

NBER WORKING PAPER SERIES

OPTIMAL TRANSPARENCY IN A  
DEALERSHIP MARKET WITH AN  
APPLICATION TO FOREIGN EXCHANGE

Richard K. Lyons

Working Paper No. 4467

NATIONAL BUREAU OF ECONOMIC RESEARCH  
1050 Massachusetts Avenue  
Cambridge, MA 02138  
September 1993

An earlier version of this paper was circulated as "Equilibrium Microstructure in the Foreign Exchange Market." I would like to thank Mark Flood, Ben Hermalin, James McCarthy, S. Natarajan, Chester Spatt (the editor), Avanidhar Subrahmanyam, and anonymous referee, and seminar participants at Berkeley, Columbia, NYU, Yale, and the Western Finance Association for helpful comments/discussions, and Merrill Lynch and Lasser Marshall for access to foreign exchange dealers and brokers during trading hours. Financial assistance from the Berkeley Program in Finance is gratefully acknowledged. Any errors are my own. This paper is part of NBER's research program in International Finance and Macroeconomics. Any opinions expressed are those of the author and not those of the National Bureau of Economic Research.

NBER Working Paper #4467  
September 1993

OPTIMAL TRANSPARENCY IN A  
DEALERSHIP MARKET WITH AN  
APPLICATION TO FOREIGN EXCHANGE

ABSTRACT

This paper addresses the issue of optimal transparency in a multiple-dealer market. In particular, we examine the question: Would risk-averse dealers prefer ex-ante that signed order flow were observable? We answer this question with the solution to a mechanism design problem. The resulting incentive-efficient mechanism is one in which signed order flow is not observable. Rather, dealers prefer a slower pace of price discovery because it induces additional risk-sharing. Specifically, slower price discovery permits additional trading with customers prior to revelation; this reduces the variance of unavoidable position disturbances, thereby reducing the marketmaking risk inherent in price discovery. We then apply the framework to the spot foreign exchange market in order to understand better the current degree of transparency in that market.

Richard K. Lyons  
350 Barrows Hall  
U.C. Berkeley  
Berkeley, CA 94720  
and NBER

# Optimal Transparency in a Dealership Market with an Application to Foreign Exchange

## Abstract

This paper addresses the issue of optimal transparency in a multiple-dealer market. In particular, we examine the question: Would risk-averse dealers prefer ex-ante that signed order flow were observable? We answer this question with the solution to a mechanism design problem. The resulting incentive-efficient mechanism is one in which signed order flow is not observable. Rather, dealers prefer a slower pace of price discovery because it induces additional risk-sharing. Specifically, slower price discovery permits additional trading with customers prior to revelation; this reduces the variance of unavoidable position disturbances, thereby reducing the marketmaking risk inherent in price discovery. We then apply the framework to the spot foreign exchange market in order to understand better the current degree of transparency in that market.

Richard K. Lyons  
350 Barrows Hall  
U.C. Berkeley  
Berkeley, CA 94720

---

\* An earlier version of this paper was circulated as "Equilibrium Microstructure in the Foreign Exchange Market." I would like to thank Mark Flood, Ben Hermalin, James McCarthy, S. Nagarajan, Chester Spatt (the editor), Avanidhar Subrahmayan, an anonymous referee, and seminar participants at Berkeley, Columbia, NYU, Yale, and the Western Finance Association for helpful comments/discussions, and Merrill Lynch and Lasser Marshall for access to foreign exchange dealers and brokers during trading hours. Financial assistance from the Berkeley Program in Finance is gratefully acknowledged. Any errors are my own.

# Optimal Transparency in a Dealership Market with an Application to Foreign Exchange

The degree to which markets aggregate information fast and effectively is partially a function of institutional design. Faster revelation is preferred in general, at least with respect to efficient resource allocation. However, faster is not necessarily preferred by all agents. And when those agents preferring slower revelation compose the microstructure of the market, institutional evolution might serve *their* definition of optimality, rather than some more general criterion.

This paper addresses the issue of optimal transparency in a multiple-dealer market. In particular, we examine the following question: Would risk-averse dealers prefer ex-ante that signed order flow were observable? By framing the question as a mechanism design problem, we are able to characterize the central trade-offs involved. The resulting incentive-efficient mechanism is typically one in which signed order flow is not observable. In other words, dealers prefer slower price discovery than would occur if order flow were observable. What drives the result is that additional trading prior to revelation provides additional risk-sharing. More specifically, because dealers incur unavoidable position disturbances, they prefer price discovery — and the attendant volatility — to occur when the variance of position disturbances is smaller. Additional trading lowers the variance of position disturbances because non-dealer participants share risk otherwise borne by dealers.

Transparency of the trading process is a distinct dimension of microstructural configuration. Biais (1993) highlights this fact in his analysis of fragmented versus consolidated markets [see also Wolinsky (1990)]. In his model the decentralized dealership market is posited as fragmented, while the centralized market is

consolidated. Though it is natural to associate fragmented markets with decentralized markets, decentralized markets are not always fragmented. For example, Leach and Madhavan (1993) model multiple dealers as observing all trades and quotes whether or not they participate. In contrast to these and other studies, the model here does not posit a degree of transparency, it derives it as a preferred informational configuration.

The mechanism design approach of this paper is not new, though the spirit of its application is different than in traditional work on mechanism design. The traditional work concentrates on underlying investors — their strategies and risk-sharing needs — in a context of asymmetric information; dealers are typically non-existent.<sup>1</sup> Here we cut into the problem at a different level, the level of the dealers themselves. Individual investors are small in our model, and therefore behave competitively. The strategic interaction occurs between dealers. Hence, our focus is the adverse selection problem among dealers, not between customers and dealers. Some justification for the approach is as follows. First, in the largest multiple-dealer market of them all — the wholesale spot foreign exchange (FX) market — inter-dealer trading accounts for more than 80% of total trading.<sup>2</sup> Second, there are far fewer dealers than non-dealer participants in most dealership markets, suggesting that intermediation can concentrate order-flow information at the dealer level [see Lyons (1993a)]. Finally, *concentrated* private information at the investor level in a market like FX is not realistic under normal circumstances.

The mechanisms we examine here are structured for direct analysis of the issue

---

<sup>1</sup> The seminal work on efficient trading mechanisms is Myerson and Satterthwaite (1983). For a model in which a role for a single marketmaker emerges in an efficient mechanism see Nagarajan and Ramakrishnan (1992).

<sup>2</sup> See NY FED (1989). Inter-dealer trading *never* occurs in the model of Biais (1993). Further, in his model agents are risk-averse but shocks are due solely to liquidity; in Wolinsky (1990), by comparison, information asymmetries exist but agents are risk-neutral. Our model includes both risk aversion and information asymmetries, the combination of which creates the central tension driving our results.

at hand: dealer preferences over order-flow observability. That is, we restrict our attention to mechanisms that gather and disseminate order-flow information only. This class of mechanisms is intended to capture the institutional constraints/realities of multiple-dealer markets like FX. The cost is that we lose the full power of the revelation principle since we do not consider all feasible mechanisms. Accordingly, it is not our claim that the incentive-efficient mechanism within the set we consider is necessarily incentive-efficient under all forms of information exchange.

The paper is organized as follows: Section I presents the trading model; Section II determines the set of communication equilibria, the elements of which are indexed by the transparency of order flow; Section III solves for the incentive-efficient mechanism from the set determined in the previous section; Section IV provides a suggestive link between the model and the spot FX market; and Section V concludes.

## I. The Model

First we outline the general features of the model. A more precise definition of its structure follows.

### A. General Features of the Model

There are two types of agents in the market for a single risky asset, dealers and customers, where customers comprise all non-dealer participants (e.g., investors, speculators, corporate treasurers, liquidity traders, central banks, etc.). Customers behave competitively, whereas dealers — of which there are  $n$  — behave strategically. In each of two periods there are four events in the following sequence:

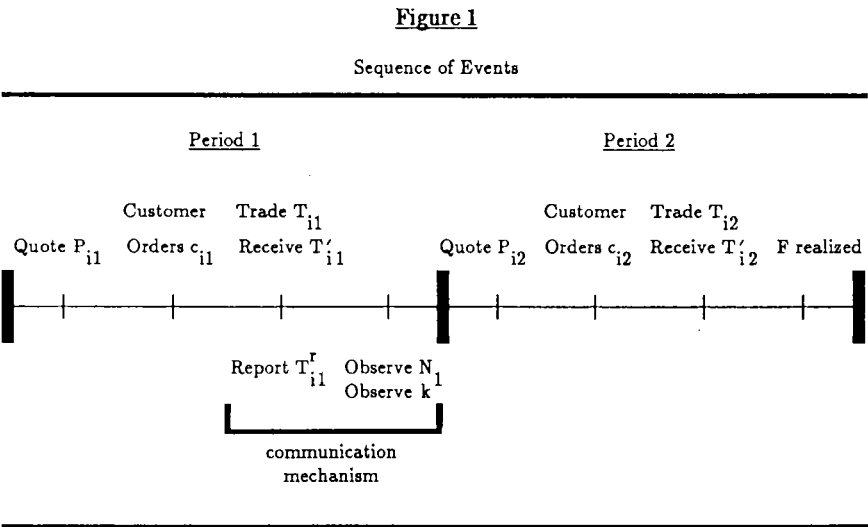
Event 1: Each dealer quotes a single firm price that is observable and available to all participants in the market.<sup>3</sup>

---

<sup>3</sup> The assumption that quotes are observable in our model is tantamount to assuming that quote search is costless.

- Event 2: Each dealer then accommodates customer market-orders individually. A distinct set of customers, representing a customer base, is linked to each dealer. The customer trades of other dealers are not observable. Because customer order flow is informative, this is the source of the asymmetric information among the dealers.
- Event 3: Dealers then trade amongst themselves. The order flow that results from inter-dealer trading in period one — to the extent that it is observable — drives the determination of period-two quotes.
- Event 4: The last event each period is public revelation of information regarding the value of the asset; in period one a signal of net inter-dealer order flow is observed, and in period two the full-information value itself is revealed.

Figure 1 provides an overview of events, and introduces some notation that is clarified below.



An important component of the model is the fact that customer order flow in period one is of a different nature than in period two. The intent is to capture the reality that customer activity in markets like FX often exhibits bursts of trading, as opposed to a smooth dribbling-in of orders over time. These bursts have effects on the variability of undesired dealer positions. Accordingly, in period one, each dealer receives customer-orders that both disturb the dealer's position and provide information. Period two, in contrast, is the tranquil period with respect to customer orders: there are no exogenous customer trades in period two. Of course, as risk-averse optimizing agents, customers are induced to trade if the risky asset is priced to yield a sufficient return conditional on public information. Under our specification of preferences, period-two customer trades are a deterministic linear function of any bias in period-two prices.

The last general feature that warrants attention is the communication mechanism. Per Figure 1, communication occurs at events three and four in period one: at event three dealers each report their period-one trades, and at event four all observe a common signal of net period-one order flow. These two events will govern the transparency of period-one trading. The approach posits a mediator, or central planner, that mediates the transmission of order-flow information between dealers in the model. Equilibrium will not, however, require any judgment on the part of the mediator: the information flows that produce equilibrium could be effected by computer; dealers in the model would no sooner cheat on the computer than contradict their equilibrium strategies in our model, which is inconsistent by definition. When it comes to *selection* of an equilibrium, on the other hand, the mediator's role is no longer trivial. We justify our focus on the incentive-efficient equilibrium with the basic tenet that institutions evolve toward optimality.



## B. Specifics of the Model

The model is a two-period game with  $n$  dealers. Each dealer has an equal-sized customer base composed of a large number of non-strategic customers. All dealers and customers have identical negative exponential utility defined over final wealth. There are two assets, one risky and one riskless, whose returns are realized at the close of period 2. The gross return on the riskless asset is normalized to one. The risky asset is in zero supply initially, and has a random full-information value  $F$ .<sup>4</sup>

### Event One: Dealer Quoting

At the outset of both periods each dealer quotes a price. Let  $P_{it}$  denote the quote of dealer  $i$  in period  $t$ . The rules governing each  $P_{it}$  are the following:

- (R1) Quotes are observable and available to all participants in the market.
- (R2) Quotes are singleton prices at which dealers agree both to buy and sell.
- (R3) Quotes are good for any size.<sup>5</sup>
- (R4) Dealers cannot refuse to quote.

This last rule is particularly relevant in period two when a dealer with a near-zero customer-order in period one — and therefore little information — would choose to exit if possible. In reality, in a market like FX those who breach the implicit contract of reciprocity in quoting are punished by other dealers (e.g., breaches are met with subsequent refusals to provide quotes or by quoting large spreads).

---

<sup>4</sup> Assuming non-zero initial supply does not affect the results; it simply requires an arbitrarily small bias in initial prices (conditional on public information) to induce agents to hold the initial supply.

<sup>5</sup> Relative to other decentralized markets, the sizes tradable at quoted prices in the FX market are very large; among the largest marketmakers, DM/\$ quotes are good for \$10 million. See also Pithyachariyakul (1986) and Mendelson (1987) for transaction prices that are constant independent of volume.

## Event Two: Customer Orders and a Definition of the Information Structure

Private information at the dealer level is determined by the customer market-orders each receives in period one. Let  $c_{i1}$  denote the *net* of the signed customer-orders received by dealer  $i$  in period one.  $c_{i1}$  is assumed independently and normally distributed about zero with known variance  $\Sigma_c$ .<sup>6</sup> At least two sources of correlation between customer orders and full-information value are plausible. First, superior information-processing by certain non-dealer participants may be present. Second, even purely liquidity-motivated customer orders may be informative. For example, in the FX market customer-orders may reveal information about firms' net export performance, which is in turn relevant for exchange rate determination.<sup>7</sup> More formally, we specify the sum of the  $n$  net orders as a signal of the full-information value of the risky asset,  $F$ :<sup>8</sup>

$$(1) \quad \sum_{i=1}^n \tilde{c}_{i1} = F + \tilde{\nu}$$

where  $\nu$  is normally distributed with mean zero and variance  $\Sigma_\nu$ . We use the convention that  $c_{it}$  is positive for net customer purchases, and negative for net sales. This specification implies that before customer-orders are received each dealer's prior expectation of the full-information value is zero; after receiving his customer-orders, each dealer has some private information.

There are no exogenous customer trades in period two. Given our specification

---

<sup>6</sup> The sequentially rational  $P_{i1}$  will be unbiased conditional on public information, so the assumption of zero mean is not imposing irrationality at the customer level.

<sup>7</sup> The information-processing interpretation is subject to the "schizophrenia" problem since customers are non-strategic here; however, the critique does not apply to the revealed net-export interpretation.

<sup>8</sup> Lyons (1993a), using transaction data, demonstrates that trading activity has significant information effects on quoted prices in the FX market.

of negative exponential utility, period-two customer-orders are a deterministic linear function of any bias in period-two prices:

$$(2) \quad c_{i2} = \zeta \left[ E[F | \Omega_{C2}] - P_{i2} \right].$$

Here,  $\Omega_{C2} = \{P_{11}, \dots, P_{n1}, N_1\}$  is the public information available to customers at the beginning of period two, where  $N_1$  is a signal of net inter-dealer volume in period one (defined under the mechanism below). The constant  $\zeta$  is common to each dealer, and corresponds to a combination of: (i) the coefficient of absolute risk aversion, (ii) the variance of the period-two return conditional on  $\Omega_{C2}$ , and (iii) the number of customers in each equal-sized customer base.  $\zeta$  approaches infinity as the number of customers in each customer base approaches infinity.

#### Event Three: Inter-Dealer Trading Protocol

Event three in both periods corresponds to inter-dealer trading. Inter-dealer trading is effected at the prices quoted at event one. Let  $T_{it}$  denote the net of outgoing inter-dealer orders placed by dealer  $i$  in period  $t$ ; let  $T'_{it}$  denote the net of incoming inter-dealer orders received by dealer  $i$  in period  $t$ , placed by other dealers. The rules governing inter-dealer trading are as follows:

- (R5) Trading is simultaneous and independent; dealers cannot condition on the realization of  $T'_{it}$  when determining  $T_{it}$ .<sup>9</sup>
- (R6) Trading with multiple partners is feasible.
- (R7) Trades in period one are allocated to the dealer on the left if there are common

---

<sup>9</sup> Simultaneous moves are consistent with the fact that transactions in the spot FX market, for example, are typically initiated electronically rather than verbally, providing the capacity for simultaneous quotes, trades, or both. Here, a period should be viewed as the time it takes to make a trade, a span measured in seconds rather than hours, days, or weeks.

quotes at which a transaction is desired (dealers are arranged in a circle).

(R8) Trades in period two are allocated to the second dealer on the left if there are common quotes at which a transaction is desired.

Rule (R5) generates an important role for  $T'_{it}$  in the model: because inter-dealer trading is simultaneous and independent,  $T'_{it}$  is an unavoidable disturbance to dealer  $i$ 's position in period  $t$  that must be carried into the following period. Rule (R8) prevents dealers from having specific information about their period-two partners, such as the level of undesired inventories.

For consistency with our previous definition of  $c_{it}$  as positive for net customer purchases, orders will always be signed according to the party that initiates the trade. Thus,  $T_{it}$  is positive for dealer  $i$  purchases, and  $T'_{it}$  is positive for purchases by other dealers from dealer  $i$ . Consequently, positive  $c_{it}$ 's and  $T'_{it}$ 's correspond to dealer  $i$  sales. Accordingly, by definition:

$$(3) \quad T_{i1} = D_{i1} + c_{i1} + E[T'_{i1} | \Omega_{i,1,3}]$$

$$(4) \quad T_{i2} = D_{i2} + c_{i2} + T'_{i1} - D_{i1} + E[T'_{i2} | \Omega_{i,2,3}]$$

where  $D_{it}$  denotes dealer  $i$ 's period  $t$  desired position in the risky asset. From equations (3) and (4) it is clear that customer-orders must be offset to establish the desired net position  $D_{it}$ . Additionally, in period two the realized period-one position must be offset, which has both an undesired component  $T'_{i1}$  and a desired component  $D_{i1}$ . Finally, to establish their desired net position dealers must factor the expected value of  $T'_{it}$  into their own trades.

The evolution of dealer  $i$ 's information set is provided in Table 1, with the following notation: the set  $\Omega_{i,t,e}$  denotes the information available to dealer  $i$  in

period  $t$  just prior to event  $e$ . That is, it defines the information available for any decision at event  $e$ .

**Table 1**

The Evolution of  $\Omega_i$

Period 1	Period 2
$\Omega_{i,1,1} = \{E(F)=0\}$	$\Omega_{i,2,1} = \Omega_{i,1,4} \cup \{N_1, k\}$
$\Omega_{i,1,2} = \Omega_{i,1,1} \cup \{P_{11}, \dots, P_{n1}\}$	$\Omega_{i,2,2} = \Omega_{i,2,1} \cup \{P_{12}, \dots, P_{n2}, c_{i2}\}$
$\Omega_{i,1,3} = \Omega_{i,1,2} \cup \{c_{i1}\}$	$\Omega_{i,2,3} = \Omega_{i,2,2}$
$\Omega_{i,1,4} = \Omega_{i,1,3} \cup \{T_{i1}, T'_{i1}, T^F_{i1}\}$	$\Omega_{i,2,4} = \Omega_{i,2,3} \cup \{T_{i2}, T'_{i2}\}$
	$\Omega_{i,Final} = \Omega_{i,2,4} \cup \{F\}$

The variables  $T^F_{i1}$ ,  $N_1$ , and  $k$  are defined in the following section which describes the communication mechanism. Henceforth, let the information set  $\Omega_i$  denote dealer  $i$ 's realization from  $\Omega$ , where  $\Omega$  is the set of all feasible histories  $\Omega_{i,t,e}$ .

### The Communication Mechanism and Dealer Optimization

In the spirit of Myerson (1985), define a *communication mechanism*  $\mu$  as a centralized communication system comprising the following two events: (i) each dealer makes a confidential report of his private information to a central mediator; then (ii) the mediator computes — possibly randomly — a signal from these reports and communicates it. Per Figure 1, the two events of the communication mechanism occur at events three and four in period one. At event three, dealer  $i$ 's report to the

mediator occurs simultaneously with inter-dealer trading (and therefore cannot be conditioned on the realization of  $T'_{i1}$ ).

Here, we focus our attention on a class of mechanisms that accords most closely with information flows in financial markets.<sup>10</sup> Specifically, the report each dealer makes is the value of  $T_{i1}$ , the period-one outgoing orders he places. We denote this report  $T_{i1}^r$ . The signal communicated publicly by the mediator at the end of period one has two components. The first is a measure of net market volume, which we denote  $N_1$ , and the second is a proportional cost to misreporting, which we denote  $k$ . More formally, we restrict our attention to mechanisms satisfying:

$$(5) \quad \mu(N_1, k | T_{i1}^r, \dots, T_{n1}^r): T_{i1}^r \in \mathbb{R}, k \in \mathbb{R}, \text{ and } N_1 \equiv \sum_{i=1}^n T_{i1}^r + \tilde{\epsilon} \in \mathbb{R}$$

where the random variable  $\epsilon$  is normally distributed with zero mean and known variance  $\Sigma_\epsilon$ . This random variable introduces the possibility that the mediator's message is a random function of the information received. In the following section, we use  $\Sigma_\epsilon$  as a means of indexing various communication equilibria.

In order to characterize more precisely the role of  $N_1$ ,  $k$ , and  $T_{i1}^r$ , we turn now to the dealers' problem, which can be expressed as:

---

<sup>10</sup> Laffont and Maskin (1990) show that a monopolistic insider can prefer a pooling equilibrium to separating equilibria. However, the feature of their model that drives their counter-intuitive result is not present in our model. Namely, in their model, the incentive constraints of the separating equilibrium create quantity constraints at low prices; in other words, at some point current prices must respond to demand. Because the monopoly insider cannot buy as much in a separating equilibrium as in a pooling equilibrium, he favors the pooling equilibrium. In our dealership model, on the other hand, current quotes have no size limits, so this advantage to pooling is not present.

$$(6) \quad \max_{S_i} E[U(W_{i2}) | \Omega_i]$$

$$\text{s.t.} \quad U(W_{i2}) = -\exp(-\theta W_{i2})$$

$$W_{i2} = W_{i0} + D_{i1}(P'_{i2} - P_{i1}) + D_{i2}(F - P'_{i2}) \\ - T'_{i1}(P'_{i2} - P_{i1}) - T'_{i2}(F - P_{i2}) - k|T^r_{i1} - T_{i1}|$$

where  $W_{it}$  denotes dealer  $i$ 's the end-of-period  $t$  wealth,  $P_{i1}$  denotes dealer  $i$ 's period-one quote, and a " ' " denotes an incoming quote or trade received by dealer  $i$ . The five choice variables over which the maximization is defined are contained in the strategy set  $S_i \equiv \{P_{i1}, P_{i2}, T_{i1}, T_{i2}, T^r_{i1}\}$ . The terms  $T'_{i1}(P'_{i2} - P_{i1})$  and  $T'_{i2}(F - P_{i2})$  in final wealth account for the effects of position disturbances in periods one and two, respectively.

The last term in the definition of final wealth,  $k|T^r_{i1} - T_{i1}|$ , is the proportional cost to the dealer of mis-reporting his period-one trade. This cost is central to incentive-compatibility since if there were no cost, each dealer would have an incentive to report a trade of the same sign but larger magnitude than the true trade, pushing the period-two price in the desired direction other things equal; in fact, there would always be a incentive to report an infinitely large trade in the preferred direction (a point we establish when characterizing equilibrium). We justify the cost in the following way, using the FX market as an example.

In many dealership markets there are two segments, one we refer to as the observable segment, and the other the unobservable segment, where observability refers specifically to net volume. In the FX market, the observable segment with respect to net volume is the brokered inter-dealer segment, whereas the unobservable

segment is the direct inter-dealer segment.<sup>11</sup> A market manipulation of the kind that corresponds to mis-reporting in our model would require a magnified trade in the observable sector that is simultaneously partially reversed in the unobservable sector. The important point is that a "report" represents an actual trade. And the fact that the inflated portion of the observable trade must be reversed implies that a round-trip transaction cost must be paid on that portion. Hence,  $k$  should be viewed as an incremental cost measured by the difference between a round-trip transaction and a one-way transaction (since in our model the cost of a one-way transaction is zero). That  $k$  is not known exactly at the time of inter-dealer trading is consistent with uncertain depth in actual markets. Further, since in reality this cost is a direct cost to any dealer using this strategy, this helps justify the fact that in our model it is not necessary for the mediator to observe  $|T_{11}^F - T_{11}|$ .

A mechanism  $\mu$  defines a *communication equilibrium* iff it is a Bayesian equilibrium for each dealer to report his outgoing period-one net trade  $T_{11}$  honestly, and to condition period-two quoting and trading on the mediator's signal under the mechanism  $\mu$ ; i.e.,  $\mu$  must be incentive compatible. Here,  $\tilde{\mu}$  defines a communication equilibrium iff:

$$(7) \quad E\left[U\left[W_{i2}(\tilde{\mu}, \tilde{s}_i)\right] \middle| \Omega_i\right] \geq E\left[U\left[W_{i2}(\tilde{\mu}, s_i)\right] \middle| \Omega_i\right], \quad \forall i \in \{1, \dots, n\}, \forall \Omega_i \in \Omega, \forall s_i: S_i \rightarrow S_i$$

where  $\tilde{s}_i$  is the strategy of dealer  $i$  described under the mechanism  $\tilde{\mu}$ , and  $S_i$  is the five-dimensional strategy set defined above.

The mechanism  $\hat{\mu}$  defines an *ex-ante incentive-efficient communication equilibrium* iff  $\hat{\mu}$  is incentive-compatible and there does not exist any other

---

<sup>11</sup> The observability of brokered trades derives from the fact that brokers typically announce when a trade has taken place, and whether it was initiated at the bid or offer. Section IV provides more detail. The potential for manipulation via brokered trades we are describing here is referred to by market participants as "painting a picture." See Stigum (1990), page 492.



incentive-compatible mechanism  $\mu$  such that:

$$(8) \quad E[U_i(\mu)] \geq E[U_i(\hat{\mu})], \quad \forall i \in \{1, \dots, n\}$$

with at least one strict inequality. Note that our definition includes only the dealers' expected utilities. This specification corresponds to the hypothesis that in an unregulated market like FX, for example, the institutional configuration will conform to the preferences of the dealers themselves. Because dealers are symmetric ex-ante, the incentive-efficient communication equilibrium maximizes the expected utility of a representative dealer. If a mechanism  $\mu$  is not incentive-efficient then it is common knowledge that all dealers would prefer to use some other incentive-compatible mechanism.

## II. Determining Communication Equilibria

To determine the communication equilibria in the above setting we first determine the properties of optimal quoting strategies. The following proposition addresses optimal period-one quotes:

*Proposition 1:* A quoting strategy is consistent with communication equilibrium only if the period-one quoted price  $P_{i1}=0 \ \forall i \in \{1, \dots, n\}$ .

Proof: The necessity of common quotes follows from quoting rules (R1)–(R4), trading rules (R5)–(R6), and risk-aversion since rational quotes must be common to avoid arbitrage. Unbiasedness is necessary to prevent non-zero expected inter-dealer trades, which are incompatible with equilibrium. Suppose not. Consider equation (3): if the common quote is biased conditional on public information, then  $E[T'_{i1} | \Omega_{i,1,3}] \neq 0$  since  $E[D_{j1} | \Omega_{i,1,3}] \neq 0$ , where  $D_{j1}$  denotes the desired dealer demand implicit in  $T'_{i1}$ . This induces each dealer to adjust  $T_{i1}$  to offset the expected effect on his position. Any across-the-board adjustment in  $T_{i1}$ , however, increases the absolute magnitude of the expected  $T'_{i1}$  one-for-one. It is therefore impossible for all dealers to expect rationally to offset the expected non-zero order. Finally,  $P_{i1}=0 \ \forall i$  since  $E(c_{i1})=0 \ \forall i$  and  $E(\nu)=0$ .

Henceforth, we denote the common period-one quote  $P_1$ . An implication of common quotes is that given the trading rule (R7), period-one orders are allocated to the first quote on the left, and each dealer receives exactly one order. In period-one, this order corresponds to the position disturbance  $T'_{i1}$  in the dealer's problem in equation (6). The next proposition addresses optimal period-two quotes:

*Proposition 2:* A quoting strategy is consistent with communication equilibrium only if the common period-two quoted price  $P_2$  is such that  $P_2 = E[F|P_1, N_1]$  and  $c_{i2} \rightarrow -E[T'_{i1} - D_{i1} | P_1, N_1] \forall i \in \{1, \dots, n\}$  as the number of customers in each customer base  $\rightarrow \infty$ .

*Proof:* The necessity of common quotes follows directly from the proof of proposition 1. The necessity that  $c_{i2} \rightarrow -E[T'_{i1} - D_{i1} | P_1, N_1] \forall i$  follows from the fact that any other level of period-two customer orders would result in non-zero expected inter-dealer trades, which are incompatible with equilibrium by the same reasoning in the proof of proposition 1. Drawing from equation (4), it is necessary that  $E[T'_{i2} | \Omega_{i,2,3}] = 0$ , which is true only if  $E[D_{j2} + c_{j2} + T'_{j1} - D_{j1} | \Omega_{i,2,3}] = 0$ , where again the subscript  $j$  denotes the dealer  $j$  components implicit in  $T'_{i1}$ . From trading rule (R8), dealer  $i$  knows nothing about the values of  $D_{j2}$ ,  $c_{j2}$ ,  $T'_{j1}$ , or  $D_{j1}$  that is not contained in the public information set  $\{P_1, N_1\}$ , so we can replace  $\Omega_{i,2,3}$  with  $\{P_1, N_1\}$ . Now, if  $P_2 = E[P_2 | P_1, N_1] \Rightarrow E[D_{j2} + c_{j2} | P_1, N_1] = 0$ ; however,  $E[T'_{j1} - D_{j1} | P_1, N_1] \neq 0$  since  $N_1$  is informative regarding period-one trading. Only a  $P_2$  such that  $E[D_{j2} + c_{j2} | P_1, N_1] = -E[T'_{j1} - D_{j1} | P_1, N_1]$  can be an equilibrium. Since  $c_{j2} \equiv \zeta \left\{ E[F | P_1, N_1] - P_2 \right\}$  from equation (2), and since  $\zeta \rightarrow \infty$  as the number of customers in each customer base  $\rightarrow \infty$ , the equilibrium  $P_2$  is necessarily such that  $P_2 = E[F | P_1, N_1] \rightarrow 0$ ,  $c_{j2} + E[T'_{j1} - D_{j1} | P_1, N_1] \rightarrow 0$ , and  $E[D_{j2} | P_1, N_1] \rightarrow 0$  as the number of customers in each customer base  $\rightarrow \infty$ .

Since we have assumed that each customer base is large in a convergence sense, henceforth we proceed under the assumption that  $P_2 = E[F | P_1, N_1]$  and  $c_{i2} = -E[T'_{i1} - D_{i1} | P_1, N_1]$ .

Proposition 2 is important for our results. The main point is that equilibrium  $P_2$  is necessarily such that customers absorb part of the position disturbance suffered by dealers as a result of period-one trading, namely  $-E[T'_{j1} - D_{j1} | P_1, N_1]$ . After

providing a complete solution to the model, we determine below a precise representation of this absorption.

The following proposition characterizes the communication equilibria that conform to the feasibility condition in equation (5):

*Proposition 3:* Each element of the set of mechanisms  $\{\mu^*: \Sigma_{\epsilon} \in [0, \infty)\}$  such that:

$$\begin{aligned} P_{i2} &= \Lambda N_1 \\ T_{i2} &= \beta_1 c_{i1} + \beta_2 T'_{i1} + \beta_3 N_1 + c_{i2} \end{aligned}$$

$\forall i \in \{1, \dots, n\}$  is a communication equilibrium if and only if  $k > \bar{k}$ , where  $\{\bar{k}, \Lambda, \beta_1, \beta_2, \beta_3\}$  are defined below.

*Proof:* The proof is in two parts: (i) first, we determine optimal trading strategies for both periods conditional on a mechanism of the proposed type assuming that dealers are induced to be honest in their reports to the mediator; (ii) then, we show that dealers' period-two actions are consistent with the mechanism and that they have no incentive to misreport their trades to the mediator if and only if the marginal cost of misreporting is greater than  $\bar{k}$ .

### (i) Optimal Trading Strategies

Determining optimal trading strategies requires dynamic programming. Appendix A shows that under the above assumptions the trading rules for each dealer can be written as:

$$(9) \quad T_{i1} = \phi c_{i1}$$

and 
$$T_{i2} \equiv D_{i2} + c_{i2} + T'_{i1} - D_{i1}$$

$$= \beta_1 c_{i1} + \beta_2 T'_{i1} + \beta_3 N_1 + c_{i2}$$

$$\text{where } \phi \equiv \left[ 1 + \frac{\theta + [(\Lambda - 1)\Sigma_F^{-1}][1 + \lambda_2(\theta - \lambda_2\Sigma_F^{-1})/(\lambda_2^2/\Sigma_F + 1/\Sigma_{N_1'})]}{(\theta - \lambda_2\Sigma_F^{-1})^2(\lambda_2^2\Sigma_{N_1'}\Sigma_F^{-1} + 1)^{-1}\Lambda\Sigma_{N_1'} - 2\theta - \Lambda\Sigma_F^{-1}} \right],$$

$$\beta_1 = \left[ \frac{1 - \lambda_1\phi}{\theta\Sigma_F} - (\phi - 1) \right], \quad \beta_2 = \left[ 1 + \frac{1/\phi - \lambda_1}{\theta\Sigma_F} \right], \quad \text{and } \beta_3 = \left[ \frac{\lambda_1 - \Lambda}{\theta\Sigma_F} \right].$$

The value of  $\Lambda$  in the period-two pricing rule appearing in proposition 3 is:

$$(10) \quad \Lambda = \frac{\phi \Sigma_c}{\phi^2 \Sigma_c + \Sigma_\epsilon/n}.$$

The derivation of these expressions establishes that dealers' period-two actions agree with the mechanism if reports are honest.

#### (ii) The Value of $k$ That Induces Truth-Telling

It remains to be shown that honesty in reporting  $T_{i1}$  is incentive compatible. As noted in the previous section, if there were no cost to misreporting then a dealer would always find it in his interest — if other dealers are reporting honestly — to report an inflated period-one trade  $T_{i1}$ . This is because over-reporting moves  $P_2$  in the direction desired by dealer  $i$ . Two facts are necessary to see this. First, the period-two quote  $P_2$  under the mechanism equals  $\Lambda N_1$ , and  $N_1$  depends linearly on dealer  $i$ 's report:  $dP_2/dT_{i1}^r = d(\Lambda N_1)/dT_{i1}^r = \Lambda > 0$ . And second,  $T_{i1}$  and  $D_{i1}$  will always have the same sign since  $T_{i1} = \phi c_i = D_{i1} + c_i$  and  $\phi > 1$ .

Now, the expected gain at the margin from misreporting is equal to:

$$dE[U(W_{i2})|\Omega_i]/dT_{i1}^r = dE\left[-\exp\left[-\theta[D_{i1}(P_2-P_1)+D_{i2}(F-P_2)-k|T_{i1}^r-T_{i1}|]\right]|\Omega_i\right]/dT_{i1}^r.$$

Under a given mechanism, variation in  $T_{i1}^r$  alters the subjective means of the two price changes, without altering the subjective second moments. Hence, the sign of the above derivative will be the same as the sign of:

$$\begin{aligned} dE\left[D_{i1}(P_2-P_1) + D_{i2}(F-P_2) - k|T_{i1}^r-T_{i1}|\right]|\Omega_i\bigg]/dT_{i1}^r &= D_{i1}dE[P_2-P_1|\Omega_i]/dT_{i1}^r \\ &+ (dD_{i1}/dT_{i1}^r)E[P_2-P_1|\Omega_i] + D_{i2}dE[F-P_2|\Omega_i]/dT_{i1}^r + (dD_{i2}/dT_{i1}^r)E[F-P_2|\Omega_i] - k. \end{aligned}$$

This expression's first eight components imply the following necessary and sufficient condition on  $k$  such that all dealers are induced to report honestly (see appendix):

$$(11) \quad k > (2\Lambda/\phi)\left[(\phi-1) - (1-\Lambda\phi)/\theta\Sigma_F\right]\left[\inf_{T_{i1}^r}|T_{i1}^r|\right] \equiv \bar{k}.$$

If  $k < \bar{k}$  then the mechanism  $\mu^*$  is not a communication equilibrium;  $\mu^*$  is incentive compatible only if the marginal cost to misreporting is greater than  $\bar{k}$ . This concludes the proof of proposition 3.

Given the solution to the model described in proposition 3, we can now solve explicitly for the equilibrium value of  $c_{i2} = -E[T'_{i1}-D_{i1}|P_1, N_1]$  described in proposition 2:

*Proposition 4:*  $P_2$  is consistent with communication equilibrium only if  $c_{i2} = -\Lambda N_1/n$   $\forall i \in \{1, \dots, n\}$ .

Proof:  $E[T'_{j1}-D_{j1}|P_1, N_1] = E[T'_{j1}|P_1, N_1] - E[D_{j1}|P_1, N_1]$  since  $T'_{j1}$  and  $D_{j1}$  are independent. But  $E[T'_{j1}|P_1, N_1] = E[T_{j1}|P_1, N_1]$  so we have  $E[T'_{j1}-D_{j1}|P_1, N_1]$

$$= \phi E[c_j | P_1, N_1] - (\phi - 1) E[c_j | P_1, N_1] = -E[c_j | P_1, N_1] = -\Delta N_1 / n \text{ since } \Delta N_1 = E\left[\sum_{j=1}^n c_{j1} | P_1, N_1\right] \text{ by construction.}$$

### III. Incentive-Efficient Communication Equilibrium

#### A. The Formal Solution

We turn now to the determination of the incentive-efficient mechanism within the set of incentive-compatible mechanisms defined above. (Recall that the elements of this set are indexed by  $\Sigma_\epsilon$ , the noise variance in the net volume signal  $N_1$ .) As defined in equation (8), the incentive-efficient mechanism maximizes the *ex-ante* utility of the representative dealer.

First, we return to the definition of final wealth from the dealer's problem in equation (6):<sup>12</sup>

$$W_{i2} = W_{i0} + D_{i1}(P_2 - P_1) + D_{i2}(F - P_2) - T'_{i1}(P_2 - P_1) - T'_{i2}(F - P_2).$$

The last two terms account for the impact of position disturbances, which represent a utility cost of marketmaking. The incentive-efficient mechanism minimizes this utility cost.<sup>13</sup> That is, it implements the  $\Sigma_\epsilon$  that solves the following problem, where we have defined the random variable  $m \equiv -T'_{i1}(P_2 - P_1) - T'_{i2}(F - P_2)$ , with density function denoted  $f_M(m)$ :

<sup>12</sup> The term reflecting the cost of misreporting has been suppressed here since we are now considering only the communication equilibria described in proposition 3, and by definition there is no misreporting under these mechanisms.

<sup>13</sup> The facts that (i)  $D_{i1}$  and  $D_{i2}$  are independent of  $T'_{i1}$  and  $T'_{i2}$ , and (ii)  $T'_{i1}$  and  $T'_{i2}$  are conditionally mean zero justifies treating the last two terms independently the maximization.

$$(12) \quad \text{Max}_{\Sigma_{\epsilon}} \int_{-\infty}^{\infty} -\exp(-\theta m) f_M(m) dm.$$

Note that each of the four components of  $m$  is normally distributed, though  $m$  is not. The first order condition of this problem, derived in appendix B, is reproduced here:

$$(13) \quad \left[ \frac{d}{d\Sigma_{\epsilon}} \right] \left[ -|\Sigma_1|^{-\frac{1}{2}} |A_1|^{-\frac{1}{2}} - |\Sigma_2|^{-\frac{1}{2}} |A_2|^{-\frac{1}{2}} \right] = 0$$

where

$$\Sigma_t \equiv \begin{bmatrix} \Sigma_{z_t} & \rho_t \\ \rho_t & \Sigma_{x_t} \end{bmatrix},$$

$$A_t \equiv \begin{bmatrix} (1/\Sigma_{z_t}) [1 + \rho_t^2 / (\Sigma_{z_t} \Sigma_{x_t} - \rho_t^2)] & -\rho_t / (\Sigma_{z_t} \Sigma_{x_t} - \rho_t^2) + \theta \\ -\rho_t / (\Sigma_{z_t} \Sigma_{x_t} - \rho_t^2) + \theta & \Sigma_{z_t} / (\Sigma_{z_t} \Sigma_{x_t} - \rho_t^2) \end{bmatrix},$$

$$z_1 \equiv -T'_{i1}, z \equiv -T'_{i2}, x_1 \equiv (P_2 - P_1), x_2 \equiv (F - P_2), \text{ and } \rho_t \equiv \text{Cov}(z_t, x_t).$$

Though this solution is not very enlightening by itself, Figure 2 below provides simulation results that evince clearly the main result: when order flow is relatively important in the process of price discovery (i.e.  $\Sigma_{\nu}$  is small), dealers prefer less order-flow transparency. The figure charts the ex-ante preferred  $\Sigma_{\epsilon}$  as a function of  $\Sigma_{\nu}$  [see equation (1)], with the number of dealers  $n$  set at 10 and the coefficient of absolute risk aversion  $\theta$  set at 0.95. The values of  $\Sigma_{\epsilon}$  and  $\Sigma_{\nu}$  are both scaled by the (endogenous) variance of the sum of the period-one trades  $\Sigma_{\Sigma T_1}$  to ease interpretation. When price discovery is wholly a function of order flow, the preferred



noise variance in the net volume signal is of the same order of magnitude as net volume itself.

### B. A Benchmark Example

To clarify what is driving the result illustrated in Figure 2 we consider a simple benchmark example. Suppose variation in  $\Sigma_\epsilon$  has no effect on the adverse selection problem inherent in marketmaking (which is very nearly true since delaying revelation does not alter the fact that each dealer is endowed with a quantum of information he can exploit in inter-dealer trading). The adverse selection problem, from the solution in equation (13), is captured by the fact that  $\rho_t \equiv \text{Cov}(-T'_{i1}, P_{t+1} - P_t) \neq 0$ . It is straightforward to show that if  $\rho_t = 0$  the dealer's problem can be expressed as:

$$\text{Max}_{\Sigma_\epsilon} - \left[ 1 - \theta^2 \Sigma_{T'_{i1}} \Sigma_{P_2 - P_1} \right]^{-\frac{1}{2}} - \left[ 1 - \theta^2 \Sigma_{T'_{i2}} \Sigma_{F - P_2} \right]^{-\frac{1}{2}}$$

where  $\Sigma_g$  denotes the unconditional variance of  $g$ ,  $g \in \{T'_{i1}, T'_{i2}, P_2 - P_1, F - P_2\}$ . From this expression it is clear that the trade-off between revelation in the first period (or  $\Sigma_{P_2 - P_1}$ ) and revelation in the second (or  $\Sigma_{F - P_2}$ ) is governed by the relative sizes of  $\Sigma_{T'_{i1}}$  and  $\Sigma_{T'_{i2}}$ . In general, the lower is  $\Sigma_{T'_{i2}}$  relative to  $\Sigma_{T'_{i1}}$  the greater the incentive to push the variance of price discovery into the second period.

First, consider the relationship between  $\Sigma_\epsilon$  and the two price-change variances  $\Sigma_{P_2 - P_1}$  and  $\Sigma_{F - P_2}$ . In the case where  $\Sigma_\epsilon = 0$ , all order-flow information will be revealed at the end of period-one trading since knowledge of  $\phi$  and an exact measure of net volume are sufficient for the sum of the  $c_{i1}$ . In this case, only non-order-flow information remains for  $F - P_2$ , i.e., the  $\tilde{v}$  from equation (1). If  $\Sigma_\epsilon > 0$ , however, all

order-flow information is not revealed by  $P_2$ ; thus,  $F-P_2$  will necessarily reflect some order-flow information, together with  $\nu$ .

Consider now the the relationship between  $\Sigma_\epsilon$  and the two position-disturbance variances  $\Sigma_{T'_{i1}}$  and  $\Sigma_{T'_{i2}}$ . These relationships are the key to the result that slower price discovery can be preferable. Specifically, the preference for slower price discovery derives from the fact that in equilibrium customer-orders in period two can reduce  $\Sigma_{T'_{i2}}$  relative to  $\Sigma_{T'_{i1}}$ .

To determine this effect precisely, note from equation (4) that:

$$T_{i2} = D_{i2} + c_{i2} + T'_{i1} - D_{i1}$$

since  $E[T'_{i2} | \Omega_{i,2,3}] = 0$  in equilibrium. Similarly, we can express the realized period-two order received by dealer  $i$ ,  $T'_{i2}$ , as:

$$T'_{i2} = D_{j2} + c_{j2} + T'_{j1} - D_{j1}$$

where dealer  $j$  is placing the order and  $T'_{j1}$  denotes the disturbance to dealer  $j$ 's position in period one. Recall that  $E[T'_{j1} - D_{j1} | \Omega_{i,2,3}] = \Delta N_1 / n \neq 0$ . From proposition 4 we know that  $P_2$  is necessarily such that  $c_{i2} = -\Delta N_1 / n \forall i$ . Thus, in equilibrium customers absorb in period two an estimate of the mean inventory imbalance among dealers from period one. Again, this follows from the fact that the conditional expectation of  $T'_{i2}$  must be zero in equilibrium. Specifically, since period-two customer orders are deterministic, we can write:

$$\Sigma_{T'_{i2}} = \Sigma_{D'_{i2}} + \text{Var}[T'_{i1} - \Delta N_1 / n] + \Sigma_{D'_{i1}}$$

which yields an equation that relates  $\Sigma_{T'_{i2}}$  directly to  $\Sigma_{T'_{i1}}$  (see appendix B):

$$(14) \quad \Sigma_{T'_{i2}} = \left[ \left( \frac{n-\Lambda(2-\Lambda)}{n} \right) \Sigma_{T'_{i1}} + (\Lambda/n)^2 \Sigma_{\epsilon} \right] + (\phi-1)^2 \Sigma_{\epsilon} + \Sigma_{D'_{i2}}.$$

Equation (14) implies that:

$$(15) \quad \Sigma_{\epsilon}=0 \Rightarrow \Lambda=1 \Rightarrow \Sigma_{T'_{i2}} = \left[ \frac{n-1}{n} \right] \Sigma_{T'_{i1}} + (\phi-1)^2 \Sigma_{\epsilon}$$

where  $\Sigma_{D'_{i2}}=0$  since all order-flow information is revealed by period-one trading.

The above also clarifies why a very high  $\Sigma_{\epsilon}$ , and therefore very little revelation, is not preferred by dealers. Customers absorb in period two what amounts to an *estimate* of the mean inventory imbalance among dealers. Thus, a high  $\Sigma_{\epsilon}$  reduces the precision of the estimate of the mean, and thereby undermines risk-sharing.

The following propositions provide comparative static results regarding the three core parameters of the model:  $n$ ,  $\theta$ , and  $\Sigma_{\nu}$ .

**Proposition 5:** The fewer the number of dealers  $n$ , the greater the incentive to slow the pace of price discovery.

**Proof:** The risk-sharing benefits of  $\Sigma_{\epsilon}>0$  derive from the fact that  $\text{Var}[T'_{i1}-\Lambda N_1/n] = \left[ \frac{n-1}{n} \right] \Sigma_{T'_{i1}} < \Sigma_{T'_{i1}}$  at  $\Sigma_{\epsilon}=0$ . This incentive to slow price discovery shrinks as  $n \rightarrow \infty$  since  $(n-1)/n \rightarrow 1$ .

Another way to view this is that the mean inventory imbalance — which is the source of the risk-sharing benefits — converges to 0 as  $n \rightarrow \infty$ .

*Proposition 6:* The greater the coefficient of absolute risk aversion  $\theta$ , the greater the incentive to slow the pace of price discovery.

Proof: From equation (9), as  $\theta \rightarrow \infty \Rightarrow \phi \rightarrow 1$  since  $D_{i1} \rightarrow 0$ . Hence, ceteris paribus, the more risk averse the dealers the lower is  $\Sigma_{T'_{i2}}$  relative to  $\Sigma_{T'_{i1}}$ , and the greater is the incentive to push price variance into period two.

*Proposition 7:* The greater the share of order-flow information in the full-information value, the greater the incentive to slow the pace of price discovery.

Proof: All non-order-flow information, i.e.  $\nu$ , is revealed after period-two trading. If  $\Sigma_\nu$  is very large, the period-two price variance will be high, implying that the marginal utility of inducing customers to absorb the mean inventory imbalance is high, generating a low preferred  $\Sigma_\epsilon$ . In contrast, if  $\Sigma_\nu$  is zero, all price variance would be realized in period one if  $\Sigma_\epsilon = 0$ . In this case, dealers prefer to slow the pace of price discovery via a higher  $\Sigma_\epsilon$  to permit absorption of mean inventory imbalances.

#### IV. Discussion

In this section we apply our framework to the spot FX market. The mapping is by no means perfect, but the application does provide some suggestive evidence. First we introduce some institutional background. Then we describe the degree of order-flow transparency that has arisen endogenously in that market.

The most important institutional feature of the spot FX market with respect to transparency is the fact that every dealer in the major currencies trades while listening to so-called "broker boxes" — intercoms over which inter-dealer brokers provide information on transaction prices, quantities, and whether trades are effected at the bid or the ask. For each of the major currencies there are typically 4 or 5 brokers who account for nearly all of the brokered trading. Hence, by listening to these brokers all dealers receive a common signal of net volume and transaction prices. This statistic is the core of dealers' high-frequency information set [see Lyons (1993a,1993b) for more detail].

To determine the overall degree of order-flow transparency, consider first the share of total trading that is brokered in the major trading centers. Table 2 provides a summary. The countries listed are the 7 with the highest turnover in 1989 whose statistics are comparable.<sup>14</sup> The average share (rounded) in 1989 is 39%, with a high of 44% and a low of 35%. Thus, on the whole, there is little variation across countries. It is worth noting that of the remaining roughly 60% of volume, direct inter-dealer trading typically accounts for about 50%, and trading between dealers and customers accounts for the remaining 10%. For this 60%, there is no transparency beyond the transacting parties.

---

<sup>14</sup> High volume countries for which a comparable statistic is not available include Singapore, Australia, and Switzerland. For Singapore, no statistic is available. For Australia and Switzerland the published statistic is biased-down since it is based exclusively on data from local brokers. Nevertheless, the numbers are still in the neighborhood of the others: Australia's reported share was 33% and Switzerland's 19% in 1989.

Table 2

Share of FX turnover arranged through brokers (%)

	1989
United Kingdom	38
United States	44
Japan	35
Hong Kong	35
France	42
Canada	40
Netherlands	41

\* Source: *BIS Survey of Foreign Exchange Market Activity* (1990); adjusted for double-counting.

Table 3 presents the brokered share for the U.S. over the three tri-annual surveys conducted to date, thereby providing some evidence regarding variation over time. Here too there is little variation. Note that the U.S. share — the highest in Table 2 — appears to have regressed toward the mean of 39%.

Table 3

Share of FX turnover arranged through brokers (%)

	1986	1989	1992
United States	44	44	39

\* Source: *The Foreign Exchange Committee Annual Report, 1989 & 1992*, Federal Reserve Bank of New York; adjusted for double-counting.

We interpret these statistics as suggestive of an equilibrium degree of transparency in this market. Two important facts support this interpretation. First, the FX market is unregulated with respect to its information structure, so speaking of an endogenous level of transparency that serves the dealers is more sensible here. Second, there is no other source of market net order-flow information than the broker intercoms. While it is true that dealers have other motivations for using brokers, that fact that brokered trading defines the degree of transparency is undeniable.

### Are the Empirical Shares Consistent with the Model?

Though the principal objective of this paper is to endogenize the degree of order-flow transparency, we can push the model further by mapping the two-period equilibrium into continual trading. This should be recognized, however, as a conjectural exercise to say the least.

Now, one might conclude from Table 2 that in reality dealers observe about 40% of the order-flow information since that corresponds to the empirical brokered share. Comparing this 40% to the shares predicted by the model, however, would be misleading in two ways. First, *every* brokered trade is not announced to the market; from the estimate that 50%–75% of them are, the 40% comes down to a range of 20%–30%.<sup>15</sup> Second, the key factor in determining the incentive-efficient equilibrium is the absorption of the average inventory imbalance by customers in *one period* (period two). We need to consider the difference between this one-period absorption and that which occurs in continual trading. Empirically, customer transactions arrive at about 1/4th the frequency of inter-dealer transactions (as inferred from the fact that roughly 80% of trading is inter-dealer). This suggests that in reality the

---

<sup>15</sup> The sale or purchase is announced only when it clears the bid or ask at the given broker. Though the likelihood of this depends on the currency, the 50% to 75% estimate was provided to me by two marketmakers that trade different currencies for different firms.

model's one-period absorption by customers occurs over a number of trading "rounds". Accordingly — using the frequency ratio of 1 to 4 — one round in the model corresponds to 4 rounds in the actual market. Hence, if dealers view 20%–30% of the order-flow information in each round for 4 rounds, that implies that the share of noise remaining is roughly  $(1-0.3)^4$  to  $(1-0.2)^4$ , or 24%–41%. This is now more directly comparable to the noise shares predicted by the model.

## V. Conclusions

The central objective of this paper was to endogenize the transparency of the trading process in a dealership market and examine the implications of that endogeneity. The model introduced a tension between the speed of revelation and the amount of risk sharing that has not been recognized elsewhere in the literature, at least to this author's knowledge.<sup>16</sup>

Methodologically, the paper further highlights the value of mechanism design for understanding institutional adaptation in context of imperfect information. In the case of the FX market, the approach is particularly applicable since in that market transparency is both endogenous and measurable. This approach also made it possible to formalize the optimal quantity of noise: unlike previous work in microstructure where exogenous noise influences revelation in equilibrium, here dealers *prefer* an amount of noise that prevents full revelation.

An important consideration in evaluating the implications of the model is the role of volume-mediated price discovery. In the first period, price discovery was wholly volume-mediated; there was no scope for communicating private order-flow

---

<sup>16</sup> A natural extension that would likely enhance this tension is introduction of a means for reducing *idiosyncratic* inventory imbalances via better matching between dealers over trading rounds.



information via quoted prices, the reason being that arbitrage insures harmonization in quotes. This characteristic of the model is much more broadly applicable in our view. That is, arbitrage pricing theory has important implications for the extent to which private information is communicated via volume versus price in decentralized quote-driven markets. Of course, introduction of a bid-ask spread provides some limited scope for unarbitragable dispersion of quotes. But the tightness of the spread in a market like FX insures that this scope is quite limited, at least relative to the dispersion of dealers' beliefs as reflected in survey data and other sources.

Finally, the cost of misreporting that induces incentive-compatibility in our model, which we interpreted as an incremental transaction cost to round-trip trades, provides an additional result. That is, in traditional models the spread is determined in a competitive environment by a zero profit condition where there are costs to providing immediacy (e.g., adverse selection, risk aversion, inventory holding costs, etc.). In contrast, in our framework with multiple strategic dealers the transaction cost that arises endogenously plays a new role: it acts as means of protecting the integrity of the information structure.

## Appendix

### A. Determining Incentive-Compatible Mechanisms

The mechanisms under consideration are those meeting the condition expressed in equation (5):

$$(5) \quad \mu(N_1, k | T_{11}^r, \dots, T_{n1}^r): T_{11}^r \in \mathbb{R}, k \in \mathbb{R}, \text{ and } N_1 \equiv \sum_{i=1}^n T_{i1}^r + \tilde{\epsilon} \in \mathbb{R}.$$

Determining the dynamic programming solution conditional on a given mechanism requires the period-two desired position for use in the period-one first order condition. Under normality and exponential utility it is well known that the period-two desired position is:

$$(A1) \quad D_{i2} = (\gamma_{iF} - P_2)(\Sigma_F)^{-1}$$

where  $\gamma_{iF} \equiv E[L | P_1, c_{i1}, T'_{i1}, N_1, k]$ . Hence, omitting terms unrelated to  $D_{i1}$  one can write the dealers' problem as:

$$(A2) \quad \text{Max}_{D_{i1}} E_{P_2, \gamma_{iF}, F} \left[ -\exp \left[ -\theta D_{i1} (P_2 - P_1) - (\gamma_{iF} - P_2)(\Sigma_F)^{-1} (L - P_2) \right] \right] \Big|_{P_1, c_{i1}}$$

From the law of iterated projections the expectation can be written as:

$$E_{P_2, \gamma_{iF}} \left[ E_F \left[ -\exp \left[ -\theta D_{i1} (P_2 - P_1) - (\gamma_{iF} - P_2)(\Sigma_F)^{-1} (F - P_2) \right] \right] \Big|_{P_1, c_{i1}, T'_{i1}, N_1, k} \right] \Big|_{P_1, c_{i1}}$$

Making use of the definition of the moment generating function for a

normally-distributed random variable  $x$   $E[\exp(tx)] = \exp(\gamma t + \sigma^2 t^2/2)$ , the inside expectation over  $F$  yields:

$$(A3) \quad E_F \left[ \cdot \mid P_1, c_{i1}, T'_{i1}, N_1, k \right] = -\exp \left[ -\theta D_{i1} (P_2 - P_1) - (2\Sigma_F)^{-1} (\gamma_{iF} - P_2)^2 \right]$$

since  $E[F - P_2 \mid P_1, c_{i1}, T'_{i1}, N_1, k] = \gamma_{iF} - P_2$  and:

$$\begin{aligned} & -\exp \left[ (\gamma_{iF} - P_2)(-1)(\gamma_{iF} - P_2)(\Sigma_F)^{-1} + (1/2)\Sigma_F \{-(\gamma_{iF} - P_2)(\Sigma_F)^{-1}\}^2 \right] \\ & = -\exp \left[ -(\gamma_{iF} - P_2)^2 (\Sigma_F)^{-1} + (1/2)[(\gamma_{iF} - P_2)^2 (\Sigma_F)^{-1}] \right] \\ & = -\exp \left[ -(2\Sigma_F)^{-1} (\gamma_{iF} - P_2)^2 \right]. \end{aligned}$$

This leaves the objective function with two remaining random variables:

$$(A4) \quad \text{Max}_{D_{i1}} E_{P_2, \gamma_{iF}} \left[ -\exp \left[ -\theta D_{i1} (P_2 - P_1) - (2\Sigma_F)^{-1} (\gamma_{iF} - P_2)^2 \right] \mid P_1, c_{i1} \right].$$

Now, the period-two price is specified by the given mechanism. If the mechanism defines a communication equilibrium then quoting this price is incentive compatible for each dealer. Under the mechanism  $\mu^*$  of proposition 3 quotes take the following form:

$$(A5) \quad P_2 = \Lambda N_1.$$

for some constant  $\Lambda$ . (We demonstrate below that  $E[F \mid N_1, P_1] = \Lambda N_1$ .) In order to determine  $\gamma_{iF} - P_2$  define the following signal extraction coefficient:

$$\lambda_1 \equiv \phi \Sigma_c / [\Sigma_T + \Sigma_c / (n-2)],$$

where  $\phi$  and  $\Sigma_T$  are determined from the optimal period-one trading rule [equation (9)]. One can then write:

$$\begin{aligned} (A6) \quad \gamma_{iF} - P_2 &= E[F - \Lambda N_1 | P_1, c_{i1}, T'_{i1}, N_1, k] \\ &= E\left[\sum_{i=1}^n c_{i1} - \Lambda N_1 | P_1, c_{i1}, T'_{i1}, N_1, k\right] \\ &= E[c_{i1} + c_{j1} + \sum_{k \neq i, j}^n c_{k1} - \Lambda N_1 | P_1, c_{i1}, T'_{i1}, N_1, k] \\ &= c_{i1} + (1/\phi)T'_{i1} + \lambda_1(N_1 - T_{i1} - T'_{i1}) - \Lambda N_1 \\ &= c_{i1} - \lambda_1 T_{i1} + (1/\phi - \lambda_1)T'_{i1} + (\lambda_1 - \Lambda)N_1 \end{aligned}$$

where  $c_{j1}$  is the customer order embedded in  $T'_{i1}$ , and the best estimate of  $c_{j1}$  conditional on  $T'_{i1}$  is  $(1/\phi)T'_{i1}$  since  $T'_{i1} = \phi c_{j1}$ . This is an important expression; it summarizes dealer  $i$ 's beliefs as a function of four key variables observable by him. Now, since  $T_{i1} = D_{i1} + c_{i1}$  the objective function expressed in equation (A4) can be written as:

$$\begin{aligned} (A7) \quad \text{Max}_{D_{i1}} E_{P_2} &\left[ -\exp\left[-\theta D_{i1}(P_2 - P_1)\right] \right. \\ &\quad \left. - (2\Sigma_F)^{-1}[c_{i1} - \lambda_1(D_{i1} + c_{i1}) + (1/\phi - \lambda_1)T'_{i1} + (\lambda_1 - \Lambda)N_1]^2 \right] \Big| P_1, c_{i1}. \end{aligned}$$

Since dealer  $i$  takes into consideration the effect of her demand on the period-two price it would not be appropriate to take the expectation over  $P_2$ . It is necessary to define a new variable,  $N'_1$ :

(A8)

$$N'_1 \equiv N_1 - T_{i1}.$$

Since  $P_2 = \Lambda N_1 = \Lambda(N'_1 + T_{i1})$  the objective function becomes:

$$(A9) \quad \text{Max}_{D_{i1}} E_{N'_1} \left[ -\exp \left[ -\theta D_{i1} (\Lambda D_{i1} + \Lambda c_{i1} + \Lambda N'_1) \right. \right. \\ \left. \left. - (2\Sigma_F)^{-1} [(1-\Lambda)c_{i1} - \Lambda D_{i1} + (1/\phi - \lambda_1)T'_{i1} + (\lambda_1 - \Lambda)N'_1]^2 \right] \right] \Big|_{P_1, c_{i1}}$$

omitting terms unrelated to  $D_{i1}$  or  $N'_1$ . In order to account for the relationship between  $T'_{i1}$  and  $N'_1$  when taking the expectation, it is necessary to express  $T'_{i1}$  in terms of  $N'_1$ :

$$E[T'_{i1} | N'_1] = \Sigma_T / [(n-1)\Sigma_T + \Sigma_\epsilon] N'_1$$

which implies that:

$$E \left[ (1/\phi - \lambda_1)T'_{i1} + (\lambda_1 - \Lambda)N'_1 \mid N'_1 \right] = \left[ (1/\phi - \lambda_1) \left[ \Sigma_T / [(n-1)\Sigma_T + \Sigma_\epsilon] \right] + (\lambda_1 - \Lambda) \right] N'_1 \\ \equiv \lambda_2 N'_1.$$

Now, recognizing that:

$$E[N'_1 | P_1, c_{i1}] = 0$$

$$\text{Var}[N'_1 | P_1, c_{i1}] = (n-1)\Sigma_T + \Sigma_\epsilon \equiv \Sigma_{N'_1}$$

one can write an objective function proportional to that in equation (A9):

$$\begin{aligned}
 (A10) \quad & \text{Max}_{D_{i1}} - \int_{-\infty}^{\infty} \exp \left[ -\theta D_{i1} (\Lambda D_{i1} + \Lambda c_{i1} + \Lambda N'_1) - \right. \\
 & \left. - (2\Sigma_F)^{-1} [(1-\Lambda)c_{i1} - \Lambda D_{i1} + \lambda_2 N'_1]^2 - (1/2)(\Sigma_{N'_1})^{-1} (N'_1)^2 \right] dN'_1 \\
 & = \text{Max}_{D_{i1}} - \int_{-\infty}^{\infty} \exp \left[ \left(-\frac{1}{2}\right) \left[ 2\theta \Lambda D_{i1}^2 + 2\theta \Lambda c_{i1} D_{i1} + (1/\Sigma_F) [(1-\Lambda)c_{i1} - \Lambda D_{i1}]^2 \right. \right. \\
 & \left. \left. + (1/\Sigma_F) [\lambda_2^2 N_1'^2 + 2(1-\Lambda)\lambda_2 c_{i1} N'_1 - 2\Lambda \lambda_2 D_{i1} N'_1] + (1/\Sigma_{N'_1}) N_1'^2 + 2\theta \Lambda D_{i1} N'_1 \right] \right] dN'_1 \\
 & = \text{Max}_{D_{i1}} - \int_{-\infty}^{\infty} \exp \left[ \left(-\frac{1}{2}\right) \left[ 2\theta \Lambda D_{i1}^2 + 2\theta \Lambda c_{i1} D_{i1} + (1/\Sigma_F) [(1-\Lambda)c_{i1} - \Lambda D_{i1}]^2 \right. \right. \\
 & \quad \left. \left. + (\lambda_2^2/\Sigma_F + 1/\Sigma_{N'_1}) \left[ N'_1 + \frac{(1-\Lambda)\lambda_2 c_{i1} - (\Lambda \lambda_2 + \theta \Lambda \Sigma_F) D_{i1}}{(\lambda_2^2 + \Sigma_F/\Sigma_{N'_1})} \right]^2 \right. \right. \\
 & \quad \left. \left. - (\lambda_2^2/\Sigma_F + 1/\Sigma_{N'_1}) \left[ \frac{(1-\Lambda)\lambda_2 c_{i1} - (\Lambda \lambda_2 + \theta \Lambda \Sigma_F) D_{i1}}{(\lambda_2^2 + \Sigma_F/\Sigma_{N'_1})} \right]^2 \right] \right] dN'_1 \\
 & = \text{Max}_{D_{i1}} - \exp \left[ \left(-\frac{1}{2}\right) \left[ 2\theta \Lambda D_{i1}^2 + 2\theta \Lambda c_{i1} D_{i1} + (1/\Sigma_F) [(1-\Lambda)c_{i1} - \Lambda D_{i1}]^2 \right. \right. \\
 & \quad \left. \left. - (\lambda_2^2/\Sigma_F + 1/\Sigma_{N'_1}) \left[ \frac{(1-\Lambda)\lambda_2 c_{i1} - (\Lambda \lambda_2 + \theta \Lambda \Sigma_F) D_{i1}}{(\lambda_2^2 + \Sigma_F/\Sigma_{N'_1})} \right]^2 \right] \right] \\
 & \quad \times \int_{-\infty}^{\infty} \exp \left[ \left(-\frac{1}{2}\right) (\lambda_2^2/\Sigma_F + 1/\Sigma_{N'_1}) \left[ N'_1 + \frac{(1-\Lambda)\lambda_2 c_{i1} - (\Lambda \lambda_2 + \theta \Lambda \Sigma_F) D_{i1}}{(\lambda_2^2 + \Sigma_F/\Sigma_{N'_1})} \right]^2 \right] dN'_1
 \end{aligned}$$

Recognizing that this integral is proportional to a cumulative normal density with a mean of  $-[(1-\Lambda)\lambda_2 c_{i1} - (\Lambda\lambda_2 + \theta\Lambda\Sigma_F)D_{i1}]/[\lambda_2^2 + \Sigma_F/\Sigma_{N_1}]$  and a variance equal to  $(\lambda_2^2/\Sigma_F + 1/\Sigma_{N_1})^{-1}$  this whole expression is equivalent to maximizing the expression in large curved brackets:

$$(A11) \quad \text{Max}_{D_{i1}} \left[ 2\theta\Lambda D_{i1}^2 + 2\theta\Lambda c_{i1} D_{i1} + (1/\Sigma_F)[(1-\Lambda)c_{i1} - \Lambda D_{i1}]^2 - (\lambda_2^2/\Sigma_F + 1/\Sigma_{N_1}) \left[ \frac{(1-\Lambda)\lambda_2 c_{i1} - (\Lambda\lambda_2 + \theta\Lambda\Sigma_F)D_{i1}}{(\lambda_2^2 + \Sigma_F/\Sigma_{N_1})} \right]^2 \right]$$

Now, to maintain linearity we derive the first order condition assuming that each dealer takes into account her impact on prices but not on the variance of prices. This yields:

$$\left[ 2\theta\Lambda + \Lambda^2\Sigma_F^{-1} - (\theta\Lambda - \Lambda\lambda_2\Sigma_F^{-1})^2/(\lambda_2^2/\Sigma_F + 1/\Sigma_{N_1}) \right] D_{i1} + \theta\Lambda c_{i1} - (1-\Lambda)\Lambda\Sigma_F^{-1}c_{i1} - \left[ (1-\Lambda)\lambda_2\Sigma_F^{-1}(\theta\Lambda - \Lambda\lambda_2\Sigma_F^{-1})/(\lambda_2^2/\Sigma_F + 1/\Sigma_{N_1}) \right] c_{i1} = 0$$

Collecting terms yields:

$$D_{i1} = \left[ \frac{\theta + [(\Lambda-1)\Sigma_F^{-1}][1 + \lambda_2(\theta - \lambda_2\Sigma_F^{-1})/(\lambda_2^2/\Sigma_F + 1/\Sigma_{N_1})]}{(\theta - \lambda_2\Sigma_F^{-1})^2(\lambda_2^2\Sigma_{N_1}^{-1}\Sigma_F^{-1} + 1)^{-1}\Lambda\Sigma_{N_1}^{-1} - 2\theta - \Lambda\Sigma_F^{-1}} \right] c_{i1}$$

Since  $T_{i1} = D_{i1} + c_{i1}$  this implies:

$$T_{i1} = \left[ 1 + \frac{\theta + [(\Lambda - 1)\Sigma_F^{-1}] [1 + \lambda_2 (\theta - \lambda_2 \Sigma_F^{-1}) / (\lambda_2^2 / \Sigma_F + 1 / \Sigma_{N_1'})]}{(\theta - \lambda_2 \Sigma_F^{-1})^2 (\lambda_2^2 \Sigma_{N_1'} \Sigma_F^{-1} + 1)^{-1} \Lambda \Sigma_{N_1'} - 2\theta - \Lambda \Sigma_F^{-1}} \right] c_{i1}$$

$$\equiv \phi c_{i1}.$$

For this trading rule, the value of  $\Lambda$  such that  $E[F|N_1, P_1] = \Lambda N_1$  is simply the signal extraction coefficient:

$$\Lambda = \frac{\phi \Sigma_c}{\phi^2 \Sigma_c + \Sigma_\epsilon / n}.$$

The period two trading rule is determined as follows:

$$T_{i2} \equiv D_{i2} + c_{i2} + T'_{i1} - D_{i1}$$

$$= \frac{\gamma_{iF} - P_2}{\theta \Sigma_F} + c_{i2} + T'_{i1} - (\phi - 1)c_{i1}$$

and using the expression in equation (A6) for the numerator of the fourth term:

$$= \left[ \frac{1 - \lambda_1 \phi}{\theta \Sigma_F} - (\phi - 1) \right] c_{i1} + \left[ 1 + \frac{1 / \phi - \lambda_1}{\theta \Sigma_F} \right] T'_{i1} + \left[ \frac{\lambda_1 - \Lambda}{\theta \Sigma_F} \right] N_1 + c_{i2}$$

$$\equiv \beta_1 c_{i1} + \beta_2 T'_{i1} + \beta_3 N_1 + c_{i2}.$$

### Determining the Threshold Value of $k$ That Induces Truth-Telling

We begin from the expression for the expected gain from misreporting in the



text:

$$\begin{aligned}
& dE \left\{ D_{i1}(P_2 - P_1) + D_{i2}(F - P_2) - k | T_{i1}^r - T_{i1} | \left| \Omega_{i,1,3} \right. \right\} / dT_{i1}^r = D_{i1} dE[P_2 - P_1 | \Omega_{i,1,3}] / dT_{i1}^r \\
& + (dD_{i1} / dT_{i1}^r) E[P_2 - P_1 | \Omega_{i,1,3}] + D_{i2} dE[F - P_2 | \Omega_{i,1,3}] / dT_{i1}^r \\
& + (dD_{i2} / dT_{i1}^r) E[F - P_2 | \Omega_{i,1,3}] - k.
\end{aligned}$$

We now evaluate each of this expression's first eight components in turn, with each component evaluated at  $T_{i1}^r = T_{i1}$ . The symbol " $\perp$ " denotes independence:

$$\begin{aligned}
\text{Term one:} \quad & T_{i1} = D_{i1} + c_{i1} = \phi c_{i1} \Rightarrow D_{i1} = \phi c_{i1} - c_{i1} = (\phi - 1) c_{i1} \\
\text{Term two:} \quad & P_1 \perp T_{i1}^r \Rightarrow dE[P_2 - P_1] / dT_{i1}^r = dP_2 / dT_{i1}^r = \Lambda \\
\text{Term three:} \quad & D_{i1} = (\phi - 1) c_{i1} \text{ and } T_{i1} = \phi c_{i1} \Rightarrow dD_{i1} / dT_{i1}^r = dD_{i1} / dT_{i1} = (\phi - 1) / \phi \\
\text{Term four:} \quad & E[P_2 - P_1] = E[\Lambda N_1] = \Lambda E \left[ \sum_{i=1}^n T_{i1}^r + \tilde{\epsilon} \right] = \Lambda T_{i1} = \Lambda \phi c_{i1} \\
\text{Term five:} \quad & D_{i2} = E[F - P_2] / \sigma_{\Sigma_F} = (c_{i1} - \Lambda T_{i1}) / \sigma_{\Sigma_F} = (1 - \Lambda \phi) c_{i1} / \sigma_{\Sigma_F} \\
\text{Term six:} \quad & F \perp T_{i1}^r \Rightarrow dE[F - P_2] / dT_{i1}^r = -dP_2 / dT_{i1}^r = -\Lambda \\
\text{Term seven:} \quad & dD_{i2} / dT_{i1}^r = d[E[F - P_2] / \sigma_{\Sigma_F}] / dT_{i1}^r = -\Lambda / \sigma_{\Sigma_F} \\
\text{Term eight:} \quad & E[F - P_2] = (1 - \Lambda \phi) c_{i1} \text{ per term five.}
\end{aligned}$$

Collecting these terms, we have:

$$\begin{aligned}
& dE \left\{ D_{i1}(P_2 - P_1) + D_{i2}(F - P_2) - k | T_{i1}^r - T_{i1} | \left| \Omega_{i,1,3} \right. \right\} / dT_{i1}^r \\
& = [(\phi - 1) c_{i1}] \Lambda + [(\phi - 1) / \phi] [\Lambda \phi c_{i1}] + [(1 - \Lambda \phi) c_{i1} / \sigma_{\Sigma_F}] (-\Lambda) + (-\Lambda / \sigma_{\Sigma_F}) [(1 - \Lambda \phi) c_{i1}] - k \\
& = (2\Lambda) \left[ (\phi - 1) - (1 - \Lambda \phi) / \sigma_{\Sigma_F} \right] c_{i1} - k
\end{aligned}$$

Given this marginal return to over-reporting is independent of  $T_{i1}^r$ , the expression implies the following necessary and sufficient condition on  $k$  such that all dealers are induced to report honestly:

$$(11) \quad k > (2\Lambda/\phi) \left[ (\phi-1) - (1-\Lambda\phi)/\theta\Sigma_F \right] \left[ \inf_{T_{i1}^r} |T_{i1}^r| \right] \equiv \bar{k}.$$

### B. Determining the Incentive-Efficient Mechanism

We begin from the expression for  $\Sigma_{T'_{i2}}$  in the text:

$$\Sigma_{T'_{i2}} = \text{Var}[T'_{i1} - \Lambda N_1/n] + \Sigma_{D'_{i1}} + \Sigma_{D'_{i2}}$$

which implies:

$$\begin{aligned} \Sigma_{T'_{i2}} &= \text{Var}[T'_{i1} - \Lambda(\Sigma T'_{i1} + \epsilon)/n] + \Sigma_{D'_{i1}} + \Sigma_{D'_{i2}} \\ &= \Sigma_{T'_{i1}} + \text{Var}[\Lambda(\Sigma T'_{i1} + \epsilon)/n] + 2\text{Cov}[T'_{i1}, -\Lambda(\Sigma T'_{i1} + \epsilon)/n] + \Sigma_{D'_{i1}} + \Sigma_{D'_{i2}} \\ &= \Sigma_{T'_{i1}} + (\Lambda^2/n)\Sigma_{T'_{i1}} + (\Lambda/n)^2\Sigma_\epsilon + 2(-1)(\Lambda/n)\text{E}[T'_{i1}, T'_{i1}] + \Sigma_{D'_{i1}} + \Sigma_{D'_{i2}} \\ &= \Sigma_{T'_{i1}} - [\Lambda(2-\Lambda)/n]\Sigma_{T'_{i1}} + (\Lambda/n)^2\Sigma_\epsilon + (\phi-1)^2\Sigma_\epsilon + \Sigma_{D'_{i2}} \end{aligned}$$

which yields equation (14) in the text:

$$(14) \quad \Sigma_{T'_{i2}} = \left[ \left[ \frac{n-\Lambda(2-\Lambda)}{n} \right] \Sigma_{T'_{i1}} + (\Lambda/n)^2\Sigma_\epsilon \right] + (\phi-1)^2\Sigma_\epsilon + \Sigma_{D'_{i2}}.$$

## The Optimization Problem

Determining the ex-ante incentive-efficient mechanism from first principles is necessary here given it involves products of normally distributed random variables. The problem is expressed in the text as:

$$\begin{aligned} & \underset{\Sigma_{\epsilon}}{\text{Max}} \quad E[-\exp(-\theta M)] \\ & \text{s.t.} \quad M = -T'_{i1}(P_2 - P_1) - T'_{i2}(F - P_2) \end{aligned}$$

We begin by simplifying notation. Let  $z_1 \equiv -T'_{i1}$ ,  $z_2 \equiv -T'_{i2}$ ,  $x_1 \equiv (P_2 - P_1)$ , and  $x_2 \equiv (F - P_2)$ , where each has an unconditional mean of zero. The problem can then be expressed as:

$$\underset{\Sigma_{\epsilon}}{\text{Max}} \quad -|\Sigma_1|^{-\frac{1}{2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp(-W_1) dz_1 dx_1 - |\Sigma_2|^{-\frac{1}{2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp(-W_2) dz_2 dx_2$$

$$\text{where} \quad W_1 \equiv \theta z_1 x_1 + \frac{1}{2}(z_1, x_1) \Sigma_1^{-1} (z_1, x_1)'$$

$$W_2 \equiv \theta z_2 x_2 + \frac{1}{2}(z_2, x_2) \Sigma_2^{-1} (z_2, x_2)'$$

$$\Sigma_1 \equiv \begin{bmatrix} \Sigma_{z_1} & \rho_1 \\ \rho_1 & \Sigma_{x_1} \end{bmatrix} \quad \text{and} \quad \Sigma_2 \equiv \begin{bmatrix} \Sigma_{z_2} & \rho_2 \\ \rho_2 & \Sigma_{x_2} \end{bmatrix},$$

with  $\rho_i \equiv \text{Cov}(z_i, x_i)$ . The first term in each of the  $W$ 's is from the utility function, the second from the normal density function. Rational expectations insures orthogonality of the  $W$ 's. Since:

$$\Sigma_t^{-1} = \begin{bmatrix} (1/\Sigma_{z_t})[1+\rho_t^2/(\Sigma_{z_t}\Sigma_{x_t}-\rho_t^2)] & -\rho_t/(\Sigma_{z_t}\Sigma_{x_t}-\rho_t^2) \\ -\rho_t/(\Sigma_{z_t}\Sigma_{x_t}-\rho_t^2) & \Sigma_{z_t}/(\Sigma_{z_t}\Sigma_{x_t}-\rho_t^2) \end{bmatrix}$$

by rearranging we can write:

$$W_t \equiv \frac{1}{2} y_t' A_t y_t$$

where

$$y_t = (z_t, x_t)'$$

$$A_t = \begin{bmatrix} (1/\Sigma_{z_t})[1+\rho_t^2/(\Sigma_{z_t}\Sigma_{x_t}-\rho_t^2)] & -\rho_t/(\Sigma_{z_t}\Sigma_{x_t}-\rho_t^2) + \theta \\ -\rho_t/(\Sigma_{z_t}\Sigma_{x_t}-\rho_t^2) + \theta & \Sigma_{z_t}/(\Sigma_{z_t}\Sigma_{x_t}-\rho_t^2) \end{bmatrix}$$

On the assumption that both of the  $A_t$  are positive definite we can write:

$$A_t = D_t D_t'$$

Using the substitution  $u_t \equiv D_t' y_t$  we have:

$$\begin{aligned} \text{Max}_{\Sigma} -|\Sigma_1|^{-\frac{1}{2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp(-W_1) dz_1 dx_1 - |\Sigma_2|^{-\frac{1}{2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp(-W_2) dz_2 dx_2 \\ = \text{Max}_{\Sigma} -|\Sigma_1|^{-\frac{1}{2}} \int_{\mathbb{R}^2} \exp(-\frac{1}{2} y_1' A_1 y_1) dy_1 - |\Sigma_2|^{-\frac{1}{2}} \int_{\mathbb{R}^2} \exp(-\frac{1}{2} y_2' A_2 y_2) dy_2 \end{aligned}$$

$$= \text{Max}_{\Sigma_{\epsilon}} -|\Sigma_1|^{-\frac{1}{2}}|A_1|^{-\frac{1}{2}} \int_{\mathbb{R}} \exp(-\frac{1}{2}u_1' u_1) du_1 - |\Sigma_2|^{-\frac{1}{2}}|A_2|^{-\frac{1}{2}} \int_{\mathbb{R}} \exp(-\frac{1}{2}u_2' u_2) du_2$$

where the  $|A_t|^{-\frac{1}{2}}$  terms account for the rescaling of the multinormal variance necessitated by the substitution. The problem can then be expressed as:

$$= \text{Max}_{\Sigma_{\epsilon}} -|\Sigma_1|^{-\frac{1}{2}}|A_1|^{-\frac{1}{2}} - |\Sigma_2|^{-\frac{1}{2}}|A_2|^{-\frac{1}{2}}$$

since  $\int_{\mathbb{R}} \exp(-\frac{1}{2}u_t' u_t) du_t$  is proportional to a cumulative multinormal density with means zero and variance-covariance I. The first order condition generates the ex-ante preferred  $\Sigma_{\epsilon}$ :

$$(13) \quad \left[ \frac{d}{d\Sigma_{\epsilon}} \right] \left[ -|\Sigma_1|^{-\frac{1}{2}}|A_1|^{-\frac{1}{2}} - |\Sigma_2|^{-\frac{1}{2}}|A_2|^{-\frac{1}{2}} \right] = 0.$$

It is straightforward to show that when  $\rho_t=0$  the problem can be expressed as:

$$\text{Max}_{\Sigma_{\epsilon}} -(1-\theta^2 \Sigma_{z_1} \Sigma_{x_1})^{-\frac{1}{2}} - (1-\theta^2 \Sigma_{z_2} \Sigma_{x_2})^{-\frac{1}{2}}$$

which provides a formal basis for the analysis in Section III.

## References

- Biais, B. (1993): "Price Formation and Equilibrium Liquidity in Fragmented and Centralized Markets," Journal of Finance, XLVIII: 157-185.
- Bray, M. (1981): "Futures Trading, Rational Expectations, and the Efficient Markets Hypothesis," Econometrica, 49: 575-596.
- Diamond, D., and R. Verrecchia (1981): "Information Aggregation in a Noisy Rational Expectations Economy," Journal of Financial Economics, 9: 221-235.
- Flood, M. (1991): "Microstructure Theory and the Foreign Exchange Market," Federal Reserve Bank of St. Louis Review, 73: 52-70.
- Flood, M. (1992): "Market Structure and Inefficiency in the Foreign Exchange Market," Federal Reserve Bank of St. Louis Working Paper #91-001C.
- Garman, M. (1976): "Market Microstructure," Journal of Financial Economics, 3: 257-275.
- Goodhart, C. (1988): "The Foreign Exchange Market: A Random Walk with a Dragging Anchor," Economica, 55: 437-460.
- Ho, T., and H. Stoll (1983): "The Dynamics of Dealer Markets Under Competition," Journal of Finance, 1053-1074.
- Kyle, A. (1989): "Informed Speculation with Imperfect Competition," Review of Economic Studies, 56: 317-356.
- Laffont, J., and E. Maskin (1990): "The Efficient Market Hypothesis and Insider Trading on the Stock Market," Journal of Political Economy, 98: 70-93.
- Leach, J.C., and A. Madhavan (1993): "Price Experimentation and Security Market Structure," Review of Financial Studies, 6: 375-404.
- Lyons, R. (1990): "Whence Exchange Rate Overshooting: Money Stock or Flow?" Journal of International Economics, November, 29: 369-384.
- Lyons, R. (1993a): "Information Intermediation in the Microstructure of the Foreign Exchange Market," Berkeley Business School mimeo, August.
- Lyons, R. (1993b): "Tests of Microstructural Hypotheses in the Foreign Exchange Market," Berkeley Business School mimeo, March.
- Mendelson, H. (1987): "Consolidation, Fragmentation, and Market Performance," Journal of Financial and Quantitative Analysis, 22: .
- Myerson, R. (1985): "Bayesian Equilibrium and Incentive Compatibility: An Introduction," in Social Goals and Social Organization, ed. by L. Hurwicz et al. Cambridge: Cambridge University Press, 229-259.

- Myerson, R. (1986): "Multistage Games with Communication," Econometrica, 54: 323-358.
- Myerson, R. and M. Satterthwaite (1983): "Efficient Mechanisms for Bilateral Trading," Journal of Economic Theory, 29: 265-281.
- Nagarajan, S. and R. Ramakrishnan (1992): "A Theory of Trading Intermediation under Multilateral Asymmetric Information," Columbia Business School mimeo.
- New York Federal Reserve Bank (1989): "The Foreign Exchange Committee Annual Report, 1989."
- Pithyachariyakul, P. (1986): "Exchange Markets: A Welfare Comparison of Marketmaker and Walrasian Systems," Quarterly Journal of Economics, 101: .
- Roell, A. (1990): "Dual-Capacity Trading and the Quality of the Market," Journal of Financial Intermediation, 1: 105-124.
- Stigum, M. (1990): The Money Market, third edition, Homewood: Dow Jones-Irwin.
- Wolinsky, A. (1990): "Information Revelation in a Market With Pairwise Meetings," Econometrica, 58: 1-25.

Figure 2  
Incentive Efficient Configurations

