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REASSESSING THE SOCIAL RETURNS TO EQUIPMENT INVESTMENT

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ABSTRACT

The recent literature on the sources of economic growth has challenged the traditional growth accounting of the Solow model, which assigned a relatively limited role to capital deepening. As part of this literature, De Long and Summers have argued in two papers that the link between equipment investment and economic growth across countries is stronger than can be generated by the Solow model. Accordingly, they conclude that such investment yields important external benefits.

However, their analysis suffers from two shortcomings. First, De Long and Summers have not conducted any formal statistical tests of the Solow model. Second, even their informal rejection of the model fails to survive reasonable tests of robustness.

We formally test the predictions of the Solow model using De Long and Summers' data. Our results cast doubt on the existence of externalities to equipment investment. In particular, we find that the empirical link between investment and growth in the OECD countries is fully consistent with the Solow model. Moreover, for De Long and Summers' full sample, the evidence of excess returns to equipment investment is tenuous.

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1. INTRODUCTION

Amid growing concern about the slow rate of improvement in living standards in the United States, considerable attention has focussed on the sources of economic growth and the role of investment in the growth process. A key question in this inquiry concerns the pattern of returns to various forms of investment. Under the competitive assumptions usually employed, and absent government intervention, investor behavior will equate the social returns to different assets. However, if some investments yield external benefits, their full social return will exceed that of other assets for which the returns accrue solely to the direct investor. In this case, competitive conditions do not ensure the optimal allocation of investment, and economic growth can be enhanced by devoting a greater share of the economy's resources to investments with the highest social returns.

In two recent papers, De Long and Summers (1991,1992) have argued that such externalities do exist and are important for investment in equipment. They base their conclusion on the finding of a strong link between equipment investment and economic growth for a broad cross-section of nations. This association, they argue, is much stronger than that predicted by a standard growth model that lacks externalities.

Many commentators already have raised doubts about De Long and Summers' claim to have found a causal link between equipment investment and economic growth. Does greater equipment investment actually boost growth, as De Long and Summers assert, or does it merely accompany the growth that is generated by other factors? This criticism, which permeates the "Comments and Discussion" following De Long and Summers (1992), reflects a concern about the ability of their simple regressions to capture the process generating economic growth. Although we share this concern, this paper raises an even more basic question about the interpretation of De Long and Summers' results. Even if we accept that De Long and Summers' regressions really do measure the causal effect of equipment spending on economic growth, we show that they have not presented convincing evidence of high social returns to equipment investment.

To carry out the analysis, we extend Solow's (1956) growth model to incorporate multiple capital goods and derive the implied effects of investment in equipment and structures on economic growth. Guided by the theory, we use De Long and Summers' data to examine the empirical link between investment and growth in the OECD countries -- those developed countries whose experience likely is most relevant for the United States. This examination reveals returns to equipment and structures investment that are fully consistent with the Solow model. Moreover for De Long and Summers' full sample, the evidence of excess returns to equipment investment is tenuous. In fact, if we remove a single country (Botswana) from the sample, we find that the effect of equipment investment on economic growth is consistent with the predictions of the traditional model.

2. *THE BASIC DE LONG-SUMMERS RESULTS*

Working with a cross-section of 61 countries, De Long and Summers (1991) estimated an equation of the form:

$$(1) \qquad DYL = c + \beta_E i_E + \beta_S i_S + \Theta * GAP + \gamma * DL + \epsilon,$$

where DYL = the average annual growth rate of real GDP per worker over 1960-85, i_E = the average share of real equipment investment in GDP over this period, i_S = the corresponding share for structures investment, GAP = the proportionate gap in real GDP per worker vis-a-vis the United States as of 1960 [$(YL_{US}-YL_m)/YL_{US}$ for country m], and DL = the average annual growth rate in the labor force over 1960-85. GAP and DL are included to control for factors other than investment rates that could influence the growth of real GDP per worker. The OLS estimates of equation (1) for the 61-country sample are[1]

$$\underset{}{DYL} = \underset{(.010)}{-.017} + \underset{(.069)}{.223\, i_E} + \underset{(.039)}{.096\, i_S} + \underset{(.009)}{.020\, GAP} - \underset{(.194)}{.023\, DL}$$

$N = 61 \quad \bar{R}^2 = .322$

The central finding is a positive and statistically significant association between the growth of real GDP per worker and the share of real GDP devoted to equipment investment. As shown above, a one percentage point increase in i_E, all else equal, is estimated to boost the average annual growth of real GDP per worker by 0.223 percentage point per year, which cumulates to nearly a 6 percent difference over a 25-year period. The estimate of β_S, though also statistically significant, is less than one-half the size of β_E.

In their 1992 paper, De Long and Summers again estimate equation (1), but they add 27 new countries to the 1991 sample and provide estimates over sample periods other than 1960-85. They also broaden the definition of equipment investment to include not only electrical and nonelectrical machinery, but transportation equipment as well.[2] As a final

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[1]Standard errors are shown in parentheses. These results represent our replication of De Long and Summers' regression using the data provided in the appendix to their 1991 paper. That listing contained incorrect values of i_s for Argentina and Chile; the estimates reported here are based on corrected data that we obtained from De Long. Our estimates are close, though not identical, to those reported by De Long and Summers using the corrected data (see De Long and Summers (1992), table 1).

[2]This change of definition is puzzling. In their 1991 paper, De Long and Summers specifically excluded transportation equipment from i_E, arguing that cross-country differences in the intensity of such investment might merely reflect "differences in the 'need' for transportation caused by differences in urbanization and population density" (p. 449). In addition, De Long and Summers' rationale for the existence of high social returns to machinery investment -- that the workers operating such machinery learn valuable skills and help fine-tune the design of existing models -- appears strained when applied to motor vehicles and other transportation equipment.

modification, their 1992 paper uses the natural log of GAP, rather than its level, on the right-hand side of equation (1). With these changes, the estimates of β_E and β_S over 1960-85 for their 88-country sample become 0.327 and 0.062, respectively -- values that are not significantly different than those shown above.[3] De Long and Summers interpret these results, and similar findings for various cuts of their data and alternative sample periods. as robust evidence of high social returns to equipment investment.

3. A *QUICK LOOK AT THE DATA*

As a first step in evaluating this claim, useful insights can be gained from an informal examination of De Long and Summers' data. Figure 1 displays the relationship between equipment investment and economic growth in De Long and Summers' original 61-country dataset. The horizontal axis plots DYL, the average annual growth rate of real GDP per worker over 1960-85, while the vertical axis plots i_E, the average share of real equipment investment in real GDP over this same period. Each mark in the plot represents the experience of a single country in the sample. This scatter plot has a noticeable upward slope, providing visual confirmation for the significant positive estimate of β_E in the regression presented above.[4]

However, two disturbing patterns are evident in the data. First there are several outlier countries, the most extreme of which is Botswana, a small country where the dominant economic activity is diamond mining. During the sample period, Botswana experienced by

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[3]We should note that the extra 27 countries may not provide much information beyond that in the original sample. De Long and Summers actually have no data on the composition of investment for these additional countries; rather, they impute i_E from data on the relative price of equipment and the volume of equipment imports.

[4]Strictly speaking, figure 1 does not display the relationship between equipment investment and economic growth in a multivariate regression, as it fails to control for the influence of other determinants of growth. We present figure 1 not as a substitute for regression results, but simply to illustrate basic features of De Long and Summers' data.

far the fastest growth in GDP per worker among the 61 countries and had the largest share of equipment investment in GDP. Second, the data appear to be bunched into two groups -- the OECD countries that have relatively high equipment shares and faster-than-average economic growth and the non-OECD countries that have relatively low equipment shares and relatively slow growth. The importance of De Long and Summers' results for economic policy in the United States would be reduced if they hinge on a few outliers or on the inclusion of countries whose economies bear little resemblance to that of the United States. Indeed, as we shall see in section 5, their results are sensitive to these factors.

4. THE PREDICTIONS OF TRADITIONAL THEORY

To demonstrate that equipment investment has external benefits, one must show not only that β_E is positive, but also that this coefficient is too large to have been generated by a standard neoclassical growth model. This section briefly describes the testable predictions of such a model, building on the analysis presented in De Long and Summers (1992) and Mankiw, Romer, and Weil (1992).

The Solow model predicts that the long-run growth rate of GDP depends on two factors: the growth rate of labor input and the rate of exogenous technological change. However, in the short run, the model specifies that GDP growth will be influenced as well by changes in the rate of capital accumulation, which arise from changes in the share of output devoted to various forms of investment. Thus, the Solow model implies a short-run correlation between the rate of GDP growth and investment shares. Accordingly, the model's predictions about the role of investment in the growth process can be tested by examining the size of these correlations across countries. But the test must be carefully specified, as the values of the predicted correlations depend on the length of the sample period and the time pattern of the deviations in investment shares.

The appendix formally derives the relation between the investment shares and the growth of GDP per worker along the transition path of a Solow model with capital disaggregated into equipment and structures. This derivation assumes that the length of the sample period is t years. We also assume that the shares of output devoted to equipment and structures investment in a given country are constant over the sample period, though these shares are allowed, of course, to differ across countries. The appendix establishes that, in a cross-section of countries, the value of β_E predicted for a sample period of t years is approximately:

$$(2) \qquad \left(\frac{1-e^{\lambda_E t}}{-\lambda_E t}\right)(r+\delta_E)$$

where $\lambda_E = -(1-\alpha_E-\alpha_S)(g+\delta_E) < 0$, α_E and α_S are the shares of equipment and structures in the economy's production function (assumed to be Cobb-Douglas), g is the sum of the labor-force growth rate and the exogenous rate of labor-augmenting technical progress, δ_E is the rate of depreciation for equipment, and r is the before-tax social rate of return to investment (net of depreciation). The predicted value of β_S has the same general form as expression (2):

$$(3) \qquad \left(\frac{1-e^{\lambda_S t}}{-\lambda_S t}\right)(r+\delta_S)$$

where $\lambda_S = -(1-\alpha_E-\alpha_S)(g+\delta_S) < 0$ and δ_S is the depreciation rate for structures.

To interpret expressions (2) and (3), consider the immediate effect on the growth of GDP per worker from any change in the share of GDP devoted to equipent investment, i_E.

This effect equals the limit of expression (2) as $t \to 0$, which equals $r+\delta_E$ by L'Hopital's rule. For a shock to the structures share, the corresponding effect on the growth of GDP per worker is $r+\delta_S$. Thus, in the very short run, an increase in either investment share causes the growth of GDP per worker to rise by an amount precisely equal to that asset's gross marginal product.

However, as we lengthen the sample period, a shock to either i_E or i_S will have a diminishing effect on the average annual growth of GDP per worker. In the limit, as $t \to \infty$, expressions (2) and (3) both approach zero. This shrinkage in the effect of changes in i_E and i_S occurs because of the assumed diminishing returns to each factor of production. With diminishing returns, the increase in (say) the equipment stock induced by a rise in i_E yields progressively smaller increments over time to output and thus to equipment investment. As a result, gross equipment investment exceeds the replacement needs for the rising equipment stock by ever smaller margins, damping the growth of this stock. Slower growth of the equipment stock feeds back to limit gains in GDP per worker. After enough time, the economy will reach a new steady state in which GDP per worker grows at exactly the same rate as in the initial steady state.

We have established that, for i_E constant over the sample period, the predicted value of β_E from the Solow model ranges from zero for an infinitely long sample period to $r+\delta_E$ for an arbitrarily short sample period; the corresponding range for β_S is from 0 to $r+\delta_S$.[5] Expressions (2) and (3) also offer predictions on the size of β_E relative to β_S. For the simplest case, in which the sample period is arbitrarily short ($t \to 0$), (2) and (3) imply that $\beta_E/\beta_S = (r+\delta_E)/(r+\delta_S)$. Thus, β_E is predicted to be larger than β_S because the higher depreciation rate for equipment implies that equipment has a higher gross marginal product. However, the higher value of δ for equipment also implies a faster transition for equipment to

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[5]If the investment shares change over the sample period, the nonlinearity of the underlying equation dictates that a slightly different, and perhaps larger, coefficient will apply to the sample average investment share. See Abel (1992) for further discussion.

the new steady state ($|\lambda_E| > |\lambda_S|$), so that β_E falls more rapidly than β_S with an increase in t. For a sufficiently large value of t, β_E/β_S will be less than unity, reversing the relative size of the coefficients predicted in the short run.

Because the basic issue is whether either type of investment has a stronger effect on growth than that predicted by the Solow model, tests of the theory should be cast in terms of the individual coefficients β_E and β_S rather than the ratio of these coefficients. The limitation of the ratio test can be easily seen: If β_E/β_S is higher than predicted because β_S is too low, this result has quite different implications than if the high ratio reflects a high value of β_E.

Before proceeding to empirical tests, we should emphasize that expressions (2) and (3) do not yield very tight predictions for the values of β_E or β_S. Not only are these predictions based on linear approximations to the underlying model, but they also require assumed values for several important parameters, including before-tax rates of return to investment, depreciation rates, and factor shares. Researchers, therefore, should be cautious about claiming that results obtained from an equation like (1) are "inconsistent with the Solow model." More precisely, certain results may be inconsistent with the Solow model combined with the numerous assumptions that must be made to generate specific predictions.

5. EMPIRICAL RESULTS

We use expressions (2) and (3) to obtain predicted values for β_E and β_S from the Solow model, subject of course to the uncertainties just noted. To conform as closely as possible with the analysis of De Long and Summers, we employ their own preferred values for the parameters that feed into (2) and (3). Given their main sample period (1960-85), we set $t = 25$. In addition, we set $g = 0.03$, $\delta_E = 0.15$, $\delta_S = 0.02$, and $\alpha_E+\alpha_S = 0.24$. Finally, although De Long and Summers consider a range of possible values for r -- the real before-tax rate of return to capital (net of depreciation) -- their preferred value appears to be about 10 percent ($r = 0.10$), which we also believe to be a reasonable estimate.[6] With these parameter values, the predicted values of β_E and β_S are 0.071 and 0.077, respectively.

Main Findings

Table 1 presents the estimates of equation (1) for six different sets of countries, using the corrected version of the data in De Long and Summers (1991). The first column pertains to the full sample of 61 countries, repeating the estimates shown above. The estimate of β_E is roughly three times larger than the predicted value of 0.071, while the estimate of β_S is close to its predicted value of 0.077. De Long and Summers (1991) also estimated equation (1) over a 25-country subsample that includes only those countries with a level of GDP per worker in 1960 that was at least 25 percent of that in the United States. This cut of the data removes the very poor countries that employ technologies vastly less sophisticated than those in the United States. As can be seen in column 2 of the table, the "high-productivity" subsample generates an estimate of β_E that is far above the value predicted by the Solow model.

When pooling data from different countries to estimate the relationship derived above from the Solow model, one implicitly assumes (1) that the pooled countries have the same underlying production function and (2) that market forces in these countries drive the private return on investment to the competitive rate. Given these assumptions, the regression coefficients are then identified by differences across countries in the investment shares and the other variables assumed to be exogenous. Although it may be legitimate to pool the United States and other developed countries, these two maintained assumptions almost surely do not hold for the "high productivity" sample. This sample includes several newly-industrialized countries (NICs) that grew rapidly over 1960-85. In large part, their rapid growth resulted from the adoption of equipment-intensive production techniques that enabled

[6]The values for r and the depreciation rates were taken from the published version of De Long and Summers (1992); the values for the remaining parameters appeared in an earlier version of their paper.

them to approach the production frontier. This importation of improved technology violates assumption (1) -- that the pooled countries shared a common production function over the entire sample period -- and could well explain why the NICs display both a high rate of economic growth and a high rate of equipment investment. The "high productivity" sample also includes several countries, most notably Argentina, that were relatively wealthy in 1960 but whose growth subsequently lagged. De Long and Summers argue that this happened because inept economic policies and political turmoil stifled the equipment investment needed to remain close to the frontier. The resulting shortage of equipment investment would have forced the marginal private return on such investment above that prevailing in economies free from these impediments, violating assumption (2). By pooling these two groups of countries, neither of which conforms well to the assumptions of the Solow model, the "high productivity" sample generates a strong positive association between equipment investment and economic growth.

To focus more specifically on the industrial nations most like the United States -- that is, market-oriented economies already at the technological frontier -- we repeat De Long and Summers' basic regression for the subsample of 18 OECD countries in their 1991 dataset. The results of this regression are shown in column 3 of the table. As can be seen, the estimates of β_E and β_S, at 0.028 and 0.057, respectively, are both somewhat smaller than the predicted values of the Solow model. Thus, De Long and Summers' main empirical finding, the high estimated value of β_E, evaporates for the OECD countries. Column 3 clearly undermines the claim that equipment investment yields important spillover benefits in industrial countries like the United States.[7]

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[7]Many other recent studies in the literature on economic growth also have focussed on the OECD countries. Among these studies, Mankiw, Romer, and Weil (1992) found that the investment share for equipment and structures combined had a smaller effect on output growth for the OECD sample than for broader samples. Our results show that this finding was not an artifact of having combined heterogeneous types of capital.

The remaining columns of table 1 indicate another weak link in De Long and Summers' case for high social returns to equipment investment: the results for the full sample and the non-OECD countries are sensitive to the inclusion of a single country, Botswana. When we exclude Botswana, the estimate of β_E for the full sample drops to 0.157 (column 5), a value barely more than one standard error above the prediction from the Solow model.[8] For the non-OECD countries, Botswana is even more influential. As can be seen by comparing columns 4 and 6, the exclusion of Botswana cuts the estimate of β_E from 0.240 to 0.114, with the latter estimate within one-half standard error of the prediction from the Solow model. Hence, for the full sample and the non-OECD countries alone, the case for externalities to equipment investment appears rather tenuous.

With these regression estimates in hand, we turn to formal tests of the implications of the Solow model for β_E and β_S. The bottom row of table 1 reports the marginal significance level (the p-value) of an F-test of the joint null hypothesis that $\beta_E = 0.071$ and $\beta_S = 0.077$.[9] A p-value smaller than 0.05 implies that the null hypothesis can be rejected at the usual five-percent level.[10] These tests confirm that De Long and Summers' key result -- that equipment investment generates important externalities -- is not a robust finding. Although the predictions of the Solow model are rejected at the five-percent level for the full sample, merely excluding Botswana raises the p-value to 0.179, a value well above any conventional significance level. This conclusion holds as well for the OECD sample and for the

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[8]Given the idiosyncratic aspects of Botswana's economy noted above, we believe it is reasonable to exclude this small country from the sample.

[9]DeLong and Summers do not report the results of any such hypothesis tests. Rather, their arguments are based solely on the point estimates of β_E and β_S.

[10]These F-tests presume that β_E and β_S are known with certainty under the null hypothesis. If, more realistically, we regard β_E and β_S as simply reasonable guesses of the true parameter values, the reported p-values will understate the true p-values. That is, the results we report are biased toward rejection of the null hypothesis because we have ignored the uncertainty about the true values of β_E and β_S.

non-OECD sample (regardless of whether we exclude Botswana). The failure to reject the model in these cases is not particularly surprising given the relatively small differences between the estimated and predicted values of β_E in columns 3, 5, and 6. Only for the "high productivity" sample do we strongly reject the predictions of the Solow model. This rejection, as we discussed above, reflects the contrasting experience of the high-growth NICs and the South American countries that performed poorly over the sample period.

A Comparison of Results for the OECD Countries

Of the countries in De Long and Summers' sample, the OECD nations -- which include Japan, Western Europe, and Canada -- have economies that are most similar to that of the United States. Accordingly, for the purpose of setting government policies for investment in the United States, we believe that the results for the OECD countries should be given the greatest weight. This view makes it crucial to reconcile our results for the OECD countries with those in De Long and Summers (1992) that appear far more favorable to their case.[11] Table 2 compares our estimate of β_E for the OECD countries with those presented by De Long and Summers. The first row of the table simply repeats the estimate of 0.028 that we obtained using De Long and Summers' original data for 1960-85, an estimate not reported by De Long and Summers. Rather, their 1992 paper reports three alternative estimates of β_E for the OECD countries. For their main sample period, 1960-85, they report an estimate of 0.151, a value well above the estimate we obtained. Their higher estimate reflects the combined effect of adding three new OECD countries to the sample, revising the values of i_E and i_S for the original sample countries, redefining i_E to include transportation equipment, and

[11]The initial version of their 1992 paper, presented at the Brookings panel meeting in October 1992, contained no results at all for the OECD countries. The OECD results in the published version are apparently a response to the first version of this paper -- Auerbach, Hassett, and Oliner (1992).

replacing the level of GAP with its natural log.[12] The two other estimates of β_E are for alternative sample periods, 1950-60 and 1985-90. As can be seen from table 2, these estimates -- 0.177 for 1950-60 and 0.114 for 1985-90 -- also are considerably higher than the estimate yielded by their original data over 1960-85. De Long and Summers argue that their point estimates of β_E provide support for the existence of high social returns to equipment investment in the OECD countries.

However, this conclusion is warranted only if the estimates of β_E for the OECD countries are significantly above the values predicted by the Solow model. Column 5 of the table shows the prediction for β_E derived from expression (2) using the parameter values listed at the beginning of this section. Note that the predicted value for β_E rises considerably when the sample period is shortened; indeed, for a five-year sample period, the predicted value jumps to 0.181. As we demonstrated in section 4, this magnification occurs in short samples because diminishing returns have yet to force the pace of capital formation back to its steady-state rate. Column 6 presents the *t* statistic for the null hypothesis that the estimate of β_E equals its predicted value. While the various changes in specification do increase the point estimates of β_E, in no case is the Solow model rejected even at the ten-percent level. These hypothesis tests cast serious doubt on De Long and Summers' assertion that their results point to "large external benefits from equipment investment even in rich countries" (De Long and Summers (1992), p. 159).

6. CONCLUSION

The search for the keys to economic growth is an important one, but our results suggest that the search is not over. We have shown that De Long and Summers' own data fail to reject the Solow model for the OECD countries. The same is true even for their full

[12]As of this writing, we have not been able to obtain the data used to produce De Long and Summers' estimates of β_E for the OECD countries and thus are unable to assess the relative importance of each of these changes.

sample of quite heterogeneous nations if we exclude just one country (Botswana). These results clearly refute De Long and Summers' claim to have uncovered robust evidence of uniformly high social returns to equipment investment.

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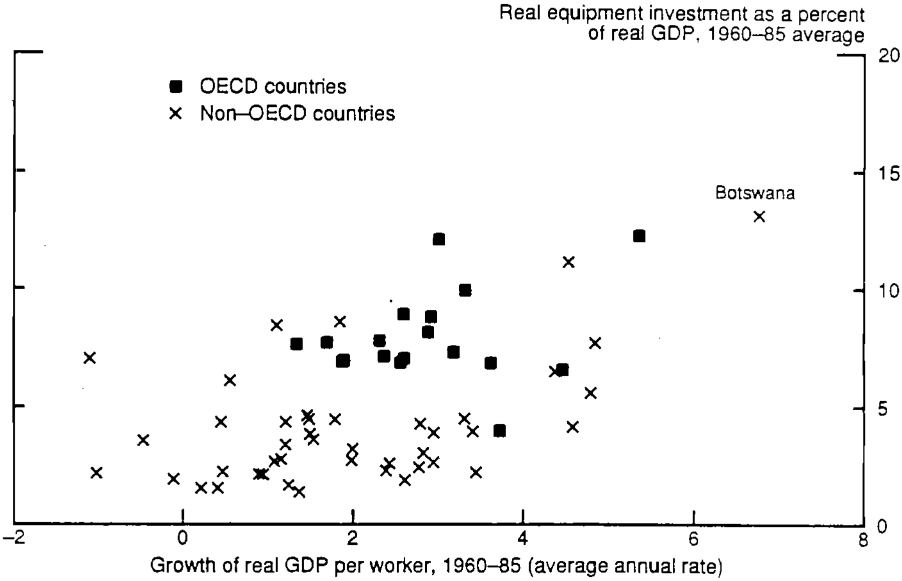
FIGURE 1

REAL EQUIPMENT INVESTMENT AND THE GROWTH OF REAL GDP PER WORKER*



* Data are from the appendix to De Long and Summers (1991).

TABLE 1

ESTIMATION OF DE LONG AND SUMMERS' BASIC EQUATION
(Standard errors in parentheses)

					Excl. Botswana	
	Full Sample	High Productivity	OECD	Non-OECD	Full Sample	Non-OECD
Variable	(1)	(2)	(3)	(4)	(5)	(6)
Constant	-.017	-.016	-.001	-.020	-.012	-.020
	(.010)	(.009)	(.008)	(.021)	(.010)	(.020)
Equipment (β_E) share	.223	.314	.028	.240	.157	.114
	(.069)	(.071)	(.083)	(.099)	(.075)	(.115)
Structures (β_S) share	.096	.020	.057	.094	.107	.099
	(.039)	(.052)	(.045)	(.053)	(.039)	(.051)
GDP Gap (Θ)	.020	.030	.035	.017	.017	.015
	(.009)	(.009)	(.008)	(.020)	(.009)	(.019)
Growth of (γ) labor force	-.023	.032	-.025	.203	-.046	.378
	(.194)	(.148)	(.274)	(.369)	(.189)	(.368)
N	61	25	18	43	60	42
$\bar{R}^2$	.322	.661	.674	.235	.262	.129
p-value	.027	.004	.632	.166	.179	.800

Note: All data are from the appendix to De Long and Summers (1991), with corrected values of the structures share for Argentina and Chile. The dependent variable is the average annual growth rate of real GDP per worker over 1960-85 (DYL). The table shows the OLS estimates of:

$$DYL = c + \beta_E i_E + \beta_S i_S + \Theta * GAP + \gamma * DL + \epsilon,$$

where i_E and i_S are the average shares of real equipment investment and real structures investment, respectively, in real GDP over 1960-85; GAP is the proportionate gap in real GDP per worker vis-a-vis the United States as of 1960; and DL is the average annual growth rate of the labor force over 1960-85. The p-values are for the F-test of the null hypothesis that $\beta_E = 0.071$ and $\beta_S = 0.077$.

TABLE 2

COMPARISON OF OECD RESULTS
(Standard errors in parentheses)

Source	Sample Period	N	Estimated β_E	Predicted β_E	t statistic
(1)	(2)	(3)	(4)	(5)	(6)
This paper	1960-85	18	.028 (.083)	.071	-.52
De Long and Summers (1992)	1960-85	21	.151 (.050)	.071	1.60
De Long and Summers (1992)	1950-60	21	.177 (.117)	.136	.35
De Long and Summers (1992)	1985-90	17	.114 (.155)	.181	-.43

Note: The estimates of β_E for De Long and Summers (1992) are from their tables 2, 3, and 6. The predicted values of β_E are derived from expression (2) in the text; see the text for details. The *t* statistic is for the null hypothesis that the estimated value of β_E equals the predicted value.

APPENDIX

This appendix sketches the derivation of the equation estimated in the paper and shows that expressions (2) and (3) are the approximate values of β_E and β_S implied by the Solow growth model. The derivation is somewhat more complicated than in the usual case with a single capital good, both because we disaggregate equipment and structures and because we allow the depreciation rates of these capital goods to differ.

We begin with a Cobb-Douglas production function characterized by constant returns to scale:

$$(A.1) \qquad Y = K_E^{\alpha_E} K_S^{\alpha_S} N^{(1-\alpha_E-\alpha_S)},$$

where Y is output, K_E and K_S are the stocks of equipment and structures, respectively, and N is effective labor input. N equals A*L, where A is an index of the quality of labor input and L denotes the labor force; A and L are assumed to grow at the constant rates a and l, so that

$$(A.2) \qquad \frac{d(\ln N)}{dt} = \frac{d(\ln A)}{dt} + \frac{d(\ln L)}{dt} = a + l \equiv g.$$

The growth rate of each capital stock follows the usual equation of motion:

$$(A.3) \qquad \frac{d(\ln K_J)}{dt} \equiv \frac{1}{K_J}\frac{dK_J}{dt} = \frac{I_J}{K_J} - \delta_J = \frac{i_J}{k_J} - \delta_J \qquad \text{(for J = E,S)}$$

where $k_J = K_J/Y$ denotes the capital-output ratio for capital of type J, $i_J = I_J/Y$ denotes the investment share of output, and δ_J is the rate of depreciation.

Taking the log of (A.1) and letting y = Y/N, the production function can be rewritten in intensive form:

$$\text{(A.4)} \quad \ln y = \left[\frac{\alpha_E}{1-\alpha_E-\alpha_S}\right] \ln k_E + \left[\frac{\alpha_S}{1-\alpha_E-\alpha_S}\right] \ln k_S.$$

Now, note that k_J can be expressed as $K_J/(yN)$, which implies that

$$\text{(A.5)} \quad \frac{d(\ln k_J)}{dt} = \frac{d(\ln K_J)}{dt} - \frac{d(\ln y)}{dt} - \frac{d(\ln N)}{dt} \qquad (\text{for } J = E,S)$$

Using (A.2), (A.3), and (A.4) to substitute for the terms on the right-hand side of (A.5), one can show that the capital-output ratios evolve over time according to the differential equations:

$$\text{(A.6)} \quad \begin{aligned} \frac{d(\ln k_E)}{dt} &= (1-\alpha_E-\alpha_S)\left(\frac{i_E}{k_E} - (g+\delta_E)\right) + \alpha_S\left[\left(\frac{i_E}{k_E} - (g+\delta_E)\right) - \left(\frac{i_S}{k_S} - (g+\delta_S)\right)\right] \\ \frac{d(\ln k_S)}{dt} &= (1-\alpha_E-\alpha_S)\left(\frac{i_S}{k_S} - (g+\delta_S)\right) + \alpha_E\left[\left(\frac{i_S}{k_S} - (g+\delta_S)\right) - \left(\frac{i_E}{k_E} - (g+\delta_E)\right)\right] \end{aligned}$$

Each equation in (A.6) can be simplified by setting the final, bracketed term to zero. This approximation is reasonable because (1) the income shares for the individual types of capital, α_E and α_S, likely are small relative to the labor share $(1-\alpha_E-\alpha_S)$, and (2) the bracketed expression contains the difference of terms that appear elsewhere as levels. Omitting the bracketed expression yields the following solution to (A.6):

$$\text{(A.7)} \quad k_J = k_J^*(1-e^{\lambda_J t}) + k_J^0 e^{\lambda_J t} \Rightarrow \frac{k_J - k_J^0}{k_J^0} = (1-e^{\lambda_J t})\frac{k_J^* - k_J^0}{k_J^0} \qquad (J=E,S)$$

where $k_J^* = i_J/(g+\delta_J)$ is the steady-state value of k_J, k_J^0 is the corresponding initial value, and

(A.8) $\lambda_J = -(1-\alpha_E-\alpha_S)(g+\delta_J) \qquad (J=E,S)$

are the (stable) characteristic roots of the system. If we difference (A.4) and use the approximation $\ln x - \ln x^0 \approx (x-x^0)/x^0$ for $x = k_E$ and k_S, equations (A.4) and (A.7) imply

$$\text{(A.9)} \quad \ln y - \ln y^0 = \sum_{J=E,S} \left[\frac{\alpha_J}{1-\alpha_E-\alpha_S}(1-e^{\lambda_J t}) \right] \ln k_J^* - \sum_{J=E,S} \left[\frac{\alpha_J}{1-\alpha_E-\alpha_S}(1-e^{\lambda_J t}) \right] \ln k_J^0.$$

Letting $\bar{\lambda}$ denote the average of λ_E and λ_S and using (A.4), we may approximate (A.9) by

$$\text{(A.10)} \quad \ln y - \ln y^0 = \sum_{J=E,S} \left[\frac{\alpha_J}{1-\alpha_E-\alpha_S}(1-e^{\lambda_J t}) \right] \ln k_J^* - (1-e^{\bar{\lambda} t}) \ln y^0.$$

To express (A.10) in terms of the investment shares i_J, note that $k_J^* = i_J/(g+\delta_J)$ implies

$$\frac{d(\ln y - \ln y^0)}{d(\ln k_J^*)} = \frac{d(\ln y - \ln y^0)}{di_J} \frac{di_J}{dk_J^*} k_J^* = \frac{d(\ln y - \ln y^0)}{di_J} i_J.$$

Accordingly, (A.10) can be rewritten as

$$\text{(A.11)} \quad \ln y - \ln y^0 = \sum_{J=E,S} \left[\frac{\alpha_J}{(1-\alpha_E-\alpha_S) i_J} (1-e^{\lambda_J t}) \right] i_J - (1-e^{\bar{\lambda} t}) \ln y^0.$$

Profit-maximization implies that the gross return to capital of type J equals its marginal product:

$$(A.12)\quad r + \delta_J = \frac{\alpha_J Y}{K_J} = \frac{\alpha_J}{k^*_J} = \frac{\alpha_J(g + \delta_J)}{i_J} \Rightarrow \frac{\alpha_J}{1-\alpha_E-\alpha_S} = \frac{i_J(r + \delta_J)}{-\lambda_J}$$

using (A.8). Substituting (A.12) into (A.11) for $\alpha_J/(1-\alpha_E-\alpha_S)$ yields:

$$(A.13)\quad \ln y - \ln y^0 = \sum_{J=E,S} \left[\frac{1-e^{\lambda_J t}}{-\lambda_J}(r + \delta_J) \right] i_J - (1-e^{\bar{\lambda} t})\ln y^0.$$

Finally, approximating the GAP variable $(y^0_{US}-y^0)/y^0_{US}$ by $\ln y^0_{US} - \ln y^0$, and dividing by t, (A.13) becomes

$$(A.14)\quad \frac{\ln y - \ln y^0}{t} = \sum_{J=E,S} \left[\frac{1-e^{\lambda_J t}}{-\lambda_J t}(r + \delta_J) \right] i_J + \frac{(1-e^{\bar{\lambda} t})}{t}\text{GAP} - \frac{(1-e^{\bar{\lambda} t})}{t} \ln y^0_{US}$$

$$= c + \sum_{J=E,S} \left[\frac{1-e^{\lambda_J t}}{-\lambda_J t}(r + \delta_J) \right] i_J + \frac{(1-e^{\bar{\lambda} t})}{t}\text{GAP},$$

where the second equality sweeps the term in $\ln(y^0_{US})$ into a constant. Because the dependent variable for our regressions is the growth of GDP per worker (Y/L) rather than growth of GDP per unit of effective labor input ($y \equiv Y/(A^*L)$), the constant term in the equation actually estimated will include the growth rate of A, i.e., $\tilde{c} = c + a$.

Expression (A.14) is essentially the equation estimated in the text, differing only by the omission of the labor-force growth rate, which has been assumed to be constant for the purposes of the derivation. The term in large brackets in (A.14) is precisely the expression for β_J shown in (2) and (3) in the text.