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## ABSTRACT

Under conventional notions about rational expectations and market efficiency, expected returns differ from the actual *ex post* returns by a forecast error that is uncorrelated with current information. In this paper, we describe how small departures from conventional notions of rational expectations and market efficiency can produce trends in excess returns. These trends are in addition to the trends typically found in the level of asset prices themselves. We report strong evidence for the presence of additional trends in excess foreign exchange and bond returns. We also estimate the additional trend component in excess returns on foreign exchange and find that it varied between -.8% and 1% for one month returns and between -6% and 8% for three month returns.

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Interpreting the behavior of expected asset price changes from observed *ex post* returns requires taking a stand on how market participants form expectations. It is now common in both the macroeconomic and finance literatures to make a joint assumption about expectations and market efficiency that ensures expected returns differ from the actual *ex post* returns by a forecast error that is uncorrelated with current information. This paper investigates whether this joint assumption holds true. First, we describe how small departures from conventional notions of rational expectations and market efficiency can produce additional trends in asset returns.[1] While empirical research has found that the levels of asset prices follow trends, our analysis shows that small departures from the standard joint assumption can produce additional trends in excess returns beyond those in asset prices themselves. We then show that there is strong empirical evidence for the presence of additional trends in foreign exchange and bond returns.

Conventional notions of both rational expectations and market efficiency have been questioned by recent research. Papers by Rogoff (1980), Lewis (1991), and Evans and Lewis (1991, 1992) show that even when market traders are rational, their forecast errors can be correlated with current information if they anticipate a discrete jump in the distribution of asset prices. In this case, forecasts of the future asset price will rationally include the expectation of a discrete jump in the process. This expectation induces the so-called "peso problem" where forecast errors can be serially correlated over small samples when the jump does not occur. Recent empirical studies have found evidence of serial correlation and small sample biases in foreign exchange and bond returns.[2] This evidence suggests that *ex post* returns can provide a biased measure of *ex ante* returns.

DeLong, Schleifer, Summers, and Waldman (1990) have questioned conventional notions of market efficiency by showing how trend-chasing feedback traders can affect equilibrium prices. While there is no single measure of expected returns if markets are comprised of heterogeneous traders, the realized return will systematically deviate from the return based upon the true value of the asset price (i.e. the price determined by fundamentals) during periods when feedback traders are active in the market.

In this paper, we show how peso problem effects arising from expected shifts in asset returns can have some of the same empirical implications as the effects of heterogeneous traders. Specifically, we show that if either type of effect is present and asset prices contain trends, then forecast errors will not only be serially correlated, but they will appear to follow trends. Although this trending behavior is a much stronger implication than the standard "peso problem" predicts, we emphasize that it will only arise for finite periods. Our analysis does *not* imply that forecast errors follow trends over indefinitely long periods of time.

We test for the presence of a small trend in *ex post* returns on foreign exchange and Euro-currencies from the beginning of 1975 to 1990. Interestingly, we find evidence to suggest that both the foreign exchange and bond markets have undergone periods when forecast errors and, hence, *ex post* returns have contained trends. When we estimate the trend component in excess foreign exchange returns we find that it has varied in annualized rates between -0.8% and 1% for one month returns and between -6% and 8% for three month returns.

At the outset, we should make clear how we view these results in relation to existing empirical studies of excess returns. In particular, our findings do *not* suggest that excess returns should be treated as non-stationary variables, a suggestion at odds with the existing literature. Even though excess returns contain a non-stationary trend component, these series are very close to stationary variables in the sense that their trend components are small. This explains why standard tests have failed to find evidence of trends in excess returns before. Moreover, Monte Carlo studies of time series that are close to being stationary suggest that they are more appropriately treated as stationary for econometric purposes.[3] From this perspective, therefore, our results are in agreement with the existing treatment of excess returns in the literature.

From another perspective, however, our results suggest that the standard econometric treatment of excess returns may be inappropriate. Empirical studies of risk premia in excess returns typically assume that forecast errors are uncorrelated with current information. By contrast, our results suggest that forecast errors have at times been highly serially correlated, a result consistent with findings based upon survey data. Furthermore, these results do not necessarily contradict rational expectations, as we will describe in more detail below.

The plan of the paper is as follows. Section 1 describes the relationship between heterogeneous trading, peso problems, and empirical trends. Section 2 discusses the empirical evidence and some Monte Carlo experiments to examine the robustness of the results. Section 3 provides estimates of the additional trends in excess returns. Concluding remarks follow.

### 1. Peso Problems, Heterogeneous Trading, and Trends

To motivate our theoretical discussion below, consider the behavior of foreign exchange returns. Figures 1 and 2 show respectively the three month forward prediction error and the forward premium for the logarithm of the dollar exchange rate against the German mark, the British pound, and the Japanese yen between 1975 and 1989. Figure 2 shows that the DM was at a premium against the dollar for the period, even though the dollar rose substantially during the early 1980s, and then fell dramatically in 1985. Figure 2 also shows a great deal of persistence in the forward premium of the pound and the yen against the dollar.

To interpret these variations, it is useful to write the forward premium as the sum of the risk premium and the expected change in the exchange rate:

$$F_t - S_t = E_tS_{t+1} - S_t + rp_t \tag{1}$$

where $F_t$ is the logarithm of the forward rate at time $t$ for delivery at time $t+1$, $S_t$ is the logarithm of the spot rate at time $t$, $E_t$ is the expectation conditional upon information available at time $t$, and $rp_t$ is the risk premium. The forward premium in Figure 1 suggests that either the risk premium or the expected change in the spot rates is highly persistent. The relationship in (1) applies to any spot rate process and we will use this relationship below to describe spot interest rates as well. In the present context, $S_t$ is the spot exchange rate.

The forward prediction error or, equivalently, the *ex post* excess return on buying foreign currency forward is:

$$S_{t+1} - F_t = S_{t+1} - E_{t+1}S_{t+1} - (F_t - E_tS_{t+1}) = S_{t+1} - E_{t+1}S_{t+1} - rp_t. \tag{2}$$

Under standard rational expectations assumptions, the market's forecast error, $S_{t+1} - E_tS_{t+1}$, is white noise and uncorrelated with all information available at time *t*. In this case, excess returns will differ from the risk premium by a white noise forecast error. Thus, movements in excess returns that are predictable from time *t* information only arise from variations in the risk premia.

Recent empirical studies indicate that the forecast errors in foreign exchange and bond returns are significantly autocorrelated. In this case, the excess returns in Figure 1 represent a combination of risk premia and forecast errors that are jointly correlated, precluding a simple decomposition. Similarly, the forward premia depicted in Figure 2 suggest that shocks to deviations between spot and forward rates are persistent. From equation (1), this evidence suggests that either risk premia or expected changes in spot rates or both are highly persistent.

In the remainder of this section, we will discuss how serially correlated forecast errors can arise from expected shifts in the returns process or from the presence of heterogeneous traders. We will show that when either effect is present and asset prices have a trend component, then forecast errors and forward premia will not only be serially correlated but will also contain a trend component. Below we begin by discussing the implications of expected shifts in the returns process before examining the implications of heterogeneous trading.

### *1.1 Peso Problems and Trends*

Consider first the case where only rational traders are in the market. In addition to the current process determining spot prices, traders also believe spot prices may follow an alternative process. Defining spot rates generated by the current process as $(S_t \mid C)$ and the alternative process as $(S_t \mid A)$, the expected future price can be written:

$$(3) \qquad E_tS_{t+1} = (1 - \lambda_t)\, E_t(S_{t+1} \mid C) + \lambda_t\, E_t(S_{t+1} \mid A),$$

where $E_t$ is the expectations operator conditional upon information available at time *t*, and $\lambda_t$ is the probability of a switch to the alternative process.

As an example, suppose that the spot price is the exchange rate. When Engel and Hamilton (1990) estimated a Markov switching model for the dollar exchange rate they found that the process could be described by two "regimes": a dollar appreciating regime and a dollar depreciating regime. The dollar appreciating regime corresponds roughly with the period, 1979 to 1985, while the dollar depreciating regime corresponds to 1975 to 1979 and 1985 to 1988. If these regimes exist, rational traders in one regime would anticipate a switch to the other regime. In terms of equation (3), suppose that the current regime, C, is a dollar appreciating regime. However, rational traders anticipate a switch to a dollar depreciating regime, the alternative regime "A". In this case, $\lambda$ denotes the transition probability of switching from the appreciating to the depreciating regime.

Expectations in the form of equation (3) can in general induce forecasts errors that are serially correlated in small samples, as is well-known from the "peso problem" literature. However, when the spot rate follows a process with a random walk component, a "peso problem" can generate stronger implications for the behavior of forecasts. Before describing these implications, we briefly summarize the relationship between random walk disturbances and stochastic trends.

An important finding of recent empirical research on asset prices and interest rates is that they have random walk components and hence contain unit roots.[4] Since shocks to the random walk component permanently affect the level of asset prices, the sum of these shocks cumulate into so-called "stochastic trend" movements in prices. Variables with these random walk components are called "I(1)" processes in the literature, and we will follow this terminology below. On the other hand, some variables are only affected by transitory shocks. These processes are covariance stationary and are called "I(0)" processes. A feature of I(1) processes is that their first-differences are I(0) stationary variables, affected only by transitory shocks.

With these definitions, we can rewrite equation (3) as,

$$(4) \qquad E_t S_{t+1} = (1 - \lambda_t)(S_t \mid C) + \lambda_t(S_t \mid A) + (1 - \lambda_t) E_t(\Delta S_{t+1} \mid C) + \lambda_t E_t(\Delta S_{t+1} \mid A),$$

$$= (1 - \lambda_t)(S_t \mid C) + \lambda_t (S_t \mid A) + I(0) \text{ terms.}$$

The second line follows from the first because the expected change in the spot rate in either regime must be I(0) since the actual changes are I(0).

Equation (4) shows the expected future spot rate decomposed into terms that contain some permanent shocks and terms that do not, (i.e. the I(0) terms). An important implication of (4) is that both the expected change in the spot rate and the forecast errors will have permanent shock components that generate trends if the current and the alternative regime for the spot rates contain distinct permanent shocks. This can be seen by rewriting the expected change in the spot price when the current regime is generating the prices:

$$(5) \qquad E_tS_{t+1} - (S_t \mid C) = \lambda_t \{(S_t \mid A) - (S_t \mid C)\} + I(0) \text{ terms.}$$

Similarly, we can write the forecast errors as:

$$(6) \qquad (S_{t+1} \mid C) - E_tS_{t+1} = -\lambda_t \{(S_t \mid A) - (S_t \mid C)\} + I(0) \text{ terms.}$$

Both expected changes in equation (5) and forecast errors in equation (6) contain a component that depends upon the difference between the two permanent shocks in the two spot rate processes. This difference will itself be subject to permanent disturbances if the permanent shocks to the individual spot rates are distinct. For example, if the current regime is a dollar appreciating regime but the market anticipates a possible switch to a dollar depreciating regime, then the difference between the two possible processes will grow over time. Therefore, both the expected change in the spot rate and the forecast errors will follow a trend during periods when the shift to the alternative process is not realized.

It is also clear from (5) and (6) that the trend components are the same in both equations. Intuitively, the current spot rate $(S_t \mid C)$ and the future realized rate conditional upon the current process, $(S_{t+1} \mid C)$, differ by a stationary process.[5] Therefore, examining the number of trends in the expected change in the spot rate is equivalent to examining the number of trends in the forecast errors. Since the expected change tends to be less noisy than the forecast errors, we will focus upon the former variable in the analysis below.

So far our discussion has focused on the case where no switch in the process takes place. However, in any given sample, there may be various switches. During some periods, the alternative process may generate spot rates, inducing forecast errors of the form,

$$(6') \qquad (S_{t+1} \mid A) - E_tS_{t+1} = -\lambda'_t \{(S_t \mid C) - (S_t \mid A)\} + I(0) \text{ terms},$$

where $\lambda'$ is the probability that the process will shift from process A to C. As long as $(S_t \mid A)$ and $(S_t \mid C)$ are subject to different permanent shocks, the forecast errors will also appear to follow a trend before the shift in regime occurs.

To illustrate how the probability of a switch in the process can induce trending behavior in excess returns, we constructed some simple numerical examples based upon different probabilities of shifts to alternative regimes, $\lambda$. Estimates of the appreciating dollar state against the pound (state 1) and the depreciating dollar state against the pound (state 2) were obtained from the estimates of the Markov-switching model of Engel and Hamilton (1990). For simplicity, the spot exchange rate was assumed to follow a random walk with different variances in each state. Thus, the exchange rates conditional upon each state were generated by the following processes:

$$(S_t \mid i) = (S_{t-1} \mid i) + \eta_{i,t} = \sum_{r=1}^{t} \eta_{i,r} \qquad i = 1, 2,$$

where $Var(\eta_{1,t}) = 16.92$ and $Var(\eta_{2,t}) = 20.25$. Based upon these generated series, the expected future exchange rate including the anticipated switch in the process was constructed as in equation (4),

$$E_tS_{t+1} = (1 - \lambda_t)(S_t \mid C) + \lambda_t(S_t \mid A) = (1 - \lambda_t) \sum_{r=1}^{t} \eta_{C,r} + \lambda_t \sum_{r=1}^{t} \eta_{A,r}$$

where the current process, C, was alternatively state 1 or state 2. Finally, using the generated series and the expected future spot rate, the forecast errors, $(S_{t+1} \mid C) - E_tS_{t+1}$, were constructed. These forecast errors were constructed assuming that, first, the current process was state 1 and, second, the current process was state 2.

Table 1 reports summary statistics from 200 replications of these series. The table reports the mean and the first, sixth, and twelfth autocorrelation of the forecast error. The parentheses report the standard deviation of the distribution of these statistics. Panel A reports the results conditional upon the current process being state 1 while Panel B conditions the process upon state 2. The first column gives the assumed probabilities of transition from the current state to the alternative process, $\lambda$.

Two main conclusions arise from the results in the table. First, there is no tendency for the forecast errors to be biased in one direction. In all cases, the mean of the forecast errors are insignificantly different than zero. Second, although the forecast errors contain a random walk component by construction, this component is only apparent when the probability of a switch in regime is quite high. When the transition probability is 50%, the first-order autocorrelation coefficient is close to 0.9. However, for more realistic probabilities less than 10%, there is little evidence of serial correlation at all. This last result suggests that powerful methods must be employed when testing for the presence of these possible trends, as we will in the following section.

Finally, it is important to recognize that "peso problems" may arise without inducing this trend behavior in excess returns or expected returns. For example, in equations (5) and (6), the processes conditioned upon each regime, $(S_t \mid A)$ and $(S_t \mid C)$, may be subject to the same permanent shocks and so share the same random walk component. Such a case would arise if the two regimes differed only by transitory disturbances. Thus, the implication that excess returns contain a random walk component is a much stronger implication of "peso problems" than previously examined in the literature.

*1.2 Heterogeneous Trading with Feedback Traders and Fundamentalists*

Above, we showed how anticipations of a shift in the process that generates asset prices can induce a random walk component in forecast errors as well as in excess returns. We motivated our discussion by assuming that all traders were rational and shared the expectation that the process generating the spot rates would switch. However, the phenomena we described above arises in more general circumstances. In the most general terms, "peso problem" phenomena will occur when *some* agents whose expectations are reflected in the forward rate expect the price process to shift. A homogeneous belief by market participants of a switch in the process of fundamental variables is just one example of this phenomenon.

Recent research examining the effects of heterogeneous traders provides another example where additional trends may appear in spot prices. According to this argument, some traders are rational and informed while others chase trend movements in asset prices. Even though rational traders are in the market, risk aversion limits the trades that they are willing to take against other less-informed traders. As a result, the price may trend away from its fundamental value for periods of time.[6]

Some anecdotal evidence serves to illustrate the intuition behind these studies. First, DeLong, Schleifer, Summers, and Waldmann (1990) describe examples based upon reports by several successful traders in the U.S. stock market.[7] According to these sources, informed traders bet against uninformed traders who become excited about trend movements in stock prices. Examples include the rise in stock prices of conglomerates during the 1960's and the boom in Real Estate Investment Trusts during the 1970's. The profit-maximizing trading strategy by informed traders who knew that these prices were overvalued was to buy with the rest of the market and then sell out before the uninformed traders discovered that the stock was overvalued. Therefore, DeLong *et al* argue that this "trend-chasing" behavior is consistent with the presence of rational traders.

Frankel and Froot (1988) provide supporting evidence that informed traders may chase trends even though they believe that prices are inconsistent with fundamentals. They examine survey data by exchange rate forecasting firms during the persistent rise in the dollar during the early 1980's. These forecasters were recommending clients to buy the dollar, even though their longer-term forecasts maintained that the dollar was overvalued. This evidence suggests that even informed traders were chasing the upward trend in the dollar.

These anecdotes suggest an interaction between informed rational traders, uninformed "feedback traders" who trade based upon the past trend, and passive investors. DeLong, *et al* theoretically analyze the equilibrium price determined by these traders assuming that at a given future horizon, the price is given by its fundamental level. They find that the price systematically trends away from its true fundamental level before the reversion of the price to its equilibrium value.[8]

Consider now how this scenario would generate additional trends in expected spot price changes and forecast errors by rational traders. Suppose that rational informed traders know that the price will eventually revert to its level implied by the fundamentals process. We will define this spot price process as $(S_t \mid F)$. Alternatively, the price may continue its current trend following a process defined as $(S_t \mid T)$. In this case, the rational traders' forecast of the future price is given by:

$$(7) \qquad E_tS_{t+1} \;=\; (1 - \lambda_t)\, E_t(S_{t+1} \mid T) + \lambda_t\, E_t(S_{t+1} \mid F),$$

where $\lambda_t$ is the rational traders' assessed probability that the price will revert to its fundamental level between $t$ and $t+1$. Thus, rational traders incorporate both the anticipation of the trend continuing with expected price, $E_t(S_{t+1} \mid T)$, or reverting, implying expected price, $E_t(S_{t+1} \mid F)$.

We may relate this expectation to the empirically observed trend by noting, as before, that financial asset prices appear to be subject to permanent shocks. Therefore, we may write,

$$(8) \qquad E_tS_{t+1} \;=\; (1 - \lambda_t)\,(S_t \mid T) + \lambda_t\,(S_t \mid F) + (1 - \lambda_t)\, E_t(\Delta S_{t+1} \mid T) + \lambda_t\, E_t(\Delta S_{t+1} \mid F),$$

$$= (1 - \lambda_t)\,(S_t \mid T) + \lambda_t\,(S_t \mid F) + I(0) \text{ terms.}$$

If the price during trend chasing, $(S \mid T)$, is subject to different permanent shocks than the price following fundamentals, $(S \mid F)$, the expected future spot price is a probability-weighted average of the two prices and will appear to trend apart from the actual spot price during the trend-chasing period.

Comparing the rationally expected future price in equation (8) with the general "peso problem" expectation in equation (4) reveals that they are observationally equivalent. If we call the process for the price during trend chasing period the "current" process and the price implied by fundamentals as the "alternative" process, then equation (8) simply says that informed traders believe a switch to an alternative process is possible. In this case, during periods when prices do not revert to their fundamental value, rational traders will underestimate the future price since they condition their expectations on this possibility. Thus, the possibility that prices may revert to their fundamental value induces a "peso problem" in rational traders' forecasts.

The difference between the situation described here and the standard "peso problem" is interpretation. In the standard "peso problem" prices are determined by expected future fundamentals. Rational agents anticipate a switch in the process of fundamentals and, hence, in the process of asset prices. By contrast, in the heterogeneous trading situation the switch need not arise from a change in the fundamentals process. Here rational traders believe that either the trend generated by "feedback" trading will continue or else a switch towards the fundamentally determined value of the asset price will occur. Unlike the standard "peso problem", the anticipation of this switch may simply arise from trading behavior by uninformed traders. Nevertheless, both situations lead to an additional trend in the expectations by rational traders.

## 2. Do Expected Future Prices Periodically Trend Apart from Actual Prices?

We have shown that expectations of a switch in the process followed by a spot rate can induce a trend in both the expected change and the *ex post* forecast error of the spot rate. This implication is clearly stronger than the standard "peso problem" result that forecast errors will be biased when observed *ex post* during periods when the switch does not materialize. We described how this behavior could arise generally whenever some traders in the market rationally anticipate a switch in the spot rate process.

In this section, we begin by describing how this relatively strong implication of anticipated shifts in the trend of asset prices may be tested empirically. We then present evidence supporting the presence of trends in the forecast errors of returns.

### *2.1 Trends in Forward Rates*

To evaluate whether expected future prices periodically trend away from actual prices, we must relate the expected future price to an observable variable. From equation (1), the expected future spot rate differs from the forward rate by a risk premium, i.e., $F_t = E_tS_{t+1} + rp_t$. The risk premium is typically treated as stationary I(0) process in both empirical and theoretical studies.[9] If the risk premium is stationary, the forward rate will differ from the expected future spot rate by a stationary I(0) variable. In this case, the relationship between the forward rate and the actual price allows identification of the trend relationships discussed above. Substituting equation (2) into equation (5) yields,

$$(9) \qquad F_t = E_tS_{t+1} + I(0) \text{ terms} = (1 - \lambda_t)(S_t \mid C) + \lambda_t (S_t \mid A) + I(0) \text{ terms}.$$

Thus, whenever some traders rationally anticipate a switch in the spot rate process, the forward rate may contain two trends: one trend arises from the current process, the other trend comes from the alternative process.

To evaluate the trend component in the two processes more carefully, we will decompose the spot price into its transitory and random walk components. It is well-known that any ARIMA process can be written as a random walk plus a covariance stationary transitory shock component. We can therefore write the spot prices conditional upon the two processes $i = C,A$ above as:

$$(10) \qquad (S_t \mid i) = u_{i,t} + e_{i,t}, \qquad \text{with } u_{i,t} = u_{i,t-1} + \eta_{i,t},$$

where $\eta_{i,t}$ is independent and identically distributed, and $e_{i,t}$ is a stationary I(0) process. Thus, in terms of our previous discussion, $\eta_{i,t}$, is the permanent shock to spot prices under regime $i$ = A or C. For expositional simplicity, we will assume that $\eta_{i,t}$, is uncorrelated with the transitory disturbances to $e_{i,t}$, although none of the empirical results depend upon this assumption.

The representation in (10) shows that the price can be written as the cumulated effects of permanent shocks, arising from the $u_{i,t}$ component, as well as mean-reverting stationary effects arising from the $e_{i,t}$ component. To write the price in terms of the past history of shocks, define an initial point in time, say $t = 0$, and set $\eta_{i,0} = 0$. Then the spot price can be written as:

(11) $(S_t \mid i) = \sum_{\tau=1}^{t} \eta_{i,\tau} + I(0)$ terms.

(11) shows that the price generated by process $i$ is driven by the cumulated effects of the permanent shocks, $\eta_{i,t}$. Above, we referred to this accumulation of permanent shocks as a trend. The spot rate process is also affected by the composite effects of the stationary I(0) components. Substituting (11) for each process $i$ into the forward rate expression in (9) yields,

(12) $F_t = E_t S_{t+1} + I(0) \text{ terms} = (1 - \lambda_t) \sum_{\tau=1}^{t} \eta_{C,\tau} + \lambda_t \sum_{\tau=1}^{t} \eta_{A,\tau} + I(0)$ terms.

Clearly, if $\eta_{C,t} \neq \eta_{A,t}$, the trend components of the current process will deviate from the trend components of the alternative process.

### 2.2 *Testing for Additional Trends*

Equation (12) demonstrates that when anticipated shifts in the trend of asset prices are incorporated into forward rates, the forward rate will contain two trends: one arising from the current trend in the spot process, the other arising from the trend in the alternative process. One test of this hypothesis would be to test whether forward rates and current spot rates are "cointegrated." Recent empirical studies have examined this relationship for a number of different financial assets and found that forward rates and spot rates appear to be cointegrated.

There are, however, at least three reasons to think that the tests for cointegration in the literature are inappropriate for our current hypothesis. First, as equation (12) shows, the importance of the trend in the alternative process depends upon the probability, $\lambda$, of a shift in the price process over the next period. If this alternative process reflects a change in policy regime, $\lambda$ is likely be low for much of a given sample. We might also expect $\lambda$ to be low if the alternative process incorporates rational traders' anticipations of a collapse of the price process back to its fundamental level. In either case, it will be very hard to the trend in excess returns, as our results in Table 1 showed when $\lambda$ was small.

Second, if the trend in the current process is close to the trend in the alternative process, the difference between the two trends may be difficult to detect. In other words, if $\eta_{C,t} \approx \eta_{A,t}$, the two trend components may be very similar.

Third, since the random walk component will arise only during periods before anticipated shifts in the spot rate process, the alternative trend movements may be difficult to find using the entire sample. There may be several shifts in the process in any given sample, leading to a shift in the trend behavior of the excess returns.

For all three reasons, our framework suggests that any additional trend in forward rates not found in spot rates is probably small empirically. On this issue, Campbell and Perron (1991) have recently shown that processes with empirically small trends appear very much like processes without trends. If the additional trends are small, it is unlikely that standard cointegration tests would find them.

We require a test capable of detecting trends that may be empirically small. To do so, we used Johansen's (1988) methodology to search for the presence of additional trends in a set of spot and forward rates for related assets. This approach allows us to exploit the fact that "peso problem" effects are likely to be correlated across markets for related assets. By combining cross-market information, the statistical efficiency of our tests is greater than if we examined individual pairs of spot and forward rates. As a result, we are more likely to be able to detect the presence of small additional trends if, in fact, they are present. To check the robustness of our results, we also conducted a number of Monte Carlo experiments. We will first describe the results of the Johansen procedure and postpone the discussion of the Monte Carlo experiments until later.

### *2.3 Data Description*

We examined forward and spot rates on foreign exchange and bond returns for the US, UK, Germany and Japan. Spot exchange rates, along with one month and three month forward rates, were sampled at the end of the month from Citicorp Database Services for the period 1975 to 1989.[10] The exchange rates examined were the U.S. dollar against the German mark, the British pound, and the Japanese yen.

We also used interest rates on Eurocurrency deposits. These series were obtained from Harris

Bank. We used one and three month spot rates on deposits, the forward rate on a one month deposit for delivery at one month in the future, and the forward rate on a three month deposit for delivery at three months in the future.[11] To correspond to the currencies above, we chose interest rates denominated in Germany marks, British pounds, Japanese yen, and U.S dollars.

*2.4 Do Foreign Exchange Excess Returns Contain Additional Trends?*

We begin by considering the null hypothesis that forward rates contain only the same trend component as spot rates arising from $\eta_C$. In other words, we test whether the other potential trend components arising from $\eta_A$ in equation (12) are identically equal to zero.

To see the relationship between the spot and forward exchange rate, define the logarithm of the exchange rate of currency *j* against the dollar as $x^j$, and, for expositional simplicity, assume that exchange rate realizations are drawn from the current process alone.[12] Thus, defining $\eta_C^{x^j}$ as the permanent shocks to the current exchange rate process, we have:

$$(13) \qquad x_t^j = \Sigma_{\tau=1}^{t} \eta_{C,\tau}^{x^j} + I(0) \text{ terms},$$

where $x^j$ is the foreign currency price against the dollar for $j$ = £, DM, and ¥. Denoting the forward rate for future delivery of exchange rate $x^j$ as $f^{x^j}$, the exchange rate version of equation (12) is:

$$(14) \qquad f_t^{x^j} = b \sum_{\tau=1}^{t} \eta_{C,\tau}^{x^j} + d \sum_{\tau=1}^{t} \eta_{A,\tau}^{x^j} + I(0) \text{ terms},$$

where $b = (1 - \lambda_t)$, $d = \lambda_t$, and $\eta_{A,\tau}^{x^j}$ are the permanent shocks to the alternative exchange rate *j* process.

Under the hypothesis that there are no "peso problems" induced by anticipated policy changes or the expected collapse of trends arising from feedback traders, $\eta_{A,\tau}^{x^j} = 0$ for all $\tau$. In these circumstances (13) and (14) show that forward rates trend only with spot exchange rates. Alternatively, if some traders condition their expectations upon an alternative process, forward rates may contain additional trend components as described above in (12).

To consider the null hypothesis of $\eta_{A,\tau}^{j} = 0$, we proceeded in two steps. First we tested for the number of trends in a vector of spot rates and a vector of forward rates individually. Then, we tested for the number of trends when the vectors of spot and forward rates were combined. If there are no additional trends in forward rates, the number of trends should not increase when we add forward rates to the system of spot rates. Since we suspected that any additional tends would be small, we included the pound/dollar, yen/dollar, and mark/dollar rates in the vectors of spot and forward rates to exploit any cross-currency information. In this way, if expectations incorporate an anticipated shift in the future value of the dollar, say, any additional trends should be correlated across currencies. So combining exchange rates will increase the power of our tests.

Row 1 of Table 2 reports the Johansen (1988) test for the hypothesis that there are three or more trends in the three spot exchange rates.[13] Both Johansen's Trace Test and Maximal Eigenvalue Test do not reject this hypothesis, indicating that each exchange rate can be written in terms of its own trend component. Rows 2 and 3 report tests for the number of trends in the three forward rates. Row 2 considers forward rates at the one month horizon and row 3 considers forward rates at the three month horizon. The results in these rows show that we cannot reject the hypothesis that there are three trends in both vectors of forward rates.[14]

Rows 4 and 5 of Table 2 report the test statistics for the hypothesis that the system of spot rates and forward rates contain at least four, five and six trends, respectively. If the null hypothesis of $\eta_{A,\tau}^{j} = 0$ for all $\tau$ holds true, spot and forward rates for currency $j$ will share the same trends. In this case, given the results in rows 1 - 3, the number of trends will remain the same at three. However, if forward rates contain additional trend components shared across currencies, then the number of trends may increase. In row 5 the hypothesis of six and five independent trend components is rejected at the 95% confidence level for both test statistics at the 3 month horizon. The hypothesis is also rejected at the one month horizon except for the trace test of five or less trends.[15] These estimates indicate the surprising result that forward rates contain an additional stochastic trend independent of the spot rate trend.

In summary, Table 2 provides evidence that forward exchange rates follow trends in addition to those followed by spot rates. Specifically, when three spot exchange rates were combined with three forward exchange rates, the system contained four stochastic trends, one more than the three spot exchange rates alone. Of course, these results are subject to the caveat that the Johansen tests may not be powerful enough to reject the presence of the additional trend. However, as we will show below with the aid of some Monte Carlo experiments, this does not appear to be so in our data. We will argue, therefore, that the surprising results in Table 2 are in fact robust.

### *2.5 Do Trends Arise in Individual Currencies?*

We now turn to consider whether the trending deviations between spot and forward rates implied by the results in Table 2 are associated with a particular exchange rate. We will exploit the fact that arbitrage ensures covered interest parity holds to combine the information contained in interest rates and exchange rates.

Let $R^j$ be the interest rate on deposits denominated in currency *j*. The covered interest parity relationship can be written in the case of dollar deposits relative to the domestic currency *j* deposits as:

$$f_t^{x^j} = R_t^j + x_t^j - R_t^{\$}. \tag{15}$$

Thus, arbitrage ensures that the forward rate is a linear combination of the current spot rate exchange rate $x_t^j$, the current interest rate on dollar bonds $R_t^{\$}$, and the interest rate on domestic currency *j* bonds $R_t^j$.

Using this parity condition, we can evaluate whether deviations between spot and forward rates implied by the results in Table 2 are associated with a particular exchange rate. If spot and forward rates share the same trend, then the forward premium $f_t^{x^j} - x_t^j$ shares this same trend. Since $f_t^{x^j} - x_t^j = R_t^j - R_t^{\$}$, $R_t^j$ and $R_t^{\$}$ can have at most one shared independent trend in addition to the trend in the spot exchange rate. Therefore, the greatest number of independent trends that the spot rate, domestic and foreign interest rates can contain under the null hypothesis of $\eta_{\Lambda,f}^{x^j} = 0$, is two.

Based upon this observation, we examined whether additional trends are present by testing for the number of trends in the vector of the domestic interest rate, the exchange rate, and US interest rate. If we find evidence of three or more trends in this vector, then the results can suggest which of the forward exchange rates contain trends in addition to their corresponding spot rates.

Table 3 reports the Johansen test statistics for the three variable systems of the dollar interest rate, the domestic interest rate, and the exchange rate against the dollar. We tested for the number of trends in systems of one and three month interest rates separately. These tests are conducted for systems of the spot rates in rows 1 and 3 and for the forward rates in rows 2 and 4. As the table shows under column A, we cannot reject the hypothesis that three trends are present in the UK rates at either maturity. For the Japanese yen in Column C, the Trace Test for three trends is not rejected at either maturity, although the Maximal Eigenvalue test is rejected for the three month rates. These findings suggest that the pound and yen forward rates contain a trend not found in the corresponding spot exchange rates. Since the results in Table 2 showed that there was an additional trend in at least one of the forward rates, these results suggest that the additional trend in the pound/dollar returns may be shared with yen/dollar returns.

### *2.6 Do Trends Arise Across Foreign Exchange and Bond Markets?*

The results in Tables 2 and 3 suggest that the effects of "peso problems" can show up in different exchange rates. This, in turn, raises the possibility that their influence can be detected in the different assets. Indeed, "peso problems" are likely to have affected both interest and exchange rates.

Consider, for example, the effects of the change in the Federal Reserve's operating procedure in 1979. Following the change, US short-term interest rates increased dramatically until a peak was reached in 1981. At the same time, the value of the US dollar began an upward trend that would continue until 1985. If market participants believed that the Fed. could not maintain the tight monetary policy that had accompanied the change in operating procedure, such beliefs would have induced a "peso problem" in both the US bond market and the foreign exchange market. The behavior of the term structure appears to support this hypothesis. Long-term interest rates were persistently below short-term interest rates suggesting that bond traders believed short-term interest rates would be lower in the future after monetary policy was relaxed. During this period, expected future short-term rates (implied by the term structure) were lower than they turned out to be *ex post*. At the same time, tight monetary policy contributed to an increase in the value of the dollar. However, since traders believed that a switch to looser monetary policy was possible, they also anticipated a switch to a weaker dollar. These expectations were reflected in forward rates that systematically predicted a weaker dollar than was realized *ex post*.

As this example illustrates, "peso problems" in one market are likely to be correlated with "peso problems" in another market when there is an anticipated switch in the fundamentals process common to both assets. Similarly, through arbitrage across markets, feedback traders that drive a price from its underlying value in one market are likely to influence other markets as well. Therefore, if additional trends exist in foreign exchange returns, as Tables 2 and 3 show, these trends are likely to be correlated with the additional trends in bond returns.

To examine whether the additional trends are detectable in foreign exchange and bond markets, we combined the spot rates on the domestic interest rate, the exchange rate, and the US interest rate ($R^j$, $x^j$, $R^\$$), with their forward rates ($f^{R^j}$, $f^{x^j}$, $f^{R^\$}$) where $f^{R^j}$ is the forward interest rates in currency *j*. Since the relationship between spot and forward rates in (13) and (14) applies equally to interest and exchange rates, we may use the same approach to test for the presence of additional trends in ($f^{R^j}$, $f^{x^j}$, $f^{R^\$}$) as we did when we considered forward exchange rates alone. Specifically, if there are no additional trends in forward interest rates, we should find the same number of trends in the systems of spot and forward rates as we did for the spot and forward rates separately.

Table 4 reports the results of the Johansen test for the number of trends in ($R^j$, $x^j$, $R^\$$, $f^{R^j}$, $f^{x^j}$, $f^{R^\$}$). The statistics for the German mark in Column B indicate that we can reject the hypothesis of three or more stochastic trends in one month returns, although the evidence is less clear in 3 month returns. Recall that there was no evidence of additional trends in foreign exchange returns for the mark in Table 3. Here we see that there is also no strong evidence of additional trends in mark bond returns. This finding appears consistent with the idea that "peso problems" should be correlated across related markets.

The results for Japanese and UK rates reported in Columns A and C of Table 4 are quite different. We would expect to find at least three stochastic trends in these systems because the results in Table 3 indicated that both the yen and the pound contained additional trends. Table 4 confirms this prediction. However, for the three month UK system, we find evidence of four stochastic trends. Since the foreign exchange forward rate, $f^s$, is a linear combination of the three spot rates, the additional trend cannot arise from this rate. Instead its presence must indicate the effects of anticipated shifts in the process for UK interest rates. For the Japanese systems, we cannot reject four trends in the one month rates, and *five* trends in the three month rates. Both results suggest that the process for Japanese interest rates was expected to shift.

To summarize, the evidence in Tables 2 - 4, indicates that exchange rates and interest rates contain trends in addition to the those assumed in the existing empirical literature. As we showed above, these additional trends would arise if traders considered shifts to alternative trends in their future expectations of asset prices. We find that these trends appear across both foreign exchange and bond markets. They appear to be most important in foreign exchange returns and in Uk and Japanese interest rates.

*2.6 Are the Results Robust?*

The results above appear to provide strong evidence that the deviation between some spot and forward rates contain statistically significant trends. These findings seems quite surprising because standard models assume that forecast errors and risk premia do not trend away from the actual asset prices. For this reason we wanted to make sure that our results were robust. In particular, we were concerned that the Johansen tests may not be powerful enough to reject the presence of additional trends. To examine this issue, and other assumptions about the data which may affect the test statistics, we conducted a number of Monte Carlo experiments.

Our Monte Carlo experiments were constructed to generate spot rate processes with the same variance in their permanent components as we observe in the exchange rate data. From these permanent trend components in spot rates, we generated systems of forward rates with different numbers of trends depending upon the experiment. Therefore, we knew by construction the true number of trends in these artificial forward rates. We then calculated both versions of the Johansen test. Repeating this process 1000 times, we generated an empirical distribution for the test statistics where the number of trends is known. The appendix gives the details of these experiments.

With the empirical distribution of the Johansen test statistics generated by these Monte Carlo experiments, we can examine the power of the tests to reject additional trends. In other words, we can ask whether these tests would fail to reject the hypothesis of a given number of trends when it should reject. Figure 3 shows the empirical distribution of Johansen's trace test for four trends when (by construction) there are only three trends in the data. Because the exchange rate data was used to parametarize this experiment, the Monte Carlo results can be compared to the test statistics in rows 4 and 5 of Table 2 where we tested for four trends.

Figure 3 answers the question: If we test for four trends, but only three are truly present in the system, how likely are we to find the test statistics in Table 2? The figure depicts two cases, representing two different assumptions about the order of the VAR used to construct the Johansen tests. Raising the number of lags in the VAR from one to three shifts the empirical distribution to left but not enough to account for the results reported in Table 2. The probability of observing 28.09, the statistic when spot exchange rates are combined with one-month forward rates, when in fact there are only three stochastic trends, is considerably less than the 1% marginal significance level of 51.5 or 53.2 for the two empirical distributions. Therefore, the Johansen test appears to have a good deal of power to reject four trends when only three trends are present.

Figure 4 illustrates the analogous empirical distribution for the Maximum Eigenvalue test. As for the trace test, raising the order in the VAR implies lower values for the empirical distribution. Again, however, the probability of finding the statistics reported in Table 2 when in fact only 3 trends were present is minuscule.

Figures 3 and 4 represent only a small fraction of the experiments we conducted. We also ran experiments allowing for: (1) different numbers of trends holding the true trend number constant; (2) different numbers of true trends; and (3) heteroskedasticity rather than homoskedasticity. One important result to emerge from these experiments was that ignoring the presence of heteroskedasticity tends to bias the test statistics upwards. This suggests that the statistics presented in Tables 2 to 4 are *biased upwards* because exchange rates and interest rates are known to be heteroskedastic.

In summary, our Monte Carlo experiments indicate that the test statistics obtained in Tables 2 to

4 based upon the Johansen distribution assuming homoskedasticity are too high. The results in these tabes are therefore biased toward finding *too few* rather than *too many* trends. We conclude that the tables provide strong evidence of statistically significant, but probably small, trends in forward rates relative to spot rates.

### 3. Estimating the Trends in Expected Returns

In this section we examine the size of the additional trends in forward rates. To do so we estimate a time series model for spot and forward exchange rates consistent with the results presented above. We then use the model estimates to identify the path of the additional trend in excess foreign exchange returns.

Our results in Table 2 indicate that the vector of three spot and three forward exchange rates, $y_{t+1} \equiv [x^{\pounds}_{t+1}, x^{DM}_{t+1}, x^{¥}_{t+1}, f^{\pounds}_t, f^{DM}_t, f^{¥}_t]'$, contain four trends. Following Stock and Watson (1988), we may therefore represent the dynamics of $y_t$ as the vector sum of four random walks and six stationary I(0) processes:

$$(15) \qquad y_{t+1} = A\,\omega_{t+1} + z_{t+1}, \qquad \omega_{t+1} = \omega_t + \upsilon_{t+1}, \qquad E[\upsilon_{t+1}, \upsilon_{t+1}'] = I_4$$

where $\omega_t \equiv [\omega_{i,t}]'$ is a $4\times1$ vector of independent random walks with $\omega_0$ a vector of constants, $z_t$ is $6\times1$ vector stationary I(0) process, $I_4$ is a $4\times4$ identity matrix, and A is $6\times4$ matrix. In order to interpret the four trends in $\omega_t$, we assume that the factor loading matrix A, has the following structure

$$(16) \qquad A = \begin{bmatrix} a_{11} & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 \\ a_{31} & a_{32} & a_{33} & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} \\ a_{51} & a_{52} & a_{53} & a_{54} \\ a_{61} & a_{62} & a_{63} & a_{64} \end{bmatrix}$$

With these restrictions imposed upon A we can interpret the first three elements of $\omega_t$ as the trends in spot rates. The fourth element in $\omega_t$ is interpreted as the additional trend found in the forward rates but not in the spot rate.[16]

We are particularly interested in examining the behavior of the additional trend, i.e. the fourth element in $\omega_t$. To do so, we use a two step procedure to estimate the model for excess returns implied by (15):

$$(17) \qquad \bar{y}_{t+1} = [I_3, -I_3]\, A\, \omega_{t+1} + \bar{z}_{t+1} \qquad\qquad \bar{z}_{t+1} \sim \text{ARMA}(n,m)$$

where $\bar{y}_{t+1} = [x^{\pounds}_{t+1}-f^{\pounds}_t,\ x^{DM}_{t+1}-f^{DM}_t,\ x^{¥}_{t+1}-f^{¥}_t]'$ and $\bar{z}_t = [I_3, -I_3]z_t$.

The first step in our estimation procedure is to obtain consistent estimates of the factor loading matrix A. This procedure is described in the appendix. We then estimate the remaining parameters in (17), $\omega_0$ and the parameters of the vector ARMA process, by maximum likelihood with the aid of the Kalman filter. In order to identify the ARMA component of excess returns, we estimated models with all combinations of $n$ and $m$ up to $n = m = 2$. The best model was selected on the basis of Akaike's information criteria. An ARMA(1,1) specification was chosen in the model for one month returns, and an ARMA(1,2) specification in the model for three month returns.[17] trends in the forward rates not contained in the spot rates. In other words, these figures plot estimates of $[I_3, -I_3]A\omega^*_t$ where $\omega^*_t = [0,0,0,\omega_{4,t}]'$, the component in $x_{t+1}$ - $f_t$ arising from the additional trend. The basic pattern of the one month and three month horizons are similar. Both trends have peaks in 1978 and 1980. They also have negative swings from 1981 through 1983 and positive swings from 1985 until essentially 1990. These general movements are consistent with a "peso problem" explanation if the dollar exchange rate switches between appreciating and then depreciating states, as we discussed in section 1.

To see why, note from equation (6) that the excess return conditional upon the current state can be written as:

$$(18) \qquad (x_{t+1} \mid C) - E_t x_{t+1} + I(0) \text{ terms}$$

$$= (x_{t+1} \mid C) - E_t(x_{t+1} \mid C) + \lambda[E_t(x_{t+1} \mid C) - E_t(x_{t+1} \mid A)] + I(0) \text{ terms}$$

$$= \lambda[E_t(x_{t+1} \mid C) - E_t(x_{t+1} \mid A)] + I(0) \text{ terms.}$$

Now suppose that the dollar were currently in an appreciating state, so that process C represents a strengthening dollar. If traders believe that a switch to a depreciating regime, A, were likely, $E_t(x_{t+1} \mid C) < E_t(x_{t+1} \mid A)$ and the trend component in excess returns will be negative. By contrast, if the dollar were depreciating representing a new regime C, and traders believe a switch to an appreciating regime were possible, then $E_t(x_{t+1} \mid C) > E_t(x_{t+1} \mid A)$. In this case, the trend component would be positive.

Indeed, from the period of appreciating dollar from 1980 until 1984, the trend component is mostly negative, while during the period of depreciating dollar from 1985 through 1989, the trend component is positive. This evidence suggests that traders conditioned their forecasts during a trend movement in the exchange rate upon a possibility that the exchange rate will shift to a process with a different trend.

## 4. Concluding Remarks

In this paper, we have shown that rationally expected future asset prices and, therefore, their forward prices can systematically trend away from the actual price. This possibility arises when rational traders incorporate the expectation that trends in asset prices induced either by policy or by feedback traders may shift over the forward rate contract horizon. We investigated this relationship by testing for the number of trends in systems of spot and forward rates. If alternative trends do not affect market expectations, then adding forward rates to a system of their corresponding spot rates should not increase the number of trends. Interestingly, we found evidence of additional trends in both foreign exchange returns and in interest rates for the Japanese yen and the British pound. We conducted a number of

Monte Carlo experiments on these results and found them to be quite robust.

When we estimated the additional trend component in excess foreign exchange returns we find that it has varied in annualized rates between -0.8% and 1% for one month returns and between -6% and 8% for three month returns. These findings are clearly inconsistent with standard models of the risk premium or the treatment of systematic forecast errors. We believe that future research should further investigate the origins of these trends.

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## Endnotes

[1]Throughout the paper, we will use the word "trends" as short hand for the more precise terminology of "stochastic trends." These stochastic trends are the cumulation of permanent disturbances upon asset prices. We do not consider deterministic trends in this paper.

[2]Using time series analysis, see Evans and Lewis (1991, 1992). For a structural approach, see Lewis (1991). Froot (1989) and Frankel and Froot (1989) find evidence of systematic forecast errors in interest rate survey data and in foreign exchange survey data, respectively.

[3]See, for example, Schwert (1989) and Campbell and Perron (1991).

[4]For a survey about unit roots, see Diebold and Nerlove (1990). Unit roots have been found in nominal interest rates by Campbell and Shiller (1987) and Mishkin (1991), and in exchange rates by Baillie and Bollerslev (1989), Meese and Rogoff (1983), and Meese and Singleton (1982). Other prices that appear to be affected by permanent disturbances include those on stocks and commodities.

[5]Technically, since $(S_t \mid C)$ is an I(1) process, then $(S_{t+1} \mid C) - (S_t \mid C)$ must be an I(0) process by definition.

[6]See Cutler, Poterba and Summers (1990) and Frankel and Froot (1986) for a discussion of how different traders interact in the market.

[7]For example, they cite stories of successful speculative strategies by the Wall Street guru, George Soros (1987).

[8]More precisely, the growing trend movement in the price away from its fundamental level occurs when there is uncertainty about the fundamental level.

[9]For example, standard models of time-varying risk premia imply that risk premia are stationary since they depend upon the time-series properties of the change in consumption. For a further discussion and examples, see Evans and Lewis (1992), Grossman and Shiller (1981) and Backus, Gregory, and Zin (1989).

[10]These data were kindly provided by Geert Bekaert and Robert Hodrick. For details, see Bekaert and Hodrick (1991).

[11]Using the linearized term structure relationship from Campbell and Shiller (1991) for the case of pure discount bonds as we have here, the forward rate on an *k* period bond contracted for trade in *n* periods is: $[(k+n)\, R^{k+n}_t - n\, R^n_t]/k$, where $R^j_t$ is the rate on a *j* period deposit at time *t*. In this paper, we only consider the case where $k = n$ for one month and three month deposits, so that $F^k_t = [2k\, R^{2k}_t - k\, R^k_t]/k$ for $k = 1, 3$. Some of these deposits were available for earlier periods than for exchange rates. In the combinations considered below, we used the longest time series of data available.

[12]As we showed above in equation (6'), we could also condition upon the alternative process. Since we will consider the null hypothesis of *no* alternative process, conditioning on the current process provides a useful benchmark.

[13]Recall that we use the term "trends" as short hand for the more precise terminology of "stochastic trends".

[14]Although these results may appear to confirm the null hypothesis, in fact they are also perfectly consistent with $\eta_{A,\tau}^{x^j} \neq 0$. To see why, note that the forward rate in (14) can also be written as

$$f_t^{x^j} = u_t^j + I(0) \text{ terms}, \qquad \text{with} \quad u_t^j = u_{t-1}^j + b\,\eta_{C,\tau}^{x^j} + d\,\eta_{A,\tau}^{x^j}.$$

Written in this form, the forward rate contains a random walk component with innovations that are a linear combination of the two permanent shocks. Judged in isolation, the forward rate will therefore appear to contain a single trend whether $\eta_{A,\tau}^{x^j} \neq 0$ or not.

[15]Percentiles of the distribution from Johansen and Juselius (1990) are provided in the appendix.

[16]Notice the conventional view that forward and spot rates share the same trend place numerous restrictions on the factor loading matrix A. Specifically, the factor loading matrix would be $[A_1', A_1']'$ where $A_1$ is the sub-matrix comprising the first three rows of A in (16).

[17]The model for three month returns has the same structure as the one month model except $y_{t+1} = [x_{t+1}^{\pounds}, {}^{DM}_{t+1}, x_{t+1}^{Y}, f_{t-2}^{\pounds}, f_{t-2}^{DM}, f_{t-2}^{Y}]'$ where $f_t^i$ are now three month forward rates.

**Table 1**

**Results of Forecast Error Experiment**

| Transition Probability | Mean (Std. Dev.) | $\rho_1$ (Std. Dev.) | $\rho_6$ (Std. Dev.) | $\rho_{12}$ (Std. Dev.) |
|---|---|---|---|---|
| *A. Conditional on Process Generated by State 1* | | | | |
| .50 | −1.01 (30.00) | .936 (.044) | .780 (.120) | .610 (.176) |
| .40 | −.59 (23.65) | .911 (.053) | .760 (.108) | .604 (.153) |
| .30 | −.27 (21.38) | .861 (.099) | .724 (.148) | .578 (.181) |
| .20 | .01 (11.89) | .752 (.112) | .623 (.150) | .485 (.174) |
| .10 | −.15 (6.93) | .465 (.188) | .401 (.186) | .321 (.188) |
| .05 | .61 (2.97) | .186 (.133) | .137 (.140) | .116 (.132) |
| *B. Conditional on Process Generated by State 2* | | | | |
| .50 | 1.61 (33.43) | .901 (.065) | .745 (.127) | .578 (.181) |
| .40 | 3.42 (25.32) | .853 (.089) | .712 (.139) | .574 (.177) |
| .30 | 2.01 (22.18) | .771 (.137) | .636 (.187) | .500 (.222) |
| .20 | −1.88 (10.85) | .558 (.203) | .476 (.212) | .380 (.207) |
| .10 | 0.71 (6.31) | .264 (.147) | .251 (.137) | .199 (.125) |
| .05 | −0.11 (3.64) | .084 (.095) | .068 (.088) | .060 (.089) |

Notes: Exchange Rate process $\Delta x = u_t^i$, where $\text{Var}(u_t^1) = 16.92$ for State 1 and $\text{Var}(u_t^2) = 20.25$ for State 2. (Variance parameters are from Engel and Hamilton (1990).)

Table 2

Johansen Tests for Number of Stochastic Trends in Exchange Rates

| Variables in Vector | Number of Stochastic Trends | Test | |
|---|---|---|---|
| | | $T_1^a$ | $T_2^b$ |
| 1. Spot Exchange Rates ($x_t^*$) | 3 | 13.23 | 8.81 |
| 2. One Month Forward Rates ($f_t^{x_i}$) | 3 | 13.13 | 8.70 |
| 3. Three Month Forward Rates ($f_t^{x_i}$) | 3 | 12.83 | 8.41 |
| 4. One Month Forward and Spot Exchange Rates ($x_t^*, f_t^{**}$) | 6 | 118.00[c] | 53.13[c] |
| | 5 | 64.91 | 36.83[c] |
| | 4 | 28.09 | 15.20 |
| 5. Three Month Forward and Spot Exchange Rates ($x_t^*, f_t^{**}$) | 6 | 115.70[c] | 49.46[c] |
| | 5 | 66.23[c] | 37.91[c] |
| | 4 | 28.31 | 15.63 |

Notes: All systems are for * = £, DM, ¥. Tests are based upon AIC information criterion choice of one lag in VAR system.

[a]Johansen "Trace Test." See the appendix for percentiles of distribution.

[b]Johansen "Maximal Eigenvalue Test." See the appendix for percentiles of distribution.

[c]Significant rejection at the 5% confidence level.

Table 3

Johansen Tests for Number of Stochastic Trends in Spot Rates and Forward Rates Individually

| Variables in Vector | Number of Stochastic Trends | Assets $i$ = | | | | | |
|---|---|---|---|---|---|---|---|
| | | A. British Pound | | B. German Mark | | C. Japanese Yen | |
| | | $T_1$[a] | $T_2$[b] | $T_1$ | $T_2$ | $T_1$ | $T_2$ |
| *One Month*[c] | | | | | | | |
| 1. Spot Rates | 3 | 22.84 | 11.66 | 34.78[d] | 28.48[d] | 24.41 | 21.18 |
| | 2 | 11.18 | 10.77 | 6.31 | 6.30 | 3.23 | 2.79 |
| 2. Forward Rates | 3 | 23.64 | 11.94 | 31.67[d] | 25.53[d] | 25.57 | 21.84[e] |
| | 2 | 11.70 | 11.31 | 6.14 | 6.14 | 3.72 | 3.25 |
| *Three Month*[f] | | | | | | | |
| 3. Spot Rates | 3 | 23.62 | 11.87 | 31.65[d] | 25.53[d] | 25.54 | 21.81[d] |
| | 2 | 11.75 | 11.36 | 6.12 | 6.12 | 3.73 | 3.26 |
| 4. Forward Rates | 3 | 22.07 | 12.24 | 19.94 | 14.57 | 19.76 | 16.91 |

Notes: Tests are based upon AIC information criterion choice of three lags in VAR system.

[a]Johansen "Trace Test." See the appendix for percentiles of distribution.
[b]Johansen "Maximal Eigenvalue Test." See the appendix for percentiles of distribution.
[c]Interest rate spots are for 1 Month maturities.
[d]Significant at the 90% confidence level.
[e]Significant at the 95% confidence level.
[f]Interest rate spots are for 3 Month maturities.

Table 4

**Johansen Tests for Number of Stochastic Trends in Spot Rates and Forward Rates Jointly**

| Variables in Vector | Number of Stochastic Trends | Assets $i$ = A. British Pound $T_1$[a] | $T_2$[b] | B. German Mark $T_1$ | $T_2$ | C. Japanese Yen $T_1$ | $T_2$ |
|---|---|---|---|---|---|---|---|
| 1. One Month[c] | 5 | 90.42[d] | 41.47[d] | 99.24[d] | 47.22[d] | 72.57[d] | 33.10[e] |
| | 4 | 48.95[d] | 27.90[d] | 52.02[d] | 24.27 | 39.47 | 22.19 |
| | 3 | 21.05 | 12.25 | 27.75 | 22.32[e] | 17.28 | 13.64 |
| 2. Three Month[f] | 6 | 146.60[d] | 71.34[d] | 195.27[d] | 105.50[d] | 125.25[d] | 66.10[d] |
| | 5 | 75.26[d] | 34.13[d] | 89.77[d] | 43.43[d] | 59.14 | 28.74 |
| | 4 | 41.12 | 20.91 | 46.34[d] | 26.21[d] | 30.41 | 14.58 |
| | 3 | 20.22 | 12.78 | 20.13 | 15.05 | 15.83 | 13.01 |

Notes: Tests are based upon AIC information criterion choice of three lags in VAR system.

[a]Johansen "Trace Test." See the appendix for percentiles of distribution.

[b]Johansen "Maximal Eigenvalue Test." See the appendix for percentiles of distribution.

[c]Systems of spot exchange rates and one month interest rates for dollar and foreign currency, together with one month forward exchange rates and one month forward interest rates for dollar and foreign currency.

[d]Significant at the 90% confidence level.

[e]Significant at the 95% confidence level.

[f]Systems of spot exchange rates and three month interest rates for dollar and foreign currency, together with three month forward exchange rates and three month forward interest rates for dollar and foreign currency.

## Appendix to Trends in Expected Returns in Currency and Bond Markets

This appendix begins by explaining how consistent estimates of the factor loading matrix A were obtained in order to estimate the time series models for excess returns in equation (17) of section 3. It then describes the Monte Carlo experiments we ran to investigate the robustness of the Johansen statistics reported in Tables 2 - 4. Figures 3 and 4 are based on some of the results of these experiments.

*Estimation of the Factor Loading Matrix*: The factor loading matrix A relates the stochastic trends $\omega_t$ to the vector of spot and forward rates $y_t$:

$$(15) \qquad y_{t+1} = A\,\omega_{t+1} + z_{t+1}, \qquad \omega_{t+1} = \omega_t + \upsilon_{t+1}, \qquad E[\upsilon_{t+1}\upsilon_{t+1}'] = I_4$$

To obtain consistent estimates of A, we exploit two properties of the process:

(i) $\alpha' A = 0$ where $\alpha$ is the matrix of cointegrating vectors for $y_t$.

(ii) $Cov_0(\Delta y_t) = AA'$ where $Cov_0$ denotes the long run covariance matrix.

Condition (i) is not sufficient to uniquely identify A given $\alpha$, for if $\alpha' A = 0$, then $\alpha' AR = 0$ for an arbitrary $4 \times 4$ matrix R. Our procedure is therefore to write $A = A^* R$, and use condition (i) to identify $A^*$. Then we use (ii) together with the prior restrictions on A described in the text to identify R.

We use the approach described by Gonzola and Granger (1991) to find $A^*$. First we regress $\Delta y_t$ on $\Delta y_{t-1}, \ldots \Delta y_{t-q+1}$ and save the residuals as $e_{0,t}$. Then we regress $y_{t-1}$ on $\Delta y_{t-1}, \ldots \Delta y_{t-q+1}$ and the save the residuals as $e_{1,t}$. Next, the eigenvalue problem $| \lambda S_{00} - S_{01}(S_{11})^{-1} S_{10} | = 0$ is solved for $\lambda_1 > \lambda_2 . > \lambda_6$ and the associated eigen vectors $[m_1, m_2, .. m_6]$ where $S_{ij} = T^{-1} \Sigma_{t=1}^{T} e_{it} e_{jt}$. Gonzola and Granger demonstrate that the estimate of $A^*$ given by $[m_3, m_4, m_5, m_6] S_{01}$ is orthogonal to the estimated matrix of cointegrating vectors for $y_t$, so that condition (i) is satisfied, i.e. $\alpha' A^* R = 0$.

To find R, we first need to estimate the long run covariance matrix for $\Delta y_t$. This is complicated by the fact that in any finite sample an unrestricted estimate of the covariance matrix will have rank 6 while the assumed presence of 4 trends in the data implies that the covariance matrix should have rank 4. We used the Newey-West estimator of the long run covariance matrix (allowing for serial correlation) to obtain a consistent estimate of the unrestricted covariance matrix, $\Omega$. Next we write $\Omega$ as $\Gamma\Lambda\Gamma'$ where $\Lambda$ is a diagonal matrix of eigen values, and $\Gamma$ is the matrix of eigen vectors. A consistent estimate of the long run covariance matrix with rank 4 is then constructed as $\Omega^{+} = \Gamma\Lambda^{+}\Gamma'$ where $\Lambda^{+}$ is equal to $\Lambda$ except that the two smallest eigen values in $\Lambda$ are set equal to zero. Note that since $\Omega$ is a consistent estimate of the unrestricted covariance matrix, $\Omega^{+}$ must be a consistent estimate when there are 4 trends in $y_t$.

Finally, we use the estimates of $A^{*}$ and $\Omega^{+}$ to find R. Condition (ii) implies that R must solve $A^{*}RR'A^{*\prime} = \Omega^{+}$, or equivalently that

$$\text{(iii)} \qquad RR' = (A^{*\prime}A^{*})^{-1}A^{*\prime}\Omega^{+}A^{*}(A^{*\prime}A^{*})^{-1}.$$

Since RR' is a symmetric $4\times4$ matrix, this condition places 10 independent restrictions on the 16 elements in R. 6 further restrictions are imposed by (16) in the text because 6 elements on the right hand side are zero. In effect, therefore, there are 6 zero restrictions imposed on $A^{*}R$. Thus, (16) and (iii) impose 16 independent restrictions on R which are sufficient to identify the matrix. Once we have found the unique matrix R that satisfies (ii) and (16) given $A^{*}$, the consistent estimate of A is formed as $A^{*}R$.

The estimates of the factor loading matrices are:

One month model

$$A = \begin{matrix} 3.658 & 0.000 & 0.000 & 0.000 \\ 3.146 & 1.074 & 0.000 & 0.000 \\ 2.526 & -2.261 & 0.724 & 0.000 \\ 3.648 & -0.054 & -0.116 & -0.112 \\ 3.151 & 1.045 & -0.040 & -0.040 \\ 2.550 & -2.337 & 0.581 & -0.140 \end{matrix}$$

Three month model

$$A = \begin{matrix} -3.629 & 0.000 & 0.000 & 0.000 \\ -3.234 & -1.132 & 0.000 & 0.000 \\ -2.486 & 2.297 & -0.187 & 0.000 \\ -3.535 & 0.037 & 0.337 & 0.932 \\ -3.221 & -1.114 & 0.052 & 0.144 \\ -2.479 & 2.358 & 0.036 & 0.621 \end{matrix}$$

Since $y_{t+1} = [x^{\pounds}_{t+1}, x^{DM}_{t+1}, x^{Y}_{t+1}, f^{\pounds}_{t}, f^{DM}_{t}, f^{Y}_{t}]'$, the fourth column of each matrix shows that the additional trend has the largest effect on the pound and the yen forward rates.

*Monte Carlo Experiments*: In the experiments we consider tests on the *d*-dimensioned vector $z_t = [z_{1,t}', z_{2,t}']'$ which contains *m* stochastic trends and *r* cointegrating vectors (hence $d = m+r$). The model generating the data on $z_t$ is:

$$z_{1,t} = v_t + \Lambda u_{1,t} \qquad E_{t-1}[u_{1,t}, u_{1,t}'] = \Sigma_{1,t} \tag{A1}$$

$$z_{2,t} = \Gamma z_{1,t} + u_{1,t} \qquad E_{t-1}[u_{2,t}, u_{2,t}'] = \Sigma_{2,t} \tag{A2}$$

$$v_t = v_{t-1} + \epsilon_t \qquad E_{t-1}[\epsilon_{1,t}, \epsilon_{1,t}'] = Q_{1,t} \tag{A3}$$

where; $v_t$ is an *m*-dimensioned random walk with innovations $\epsilon_t$, $u_{1,t}$ is an *m*-dimensioned vector of serially uncorrelated errors, and $u_{2,t}$ is an *r*-dimensioned vector of serially uncorrelated errors. We assume that the conditional covariance matrices $Q_t$, $\Sigma_{1,t}$, and $\Sigma_{2,t}$ are diagonal and that $\epsilon_t$, $u_{1,t}$ and $u_{2,t}$ are uncorrelated with one another. $\Lambda$ is a diagonal matrix of dimension *m* and $\Gamma$ is an $r \times m$ matrix of cointegrating vectors.

We considered the sampling behavior of the Johansen statistics using several different versions of the data generation process described in (A1) - (A3). To conform with our results in Tables 2 - 4, we ran two sets of experiments; one where $z_t$ contained 3 variables, the other where $z_t$ contained 6 variables. In each set of experiments we varied the number of stochastic trends generated in $z_t$ from 1 to *d* to examine the power and size of the test statistics. We also compared the test statistics on $z_t$ generated with homoskedastic error terms (i.e. $Q_t = Q$, $\Sigma_{1,t} = \Sigma_1$, and $\Sigma_{2,t} = \Sigma_2$), against those on $z_t$ generated with heteroskedastic terms.

To parametarize the data generation process in (A1) -(A3) we use actual data on spot and forward exchange rates. In the experiments where $d = 3$, define Y as a data matrix with rows $[\Delta S_t^{\pounds}, \Delta S_t^{DM}, \Delta S_t^{Y}]$, and when $d = 6$, define Y as a data matrix with rows $[\Delta S_t^{\pounds}, \Delta S_t^{DM}, \Delta S_t^{Y}, \Delta f_t^{\pounds}, \Delta f_t^{DM}, \Delta f_t^{Y}]$ where $\Delta f_t^j$ are the one month forward rates. In both cases we assume that $\Lambda = I_m$ and that $\Gamma = [I_m, \ell']'$ where $\ell$ is $(r\text{-}m) \times m$ matrix of ones. When we ran experiments where $\Lambda = I_m \times 0.1$, we found the results to very similar to those reported below. (These results are available upon request.)

*Homoskedastic Experiments:* The following steps are repeated 1000 times.

1. Sample from the first $m$ columns of Y to create $\{\epsilon_t\}_0^T$, from which we construct $\{v_t\}_0^T$. $\{v_t\}_0^T$ is therefore an $m$-dimensioned random walk equal to the sample length T, with innovations that are a random sample of Y. Next, re-sample from the first $m$ columns of Y to create $\{u_{1,t}\}_0^T$ and use (A1) to combine $\{u_{1,t}\}_0^T$ with $\{v_t\}_0^T$ to form $\{z_{1,t}\}_0^T$.
2. Sample from the last $r$ columns of Y to create $\{u_{2,t}\}_0^T$, and then use (A2) together with $\{z_{1,t}\}_0^T$ to calculate $\{z_{2,t}\}_0^T$.
3. Form $\{z_t\}_0^T$ where $z_t = [z_{1,t}', z_{2,t}']'$. Calculate the Johansen Trace test and Maximum Eigenvalue test for $j$ trends where $j = 1,2,...d$ using (a) a VAR with 1 lag, and (b) a VAR with 3 lags. Record the results.

*Heteroskedastic Experiments:* The following steps are repeated 1000 times.

1. Estimate ARCH models for the elements in Y. Then re-scale the elements of Y by dividing each observation by its estimated standard deviation from the ARCH models. Sample from the first $m$ columns of scaled matrix $Y^*$ to create $\{\epsilon_t^*\}_0^T$, and then re-scale $\{\epsilon_t^*\}_0^T$ using the ARCH predictions to form $\{\epsilon_t\}_0^T$. These innovations are used to calculate $\{v_t\}_0^T$. Next, re-sample from the first $m$ columns of $Y^*$ to create $\{u_{1,t}^*\}_0^T$, and then re-scale using the ARCH predictions to form $\{u_{1,t}\}_0^T$. Finally use (A1) to combine $\{u_{1,t}\}_0^T$ with $\{v_t\}_0^T$ to form $\{z_{1,t}\}_0^T$.
2. Sample from the last $r$ columns of the $Y^*$ to create $\{u^{*,t}\}_0^T$, and then re-scale using the ARCH predictions to form $\{u_{2,t}\}_0^T$. Use (A2) together with $\{z_{1,t}\}_0^T$ to calculate $\{z_{2,t}\}_0^T$.
3. As step 3 above.

Tables B and C report a sub-set of the empirical sampling distributions of the test statistics. The complete set of results are available upon request.

**Table A**

**Percentiles of Johansen Test Statistics Distribution**

| No. of Stoch. Trends | 50% | 80% | 90% | 95% | 97.5% | 99% | mean | var |
|---|---|---|---|---|---|---|---|---|
| | | | | *Maximal eigenvalue* | | | | |
| 1. | 2.415 | 4.905 | 6.691 | 8.083 | 9.658 | 11.576 | 3.030 | 7.024 |
| 2. | 7.474 | 10.666 | 12.783 | 14.595 | 16.403 | 18.782 | 8.030 | 12.568 |
| 3. | 12.707 | 16.521 | 18.959 | 21.279 | 23.362 | 26.154 | 13.278 | 18.518 |
| 4. | 17.875 | 22.341 | 24.917 | 27.341 | 29.599 | 32.616 | 18.451 | 24.163 |
| 5. | 23.132 | 27.953 | 30.818 | 33.262 | 35.700 | 38.858 | 23.680 | 29.000 |
| | | | | *Trace* | | | | |
| 1. | 2.415 | 4.905 | 6.691 | 8.083 | 9.658 | 11.576 | 3.030 | 7.024 |
| 2. | 9.335 | 13.038 | 15.583 | 17.844 | 19.611 | 21.962 | 9.879 | 18.017 |
| 3. | 20.188 | 25.445 | 28.436 | 31.256 | 34.062 | 37.291 | 20.809 | 34.159 |
| 4. | 34.873 | 41.623 | 45.248 | 48.419 | 51.801 | 55.551 | 35.475 | 56.880 |
| 5. | 53.373 | 61.566 | 65.956 | 69.977 | 73.031 | 77.911 | 53.949 | 84.092 |

**Notes:** Reproduced from Johansen and Juselius (1990).

Simulations based upon 6,000 replications of 400 step random walks.

## Table B
## Percentiles of the Johansen Trace Test

| VAR Lags | VAR Dim. | No. of Trends Simulated | No. of Trends Tested | 1% | 2.5% | 5% | 10% | 20% | 50% | 80% | 90% | 95% | 97.5% | 99% | mean | var |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 3 | 3 | 3 | 8.512 | 9.431 | 11.074 | 12.452 | 14.497 | 18.951 | 24.964 | 28.829 | 31.241 | 35.518 | 38.764 | 19.970 | 41.041 |
| 1 | 3 | 3 | 3 | *7.950* | *9.886* | *11.072* | *12.841* | *14.699* | *19.772* | *25.100* | *28.462* | *31.915* | *33.991* | *38.421* | *20.267* | *40.868* |
| 3 | 3 | 3 | 3 | 8.338 | 9.265 | 10.450 | 12.23 | 14.49 | 18.608 | 24.557 | 27.948 | 31.440 | 34.180 | 36.474 | 19.638 | 39.937 |
| 3 | 3 | 3 | 3 | *8.149* | *9.300* | *11.038* | *12.683* | *14.737* | *19.543* | *24.846* | *28.209* | *31.261* | *34.301* | *36.750* | *20.099* | *39.011* |
| 1 | 3 | 2 | 3 | 61.036 | 63.267 | 66.344 | 69.272 | 73.357 | 83.091 | 92.477 | 98.354 | 103.421 | 107.775 | 112.559 | 83.480 | 129.149 |
| 1 | 3 | 2 | 3 | *59.911* | *62.247* | *65.219* | *69.002* | *73.940* | *82.885* | *91.353* | *96.943* | *101.535* | *105.354* | *110.532* | *83.013* | *117.859* |
| 3 | 3 | 2 | 3 | 41.517 | 43.646 | 46.421 | 49.104 | 53.130 | 61.528 | 70.279 | 75.169 | 78.952 | 82.747 | 87.671 | 61.972 | 103.029 |
| 3 | 3 | 2 | 3 | *40.824* | *43.474* | *45.831* | *49.419* | *53.746* | *61.771* | *70.287* | *74.258* | *78.513* | *81.478* | *83.921* | *62.062* | *93.271* |
| 1 | 6 | 6 | 6 | 53.457 | 58.103 | 60.960 | 63.802 | 67.976 | 76.509 | 86.253 | 92.609 | 97.619 | 102.069 | 107.853 | 77.482 | 124.973 |
| 1 | 6 | 6 | 6 | *53.171* | *56.254* | *61.130* | *63.723* | *67.996* | *77.034* | *87.407* | *94.484* | *101.327* | *104.983* | *110.152* | *78.134* | *144.437* |
| 3 | 6 | 6 | 6 | 50.433 | 55.302 | 58.956 | 61.871 | 65.896 | 74.962 | 84.883 | 90.338 | 95.626 | 99.384 | 102.211 | 75.768 | 125.012 |
| 3 | 6 | 6 | 6 | *51.485* | *56.321* | *59.228* | *62.076* | *66.354* | *75.127* | *85.393* | *90.834* | *96.050* | *101.463* | *106.827* | *76.179* | *131.803* |
| 1 | 6 | 3 | 4 | 63.991 | 66.819 | 69.769 | 72.812 | 76.754 | 84.294 | 92.287 | 97.465 | 100.825 | 103.812 | 107.800 | 84.637 | 89.544 |
| 1 | 6 | 3 | 4 | *64.279* | *66.516* | *69.783* | *73.226* | *76.180* | *83.948* | *92.419* | *96.213* | *99.985* | *103.154* | *106.086* | *84.399* | *86.341* |
| 3 | 6 | 3 | 4 | 48.256 | 49.969 | 52.510 | 54.921 | 58.060 | 64.743 | 72.497 | 76.495 | 79.973 | 82.513 | 86.351 | 65.414 | 70.894 |
| 3 | 6 | 3 | 4 | *48.668* | *50.557* | *52.261* | *54.678* | *58.306* | *65.541* | *73.012* | *77.059* | *80.323* | *83.216* | *85.892* | *65.815* | *72.045* |

Notes: Upper entry generated assuming homoskedastic errors, lower entry generated assuming conditionally heteroskedastic errors.

## Table C
## Percentiles of the Johansen Maximal Eigenvalue Test

| VAR Lags | VAR Dim. | No. of Trends Simulated | No. of Trends Tested | 1% | 2.5% | 5% | 10% | 20% | 50% | 80% | 90% | 95% | 97.5% | 99% | mean | var |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 3 | 3 | 3 | 4.859 | 5.992 | 6.894 | 7.757 | 9.240 | 12.245 | 16.278 | 19.356 | 22.260 | 24.780 | 27.543 | 13.079 | 22.253 |
| 1 | 3 | 3 | 3 | 5.143 | 5.914 | 6.685 | 7.897 | 9.429 | 12.723 | 16.967 | 19.239 | 21.845 | 24.139 | 26.905 | 13.334 | 21.792 |
| 3 | 3 | 3 | 3 | 4.939 | 5.944 | 6.516 | 7.525 | 8.954 | 12.137 | 16.381 | 19.243 | 21.968 | 24.501 | 27.144 | 12.919 | 22.245 |
| 3 | 3 | 3 | 3 | 4.916 | 5.693 | 6.705 | 7.869 | 9.330 | 12.777 | 16.919 | 19.575 | 21.985 | 24.059 | 26.120 | 13.314 | 21.863 |
| 1 | 3 | 2 | 3 | 54.426 | 57.048 | 59.312 | 62.249 | 66.468 | 74.600 | 84.032 | 88.846 | 93.639 | 96.934 | 101.043 | 75.313 | 108.521 |
| 1 | 3 | 2 | 3 | 51.906 | 56.089 | 58.527 | 61.456 | 66.242 | 73.880 | 82.587 | 86.708 | 91.851 | 94.277 | 100.429 | 74.466 | 99.946 |
| 3 | 3 | 2 | 3 | 34.942 | 37.717 | 39.820 | 42.389 | 46.204 | 53.590 | 61.251 | 66.171 | 69.329 | 72.657 | 76.842 | 54.019 | 83.181 |
| 3 | 3 | 2 | 3 | 34.692 | 37.553 | 39.329 | 42.564 | 45.854 | 53.381 | 61.264 | 65.753 | 68.798 | 71.050 | 76.465 | 53.628 | 78.322 |
| 1 | 6 | 6 | 6 | 19.238 | 20.015 | 21.487 | 23.174 | 25.262 | 29.915 | 35.250 | 39.165 | 42.045 | 45.867 | 49.153 | 30.594 | 40.325 |
| 1 | 6 | 6 | 6 | 18.664 | 20.054 | 21.450 | 22.962 | 25.118 | 30.229 | 36.517 | 40.299 | 43.788 | 46.880 | 51.445 | 31.055 | 48.556 |
| 3 | 6 | 6 | 6 | 18.587 | 19.456 | 20.517 | 22.213 | 24.196 | 28.750 | 34.338 | 37.482 | 40.156 | 43.240 | 46.046 | 29.467 | 37.302 |
| 3 | 6 | 6 | 6 | 18.070 | 19.216 | 20.485 | 22.172 | 24.302 | 29.118 | 34.872 | 37.630 | 40.408 | 43.453 | 45.521 | 29.707 | 37.992 |
| 1 | 6 | 3 | 4 | 48.114 | 50.431 | 52.559 | 54.859 | 58.404 | 64.509 | 70.487 | 73.779 | 77.094 | 79.051 | 80.955 | 64.538 | 52.425 |
| 1 | 6 | 3 | 4 | 47.550 | 49.959 | 52.633 | 54.877 | 58.459 | 64.200 | 69.913 | 72.840 | 75.494 | 77.548 | 80.296 | 64.125 | 48.651 |
| 3 | 6 | 3 | 4 | 32.027 | 35.034 | 36.652 | 38.709 | 40.859 | 45.656 | 50.421 | 53.202 | 55.599 | 57.179 | 58.898 | 45.749 | 32.941 |
| 3 | 6 | 3 | 4 | 32.719 | 34.838 | 36.581 | 38.160 | 40.831 | 45.658 | 50.800 | 53.236 | 56.074 | 58.031 | 61.310 | 45.836 | 35.597 |

Notes: Upper entry generated assuming homoskedastic errors, lower entry generated assuming conditionally heteroskedastic errors.

Figure 1: Three Month Excess Returns from Buying Dollars Forward

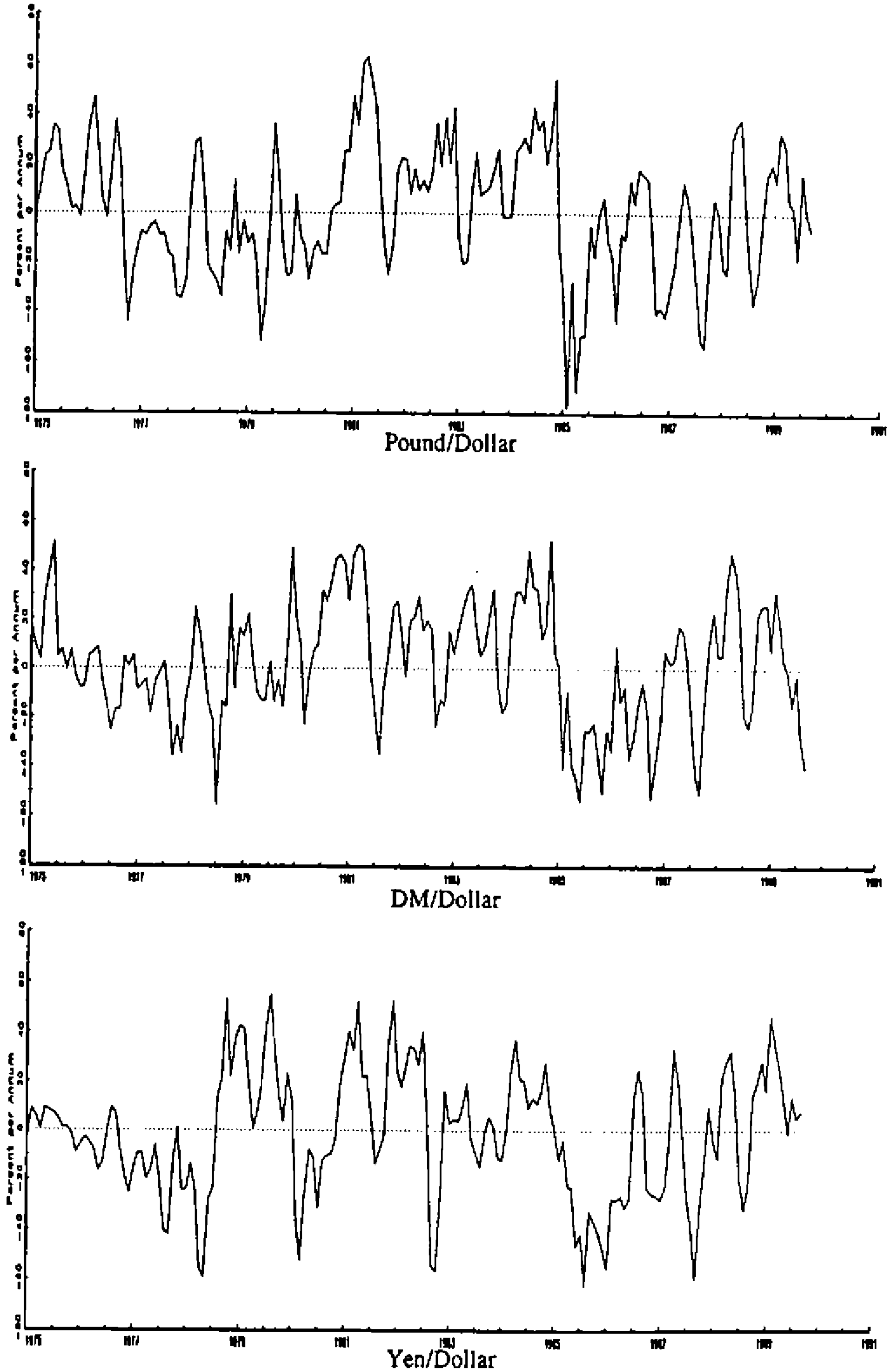
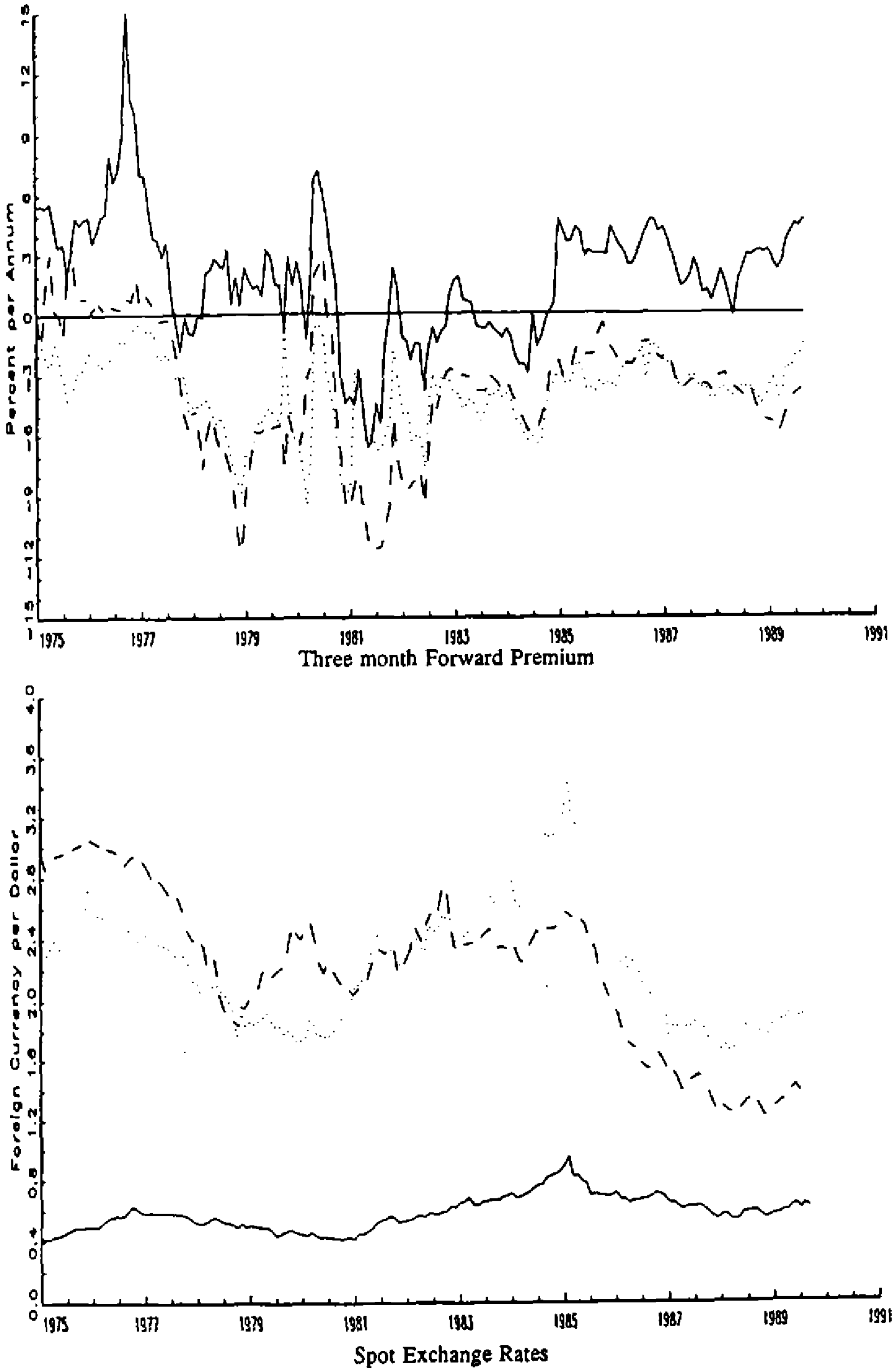
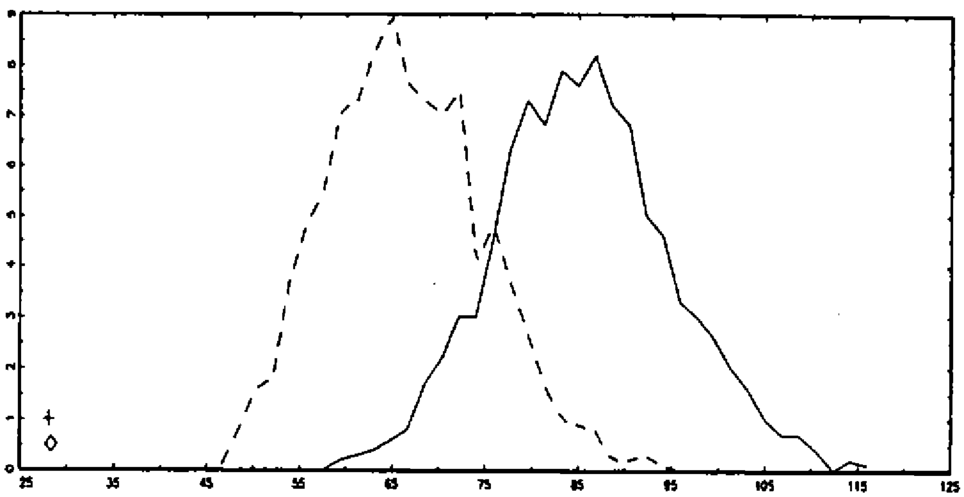


**Figure 2: Forward Premia and Spot Exchange Rates**

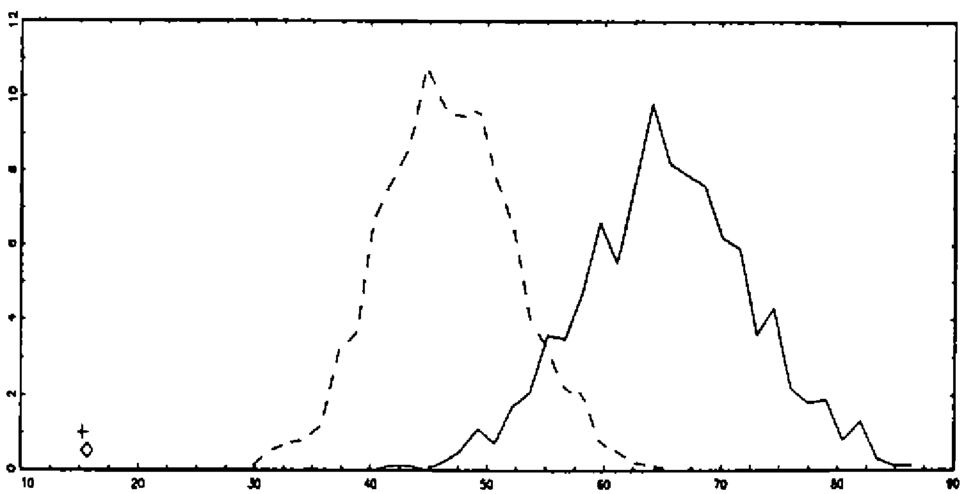



Key: ———— £/\$, ....... DM/\$, ------ ¥/\$, (measured as 100 ¥ per \$)

## Figure 3: Monte Carlo Distribution of the Trace Test




## Figure 4: Monte Carlo Distribution of the Maximal Eigenvalue Test

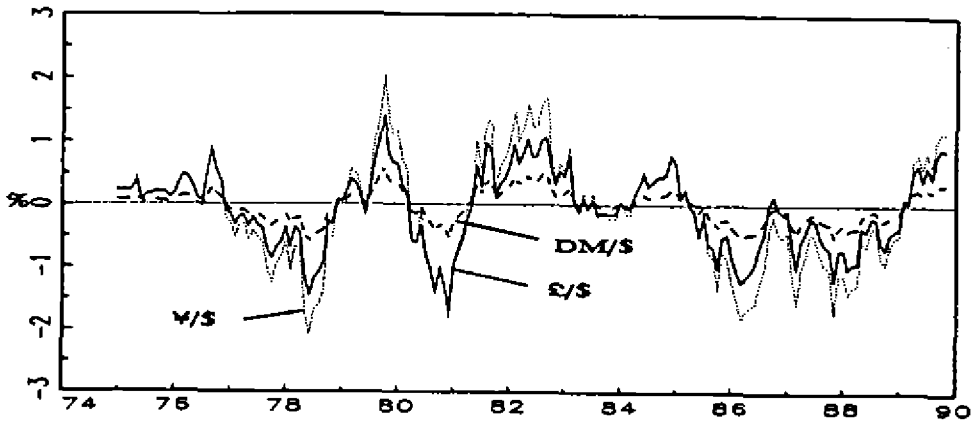



The statistics test for the presence of 4 trends in the 6-dimensioned vector of forward and spot exchange rates when in fact there are 3 in the generated data.

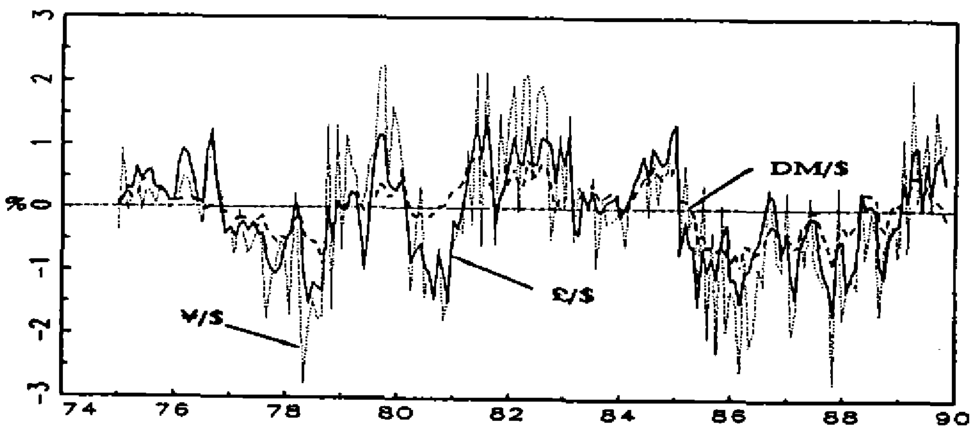
| | |
|---|---|
| ______: | empirical distribution of test statistics based on a VAR of order 1 |
| – – –: | empirical distribution of test statistics based on a VAR of order 3 |
| + | test statistics for 1-month forward rates reported in Table 2. |
| ◇ | test statistics for 3-month forward rates reported in Table 2. |

## Figure 5

Trends in Forward Premia (One Month Horizon)




Predicted Forward Premia (One Month Horizon)




"Additional" Trend in Forward Premia (One Month Horizon)

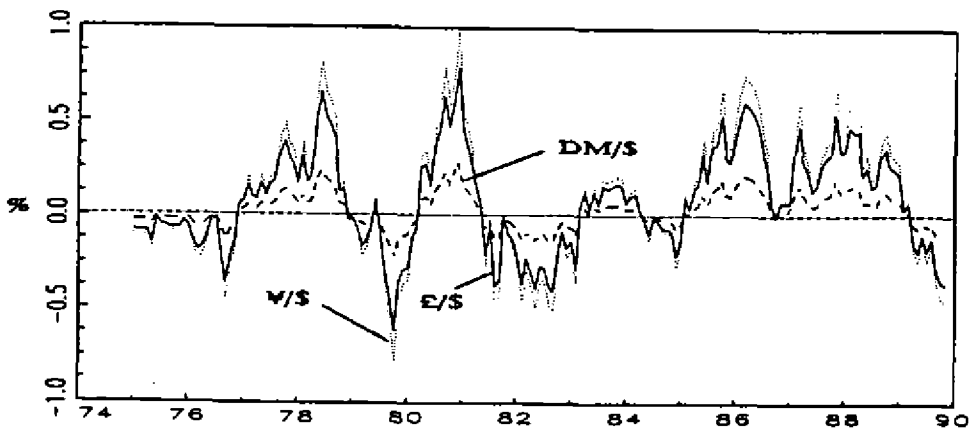
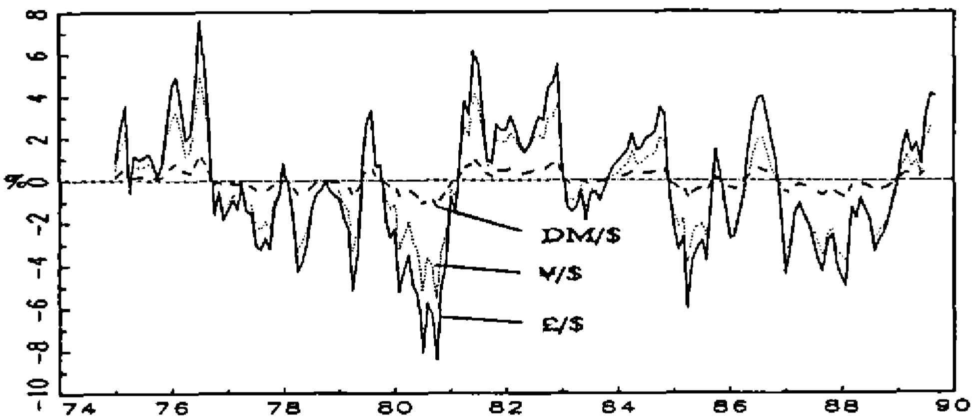
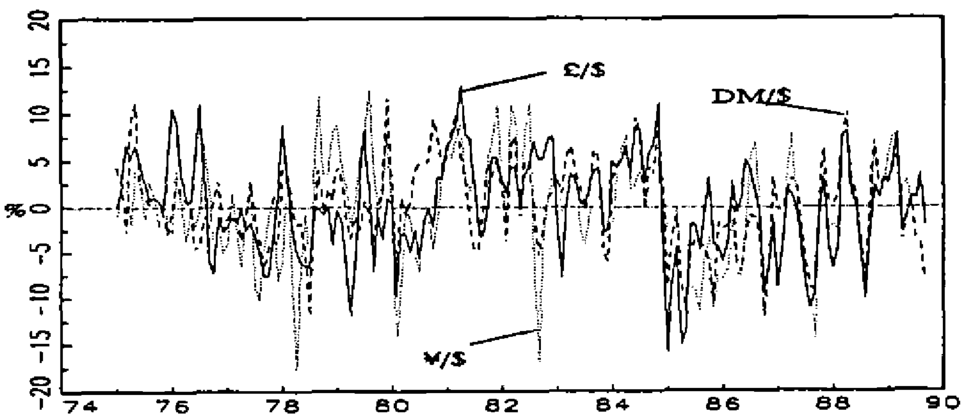


**Figure 6**

Trends in Forward Premia (Three Month Horizon)




Predicted Forward Premia (Three Month Horizon)




"Additional" Trend in Forward Premia (Three Month Horizon)