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ABSTRACT

We apply the method of constrained asset share estimation (CASE) to test the mean-variance efficiency (MVE) of the stock market. This method allows conditional expected returns to vary in relatively unrestricted ways. The data estimate reasonably the price of risk, and, in some cases, the MVE model is valuable in explaining expected equity returns. Unlike with most tests of MVE, we can put an explicit interpretation on the alternative hypothesis -- a general linear Tobin portfolio choice model. We reject the restrictions implied by MVE.

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1. Introduction

This paper uses a technique that we call Constrained Asset Share Estimation (CASE) to test the conditional mean-variance efficiency (MVE) of the U.S. stock market. The technique is useful in time-series tests of simple asset pricing models because it allows estimated expected returns to vary in an unrestricted way. It was first applied in a macroeconomic context in which the "market" portfolio included not only equities, but also money, bonds and real estate.[1] It has since been applied more widely to other portfolios and has been extended to allow for variation in conditional second as well as first moments.[2]

The CASE technique nests MVE in a more general, but economically meaningful, theory of portfolio determination. In contrast, most tests of the null hypothesis of MVE have no clear alternative hypothesis. This feature is particularly important because many tests do in fact reject MVE. When one rejects the null hypothesis, it is crucial to have some idea of what the alternative is. In the central tests below, the alternative to MVE is that investors' portfolio shares are linearly related to expected returns, but that investors' asset demands are not determined in the precise way that MVE would imply they should. The alternative hypothesis is the more general portfolio-balance approach to asset demands that was first introduced by Tobin (1958, 1969). The problem he was addressing was the relationship between expected returns and the demand for bonds and other assets. Sharpe's (1964) CAPM grew out of an attempt to place more structure on Tobin's portfolio balance model by modelling the behavior of individuals as mean-variance

[1] See Frankel (1982, 1985a), Frankel and Dickens (1984), and Frankel and Engel (1984).

[2] Ferson, Kandel and Stambaugh (1987) estimate the constant-variance version for stock portfolios. Bodurtha and Mark (1991), Bollerslev, Engle and Wooldridge (1987) and Engel and Rodrigues (1989) estimate a version which allows for changing conditional second moments.

optimizers (as in Markowitz (1952) and Tobin (1958)). However, most modern testing of CAPM has departed from this original context.

The CASE method does not impose the condition that expected returns are constant over time. It allows expected returns to vary freely, as they must, for example, whenever new information which may not be observed by the econometrician becomes available to the investor. In addition, in some of the tests below we allow second moments to vary according to an ARCH or GARCH process.[3] Allowing for such variation in conditional moments is essential for a properly specified test of MVE. There is considerable evidence that both the conditional expectation and conditional variance of excess returns contain important predictable components.[4]

Our tests below emphasize the nested nature of the hypotheses we consider. We pay special attention to the importance of ARCH vs. MVE vs. the asset shares themselves in explaining risk premia. The broad findings can be summarized as follows. First, we find that stock-market shares by themselves have statistically significant power in predicting monthly excess stock returns. This is what we would expect if the stock market is mean-variance efficient and if required returns change over time. However, we reject the restrictions implied by constant-variance MVE. Moreover, the predictive ability of the asset shares disappears when the constant variance version of MVE is imposed.

However, the constant variance version of MVE can be rejected in favor of a version

[3] The ARCH process does not allow second moments to vary freely. It is analogous to estimating the first moments by an ARIMA process, in which this period's expectation is related to recent realizations, rather than by the CASE technique, in which expectations can vary freely.

[4] See for example, Fama and French (1988) and Poterba and Summers (1988) for evidence on the predictability of stock market returns, and Bollerslev (1987) and Bollerslev, Engle and Wooldridge (1988) for evidence on the predictability of conditional variances of excess returns. These findings coupled with the results of Hansen and Richard (1987), who show that the conditionally and unconditionally mean-variance efficient frontiers are generally different, suggest that such variation in conditional moments is important for tests of MVE.

of MVE in which the covariance of the asset returns follows a GARCH process. Furthermore, that version of MVE does have statistically significant ability to predict stock prices. This model produces an estimate of the coefficient of risk aversion of about 3.0, with a standard error of about 1.4. So, a version of MVE in which market betas vary conditionally both because of changes in asset shares and time variation in the covariances of individual asset returns has explanatory power and produces plausible parameter estimates. This finding may be relevant to the recent findings of Fama and French (1992) that betas based on unconditional covariances have no predictive ability once size is included as an explanatory variable. Although we do not address that issue directly, the suggestion of our findings is that the CAPM might have performed better in the Fama and French setting if the betas were conditional on contemporaneous information. Nonetheless, we find the restrictions that this version of MVE imposes on a GARCH version of the Tobin portfolio-balance model are rejected.

In short, the unrestricted linear Tobin asset pricing model has predictive power under both of our specifications (with the GARCH specification doing better than the constant-variance version). The MVE-constrained model itself also has predictive power under the GARCH specification. However, the restrictions that MVE places on the Tobin model can be rejected in all cases.

Sections 2 and 3 briefly describe the model and the data, respectively. Section 4 tests for constant-variance MVE. We introduce our ARCH specifications in section 5. Section 6 summarizes our general nesting procedure for the hypotheses of interest and offers our conclusions.

2. The model

Mean-variance efficiency implies that the vector of conditional risk premia is a linear combination of the asset shares in the portfolio, with the weights proportional to the conditional variance of asset returns:

$$E_t(r_{t+1}) = \rho_t \Omega_t \lambda_t, \tag{1}$$

where $E_t(r_{t+1})$ is the expected return above the riskless rate on an N x 1 vector of assets conditional on all information available at time t, Ω_t is the conditional variance of returns between t and t+1, λ_t is the N x 1 vector of portfolio weights, with $\sum_{i=1}^{N} \lambda_{t,i} = 1$, and ρ_t is the price of risk equal to $E_t(m_{t+1})/Var_t(m_{t+1})$, where m_{t+1} is the return on the aggregate portfolio. If the aggregate stock portfolio is the "market" portfolio, MVE is equivalent to CAPM, and the parameter ρ is to be interpreted as the coefficient of relative risk aversion. Note that the right-hand side of (1) is equivalent to the risk-adjusted conditional expected return on the aggregate (or market) portfolio,

$$E_t(r_{t+1}) = \beta_t E_t(m_{t+1}) \tag{2}$$

where

$$\beta_t = \frac{cov_t(m_{t+1}, r_{t+1})}{var_t(m_{t+1})} = \frac{\Omega_t \lambda_t}{var_t(m_{t+1})}.$$

This expression makes it clear that the vector of sub-portfolio β_ts varies both with the shares of assets in the portfolio, λ_t, and the conditional covariance matrix, Ω_t, and thus may move substantially over short time intervals.

Under rational expectations, we can replace the vector of expected excess returns with the actual returns by including a prediction error that is orthogonal to all information at time t:

$$r_{t+1} = \rho_t \Omega_t \lambda_t + \varepsilon_{t+1}, \tag{3}$$

where $\varepsilon_{t+1} = r_{t+1} - E(r_{t+1})$. The insight in Frankel (1982) was that information about the conditional covariance matrix of returns can be obtained from the error terms, since under MVE:

$$\Omega_t = E_t(\varepsilon_{t+1}\varepsilon'_{t+1}). \tag{4}$$

MVE therefore imposes a set of restrictions that are highly nonlinear in that they constitute proportionality between the coefficient matrix and the variance-covariance matrix of the error term in (2).

To evaluate (4), we must take a position on how Ω_t changes over time. In sections 4 and 5 below, we assume that Ω_t is constant and that it follows an ARCH or GARCH process, respectively. We test the hypothesis that MVE holds against more general alternatives in which investors forecast excess returns as a function of asset shares and past prediction errors (as in the Tobin model).

The portfolio-balance model of Tobin (1958,1969), representing a general relationship between asset demands and expected returns, can be written as:

$$\lambda_t = \mathbf{B}_t E_t(r_{t+1}), \tag{5}$$

where **B** is an N x N matrix of coefficients. By inverting the system of equations in (5), we obtain an expression for expected excess returns,

$$E_t(r_{t+1}) = \mathbf{A}_t\lambda_t, \tag{6}$$

where $\mathbf{A}_t = \mathbf{B}_t^{-1}$. This system of equations representing the portfolio-balance model is a generalization of MVE. MVE imposes the restriction that the matrix of coefficients $\mathbf{A}_t$ be proportional to the variance of the forecast error, ε_{t+1}.

Hence an insight of the CASE method: MVE can be viewed as the null hypothesis in a test where the alternative hypothesis is the more general unconstrained portfolio balance model. Using ex post returns, (6) can be written:

$$r_{t+1} = \mathbf{A}_t\lambda_t + \varepsilon_{t+1}. \tag{7}$$

Although the values of the equities are endogenous variables in an economic sense, they are still uncorrelated with the prediction errors, which under rational expectations are uncorrelated with all information available at time t.[5]

We also test the MVE hypothesis above, as well as the more general alternatives, against an even more restrictive null hypothesis: that investors expect conditional excess returns to be zero. The results of our tests are discussed in sections 4 and 5. Section 6 presents a diagram which makes it easy to see the results of our nested hypothesis tests.

It is interesting to contrast our test of MVE with two recent closely related tests. A detailed comparison of our test to that of Harvey (1989) would consume much space, but the essence of the comparison is simple. The MVE of equation (1) or (2) implies a relation between expected returns and covariances of returns. The model is not testable until some additional auxiliary assumption is imposed. Our auxiliary assumption is in the form of a model for the covariance matrix Ω_t -- it is alternatively modeled to be constant, or to follow a GARCH process. Once the model for Ω_t is chosen, the MVE model determines the behavior of expected returns. Harvey, alternatively, makes the auxiliary assumption on the expected returns, rather than the covariances. He assumes that expected returns are linear in observable economic data. This model, combined with MVE, then determines the behavior of the covariance of returns. The two approaches are similar in that they test the cross-equation restrictions imposed by MVE while maintaining the auxiliary assumption. One advantage that arises from the approach of our paper, in making the auxiliary assumption about the covariances rather than the means, is that our alternative to MVE is explicit and economically meaningful.

Ng (1991) estimates a constrained version of MVE that is in most respects identical to our constrained model. However, her alternative hypothesis differs from ours. She

[5] Note that the N asset shares, $\lambda_{t,1}...\lambda_{t,N}$ are perfectly collinear because they sum to 1. This does not pose a problem for the estimation of (7), however, because the equations do not include a constant term.

tests the restriction that the intercept term in equation (1) is zero. As mentioned, our test is more in line with Harvey (1989), in the sense that we test cross-equation restrictions.

So, the test proposed here is complementary to the tests of Ng and Harvey, and perhaps has some advantages.

3. The data

Our tests use monthly stock returns from the New York and American Stock Exchanges from January 1955 to December 1984. To ease the computational burdens in estimating (3) we aggregate the stocks into N = 11 (and sometimes 7) industry portfolios.[6]

Table 1 describes the aggregation of stocks into industry portfolios. The returns for each portfolio are value-weighted average returns. The N x 1 vector of portfolio shares, λ_t, is the value of the stocks in the portfolios as a fraction of the total value of all stocks. Because it is desirable to group together equities that have highly correlated returns, we tried to put similar industries into the same portfolio.[7] Stambaugh (1982) aggregates into 20 industries, roughly by type of final output. We further aggregate into 11 industries, combining some of Stambaugh's categories. Table 1 shows Stambaugh's 20 industries, as well as the 11-industry aggregation that we use to

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[6] If there are N assets, the computation involves a parameter matrix of dimension N(N-1)/2 x N(N-1)/2 that must be repeatedly inverted. Engel and Rodrigues (1992) offer a Wald test version of the CASE test that is less computationally difficult. We apply it in Section 5 below.

It is easy to demonstrate that if the returns on the industry portfolios are computed using the asset shares as weights, that the MVE model of equations (1) or (2) holds for returns on industry portfolios.

[7] On the other hand, we would not want to include together the suppliers of intermediate products and the producers of final output in the same industry. When steel prices rise, the cost of producing autos increases so that it is possible that steel producers' profits rise when auto manufacturers' profits decline.

perform our maximum likelihood tests of MVE. Table 1 also reports a 7-industry aggregation that we use for the ARCH estimation in section 4.

The value shares, λ_t, are used to predict excess returns between time t and t+1. The shares are measured monthly from the last day of January 1955 to the last day of November 1984 (359 observations), while the returns are calculated as the dividend plus appreciation over the previous month beginning the last day of February 1955 and ending the last day of December 1984. All returns are nominal excess returns above the return on the one-month Treasury bill recorded by Ibbotson Associates (1986).

4. Tests of MVE with constant conditional variances

Table 2 reports the results from estimating the unconstrained system of equations (7), when the matrix A is treated as constant over time. Few of the coefficients individually are significantly different from zero. Not surprisingly, the R^2s are not very high, and none exceeds .10. We can, however, reject at the 95 percent level the hypothesis that the asset shares have no explanatory power for excess stock returns. The value of the chi-square statistic (121 d.f.) is 233.56 compared to a critical value of 147.39.[8,9]

Under the MVE hypothesis, this unconstrained system of inverted asset demand equations is not estimated efficiently. If we impose more structure on the system we can hope to improve the precision of our parameter estimates. So we will estimate the system

[8] The 99 percent critical value is 159.32.

[9] The only prior belief we have about the coefficients is that the return on asset j is likely to be positively related to the share of asset j in the total portfolio. If we think of the market portfolio as comprised only of stocks, then in equilibrium investors will demand a higher return from a given stock portfolio the more of it they are required to hold. Table 2 shows that in 8 out of the 11 regressions this own-coefficient is negative (and significantly negative for industries 2 and 7). It is not significantly positive in any of the regressions.

of equations (3) which impose the MVE constraints that $A = \rho\Omega$. In this section, the variance matrix Ω is assumed constant over time.

The N equation system (3) must be estimated by maximum likelihood techniques, imposing cross-equation restrictions between the matrix of coefficients in the regressions and the variance matrix of the regression errors. Note that the assumption that Ω is constant is not the same as the assumption tests of constant betas and expected returns. As we saw in the previous section, even with a constant covariance matrix, the betas, and hence the expected returns on all securities including the aggregate or "market" portfolio, will vary over time in a general way. Table 3 reports the maximum likelihood results of (3).

We can report a chi-square statistic for the restrictions implied by (3). This is the CASE test of the MVE hypothesis against the more general portfolio-balance model. We impose 120 restrictions on the unconstrained system (121 coefficients are constrained to be proportional to their corresponding elements in the variance matrix). The test statistic has a value of 231.34, so we easily reject the hypothesis of MVE at the 99 percent level.

If one were willing to accept the MVE estimates on the basis of prior beliefs, they yield in some ways much more plausible asset pricing equations. We noted that in the unconstrained regressions we frequently found that an increase in an asset share would actually decrease that asset's expected return. That is not possible with the constrained MVE estimates. Also, the point estimate of ρ, which can be interpreted as the coefficient of relative risk aversion under the assumption that λ_t are shares of the complete market portfolio, is very plausible -- 2.03. It is very close to the "Samuelson presumption" of a likely value for average risk aversion. The coefficient is not estimated precisely, however, as it is not statistically different from zero at the 95 percent level. But its 95 percent confidence interval ranges only up to about 5.3 --

still a believable estimate for average risk aversion.

On the other hand, the constrained model does a very poor job of predicting excess returns. The failure to reject the hypothesis that $\rho = 0$ implies that asset shares provide no statistically significant explanatory power for risk premia under the MVE restrictions, because the coefficients on the shares are all multiples of ρ.[10] In other words, MVE vitiates the predictive power of the asset shares alone.

The estimates reported in Tables 2 and 3 calculate the shares as a fraction of total equity investment. If, however, there are positive net holdings of the riskless asset, then the shares should properly be calculated as a fraction of total equity investment plus the total net value of the riskless asset. The riskless asset could have a positive net value if the government issues riskless short-term bonds, and investors consider government bonds to be additions to net wealth (so that they do not fully discount future tax liabilities) or if the government issues money. We estimated the model under the assumption that the relevant measure of the net supply is the value of all government bonds (which is calculated by Cox (1985)), and again under the assumption that the value of outstanding Treasury bills measure the net supply of the riskless asset. In both cases, there was almost no change in the estimates.

We considered two other formulations which apply when ρ is interpreted as the coefficient of relative risk aversion, besides assuming that it is constant. In the first, we assumed constant *absolute* risk aversion. In that case, $\rho_t = bW_t$ where b is the coefficient of absolute risk aversion and W_t is the value of all equities at time t. In the second, we considered a more general formulation consistent with the HARA class of utility functions, $\rho_t = a + bW_t$. If $b = 0$, we have the constant relative risk aversion case, and if $a = 0$ we have the constant absolute risk aversion case. Again, however,

[10] Under the MVE restrictions, constraining r to be zero lowers the log-likelihood value by 1.1.

these versions of the model failed to improve the constrained model's performance.[11]

Maximum likelihood estimation of MVE is a difficult problem because of the constraints imposed between the coefficients and the variance. The entire system must be estimated simultaneously, which in the case of the 11-asset system means simultaneously estimating 122 coefficients.

If we are interested in testing MVE, but not in actually obtaining the constrained coefficient estimates, we do not need to estimate the constrained set of equations. A Wald test can be performed using only the unrestricted model. In this case, the unconstrained model (6) is particularly easy to estimate, because it requires only equation-by-equation ordinary least squares. Engel and Rodrigues (1992) provide an expression for the Wald statistic for the MVE restrictions. The Wald statistic is not difficult to compute even for large collections of assets. We tested the MVE restrictions for the entire set of 20 industry portfolios composed by Stambaugh. We again reject MVE restrictions easily. The test statistic is distributed chi-square (19 d.f.), and has a value of 58.99, well above the 99 percent critical value.[12]

The estimates of this section provide little support for MVE of the stock market. In all of the tests performed, the restrictions that MVE places on a more general asset demand model are strongly rejected.

5. Tests of MVE with ARCH conditional variances

In the estimates reported in section 4, we assumed that the return covariance

[11] In order to save space, we do not report these results.

[12] The comparable Wald test for the 11-asset aggregation yields a statistic distributed as χ^2_{10} equal to 22.76. This also rejects the MVE restrictions at the 99 percent level. These particular tests restrict only the diagonal elements of the return covariance matrix, and yet they reject easily.

matrix, Ω_t, was constant over time. Because it has become clear in recent years that conditional variances of financial variables show a considerable amount of variation, we turn to a model of time-varying conditional variances.

In simple regression models, the presence of heteroskedasticity often does not affect the consistency of coefficient estimates, although it does cause standard calculations of test statistics to be inconsistent. When the MVE restrictions are imposed, however, changes in variances imply changes in coefficient estimates, which in turn imply changes in expected excess returns. The coefficient on the asset shares in the constrained model must move over time if Ω_t does, so holding Ω_t constant leads to inconsistent coefficient estimates.

Inspection of (2) makes it easy to see why it is important to allow for variation in Ω_t. There are two possible sources of variation in expected returns if ρ is constant: changes in asset shares, λ_t, and changes in Ω_t. Suppose, for example, that favorable news about a stock is announced. One could easily think of cases in which the price is pushed up, increasing the stock's share in the aggregate portfolio, even though its expected return is now lower with the news. If the market is mean-variance efficient, this can happen when the riskiness of the asset declines -- its own variance falls, or its covariance with other assets decline. But, for the *j*th asset, this is exactly a change in the *j*th row of Ω_t.

We choose to model variances empirically following Engle's (1982) ARCH process. The ARCH takes the conditional variance of this period's forecast error to be a function of past forecast errors. It is not based on any theoretical notion of how the general equilibrium of the economy works. It is an *ad hoc* model that seems to work well in practice.

In this section, we apply a multi-equation version of ARCH to the MVE problem. Because of the difficulty in estimating large ARCH systems, we have further aggregated the assets into the 7 portfolios described in table 1. Even with only 7 equations to estimate, the dimension of the ARCH problem can be quite large. For example, even if we restrict ourselves to first-order ARCH in which the variances and covariances this period are related only to the squares and cross-products of forecast errors in the previous period, the problem is unmanageably large. There are 28 independent elements in the covariance matrix. If each element were linear related to the 28 lagged squares and cross products of the forecast errors, there would be 812 parameters to estimate.

Given the complexity of estimating the MVE-ARCH system, and given the limited amount of data, it is helpful to lower the number of ARCH coefficients. Our test of MVE uses a parsimonious version of ARCH, in which the model, has return variances given by

$$\Omega_t = P'P + G\varepsilon_t\varepsilon_t'G. \tag{8}$$

We treat as parameters the upper triangular matrix P, and the diagonal matrix G. Under this formulation, each element of Ω_t is linearly related to its corresponding component in the matrix of cross-products of lagged forecast errors. There are only 35 coefficients to estimate. This formulation enforces positive semi-definiteness on the covariance matrix Ω_t.

The unrestricted form of the inverted system of asset demand equations is given by equation (7). MVE imposes the restriction that $A_t = \rho\Omega_t$, where Ω_t is the conditional variance of r_{t+1}. In practice, if MVE is to be nested in the general system of asset demands, then the elements of A_t in the general system might be related to the same variables that Ω_t is assumed to be related to. More specifically, we assume that in the unrestricted model, the coefficient matrix A_t evolves according to:

$$A_t = Q'Q + F\varepsilon_t\varepsilon_t'F, \tag{9}$$

where Q is upper triangular and F is diagonal, and the conditional covariance matrix of returns, Ω_t, is given by (8). The MVE constraint, that $A_t = \rho\Omega_t$, imposes 34 constraints on the unconstrained asset demand equations in (7).

Before turning to the results of the ARCH estimates, it is useful to examine the constrained MVE estimates on the 7 equation system when Ω_t is constrained to be constant, as in the previous section. Table 4 shows that the 7-equation system performs much like its 11-equation counterpart. The estimate of ρ is close to 2.0. However, it is still not statistically different from zero, which indicates that the asset share data with the MVE constraints imposed do a poor job of explaining expected returns. In this case, MVE imposes 27 constraints on the general system. The test statistic is distributed chi-square (27 d.f.) and is estimated to be 70.00. The MVE constraints can be rejected strongly at the 99 percent level.

Table 5 reports the results of the MVE restrictions imposed on the ARCH system. There are two hypotheses to test here. The first asks whether we can reject the constant-variance MVE model in favor of the ARCH-MVE. A rejection would imply that time-varying variances statistically reduce the distance between the stock-market portfolio and the mean-variance efficient frontier. Such a rejection would lead us to the other interesting question: can we reject the restrictions implied by MVE on the unrestricted ARCH cum portfolio-balance system in (7) and (9)?

The constant-variance version of MVE is a special case of the ARCH-MVE model, in which the G matrix from (8) is constrained to be zero. This imposes 7 constraints on the ARCH system. Our test statistic is 30.82 and is distributed chi-square (7 d.f.) We reject the constant-variance restrictions at the 99 percent level. ARCH therefore improves significantly on the constant-variance form of MVE.

However, only four of the 7 ARCH coefficients (elements of the G matrix) are significantly different from zero at the 95 percent level. These coefficients are all quite small in magnitude. The square of each element gives the coefficient relating the variance in each equation to its own lagged squared forecast error. Only one of the squared components of G is greater than .10.

The point estimate of ρ is 1.91 -- again close to the Samuelson value of 2.0. Once again, the estimate is not statistically different from zero at the 95 percent level (although it is now significant at the 80 percent level).

The next step would be to compare the performance of the ARCH model with the MVE constraints imposed (equations (1) and (8)) to the ARCH model, with the more general, Tobin model of asset demands (equations (7) and (9)). However, given the unsatisfactory performance of the ARCH-MVE model in forecasting returns, we instead first see if the ARCH model of equation (8) can be improved. Specifically, we replace (8) with a multivariate GARCH specification, based on the model of variances proposed by Bollerslev (1986). We have

$$\Omega_t = P'P + G\varepsilon_t\varepsilon_t'G + H\Omega_{t-1}H. \tag{10}$$

This formulation modifies equation (8) by adding the term $H\Omega_{t-1}H$, where H is diagonal, to the model of the variance.

There are several interesting aspects to these GARCH estimates, which are reported in Table 6. First, since the ARCH model is nested in the GARCH model, we can test for the joint significance of the GARCH coefficients in the matrix H. That test statistic is chi-square (7 d.f.) and its value is 76.22. The hypothesis that H is zero is overwhelmingly rejected.

In fact, the elements of the matrix H are all quite large, as opposed to the elements of the G matrix. They all exceed .9, and are statistically significant individually. The square of these coefficients would serve as a measure of the persistence of the diagonal elements of the variance matrix. There is evidently a great deal of persistence, which in turn implies that the risk premia on these assets are highly serially correlated.

The coefficient ρ is estimated to be 3.04, with a standard error of 1.41. If we make the additional assumptions required to obtain CAPM from MVE, ρ has the interpretation of being the coefficient of relative risk aversion. A value of 3 seems quite plausible, and does not imply excessive risk aversion as some other asset pricing models require in order to accord reasonably well with the data (see, for example, Mehra and Prescott (1985)).

Moreover, the fact that the t-statistic (= 2.16) is significantly different from zero implies that the constrained MVE model with GARCH has a statistically significant power in explaining equity returns ex ante. That is, the model is useful in predicting the excess returns on equities.

The next step is then to compare the GARCH model with the MVE constraints imposed to the Tobin portfolio balance model given by equation (7). We modify the model of A_t from equation (9) by adding terms relating A_t to lagged values of the variance matrix:

$$A_t = Q'Q + F\varepsilon_t\varepsilon_t'F + K\Omega_{t-1}K. \tag{11}$$

Here, the matrix K is diagonal.

Rather than estimate the full-blown unconstrained model, consisting of equations (7) and (11), we test the restrictions that the MVE system, (1) and (10) put on this model using a Lagrange multiplier (LM) test. The LM test is useful in this context because it requires estimation only of the constrained model by maximum likelihood. The hypothesis that A_t is proportional to Ω_t imposes 42 constraints on the general, Tobin portfolio-balance model. The test statistic is distributed chi-square (42 d.f.) and has a value of 81.90. The constrained M.V.E. model is easily rejected at the 99 per cent level.[13]

We conclude that while letting the variance change over time is important in improving the explanatory power of MVE, it does not improve it enough relative to an

[13] Equation (11) imposes symmetry on the portfolio balance model. Testing the GARCH-MVE against a version of the portfolio balance model in which the constant term in equation (11) is allowed to be asymmetric produces an LM statistic of 335.15. This is drawn from a chi-square (62 d.f.) distribution, and is highly significant.

unconstrained system of asset-demand equations.

6. Summary of conclusions

Figure 1 provides a graphical summary of our nested hypothesis tests. At the top of the figure is the most unrestricted model we consider, the unrestricted GARCH model in equations (7) and (11). At the bottom of the figure is the most restrictive model, that asset shares are of no help in explaining required returns, or equivalently, that risk aversion is zero. For each pair of models, the line connecting them reports the results of a test of whether the lower model (the null hypothesis) can be rejected in favor of the upper model (the alternative hypothesis). It is easy to see that both of the MVE formulations -- the constant-variance case and the GARCH case -- are rejected when compared with any more general alternative hypothesis.

It is also apparent that allowing variances to be time-varying significantly improves the explanatory power for both the constrained and unconstrained models. In section 4, we reported that the ARCH-MVE model significantly outperformed the constant variance MVE model, but that the ARCH-MVE model was in turn bettered by the GARCH-MVE model. It is also the case that the unconstrained portfolio balance model with GARCH coefficients significantly outperforms the portfolio balance model with constant coefficients.[14]

All of the models estimated, with the exception of the constant-variance version of MVE have significant predictive power for expected excess returns. That is, the model of investor risk-neutrality ($A_t = 0$) can be rejected.[15]

[14] An LM test for that proposition is distributed chi-square (28 d.f.), and takes on a value of 359.0, rejecting the null extremely strongly.

[15] More accurately, the ARCH-MVE does not significantly improve on the risk neutral model, but the GARCH-MVE model does.

In particular, it is interesting that the GARCH version of MVE has power in explaining equity returns. Allowing the covariances of asset returns to be time-varying significantly improves the predictive power of the constrained MVE model. Fama and French (1992) suggest that covariances, as reflected in a measure of an assets unconditional beta, essentially have no power to predict returns when size is included as an explanatory variable. It seems unlikely that the predictive power in our model occurs because our betas are correlated with size, because it is unlikely that the GARCH effects are related to firm size. Though we do no formal test of our model with size included as an explanatory variable, it appears that the covariance of asset returns does help predict the mean of asset returns, as CAPM would have it.

Still, we always reject MVE in favor of the Tobin portfolio balance model. There are several ways to rationalize this rejection. One would be that the true asset pricing model is not the CAPM, but rather a multi-factor CAPM, the APT, a version of the intertemporal CAPM, or perhaps a version of the one-period CAPM that allows for more investor heterogeneity in either tastes or information sets. A second explanation for the results would rely on the Roll (1977) critique. If the stock market is very unlike the true "market" portfolio, we would not expect to find MVE, even if the CAPM holds.[16] Indeed, under this explanation, the asset shares and ARCH processes cannot be accurately observed.

A third explanation of the results would be that the residuals in (2) lead to poor measures of the conditional variances. If "peso problems" affect stock market returns, the estimated residuals will be biased. Imposing the MVE restrictions only compounds the problems. For example, it is well known that in the five years following the stock-

[16] Similar results were found, however, when money, bonds, and real estate were allowed into the portfolio (Frankel, 1985a,b, and Frankel and Dickens, 1984) and when foreign assets were allowed (Frankel, 1982, and Frankel and Engel, 1984.)

market boom of August 1982, the market rose at an average annual rate of 22 percent. Few would argue in retrospect that it is possible to obtain from this period *ex post*, valid measures of *ex ante* expected risk and return.

One could imagine other reasons as well why the MVE model may fail to describe the asset price movements of a given sample as well as the generalized Tobin portfolio balance model. The CASE approach allows us to see how the MVE model, while successful in its GARCH formulation at predicting excess returns, is still not as successful as the unrestricted model.

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TABLE 1

Industry Portfolios and S.E.C. Codes

	Industry	S.E.C. Codes
1.	Mining	10,11,12,13,14
2.	Food and Beverages	20
3.	Textile and Apparel	22,23
4.	Paper Products	26
5.	Chemical	28
6.	Petroleum	29
7.	Stone, Clay and Glass	32
8.	Primary Metals	33
9.	Fabricated Metals	34
10.	Machinery	35
11.	Appliances, Electric Equipment	36
12.	Transportation Equipment	37
13.	Miscellaneous Manufacturing	38,39
14.	Railroads	40
15.	Other Transportation	41,42,44,45,47
16.	Utilities	49
17.	Department Stores	53
18.	Other Retail Trade	50-52, 54-59
19.	Banking, Financial, Real Estate	60-67
20.	Miscellaneous	1,4,15-17,21,24,25,27,30,31, 46,48,70,73,75,78-80,82,89,99

11 Portfolios (combinations of the 20 portfolios)

Portfolio	Industry Portfolios
1	1, 20
2	2, 3, 4
3	5
4	6
5	7, 8, 9
6	10
7	11
8	12-15
9	16
10	17, 18
11	19

7 Portfolios (combinations of the 20 portfolios)

Portfolio	Industry Portfolios
1	1, 2, 3, 4, 20
2	5, 7, 8, 9
3	6
4	10, 11
5	12-15
6	16
7	17-19

TABLE 2

Estimated Coefficients from Unconstrained OLS Regressions

Dependent Variable: Excess rate of return on asset j
Independent variable: Shares of asset j in total portfolio

λ^1	λ^2	λ^3	λ^4	λ^5	λ^6	λ^7	λ^8	λ^9	λ^{10}	λ^{11}
Equation 1										
-0.14	0.19	0.26	-0.06	-0.11	0.14	-0.70	0.08	0.21	-0.35	0.26
(0.12)	(0.82)	(0.30)	(0.26)	(0.32)	(0.25)	(0.44)	(0.22)	(0.32)	(0.25)	(0.44)
R^2 = .023				Breusch-Godfrey statistic (20 lags) = 42.79*						
Equation 2										
-0.11	-2.29*	0.64*	-0.29	-0.28	0.44	-1.12*	0.16	0.59*	0.83	2.06
(0.13)	(0.86)	(0.32)	(0.27)	(0.34)	(0.26)	(0.46)	(0.23)	(0.22)	(0.57)	(1.24)
R^2 = .050				Breusch-Godfrey statistic (20 lags) = 23.38						
Equation 3										
-0.20	-1.05	0.12	-0.04	-0.32	0.14	-1.20*	-0.02	0.46*	1.16	2.05
(0.13)	(0.89)	(0.33)	(0.28)	(0.35)	(0.27)	(0.47)	(0.24)	(0.23)	(0.59)	(1.29)
R^2 = .047				Breusch-Godfrey statistic (20 lags) = 16.99						
Equation 4										
0.15	-0.55	0.74	-0.82*	-0.81*	0.14	-1.01	0.44	-0.01	-0.60	2.79
(0.16)	(1.09)	(0.40)	(0.34)	(0.43)	(0.33)	(0.57)	(0.29)	(0.28)	(0.72)	(1.57)
R^2 = .027				Breusch-Godfrey statistic (20 lags) = 21.74						
Equation 5										
-0.25	-1.00	0.83*	-0.25	-0.81	0.18	-1.68*	0.50	0.41	-0.02	2.20
(0.16)	(1.07)	(0.39)	(0.34)	(0.42)	(0.33)	(0.57)	(0.29)	(0.28)	(0.71)	(1.55)
R^2 = .044				Breusch-Godfrey statistic (20 lags) = 30.71						
Equation 6										
-0.10	-0.19	0.46	-0.40	-0.68	-0.45	-0.28	0.37	0.18	-0.06	1.99
(0.15)	(1.04)	(0.38)	(0.33)	(0.41)	(0.32)	(0.56)	(0.28)	(0.27)	(0.69)	(1.51)
R^2 = .046				Breusch-Godfrey statistic (20 lags) = 20.41						

Table 2 (continued)

Equation 7

-0.17	-2.72*	0.83*	-0.26	0.71	0.44	-2.15*	0.37	0.75*	1.21	3.15
(0.17)	(1.13)	(0.41)	(0.36)	(0.44)	(0.35)	(0.60)	(0.30)	(0.29)	(0.75)	(1.63)

R^2 = .066 Breusch-Godfrey statistic (20 lags) = 17.38

Equation 8

-0.14	-0.85	0.25	-0.10	-0.43	0.08	-1.41*	-0.04	0.62	0.94	1.80
(0.14)	(0.93)	(0.34)	(0.39)	(0.36)	(0.29)	(0.49)	(0.25)	(0.24)	(0.62)	(1.34)

R^2 = .067 Breusch-Godfrey statistic (20 lags) = 21.10

Equation 9

-0.09	-0.77	0.50	-0.10	-0.12	0.18	-0.64	-0.04	0.30	0.07	0.82
(0.12)	(0.80)	(0.30)	(0.25)	(0.31)	(0.25)	(0.43)	(0.21)	(0.21)	(0.53)	(1.16)

R^2 = .032 Breusch-Godfrey statistic (20 lags) = 35.07*

Equation 10

-0.11	-0.38	0.20	-0.10	-0.27	0.01	-0.56	0.06	0.41	-0.02	1.05
(0.16)	(1.06)	(0.39)	(0.33)	(0.42)	(0.33)	(0.56)	(0.28)	(0.28)	(0.70)	(1.53)

R^2 = .027 Breusch-Godfrey statistic (20 lags) = 44.68*

Equation 11

-0.04	-0.25	0.13	0.19	0.09	0.24	0.13	0.19	0.54	-0.20	-1.31
(0.14)	(0.95)	(0.35)	(0.30)	(0.37)	(0.29)	(0.50)	(0.25)	(0.25)	(0.63)	(1.37)

R^2 = .027 Breusch-Godfrey statistic (20 lags) = 42.42*

* = significant at 5% level

(standard errors in parentheses)

TABLE 3

CAPM Estimation, constant Ω, 11 assets

$$r_{t+1} = \rho(P'P)\lambda_t + \varepsilon_{t+1}$$

$$\mathrm{Var}_t(\varepsilon_{t+1}) = P'P$$

The estimate of the coefficient ρ:

2.0319
(1.6130)

The estimate of the upper triangular matrix P:

.0398	.0322	.0334	.0385	.0411	.0346	.0404	.0331	.0257	.0317	.0374
(.0018)	(.0021)	(.0023)	(.0028)	(.0026)	(.0026)	(.0030)	(.0025)	(.0022)	(.0029)	(.0023)
	.0274	.0197	-.0033	.0166	.0198	.0223	.0189	.0089	.0252	.0047
	(.0011)	(.0015)	(.0023)	(.0019)	(.0025)	(.0022)	(.0019)	(.0017)	(.0025)	(.0015)
		.0204	.0042	.0044	.0097	.0097	.0078	-.0046	.0015	.0000
		(.0008)	(.0024)	(.0015)	(.0019)	(.0019)	(.0014)	(.0018)	(.0018)	(.0018)
			.0360	-.0029	-.0019	-.0032	-.0017	.0018	-.0073	.0125
			(.0014)	(.0016)	(.0021)	(.0019)	(.0015)	(.0018)	(.0019)	(.0016)
				.0276	.0058	.0102	.0090	-.0046	.0003	.0051
				(.0011)	(.0019)	(.0019)	(.0013)	(.0017)	(.0017)	(.0016)
					.0304	.0068	.0050	-.0025	.0021	-.0009
					(.0011)	(.0016)	(.0015)	(.0017)	(.0018)	(.0014)
						.0272	.0063	.0000	.0063	.0020
						(.0011)	(.0014)	(.0017)	(.0019)	(.0017)
							.0214	.0020	.0094	.0027
							(.0010)	(.0018)	(.0018)	(.0017)
								.0272	.0032	.0050
								(.0011)	(.0017)	(.0014)
									.0287	.0006
									(.0013)	(.0013)
										.0219
										(.0007)

(standard errors in parentheses)

TABLE 4

CAPM Estimates, Constant Ω, 7 assets

$$r_{t+1} = \rho(P'P)\lambda_t + \varepsilon_{t+1}$$

$$Var_t(\varepsilon_{t+1}) = P'P$$

The estimate of the coefficient ρ:

2.028
(1.466)

The estimate of the upper triangular matrix P:

.03842 (.00150)	.03935 (.00185)	.03711 (.00246)	.04015 (.00213)	.03640 (.00206)	.02695 (.00190)	.03782 (.00197)
	.02075 (.00075)	-.00471 (.00225)	.01708 (.00140)	.01571 (.00149)	-.00389 (.00153)	.00485 (.00123)
		.03757 (.00115)	-.00296 (.00143)	-.00242 (.00120)	-.00033 (.00166)	.00140 (.00117)
			.02435 (.00092)	.00762 (.00137)	-.00136 (.00156)	.00342 (.00117)
				.02206 (.00087)	.00269 (.00149)	.00735 (.00108)
					.02801 (.00103)	.00408 (.00109)
						.02034 (.00075)

(standard errors in parentheses)

Table 5

CAPM Estimates, ARCH, 7 assets

$$r_{t+1} = \rho\Omega_t\lambda_t + \varepsilon_{t+1}$$

$$Var_t(\varepsilon_{t+1}) = \Omega_t = P'P + G\varepsilon_t\varepsilon_t'G$$

The estimate of the coefficient ρ:

1.912
(1.477)

The estimate of the upper triangular matrix P:

.03714 (.00152)	.03883 (.00189)	.03364 (.00274)	.04036 (.00213)	.03738 (.00204)	.02700 (.00191)	.03700 (.00200)
	.02050 (.00077)	-.00278 (.00233)	.01648 (.00150)	.01486 (.00158)	-.00395 (.00160)	.00494 (.00130)
		.03541 (.00116)	-.00285 (.00157)	-.00127 (.00124)	.00084 (.00182)	.00082 (.00128)
			.02405 (.00095)	.00687 (.00138)	-.00140 (.00160)	.00308 (.00122)
				.02118 (.00096)	.00253 (.00158)	.00747 (.00109)
					.02779 (.00109)	.00391 (.00112)
						.01971 (.00082)

The estimates of the diagonal elements of G:

.19819 (.03953)	.13305 (.04684)	.31874 (.06162)	.06267 (.04517)	-.03718 (.04355)	.15481 (.09843)	.17706 (.04668)

(standard errors in parentheses)

Table 6

CAPM Estimates, GARCH, 7 assets

$$r_{t+1} = \rho\Omega_t\lambda_t + \varepsilon_{t+1}$$

$$Var_t(\varepsilon_{t+1}) = \Omega_t = P'P + G\varepsilon_t\varepsilon_t'G + H\Omega_{t-1}H$$

The estimate of the coefficient ρ:

3.043
(1.407)

The estimate of the upper triangular matrix P:

.01075	.01314	.01078	.01227	.01039	.00716	.00824
(.00126)	(.00175)	(.00172)	(.00156)	(.00106)	(.00112)	(.00088)
	.00625	-.00006	.00424	.00338	-.00099	.00102
	(.00073)	(.00123)	(.00088)	(.00100)	(.00071)	(.00085)
		.01087	-.00050	.00011	.00046	.00084
		(.00148)	(.00072)	(.00070)	(.00077)	(.00071)
			.00679	.00134	-.00045	-.00020
			(.00110)	(.00074)	(.00053)	(.00073)
				.00519	.00057	-.00004
				(.00076)	(.00069)	(.00077)
					.00663	.00071
					(.00092)	(.00067)
						.00194
						(.00102)

The estimates of the diagonal elements of G:

.18916	.13561	.25085	.11798	.08263	.22818	.12647
(.02024)	(.02459)	(.04273)	(.02997)	(.02781)	(.03748)	(.01812)

The estimates of the diagonal elements of H:

.94234	.93786	.92592	.95185	.96264	.94444	.97392
(.01117)	(.01424)	(.01969)	(.01252)	(.00724)	(.01272)	(.00328)

(standard errors in parentheses)

Tests of the Model

$$r_{t+1} = A_t\lambda_t + \varepsilon_{t+1};\ E_t(\varepsilon_{t+1}\varepsilon'_{t+1}) = \Omega_t$$

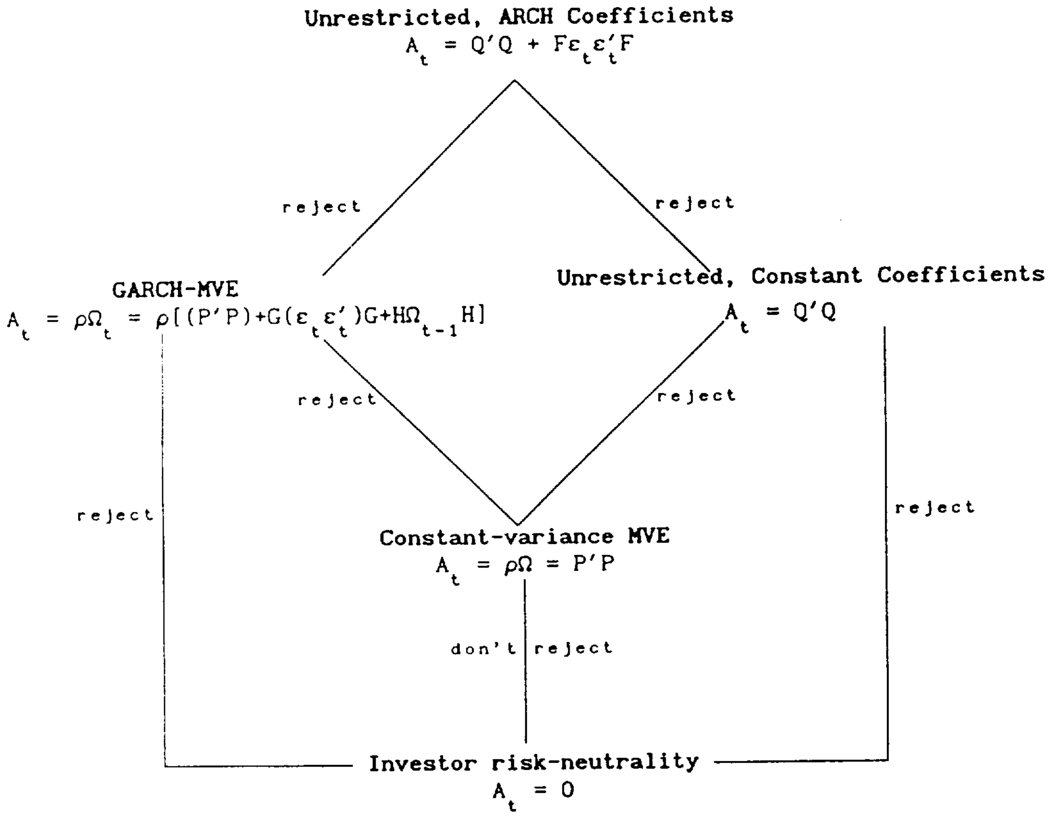

Figure 1