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PARTIAL- VS GENERAL-EQUILIBRIUM MODELS  
OF THE INTERNATIONAL CAPITAL MARKET

Bernard Dumas

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ABSTRACT

In this essay, I discuss and compare two ways of modelling international capital market equilibrium: the orthodox, general-equilibrium approach and the heterodox, partial-equilibrium CAPM (Capital Asset Pricing Model) approach. The benchmark for this comparison is the model's ability to provide an explanation for, or take into account, a number of stylized facts of international finance: UIRP deviations, home-equity preference, PPP deviations and their persistence, consumption behavior in relation to wealth. In addition, I ask which approach is more likely in future research to help us identify the relevant state variables of the economy. None of the models satisfactorily explains the stylized facts but the CAPM approach affords the most productive avenue for empirical research in the immediate future.

Bernard Dumas  
HEC School of Management  
78351 Jouy-en-Josas Cedex  
FRANCE  
and NBER

As a representation of international capital market equilibrium, one model has clearly failed. It is the model featuring independently identically distributed (IID) random shocks and a homogeneous, or quasi-homogeneous population of consumer-investors, differing at most by their risk aversions and endowments.<sup>2</sup> We review in Section 1 the stylized facts which lead one to believe that the predictions of the model are incorrect or that its assumptions are so grossly violated that its predictions could not possibly be correct.

In that IID/homogenous investors model, financial markets are essentially Arrow-Debreu complete in the sense that an unconstrained Pareto optimum prevails. Investors are able to insure all the risks that they wish to insure. Surprises are hedged away. Hence, along the equilibrium path, the economy is guaranteed a very tranquil ride. This is the main stylized fact that is contradicted by the data.

The major issue is how to search for, and how to identify, the right model that will replace the failed one. Apart from some attempts at modifying the form of the investors' utility function,<sup>3</sup> the main directions of research aim to relax the assumption that the investor population is homogeneous or that investors can hedge away all surprises. Markets may be incomplete, segmented or imperfect.

The financial market is incomplete when some securities that at least some investors would like to trade (or to hold), are not available for trading to anyone. The incompleteness may be postulated (exogenous) or endogenous. When it is endogenous, it may arise from the inability to write contracts for which the outcome (the state or the state variable) is observable by the parties to the contract or verifiable by third parties in charge of enforcing the contract, such as members of the judicial system. Or it may arise from the absence of law or enforcement mechanism, public or private, as is often the case when contracts involve parties residing in different countries.

Financial markets are segmented when some individuals (e.g., nationals of some countries) are prevented from trading (or holding) some securities. The distinction between incompleteness and segmentation is a fine one; it may

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<sup>2</sup>The assumption is also frequently made that the investors' lifetime utility functions are time additive.

<sup>3</sup>Moving away, that is, from the assumption of time separability or the von Neumann-Morgenstern property of state separability.

turn out to be a vacuous one. Ultimately, when incompleteness or segmentation is fully endogenized, models will be classified according to the assumed cause of either phenomenon not according to which one is being considered.

The financial market is imperfect when frictions, such as transactions costs or holding costs, cause the equilibrium to differ from the perfect market outcome. It seems plausible that a financial market bridging several countries is less perfect than a financial market designed by one country. Imperfections would seem particularly relevant to the modelling of the multi-country economy.<sup>4</sup> Imperfections in the financial markets are themselves a source of incompleteness and/or segmentation, since transactions costs prevent some individuals from holding or trading securities that they otherwise would hold or trade.

Most forms of incompleteness, segmentation or imperfection in the financial market generate a form of heterogeneity in the investor population. For instance, transactions costs cause investors to differ from each other according to their inventory of securities. Heterogeneity of the investing public may also arise from imperfections that exist *outside* the financial market. For example, when the Law of One Price does not hold worldwide in the goods market, the prices of commodities to which investors of different countries have access are unequal. When the price differences are stochastic, they create a form of heterogeneity among investors that has been the focus of a generalization of the Capital Asset Pricing Model which we review in Section 2 below.<sup>5</sup>

Furthermore, the form of incompleteness/segmentation/imperfection or heterogeneity postulated in a model dictates the relevant set of state variables that is needed to describe the evolution of the economy. For

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<sup>4</sup> At the same time, the cross-border segment of the world financial market may be less imperfect than any of the purely national market segments.

<sup>5</sup> Another departure from the basic model can be formulated by postulating that the shocks external to the system are not IID and follow, instead, some given stochastic process. However, if at least some external shocks are not observable to the econometrician, a theory based on a postulated process for these shocks quickly becomes tautological, in the sense that it may always be possible to postulate the right process which will account for almost any observed behavior of financial prices. As a research strategy it would seem preferable to first try IID external shocks with a design of the economy such that financial markets returns will not be IID in equilibrium, to match observation. If that line of research fails, it will then be time to investigate non IID outside shocks.

instance, in the presence of transactions costs, the wealths of investors, which are traditional state variables in the frictionless investment-consumption problem, no longer suffice to describe the state of the economy. Because the several forms of wealth are no longer perfect substitutes, each investor's detailed holdings must be considered as separate state variables.

Much of current research activity aims to tie equilibrium rates of return on the financial market to a number of state variables. Since returns are not IID, their conditional probability distribution has become the focus of research. This is a twofold endeavor. First, one must identify the state variables that can serve to condition returns (i.e., that have some power to predict returns). Second, one must verify whether the conditional distribution satisfies some asset pricing restrictions. For instance, can the first-moments of returns be made to match *time-varying risk premia* built on second moments, as the conditional form of the CAPM would suggest they should? The search for the relevant state variables, which will account for the time variability of asset returns, is also a search for the relevant model specification.

When searching for the correct model type, the orthodox method in economics calls for General-Equilibrium (GE) models of each type to be developed and empirically tested. GE models are superior guides to policy because they permit comparative statics and dynamics. From the empirical standpoint, they have the decisive advantage that each GE model dictates the list of state variables that describe the state of the economy. The only practical question that remains when setting out to test a GE model is that some state variables may not be observable; a search for proxy variables must then be undertaken. But there is no interrogation as to which state variable ought to be used as instruments.

However, GE models of the international capital market appear very complex to solve. One can run, instead, tests of somewhat aggregated first-order conditions which make up partial equilibrium models. The various Capital Asset Pricing Models (CAPMs) are examples of these. State variables are found by means of trial and error. I will regard the empirical analysis of partial-equilibrium models as heterodox but, possibly, more pragmatic and useful than would be the orthodox method of testing GE models after they have been fully worked out.

The question that I would like to discuss in this essay is the following: which of two different lines of attack (the orthodox GE method or the heterodox CAPM method) is more likely to help us identify the relevant state

variables, in order to have a clue as to the correct model type? I want to review the kit of models developed so far and assess the potential for success.

The remainder of the essay is organized as follows. In section 1, I review briefly the stylized facts of international financial life which ought to be explained or must, at the very least, be incorporated in models of the international economy. In section 2, I review International CAPMs that take into account deviations from Purchasing Power Parity (PPP) between consumers of various countries of residence. In section 3, I survey the empirical tests of CAPMs and their results. In section 4, I describe some General Equilibrium models. Section 5 concludes as far as GE models are concerned.

### 1. The stylized facts

Several anomalies are listed below. These are unlikely ever to be explained by the neoclassical model featuring a homogeneous investor population and IID shocks. Future models of the international economy must, at the very least, take these anomalies into account. The final goal would be to explain them with the right mix of incompleteness/segmentation/imperfection.

(i) Violations of Uncovered Interest Rate Parity (and attendant predictability of spot exchange rates changes) are well documented. I review this evidence briefly in section 3.1 below. The violations are such that they are unlikely to be explainable as risk premia.

(ii) Currencies remain under/overvalued relative to their Purchasing Power Parity level for very long periods of time. Deficits/surpluses of balances of trade are also persistent. The long-term reversion of currencies toward PPP is so slow that they are barely detectable by statisticians. In fact, the hypothesis was entertained and not rejected that real exchange rates may follow a martingale process (Adler and Lehmann (1983)). At the very least, we can say that changes in real exchange rates have a large permanent component (Campbell and Clarida (1987), Huizinga (1987)). Volatile changes in deviations from PPP translate into large differences in real interest rates between countries.

Can PPP deviations of this magnitude be relative price deviations? Or do they represent violations of the Law of One Price (LOP)? Giovannini (1988), Marston (1990), and Engel (1991) among others have shown that the LOP is grossly violated, for the reason that competition between firms is imperfect.

(iii) Studies of the U.S. economy have revealed that the time series of

consumption is too smooth for consumption to be determined by wealth and for wealth to be the present value of future consumption. Stock market studies have shown that consumption is also too smooth to serve as a basis for pricing financial assets, as the neoclassical theory of marginal utility says it should. I return to that issue in section 4.1. On the international scene, the evidence is that the correlation of consumption across countries is extremely low, whereas the theory of complete, integrated markets says it should be equal to one.<sup>6</sup>

(iv) Investors' portfolios exhibit strong home equity preference. French investors devote almost all their portfolio to French securities, whereas traditional portfolio theory would have them hold the world market portfolio. The evidence on this issue is presented in Eldor, Pines and Schwartz (1988), French and Poterba (1991), Cooper and Kaplanis (1991), Howell and Cozzini (1990, 1991), Broadgate Consultants (1991), Tesar and Werner (1992). Because extant economic theory overrates the investors' willingness to invest abroad, it is not easily capable of explaining savings in relation to terms of trade. Neither does it explain well the high degree of correlation between savings and investment noted by Feldstein and Horioka (1980). One issue of interest is the following. Have PPP deviations and exchange rate behavior, noted under (ii) above, something to do with the observed home equity preference?

## 2. International CAPMs and Uncovered Interest Rate Parity

2.1 Heterogeneity created by consumption goods prices; the International CAPM

Siegel (1972) first raised the question of asset pricing when investors live in different countries and deviations from Purchasing Power Parity is the source of heterogeneity among them. Under risk neutrality, he reasoned, forward prices are equal to expected future spot prices.<sup>7</sup> But, if a forward exchange rate, expressed in dollars per franc, is equal to the expected value of the future spot exchange rate, also in dollars per franc, how then can the forward exchange rate in terms of francs per dollar be equal to the expected value of the future spot in francs per dollar? The answer is: for as long as

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<sup>6</sup> See Grossman and Shiller (1982), Leme (1984), Scheinkman (1984), Obstfeld (1986, 1989).

<sup>7</sup> Equivalently, since Covered Interest Rate Parity always holds, this is a statement that the rates of interest on two currencies differ by the expected exchange rate change, which is Uncovered Interest Rates Parity (UIRP).

the future spot rate is random, the two equalities cannot both be true, because of Jensen's inequality. This simple observation was called "the Siegel paradox".

For two decades, international macroeconomists eschewed Siegel's question and went to the extreme of postulating, in an *ad hoc* way, that the logarithm of the forward exchange rate was equal to the expected value of the logarithm of the future spot rate. No known utility function, with which to endow consumer investors, risk neutral or otherwise, can produce such a result at equilibrium.<sup>8</sup> The goal was to do away with Siegel's paradox. Empiricists and statisticians (McCulloch (1975)) pointed out that, for the real-world degree of volatility of exchange rates, the "Jensen inequality term" was of small magnitude anyway.

In 1974, Solnik (1974) confronted directly the problem of deriving restrictions on equilibrium rates of return, that would prevail in a world capital market populated by risk averse investors who would differ in their consumption prices. A logical antecedent to the Solnik model is the closed-economy CAPM expressed in nominal terms which has been derived by Friend, Landskroner and Losq (1976). Calling  $\rho_i$  the rate of return on security  $i$ , over a short holding period, expressed in real terms (i.e., adjusted for inflation), the classic CAPM of Sharpe (1964), Lintner (1965) and Mossin (1966) says that, in equilibrium, there must exist two numbers, common to all securities,  $\eta$  and  $\theta$ , such that, for all securities (all  $i$ ):

$$E(\rho_i) = \eta + \theta \text{cov}(\rho_i, \rho_m), \quad (2.1)$$

where  $\rho_m$  is the real rate of return on the market portfolio. The two numbers  $\eta$  and  $\theta$  may be interpreted as the (possibly shadow) real rate of return and as the market average degree of risk aversion, respectively. Observing that the real rate of return,  $\rho_i$ , is given by:

$$\rho_i = (1 + R_i)/(1 + \pi) - 1, \quad (2.2)$$

where  $R_i$  is the nominal rate of return and  $\pi$  the rate of inflation, one can

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<sup>8</sup> It is not wrong to compare the log of the forward rate to the expected value of the log of the future spot rate. But then, some other term -- a sort of premium -- must also enter the equation, if compatibility with equilibrium is to be maintained.



substitute (2.2) into (2.1) and apply the rules of stochastic calculus (Itô's lemma) to obtain:

$$E(R_1) = E(\pi) + \text{var}(\pi) - \text{cov}(R_1, \pi) = \eta + \theta \text{cov}(R_1 - \pi, R_m - \pi) \quad (2.3)$$

or, rearranging terms:

$$E(R_1) = \eta + E(\pi) - (1 - \theta)\text{var}(\pi) + (1 - \theta)\text{cov}(R_1, \pi) + \theta \text{cov}(R_1, R_m). \quad (2.4)$$

In equation (2.4), the first three terms of the right-hand side sum to nominally riskless rate of return,  $r$ , if one is available.<sup>9</sup> Hence, we can rewrite (2.4) in the following form:

$$E(R_1) = r + (1 - \theta)\text{cov}(R_1, \pi) + \theta \text{cov}(R_1, R_m). \quad (2.5)$$

This simple, nominal CAPM provides us with a simple lesson. Risky inflation produces a separate premium in nominal returns. This premium receives a coefficient equal to one minus the market risk aversion; it would, therefore, be present even if investors were risk neutral ( $\theta = 0$ ). Another version of the same lesson is that the coefficient on the covariance with inflation and the coefficient on the covariance with the market sum to one. This restriction on the coefficients of the CAPM comes from the fact that investors evaluate returns in real terms (i.e., in terms of consumption units). They do not suffer from "money illusion".

As does portfolio theory generally, the work of Solnik (1974) produced two insights concerning the international equilibrium. One was a CAPM, or a restriction on securities returns; the other was a description of investors' holdings in equilibrium.

A special case of some relevance is the one in which,<sup>10</sup> in each of the respective local currency, the inflation rate is non random: the French inflation rate, measured as a percentage increase in the francs consumer index, is non random; so is the U.S. inflation rate measured in dollars etc... We call that case "the Solnik special case" and we consider its implications in various respects.

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<sup>9</sup>If none is available, call  $r$  the shadow nominally riskless return.

<sup>10</sup>This was the assumption made in Solnik's work.

Observe that the rate of inflation over a period in any country may be measured in any currency. For instance, we may measure the French inflation rate in dollar units, using the following translation formula:

$$\pi_{FRA}^{\$} = (1 + \pi_{FRA}^{FF}) \cdot (1 + \nu_{FF}^{\$}) - 1, \quad (2.6)$$

where  $\nu_{FF}^{\$}$  is the relative change in the dollar/franc spot exchange rate over the period. Equation (2.6) is a formula used merely to change units of measurement. It contains no economic assumption.<sup>11</sup> In Solnik's special case, the rate of inflation in France measured in francs is non random, but the rate of inflation in France measured in dollars is random, reflecting, as it does then, exclusively the randomness in the exchange rate.

Consider now the rates of return,  $R_i$ , of all securities and all the country rates of inflation, all expressed in the same unit, say the current dollar. If need there be, the securities rates of return that are expressed in foreign currency units may be translated into dollars using the following formula, which is analogous to the formula used for inflation rates:

$$R_i = (1 + R_i^*) \cdot (1 + \nu) - 1, \quad (2.7)$$

where  $R_i^*$  is the rate of return on security  $i$  expressed in the non dollar currency and  $\nu$  is the rate of change of the spot exchange rate expressed in dollars per unit of non dollar currency.

In the absence of Purchasing Power Parity, the rates of inflation in the various countries, all expressed in dollars, are unequal, and their differences are random. That is the kind of heterogeneity that we must have among investors, in order realistically to represent an international world in which people appreciate differently the real returns from the same securities. Each country or each national group of consumer-investors, of country  $i$  say, has its own rate of inflation,  $\pi_i^{\$}$ . In Solnik's special case, it reflects entirely the random fluctuations of each currency against the dollar; the

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<sup>11</sup>The following relationship would be a statement of relative Purchasing Power Parity:

$$\pi_{US}^{\$} = (1 + \pi_{FRA}^{FF}) \cdot (1 + \nu_{FF}^{\$}) - 1.$$

Note the difference with (2.6).  $\pi_{US}^{\$}$  is the dollar rate of inflation in the U.S.

U.S. rate of inflation is the only one that is not random in dollar units.

The international (nominal) CAPM, expressed in dollars, may now be derived in the following way. For each national group of investors, a first-order condition similar to equation (2.5) holds:

$$E(R_i^l) = r + (1 - \theta^l) \text{cov}(R_i^l, \pi^l) + \theta^l \text{cov}(R_i^l, R_p^l), \quad (2.8)$$

where  $r$  is the dollar, nominally riskless interest rate and  $R_p^l$  is the dollar rate of return on the optimal portfolio held by the investors of country  $l$ :  $R_p^l = \sum_i x_i^l R_i^l$  ( $x_i^l$  being the weight allocated by investors of country  $l$  to security  $i$ ). In order to aggregate the first-order conditions (2.8) over all the investor groups, we divide both sides of (2.8) by  $\theta^l$ , multiply them by  $W^l$  (each country's wealth), sum them over all national investor groups and finally divide them by  $\sum_l W^l / \theta^l$ , to get:

$$E(R_i) = r + \theta \sum_l (1/\theta^l - 1) W^l \text{cov}(R_i^l, \pi^l) / W + \theta \text{cov}(R_i, R_m), \quad (2.9)$$

$$W = \sum_l W^l, \quad 1/\theta = (\sum_l W^l / \theta^l) / W.$$

The international, nominal CAPM (2.9) now contains as many "inflation premia" as there are national investor groups.

In Solnik's special case, U.S. investors do not contribute a term to the dollar CAPM because for them  $\pi^l = 0$ . The inflation premia that do exist simply reflect covariances with exchange rates ( $\text{cov}(R_i^l, \pi^l) = \text{cov}(R_i^l, \nu^l)$ ). They are premia for bearing the foreign exchange risk which is statistically contained in any investment.

Relaxing Solnik's assumption only causes the various inflation rates,  $\pi^l$ , to reflect not only the random fluctuation in exchange rates but also the randomness in the various local currency inflation rates, in accordance with formula (2.6). Furthermore, the inflation premium corresponding to U.S. investors would not be zero as their  $\text{cov}(R_i, \pi^l)$  would generally differ from zero.<sup>12</sup> Observe that the coefficients of all the premia (inflation and market premia) sum to one, as they did in (2.5), because of the absence of money illusion.

The dollar rate of return from a foreign currency deposit is given by a

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<sup>12</sup> See Adler and Dumas (1983).

formula which is a special case of (2.7); it is equal to:  $(1 + r_j^*)(1 + \nu_j) - 1$ , where  $r_j^*$  is the nominal rate of interest on foreign currency  $j$ . Applying (2.9) to foreign currency deposits gives:

$$r_j^* + E(\nu_j) = r + \theta \sum_L (1/\theta^L - 1) W^L \text{cov}(\nu_j, \pi^L)/W + \theta \text{cov}(\nu_j, R_m). \quad (2.10)$$

This equation is the form taken under risk aversion by the well-known Uncovered Interest Rate Parity (UIRP) relation which relates the interest rates on two different currencies.

Consider now a change of currency unit. We switch to a different currency in terms of which to express the CAPM. The translation formula for rates of return is equation (2.7), where  $\nu$  is the rate of change of the dollar value of the currency into which one switches. Substituting (2.7) into (2.9) and using the rules of stochastic calculus, we get:

$$\begin{aligned} E(R_1^*) + E(\nu) + \text{cov}(R_1^*, \nu) = r + \theta \sum_L (1/\theta^L - 1) W^L \text{cov}(R_1^* + \nu, \pi^{*L} + \nu)/W \\ + \theta \text{cov}(R_1^* + \nu, R_m^* + \nu), \end{aligned}$$

or:<sup>13</sup>

$$\begin{aligned} E(R_1^*) = r - E(\nu) + \theta \sum_L (1/\theta^L - 1) W^L \text{cov}(\nu, \pi^{*L} + \nu)/W + \theta \text{cov}(\nu, R_m^* + \nu) \\ + \theta \sum_L (1/\theta^L - 1) W^L \text{cov}(R_1^*, \pi^{*L})/W + \theta \text{cov}(R_1^*, R_m^*). \end{aligned} \quad (2.11)$$

By virtue of (2.10), the first four terms of the right-hand side of (2.11) sum to  $r^*$ , the rate of interest on the currency one translates into. Hence we have:

$$E(R_1^*) = r^* + \theta \sum_L (1/\theta^L - 1) W^L \text{cov}(R_1^*, \pi^{*L})/W + \theta \text{cov}(R_1^*, R_m^*), \quad (2.12)$$

which is an international CAPM exactly identical to (2.9), only with a different intercept,  $r^*$ , instead of  $r$ . Changing currency unit preserves the CAPM, as would be expected in the absence of money illusion.

Adler and Dumas (1983) observed that the international CAPM (2.9) may be

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<sup>13</sup> We use the fact that the coefficients of all the premia sum to one to cancel  $\text{cov}(R_1^*, \nu)$  between the two sides of the equation.

reduced to the ordinary nominal CAPM (2.5), as far as equities are concerned. This reduced international CAPM applies to equity rates of return "hedged against exchange risk". Consider supplementing each equity investment with a basket of currencies, held short, such that the net return -- equity rate of return plus or minus the returns from currencies -- is rendered independent of PPP deviations:

$$R_i = \sum_j \gamma_{ij} (\pi_j^* + \nu_j) + \zeta_i, \quad \text{all } i \quad (2.13)$$

$$\sum_j \gamma_{ij} = 1, \quad E(\zeta_i) = 0,$$

$$\text{cov}[\zeta_i, (\pi^L - \pi^{\text{US}})] = 0, \quad \text{all } i.$$

The coefficients  $\gamma_{ij}$  are chosen in such a way that  $\zeta_i$  is independent of all PPP deviations,  $\pi_j^L - \pi^{\text{US}}$ .<sup>14</sup> The coefficients  $\gamma_{ij}$  may be interpreted as "exposures" of the equities to exchange risk.<sup>15</sup> The net returns,  $\zeta_i$ , are equity returns hedged against exchange risk. Adler and Dumas show that the following restriction holds for all  $i$  and any  $l$ :

$$E(\zeta_i) = (1 - \theta) \text{cov}(\zeta_i, \pi^L) + \theta \text{cov}(\zeta_i, R_m). \quad (2.14)$$

The CAPM (2.14) prices equity rates of return relative to all currency rates of return.

## 2.2 Portfolio holdings in equilibrium

Sercu (1980)<sup>16</sup> has given a lucid description of equilibrium portfolio holdings in the Solnik model.<sup>17</sup> Consider inverting the first-order condition system (2.8) to obtain the composition,  $x^L$ , of the portfolio held by investors

<sup>14</sup>In the Solnik special case, this reduces to a requirement that  $\zeta_i$  be made independent of all exchange rate movements,  $\nu_j$ .

<sup>15</sup>The set of coefficients  $\gamma_{ij}$  is invariant under a translation of returns from one currency of reference to the other.

<sup>16</sup>As well as Ross and Walsh (1983).

<sup>17</sup>Sercu (1980) also relaxed an assumption made by Solnik unnecessarily, namely that securities rates of return expressed in their respective currencies would be independent of exchange rates (which is:  $\gamma_{ii} = 1$ ;  $\gamma_{ij} = 0$ ,  $j \neq i$ ).

of country  $l$  taken as a whole. Observe that, for  $\theta^l = 1$ , the portfolio composition is independent of the investor's country of residence, since  $\pi^l$  disappears from the first-order conditions. An investor with unit risk aversion<sup>18</sup> is nationless, as it were. His portfolio is universally efficient.

Black's (1976) "Separation Theorem" states that, assuming short sales are allowed, the set of mean-variance efficient portfolios is included in the set of linear combinations of just two efficient portfolios. The idea then comes of decomposing the portfolio of the investor of any country (with any value of risk aversion,  $\theta^l$ ) into two components. We already know that the unit risk aversion portfolio is universally efficient. The other component will have to be specific to the investors of each country. Call  $\Omega$  the variance-covariance matrix of nominal dollar rates of return [ $\Omega_{i,j} = \text{cov}(R_i, R_j)$ ] and  $\omega^l$  the vector of covariances of dollar rates of return with country  $l$ 's nominal inflation measured in dollars [ $\omega_j^l = \text{cov}(R_j, \pi^l)$ ]. Finally, call  $\mu$  the vector of dollar excess returns ( $\mu_i = E(R_i) - r$ ). Then, from (2.8), the portfolio composition,  $x^l$ , may be written:<sup>19</sup>

$$x^l = (1/\theta^l)\Omega^{-1}\mu + (1 - 1/\theta^l)\Omega^{-1}\omega^l. \quad (2.15)$$

In equation (2.15), the first portfolio component,  $\Omega^{-1}\mu$ , is identifiable as the unit-risk-aversion portfolio; we verify that its composition is independent of  $l$ , the country of the investor. The composition of the second one,  $\Omega^{-1}\omega^l$ , is formally identical to the vector of coefficients of a multiple regression of  $\pi^l$  on all the securities' returns,  $R_i$ . This makes it easy to interpret. This portfolio is designed as a hedge of the investor's home inflation, since the correlation of its nominal return with home inflation is maximum. In the Solnik special case, in which home inflation is non stochastic when expressed in the investor's currency, the hedge portfolio is simply made up 100% of the investor's home currency deposit.

This simple decomposition gives us a picture of securities holdings in world capital market equilibrium. We are aided in this description by Figure 1. All investors in the world hold the unit-risk-aversion portfolio, RAL,

<sup>18</sup>His von Neumann-Morgenstern utility function would be the logarithmic function. We could call his portfolio "the logarithmic portfolio".

<sup>19</sup>The holding of dollar deposits is equal to the fraction  $1 - \sum_i x_i^l$  of the investor's wealth.

which generally contains equities<sup>20</sup> as well as currency deposits. Each country devotes to it a fraction  $1/\theta^L$  of its wealth.

In the Solnik special case, they devote the remainder,  $1 - 1/\theta^L$ , of their wealth to their home currency deposit exclusively. Hence, equities are held as part of the RAL portfolio only. In equilibrium, all the equity holdings must sum to the world market portfolio of equities. The relative composition of the equity part of the RAL portfolio, which is also the relative composition of the equity part of any investor's portfolio, must, therefore, be identical to the composition of the world market portfolio of equities, no matter what may be the structure of the variances and covariances of returns. At equilibrium, all investors hold equities in the same proportions.<sup>21</sup>

Currency deposits, on the other hand, are forms of borrowing (when they are held negatively) and lending. For every borrower there must be a lender; they net out to zero. If French people invest a fraction  $1 - 1/\theta^{FRA}$  of their wealth into franc deposits, as a "hedge portfolio", then the world at large must be borrowing the same amount, as part of their RAL portfolio. This observation provides the equilibrium composition of the currency part of the RAL portfolio; it provides also the relative size of the equity and currency parts of this portfolio.

In the typical case in which every country's risk aversion is greater than 1, the people of the various countries hold their home currency deposits positively ( $1 - 1/\theta^L > 0$ ). The RAL portfolio, therefore, typically contains currencies negatively. This portfolio, which is universally efficient, has the same composition as the world market portfolio, as far as equities are concerned, but contains some currency financing.<sup>22</sup>

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<sup>20</sup>I include under "equities" all securities which are in positive net supply.

<sup>21</sup>When the Solnik special case does not prevail, investors also hold equities as part of their hedge portfolios. There is no simple relationship between each investor's portfolio and the world market portfolio.

<sup>22</sup>Currencies in the RAL portfolio are an investment, albeit a negative one, as much as equities. The best proof is that the amount invested in them depends on risk aversion (and on expected returns). Naturally, the diversification motive also plays a role in the currency investment but that is not a sufficient reason to regard the currency financing in the portfolio purely as an exchange-risk hedge of the equity investment. For descriptive purposes, it could be helpful to decompose the RAL portfolio into an equity component hedged against exchange risk and a pure investment in currencies [this would be done by partitioning the matrix  $\Omega$  and then inverting it]. But

### 2.3 The lessons of international portfolio theory

International portfolio theory has produced two results which are, in principle, testable: a CAPM equation and a detailed description of portfolio holdings in equilibrium. Section 3 will be devoted to empirical tests of various CAPMs applied to international securities returns. We now discuss the implication for UIRP and the verisimilitude of portfolio holdings.

If the financial market is integrated, the CAPM (2.9) applies, of course, to all securities. When applied to currency deposits, it takes the form (2.10) which is a relationship between short-term nominal interest rates quoted on two different currencies, or, equivalently, a relationship between the short-maturity forward premia and the expected spot exchange rate. It provides the deviation from the traditional UIRP which prevails when investors are risk averse and PPP does not hold. In section 3 below, we examine the empirical evidence on UIRP to see whether it is likely to be compatible with (2.10) written in various ways.

We can now return to Siegel's paradox by taking the limit  $\theta^L \rightarrow 0$  in (2.10) to reach risk neutrality. It is obvious from (2.10) or (2.9) that the equilibrium collapses. When investors are risk neutral and PPP does not hold, they disagree infinitely about the required returns; no equilibrium exists.

If PPP prevails, however, the correct CAPM is (2.5) which, when applied to currency deposits, gives:

$$r_i^* + E(\nu_i) = r + (1 - \theta)\text{cov}(\nu_i, \pi) + \theta\text{cov}(\nu_i, R_m), \quad (2.16)$$

where  $\pi$  is world inflation measured in the same currency unit as exchange rates and rates of return. If  $\theta = 0$ , we still have:  $r_i + E(\nu_i) = r + \text{cov}(\nu_i, \pi)$ . There exists an equilibrium but the equilibrium relationship between interest rates incorporates an inflation premium which is a deviation from nominal UIRP, the reason being that investors care about real returns. This inflation premium resolves the Siegel paradox.<sup>23</sup> Whether  $\theta = 0$  or not, equation (2.16) is perfectly symmetric; the expected rate of return of

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such a decomposition would be uninteresting, since all investors would hold the two subparts in the same proportion.

<sup>23</sup>This was first noted in print by Frenkel and Razin (1980).



investing in dollars measured in currency  $i$  is:<sup>24</sup>

$$r = E(\nu_1) + \text{var}(\nu_1) = r_1^* + (1 - \theta)\text{cov}(-\nu_1, \pi - \nu_1) + \theta\text{cov}(-\nu_1, R_m - \nu_1), (2.17)$$

which is equivalent to (2.16) itself.

Turning now to equilibrium portfolio holdings, the lesson of international portfolio theory is rich in precision. But is it compatible with casual empiricism? In section 1, we pointed out that investors' actual portfolios exhibit strong home equity bias. The theory predicts that investors would exhibit strong home currency preference in the sense that they would hold their home currency deposits as a hedge, or a riskless asset. But the theory also says that they all hold equity in the same relative composition as in the world market portfolio. This is true strictly only in the Solnik special case, which is, likely, not far from the truth. It would not be true if home currency inflation in the investors' country of residence were random. Although this should be settled empirically, home inflation risk is unlikely to be a convincing justification for home equity preference.

Exchange risk and PPP deviations turn out not to be a relevant explanation for home equity bias.<sup>25</sup> This is because people, in the model, can hedge away exchange risk. The availability of currency deposits in all currencies (or forward exchange contracts) renders the investor population homogeneous once again.<sup>26</sup>

Evidently, a number of features of the actual international capital market are missing from the model, that would help explain home equity bias.

<sup>24</sup> The inflation premium serves to resolve the paradox even if world inflation measured in dollars,  $\pi$ , happens to be non random (so that the premium from the dollar point of view happens to be zero). If PPP prevails, world inflation, when measured in non dollar currency  $i$ , is random and a premium appears in the translated equation (2.17). That premium does the job of explaining away the Siegel paradox.

<sup>25</sup> In particular, differences in consumption baskets *per se* do not explain home equity bias. I supplement this discussion in section 4.2 below. There, I also briefly consider nontraded goods.

<sup>26</sup> Black (1990) pointed out that Japanese investors provide cheap hedging of Yen risk to American investors and vice versa. If his risk aversion is greater than 1, each investor wishes to hold his home currency positively as part of his hedge portfolio. Foreigners can, therefore, short the home currency at little cost.

Capital market segmentation, in various forms, is an obvious candidate.<sup>27</sup> Research should be undertaken to identify the nature and sources of segmentation. Sovereign risk has been studied in papers on the Less Developed Country debt crisis but has not been modeled as a source of capital market segmentation. Asymmetric information, along the line of Admati (1985) or Diamond and Verrechia (1981), should find an application in International Finance. French people may possess more direct information on the state of the French economy and French firms than do American investors, and vice versa. Local information would be a form of insider information leading investors to invest less abroad, and possibly also to modify the composition of their foreign portfolio.

Considering the paucity of data on actual portfolio holdings, it would be useful to draw the implications of international portfolio theory for international capital flows, as these are recorded in the balance of payments.<sup>28</sup> If securities rates of return are assumed to be Identically Independently Distributed over time (IID), the theory predicts a modicum of transactions in securities. As in the classic, one-country theory, rebalancing occurs following the realizations of rates of return. Since investors wish to maintain fairly fixed fractions of wealth invested in the various equity securities, a high realization of return on a security produces an increase in the value of portfolio holdings of that security, leading to a desire to sell it. At that point, explicit treatment of physical investment by firms and households becomes necessary. If physical capital is perfectly flexible, the desire to sell is accommodated by a reduction (or a reduced increase) in the firm's capital stock (by means of dividend distribution and/or share repurchase). The investor hardly needs to transact.

In the international model, portfolios differ across investors in their currency components. This gives some play to exchange rates. Following the realization of a large increase in the value of the franc, French people wish

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<sup>27</sup> The main academic contribution to the theory of capital market segmentation is Errunza and Losq (1985). Most of the literature on segmentation takes segmentation as a given and works out its consequences. At best, it classifies the various forms of segmentation. It does not explain it. The drawback is that no guidance is provided as to which form of segmentation constitutes a reasonable assumption.

<sup>28</sup> Unfortunately, international portfolio theory does not recognize the distinction, which is made in balance-of-payments statistics, between capital flows of a portfolio nature and direct investment.

to sell it to buy other currencies and equities. The foreigners' franc debt has increased in value; they want to pay some back. These two desires are compatible.

There would be more action on the capital-flow front if rates of returns were not IID. Strictly speaking, the model that we have explicated so far is not compatible with non IID returns, because we have not incorporated in the investors' first-order conditions (2.8) intertemporal hedging terms à la Merton (1971, 1973). If we were to consider the model, nonetheless, as an approximation of a dynamic model and were to make use of it as workhorse, simply by replacing the expected values, variances and covariances of returns by their conditional counterparts, the capital flow issue would not be clarified until after the joint stochastic process for rates of return and other "information" variables has been identified. For instance, an increase in a security's conditionally expected rate return would cause investors to buy that security and the issuing firm simultaneously to increase its physical capital. But a temporary increase in return would induce a smaller value of this flow than would a permanent one.<sup>29</sup>

Soon, it appears that a proper study of international capital flows requires a worked out General Equilibrium model with explicit treatment of monetary and fiscal policy, capital formation by firms, price and wage rigidity etc.. so that one knows which variables drive the stochastic process for rates of return.

#### 2.4 Other CAPMS applicable to international returns

In the previous sections, the traditional, one-country CAPM has been shown not to survive, without amendment, the transfer to a world of heterogeneous investors. Other CAPM types are different in this respect. The Consumption CAPM of Breeden (1978) or the Arbitrage Pricing Theory (APT) of Ross (1976) may be applied to international data without change.

Consider first the Consumption CAPM. Breeden (1978)<sup>30</sup> has shown that securities' real rates of return satisfy a first-order condition involving any investor's rate of consumption. This condition holds generally, in a multiperiod as much as in a one-period setting, in a complete or an incomplete

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<sup>29</sup>In section 3 below, I discuss empirical work in which the process for rates of return is taken into account.

<sup>30</sup>See also Lucas (1978), equation (6).

market for as long as the investor optimizes his portfolio and his consumption. It only requires that the investor have access to the securities to which the condition is applied. According to this condition, there exist two numbers  $\eta$  and  $\theta$  such that:

$$E(\rho_1) = \eta + \theta \text{cov}(\rho_1, \chi), \quad (2.18)$$

where  $\chi$  is the investor's real consumption rate. Translation of this real CAPM into a nominal one gives:

$$E(R_1) - E(\pi) + \text{var}(\pi) - \text{cov}(R_1, \pi) = \eta + \theta \text{cov}(R_1 - \pi, c - \pi), \quad (2.19)$$

where  $c$  is the investor's nominal consumption rate. Finally, interpreting some terms as summing to the nominal rate of interest, we get:

$$E(R_1) = r + (1 - \theta) \text{cov}(R_1, \pi) + \theta \text{cov}(R_1, c). \quad (2.20)$$

This condition holds from the point of view of any national group of investors.<sup>31</sup> It can be used provided that group's rate of consumption is observable, or under some other auxiliary assumptions that give the model some empirical content. Equation (2.20) would be the ideal device to determine to which securities each category of investors has access and thereby track down segmentations. However, the Consumption CAPM has failed empirically as a pricing device<sup>32</sup> mostly because consumption is observed infrequently and its observed behavior is so much smoother than that of wealth. The variable,  $c$ , has little variation over time and the term  $\text{cov}(R_1, c)$  fails to explain the cross-section of returns. Using the consumption CAPM, Wheatley (1988) has not detected segmentation in the world equity market. Hansen and Hodrick (1983) have used (2.20) as a point of departure but have found a way, under some auxiliary assumption, to avoid the need to measure consumption. We return to their work in section 3.3.

Ross' (1976) APT may be applied to international data even if the world

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<sup>31</sup>Stulz (1981) has examined the aggregation of this condition over all national investor groups, in the absence of PPP.

<sup>32</sup>See Hansen and Singleton (1982), or Breeden, Gibbons and Litzenberger (1989).

population is made heterogeneous by PPP deviations. Ross postulates that securities rates of return satisfy a factor structure such as:

$$R_i = E(R_i) + f \cdot \beta_i + \epsilon_i, \quad (2.21)$$

where  $\beta_i$  is a vector of coefficients and  $f$  a vector of  $K$  common random factors;  $E(f) = 0$ ;  $E(\epsilon_i) = 0$ ;  $E(f \cdot \epsilon_i) = 0$ . Some structure (often a diagonal structure) is imposed on the variances and covariances of the security specific residuals,  $\epsilon_i$ s, while the number of securities in the market is increased to infinity, and the number of factors remains fixed at  $K$ . By virtue of the Law of Large Numbers, any portfolio made up of the large number of securities will have a negligible  $\epsilon$  risk. Only the factor risks are "non diversifiable". Ross shows that, in order to bar arbitrage, there must exist a pricing relationship between expected returns and the non diversifiable risks,  $\beta$ :

$$E(R_i) = r + \lambda \cdot \beta_i, \quad (2.22)$$

where  $\lambda$  is a vector of market prices of risk applicable to all securities.

Solnik (1983a) and Levine (1989) have shown that a relationship such as (2.22) still holds after a change of reference currency.<sup>33</sup>

### 3. Comparing empirical methods

Because the CAPM is a partial-equilibrium, incompletely specified model, it is not directly testable without some auxiliary assumptions. Mostly, what is needed is the specification of the external state variables that move the probability distribution of returns period after period. Since the model by itself, in contrast to a fully specified General Equilibrium model, contains no clue as to the choice of these instrumental variables, one uses it purely as a "fishing net". The test of performance for a particular set of variables is how well they help to predict returns and to what extent the conditional expected values, variances and covariances so obtained satisfy the CAPM

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<sup>33</sup> For the dissenting view, see Ikeda (1991).

restriction.<sup>34</sup>

### 3.1 Constant risk premia

Many tests of Uncovered Interest Rate Parity have been conducted by regressing spot exchange rate movements on earlier interest rate differentials (or forward premia). For instance, Cumby and Obstfeld (1984) tested the following equation:

$$\ln S_{t+1} - \ln S_t = a + b(\ln F_t - \ln S_t) + u_{t+1}, \quad (3.1)$$

where  $S_t$  and  $F_t$  are a pair of spot and forward exchange rates at time  $t$ . Under the strict UIRP hypothesis,<sup>35</sup> one would have:  $a = 0$  and  $b = 1$ . Amending the strict UIRP to include a risk premium, and adding the hypothesis that that risk premium is constant over time, one should still have:  $b = 1$ . For most currencies over the floating-rate period, Cumby and Obstfeld found estimates of beta that were much smaller, and significantly smaller, than 1 and many times also smaller than 0. Moreover, they found evidence of heteroskedasticity in the residuals,  $u$ . For this reason, it is hard to justify the assumption that risk premia are constant.

Fama (1984) asked the question: would a justifiable risk premium be sufficiently variable to account for the finding of  $b \ll 1$ ? To throw some light on this question, he broke the right-hand side of equation (3.1) in the following way:

$$\ln S_{t+1} - \ln S_t = a + (\ln F_t - \ln S_t) + (b - 1) \cdot (\ln F_t - \ln S_t) + u_{t+1}. \quad (3.2)$$

The terms of the right-hand side of (3.2) can be interpreted: the second term is, of course, the forward premium; the sum of the first three terms equals the conditionally expected change in the (log) spot exchange rate,  $E_t[\ln S_{t+1}] - \ln S_t$ , while the sum of the first and third term,  $a + (b - 1) \cdot (\ln F_t - \ln S_t)$ , is the conditional risk premium. In this interpretation, a finding of  $b < 0.5$  implies that the conditional risk premium varies over time more than the

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<sup>34</sup>We have already pointed out that, when returns are not IID, for consistency one should really be using a multiperiod CAPM in the manner of Merton (1971, 1973).

<sup>35</sup>Or, rather, a logarithmic version of it, in order to avoid the Siegel paradox.

conditionally expected change in the exchange rate, or that the unconditional variance of the risk premium is greater than the unconditional variance of the expected change.<sup>36</sup> For instance, if  $b = 0.4$ , the conditional expected change varies as much as 0.4 times the forward premium, while the conditional risk premium varies as much as -0.6 times the forward premium. In absolute value, 0.6 is greater than 0.4. Also, note the signs: the conditional risk premium would have to covary negatively with the conditional expected change. These features of a risk premium are unlikely to be found in a General Equilibrium model.

The finding that  $b \neq 1$  implies that exchange rate changes are predictable from the knowledge of forward premia. D. Hsieh has pointed out, however, that this in-sample predictability of exchange rates frequently does not translate into the ability to predict out of sample. Some assumptions of the statistical theory underlying the tests must, therefore, be violated. Hsieh (1991) suspects that the assumption of linearity, built into equation (3.1), ought to be investigated. He uses non linear methods in an attempt to achieve out-of-sample predictability.

One alternative to the constancy of the risk premium has been suggested by Frankel (1982). He assumed that the variance-covariance matrix of exchange rates is constant over time. This assumption does not imply a constant risk premium because the supply of assets denominated in the various currencies varies over time. Frankel (1982), and later Engel and Rodrigues (1988) as well as Engel, Frankel and Froot (1991), test the following equation:

$$R_{t+1} - r_t = \theta_t \Omega x_t + u_{t+1}, \quad (3.3)$$

where the left-hand side term is a vector of excess return on several securities over the time period  $t$  to  $t+1$ ,  $x_t$  is a vector of asset supplies at time  $t$  and  $u$  a vector of unanticipated returns. Equation (3.3) is tested under the restriction that the matrix  $\Omega$  is constant and is equal to the variance-covariance matrix of the unanticipated returns,  $u$ . The advantage of the method is that no restriction is imposed on the price of risk,  $\theta_t$ . The vector of asset supplies is identified with the supplies of government bonds denominated

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<sup>36</sup> To learn about the connection between regression tests such as (3.1) and variance bounds tests, see: Frankel and Stock (1987), Froot (1987) and Cochrane (1991).

in the various currencies.<sup>37</sup> The test rejects the restriction.

### 3.2 Instrumental variables

In this subsection, I try to establish an inventory of the variables that have been used by researchers to predict returns, in the one-country as well as in a multi-country setting.

Stambaugh (1986), Fama and French (1988a,b) among others pointed out that stock market returns are predictable (which means "non IID") on the basis of past returns and on the basis of a few "information variables". Figure 2, which is a diagram borrowed from Kandel and Stambaugh (1988) summarizes the empirical dependence on past returns; the diagram shows the serial correlation in relation to the length of the holding period. The serial dependence is particularly strong for rates of return calculated over longer holding periods.<sup>38</sup> As much as 15 to 25 percent of the variation of 3-5 year stock returns is predictable from past returns.<sup>39</sup> Notice that the serial correlation is mostly positive over shorter holding periods (below and of the order of one year) and negative for longer holding periods, reflecting reversion in stock prices.<sup>40</sup>

In the one-country context, Fama and Schwert (1977), Rozeff (1984), Keim and Stambaugh (1986), Fama and French (1988b) and Campbell and Shiller (1988) have also shown empirically that stock returns are predictable on the basis of one or several of the following variables: the dividend yield,<sup>41</sup> the short-

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<sup>37</sup>For the method to be correct, however,  $x_t$  must include all assets.

<sup>38</sup>For a methodological critique of the Fama-French approach, see Richardson (1990). The reader is cautioned that the successive points of the curve of Figure 2 are not observed independently; they are calculated from the same data; the pattern displayed by the curve may, therefore, be somewhat misleading.

<sup>39</sup>Serial dependence in a single series may be captured not only by running distributed-lag regressions, but also by a Lo and MacKinlay (1988) test, which is based on a ratio of two variances measured on various period lengths. The approach can be extended to a multivariate context.

<sup>40</sup>Please, note that reversion does not necessarily result from reversion in the expected value (mean reversion). It can also result from changes in variance; reversion is induced when variance becomes larger as one moves away from the reversion point.

<sup>41</sup>The Fama-French study is concerned with the behavior of stock indexes. I am referring here to the dividend yield on the index.



term rate of interest, the spread between long and short bond yields (which is a term-structure premium), the spread between corporate and government bonds (which is a default-risk spread), and a number of dummy variables such as one for the month of January and for some days of the week. Figure 3, also borrowed from Kandel and Stambaugh (1988), summarizes the empirical dependence of longer-holding-period rates of return on these variables.<sup>42</sup>

Cutler, Poterba and Summers (1991)<sup>43</sup> proceeded to verify whether other variables had any forecasting ability for stock returns in the international context. In view of the fact that the historical record for stock markets of countries other than the U.S. is typically comparatively short (of the order of 20-30 years), international studies cannot very well consider rates of return over very long holding periods. Cutler, Poterba and Summers examined one-month holding period returns and cumulated them to holding periods of up to 48 months. Their sample included stock market indexes observed over the period 1960-1988 for thirteen countries and ten bilateral foreign exchange rates from 1974 to 1988, as well as commodities, such as gold and silver, and real assets. In most cases, they found positive autocorrelation of rates of return over holding periods shorter than or equal to one year and negative autocorrelation for longer holding periods (one to four years). The stock returns were predictable, but weakly so, on the basis of own dividend yields.<sup>44</sup>

As far as foreign exchange rates are concerned, the results summarized in section 3.1 above imply predictability on the basis of forward premia. Predictability has also been found on the basis of past forecast errors of the forward rates.<sup>45</sup>

Bekaert and Hodrick (1992) used a vector-autoregressive (VAR) framework to study simultaneously stock returns and foreign-exchange returns in relation to some predictive variables. They looked at monthly observations of the stock markets of four countries (USA, Japan, UK, Germany) and three foreign exchange rates over the period January 1981 to December 1989. They also

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<sup>42</sup>For a critique, see Forster and Smith (1992). Again, the reader is cautioned that the successive points of the curve are not statistically independent.

<sup>43</sup>Earlier, Gultekin (1983) and Solnik (1983) had examined whether short-term interest rates predict stock returns in each country separately.

<sup>44</sup>See also Solnik (1992).

<sup>45</sup>See Hodrick (1987) for a survey of this evidence.

examined rates of return of equities, from the DM, the Yen and the Pound points of view. The predictive variables were dividend yields and forward exchange premia. VARs were estimated for two countries at a time; the corresponding two dividend yields were included in the VAR. Bekaert and Hodrick concluded, with confidence level of the order of 0.99, that the VAR is capable of predicting monthly rates of return. They also calculated the autocorrelations of longer-holding period returns implied by the monthly VAR model with its estimated coefficients, in order to study the link between short-term and longer-term predictability. Interestingly, for some countries at least, they were able to replicate in this way patterns of autocorrelations such as the ones described above: positive autocorrelations over a few months, negative autocorrelations over periods of more than one year.<sup>46</sup>

Using a number of combinations of variables that include the U.S. and Japanese dividend yields, short-term rates, and long-term/short-term rate spreads, Campbell and Hamao (1992) report an  $R^2$  equal to 0.065 for Japan and an  $R^2$  equal to .10 for the U.S. over the period 1971-1987.

Harvey (1991) affords a comparison between various sets of predictive variables. The study covers seventeen countries over the period December 1969 to May 1989. One set of predictive variables is "the common set" which includes: the lagged world stock market rate of return,<sup>47</sup> a dummy variable for the month of January, the U.S. dividend yield, the U.S. term structure premium and a U.S. default risk spread. But several "local instrument sets" are also examined which reflect own country effects. Harvey concludes that: "the common information variables appear to capture the bulk of the predictable variation in the country returns." Comparing the common information regressions to the completely local information regressions, almost all countries show a lower  $R^2$  using the pure local information. The two local information variables that are the most helpful (but only marginally so) are the lagged own-country returns and the local dividend yields.

The various empirical asset pricing studies that will be reviewed in the next subsections use similar sets of instrumental variables.

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<sup>46</sup>This was achieved without taking into account the variability of the conditional variance of monthly returns.

<sup>47</sup>Harvey's purpose was to exploit parsimoniously the serial dependence found in Fama and French (1988a).

### 3.3 Latent variables models

Hansen and Hodrick (1983) with foreign-exchange market data and Gibbons and Ferson (1985) with stock market data, were the first to use a statistical model called the "latent variable model". To explain this statistical model, let me use as a point of departure the consumption CAPM which we rewrite in its conditional form:

$$E_{t-1}(R_i) = r_{t-1} + (1 - \theta_{t-1})\text{cov}_{t-1}(R_i, \pi) + \theta_{t-1}\text{cov}_{t-1}(R_i, c). \quad (3.4)$$

The subscript  $t-1$  in equation (3.4) refers to the values of moments and parameters conditioned on the values of the information variables at time  $t-1$ . In most tests, the inflation premium,  $\text{cov}_{t-1}(R_i, \pi)$ , is neglected and some portfolio,  $p$ , is used to interpret  $\theta$ , leading to:

$$E_t(R_i) - r_t = [E_t(R_p) - r_t] \cdot \beta_i, \quad (3.5)$$

where:  $\beta_i = \text{cov}_{t-1}(R_i, c) / \text{cov}_{t-1}(R_p, c)$ . The assumption is made that  $\beta_i$  is constant over time. Equation (3.5) then says that the time variations of all securities' expected returns have one factor in common:  $E_{t-1}(R_p) - r_t$ . Some way must be found of deciding empirically whether one factor accounts for expected returns. To this end, postulate that the excess return on portfolio  $p$  is linearly related to a number of instrumental variables,  $z$ :

$$R_{p,t} - r_t = \sum_j \delta_j \cdot z_{j,t} + u_t, \quad (3.6)$$

with the condition:

$$E(u | z) = 0, \quad (3.7)$$

which implies:

$$E(u \cdot z) = 0. \quad (3.8)$$

Substituting (3.6) into (3.5) gives:

$$E(h \mid z) = 0. \quad (3.9)$$

where  $h$  is defined by:

$$R_{i,t} - r_{t-1} = \beta_i \cdot [\sum_j \delta_j \cdot z_{j,t-1}] + (u_t \cdot \beta_i + h_{i,t}), \quad (3.10)$$

Equation (3.9) implies:

$$E(h_i \cdot z) = 0. \quad (3.11)$$

Equations (3.8) and (3.11) put together, of course, imply:

$$E[(u_t \cdot \beta_i + h_{i,t}) \cdot z] = 0, \quad (3.12)$$

which means that the residual of (3.10) is orthogonal to the instruments.

The latent variable method proceeds as follows. First, estimate (3.10) as an unconstrained regression:

$$R_{i,t} - r_{t-1} = \sum_j \gamma_{ij} \cdot z_{j,t-1} + \epsilon_{i,t}. \quad (3.13)$$

Then, reestimate (3.10) constraining the coefficients  $\gamma_{ij}$  to be of the form:  $\gamma_{ij} = \beta_i \cdot \delta_j$ . Finally, compare the goodness of fit (the  $\chi^2$ s) of the two regressions. If their difference is significant statistically, the pricing model is rejected.

Using this method, Hansen and Hodrick (1983) rejected the consumption CAPM on foreign exchange rates, using lagged forward premia as instruments. But Hodrick and Srivastava (1984) accepted it on the same data, using past forecast errors of forward rates as instruments.

Giovannini and Jorion (1987, 1989) used this method also to run tests of adequacy of the consumption CAPM, applied jointly to foreign exchange and stock markets. They rejected the model on the basis of weekly observations of spot exchange rates of seven currencies, one-week Eurocurrency interest rates and U.S. stock market returns on a value weighted index, over the period Aug 3, 1973-Dec 28, 1984. They also tried to relax the assumption that  $\beta$ s are constant, by allowing them to vary linearly with rates of interest. The model was no longer rejected. Variation in betas may be worth investigating. We return to that issue in the next sections.

Campbell and Hamao (1992) and Bekaert and Hodrick (1992) report results of models with one, two or three latent variables, pertaining to pairs of countries. The data on U.S.-Japan reject the model while the data on U.S.-UK and U.S.-Germany do not. Three-country systems also reject the one- and two-latent variable models.

In order to allow it to vary over time, Cumby (1987) explicitly models covariance with consumption in a linear relation with a set of instrumental variables.<sup>48</sup> He also rejects the consumption CAPM on international data.

The latent variable method begs one issue. Suppose that one finds that the model is accepted with the meaning that some time variation of  $\theta_t$  or  $E_t(R_p) - r_t$  can account for risk premia. The issue that is not addressed is whether the implied variance of this variable is or is not excessive relative to the variance one would expect from a fully specified portfolio optimization model based on risk aversion.<sup>49</sup>

### 3.4 The Arbitrage Pricing Theory and multifactor CAPMs

The difference is subtle between latent variable models and empirical implementations of the Arbitrage Pricing Theory. Ross' (1976) original intent in constructing the APT was to allow more degrees of freedom than there were in the classic, one-premium CAPM. To this end, he proposed the introduction of several factors which would generate as many risk premia.

The factors could either be prespecified, observable variables, in which case the factor loadings were to be estimated by regression; or they could be non specified, or latent, in which case the factor structure and loadings were to be estimated by some form of factor analysis.

Just as the CAPM may be stated in terms of unconditional or conditional moments, so does the APT. It suffices to rewrite equation (2.22) as:

$$E_{t-1}(R_i) = r_{t-1} + \lambda_{t-1} \cdot \beta_i, \quad (3.14)$$

where the conditional expected excess returns on assets,  $E_{t-1}(R_i) - r_{t-1}$ , and the prices of risk,  $\lambda$ , vary over time, perhaps in relation to a number of instrumental variables,  $z$ . Connor and Korajczyk (1991) have shown that the

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<sup>48</sup> This is in the same spirit as Giovannini and Jorion who allowed  $\beta_s$  to vary in relation to the rate of interest.

<sup>49</sup> See section 3.7 below.

variable risk premia,  $\lambda_{t-1}$ , may be interpreted as conditionally expected excess returns on portfolios whose rates of return mimic the factors,  $f$ .<sup>50</sup>

As in the latent variable approach, the factor loadings,  $\beta_{1s}$ , are always assumed constant. When the factors are unspecified variables, therefore, there is practically no difference conceptually between the APT in its conditional form, and the latent-variable approach with several latent variables.<sup>51</sup>

When the factors are prespecified, observable variables, it is important to distinguish two sets of variables which play different roles in the statistical analysis. Set # 1 is the set of variables,<sup>52</sup>  $f$ , which are priced factors in the pricing equation. These need not predict returns over time; they need only serve as common variables that explain contemporaneously a wide cross section of returns. Alternatively, an optimization-based theory may point to the priced factors; for instance, in the classic CAPM, the single priced factor is the market. Set # 2 is the set of predetermined, or instrumental variables,  $z$ , to which the variations of  $E_t(R_1) - r_t$  and  $\lambda_t$  will be related. In some CAPMs,<sup>53</sup> it would be stipulated that the  $z$ s are lagged versions of the  $f$ s; but that is not always so. In point of fact, many of the variables that we have identified in section 3.2 as useful instruments were also recognized by APT investigators Chen, Roll and Ross (1986) as useful factors.

In the case of prespecified factors, empirical implementations of the APT rely on a two-pass method. First, to estimate the betas, deviations of rates of returns from their conditionally expected values are regressed on factors in a times series fashion. Second, the conditionally expected returns are

<sup>50</sup> Consider a portfolio,  $j$ , such that  $\beta_{jj} = 1$  and  $\beta_{jk} = 0$  ( $k \neq j$ ). Equation (2.22) applied to that portfolio  $j$  reads:

$$E(R_j) = r + \lambda_j.$$

<sup>51</sup> However, different researchers utilize different estimation techniques. The latent variable model is frequently estimated with the Generalized Method of Moments (GMM), which places severe limitations on the number of assets that can be considered simultaneously, whereas the APT model is usually estimated by means of a factor analytic technique, which is a two-step method. First, the factor analytic structure is estimated, then the expected returns are compared cross-sectionally to the loadings (betas).

<sup>52</sup> In the notation of equation (2.21).

<sup>53</sup> Such as the multiperiod CAPM of Merton (1971, 1973) which can also be written in the form (3.14).

confronted in a cross section with the calculated betas.

Gultekin, Gultekin and Penati (1989) is a study which relies on the unconditional version of the APT, using both prespecified as well as latent factors. The data concern the stock markets of the United States and Japan over two periods of time: January 5, 1977 to December 31, 1980 and January 7, 1981 to December 26, 1984. Trading of Japanese securities by foreigners was liberalized on January 1, 1981. Gultekin, Gultekin and Penati formed twenty two portfolios of U.S. and Japanese stocks and estimated the risk coefficients,  $\lambda$ , for these two categories of securities separately, based on the same set of prespecified factors. Their main test was a test of equality of the  $\lambda$ s between the two categories of stocks. Rejection of the null hypothesis of equality indicates capital market segmentation between the U.S. and Japan. For most testing procedures used, the null is clearly rejected over the first period and not rejected over the second period. The technique has been able to detect segmentation during a period where we know it is present.

Korajczyk and Viallet (1991) studied rates of return on foreign currency investments in relation to latent, common factors that account for a large part of the variance of returns across the foreign exchange and equity markets. If currencies and equities are priced on an integrated market, both the APT and, e.g., the CAPM (2.10) indicate that the changes in a common set of risk premia should be able to account for changes over time in expected returns, to the exclusion of any other variable. Therefore, if excess returns on foreign currencies are regressed on a set of regressors which includes forward premia (interest rate differentials), as in Cumby and Obstfeld (1984),<sup>54</sup> but include also the excess returns on the factor mimicking portfolios, the forward premia should receive a coefficient that is not significantly different from zero; such is their null hypothesis. They used monthly observations of eight foreign exchange rates against the dollar over the period January 1974 to December 1988. They built their factor structure from these eight currencies and as many as 8000 to 11000 equity securities traded in various stock exchanges of the world.<sup>55</sup> They reject the null

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<sup>54</sup> See section 3.1.

<sup>55</sup> All returns are translated in dollars. Currency translation effects are common to all stocks. There is a strong suspicion that the factor mimicking portfolios reflect mostly currencies.

hypothesis. The APT with constant betas does not adequately explain excess returns on currencies.

Ferson and Harvey (1991) investigate a CAPM/APT model in which the factors include the world market portfolio and a number of prespecified common variables,  $f$ , such as the price of oil. They call them "The World Economic Risk Factors". Separately, they introduce "Aggregate Information Variables",  $z$ , such as the ones listed in section 3.2 above. They first estimate the betas with respect to the world market and the economic risk variables. The originality of the work of Ferson and Harvey<sup>56</sup> is that they allow for variations in these betas. They simply recompute them at the end of each 60-month period. The betas, that is, are assumed not to vary over the subperiods, in the tradition of Fama and MacBeth (1973). They also demonstrate that the information variables have predictive power for rates of return. The second step would be to test a conditional CAPM/APT. The difficulty with the technique is that there is no way of knowing whether a 60-month updating frequency for betas is an adequate one. The only way of answering that question is to explicitly account for variations of the second moments.

### 3.5 AutoRegressive Conditional Heteroskedasticity (ARCH)

ARCH is a statistical model that serves to estimate a process for the second moments. It implies persistence in the second moment, a feature which seems present in many economic time series. The simplest ARCH model for the univariate series  $(y_t)$  is written:

$$y_t = \phi y_{t-1} + \epsilon_t$$

$$\epsilon_t \sim N(0, h_t)$$

$$h_t = \omega + \alpha \epsilon_{t-1}^2$$

where  $|\phi| < 1$ ,  $\omega > 0$  and  $\alpha \geq 0$ . A more sophisticated version (called Generalized ARCH in Mean) is:<sup>57</sup>

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<sup>56</sup> See Ferson and Harvey (1991a) for their work on U.S. securities returns.

<sup>57</sup> The adjective "Generalized" refers to the presence of the  $h_{t-1}$  term in the equation for  $h_t$  and the expression "in Mean" refers to the presence of the  $h_t$  term in the "mean equation", the first equation of the system.



$$y_t = \phi y_{t-1} + \gamma h_t + \epsilon_t$$

$$\epsilon_t \sim N(0, h_t)$$

$$h_t = \omega + \alpha \epsilon_{t-1}^2 + \beta h_{t-1}$$

ARCH models are typically estimated by means of maximum likelihood.

Chan, Karolyi and Stulz (1992) have used<sup>58</sup> a multivariate version of GARCH to test whether the covariance with foreign markets generates a significant risk premium in the CAPM applied to domestic stocks. They implement the following equation system:

$$r_{dt+1} = \alpha_d + \beta_{dv} w_{dt} h_{dt+1} + \beta_{dc} (1 - w_{dt}) h_{ct+1} + \theta_{d1} \epsilon_{dt} + \theta_{d2} \epsilon_{dt-1} + \epsilon_{dt+1} \quad (3.15)$$

$$r_{ft+1} = \alpha_f + \beta_{fv} w_{ft} h_{ft+1} + \beta_{fc} (1 - w_{ft}) h_{ct+1} + \theta_{f1} \epsilon_{ft} + \theta_{f2} \epsilon_{ft-1} + \epsilon_{ft+1} \quad (3.16)$$

$$\epsilon_t = \begin{bmatrix} \epsilon_{dt+1} \\ \epsilon_{ft+1} \end{bmatrix} \sim N(0, H_{t+1}) \quad H_{t+1} = \begin{bmatrix} h_{dt+1} & h_{ct+1} \\ h_{ct+1} & h_{ft+1} \end{bmatrix} \quad (3.17)$$

$$H_{t+1} = P'P + F'H_t F + G' \epsilon_t \epsilon_t' G \quad (3.18)$$

In equations (3.15) and (3.16),  $r_d$  and  $r_f$  denote domestic or foreign equity return;  $w_d$ ,  $w_f$  denote the composition of the benchmark portfolio as between domestic and foreign stocks and the  $h$ s are time-varying, conditional variances and covariances. The weights,  $w$ , are observable, being the relative capitalizations of the various markets. They use daily stock returns on indexes from the United States, and Japan (Nikkei) or Europe, Asia and the Far East (Morgan Stanleys' EAFE). They find that the conditional covariance of the U.S. return with the Nikkei return has a significant positive effect on the U.S. conditional expected returns, but that the conditional variance of the

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<sup>58</sup> Hamao, Masulis and Ng (1990) also use a GARCH formulation to model returns in several countries. Although they use an ARCH-in-Mean formulation, their main goal is not to test for the significance of risk premia but to describe spillover effects from one market to another. Engel and Rodrigues (1988) and Engel, Frankel and Froot (1991) use ARCH to allow variations in the variance-covariance matrix of the Frankel (1982) asset supply CAPM, discussed above in section 3.1.

U.S. return has no effect. In another test, they find no evidence that the prices of risk,  $\beta$ s in the notation of (3.15) and (3.16), differ as between U.S. and Japanese shares or that segmentation exists between the two markets.

### 3.6 The full conditional version of the CAPM

Harvey (1991) has discovered a method which allows one to remain agnostic concerning the time variation of conditional second moments, so that they can vary in an unspecified way in tests of the classic CAPM. He first specifies a model for the conditional first moments. He assumes that investors process information using a linear filter:

$$R_{jt} - r_{t-1} = z_{t-1} \delta_j + u_{jt} ; \quad (3.19)$$

$$E[u_{jt} | z_{t-1}] = 0. \quad (3.20)$$

Here  $u_{jt}$  is the investor's forecast error for the return on asset  $j$ ,  $z_{t-1}$  is a row vector of predetermined instrumental variables which are known to the investor and  $\delta_j$  is a column vector of time-invariant forecast coefficients. A similar assumption is made concerning  $R_{mt} - r_{t-1}$ , the excess return on the market portfolio. Given the assumptions, the classic CAPM (without inflation premium) may be written:

$$z_{t-1} \delta_j = \theta_{t-1} E[u_{jt} u_{mt} | z_{t-1}]; \quad (3.21)$$

$$\theta_{t-1} = z_{t-1} \delta_m / E[u_{mt}^2 | z_{t-1}]. \quad (3.22)$$

Equation (3.22) illustrates that  $\theta_{t-1}$  is an unspecified function of  $z_{t-1}$ . Now, define:

$$h_{jt} = u_{mt}^2 z_{t-1} \delta_j - u_{jt} u_{mt} z_{t-1} \delta_m, \quad (3.23)$$

and notice that the CAPM equation (3.21) can now be written:

$$E[h_{jt} | z_{t-1}] = 0. \quad (3.24)$$

In this form, the "market price of covariance risk",  $\theta_{t-1}$ , has been substituted out between the pricing equations for individual assets and the

pricing equation for the market. At no time has it been necessary to specify the behavior of second moments. Equations (3.20) and (3.24) constitute an overidentified system of moment conditions which can be optimized with respect to parameter choice and tested, using the Generalized Method of Moments. On the basis of data that we have already described (section 3.2), Harvey finds, in individual country tests, that the classic CAPM is rejected

Dumas and Solnik (1991) follow the lead of Harvey but test the International, nominal CAPM, equation (2.9):<sup>59</sup>

$$E_{t-1}(R_i) - r_{t-1} = \sum_l \lambda_{t-1}^l \text{cov}_{t-1}(R_i, \nu_t^l) + \theta_{t-1} \text{cov}_t(R_i, R_m), \quad (3.25)$$

where, as we recall,  $\nu_t^l$  stands for the change in exchange rate  $l$  over the period  $t$  to  $t+1$ . The  $\lambda^l$ 's are baptized "market prices of foreign exchange risk" while  $\theta$  is the market price of covariance risk in the terminology of Harvey. Because the International CAPM is a multi-factor, or multiple-risk premia, CAPM, the substitution approach of Harvey, to eliminate  $\theta_t$ , is not applicable. Dumas and Solnik retain Harvey's assumption (3.19) concerning the conditional first moments of returns but also assume that the market prices of risk are related to the instrumental variables,  $z$ :<sup>60</sup>

$$\lambda_{t-1}^l = z_{t-1} \phi^l; \quad \theta_{t-1} = z_{t-1} \phi_m. \quad (3.26)$$

Here the  $\phi$ s are time-invariant vectors of weights.

Now, define:

$$h_{jt} = z_{t-1} \delta_j - z_{t-1} \sum_{l=1}^L \phi^l u_{jt} u_{n+l,t} - z_{t-1} \phi_m u_{jt} u_{mt}. \quad (3.27)$$

$h_{jt}$  is a disturbance that is unrelated to the information  $z_{t-1}$  under the null hypothesis that (3.25) holds. We form the vector of residuals:  $\epsilon_t = (u_t, h_t)$  and impose the overidentified moment conditions:  $E[z_t \epsilon] = 0$ .

Dumas and Solnik examine simultaneously monthly data concerning four

<sup>59</sup> Dumas and Solnik make the approximation that the Solnik special case applies. The replacement of  $\pi$  by  $\nu$  in the following equation is due to that.

<sup>60</sup> In contrast to the latent variable model, they are forced to assume an exact relationship.

national stock market indexes and three exchange rates over the period 1970-1991. The main result is that the overidentifying restrictions of the classic CAPM are rejected while those of the international CAPM (3.25) are not. In a test of (3.21) against (3.25), rejection occurs, which means that the foreign-exchange risk premia in (3.25) are significant.

### 3.7 Implied pricing kernels

All existing CAPMs, whether static or intertemporal, may be written in the form:

$$E_t[m_{t+1} \cdot (R_{t+1} - r_t)] = 0, \quad (3.28)$$

where  $R_{t+1} - r_t$  is a vector of excess returns and  $m_{t+1}$  is a marginal rate of substitution between consumption at time  $t$  and consumption at time  $t + 1$ . Hansen and Jagannathan (1991) have pointed out that there always exists a linear combination,  $m_{t+1}^*$ , of available returns such that:

$$E_t[m_{t+1}^* \cdot (R_{t+1} - r_t)] = 0. \quad (3.29)$$

$m_{t+1}^*$  is called a "pricing kernel". The various CAPMs are economic theories capable of restricting the dimension of the rate of return basis in terms of which  $m_{t+1}^*$  is expressed. Hansen and Jagannathan also show that the variance of  $m_{t+1}^*$  is a lower bound of the variance of  $m_{t+1}$  that would be capable of rationalizing the observed variance of asset returns.

De Santis (1991), Bekaert and Hodrick (1992) and Bansal, Hsieh and Visvanathan (1992) apply this idea, or generalizations thereof, to international stock returns. De Santis finds that adding securities of a capital market to the pricing kernel projection space helps in explaining returns in another capital market. This may be indicative of market segmentation.

### 3.8 Conclusion: what have we learned from empirical CAPMs?

From the standpoint of economic science, CAPM tests are somewhat heterodox. Because the CAPM is a partial-equilibrium model, most CAPM tests require auxiliary, *ad hoc* assumptions, i.e. assumptions that are neither dictated by the economic theory being tested nor separately testable. For instance, the test of the International CAPM by Dumas and Solnik (1991),

described in section 3.6, relies on the assumption that the prices of risk are linearly related to the instrumental variables. Or, the latent-variable method, described in section 3.3, is based on the assumption that the betas of individual securities relative to consumption,  $c$ , are constant.

Wheatley (1989) has cogently illustrated the potential pitfalls of *ad hoc*, untestable assumptions in his critique of latent variable tests. He pointed out, first, that the assumption of constant betas cannot be verified separately since the benchmark,  $c$ , is unobserved. As in any joint test, if the *ad hoc* assumption of beta constancy is actually false, the test could mistakenly reject the theoretical restriction that a single unobserved benchmark is the basis for pricing when it is in fact true. More striking is Wheatley's argument that the test could mistakenly accept the hypothesis. He argues that, for any dataset whatsoever, there exists an infinity of portfolios,  $p$ , that verify exactly a CAPM restriction similar to (3.5), with  $\beta_1 = \text{cov}(R_1, R_p) / \text{var}(R_p)$ . If one of these portfolios happens to yield constant betas for all securities, then the econometrician mistakenly concludes that the validity of (3.5) implies a single unobserved benchmark.

Similar arguments can presumably be made concerning tests of any partial-equilibrium relationship which, by construction, is incompletely specified. It is frequently impossible, strictly speaking, to test a part of the economic system without testing the whole. In principle, a GE model should be specified and tested. However, the assumptions of existing GE models may be utility theoretic (not *ad hoc*) but they are, at present, many times more restrictive than the *ad hoc* assumptions mentioned so far. Given their heroic assumptions, the likelihood of available GE models being close to the truth is so small that it is hardly worth the effort of testing them at this early stage.

In the meantime, if one accepts some level of pragmatism, CAPMs may afford a glimpse at the truth. The results reported above have given us some information concerning the degree of integration between stock markets of the world. Except for Gultekin, Gultekin and Penati (1989), little evidence of segmentation has been found in stock returns.<sup>61</sup> The low degree of correlation in consumption between countries [stylized fact (iii)] could, therefore, be due not to segmentation between countries but to worldwide market

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<sup>61</sup>Roll (1992) finds that different behaviors of twenty four national equity markets can largely be explained by industry specialization and exchange rate fluctuations.

incompleteness. The counterpoint to such a hypothesis is that home equity preference [stylized fact (iv)] would seem to be a form of segmentation *per se*. Theoreticians should perhaps take that information into account when building future models.

We have also learned that PPP deviations, likely, play a significant role in pricing assets on the international capital market. However, we have seen theoretically that, provided investors are willing to hedge exchange risk, PPP deviations, in principle, should not be an explanation for home equity preference.

Finally, we have learned that existing CAPMs can be made compatible with the evidence on the risk premium in the foreign-exchange market. Several tests reported above<sup>62</sup> are able to fit the same CAPM simultaneously to stock-market and foreign-exchange market data. However, it may be that the degree of time variation of market prices of risk necessary to achieve that goal is not compatible with any known General Equilibrium model. Fama's (1984) question is not yet answered.

An argument can be made in favor of continuing to test CAPMs at the same time that GE models become richer. Indeed, when a CAPM is rejected, there may be no point building a GE model around it. On the other hand, some empirical paradoxes become truly apparent only in empirical tests of, or by reference to, GE models. Such is the case for the equity premium puzzle of Mehra and Prescott (1985),<sup>63</sup> or the foreign-exchange risk premium puzzle.<sup>64</sup>

#### 4. General-equilibrium models without money

In discussing international capital flows, in section 2.3 above, we rapidly came to the conclusion that a partial-equilibrium, purely financial model such as the one of portfolio theory would, likely, not be able to account for international capital flows. For that purpose, we need to know the behavior of physical variables (consumption, investment, employment...). As

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<sup>62</sup> e.g. Giovannini and Jorion (1987, 1989) or Dumas and Solnik (1991). But see Korajczyk and Viallet (1992).

<sup>63</sup> See section 4.1.

<sup>64</sup> Backus *et al.* (1990) and Sibert (1989) have built GE models of the the foreign exchange risk premium and calibrated it. They have done for the exchange risk premium what Mehra and Prescott had done for the equity premium. As Fama suspected, they find that the theoretical premium is much too small to account for the empirical results of Cumby and Obstfeld (1984).

Frankel (1992) pointed out in this context, "Bonds are not perfect substitutes for equities or for factories, and the latter are clearly not perfect substitutes across countries." The same conclusion would be reached by someone attempting to understand the role of financial markets in the international transmission of shocks and business cycles.

#### 4.1 The difficulty with General Equilibrium models

It may be argued that the construction of international GE models is premature at this point. In a one-country setting, stochastic GE models, that have been built on existing financial models, are as yet unable to account for the relationship between financial and real variables.

Consider, for instance, the relationship between consumption and wealth, as measured in the stock market. An investor-consumer, who maximizes the expected utility of his lifetime consumption stream, would smooth his consumption. He would let his consumption rate depend on his lifetime anticipated income stream, or, equivalently, on his wealth. The volatility of his consumption should be of the same order of magnitude as that of his wealth, and both volatilities should be no greater than that of income. The data, however, tell a different story. The volatility of wealth, measured in the stock market, is several times larger than the volatility of consumption which is, in turn, approximately equal to the volatility of income.<sup>65</sup>

This observation explains the empirical failure of the consumption CAPM. It is also at the heart of the "equity premium puzzle" pointed out by Mehra and Prescott (1985). Given the empirically observed stochastic behavior of consumption, which is very smooth, the excess expected return of equities relative to riskless assets seems excessive. The risk in the economy, as measured by the volatility of consumption, is not large enough to account for it, unless one assumes that relative risk aversion is as large as forty.<sup>66</sup>

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<sup>65</sup> Shiller (1981) made the same, and largely equivalent, point concerning the volatility of wealth compared to the volatility of dividends.

<sup>66</sup> Weil (1989) suggested that this paradox arises from the assumed time-additivity of lifetime utility. He used instead the Epstein-Zin (1989) lifetime utility. But, with that utility specification, it is still true that consumption at any given time is proportional to wealth. Constantinides (1990) suggested a lifetime utility with habit formation. Past consumption serves as a standard below which people hate to let their current consumption fall. In that case, current consumption is a function of wealth and past consumption. This is equivalent to introducing a friction which slows down consumption

The GE models that have been built in the international context have had two goals. They aim to explain the volatility of real exchange rates and the relationship between them and trade/capital flows. They all suffer from the shortcoming that they yield theoretical risk premia that are too small compared to empirical ones. We can classify international GE models into two categories: models without frictions and models with frictions.

#### 4.2 GE Models without frictions

In these models, traded goods prices satisfy the Law of One Price between countries. Deviations from PPP are accounted for by the existence of non traded goods that are produced and consumed in each country specifically. The initial work in this area was done by Dornbusch (1983), Stulz (1987), Devereux (1988), Stockman and Dellas (1989) and Stockman and Tesar (1990). A stochastic model with nontraded goods is exposted in another chapter of the *Handbook*.<sup>67</sup>

To avoid repetition within the book, I simply summarize the conclusions that emerge from this literature: (i) non traded goods stochastic equilibrium models are capable of explaining a relatively low degree of cross correlation in consumption between countries and high correlations between saving and investment within countries; (ii) they are capable of explaining persistent deviations from PPP and comparatively volatile real exchange rates; (iii) they are capable of explaining home equity preference.<sup>68</sup>

I dwell on this third point, first to explain it, second to relate it to a similar point made in the Finance literature. The explanation goes thus: investors living in one country know that they will have to consume the home good which they alone consume. They, therefore, hold the equity of the firms producing the home good as a consumption hedge.

The reasoning is a special case of the observation, made by Adler and Dumas (1983) and repeated above in section 2.2, that investors would generally hold equities as part of their hedge portfolio, which is, as we recall,

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adjustments.

<sup>67</sup> See Backus *et al.* (1992).

<sup>68</sup> The Backus *et al.* paper in this book shows that the model is also capable of rationalizing empirical patterns in the relationship between real exchange rates and balances of trade.



specific to each national investor group.<sup>69</sup>

While the point is evidently correct, theoretically speaking, I cannot see how the data on home equity preference (reviewed in section 1) can be matched that way. The Adler and Dumas (1983) paper contains some empirically based calculations of optimal hedge portfolios for developed country investors.<sup>70</sup> These reflect the actual behaviors of equity prices and inflation in each country during the 1970s. The inflation index used is the Consumer Price Index which contains a large share of nontraded goods. In practically all cases, the optimal hedge portfolio is 99% made up of the home currency short-term bank deposit. Empirically speaking, that is, the hedge-portfolio weights falling on equities are not of the order of magnitude capable of explaining the strong home-equity preference observed in the real world.

Pure inflation risk (measured in the investor's home currency), as distinct from exchange risk, is too small -- and is too little correlated with exchange rates and equities -- to be a factor in the portfolio choices of investors of the countries considered. Solnik's special case was very close to the truth in these countries.<sup>71</sup> There is every reason to think that the same empirical observation could be made today, inflation risk having been further reduced.

#### 4.3 GE Models with frictions

The empirical work of Giovannini (1988), Marston (1990), Engel (1991) and others, quoted in section 1, made clear that traded goods do not sell for the same price in different countries; the LOP is not satisfied in the goods markets.

Dumas (1992) set out to build a stochastic model that endogenizes deviations from LOP. The device that is used for this purpose is a "shipping

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<sup>69</sup>This is an example of a point that can be made in the simple context of portfolio theory without recourse to a full-fledged GE model.

<sup>70</sup>The article gives the time average of hedge portfolios calculated monthly, conditional on the various currencies' interest rates.

<sup>71</sup>Recall that, when that special case is valid, investors hold no equity whatever as part of the hedge portfolio.

cost".<sup>72</sup> The world economy is made of two countries, each with its own capital stock,  $K$  and  $K^*$ . Shipping physical resources from one country to the other is costly. The cost is assumed proportional to the physical amount transferred; a fraction  $1 - s$  of physical resources is expended in transportation. In this model, the single good is sometimes traded, sometimes not traded.

In each country, the physical resources may at will be consumed, invested in the local production process or transferred abroad. The local production process is risky, with constant risk and constant returns to scale. The correlation between the output shocks in the two countries is zero. Households only consume the physical resources currently available in their country, an assumption which is inherited from the International CAPM model of section 2.1 above. Households' utility functions are isoelastic.

The shipping cost is the only friction present in the model. Financial markets, are fully integrated and frictionless. Perfect competition is assumed in all markets. Equilibrium is calculated as a central planning problem.

The price variable of interest in the model is the price,  $p$ , of physical resources currently located in one country relative to physical resources currently located in the other; this price is the deviation from the LOP and is called "the real exchange rate". It is at once the relative price of goods and the relative price of the equities of the firms operating in the two countries. The exchange rate fluctuates around the value one (parity); it stays within a band, the width of which is determined by the transportation loss factor. Figure 4 shows the relationship between the logarithm of the exchange rate,  $\ln p$ , and the logarithm of the physical imbalance:  $\omega = \ln K - \ln K^*$ .

The author's main purpose in building the model was to derive endogeneously the stochastic process for LOP deviations within the band, that would be compatible with financial market equilibrium. The process obtained is not a martingale; it displays reversion in the mean. It also displays a strong degree of heteroskedasticity;<sup>73</sup> the conditional variance is equal to zero at the edges of the band and is largest at the central point. The behavior of variance causes the real exchange rate process to be centrifugal,

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<sup>72</sup>This device is analogous to the "installation costs" that have been invoked by Abel (1979) to endogenize Tobin's  $q$ , the ratio of the equity value of installed capital to its replacement value.

<sup>73</sup>Note that the heteroskedasticity in the exchange rate is endogenous. Output shocks in each country are assumed homoskedastic.

probabilitically speaking, despite reversion in the mean; the chance of a move away from parity is greater than that of a move towards parity. Figure 5 shows a simulated sample path of the process.

The rationale for the centrifugal behavior is the following. Consider a positive output shock in country #1 which increases the amount of physical resources available,  $K$ . In the absence of frictions, capital balance between the two countries would be reestablished immediately and consumption in both countries would be increased in proportion to the increase in the sum,  $K + K^*$ . In the presence of frictions, capital balance is not reestablished immediately;  $K$  increases but not  $K^*$ . Consumption in country #1,  $c$ , increases more than it would have in the absence of frictions. Consumption in country #2,  $c^*$ , increases because country #2 households have a financial claim on country #1 firms' output but it increases less than it would have in the absence of frictions. Hence, short-term consumption imbalance further favors the increase of country #1 physical resources. This effect makes the next shipment from country #1 to country #2 nearer in time. At that time only, country #2 households will collect their claim.

The important consequence of the centrifugal behavior is persistence. The real exchange rate is away from parity, and the transfers of physical resources that occur between the two countries remain of the same sign, for periods of time that are "very long".<sup>74</sup> The variance of the exchange rate is persistent as well. These features accord well with facts. In particular, they imply positive serial correlation in observations of the real exchange rate over the shorter run. Over the longer run, shipments "push back" the real exchange rate to the inside of the band and the serial correlation becomes negative.

The Dumas (1992) paper also derives an expression for the real-interest rate differential, and the risk premia that it incorporates, that is entirely based on the slope of the  $\ln p$  function of Figure 4. The expression accounts for the Cumby-Obstfeld (1984) empirical finding, reviewed above, that exchange rate changes are related to the interest-rate differential by a coefficient that is less than one. But the coefficient is not sufficiently far from one to

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<sup>74</sup>With proportional transactions costs, trade is choppy. That is an unpleasant aspect of the model but one that washes out with some degree of time aggregation.

account fully for the empirical findings.<sup>75</sup>

##### 5. Issues and conclusions regarding General Equilibrium models

The two GE models that have been briefly discussed are embarrassingly rudimentary. No doubt, their extreme simplicity has something to do with their inability to resolve the puzzles of international finance. Therefore, a laundry list of what is missing in these models may be appropriate at this point. Such a list could conveniently be summarized by three words: imperfectly adjustable prices, imperfect competition and money.

Imperfectly adjustable nominal prices of goods are the hallmark of the barriers that exist between countries. The role of each country's money as a unit in terms of which prices are sticky cannot be overemphasized. Mussa (1986) compared the behavior over time of the prices of goods in U.S. and Canadian cities. The behavior of prices in Toronto was closer to their behavior in Vancouver than to their behavior in Detroit. As a matter of fact, the behavior of Canadian prices expressed in Canadian dollars was more similar to the behavior of U.S. prices expressed in U.S. dollars than to the behavior of U.S. prices translated into Canadian dollars.

Sticky prices would go a long way toward "explaining" the wide PPP deviations that accompany the daily movements in nominal exchange rates. Even if prices are simply posited to be sticky, some device must still be invoked to prevent commodity arbitrage between countries; otherwise no deviations from the Law of One Price would materialize. The "shipping costs" of Dumas (1992) could serve as such a device. It would be useful to study the dynamics of the real exchange rate in the presence of sticky prices and shipping costs.

The reasons why prices are sticky have long been debated in macroeconomics. They have to do with consumer search, imperfect competition, locational advantage and corporate firms' strategies. These factors take special meanings in the international setting.<sup>76</sup> The evidence is that import/export firms do not pass through to their customers the exchange rate movements that they suffer or benefit from. They prefer to smooth them out and

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<sup>75</sup> Finally, Uppal (1992) has shown that the same model is capable of generating some degree of home-equity preference.

<sup>76</sup> See H. Dixon's chapter in this *Handbook*.

to price their goods "to the market".<sup>77</sup> International trade is an intertemporal corporate decision.<sup>78</sup>

As we have seen, the segmentation in the goods market coincides with the borders of monetary zones (Mussa (1986)). Some recent evidence (Bayoumi (1990), Bayoumi and Rose (1992), Sinn (1991)) seems to indicate that the same is true for capital markets. The "home equity preference" mentioned in section 1 may be, in fact, a preference for the equities issued in the monetary zone (not specifically the country) that the investor lives in.<sup>79</sup> If this evidence is right, it makes it essential to have at our disposal a General Equilibrium monetary model of the international economy, with several currencies.

There are at least two obstacles on the way to this goal. First, the microeconomics of money is at a standstill. There is no consensus as to which form of market incompleteness, or restriction to market access, is the main motivation for holding money. Some of the existing GE models (Lucas (1982) or Svensson (1985)) use the Cash-in-advance formulation, with its somewhat artificial timing for the access to markets. Others (e.g., Labadie (1986), Sibert (1989)) have recourse to the overlapping-generation formulation. Others still enter money in the utility function (Stulz (1987)). Most of the existing models let prices adjust freely;<sup>80</sup> the Law of One Price prevails between countries. There is need for a microeconomic theory of money that would give a role to money as a unit of account, in terms of which local goods prices are kept rigid.

The second obstacle is a technical one. When money balances are present in portfolios, an equilibrium is not a Pareto optimum. Hence, the equilibrium must be calculated directly, without recourse to the equivalent central-planning program. The solution of monetary models is a formidable

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<sup>77</sup>Krugman (1987), Marston (1990).

<sup>78</sup>Dixit's (1989a, b) "hysteresis" model of entry into a foreign market (which takes after Baldwin (1986) and Baldwin and Krugman (1989)) has been extended to a general-equilibrium setting by Dumas (1988). Dixit (1989b) and Dumas both find that equilibrium prices are specific to the local market and differ from production cost even though perfect competition is assumed.

<sup>79</sup>There is presumably a link between the segmentation in the goods market along monetary borders and the segmentation of the capital market. But we have seen in section 2 that exchange risk, real or nominal, is not likely to be the source of home equity preference, as long as investors are able to hedge it by borrowing and lending currencies.

<sup>80</sup>An exception is Svensson and van Wijnbergen (1989).

computational problem. Lucas (1982), for instance, has been forced to assume a high degree of similarity between the households of different countries; essentially, they differ only in the currency that they wish to hold.<sup>81</sup> Thanks to this simplifying assumption, Lucas' equilibrium is a "pooling equilibrium", in which, except for money, households' portfolios are identical across countries.

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<sup>81</sup>In addition, claims to future monetary transfers are assumed tradable.

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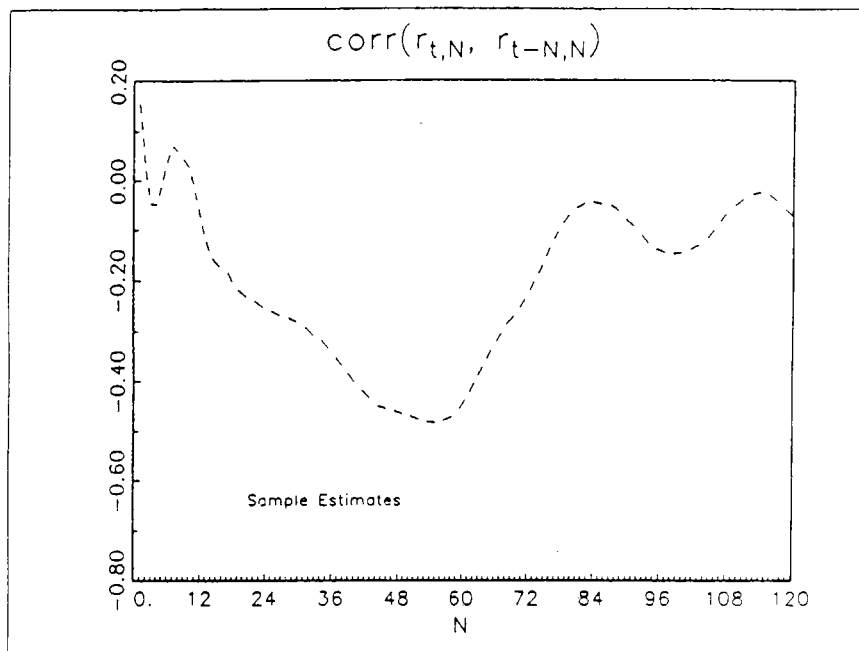
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FIGURE 1

## \$ AMOUNTS

	RA1 portfolio	Hedge portfolios FRA UK .....USA			World market portfolio
Equities	$W \cdot x_m$	0	0	0	$W \cdot x_m$
<b>Currencies</b>					
FRF	$-(1-1/\theta^{\text{FRA}})W^{\text{FRA}}$	$(1-1/\theta^{\text{FRA}})W^{\text{FRA}}$			0
GBP	$-(1-1/\theta^{\text{UK}})W^{\text{UK}}$	$(1-1/\theta^{\text{UK}})W^{\text{UK}}$ 0			0
.		0			
USD	$-(1-1/\theta^{\text{US}})W^{\text{US}}$	$(1-1/\theta^{\text{US}})W^{\text{US}}$			0
Total	$W/\theta$	$W(1 - 1/\theta)$			$W$

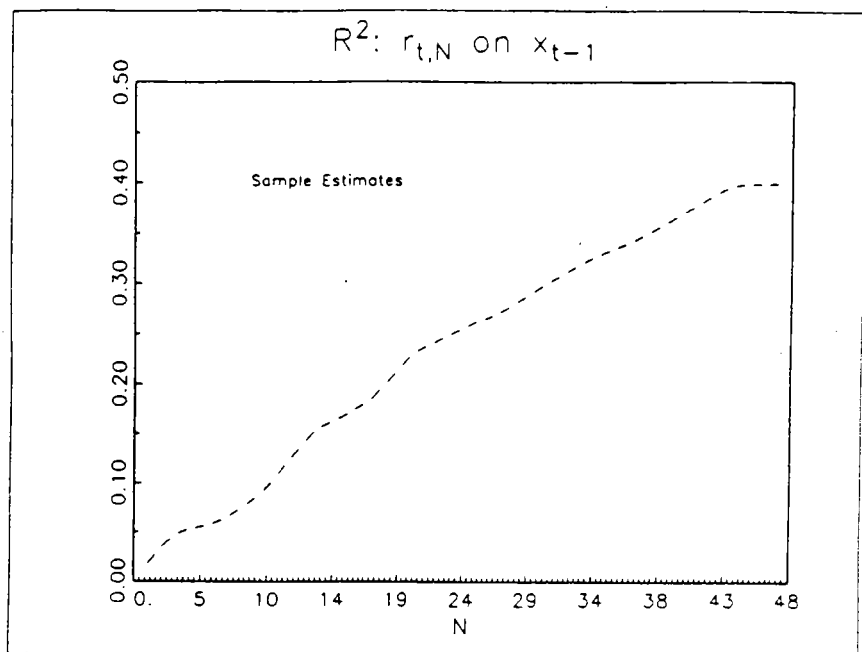


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Figure 1. Sample estimates of first-order autocorrelations of N-month real returns (continuously compounded) on the equally weighted NYSE portfolio. The estimates are obtained by regressing the N-month return on its lagged value, using monthly observations with overlapping return horizons.

Source: Kandel and Stambaugh (1982)



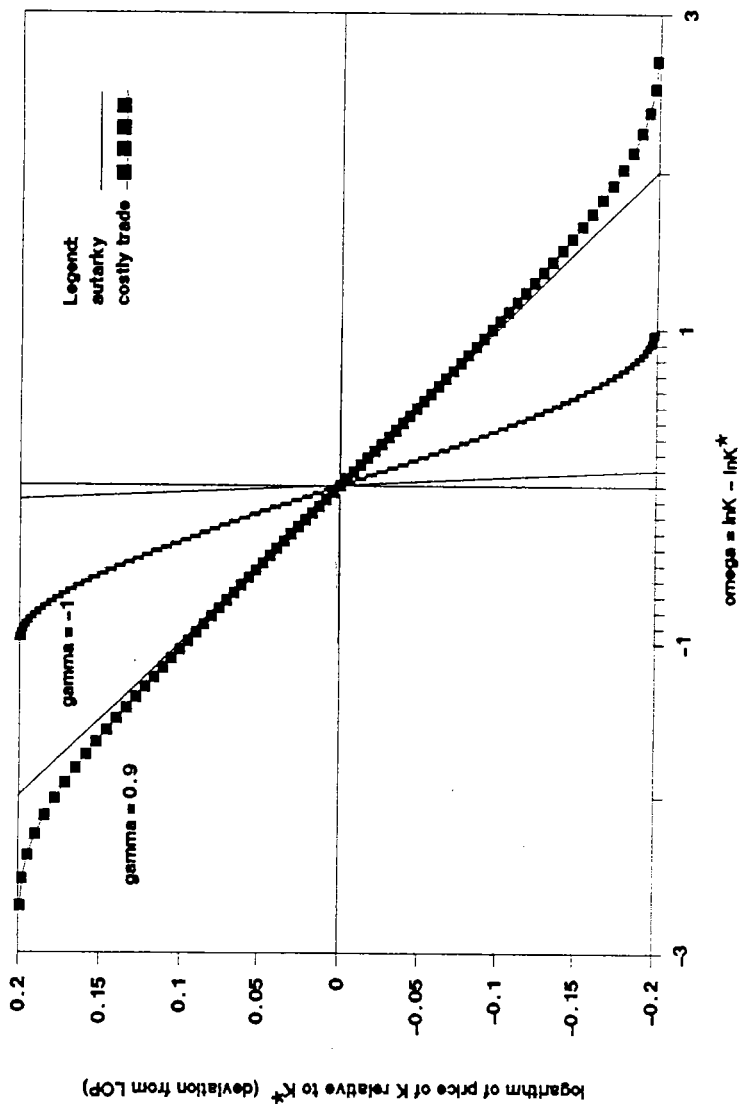


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Figure 2. R-squared values in regressions of N-month real returns (continuously compounded) on the equally weighted NYSE portfolio on three predictive variables (the Baa yield minus the Aaa yield, the Aaa yield minus the T-Bill yield, and the dividend-price ratio). For each N, the R-squared value is obtained in a regression using monthly observations with overlapping return horizons.

Source: Kandel and Stambaugh (1988)

Figure 4: THE FUNCTION  $\ln p(\omega)$



1 -  $\gamma$  is the investors' risk aversion

Fig. 5: SAMPLE PATH OF THE REAL EXCHANGE RATE

