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A DYNAMIC SPATIAL MODEL

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ABSTRACT

Any interesting model of economic geography must involve a tension between "centripetal" forces that tend to produce agglomerations and "centrifugal" forces that tend to pull them apart. This paper explores one such model, and shows that the model links together a number of themes in the geography literature. These include: the role of market access, as measured by a measure of "market potential", in determining manufacturing location; the role of forward and backward linkages in producing agglomerations; the potential for "catastrophes", i.e. , discontinuous changes in location in response to small changes in exogenous variables: and the idea that the economy is a "self-organizing system" that evolves a self-sustaining locational structure.

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After a generation of neglect, the last few years have seen a broad revival of interest in regional economics. Economists interested in international trade, in growth, and in macroeconomics have all turned to regional models and regional data. This revival of interest in matters regional has several sources.

First, regional economic issues, though they faded from mainstream economics for more than twenty years, are of considerable importance in their own right. The importance of the regional dimension has been driven home by such events as the boom-bust cycle in New England and the difficulties of German reunification. So-called "spatial aspects" -- a euphemism for the continuing poverty of much of Europe's periphery -- are a major issue in the discussions of economic and monetary union in Europe (see Commission of the European Communities, 1990).

Second, regional experience provides a valuable laboratory for empirical work. Thus Blanchard and Katz (1992) use regional evidence to investigate the process of macroeconomic adjustment, Barro and Sala-i-Martin (1991) use it to investigate the characteristics of the growth process, and Glaeser et.al (1990) use it to shed light on the nature of external economies.

Finally, regional economics, or more broadly economic geography, is a subject of considerable intellectual interest. In particular, it is one place in which economic theory can make contact with trends in other fields of research. A number of economists, notably Brian Arthur (1990a, 1990b) have argued that increasing returns and external economies mean that real economies are characterized by strongly nonlinear dynamics. They thus argue that economic analysis should draw on the ideas of scientists like Prigogine (Nicolis and Prigogine 1989), who emphasize the possibilities of multiple equilibria, catastrophic change, and self-organization. (See Waldrop (1992) for an entertaining description of the attempts, centered on the Santa Fe Institute, to make "complexity" into an interdisciplinary organizing principle). It is debatable how useful these ideas will turn out to be in economics at large, but it seems clear that they have their most natural application in spatial and regional economics. Indeed, geographers have been among the first social scientists to attempt to make use of trendy new concepts in nonlinear dynamics (e.g. Wilson (1981)).

This paper is an exploration of the

dynamic implications of a simple model of economic geography. I have presented versions of this model in two earlier papers (Krugman 1991, 1992). Those papers, however, restricted themselves to static analysis, asking under what conditions particular spatial economic structures *were*, in fact, equilibria. This paper is explicit about the dynamics. It also extends the two-region or one-city analysis of the earlier papers to the case of multiple agglomerations.

While the model is simple in conception, and the results we get are quite intuitive, this dynamic analysis defies paper-and-pencil analytics. Thus the paper relies heavily on numerical examples. This is currently an unfashionable theoretical technique, but as we will see it is highly productive in this case.

The paper is in seven parts. Part 1 describes the basic approach, and briefly surveys some theoretical antecedents. Part 2 lays out the assumptions of the model. Part 3 shows how short-run equilibrium is determined. Part 4 uses a static analysis of a two-region case to illustrate the nature of the "centripetal" and "centrifugal" forces in the model. Part 5 then examines dynamics in the two-region case, while Part 6 analyzes the evolution of a multi-region economy. Finally, Part 7 draws some conclusions.

1. The basic approach

Any interesting model of economic geography must exhibit a tension between two kinds of forces: "centripetal" forces that tend to pull economic activity into agglomerations, and "centrifugal" forces that tend to break such agglomerations up or limit their size.

There is a well-developed literature in urban economics, largely deriving from the work of Henderson (1974), in which a system of cities evolves from such a tension. In Henderson-type models, the centripetal force arises from assumed localized external economies in production; the centrifugal force is urban land rent. Together with assumptions about the process of city formation, Henderson's approach yields a model of the number and sizes of cities (though not of their location relative to one another).

There is a variant of this approach, represented for example by Fujita (1988), in which external economies are not assumed but instead derived from increasing returns in a monopolistically competitive industry producing nontraded inputs. This leaves the basic approach unchanged, and still leaves the spatial relationship of cities to each other undetermined.

In my own work, I have tried a somewhat different approach. No special assumptions are made either about localized external economies or nontradeability. Indeed, cities are not primitive concepts in the model. Instead, agglomerations emerge from the interaction between increasing returns at the level of the individual production facility, transportation costs, and factor mobility. Because of increasing returns, it is advantageous to concentrate production of each good at a few locations. Because of transportation costs, the best locations are those with good access to markets **(backward** linkage) and suppliers (forward linkage). But access to markets and suppliers will be best precisely at those points at which producers have concentrated, and hence drawn mobile factors of production to their vicinity.

But not all factors are mobile, and the presence of immobile factors provides the centrifugal force that works against agglomeration. In principle, one should include urban land rents as part of the story: in the models I have worked out so far, however, this force is disregarded. Instead, the only force working against agglomeration is the incentive to set up new facilities to serve a dispersed agricultural hinterland.

Many of the elements of this story have been familiar to geographers for some time. (Useful surveys of the geography literature may be found in Dicken and Lloyd (1990) and Chisholm (1990)). At the risk of oversimplifying a rich tradition, we may identify three main strands of literature that bear on the approach taken here.

Closest in spirit to my model is the literature on "market potential", begun by Harris (1954). This literature argues that the desirability of a region as a production site depends on its access to markets,. and that the quality of that access may be described by an index of "market potential" which is a weighted sum of the purchasing power of all regions, with the weights depending inversely on distance. Thus if Y_k is the income of region k, and D_{jk} is the distance between j and k, then the market potential of region j would be determined by an index of the form

$$M_j = \sum_k Y_k g(D_{jk}) \qquad (1)$$

where g(.) is some declining function.

Harris showed that the traditional manufacturing belt in the United States was, for a variety of g(.) functions, the area of highest market potential: while he did not have an explicit model, he noted informally that the persistence of that belt could be attributed to the circular relationship in which industrial concentration tended both to follow and to create market access: "[M]anufacturing has developed partly in areas or regions of largest markets and in turn the size of these markets has been augmented and other favorable conditions have been developed by the very growth of this industry." (Harris 1954, p. 315, quoted by Chisholm 1990).

Market potential analyses have been a staple of geographical discussion, especially in Europe (see, for example, Keeble et. al. 1982). The main theoretical weakness of the approach is a lack of microeconomic foundations: while it is plausible that some index of market potential should help determine production location, there is no explicit representation of how the market actually works.

A second, closely related literature emphasizes the role of cumulative processes in regional growth. Pred (1966), drawing on the ideas of Myrdal (1957), suggested that agglomerations, by providing a large local market, are able to attract new industries, which further enlarges their local market, and so on. Other authors, such as Dixon and Thirlwall (1975), have proposed alternative motives for agglomeration but similar dynamics. Such cumulative causation suggests that initial advantages due to historical accident may play a major role in explaining the pattern of location. Like the market potential literature, however, the cumulative process literature lacks microfoundations.

Finally, we must mention central place theory. Developed by Christaller (1933) and Lösch (1940), this theory emphasizes the tradeoff between economies of scale and transportation costs. Central place theory suggests that the attempts of firms to make the best of this tradeoff should lead to the emergence of a lattice of production sites roughly evenly spaced across the landscape, perhaps in a hierarchical structure in which activities with larger scale economies or lower transport costs are concentrated in a smaller number of higher-level sites. Central place theory has been a powerful organizing principle for research, even though it has well-known weaknesses. Most notably, not only does it not have **any** explicit microfoundations: it also neglects the circular causation that is such a central theme in both the market potential and the cumulative process literatures. Nonetheless, one would like a geographical model to exhibit at least some central-place features.

In summary, then, the urban economics literature offers clear and explicit analysis, but does not model the spatial relationship of cities to each other. The geographical tradition, while rich in insight, lacks a microeconomic foundation and as a result lacks the sharp edges we want from a theoretical analysis. What we want to do next is introduce a formal model, with complete microfoundations, that captures and clarifies the insights of the geography tradition.

2. Assumptions of the model

In any model in which increasing returns play a crucial role, one must somehow handle the problem of market structure. Traditional urban models do so by assuming that increasing returns are purely external to firms, allowing the modeler to continue to assume perfect competition. The approach taken here, however, involves avoiding any direct assumption of external economies: externalities emerge as a consequence of market interactions involving economies of scale at the level of the individual firm. Thus we must somehow model an imperfectly competitive market structure. The workhorse model of this kind is, of course, the Dixit-Stiglitz (1977) model of monopolistic competition. Dixit-Stiglitz monopolistic competition is grossly unrealistic, but it is tractable and flexible; as we will see, it leads to a very special but very suggestive set of results.

We assume, then, an economy in which there are two sectors, manufacturing and agriculture. Everyone shares the same Cobb-Douglas tastes for the two types of goods:

$$U = C_M^{\mu} C_A^{1-\mu} \tag{2}$$

where μ is the share of manufactured goods in expenditure.

We assume that there is a single, homogeneous agricultural good. Manufactures, however, is a composite of a large number of symmetric product varieties, with a constant elasticity of substitution σ between any two varieties:

$$C_M = \left[\sum_i c_i^{\frac{a-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{3}$$

In setting up the production side of this economy, we want to make allowance for both mobile and immobile factors of production, which at any given time are distributed across a number of locations j = 1,...,J. One might suppose that the natural thing would be to assume that labor and possibly capital are the mobile factors, while land is the immobile factor; and that both mobile and immobile factors are used in both sectors. To do this, however, we would have to take account of land-labor substitution in both sectors, a major complication of the model. We would also have to worry about where landowners live. It turns out to be much simpler, if even less realistic, to assume that the two factors of production are both "labor": mobile "workers" who produce manufactured goods and immobile "farmers" who produce the agricultural good.

Farming is an activity that takes place under constant returns to scale; thus the farm labor used in producing any given quantity of the agricultural good at location j can, by choice of units, be set **equal** to production:

$$L_{Aj} = Q_{Aj} \tag{4}$$

Manufacturing, however, we assume to involve economies of scale, with a fixed cost for **any** variety produced at any given location:

$$L_{Mij} = \alpha + \beta Q_{Mij} \tag{5}$$

Let $\mathbf{L_A}$ and $\mathbf{L_M}$ represent the economy-wide supplies of the two factors "farmers" and **"workers"** respectively. We will assume that these supplies are fixed. They are, however, allocated across locations. A share ϕ_j of the farm labor force is in location j; we take these shares as exogenous. At any point in time, a share λ_j of the manufacturing labor force is also in location j; these shares will evolve over time in a fashion specified below.

At any point in time, then, there will be location-by-location full employment equations for both factors/sectors:

$$L_{Aj} = \phi_j L_A \tag{6}$$

$$\sum_i L_{Mij} = \lambda_j L_M \tag{7}$$

Next we introduce transportation costs. For simplicity, we make some completely unrealistic assumptions about these costs. First, we assume that they apply only to manufactured goods. Second, we assume that they take the "iceberg" form introduced by Paul Samuelson: instead of modeling a separate transportation industry, we simply assume that a fraction of any manufactured good shipped melts away en route. Specifically, let x be the amount of some good shipped from j to k, and let z be the amount that arrives; then we assume

$$z_{ijk} = e^{-\tau D_{jk}} x_{ijk} \tag{8}$$

where τ is the transportation cost and D_{jk} is the distance between the two locations.

Finally, we turn to factor mobility. Farmers are assumed completely immobile.

Workers are assumed to move toward locations that offer them higher real wages. (No attempt is made here to model the moving decision explicitly). As we will see in the next section, it is possible to solve the model at any point in time for the real wages ω_j paid to workers at each location. Let us define the average real wage as

$$\overline{\omega} = \sum_j \lambda_j \omega_j \tag{9}$$

Then the assumed law of motion of the economy is

$$\frac{d\lambda_j}{dt} = \rho \lambda_j (\omega_j - \overline{\omega}) \tag{10}$$

That is, workers move away from locations with below-average real wages and toward sites with above-average real wages.

We have now specified a complete dynamic model of geographic dynamics. The inputs into this model are the parameters μ, τ, and σ (which turn out to be the only parameters that cannot be eliminated by choice of units); a given allocation of farm labor across locations; a matrix of distances between locations; and an initial allocation of workers across, locations. These inputs determine equilibrium at a point in time, and in particular the vector of real wages, which dictates the changes in the allocation of workers, leading to an evolution of that equilibrium over time.

This sounds pretty abstract. Our next step is to describe some of the features of short-run equilibrium.

3. Short-run equilibrium

As a preliminary step to the description of short-run equilibrium, it is useful to recall two basic points about Dixit-Stiglitz-type models.

First, in these models, the producer of any one manufactured variety faces a constant elasticity of demand σ. Her profit-maximizing strategy is therefore to set price as a fixed markup over marginal cost:

$$p_{ij} = \frac{\sigma}{\sigma - 1} \beta w_j \qquad \textbf{(11)}$$

where w_j is the wage rate of workers at location j.

By choice of units we can simply say that the f.o.b. price of manufactured goods at j is equal to the wage rate:

$$p_j = w_j \qquad \textbf{(12)}$$

Second, if firms are free to enter until profits are zero, there is a unique zero-profit output of any manufactured variety, which can be shown to be

$$Q_{Mi} = \frac{\alpha}{\beta}(\sigma-1) \tag{13}$$

Since all varieties are produced at the same scale, the number of varieties produced at any given location is simply proportional to that location's manufacturing labor force. In particular, let n be the number of manufactured varieties produced in the economy as a whole, and let n_j be the number produced at location j. Then we have

$$n_j/n = \lambda_j \tag{14}$$

Equation (14) plays a crucial role in the whole analysis in this paper. The logic of the model depends crucially on increasing returns, yet as we write out the equations of short-run equilibrium these increasing returns will not be *very* visible. Where did they go? The answer is that they are hidden in (14). What increasing returns do is to make it profitable to produce each variety in only one location, so that different locations do not produce the same set of goods but differentiated bundles of products. When a location gains labor it does not produce more of the same mix of products, but adds new products. This "quantization" of production is the only way in which increasing returns actually enter the solution, but it is enough: as we will see, the micro assumption does indeed have major macro effects.

There are now several ways to proceed. The one that seems easiest represents short-run equilibrium as the solution of four sets of equations.

First, we determine income at each location. Given our assumption of zero transport costs on agricultural goods, the wage rate of farmers is the same at all locations. Let there be μ workers and $1-\mu$ farmers, a normalization that will set economy-wide income equal to 1; and let us measure all prices and wages in terms of the agricultural good. Then we have

$$Y_j = (1-\mu)\phi_j + \mu\lambda_j w_j \qquad (15)$$

Next, we find the true or ideal price index of the manufactures aggregate to consumers at each location. To do this, we note that in order to have one unit of a manufactured variety make it from k to j, $\exp(\tau D_{jk})$ units must be shipped, so the c.i.f. price on arrival is $w_k \exp(\tau D_{jk})$. Given the

CES function **(3)**, the true price index of manufactures at j is therefore

$$T_j = \left[\sum_k \lambda_k \left(w_k e^{\tau D_{jk}}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \qquad \textbf{(16)}$$

Given these true price indices, we can solve for equilibrium wage rates. It is shown in the appendix that

$$w_j = \left[\sum_k Y_k \left(T_k e^{-\tau D_{jk}}\right)^{\sigma-1}\right]^{\frac{1}{\sigma}} \qquad \textbf{(17)}$$

It is worth stopping briefly at this point, to note that the right hand side of (17) bears a family resemblance to the market potential index (1). Like that index, it depends on a weighted sum of purchasing power at all locations, with the weights inversely related to distance. The difference is that the true price indices also enter into the index: essentially this reflects the effect of competition from producers in other locations, which is missing from the usual market potential approach. But there is a definite affinity between the workings of this model and the market potential tradition in geography.

Equation **(17)**, however, only determines wage rates in terms of agricultural goods. Workers are interested in real wages in terms of a consumption basket that includes manufactures as well. Thus the real wage depends on both the wage in terms of the agricultural good and on the manufactures price index:

$$\omega_j = w_j T_j^{-\mu} \tag{18}$$

We now have a soluble set of equations for short-run equilibrium. Equations (15)-(17) need to be solved simultaneously for the vectors **Y**, T, and w; given these one can then solve (18).

These equations are easily solved on the computer -- in the numerical examples below, I simply started with an initial guess at w and then cycled (with some damping) over **(15)-(17)** until convergence. In general, however, they cannot be solved with pencil and paper. Yet we would like to get some intuition about the forces in our model before going over to numerical methods.

In order to do this, we examine a limited question for a special case, before moving on to more general problems.

4. Centripetal and centrifugal forces

In this section of the paper I ask a question originally posed in Krugman (1991), but which we can now restate in terms of our more general framework.

Consider an economy with only two locations, each of which has the same number of farmers ($\phi_1 = \phi_2 = 0.5$). Under what conditions is concentration of all manufacturing in one location ($\lambda_1 = 1$ or 0) an equilibrium? By answering this question, we get some useful insights into how the parameters of the model affect the relative strength of centripetal and centrifugal tendencies.

What we do is solve (15)-(18) on the assumption that $\lambda_1 = 1$, $\lambda_2 = 0$ (the case where $\lambda_1 = 0$ is symmetric). We ask whether, in that case, the real wage that workers would earn at location 2 is less than that at location 1. Concentration of manufacturing at 1 is an equilibrium if and only if $\omega_2 < \omega_1$ in that case.

To save notation, let's normalize the distance between the two locations to 1. Then we immediately find from (15)-(18) that

$$w_1 = T_1 = \omega_1 = 1 \tag{19}$$

and, substituting, that

$$W_2 = \left[\frac{1+\mu}{2}e^{-\tau(\sigma-1)} + \frac{1-\mu}{2}e^{\tau(\sigma-1)}\right]^{1/\sigma} \qquad \textbf{(20)}$$

and

$$\omega_2 = e^{-\tau\mu}\left[\frac{1+\mu}{2}e^{-\tau(\sigma-1)} + \frac{1-\mu}{2}e^{\tau(\sigma-1)}\right]^{1/\sigma} \qquad \textbf{(21)}$$

The condition for sustainability of concentrated manufacturing, then, is that the right hand side of (21) be less than one.

In the intuitive discussion of agglomeration in Part 1 of this paper, it was argued that agglomeration is possible because of the circular relationship between the location of the market and the location of manufacturing. We can see this intuition borne out in this model by considering what would happen if manufacturing were a very small part of the economy, μ close to zero. Then (21) would reduce to

$$\omega_2 = \left[\frac{1}{2}e^{-\tau(\sigma-1)} + \frac{1}{2}e^{\tau(\sigma-1)}\right]^{\frac{1}{\sigma}} < 1 \qquad \textbf{(22)}$$

which is always less than one because of Jensen's inequality. In this case, in which firms sold only to the agricultural market, it would always be advantageous to move <u>away</u> from any concentration of manufacturing in order to get away from competitors.

This desire to get away from competition represents the centrifugal force in this model, the force that works against agglomeration. By examining (21), however, we see that when the manufacturing sector is a significant part of the economy there are two centripetal forces working to hold an agglomeration together. First, the first term in (21) becomes less than one. By referring back to (18), we see that this term is there because of the role of manufacturing firms as suppliers of goods to manufacturing workers; in effect, this is a kind of Hirschmann (1958)-type <u>forward linkage</u>. Second, the expression inside the brackets involves a higher weight on the component that is less than one and a lower weight on the component that is greater than one. This reflects the point that the region in which manufacturing is concentrated has a higher income than the other location. Thus there is also a <u>backward linkage</u> in which manufacturing wants to be close to the market that manufacturing itself creates.

An economy with a large μ, then, may have a self-sustaining manufacturing concentration due to forward **and** backward linkages, and we **may** presume that concentration is more likely, the larger is μ. What about the other parameters?

The parameter whose effect may seem counter-intuitive to some readers is the transportation cost τ. Concentration is more likely when transport costs are <u>low</u>. To see why, we note the following:

First, when $\tau=0$, $\omega_2 = 1$. No surprise here: in the absence of transport costs, location **doen't** matter.

Second, in the vicinity of $\tau=0$, we find that

$$\frac{\partial\omega_2}{\partial\tau} = -\mu - \frac{\sigma-1}{\sigma}\mu < 0 \qquad \textbf{(23)}$$

Finally, we note that (21) may. be rewritten as

$$\omega_2 = \left[\left(\frac{1+\mu}{2}e^{-\tau(2\sigma-1)} + \frac{1-\mu}{2}e^{\tau[(\sigma-1)-\sigma\mu]}\right]^{1/\sigma}$$

If

$$\frac{\sigma-1}{\sigma} > \mu \qquad \textbf{(25)}$$

then **as** τ grows the real wage in location 2 eventually must exceed 1. In that case the relationship between transport costs and the real wage must have the shape illustrated by the curves in Figure 1 (which are calculated for $\tau = .2$, $\sigma = 4$, and $\mu = .2$ and $.3$). At high transport costs a concentration of production is not sustainable; there is a range of low transport costs for which such a concentration is not sustainable.

If (25) is not satisfied, the curve lies below 1 for <u>all</u> values of τ. To understand this case, we note that $\sigma/(\sigma-1)$ is the ratio of price and hence average cost to marginal cost, a measure of equilibrium economies of scale. Thus (25) amounts to saying that neither the share of manufacturing in the economy nor economies of scale are too large. If scale economies and the manufacturing share are sufficiently large, workers will prefer to cluster together even with prohibitive transport costs.

Returning to the case where (25) is satisfied, we note that what we have defined is a critical value of τ, τ^*, below which concentration is an equilibrium. We may offer some rough intuition here by stepping a bit outside the formal model. Basically, when transport costs are sufficiently low it is worthwhile for manufacturers to concentrate their production geographically so as to realize economies of **scale.** Once they have decided to concentrate production, however, the optimal location is one that other producers have also chosen. So low transport costs foster agglomeration.

One might expect that concentration would also be more likely the higher is μ. Indeed, in Figure 1 the lower curve corresponds to the higher value of μ. It is straightforward to show (see Krugman 1991) that

$$\frac{\partial \tau^*}{\partial \mu} > 0 \qquad (26)$$

Let us also bear in mind that σ, the elasticity of substitution, is inversely related to the equilibrium degree. of economies of scale. Thus we would expect to find that a high elasticity of substitution works against agglomeration, and we can indeed show (again see Krugman 1991) that

$$\frac{\partial \tau^*}{\partial \sigma} < 0 \qquad (27)$$

What we get from this static exercise is an indication of how the parameters of the model ought to affect the balance between the centripetal forces that favor agglomeration and the centrifugal forces that oppose it. Agglomeration is favored by low transport costs (low T), a large share of manufacturing in the economy (high μ) and strong economies of scale at the level of the firm (low a).

This is, however, only an analysis of a static equilibrium. We must turn next to dynamics.

5. Dynamics in the two-resion case: stabilitv, instabilitv, and catastrophes

Let us now consider dynamics while still restricting ourselves to the case of two locations. We are interested both in the types of equilibrium that might occur and in the process of transition between equilibria when exogenous variables change.

How should we go about examining dynamics? It might be possible, with great difficulty, to prove some theorems. It is much easier, however, simply to rely on simulation. Given the small number of parameters, it is possible to explore the model quite thoroughly on the computer. Thus from this point on we will rely on numerical methods.

As a **starting** point, consider Figure 2. This figure corresponds to the case analyzed in the preceding section, where $\phi_1 = \phi_2 = 0.5$, $\mu=.2$, $\sigma=4$. Now, however, we plot the share of location 1 in manufacturing, λ_1, against the real wage difference $\omega_1 - \omega_2$. Note that we can rewrite (10) in the two-region case as

$$\frac{d\lambda_1}{dt} = \rho\lambda_1 (1-\lambda_1)(\omega_1-\omega_2) \tag{28}$$

so this diagram allows us readily to identify stable and unstable equilibria.

In the figure, we show three cases: high transport costs (τ = .385), low transport costs (τ = .285), and an intermediate case (τ = 335). In the high cost case, there is a unique stable equilibrium with'manufacturing equally divided between the two locations. In the low cost case, there are two stable equilibria with manufacturing concentrated in either location, and an unstable equilibrium with manufacturing evenly divided. In both cases the relationship between λ_1 and the wage difference is monotonic.

But in the intermediate case this relationship is not monotonic. In the case illustrated there is a unique and stable equilibrium, but with slightly lower transport costs there are <u>three</u> stable equilibria: two equilibria with all manufacturing concentrated, one with it equally split between the locations.

On reflection, it must be the case that when transport costs are near the critical level τ^* identified in Part 4, there is a non-monotonic relationship between λ and the wage differential. Suppose that transport costs were just at the critical level. Then the curve in Figure 2 would have to reach zero at either end. It could thus only be monotonic if it were linear and indeed flat, which is impossible given the non-linearity of the model. This suggests why there is such an affinity between models in which increasing returns give rise to multiple equilibria and the methods of nonlinear dynamics; I will not try to pursue this any further, but simply note that this model seems to illustrate a more general principle.

Now that we have a way to look at the dynamics of the model, let us consider how the model behaves when exogenous variables are shifted. To do this, we must choose some variables to shift. There are actually only four parameters in this two-region model: the elasticity of substitution (which determines the degree of scale economies), the share of manufacturing, the level of transport costs, and the.agricultural share of region 1. Given the structure of this model both the degree of economies of scale and the share of manufacturing are parameters of tastes rather than technology or resources, which makes it somewhat problematic to think about what it means when they change, so we focus on agriculture and transport costs.

Figure 3 shows a calculation that illustrates how the set of equilibria change when we start with an equal allocation of agriculture between the regions (ϕ_1=0.5) and gradually increase that share to 0.5?. As in Figure 2, we set μ=.2, σ = 4; the transportation cost is set sufficiently high (τ = .335) so that the unique equilibrium is initially with equal manufacturing shares, but not high enough to make the curve monotonic.

What we see is that as ϕ_1 rises, we first see the equilibrium share of location 1 in manufacturing rise. At a certain point two more equilibria emerge: a stable equilibrium with all manufacturing in location 1, and an unstable interior equilibrium. This latter equilibrium and the stable interior equilibrium first converge, then vanish.

We can illustrate the equilibria as we

vary ϕ_1 schematically with Figure 4. In that figure,. solid lines represent stable equilibria, broken lines unstable equilibria. In parts of the range there exist both a concentrated and an interior equilibrium; in others only one equilibrium exists.

Clearly, there is a potential for "catastrophes@': situations in which small changes in the exogenous location of agricultural production make an equilibrium untenable and lead to large changes in the location of manufacturing. Such a catastrophe is illustrated in Figure 4. Suppose that the economy is initially in equilibrium at point **1,** with all manufacturing concentrated in region 1. Then suppose the center of gravity of agricultural production gradually shifts to region 2. At first, this will leave the basic locational structure unchanged, as the economy moves from 1 to 2. Any further reallocation of agriculture, however, will lead to a collapse of region **1's** dominance of manufacturing, and the economy will move to point 3.

We can give this catastrophe a name: call it the "California catastrophe". It has been argued by a number of historians that the rapid growth that began in California after about 1900 was a critical mass phenomenon: with the discovery of oil -- an immobile sector that would be part of "agriculture" in this model -- the resource-exporting state finally offered a sufficiently large local market to attract manufacturing, which further enlarged the market, and so on. That is almost exactly what we see happening as we move from 2 to 3.

Next consider what happens as we change transportation costs. Figure 5 shows what happens as we gradually reduce τ starting from an initial position in which $\tau = .335$ and $\phi_1 = .504$, i.e. in which region 1 has the larger agricultural base. At the starting value of τ, there are two stable equilibria, in one of which region 2 has some manufacturing. As transport costs decline, it becomes increasingly attractive to concentrate production near the larger market. Thus this interior equilibrium has a rising share of manufacturing in region 1. Eventually the interior equilibrium disappears. Figure 6 shows schematically how the set of equilibria depends on τ; again, solid lines represent stable equilibria, broken lines unstable equilibria.

Once again we have the possibility of a catastrophe. Suppose that we start at point 1 in Figure 6, and that τ falls gradually.

Initially this will reduce but not eliminate manufacturing in region 2. Once the critical point 2 is passed, however, a cumulative process of decline sets in, even if there is no further fall in τ, until region 2's manufacturing is eliminated.

We can call this the "Mezzogiorno catastrophe". Some economic historians have argued that the coming of the railroads, by exposing the nascent industrial base of Southern Italy to competition from the North, led to that region's industrial collapse. Whether or not the story is true, this model shows that it makes sense.

These exercises do not fully exhaust the possibilities of the two-location case. Rather than explore all of these possibilities, however, we now move beyond the two-region case to study the dynamics of a multi-region example.

6. Dvnamics in a multi-reaion model: the economy as a self-orsanizins svstem

We now turn to a multi-region model. There are at least two reasons for moving beyond the two-region case. First, the tradition of geography strongly emphasizes the need to model equilibrium in space; while one may learn something about this from **two-**location models, eventually one wants to be able to talk about a true spatial structure. Second, and more immediately, we would like to talk about multiple agglomerations and their spatial relationship to each other; this is impossible in a two-location model.

We assume, then that there are **J>2** locations, and we return to the assumption that agricultural workers are equally distributed among the regions, with a share 1/J in each.

In a many-region model it is necessary to specify the matrix of distances between locations. I choose the simplest setup that preserves symmetry: the locations are equally spaced around a circle, with transportation possible only along the circle's circumference. We let the distance between any two neighboring locations **equal** 1. In the numerical examples described shortly, we consider in particular the case of 12 locations, laid out like a clock face. (The number 12 was chosen because it is a fairly small number with a large number of divisors). In this case, the distance between location 2 and location 7 is 5; the distance between location 2 and location 11 is 3.

How can we explore this economy? I have

adopted what we might call a Monte Carlo approach: start the economy with a random allocation of manufacturing workers across locations, and then let it evolve until it converges. We get insight into the model by performing this experiment repeatedly with various parameter values.

Consider first a base case (chosen after some experimenting) in which $\mu = .2$, $\tau = .2$, and $\sigma = 4$. Figure 7 shows what happens on a typical run of this case. The first set of bars show the initial, random allocation of workers across locations. The second set shows the allocation to which the economy converges. All workers end up in two concentrations, at locations 1 and 7 -- opposite one another on the circle.

There are several interesting points to notice here. First, it is evident that there is a process of reinforcement of initial advantage. Thus location 7, which starts with the largest share of workers, is able thereby to attract still more workers and eventually take half of the total. This is exactly the kind of cumulative process described by Pred (1966).

The process is not, however, simply One in which locations with larger initial work forces grow. A second city emerges at location 1. Now while 1 had a large initial labor force, it was actually smaller **than** that of other locations, for example location location 8. But location 8 was too close to the winning location 7, and fell under its "agglomeration **shadow"**, whereas 1 was able to match **7's** eventual status thanks to its relative lack **of rivals** for its agricultural hinterland. This is why the two emergent cities are opposite one another -- and therefore why the eventual pattern is one of two central places symmetrically placed.

Does this case always produce the same result? Not exactly. In many cases the cities were not exactly opposite one another, ending up 5 rather than 6 apart. Also, in a few experiments the economy ended up with three cities, exactly evenly spaced around the circle.

In a controlled set of 20 runs, however, the outcome was always a two-city economy. In 12 of the 20 **runs** the two cities were 5 locations apart, e.g. at 3 and 8; in the other 8 runs the two cities were exactly opposite. Clearly the model economy shows multiple equilibria both in terms of which locations play which role and to some extent even in terms of the equilibrium spatial structure. Nonetheless, it is also clear that there is a systematic tendency toward formation of central places roughly evenly spaced across the landscape.

What happens if we change the parameters? I have tried 10 runs on each of three alternative cases:

(i) *Less differentiated products* (σ = 2, τ = .2, μ = .2): In this case (in which firms have more market power, and in which the equilibrium degree of scale economies is also larger), all runs produced only a single city.

(ii) *A larger manufacturing share* (a = 4, τ = .2, μ = .4): In this case, in which one would expect the backward and forward linkages driving agglomeration to be stronger, we also consistently get only a single city.

(iii) *Lower transport costs* (a = 4, τ = .1, μ = .2): In this case we would expect there to be less incentive to set up multiple urban centers, and again all ten runs produce only a single city.

What do we learn? We have already seen, earlier in the paper, how both the market potential and cumulative process approaches are more or less validated in this model. Now we see not only that the same approach can produce multiple agglomerations, but that something resembling central place theory also emerges, because the dynamic forces do tend to produce agglomerations that are roughly evenly spaced across the landscape.

We may also, on a more mystical note, observe that these examples illustrate, albeit in a very simplistic way, how the economy may be regarded as a self-organizing system of the kind emphasized by Prigogine. We start each run with no structure, only a random allocation of workers across a featureless agricultural surface. Yet the model consistently evolves into a large-scale structure with roughly evenly spaced agglomerations. One need not be pompous about this, but this model does offer a pretty nice example of emergent structure in which the assumptions are not too close to the conclusions.

7. Conclusions

The study of economic geography is important in its own right, and regional economic data shed useful light on important issues in other parts of economics. Beyond this, however, the study of economic geography can be a useful proving ground for some ideas that may be useful in the rest of economics. These include the importance **of** multiple equilibria and nonlinear dynamics, and the usefulness of the computer as a theoretical as well as an empirical tool.

In this paper I have offered some exercises using a simple theoretical model of location. The model is patently unrealistic, yet the results are strongly suggestive. We see how economies of scale at the individual level may aggregate to external economies, how these external economies may lead to multiple equilibria, and how the character of equilibria depend in a systematic and intuitive way on such factors as transportation costs and economies of scale. On a grander note, we see fashionable "nonlinear" ideas like catastrophic dynamics and self-organizing systems emerge as natural consequences of the interaction of rational economic agents, in a full general equilibrium model with all the trimmings. Thus it may be hoped that the study of economic geography will help bring these ideas into wider use.

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APPENDIX: THE "MARKET POTENTIAL" FUNCTION

In this appendix we show how equation **(17)**, which determines wage rates in each location, may be derived.

Let us refer to the c.i.f. price of a good i at location k as $p'_{i,k}$. If the good was produced at location, this c.i.f price is (given our choice of units) equal to the wage rate at j, multiplied by a transportation cost factor that depends on the distance:

$$p'_{i,k} = w_j e^{\tau D_{jk}} \qquad \textbf{(A.1)}$$

We know that residents of k will spend a share μ of their income on manufactured goods, implying

$$\sum_i p'_{i,k} c_{i,k} = \mu Y_k \qquad \textbf{(A.2)}$$

We also know that the relative demand for any two manufactured goods depends only on their relative price. Let $c_{i,k}$ be the consumption of good i at location k. Then the consumption of one good relative to another -- say, good i relative to good 1 -- is

By substituting (A.2) into **(A.3)**, we can

$$\frac{c_{i,k}}{c_{1,k}} = \left(\frac{p'_{1,k}}{p'_{i,k}}\right)^{\sigma} \qquad \text{(A.3)}$$

derive an expression for the **expenditure** of k residents on product 1:

$$c_{1,k}p_{1,k} = \mu Y_k \frac{{p'_{1,k}}^{1-\sigma}}{\sum_i {p'_{i,k}}^{1-\sigma}} \qquad \text{(A.4)}$$

Now consider the total sales of goods manufactured at a particular location, say location 1, at location k. (Because of the "iceberg" assumption on transportation costs, these sales will include the full value of the expenditure of k residents on these goods). There are n_1 such goods, competing with n_j goods from each region j, all selling at the c.i.f. prices described by (A.1). It is thus clear from (A.4) that the total expenditure by k residents on j products is

$$S_{1k} = \mu Y_k \frac{n_1 w_1 e^{\tau D_{1k}}{}^{1-\sigma}}{\sum_j n_j \left[w_j e^{\tau D_{jk}}\right]^{1-\sigma}} \qquad \text{(A.5)}$$

Divide both the numerator and the denominator of (A.5) by the total number of products n, and recall that $n_j/n = \lambda_j$. Then we have

$$S_{1k} = \lambda_1 \mu Y_k \frac{\left[w_1 e^{\tau D_{1k}}\right.{}^{1-\sigma}}{\sum_j \lambda_j \left[w_j e^{\tau D_{jk}}\right]^{1-\sigma}} \qquad (6)$$

Using our definition of the true price index at k, given in text equation (16), this may be rewritten

$$S_{1k} = \lambda_1 \mu Y_k \left[w_1 e^{D_{1k}} \right]^{1-\sigma} T_k^{\sigma-1} \qquad \text{(A.7)}$$

Now consider the short-run market clearing condition for workers at location 1. One way to write this condition is that economy-wide expenditure on these workers' products must equal their income. Recall that we have normalized supplies of labor so that there are μ workers in the economy as a whole. The income of workers at location 1 is therefore $\lambda_1 \mu w_1$. Setting this **equal** to the sum of their sales at all locations k, we have

$$\lambda_1 w_1 \mu = \lambda_1 \mu w_1^{1-\sigma} \sum_k Y_k \left[e^{-\tau D_{1k}} T_k \right]^{\sigma-1} \qquad \text{(A.8)}$$

or, finally,

$$w_1 = \left[\sum_k Y_k \left(e^{-\tau D_{1k}} T_k \right)^{\sigma-1} \right]^{\frac{1}{\sigma}} \qquad \text{(A.9)}$$

The same relationship holds for each location, giving us the expression (17) in the text.

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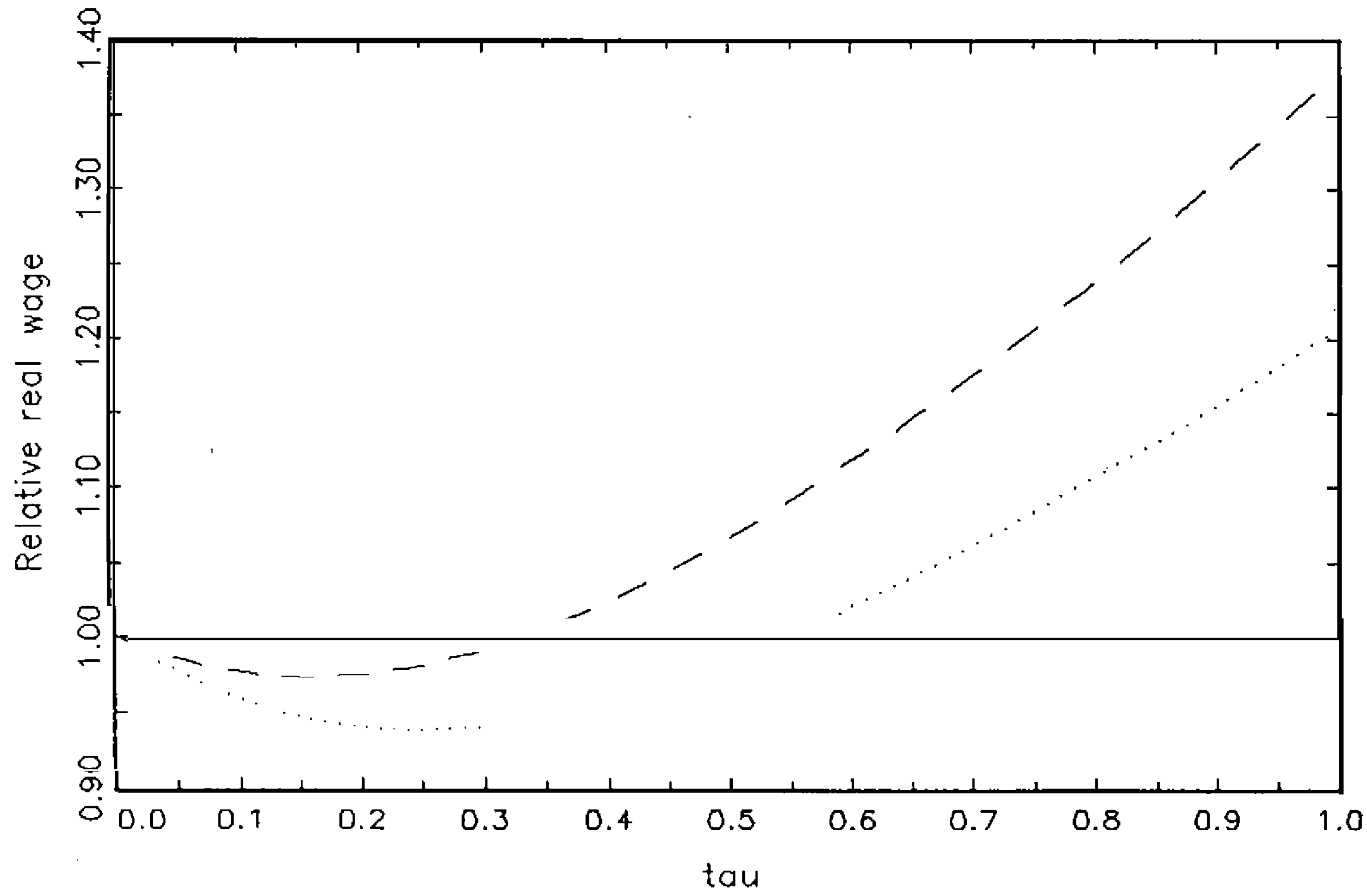
Figure 1



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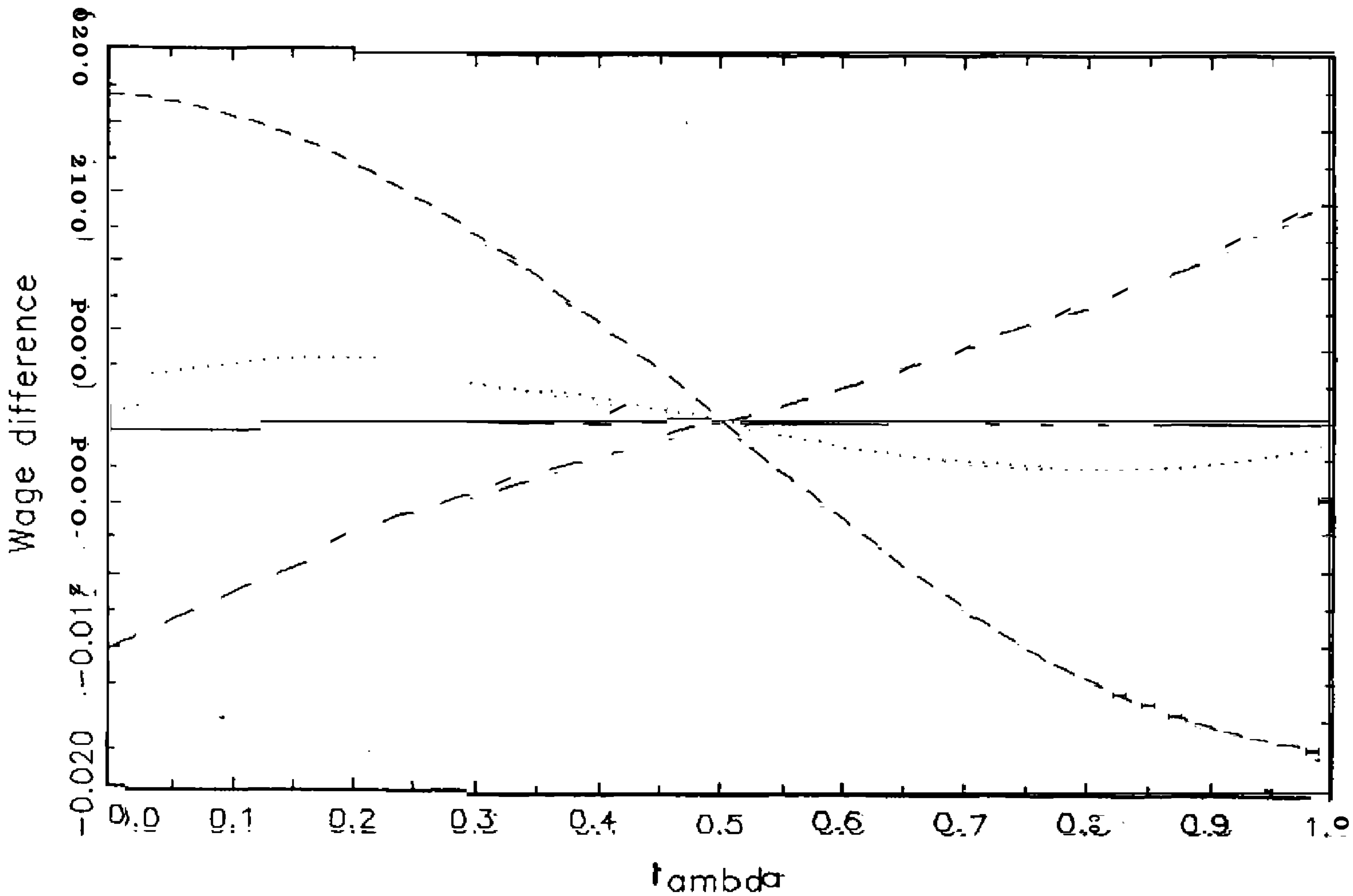

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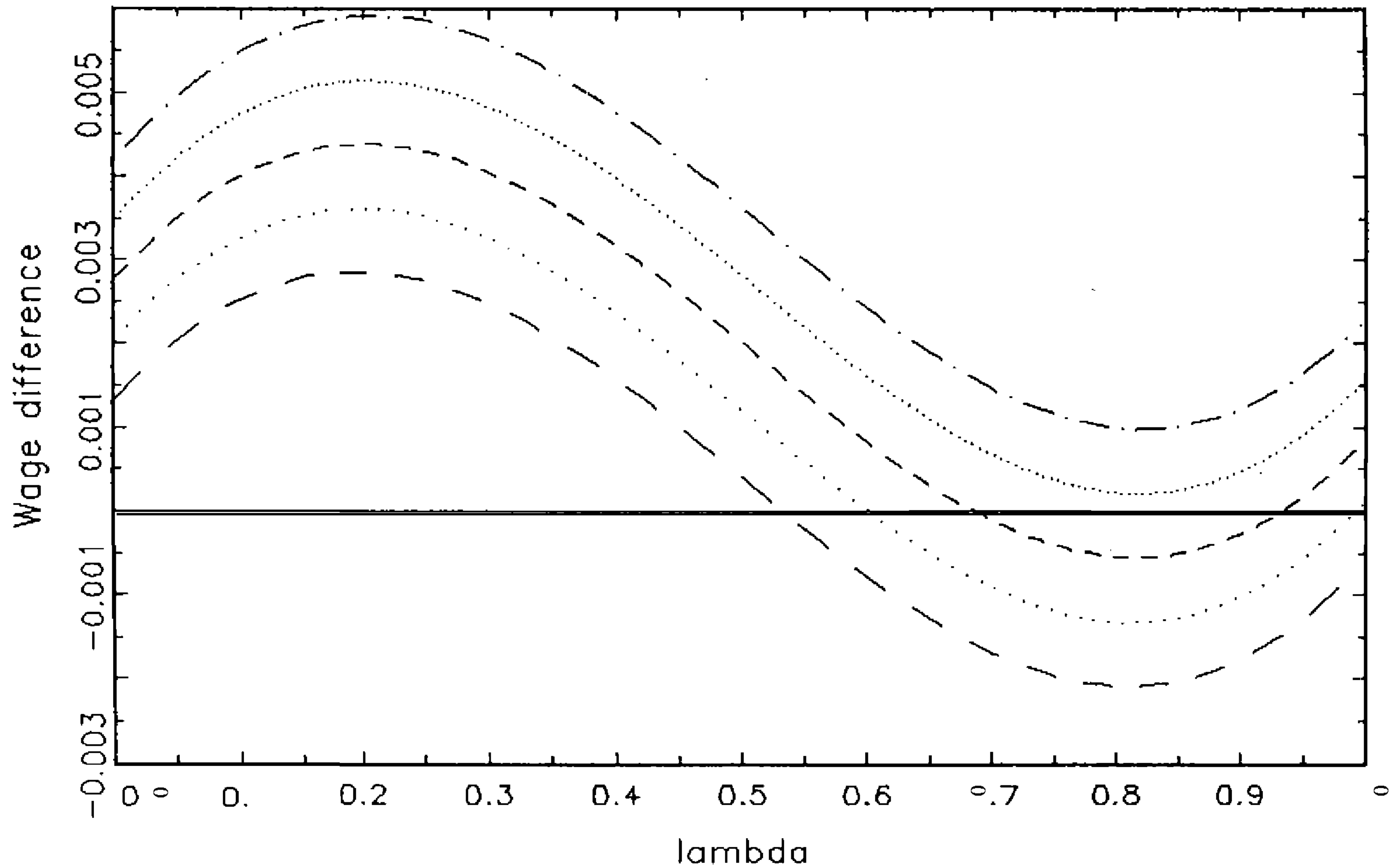
Figure 3



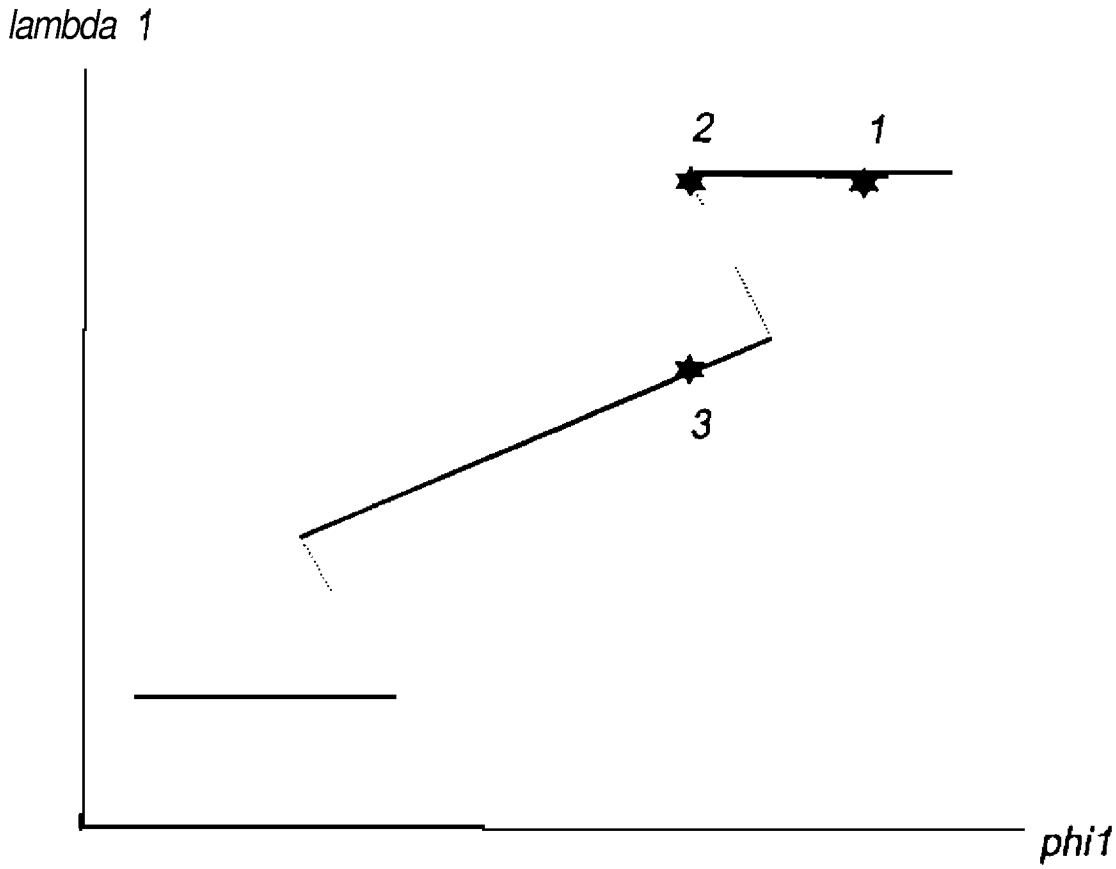


Figure 4

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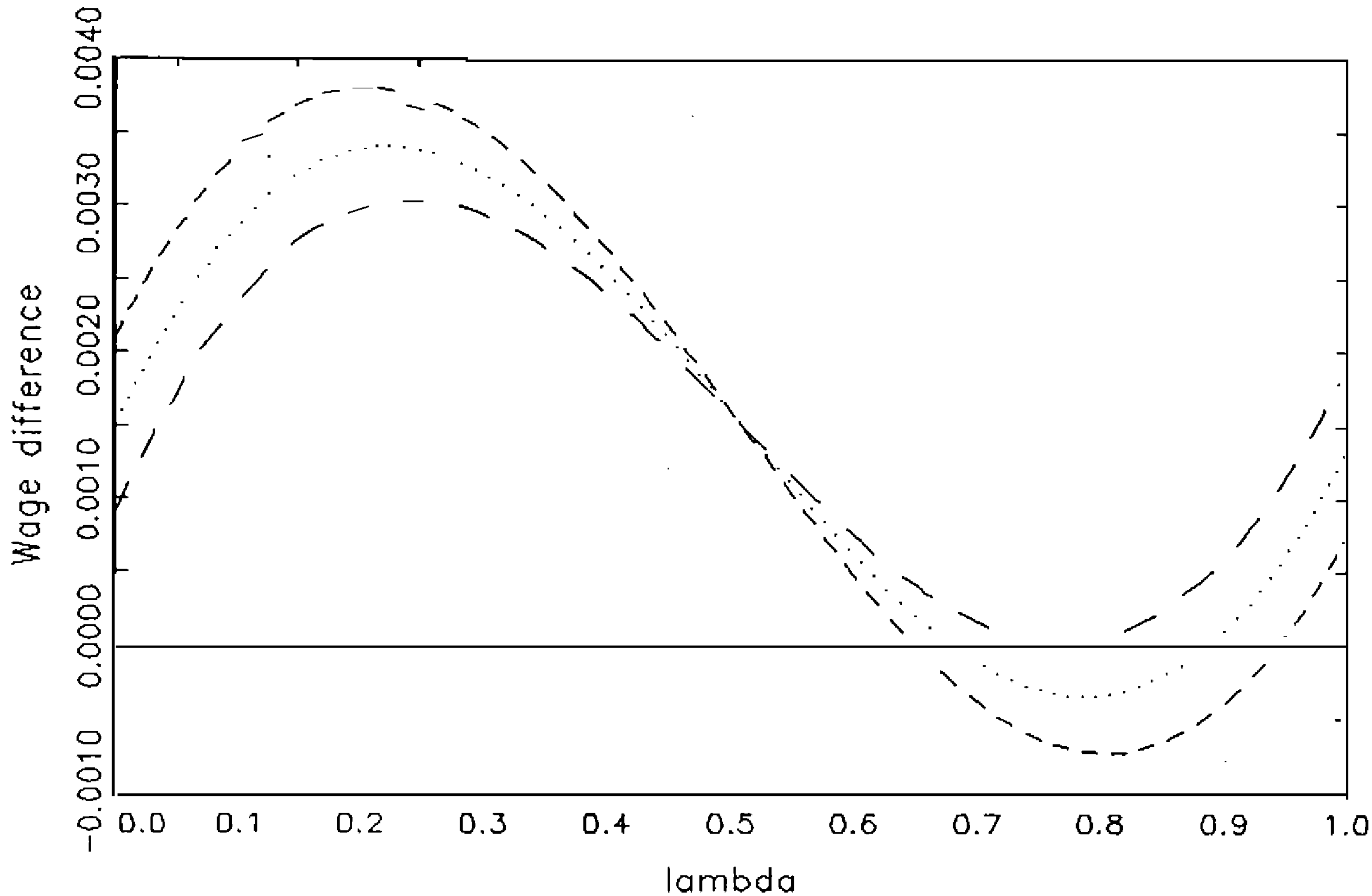
Figure 5



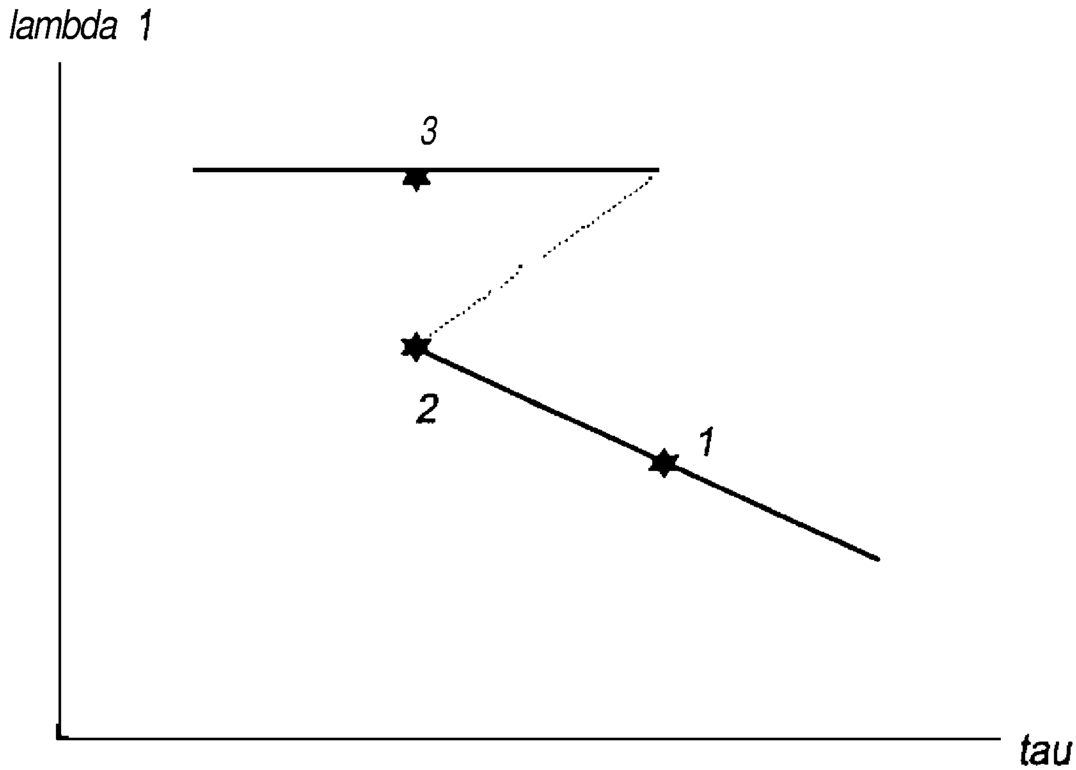


Figure 6

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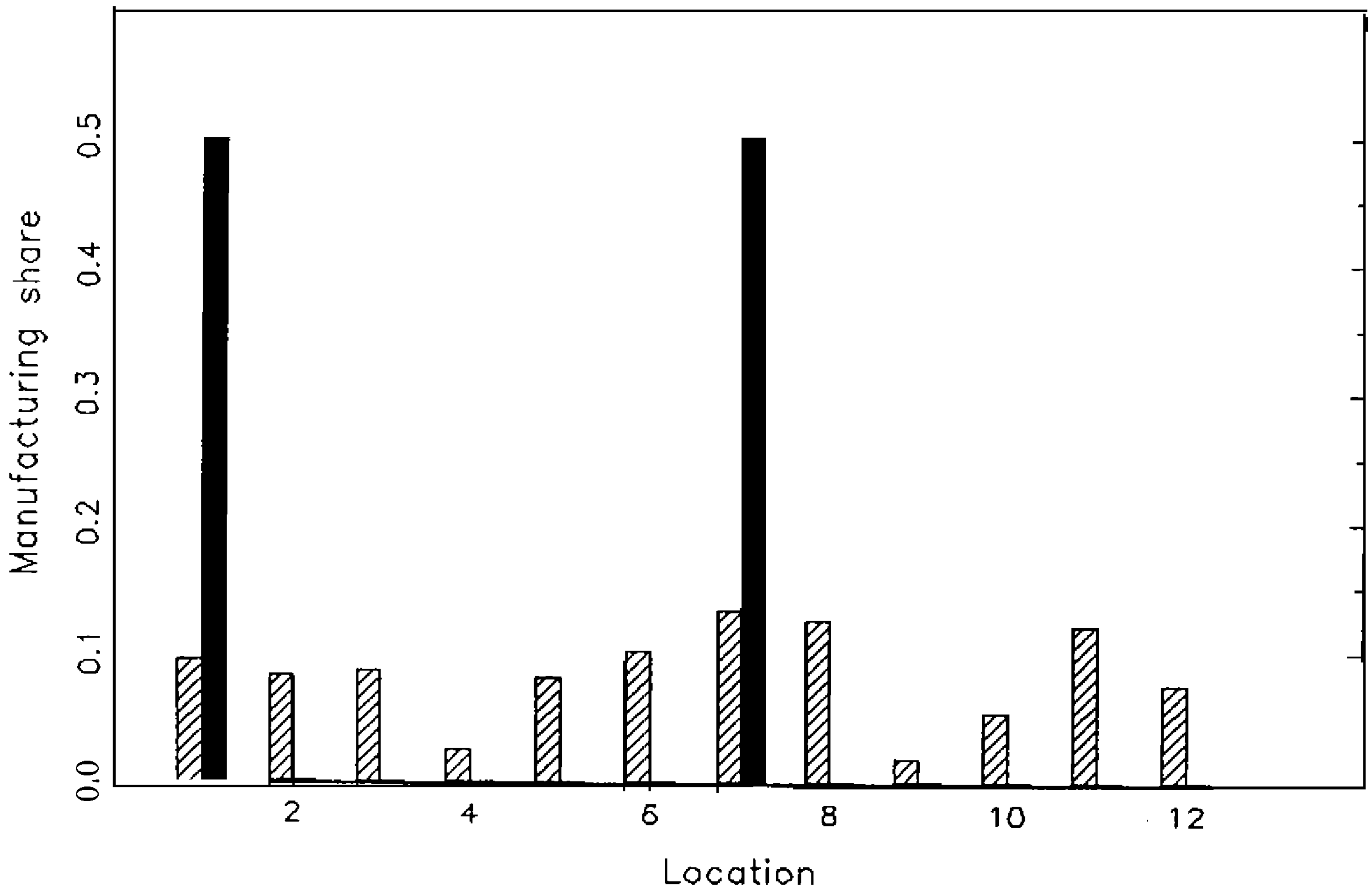
Figure 7



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