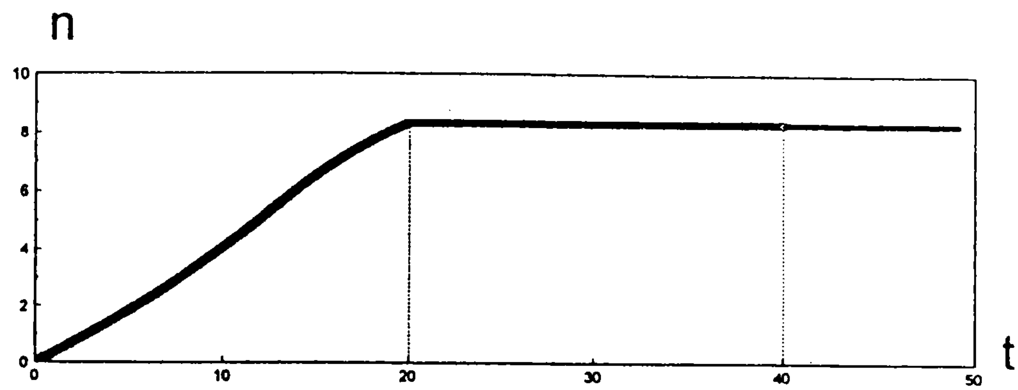
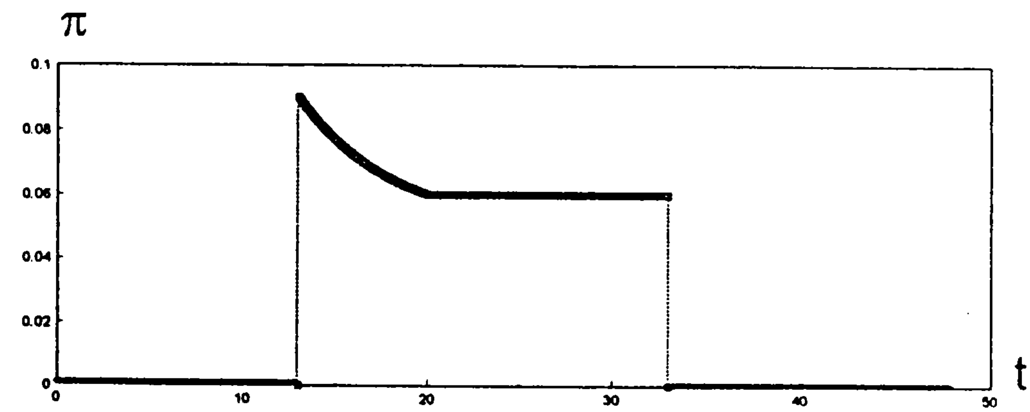


Figure 4



Panel a



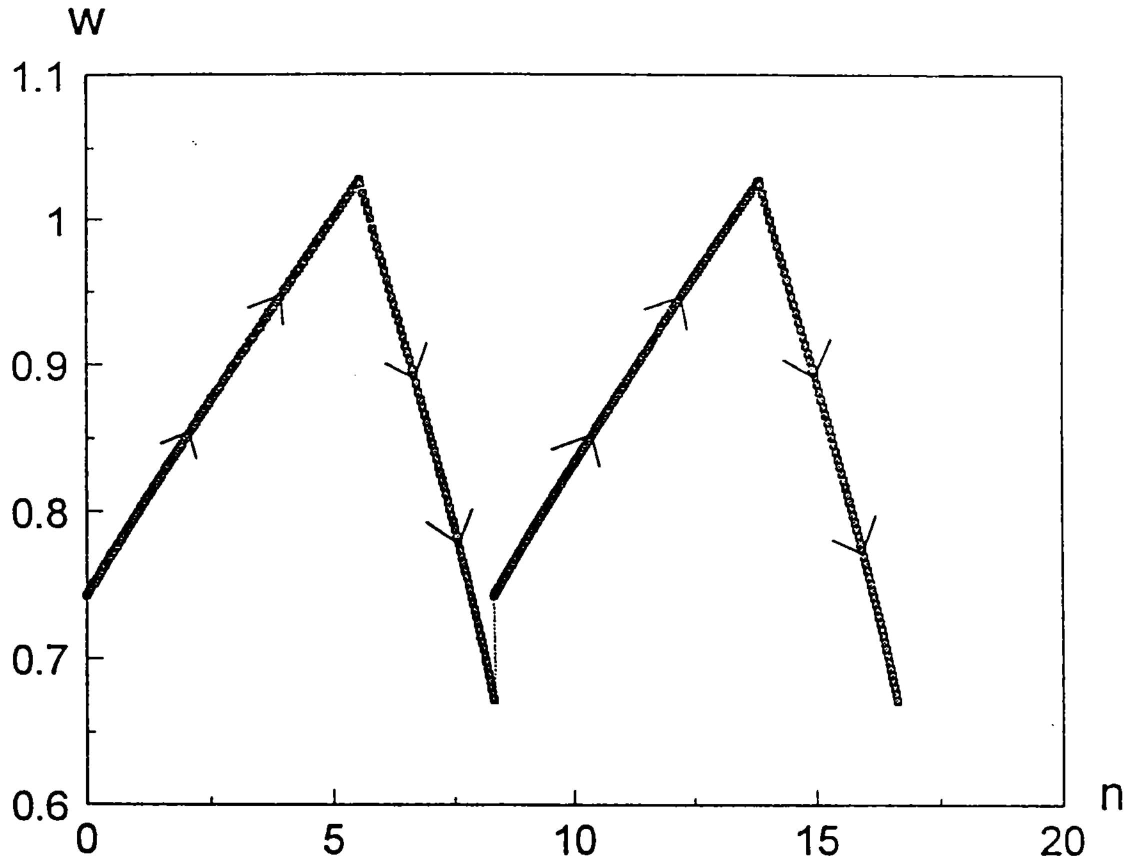
Panel b

$\lambda=1.5$

$\alpha=0.5$

$\rho=0.025$

Figure 5

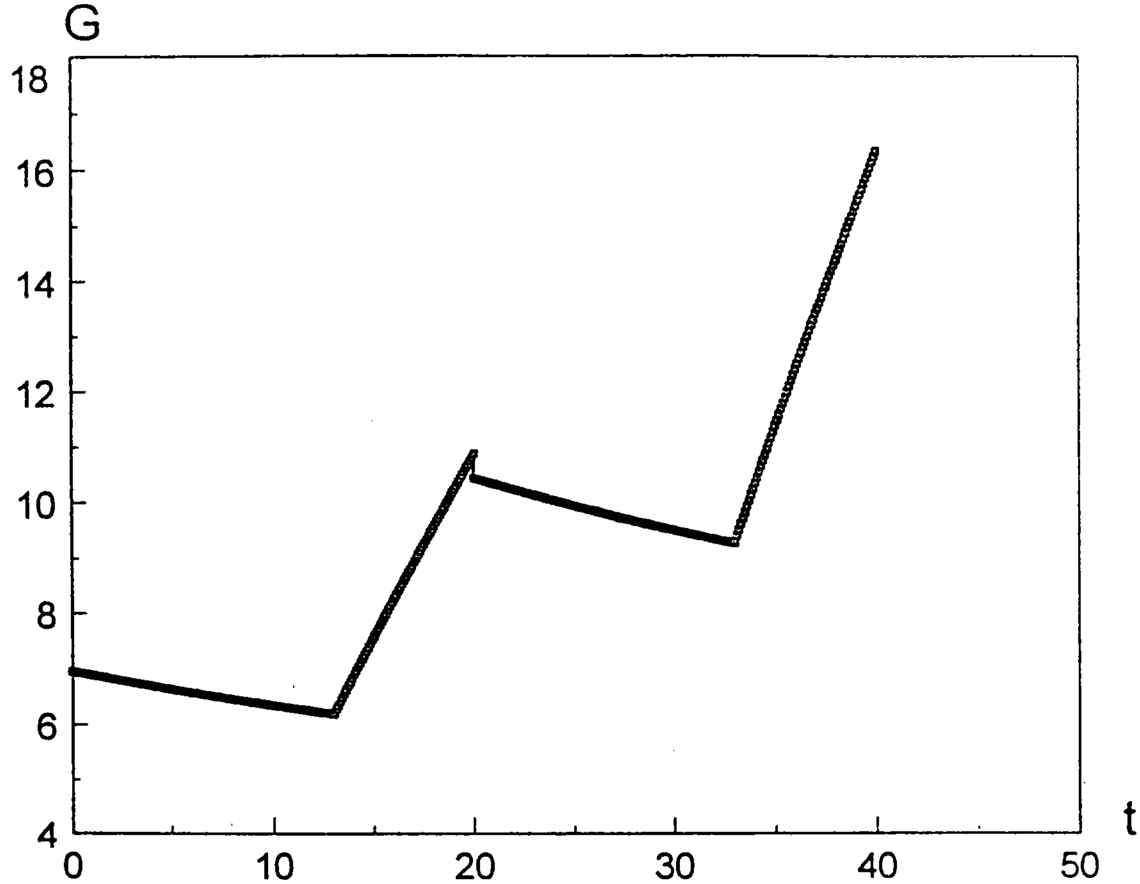


λ=1.5

α=0.5

ρ=0.025

Figure 6

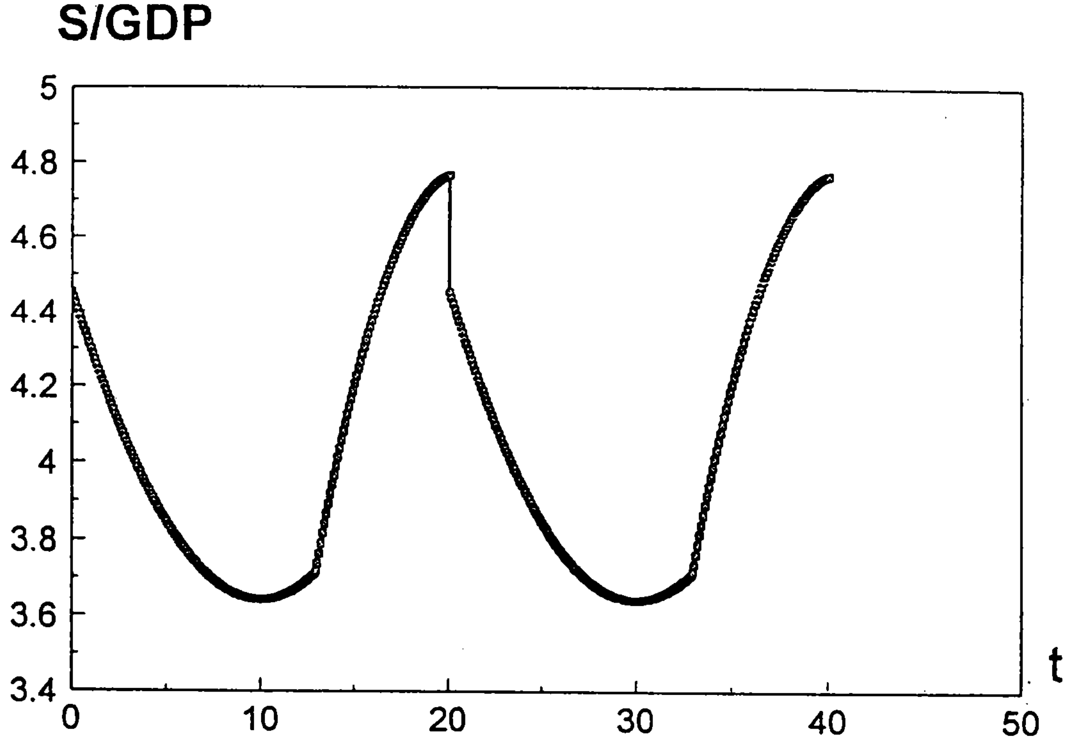


λ=1.5

α=0.5

ρ=0.025

Figure 7

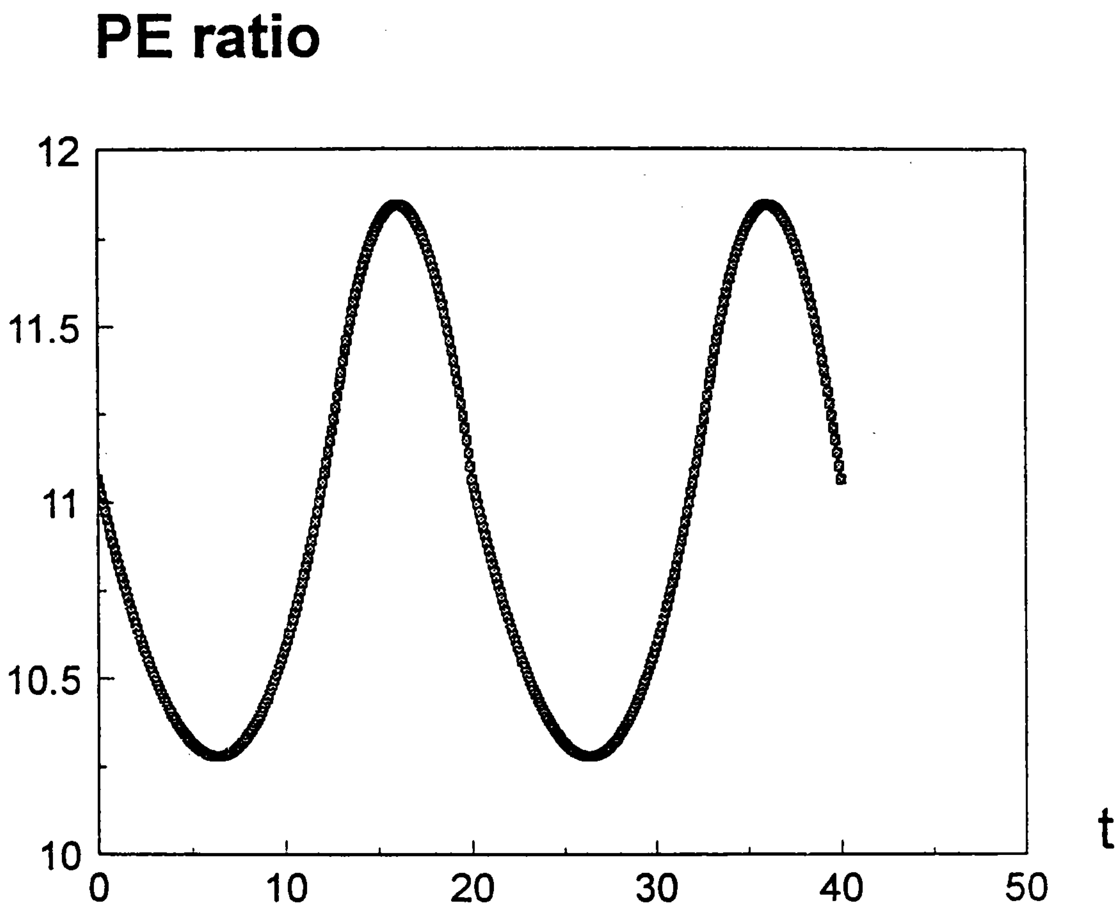


λ=1.5

α=0.5

ρ=0.025

Figure 8

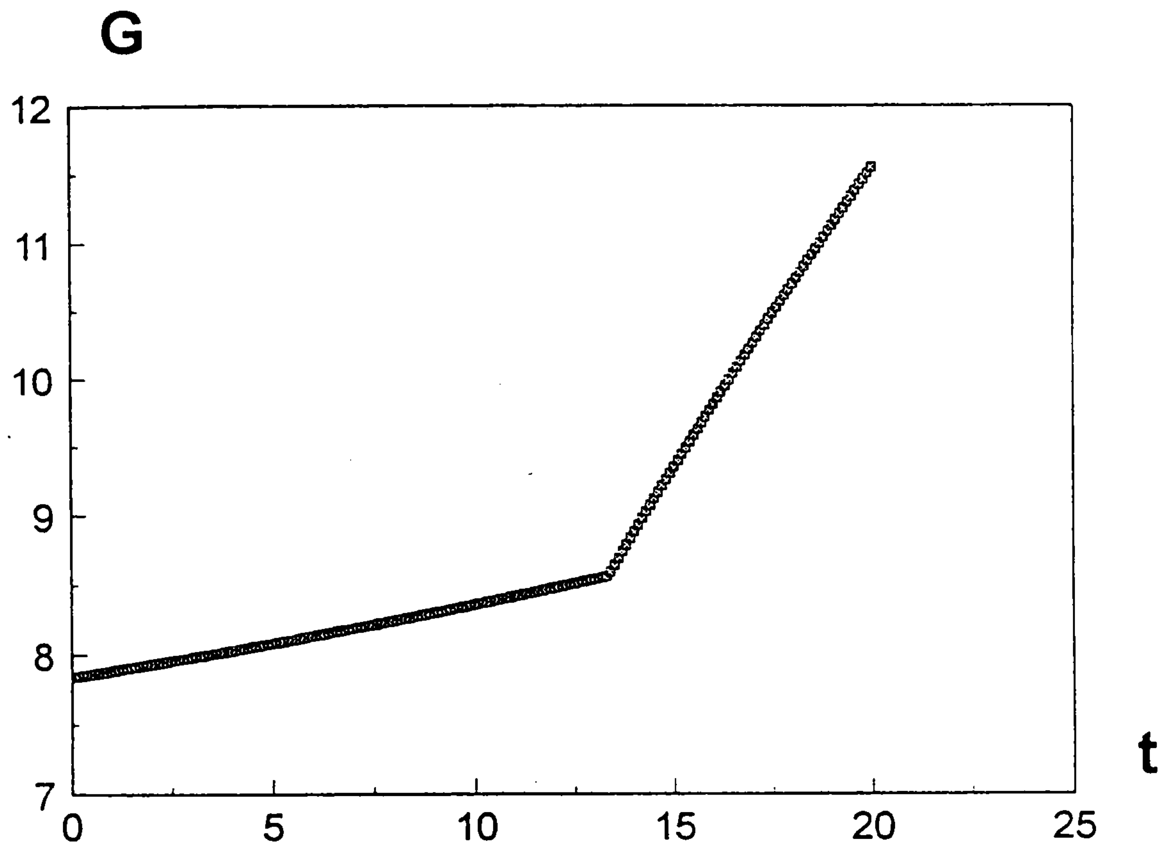


λ=1.5

α=0.5

ρ=0.025

Figure 9



λ=1.5 L=1

α=0.5 H=0.3

ρ=0.025