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ABSTRACT

In this paper, we analyze the effects of changes in monetary growth rates in the context of models of endogenous growth when the demand for money comes from a cash-in-advance constraint. We explore two alternative avenues through which the rate of inflation could affect the overall long-run rate of growth of the economy. The first of these is through nominal rigidities in the tax code. The particular rigidity that we examine is for depreciation allowances that are fixed in nominal terms. The second avenue that we examine is a distortion of the labor-leisure choice when a Lucas-style model of effective labor is used.

In both cases, the welfare costs and growth effects of various monetary growth rules relative to a constant money supply are studied. It is found that both the welfare costs of inflation and its growth effects are quite small at low to moderate levels of inflation. However, at rates of inflation that are high by U.S. standards but not uncommon in developing countries, the magnitude of both the growth effects and the welfare costs of inflation depend on the specification of the model. If cash and credit goods are substitutes there are no growth effects and moderate welfare effects. If the two goods are complements there are sizable growth effects and large welfare effects.

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Growth and the Effects of Inflation

by

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1. Introduction

Much of the recent literature on the differences in observed growth rates across countries has concentrated on differences in fiscal policies (in particular, differential tax rates on capital income). Examples of this include Jones and Manuelli (1990), King and Rebelo (1990), Barro (1990), Rebelo (1991), Easterly (1989) and Jones, Manuelli and Rossi (1993). Thus, different growth experiences across countries arise due to differences in policies that respective governments employ.

In contrast to this, the effects of monetary policy have been given much less attention. This is in spite of the fact that, in empirical work, a number of studies (see Kormendi and Meguire (1985), Cardoso and Fishlow (1989), De Gregorio (1991), Fischer (1991) and Rubini and Sala-i-Martin (1992)) have identified the rate of inflation as an important determinant of the rate of economic growth.

Of course, the study of the relationship between money and growth is a classic topic in monetary theory in the more conventional exogenous growth setting. Initially, the literature emphasized the 'portfolio' or Tobin effect. In his pioneering paper, Tobin (1965) argued that an increase in the growth rate of the money supply results in higher inflation and hence in an increase in the opportunity cost of holding cash balances. This results in a reallocation of saving from money into capital and - given the assumed constancy of the saving rate- results in an increase in the stock of capital per worker. However, Sidrausky (1967) and Brock (1974) study a similar problem in the context of a fully optimizing general equilibrium framework and find that money is superneutral in the steady state, as the stock of capital per worker is independent of the growth rate of the money supply.[1] More recently, Stockman (1981) studied a model in which there is a cash-in-advance constraint. He shows that if this constraint applies to consumption, then money is superneutral. On the other hand, if investment purchases are subject to the cash-in-advance constraint there is a negative relationship between the growth rate of the money supply and the stock of capital per capita.

In this paper, we extend this work on the effects of monetary policy to models of endogenous growth. Our aim in this endeavor is to begin to identify the mechanisms through which changes in monetary policy might

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[1]The introduction of money in the utility function opens the possibility of a rich dynamic behavior (see Matsuyama (1991)). With elastic labor supply, the Sidrausky-Brock model implies that although capital per worker (and hence the real interest rate) is independent of the growth rate of the money supply, the supply of labor is not. Depending on the specific utility function, this may result in a positive or a negative relationship between the growth rate of money and the stock of capital per capita. Finally, Danthine, Donaldson, and Smith (1987), show that in a stochastic version of the Sidrausky-Brock model, money is no longer superneutral. For a recent survey, see Orphanides and Solow (1990).

affect the long run growth rate of the economy and estimate the quantitative effects of these changes. To this end, we follow Lucas (1982), Lucas and Stokey (1987), and Cooley and Hansen (1989) and (1992) and assume that a demand for money is generated for transactions purposes through a cash-in-advance constraint on the purchases of consumption goods.

We consider two models of endogenous growth which differ in their formulation of the supply of effective labor offered to firms by workers. In the first, there is no human capital (alternatively, human capital need not be jointly supplied with labor) and, as a result, labor supply is zero asymptotically. In this version of the model, inflation affects welfare through its impact on the effective relative price of the two consumption goods. However, because the inflation rate has no impact on the limiting rate of interest paid on capital income, there is no effect on the asymptotic rate of growth of the economy. Thus, in this version of the model, inflation has level, but not growth, effects.

However, if the rate of inflation does affect investment decisions, then, even in this simple model of endogenous growth, it follows that changing the rate of monetary expansion does have growth effects. One plausible mechanism for this, is through the presence of nominal rigidities in the system. The particular case that we study is to include nominally denominated depreciation allowances included in the tax code. In this case, the effective real marginal tax rate on investment income is altered by a change in the rate of monetary expansion. Thus, different rates of monetary expansion are associated with different after tax real rates of return on investment. This results in a change in equilibrium investment decisions giving growth effects. (For related analysis of the impact of inflation in the presence of nominal rigidities see Auerbach (1979), Abel (1981) and Judd (1990).)

In the second model of endogenous growth that we analyze, we adopt a Lucas-style ((1988) and (1990)) nonconvex technology which combines hours and human capital to produce effective labor. In this case, the steady state level of effort (i.e., number of hours supplied to the market) is determined by the relative prices of consumption and leisure and this margin is distorted by inflation. This has a direct impact on the long run growth rate of the economy (through an effect on the marginal product of capital).

Although these two results provide a theoretical basis for the negative correlation between inflation and growth found in the empirical studies cited above, the question of the quantitative importance of these effects is relevant. To study this issue, we calculate the steady state growth and welfare effects of changes in the rate of monetary expansion for a series of parameterized examples. We find that in general, the size of the growth effects of inflation are quite modest even for relatively large rates of monetary expansion. For example, in the model with nominal depreciation allowances, the growth rate falls from 2% to 1.9% as money growth is increased from 0% to 50% and then to 1.7% when the rate of monetary expansion is 900%. In the model with Lucas' effective labor technology, the growth effects depend on whether cash and credit goods are substitutes or complements. In the complements case, at extreme (but not unheard of) inflation rates, the size of the growth effects can be significant. Specifically, when money growth is 200%, the growth rate falls to 1.1%. When cash and credit goods are substitutes, the relationship between the growth rate and inflation is non-monotonic, falling to 1.8% at 50% money growth and then increasing back to 2% as the rate of expansion is increased further.

The effects on welfare that we find are similar to those found in the exogenous growth literature (see Lucas (1981) and (1993), Cooley and Hansen (1989), Eckstein and Leiderman (1992) and Imrohoroglu (1992)) and vary significantly across the three models that we analyze. The welfare costs, which we measure as the percentage change in GNP necessary to make individuals indifferent between monetary stability and the given rate of expansion, of 10% money growth range from a low of .08% in the model with no growth effects to 1.5% in the model with Lucas style labor supply. The largest welfare losses that we find (56% at money growth rates of 900%) correspond to the case in which the growth effect is also largest (i.e., the Lucas model with complementarity between cash and credit goods). Moreover, the results suggest that the welfare costs of increasing inflation decrease as the level of inflation is increased.

In related independent work, Gomme (1991) studies similar problems in a growth context with the Lucas effective labor technology. He examines the relationship between the level of uncertainty in monetary expansion and average growth rates and finds that these effects are typically small as well.

The remainder of the paper is organized as follows. In section 2, we lay out the basic one sector model of endogenous growth and show that monetary expansion by itself has no effects on growth in this setting. Section 3 is devoted to the interaction of monetary policy and fiscal policy when depreciation allowances are nominally denominated. In section 4 we analyze the variant of the model with the Lucas effective labor supply and show that again inflation can have growth effects. In section 5, we present some calibrated calculations on the size of the growth and welfare effects resulting from inflation. Finally, conclusions are contained in section 6.

2. The Basic Model

Consider a simple representative agent model in which the household maximizes the present discounted value of utility subject to a sequence of budget constraints and to a cash-in-advance constraint on the purchases of a 'cash' good. Formally, the household solves:

[P1] $\text{Max } \Sigma_t \beta^t u(c_{1t}, c_{2t}, 1-n_t)$ subject to

(a) $m_t + b_{t+1} \leq v_t$

(b) $p_{1t} c_{1t} \leq m_t$

(c) $$v_{t+1} \leq (v_t - m_t - b_{t+1}) + (m_t - p_{1t} c_{1t}) - p_{2t} c_{2t} - p_{1t} x_t + p_{1t} w_t n_t + p_{1t} r_t k_t + (1+R_{t+1}) b_{t+1}$$

$$+ T_t$$

(d) $$k_{t+1} \leq (1-\delta)\, k_t + x_t$$

where k_0 and m_0 are given and

c_{1t} is the amount of the first (cash) consumption good

c_{2t} is the amount of the second (credit) consumption good

n_t is the amount of labor supplied to the market

m_t is the amount of cash held in period t

b_t is the amount of bonds purchased

v_t is wealth at time t

k_t is the individual's physical capital stock

x_t is investment in new physical capital

T_t is a cash transfer received from the government

p_{it} is the (nominal) price of good i, i=1,2

w_t is the wage rate at time t

r_t is the rental price of capital and

R_t is the nominal interest rate paid on bonds

The first constraint indicates that at the beginning of a period, the household's nominal wealth, v_t, can be used either to purchase cash, m_t, to finance purchases of the first consumption good, c_1, or to purchase one period nominally denominated bonds. The second equation is the cash-in-advance constraint. Equation (c) is the law of motion of nominal wealth. It specifies that wealth at t+1 equals unspent (in financial markets) nominal wealth at t, v_t -

$m_t - b_{t+1}$, plus unspent cash balances, $m_t - p_{1t}\, c_{1t}$, minus purchases of the credit good and the investment good plus labor, $p_{1t}\, w_t\, n_t$, and capital income, $p_{1t}\, r_t\, k_t$, plus nominal transfers from the government, T_t.

As is standard in this literature, it is essential that labor and capital income cannot be immediately used to purchase the cash good. That is, income from labor and capital services are received after the cash good is purchased. This restriction generates a well defined transactions demand for currency and makes the existence of a monetary equilibrium possible (but does not guarantee it). Finally, (d) is the standard law of motion for the capital stock.

Firms face static problems. They maximize profits, π_t, at each time t, where

$$\pi_t = p_{1t}\, c_{1t} + p_{2t}\, c_{2t} + p_{1t}\, x_t - p_{2t}\, r_t\, k_t - p_{1t}\, w_t\, n_t$$

subject to $c_{1t} + c_{2t} + x_t \leq F(k_t,n_t)$. Thus, the two consumption goods and the investment good are perfect substitutes in output. We will assume that F is concave, homogeneous of degree one and that both capital and labor have everywhere strictly positive marginal products.

There are two properties that follow immediately from the description of the firms' problem. First, since F( ) is homogeneous of degree one, profits are zero. Second, if both c_{1t} and c_{2t} are produced, it must be the case that $p_{1t} = p_{2t}$. We will assume that the output of both goods is positive in what follows and will denote the common price by p_t.

Finally, we assume that the money supply evolves according to the rule

(1) $$M_{t+1} = \mu M_t$$

and that the increases are distributed lump sum to the households. It follows that $T_t = M_{t+1} - M_t$.

An equilibrium in this economy is a sequence of prices, $(p_t^*, r_t^*, w_t^*, R_t^*)$, a real allocation, $(c_{1t}^*, c_{2t}^*, x_t^*, k_t^*)$ and stocks of financial assets, (m_t^*, b_t^*), such that:

(a) Given prices, the real allocation and the stocks of financial assets solve the consumers maximization problem [P1].

(b) Given prices, the allocation solves, for each t, the firms' maximization problem.

(c) $m_t^* = M_t$, $b_{t+1}^* = 0$ $t \geq 0$.

The basic ideas in Jones and Manuelli (1990) can be used to establish equilibrium existence in more general settings. We will simply explore the necessary conditions that any equilibrium must satisfy in this section. In what follows, we will assume that all quantities are interior.

From the household's problem, it follows that:

(2) $$\frac{u_1(t) F_2(t)}{u_3(t)} = 1 + R_{t+1}$$

(3) $$\frac{u_1(t)}{u_1(t+1)}=\frac{(1+R_{t+1})}{(1+R_{t+2})}\beta[1-\delta+F_1(t+1)]$$

(4) $$\frac{u_1(t)}{u_2(t)}=1+R_{t+1}\ .$$

(5) $$1+R_{t+2}=\frac{p_{t+1}}{p_t}[1-\delta+F_1(t+1)]$$

(6) $$p_t\ c_{1t} = M_t$$

where $u_i(t)$ is the partial derivative of the instantaneous utility function with respect to its ith argument evaluated at time t quantities.

Equations (2)-(6) describe the equilibrium under the assumption that the cash-in-advance constraint is binding (this will be true if $R_t>0$). We will assume that this holds in what follows.

Consider the special case $F(k,n) = Ak + A_2k^{\alpha}n^{1-\alpha}$ with $0<\alpha<1$. It follows that, for all n, $\lim_{k\to\infty} F_1(k,n) = A$.

If there is a steady state in which $R_t = R$, equation (3) simplifies to

$$\lim_{t\to\infty}\frac{u_1(t)}{u_1(t+1)}=\beta[1-\delta+A] \text{ if } k_t \to \infty.$$

Following Jones and Manuelli (1990), if $\beta(1-\delta+A)>1$, the long run growth rate of consumption is positive.

Consider the special case for utility given by:

$$u(c_1,c_2,1-n)=\left[(c_1^{-\lambda}+\eta c_2^{-\lambda})^{-1/\lambda}(1-n)^{\psi}\right]^{1-\sigma}/(1-\sigma).$$

In this case, $u_1/u_2 = (c_2/c_1)^{1+\lambda}/\eta$, hence, (4) becomes

(7) $$1 + R_{t+1} = (c_{2t}/c_{1t})^{1+\lambda}/\eta.$$

Note that in a nonmonetary version of the model, the analog of (7) corresponds to R=0 and hence η corresponds to relative expenditure shares. It follows that the presence of the cash-in-advance constraint artificially increases the consumption of the second good (i.e., the credit good) relative the first.

Consider next equation (2). Given our choice of utility function, it is given by:

(8) $$1+R_{t+1}=\frac{(1-n_t)w_t}{\psi c_{1t}[1+\eta(c_{1t}/c_{2t})^{\lambda}]}.$$

Since c_{1t} goes to infinity and $c_{1t}/c_{2t} \rightarrow [\eta(1+R)]^{[1/(1+\lambda)]}$, the denominator converges to infinity at the rate of c_{1t}. Therefore, the numerator must grow at the same rate. Because of the form of w_t, it follows that $\lim_{t\rightarrow\infty} n_t = 0$. To see this, note that $w_t = (1-\alpha)\ A_2\ k_t^{\alpha}\ n_t^{-\alpha}$. Since k_t^{α}/c_{1t} converges to zero (k_t and c_{1t} grow at the same rate), it must be that $(1-n_t)\ n_t^{-\alpha}$ converges to infinity. It follows that $n_t \rightarrow 0$.

The long run behavior of the model can be summarized by the following equations:

(9) (a) $\gamma^\sigma = \beta[1-\delta+A]$ (from (3))

(b) $\pi\gamma = \mu$ (from (1) and (6))

(c) $1+R = \pi\ [1-\delta+A]$ (from (5))

(d) $c_2/c_1 = [\eta(1+R)]^{1/(1+\lambda)}$ (from (4) and (7))

where γ is the rate of growth of output (and the two types of consumption) and π is the inflation rate.

It can be readily seen from equation (a) that the growth rate of consumption is determined solely by the parameters of taste and technology and is not affected by changes in the rate of growth of the money supply (i.e., μ). It follows from (b)-(d) that the limiting marginal rate of substitution between cash and credit goods is affected by changes in monetary rules. Thus, as noted above, different monetary policies have effects on the make-up of consumption (i.e., the split between c_1 and c_2) and consequently on the welfare of the representative agent, but not on the asymptotic rate of growth of the economy.

Note that in this economy, a version of the Fisher equation holds. As equation (c) shows, higher inflation rates are associated with higher nominal interest rates and constant real interest rates (in the steady state). Thus, it follows that the real rate of interest is independent of μ.

3. Nominal Rigidities in the Tax Code

In the previous section, we saw that in the absence of other distortions, inflation has no effect on growth in the simplest of all endogenous growth models. In this section, we will see how this conclusion is changed by the introduction of other distortions (e.g., taxation) in the model. As it turns out, whether or not inflation has growth effects depends on the type of distortion present. There are several possible routes to explore. The simplest of these is to include a cash in advance constraint on investment purchases. In this case, the argument of Stockman (1981) as extended to an endogenous growth setting applies: the increase in the effective relative price of capital reduces the growth rate. Alternatively, reserve requirements on a banking sector also has as an implication that inflation has growth effects. These derivations are fairly straightforward and hence are not included here. (Details available from the authors upon request.)

One plausible mechanism through which inflation affects investment decisions is through changing the real value of nominally denominated rigidities in the tax code. Examples include nominally denominated depreciation allowances, imperfectly indexed tax bracketing and nominally denominated investment tax credits. To study these possibilities, the perturbation on the model presented in section 2 that we will explore here is to include nominally denominated depreciation allowances in the presence of capital income taxation. Throughout, we will hold the capital income tax rate constant. It is straightforward to show that the presence of capital taxation alone does not give rise to the conclusion that inflation affects growth. That is, although changing the rate of taxation of capital income does have effects on growth, changing the rate of money growth does not if depreciation allowances are indexed (or are zero) and the capital income tax is unchanged.

In order to isolate the effects of nominally denominated depreciation allowances, we will simplify the model of the previous section even further. We will remove labor supply from the model entirely and use the simple 'Ak' model of endogenous growth. This is the simplest form of one sector model that delivers an endogenously determined rate of growth of the economy.

The representative consumer in this economy solves:

[P2] Max $\Sigma_t \beta^t u(c_{1t}, c_{2t})$ subject to

(a) $m_t + b_{t+1} \leq v_t$

(b) $p_t c_{1t} \leq m_t$

(c) $$v_{t+1} \leq (v_t - m_t - b_{t+1}) + (m_t - p_t c_{1t}) - p_t c_{2t} - p_t x_t + p_t r_t k_t + (1+R_{t+1}) b_{t+1} + T_t - \tau_t [p_t r_t k_t + R_{t+1} b_{t+1}] + \tau_t [p_{t-1} \delta_\tau x_{t-1} + p_{t-2} (\delta_\tau)^2 x_{t-2} + \dots]$$

(d) $k_{t+1} \leq (1-\delta) k_t + x_t$

where k_0 and m_0 are given. The notation is that of the previous section with the addition that:

τ_t is the tax rate on investment income in period t (note that we have assumed that bonds bought in period t, b_{t+1}, pay interest in period t)

δ_τ is the depreciation rate in the tax code.

Note that we have modelled depreciation allowances as geometrically declining at a rate δ_τ, not necessarily equal to the true economic depreciation rate, δ. Similar effects are obtained if one assumes the more standard linear depreciation schedules common in tax codes. (Details available from the authors on request.)

Firms face static problems. They maximize profits, π_t, at each time t, where

$$\pi_t = p_t\, c_{1t} + p_t\, c_{2t} + p_t\, x_t - p_t\, r_t\, k_t$$

subject to $c_{1t} + c_{2t} + x_t \leq F(k_t)$. Thus, the consumption good and the investment good are perfect substitutes in output.

Given our assumption about the form of F, it follows that $r_t = A$ for all t.

To be consistent with steady state growth, we will further specialize to the case that $u(c_1,c_2,1-n) = \left[(c_1^{-\lambda}+\eta c_2^{-\lambda})^{-1/\lambda}\right]^{1-\sigma}/(1-\sigma)$.

Characterization of the first order conditions of the consumers problem along with the assumption that the cash in advance constraint is binding implies that in equilibrium (after some algebraic manipulation) we must have that:

(10) $$\gamma^\sigma = \beta\ [\ b(1-\tau) + (1-\delta)\ ] + \left[\ \gamma^\sigma - \beta(1-\delta)\ \right] \times \frac{\tau\beta\delta_\tau}{\mu\gamma^{\sigma-1} - \beta\delta_\tau}$$

There are several interesting effects that the introduction of nominally denominated depreciation allowances has on the growth rate. First, it is possible to show that the limit as μ converges to β corresponds to an equilibrium in which there is no money in the system. (This corresponds to Friedman's optimal quantity of money- see Friedman (1969).) In this benchmark case, it is clear from (10) that $\delta_\tau>0$ has the effect of increasing the growth rate (relative to $\delta_\tau=0$) as this effectively reduces the tax rate on capital income. Second, an increase in the rate of growth of the money supply reduces the growth rate. The intuition underlying this result is as follows: an increase in μ results in an increase in the nominal interest rate which, in turn reduces the present value of tax credits corresponding to the depreciation allowance with the corresponding increase in the cost of capital.

With two capital goods (e.g., human and physical), an additional growth effect of inflation is also present. Changes in the rate of inflation will also affect the equilibrium mix between the two capital goods. Here, an increase in the rate of inflation decreases the real value of the depreciation allowance on physical capital. Since tax codes do not generally allow for the depreciation of human capital, this change will generally realign the shares of expenditures of human to physical capital towards their undistorted levels. Although the exact sizes of these effects depend on parameter values, this will be growth enhancing in general. Thus, in a two capital good world, the growth effects of inflation discussed here are likely to be tempered to some extent.

Quantitative estimates of the sizes of the effects discussed here are presented in Section 5 below.

4. A Model with Human Capital Utilization

One of the key effects of inflation is its distortionary impact on the labor-leisure choice. This is present even in the simple model examined in section 2. In some models of endogenous growth, distorting this choice directly effects the rate of growth of the economy through its impact on the effective utilization of human capital. In this section, we will explore the ramifications of an alteration of the model outlined in section 2 along these lines. To do this requires two changes to the model. First, we will introduce human capital. Second, we will alter the form that labor supply takes.

The problem that the consumer must solve is to:

[P3] maximize $\Sigma_t \beta^t u(c_{1t}, c_{2t}, 1-n_t)$ subject to

(a) $m_t + b_{t+1} \leq v_t$

(b) $c_{1t}\, p_t \leq m_t$

(c) $v_{t+1} \leq (v_t - m_t - b_{t+1}) + (m_t - p_t\, c_{1t}) - p_t\, c_{2t}$

$-p_t\, x_{kt} - p_t\, x_{ht} + p_t\, w_t\, n_t\, h_t + p_t\, r_t\, k_t$

$+ (1 + R_{t+1})\, b_{t+1} + T_t$

(d) $k_{t+1} \leq (1-\delta_k)k_t + x_{kt}$

(e) $h_{t+1} \leq (1-\delta_h)h_t + x_{ht}$

where the meaning of the variables is as in section 2 with the addition of h_t as the stock of human capital and x_{ht} as investment in human capital stated in terms of consumption good equivalents. Finally, note that labor income is given by $p_t\, w_t\, n_t\, h_t$. This is interpreted as the nominal wage, $p_t\, w_t$, applied to the quantity of 'effective labor' supplied to the market, $n_t\, h_t$. Thus, the household uses raw labor, n_t (hours), in combination with human capital, h_t, to form effective labor, z_t, according to $z_t = n_t\, h_t$. Note that this gives rise to a nonconvexity in the consumer's problem. The meaning of the constraints mirrors those given in sections 2 and 3.

As above, the firms face static problems, maximizing profits, π_t, in each period, where

$$\pi_t = p_t\,(c_{1t} + c_{2t} + x_{kt} + x_{ht}) - p_t\, r_t\, k_t - p_t\, w_t\, n_t\, h_t$$

subject to $c_{1t} + c_{2t} + x_{kt} + x_{ht} \leq F(k_t, n_t h_t)$ where F(,) is concave, strictly increasing and homogeneous of degree one in its two arguments.

Again, we will examine the necessary conditions describing an equilibrium under the assumption that all quantities are interior.

Specializing to the case that

$$u(c_1,c_2,1-n) = \left[(c_1^{-\lambda}+\eta c_2^{-\lambda})^{-1/\lambda}(1-n)^{\psi}\right]^{1-\sigma}/(1-\sigma)$$

and $F(k,nh) = A\, k^{\alpha}\, (nh)^{(1-\alpha)}$ and assuming that the equilibrium converges to a steady state growth path, the equations describing the system can be written as:

(11) $$\left(\frac{c_2}{c_1}\right)^{1+\lambda} = \eta\ (1 + R)$$

(12) $$(1-n)\ \frac{h}{k}(1-\alpha)A(\frac{k}{nh})^{\alpha} = \frac{c_1}{k}\psi(1+\eta(c_2/c_1)^{-\lambda})(1+R)$$

(13) $$\gamma^{\sigma} = \beta[1-\delta_h + (1-\alpha)\ A\ n^{1-\alpha}\ (h/k)^{-\alpha}]$$

(14) $$\gamma^{\sigma} = \beta[1-\delta_k + \alpha\ A\ n^{1-\alpha}\ (h/k)^{1-\alpha}]$$

(15) $$1+R = \pi\ [1-\delta_k + \alpha\ A\ n^{1-\alpha}\ (h/k)^{1-\alpha}]$$

(16) $$\pi\ \gamma = \mu$$

(17) $$c_1/k + c_2/k + x_k/k + x_h/k = A\ n^{1-\alpha}\ (h/k)^{1-\alpha}$$

(18) $$\gamma = 1 - \delta_k + x_k/k$$

(19) $$\gamma = 1 - \delta_h + x_h/h$$

where π is the limiting value of p_t/p_{t-1}, R is the limiting value of R_t, $n_t \rightarrow n$, $c_{1t}/c_{1t-1} \rightarrow \gamma$, $c_{2t}/c_{1t} \rightarrow c_2/c_1$, $h_t/k_t \rightarrow h/k$, $h_t/c_{1t} \rightarrow h/c_1$, $x_{kt}/k_t \rightarrow x_k/k$ and $x_{ht}/h_t \rightarrow x_h/h$.

This gives a system of nine nonlinear equations describing the nine, endogenous, limiting variables of the model. However, since we are primarily interested in the growth impacts of inflationary monetary policy in this framework, most of these variables and equations can be eliminated.

From (13) and (14), we obtain that the ratio h/k satisfies

$$\left(\frac{h}{k}\right)^{-\alpha}\times\left[\alpha\left(\frac{h}{k}\right)-(1-\alpha)\right]=\frac{\delta_k-\delta_h}{An^{1-\alpha}}.$$

It follows that if $\delta_h = \delta_k$, h/k is independent of μ and equal to $(1-\alpha)/\alpha$. If $\delta_h \neq \delta_k$, this expression shows that if different monetary policies have no effect upon n, then they have no effect on h/k either and hence, using (13) and (14), no impact on the long-run growth rate.

Assuming that $\delta_h = \delta_k$ and substituting into (13) gives

(13A) $\gamma^{\sigma} = \beta\ [1 - \delta + \alpha\ A\ n^{1-\alpha}\ [(1-\alpha)/\alpha]^{1-\alpha}]$.

From this, it can be seen immediately that the limiting growth rate of consumption in this version of the model is completely determined by the parameters of tastes and technology in conjunction with the (endogenously determined) asymptotic level of labor supply, n. (In a version of the model with non-trivial fiscal policy, tax rates also enter this expression.) It follows that monetary policies will have growth effects in this model <u>if and only if</u> changes in the rate of growth of the money supply give rise to an adjustment in the asymptotic level of n.[2]

Since this last possibility is our primary concern here, we will simplify

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[2]It can be shown that in the case $\delta_k \neq \delta_h$, n and h/k move in opposite directions if $\delta_k > \delta_h$ and in the same direction otherwise. Hence, our example - which implies h/k constant - will tend to overstate the growth effects of inflation if $\delta_k > \delta_h$ and understate them otherwise.

this system of equations so as to isolate the two variables, γ and n.

The assumption that $\delta_h = \delta_k$, and some simple manipulation gives:

$$(20) \qquad \gamma = 1 - \delta + Bn^{1-\alpha}\left(1 - \frac{(1-\alpha)(1-n)}{\psi n f(\mu,\gamma)}\right)$$

$$(21) \qquad \gamma^{\sigma} = \beta\ [1-\delta + B\ n^{1-\alpha}\ ]$$

where $B = A\ (1-\alpha)^{(1-\alpha)}\ \alpha^{\alpha}$ and

$$f(\mu,\gamma) \quad 1 \quad \frac{\mu\beta \quad {}^{1}\gamma^{(\sigma \quad 1)} \quad 1}{1 \quad \eta^{1/(1 \quad \lambda)}(\mu\beta \quad {}^{1}\gamma^{(\sigma \quad 1)})^{1/(1 \quad \lambda)}}.$$

Note that in contrast to the Fisherian result obtained with the model in section 2, here $(1+R) = \pi[1-\delta+B_2 n^{1-\alpha}]$. Since n is affected by μ, the steady state real rate of interest depends on the rate of monetary expansion.

Equation (20) implicitly defines a relationship between γ and n. To guarantee that equilibria exist, it must be true that $1 > \beta[1-\delta+B]^{(1-\sigma)}$. Without this condition, is is possible that infinite utility is feasible.

Finally, it can be shown that the relationship between the rate of money growth and the growth rate depends critically on the elasticity of substitution between cash and credit goods. If cash and credit goods are complements, $\lambda>0$, high values of μ eliminate growth altogether. Asymptotically, as μ goes to infinity, the growth rate converges to $1-\delta$. On the other hand, if the two goods are substitutes, $\lambda<0$, high values of μ have exactly the opposite effect. Asymptotically, the rate of growth of output is the same as that in a non-monetary version of this economy.

These two results have a simple intuition. An increase in the effective price of cash goods reduces the cash to credit goods ratio in the economy. If these two goods are substitutes, relative spending on the cash ggod (priced at 1+R) converges to zero as $\mu \to \infty$. This reduction in the relative consumption of cash goods implies that the 'consumption price index' (which takes into account the relative prices of cash and credit goods as well as the composition of spending) converges to one (its level when only credit goods are purchased). Thus, in the limit, the economy resembles one with only one, the credit, consumption good. Since the composition of output across two consumption goods with the same production function does not affect the balanced growth path of an economy with no money, the growth rate converges to that of a non-monetary, one consumption good economy as μ converges to infinity. This gives rise to a non-monotonicity of the growth rate in μ in this case, first decreasing and then, ultimately increasing as μ is increased.

On the other hand, when cash and credit goods are complements, relative spending on cash goods converges to one as $\mu \to \infty$. Because of this, the consumption price index goes to infinity with μ when $\lambda>0$. This increase in the relative price of overall consumption increases the demand for leisure which, it turn, reduces the growth rate of the economy (see equation (21)).

For this model, it is possible to show that optimal monetary policy follows a version of the Friedman rule: Choose the growth rate of the economy so that the nominal interest rate is zero. When the government follows this rule, the resulting growth rate is equal to that of the economy with no cash-in-advance constraint. This result coupled with our previous characterization of the behavior of growth rates when μ is large shows that there is widely different behavior depending on λ: If cash and credit goods are substitutes, the growth rate decreases initially as μ is increased beyond the level given by the optimal policy. However, for higher values of μ, the overall growth rate of the economy converges to the level achieved under the optimal policy.[3] In contrast, if the goods are complements, γ falls as μ is increased.

We summarize these results in the following proposition.

<u>Proposition 1</u>: Let (γ^*,n^*) be the equilibrium levels of growth and labor supply along the balaced growth path of a non-monetary version of the model (i.e., with no cash-in-advance constraint). Then,

(i) Under the optimal policy, $\mu^* = \beta\gamma^{*(1-\sigma)}$, $\gamma(\mu)$- the equilibrium growth rate as a function of μ- is equal to γ^*.[4]

(ii) If cash and credit goods are complements ($\lambda \geq 0$), $\lim_{\mu\to\infty}$

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[3]Notice that although the growth rate of the economy eventually increases to its first best level (when the goods are substitutes) as μ is increased, this does not mean that welfare converges to its level at the first best. The distortion in the compositition of consumption implies that the level of welfare is lower.

[4]In this case, the rate of decrease of the money supply under the optimal policy is faster than that in a no growth version of the model if $\sigma>1$. Effectively, the discount rate with growth is given by $\beta\gamma^{*(\sigma-1)}$.

$\gamma(\mu) = 1\text{-}\delta < 1.$

(iii) If cash and credit goods are substitutes ($\lambda<0$), $\lim_{\mu\to\infty} \gamma(\mu) = \gamma^*$.

Proof: See Appendix.

5. Some Numerical Calculations

To complete the discussion of the type of effects that we are considering in this paper, we will present the results of some calculations of the steady state effects of inflation on growth rates and welfare. In sub-sections A, B, and C below, we present these calculations for the models in sections 2, 3 and 4, respectively. For these calculations, we use combinations of parameter values so that the rate of growth of output is 2% and the ratio of cash goods to output is 10% in the steady state when income tax rates are 20% and there is no monetary expansion.

To calculate the growth effect of inflation, we solve the system of steady state equations for the calibrated examples for a variety of levels of the rate of monetary expansion, μ.

In all cases, we use as the basis for welfare comparisons the relative size of the (aggregate) capital stock that would be required to give the same level of utility (starting from the steady state) that would be received in the calibrated (μ=1) steady state. That is, if $V(k,\mu)$ is the level of utility if the initial capital stock is k and the rate of money growth is μ, $k(\mu)$ is defined by:

$$V(1,\mu) = V(k(\mu),1).$$

Thus, ΔU is given by $1-k(\mu)$.

It can be shown that:

$$(22) \qquad \ln(k(\mu)) = \quad [\ln(c_1/k(\mu)) - \ln(c_1/k(1))]$$
$$-(1/\lambda) \times [\ln(1+\eta(c_1/c_2(\mu))^\lambda) - \ln(1+\eta(c_1/c_2(1))^\lambda]$$
$$+\psi \times [\ln(1 - n(\mu)) - \ln(1 - n(1))]$$
$$-(1/(1-\sigma)) \times [\ln(1-\beta(\gamma(\mu))^{1-\sigma}) - \ln(1-\beta\gamma(1)^{1-\sigma})]$$
$$-[\ln(1+h/k(\mu)) - \ln(1+h/k(1))]$$

where the variables $c_1/k(\mu)$ and $c_1/k(1)$, etc. are the steady state values of the cash good to capital stock evaluated when the rate of monetary expansion is μ and 1, respectively.

From this, we can see that there are five terms to the steady state welfare loss of inflation: The first term represents a partial 'savings effect' measured by the change in the ratio of cash goods to capital. The second term (the 'consumption composition' effect) measures the change in welfare due to the change in the composition of consumption. This is distorted by the effect of inflation on the relative prices of cash and credit goods.[5] Note that although decreases in c_1/k increase the welfare loss, decreases in c_1/c_2 holding

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[5]Note that is not possible to completely isolate the savings and relative price effects. If we ignore changes in labor supply, output is proportional to the size of the capital stock and thus the saving rate is proportional to $(c_1 + c_2)/k = (c_1/k)(1 + c_2/c_1)$. It follows that to interpret changes in c_2/c_1 as pure price effects (with no impact on savings) it is necessary that c_1/k adjust as well.

c_1/k constant have the opposite effect. The third term (a 'labor supply' effect) captures the change in welfare due to adjustments in the consumption of leisure as μ is changed. Both the saving and labor supply effects have consequences for the long-run growth rate of the economy. The fourth term (the 'growth effect') measures the welfare loss due to the distortion the inflation causes to the growth rate. Finally, in general settings, inflation will also affect the composition of the capital stock. This 'capital composition' effect makes up the last term of (22).

In the model of section 2, inflation has no growth effects. For this to be the case, the saving/GNP ratio must remain unchanged. Thus, in this case, the only non-zero terms are the first two: Inflation distorts the choice of c_2/c_1, but c_1 adjusts so as to keep the saving rate constant.

When nominally denominated depreciation allowances are included, in addition to the first terms, the growth effect (i.e., the fourth term of (22)) also differs from zero.

Finally, in the model of section 4, all terms except the last are non-zero. (That the last is zero follows only because of our assumptions that the two depreciation rates are equal and that the production function is Cobb-Douglas.)

5A. Inflation Effects in the Simple Ak Model

We begin by giving the results of calculations based on the model of section 2. Although there is no growth effect here, the calculated welfare costs for this model will give us a basis for comparison with the other two cases to be examined. We studied this model for a range of values for the two parameters σ and λ. Since the results we obtained were fairly insensitive to the choice of σ, we will present the results only for the case σ=2, δ=.1, β=.98 and the two values for λ, λ=1.0 and λ=-.5. These require that A=.20204 and η is given by 9.0 (if λ=1.0) and 1.68 (if λ=-.5) if the growth rate is to be 2% and cash consumption purchases are 10% of GNP when μ=1.

The two cases presented correspond to different qualitative properties for the demand for money. When λ=1, the implied elasticity of substitution between cash and credit goods is .5 and hence they are complements. When λ=-.5, they are substitutes with an elasticity of substitution equal to 2.

Table I

	μ	1.00	1.05	1.10	1.50	2.00	3.00	10.00
	γ	1.02	1.02	1.02	1.02	1.02	1.02	1.02
λ=1.0	ΔU	0.00	.0003	.0008	.009	.024	.059	.224
λ=-.5	ΔU	0.00	.0005	.0014	.016	.040	.090	.282

As can be seen from this table, the welfare losses from inflation are quite small except at extreme values for μ. Moreover, when the elasticity of substitution between cash and credit goods is increased (i.e., λ is decreased), the welfare effect is increased, as expected. The intuition for this is quite simple: When labor is inelastically supplied, the more substitutable cash and credit goods are, the larger is the welfare loss associated with distorted relative prices. In the extreme case when the two consumption goods are perfect complements (i.e., $\lambda=\infty$), it can be shown that there is no welfare loss whatsoever.

5B. The Effects of Nominally Denominated Depreciation Allowances

Here, we will present the results of using equation (10) of section 3 to calculate the predicted growth effects for a simple calibrated example. Again, we explored a variety of different combinations of parameter values. For purposes of comparison, we present the cases where $\sigma=2.0$, $\delta=.1$, $\delta_\tau=.2$, $\beta=.98$, and $\tau=.20$ and, as above, show the results when $\lambda=1$ and $\lambda=-.5$. These require that A=.1924 and give values of η of 7.34 and 1.60, respectively. The results that we obtain are:

TABLE II

	μ	1.00	1.05	1.10	1.50	2.00	3.00	10.00
$\lambda=1.0$	γ	1.02	1.02	1.02	1.019	1.018	1.017	1.017
	ΔU	.000	.002	.005	.025	.048	.089	.262
$\lambda=-.5$	γ	1.02	1.02	1.02	1.019	1.018	1.017	1.017
	ΔU	.000	.003	.007	.046	.090	.151	.259

As can be seen from this table, the effects on growth of inflation in this environment are quantitatively quite small. This holds in spite of the fact that we have chosen the relatively generous value of $\delta_\tau=.2$ which is quite high relative to actual figures based on US tax law. Thus, these figures should be considered as upper estimates. Moreover, note that at high levels of inflation, the effects are even smaller. It can be shown that as $\mu \to \infty$, γ converges to the growth rate that would hold if δ_τ was zero. Thus, although the inflation effects of growth are theoretically important here, it seems as a practical matter that they are not.

The welfare effects are similarly small at reasonable values for μ, but can be quite large when μ is extreme.

When δ_τ is reduced to .05, both the growth and welfare effects are reduced accordingly.

5C. Quantitative Impacts of Inflation through the Lucas Labor Supply Effect

In this section we will present results on the quantitative effects of the changes in the rate of monetary expansion on the steady state levels of γ and n based on the model examined in section 4 analogous to those given above. Here, we present four sets of results. As above, to facilitate comparisons, we use σ=2.0, β=.98, δ_k=δ_h=.1 and α=.36. Then, we give calculations for both λ=1.0 and λ=-.5 and ψ=4.0 and 8.0. For the preferences we use, an increase in ψ is equivalent to increasing the static elasticity of labor supply. The values of the other parameters necessary to give 2% growth and a cash good to GNP ratio of .1 are:

	λ=1.0, ψ=4.0	λ=1.0, ψ=8.0	λ=-.5, ψ=4.0	λ=-.5, ψ=8.0
A	.974	1.399	.974	1.399
η	9.000	9.000	1.681	1.681.

Table III

	μ	1.00	1.05	1.10	1.50	2.00	3.00	10.00
λ=1.0, ψ=4.0	γ	1.02	1.02	1.019	1.017	1.015	1.012	1.003
	ΔU	.000	.007	.014	.068	.128	.225	.546
λ=1.0, ψ=8.0	γ	1.02	1.02	1.019	1.017	1.014	1.011	1.002
	ΔU	.000	.008	.015	.074	.137	.239	.568
λ=-.5, ψ=4.0	γ	1.02	1.02	1.019	1.018	1.018	1.018	1.019
	ΔU	.000	.007	.014	.062	.102	.148	.222
λ=-.5, ψ=8.0	γ	1.02	1.02	1.019	1.018	1.018	1.018	1.019
	ΔU	.000	.008	.015	.065	.106	.151	.222

As can be seen from this table, the growth effects of moderate inflation are quite small. Moreover, as expected, when the cash and credit goods are substitutes, the relationship is not monotone and, when μ is large, there are no growth effects of inflation. However, the welfare costs of this distortion continue to increase as the equilibrium composition of consumption moves farther from the optimal level (i.e., when the relative price is one).

When the cash and credit goods are complements and the rate of inflation is high, we find significantly higher effects on both growth and welfare. These two are related. When the two goods are complements an increase in the rate of inflation increases the full price of consumption decreasing labor supply. This has two opposing effects on welfare. First, this increase in leisure increases welfare (this is the third term in equation (22)). Offsetting this is the negative impact of lower utilization of the stock of human capital. This reduces the rate of return on investment resulting in a lower rate of growth (this corresponds to the fourth term in equation (22)). In this case, the growth effect dominates the welfare calculation. (The size of this 'growth effect' is the reason that the relationship between complementarity and welfare loss is reversed from that seen in Table I.)

Increasing the value of leisure (through increasing ψ) gives rise to a lower rate of growth, ceteris paribus. It is because of this that A must be dramatically increased to keep the rate of growth at 2% when μ=1. The results reported in Table III show that different static elasticities of labor supply consistent with the same initial steady state growth path result in similar effects on growth and welfare from changes in monetary policy.

In addition to the experiments presented here, different values of σ were explored (σ=1.1 and 3.0). Similar results were obtained in these cases.

As a final point, note that it is possible to stifle growth altogether (and even make it negative) in this model if μ is increased to extreme (but not unheard of) levels. In this case, the effect of increasing μ is to lower the marginal productivity of the factors to the point where it is no longer worthwhile to save.

From this exercise, it can be seen that the relative importance of the various parameters in determining the growth and welfare effects of inflation differ substantially. In particular, the parameters describing the intertemporal elasticity of substitution in consumption and the static elasticity of labor supply have a minor impact on these calculations. In contrast, the results are highly sensitive to the elasticity of substitution between cash and credit goods which determines the properties of the money demand function.

6. Remarks

(1) The route to a monetary impact on growth in the simple model of endogenous growth analyzed in section 4 is interesting and somewhat subtle. The effect of changes in monetary policy is to change the distortionary impact on the (static) relationship determining labor supply (i.e., n) and consumption. This effect is present in both versions of the model as evidenced in equations (2) of section 2 and (12) of section 4. In general, increasing μ causes n to change as the effective relative price of leisure is altered. However, the relationship need not be monotone as can be seen in Proposition 1.

This change in n directly gives rise to a change in γ with the Lucas labor supply technology because n corresponds to the rate of utilization of human capital (see equation (21)). This is due to the change in the interest rate (i.e., the marginal product of capital) that results from a change in n in this model.

In contrast to this, since the limiting rate of interest is independent of n in the model of sections 2, there is no growth effect.

This points to an unusual property of the model in section 4. This is that one of the key determinants of the steady state growth rate is the steady state level of effort, n. That is, in this model, an increase in effort, ceteris paribus, gives rise to an increase in the growth rate of the economy. This seems to be a direct result of the form of the Lucas effective labor technology.

Note that this gives a potential explanatory variable for the observed cross country differences in growth rates in addition to those that have been identified previously. This is the difference across countries in leisure preferences (as measured by ψ). This is in addition to the pure inflation effect outlined in section 4. (And in addition to the variables identified earlier-patience, intertemporal substitutability, parameters of technology and fiscal variables.) Moreover, because of this fact, it follows that other fiscal variables that affect the equilibrium level of effort will have growth effects. Examples of this include consumption taxes (as seen in Jones, Manuelli and Rossi (1993)) and tariffs.

(2) It is of interest to conduct an experiment similar to the one explored here in a two sector model of endogenous growth as described in Rebelo (1991), Jones and Manuelli (1992) and Fisher (1992). This will permit us to

differentiate between the effects of a nontrivial role for labor supply (as opposed to the model of section 2) from the labor supply impact on the rate of utilization of human capital and study the role that these two differences play in the qualitative characteristics of steady state growth.

To this end consider a model of capital formation in which the consumption and investment goods are produced from two different technologies:

$$c_t = A\, n_t^{1-\alpha}\, (k_{1t})^{\alpha}$$

$$k_{t+1} = (1-\delta)\, k_t + b\, k_{2t}$$

$$k_t = k_{1t} + k_{2t}$$

In this formulation, it follows that $\gamma_c = (\gamma_k)^{\alpha}$ where γ_c and γ_k are the growth rates of consumption and capital respectively. In this case, it can be shown that, as in the model presented in section 2, monetary policy, although it does affect the steady state supply of labor, does not effect the long run interest rate. Because of this, changes in μ do not effect γ_k and hence γ_c is independent of the rate of inflation.

It follows from this that the effects that we find in the model of section 4 are from the specific form of the Lucas labor supply technology and not the change in the role played by n. Of course, any other multiplicative form for effective labor (e.g., z=g(n)h) will have similar effects.

(3) Our numerical results show that for low levels of monetary expansion, both the growth and welfare effects are small. This is true independent of the

specification of the model analyzed. This is in keeping with the results from studies conducted using a variety of models (see Lucas (1981) and (1993), Cooley and Hansen (1989), Eckstein and Leiderman (1992) and Imrohoroglu (1992)).

For high rates of monetary expansion, the size of the growth and welfare effects are very sensitive to the details of the model. In particular, the nature of the demand for money (as determined by the elasticity of substitution between cash and credit goods) and the effect of labor supply on human capital utilization are crucial. When cash and credit goods are complements and a Lucas style effective labor technology is used, both the growth and welfare effects of high inflation are quite large.

In all of the experiments that we conduct we find that at low levels of inflation the welfare loss is approximately linear. However, as the rate of inflation is increased, the welfare loss increases less than proportionately.

(4) In all of the experiments on monetary policy that we conduct in this paper, we are simultaneously changing fiscal policy. This can be seen by the fact that T_t is, in equilibrium, adjusted so that the government's budget constraint still holds. There are alternative experiments that could be explored. Other possibilities include holding the level of spending fixed in real terms throughout time and adjusting other fiscal variables at the same time as μ is changed, holding the fraction of government spending relative to output fixed, etc. For these alternatives, any change in policy that has revenue effects (e.g.,

changing bidding rules for off-shore oil parcels) can have growth effects even in the simple model that is explored in section 2. This is due to the fact that counterbalancing changes in taxes on investment income will always have growth effects. For example, if a policy change which generates an increase in revenue is accompanied by a reduction in capital income taxes, the effect on growth rates will necessarily have a positive component.

This is the essence of the reason for the growth effect of monetary policy found in Hartman (1988). In that paper, the government must finance a fixed stream of expenditures through a combination of an inflation tax and a tax on capital income while satisfying period-by-period budget balance. Given this, it follows that there is a possibility of substitution between the two instruments. Because of this, associated with an increase in the rate of inflation is a reduction in the tax on capital income. This raises the effective after tax rate of return on capital income. It is this last property that gives rise to the growth effects in Hartman's model (cf. Jones and Manuelli (1990) and Rebelo (1991)) and the differences between his results and those obtained in section 2 above).

(5) In the Kormendi and Meguire (1985) study cited in the introduction, they find two distinct effects of monetary policy in the sense of the model described here. The first is a negative impact of inflation on growth while the second is a positive effect of monetary expansion on growth. Of course, these are the same in the context of model 2. It would be of interest to develop a

model in which the link between inflation and monetary policy is weaker in order to see which gives rise to the negative relationship between inflation/expansionist monetary policy and growth. Although, we are not sure of this, we conjecture that in such a model it is the inflationary effect which is the cause of the impact on growth. (Because it is this that enters the consumer's problem directly.)

(6) The results that we have outlined in the examples to this point show that there is a relationship between the rate of growth of the money supply and the rate of growth of output in a simple model in which there is only a transactions reason for holding money. It would be of interest to explore this connection in other models of money. For example, if a part of monetary policy is the issuance of bonds in an uncertain environment, monetary policy may have growth effects for quite different reasons. If, by issuing debt, the government changes the perceived portfolio of risky assets available to investors (so that there is a precautionary demand for 'money') a direct growth effect may be felt. This could be done by developing a model in which all intrinsic economic investment opportunities involve a high degree of uncertainty and yet the government issues debt with a certain payoff which is financed through a stochastic policy of taxation and/or increases in the money supply. This change in the portfolio opportunities available to investors could very well reduce investment in the (productive) highly risky production activities with a high average payoff resulting in a reduction in the growth rate

of the economy.

For a paper that explores the effects of inflation in a setting in which money and real capital are held for portfolio reasons in a turnpike environment, see Mitsui and Watanabe (1989).

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Appendix

Proof of Proposition 1:

(i) It is straightforward to calculate the balanced growth path for a version of the economy with no cash-in-advance constraint. It is given by (11) (with R=0), (12) (with R=0), (13), (14) and (17)-(19). These equations can be summarized by:

(A.1) $$\gamma = 1 - \delta + B\, n^{1-\alpha} [\, 1 - \{(1-\alpha)(1-n)\}/\psi n\,]$$

(A.2) $$\gamma^{\sigma} = \beta\, [\, 1 - \delta + B\, n^{1-\alpha}\,].$$

Note that (A.1) is just (20) when $f(\mu,\gamma)=1$ and (A.2) is the same as (21). Denote the solution to (A.1) and (A.2) by (γ^*,n^*) and suppose the parameters are such that $\gamma^*>1$. (This can always be guaranteed for B sufficiently large.)

Consider the monetary version of the model when $\mu=\mu^*=\beta\ \gamma^{*(1-\sigma)}$. It is easy to verify that $f(\mu^*,\gamma^*) = 1$ and, hence, that (γ^*,n^*) solve (20) and (21) for this choice of monetary policy. Since (γ^*,n^*) is the unconstrained optimum (i.e., in the absence of the cash-in-advance constraint), it follows that it is also the constrained optimum.

This is the standard result from cash-in-advance economies that the optimal policy 'undoes' the cash-in-advance constraint.

Before we analyze cases (ii) and (iii), it is necessary to describe an equilibrium that is not interior. If μ is such that the solution to (20) and (21) satisfies $\gamma\geq 1-\delta$, then this solution truly describes the equilibrium behavior of the system. On the other hand, if μ is such that the solution to (20) and (21) gives $\gamma<1-\delta$, then (20) and (21) are not the appropriate first order conditions (since this requires negative investment) to describe the equilibrium. In this case, it follows that $\gamma=1-\delta$ and investment is zero. In this case, (21) need not hold at equality and the equilibrium value of n is given by the solution of (20) with $\gamma=1-\delta$.

Rewrite (20) and (21) as $\gamma = h^1(n,\mu)$ and $\gamma = h^2(n)$ where h^1 is implicitly defined by (20) and $h^2(n) = \{\beta[1-\delta+Bn^{1-\alpha}]\}^{1/\sigma}$. Note that $R\geq 0$ (as required by equilibrium) if and only if $f(\mu,\gamma)\geq 1$. This implies that for all n and μ,

$$h^3(n)\equiv 1-\delta+Bn^{1-\alpha} \geq h^1(n,\mu) \geq h^4(n)\equiv 1-\delta+Bn^{1-\alpha}[1-\{(1-\alpha)(1-n)\}/\psi n].$$

The equilibrium values of γ and n are given by the solution to the system of equations determined by h^1 and h^2 for values of n such that $\gamma\geq 1-\delta$. Since h^2 is increasing in n, it follows that, in any equilibrium, $\gamma\leq\gamma^*$.

It can be seen that $h^1(1,\mu) = 1-\delta+B$ and that for every $\gamma\in[1-\delta,1-\delta+B]$, there is an $n\in(0,1]$ such that $\gamma=h^1(n,\mu)$. Define $n(\mu)$ so that $1-\delta = h^1(n(\mu),\mu)$. It follows that $n(\mu) = (1-\alpha)/(1-\alpha + \psi\, f(\mu,1-\delta))$.

(ii) If $\lambda\geq 0$, it can be shown that $\lim_{\mu\to\infty} f(\mu,\gamma)=\infty$. Hence, it follows that $h^1(n,\mu)$ converges to $h^3(n)$ pointwise as $\mu\to\infty$. Since $h^3(n) > h^2(n)$ for all n, $h^3(0)=1-\delta$, and $h^2(0)=\{\beta(1-\delta)\}^{1/\sigma}<1-\delta$, it follows that there is some μ^* such that when $\mu>\mu^*$, the solution to (20) and (21), if it exists, has the property that $\gamma<1-\delta$. Hence, when $\mu>\mu^*$, the equilibrium path is given by $\gamma=1-\delta$ and

$n=n(\mu)$. Moreover, it follows that $\lim_{\mu\to\infty} n(\mu)=0$.

(iii) If $\lambda<0$, it can be shown that $\lim_{\mu\to\infty} f(\mu,\gamma) = 1$. It follows that $h^1(n,\mu)$ converges pointwise to $h^4(n)$ (the right hand side of (A.1)) for $n>0$. Two things follow from this fact. First, it follows that when μ is sufficiently large, the solution to (20) and (21) has $\gamma\geq 1-\delta$. Thus, when μ is large, the equilibrium is given by the solution to (20) and (21). Second, it follows that as $\mu\to\infty$, the rate of growth along the equilibrium path converges to the solution to (A.1) and (A.2), γ^*.